Problem 1 The SNR of an Analog to Digital Convertor

You are working on a sensor system that samples an input signal with an ADC. Your boss tells you that you need to be able to sample a sinewave signal of 2 volts peak to peak and get a result that has a SNR of at least 55 dB. The input range of the ADC is 3 volts full scale. They want to use an 8-bit ADC to keep down the cost of the system. Your boss asks you to analyze the performance of the ADC to see if it can meet the SNR requirement.

Answer the following questions about the ADC system.

a. How many quantization levels does the ADC have?

In general, an N-Bit ADC has 2^N quantization levels. Therefore, the 8-bit ADC has $2^8 = 256$ quantization levels

b. If the full-scale input range of the ADC is 3 what is the value in volts of one quantization level which is also referred to as a code value (1CV)?

The value of one quantization level is

$$1CV = \frac{V_{fullscale}}{2^N - 1}$$

For this ADC

$$1CV = \frac{3V}{2^8 - 1} = 11.76mV$$

c. What is the standard deviation of the quantization noise?

The standard deviation of the quantization noise is found using

$$\sigma_q = 1CV \times 0.29$$

$$\sigma_q = 11.76mV \times 0.29 = 3.412mV$$

d. What is the standard deviation of the sinewave signal?

The standard deviation of the sinewave can be found by computing the RMS value from the peak to peak value of the sinewave

$$\sigma_{sine} = \frac{V_{pp}}{2\sqrt{2}}$$

In this case the standard deviation of the sine wave is then

$$\sigma_{sine} = \frac{2}{2\sqrt{2}} = .707V$$

e. What is the signal to noise ratio where the signal is the sinewave and the noise is the quantization noise of the ADC? Use the ratio of variances to compute the signal to noise ratio in decibels.

The signal to noise ratio expressed as the ratio of the variances is

$$SNR = \frac{\sigma_{sine}^2}{\sigma_q^2} = \frac{.707^2}{(3.412 \times 10^{-3})^2} = 42.95 \times 10^3$$

In decibels this is

$$SNR_{dB} = 10\log_{10}(42.95 \times 10^3) = 46.33\ dB$$

f. Does this meet the SNR requirement?

Not this does not meet the SNR requirement of 55 dB

g. If you increase the number of bits in the ADC by 1 how many decibels does the SNR change?

Since each time a bit is added to the ADC, the value of 1CV is reduced by nearly ½.

$$1CV = \frac{V_{fullscale}}{2^N - 1}$$

The quantization noise is then reduced by ½ as well

$$\sigma_q = 0.29 \times 1CV$$

This then reduces the power of the noise by a factor of 4 because the power is expressed as variance and the variance is the square of the standard deviation. Decreasing the power by a factor of 4 increases the numerical SNR by a factor of 4. Increasing the numerical SNR by a factor of 4 increases the SNR in decibels by 6.

$$10 \log_{10} 4 = 6\ dB$$

Therefore, every bit that is added increases the SNR of the ADC by 6 dB.

h. How many bits will it then take to meet the SNR requirement of 55 dB?

The SNR in decibels of the signal with an 8-bit ADC is 46.33. To achieve the requirement of 55 dB you would need to add 2 bits increasing the SNR by 12 dB to 58.33 dB

Problem 2 How many bits do I need?

Specify how many bits are needed to appropriately digitize each of the following signals. The ADC chosen must not degrade the SNR of the input signal by more than 20%. The full-scale input range of the ADC is 3 volts.

Choose from: 6 bits, 8 bits, 10 bits, 12 bits, 14 bits, or 16 bits

a. A DC signal with noise. The DC value of the signal is 1 volt and the rms noise is 1.5 millivolts. Hint: Determine the maximum output noise, then solve for the maximum amount of quantization noise that can be added. Based on that determine the quantization level and then the resolution of the ADC.

SOLUTION:

The process of digitization will add quantization noise. The amount of quantization noise added should make the total noise increase by no more than 20%.

The standard deviation (RMS) of the noise on the input 1.5 mV. Since the standard deviation of the output noise will be the quadrature sum of the input noise and the quantization noise we can solve for the amount of quantization noise knowing that the total can be no more than 20% greater than the input noise.

The total output noise should be no greater than

$$Output\ Noise\ = 1.5mV(1.2) = 1.8mV$$

Solve for the level of quantization noise. The standard deviations for noise add in quadrature so the standard deviation of the total noise can be calculated as:

$$\sigma_{total} = \sqrt{\sigma_{input}^2 + \sigma_q^2}$$

Where σ_q is the quantization noise. Then

$$1.8mV = \sqrt{(1.5 \times 10^{-3})^2 + \sigma_q^2}$$

Solving for the standard deviation of the quantization noise gives:

$$\sigma_q = \sqrt{(1.8 \times 10^{-3})^2 - (1.5 \times 10^{-3})^2} = 9.95 \times 10^{-4} = .995\ mV$$

Then since the quantization noise is 0.29 times the value of 1 LSB. The value of 1 LSB of code value must be

$$\sigma_q = 0.29(1CV)$$

$$1CV = \frac{\sigma_q}{0.29}$$

$$1CV = \frac{995mV}{0.29} = 3.43mV$$

Then with 3V being the full scale of the ADC the number of levels required is:

$$levels = \frac{3}{3.43 \times 10^{-3}} = 874.6\ levels$$

To have 874.6 levels, 8 bits is insufficient ($2^8 - 1$ = 255 levels) so a total of at least 10 bits is required.

b. A sinusoid with a peak to peak voltage of 2 volts and a signal-to-noise ratio of 45 dB. Hint determine the amount of noise on the input signal then compute the maximum amount of quantization noise allowed to increase that noise by no more than 20%

Solution:

The numerical value of the signal to noise ratio is

$$SNR = 10^{\frac{45}{10}} = 31623$$

The input signal value has a peak to peak value of 2 volts. The standard deviation of the signal is:

$$\sigma_{signal} = \frac{V_{pp}}{2\sqrt{2}} = 707.1mV$$

If the signal to noise ratio is 31623 compute the standard deviation of the noise.

$$SNR = \frac{\sigma_{signal}^2}{\sigma_{noise}^2}$$

$$\sigma_{noise} = \sqrt{\frac{\sigma_{signal}^2}{SNR}} = \sqrt{\frac{.7071^2}{31623}} = 3.976\ mV$$

Then the total noise cannot increase by more than 20% or cannot be greater than

$$\sigma_{total} = \sigma_{noise}(1.2) = 4.772\ mV$$

Solve for the quantization noise using the fact that noise adds in quadrature

$$\sigma_q = \sqrt{\sigma_{total}^2 - \sigma_{noise}^2} = \sqrt{.004772^2 - .003976^2} = 2.638mV$$

Then since the quantization noise is 0.29 times the value of 1 LSB. The value of 1 LSB or code value must be

$$\sigma_q = 0.29(1CV)$$

$$1CV = \frac{\sigma_q}{0.29}$$

$$1CV = \frac{2.638mV}{0.29} = 9.1mV$$

The full-scale voltage range of the ADC is 3 volts. Then the number of quantization levels is then

$$levels = \frac{V_{fullscale}}{1CV} = \frac{3}{9.1mV} = 329.8$$

Then the number of bits required is

$$N = \log_2 levels = \log_2 329.8 = 8.4$$

The closest sized ADC converter is then 10 bits.

Problem 3 Improving ADC Resolution using OSA

An engineer designs a microprocessor controlled ADC board that can acquire an 8 bit sample every 10 microseconds. Her boss walks in and says: "You'll get a raise if the system can be modified to acquire a 12 bit sample every 100 milliseconds- but it needs to be done by tomorrow!”.

The first thing the engineer does is to measure the noise on the analog signal entering the ADC chip. She then smiles and plans how to invest the extra money.

Explain how the engineer can make the modification to increase the resolution of the system. What will the system performance be? What was she looking for in the noise measurement? Consider the following in constructing your answer.

How does the SNR of the ADC change for each increase in the number of bits of resolution of the ADC increases (refer to problem 1)?

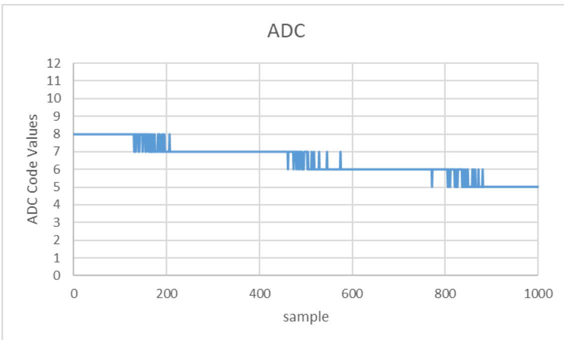
The SNR of a sampled system may be improved by using Over Sample Averaging (OSA) of the ADC output. Recall that when we take N samples of a random variable and average them the standard deviation (typical error) of the estimate becomes

$$\sigma_{estimate} = \frac{\sigma_{signal}}{\sqrt{N}}$$

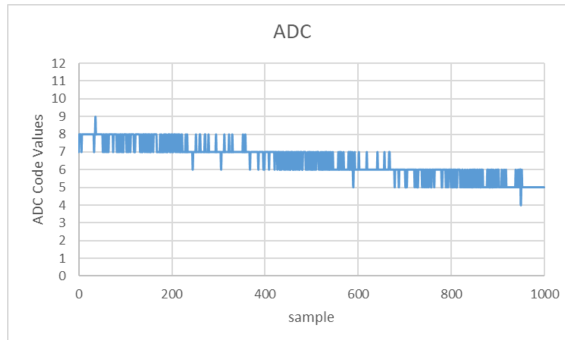
thus increasing the signal to noise ratio. For example, an 8-bit system can deliver 10-bit results (a 4 times reduction in the quantization noise) by taking $16 = 4^2$ samples and averaging them.

For OSA to be effective, the input signal must have sufficient signal and/or noise content to traverse multiple quantization levels so that the averaging is effective.

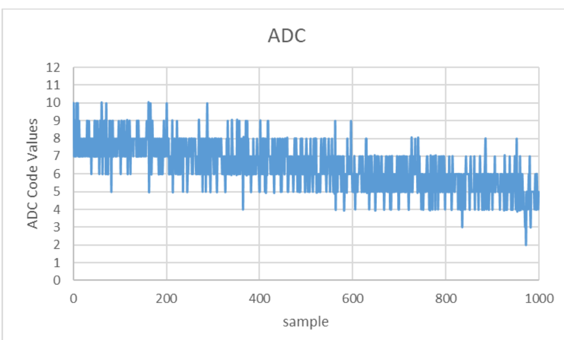
Consider the following four experimental outputs the engineer might have observed. Which is the best case scenario and why?



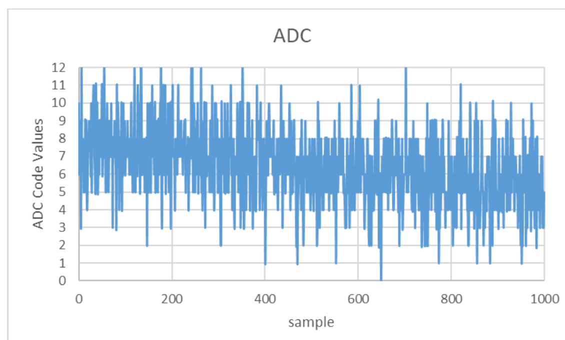
(a)



(b)



(c)



(d)

SOLUTION:

There is a lot of detail provided in these hints, however what it comes down to is, is there enough analog noise coming into the system to allow an oversample and average approach to work. Since the system is capable of sampling at a rate of 10 μSec and a sample rate of only 100 $mSec$ is required an oversample rate of up to

$$\frac{100mSec}{10\mu Sec} = 10{,}000$$

is possible.

This OSA rate would decrease the standard deviation of the samples by a factor of

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{10000}} = .01$$

The improvement increases the numerical SNR by one over this value

$$SNR\ improvement = \frac{\sigma_{signal}}{\sigma_{q_osa}} \Big/ \frac{\sigma_{signal}}{\sigma_{q_orig}} = \frac{\sigma_{qorig}}{\sigma_{qosa}} = \frac{1}{.01} = 100$$

This improvement would result in an increase in the effective number of bits by

$$N = \frac{20\log_{10}100}{6} = 6.667\ or\ 6\ bits$$

As long as the analog noise was large enough to cross the quantization levels in the 8-bit ADC then oversample averaging could be used to improve the resolution of the ADC by as much as 6 bits. The analog input noise should be approximately 1 CV in standard deviation for the input signal to cross quantization levels sufficiently.

Looking at the plots for the input noise, the first two plots indicate that the input noise amplitude isn't large enough such that the output samples cross the quantization noise levels sufficiently for averaging to be effective. The 3rd and 4th graphs both have sufficient noise, but graph 4 has more noise than is required and the signal SNR would be lower than in graph 3. Graph 3 shows the best case scenario. Just enough noise for OSA to be effective and still having a large SNR.

Problem 4 Over Sampling and Averaging

The noise level of a sensor as measured by its standard deviation is 2.7 mV. The sensor is being read by an ADC with a full-scale range of 3.3 V and 10 bits of resolutions. I only need to sample the input at a rate of 1kHz, however the ADC system can be sampled as fast as 1 MHz.

If I use oversampling and averaging how fast should I sample the signal to make sure that the total noise of my signal is less than an equivalent value of 0.75 mV?

Find the quantization noise of the ADC

$$\sigma_q = 0.29 \times \left(\frac{V_{fs}}{2^N - 1}\right) = 0.29 \times \frac{3.3}{2^{10} - 1} = 0.94\ \mathrm{mV}$$

The total equivalent noise of my sampled signal is:

$$\sigma_{total} = \sqrt{\sigma_{sensor}^2 + \sigma_q^2}$$

$$\sigma_{total} = \sqrt{2.70\ \mathrm{mV}^2 + 0.94\ \mathrm{mV}^2} = 2.86\ \mathrm{mV}\}$$

In order to reduce the total noise to less than and equivalent value of 0.75 mV I need to oversample and average the signal. Find the number of samples required for each desired signal sample by finding N in the typical error equation.

$$\sigma_{avg} = \frac{\sigma_{signal}}{\sqrt{N}}$$

Solving for N:

$$N = \left(\frac{\sigma_{total}}{\sigma_{avg}}\right)^2 = \left(\frac{2.86\ \mathrm{mV}}{0.75\ \mathrm{mV}}\right)^2 = 14.52$$

This indicates that the system must sample 15 times faster or a rate of 15.00 kHz.

Problem 4 Sampling

Answer the following questions about sampling

1. A signal with a maximum frequency content of 11 kHz is being sampled. What is the minimum theoretical sample rate required to sample the signal without aliasing?

SOLUTION:

The minimum theoretical sampling rate would be twice the maximum frequency content resulting in a minimum sample rate of 22 kHz. In practical terms one would want to sample at a higher rate than this. A practical number would be a sample rate such that the maximum frequency content is 80% of the maximum frequency content. For this case

$$f_{sample} = \frac{f_{max}}{0.8} * 2 = 27.5\ kHz$$

2. A sine wave of 3 kHz is being sampled at a rate of 8 kHz. After sampling, at what frequencies will the sampling components lie?

SOLUTION

When sampling, the positive and negative frequency components are repeated at the sample rate. The continuous sinewave of 3 kHz has a positive component at 3 kHz and a negative component at -3 kHz. The positive component after sampling results in frequency components at 3kHz (the original signal), 11 kHz and 19 kHz and so forth. The negative frequency component at -3kHz after sampling has positive components at 5 kHz, 13 kHz and 21 kHz and so forth repeating every 8 kHz

3. A sine wave of 7 kHz is being sampled at a rate of 10 kHz. After sampling, at what positive frequencies will the sampling components lie? Consider the frequency range from 0 Hz to 20 kHz.

SOLUTION

When sampling, the positive and negative frequency components are repeated at the sample rate. The continuous sinewave of 7 kHz has a positive component at 7 kHz and a negative component at -7 kHz. The positive component after sampling results in frequency components at 7kHz (the original signal), 17 kHz and 27 kHz and so forth. The negative frequency component at -7kHz after sampling has positive components at 3 kHz, 13 kHz and 23 kHz and so forth repeating every 10 kHz.

The frequency component at 3 kHz represents an aliased version of the original signal because the sampling rate is not high enough to accurately sample and be able to reconstruct the signal.

Problem 5 Precision and Accuracy

This past Thanksgiving, I was cooking a turkey for dinner. So that I was assured a delicious meal I wanted to make sure that my oven temperature was accurate. I took 50 samples of the temperature of the oven. I calculated the mean and standard deviation of the samples and got a mean of 332.5 degrees Fahrenheit and a standard deviation of 5.3 degrees Fahrenheit. I later found out that the exact temperature of the room was 329.2 degrees. What is the accuracy and precision of my estimate?

SOLUTION:

The accuracy is the difference between the estimate of the mean and the true mean. The Accuracy is then:

$$Accuracy = \mu_{est} - \mu_{true}$$

$$Accuracy = 332.5 - 329.2 = 3.3\ degrees\ F$$

The precision can be reported as either the standard deviation of my estimate or as the coefficient of variation. The standard deviation of my estimate is 5.3 degrees. The coefficient of variation is:

$$CV = \frac{\hat{\sigma}}{\hat{\mu}} \times 100 = \frac{5.3}{332.5}(100) = 1.59\%$$