Satisfiability Of Modal Logic Formulas

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1 Formula Representation

Let $\mathbb{V}ar$ be the set of variables:

$$\mathbb{V}ar = \{p_0, p_1, p_2 ...\}$$

Let $\mathbb{C}_t$ be the set of Term constants:

$$\mathbb{C}_t = \{0, 1\}$$

Term recursive definition

- $a \in \mathbb{C}_t$ is a term
- $p \in \mathbb{V}ar$ is a term
- If x is a term, then $\bar{x}$ is a term as well
- If x and y are terms, then $x\sigma y$ is a term as well, where $\sigma \in \{\sqcap, \sqcup\}$

Let $\mathbb{C}_f$ be the set of formula constants:

$$\mathbb{C}_f = \{T, F\}$$

Formula recursive definition

- $a \in \mathbb{C}_f$ is a formula
- If x and y are terms, then C(x, y) is a formula
- If x and y are terms, then $x \leq y$ is a formula
- If x and y are terms, then $x \leq_m y$ is a formula
- If ϕ is a formula, then $\neg\phi$ is a formula as well
- If ϕ and ψ are formulas, then $\phi\,\sigma\,\psi$ is a formula as well, where $\sigma \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$

2 Tableaux Process

The Tableaux process is decision procedure, which recursively breaks down a given formula into basic components based on which a decision can be concluded. The recursive step which breaks down a formula creates one or two new formulas, which in terms of their structure are simpler then the initial formula. Since the recursive step can create at most two new formulas, this means the recursive step will create at most two branches or a binary tree, where the nodes are the formulas and the links represent the recursive step.

Contradiction may arise when in the same branch, on some step there exists a formula and the negation of the same formula. If in some branch there exists a contradiction, then that branch closes. If all branches close then the, then the proof is complete.