# Digital Signal Processing

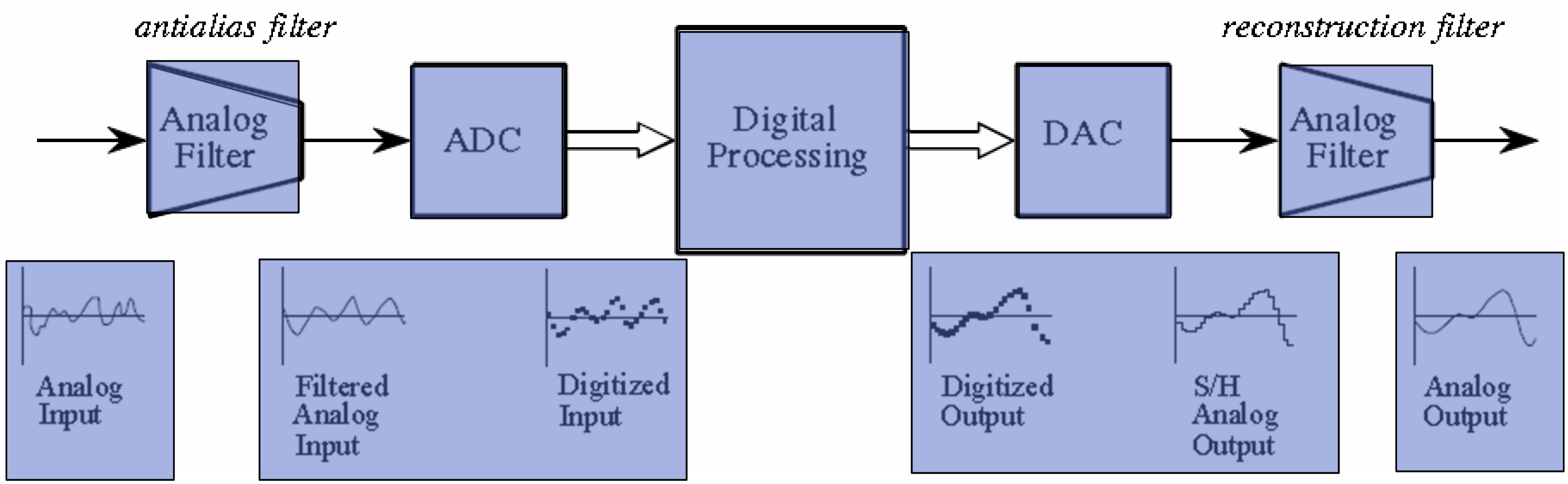
## Analog to Digital Conversion

## Sampling

# Key Points For Today

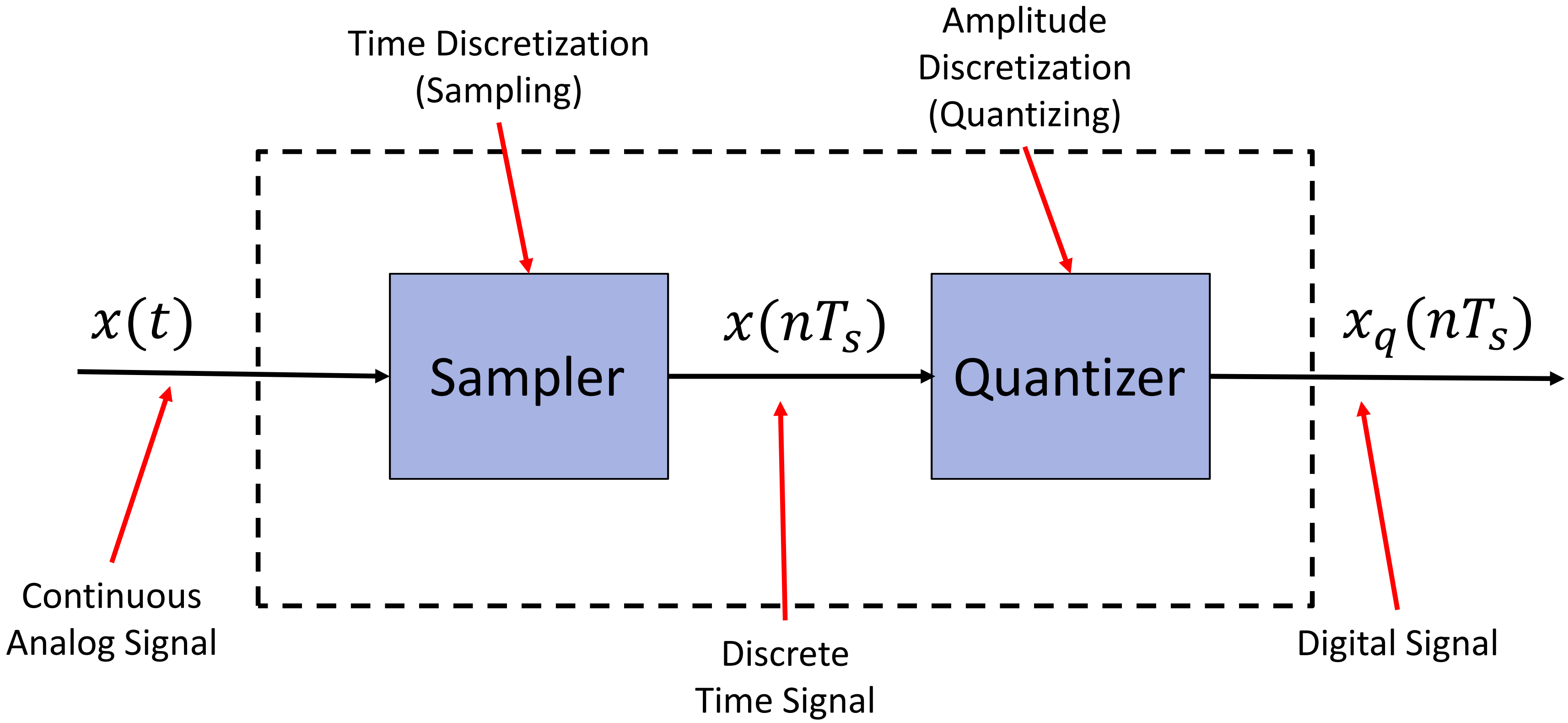
- Analog to digital conversion consists of sampling and quantizing
- Sampling must be done properly to retain the information in the signal
- Sampling at too low of a rate can distort the information in the captured signal (aliasing)

# Overview of ADC/DSP/DAC System



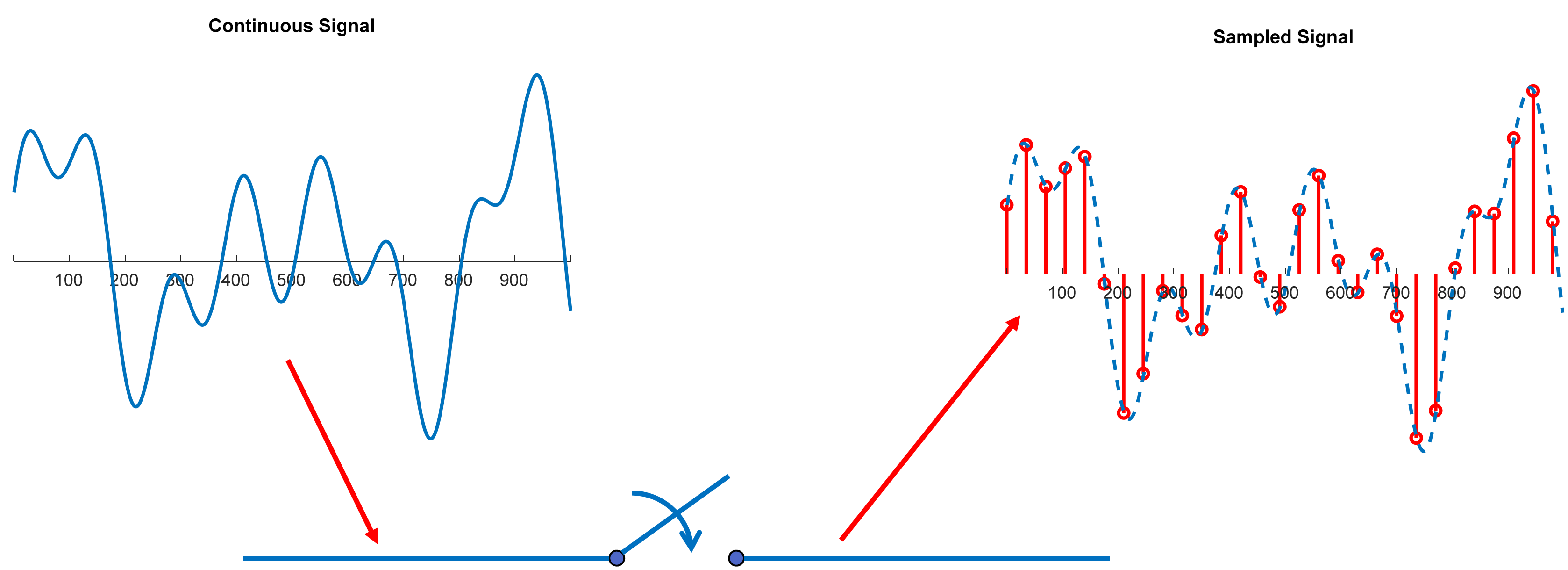
# Analog to Digital Conversion

- The conversion from analog to digital requires two forms of discretization: Time and Amplitude



# Sampling

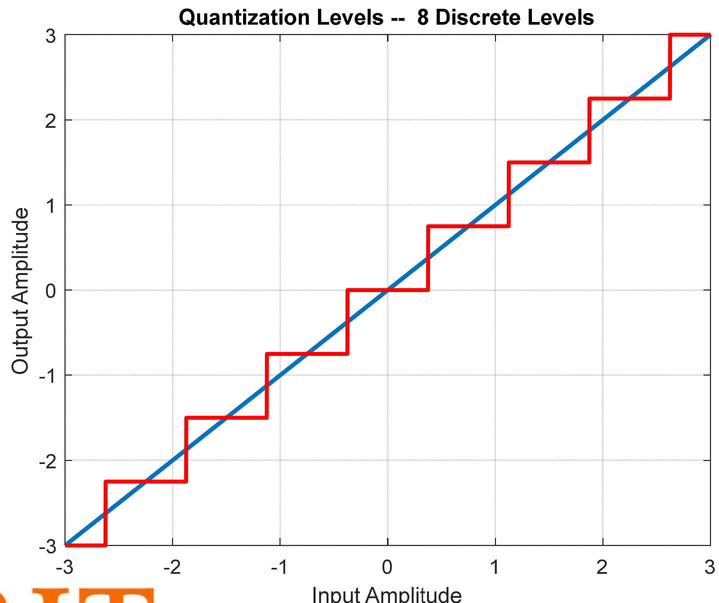
- The continuous analog signal is “viewed” or “sampled” at a periodic time, $T_s$
- $T_s$ is the sampling interval. $F_s = 1/T_s$ is the sample rate

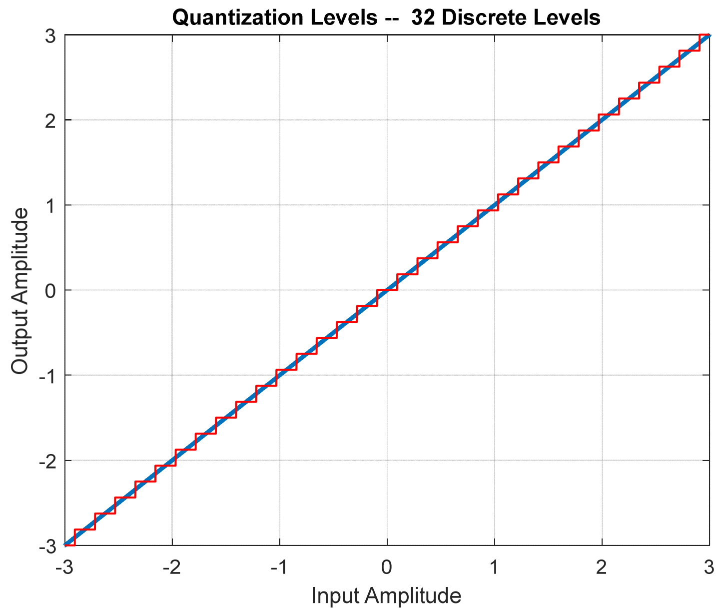



$$F_s = 1/T_s$$

# Quantizing

- When represented digitally, the amplitude of the signal is limited to discrete levels
- Amplitudes in between the discrete levels are rounded to the nearest discrete level



# ADC Conversion Impairments

- If sampling is done properly, the sampled signal contains all of the original information
  - The continuous signal can be reproduced exactly
- Quantizing can cause some loss in information.
  - Usually in the form of additive noise

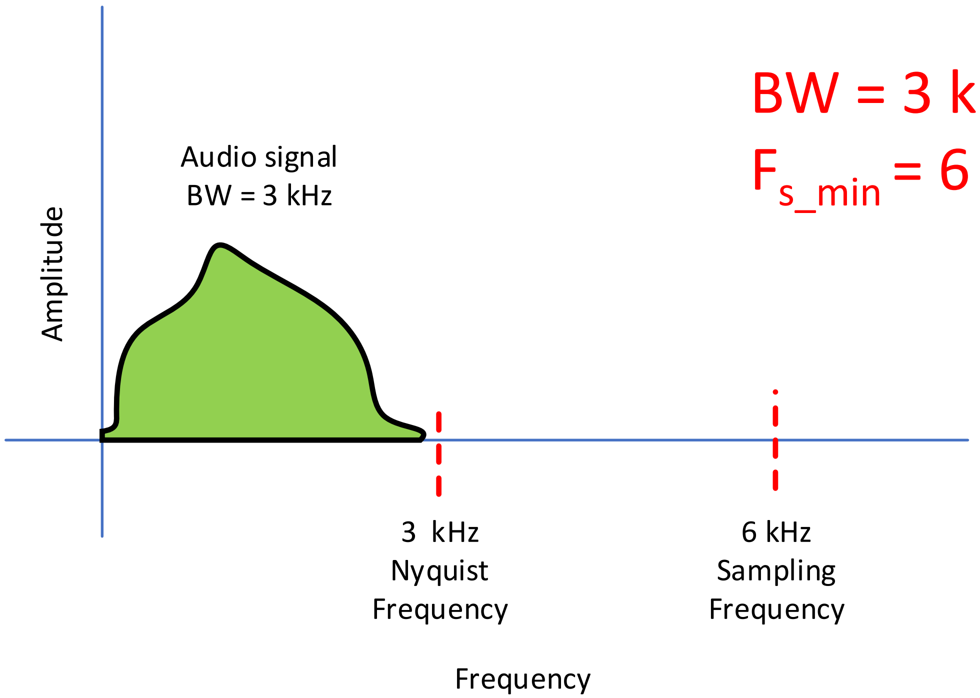
# What is Proper Sampling?

- If a signal has a bandwidth B, in order to retain all of the information, it must be sampled at a rate of at least $2 \times B$

- Example – An audio signal has a bandwidth of 3 kHz
  - To retain the original information, it must be sampled at least 6 kHz. That is 6000 samples per second or 6kSPS.

# What is Proper Sampling?

- Theoretical minimum sampling frequency to sample the original signal without distortion is >= 2X the bandwidth (at least 2x the highest frequency)




BW = 3 kHz

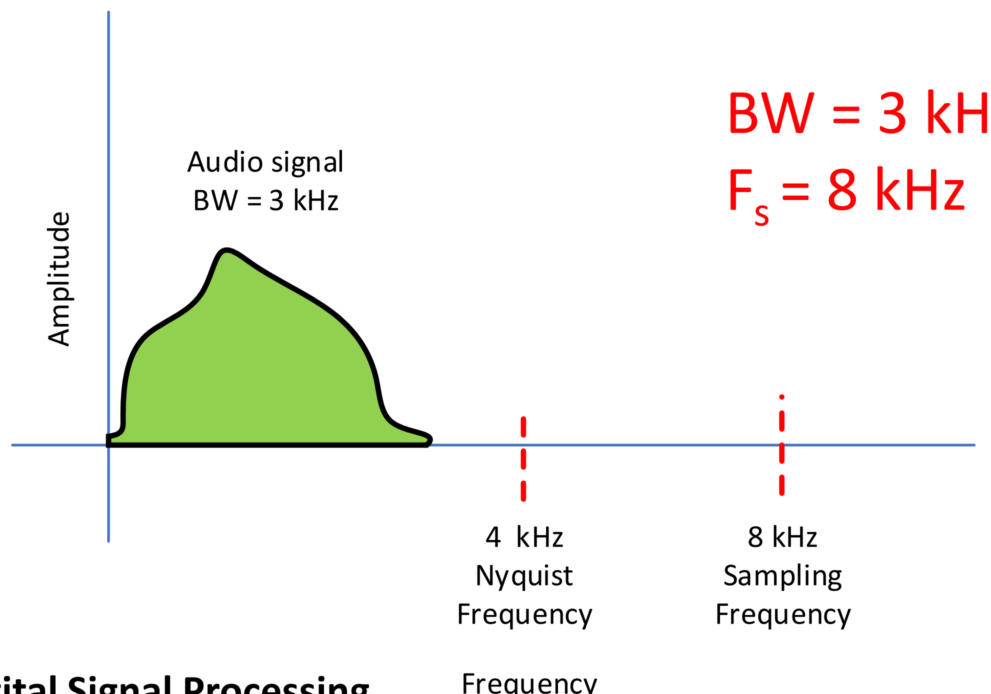
$F_{s\_min}$ = 6 kHz

# What is Proper Sampling?

- Typically, the sampling frequency is greater than that required for easier signal processing
  - Simplifed analog filtering to prevent aliasing

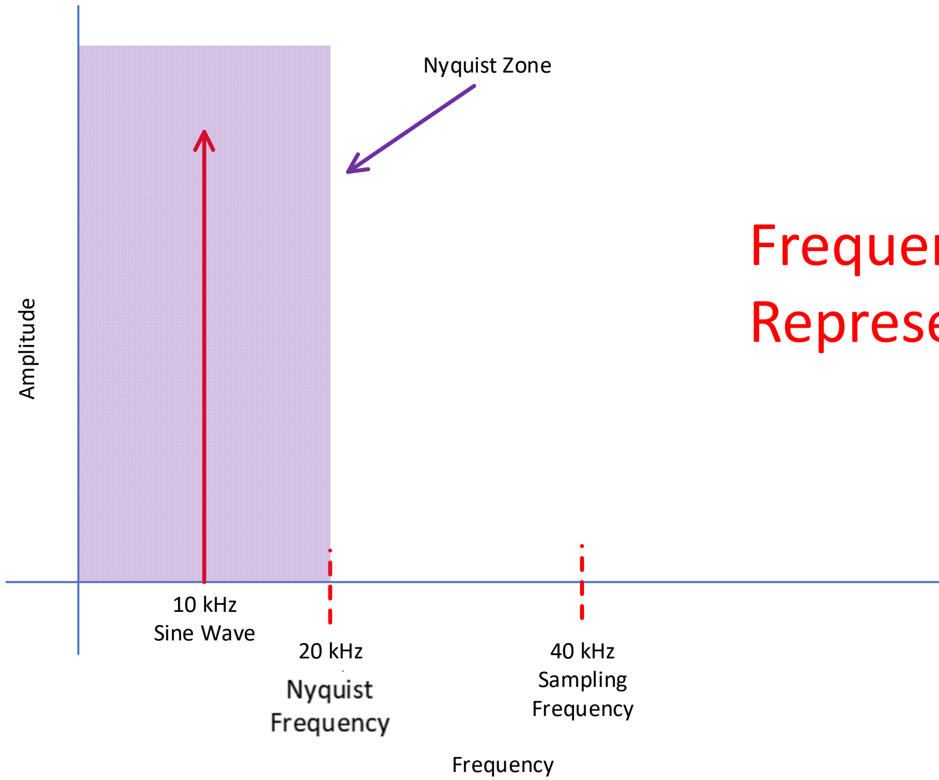
BW = 3 kHz

$F_s$ = 8 kHz



# Example – 10 kHz Sine Wave

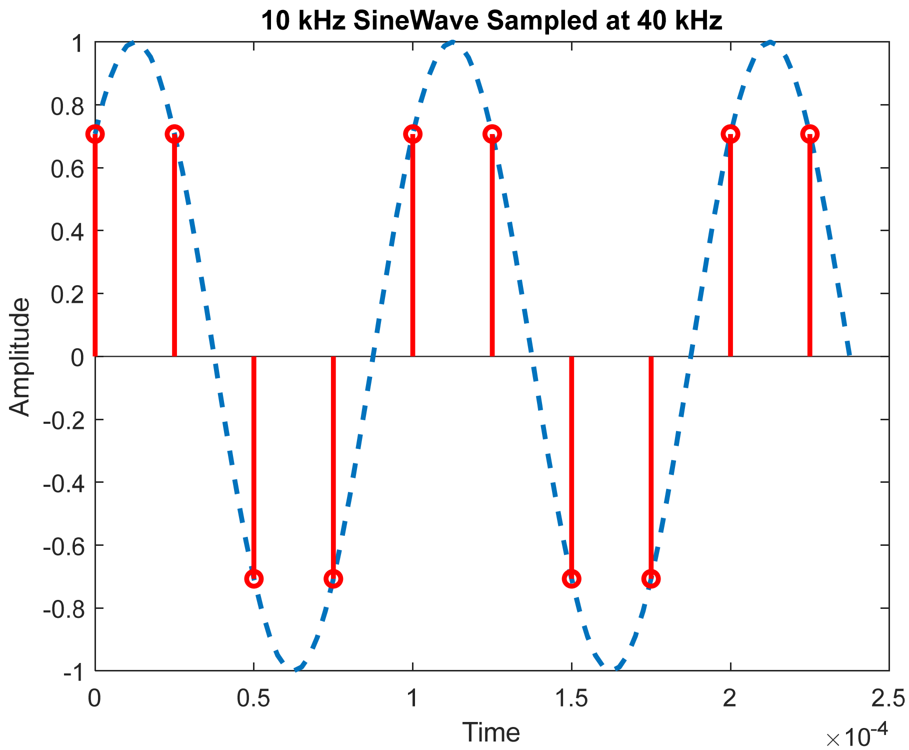
- Given a 10 kHz sine wave, sample the signal at a rate of 40 kHz (40kSPS)




Frequency Domain Representation

# Example – 10 kHz Sine Wave

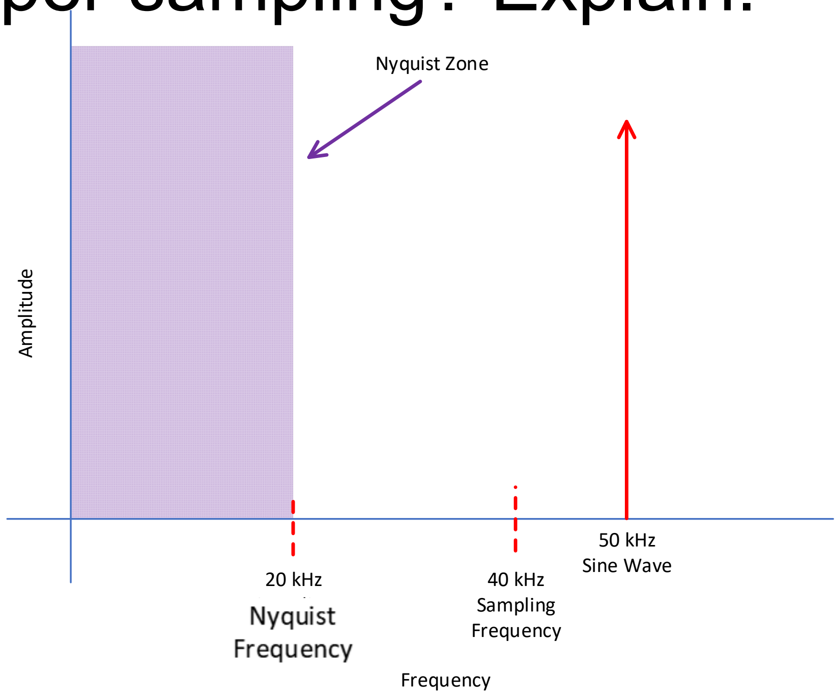
- Given a 10 kHz sine wave, sample the signal at a rate of 40 kHz (40 kSPS)




Time Domain Representation

# Consider a 50 kHz Sine Wave w/Sample Rate = 40 kHz

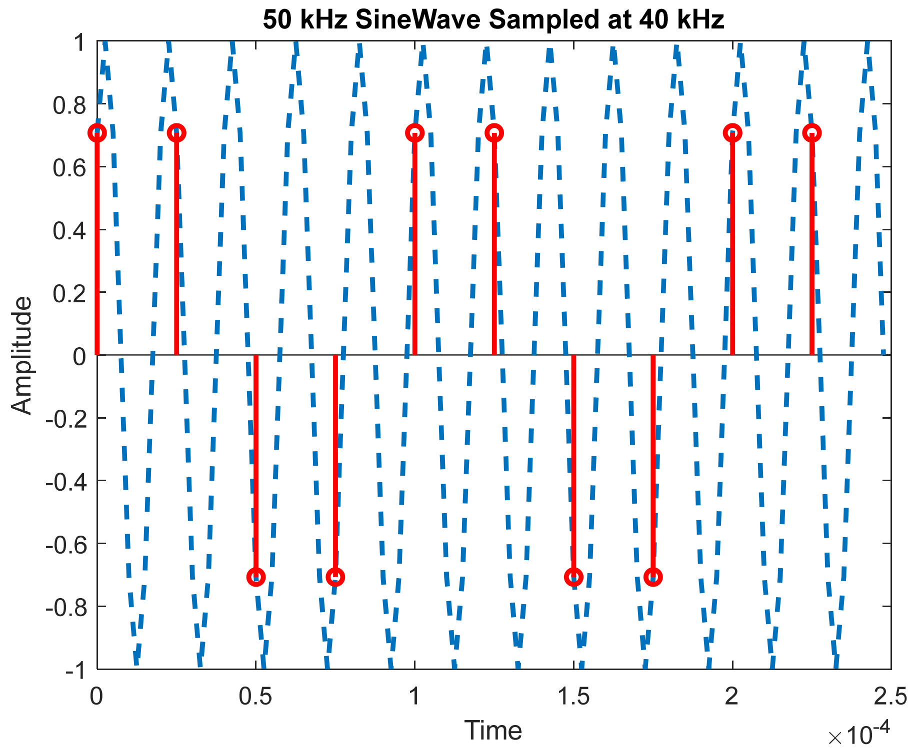
- Here, a 50 kHz sine wave is sampled at 40 kHz
- Is this proper sampling? Explain.




- What will happen?

# Consider a 50 kHz Sine Wave w/Sample Rate = 40 kHz

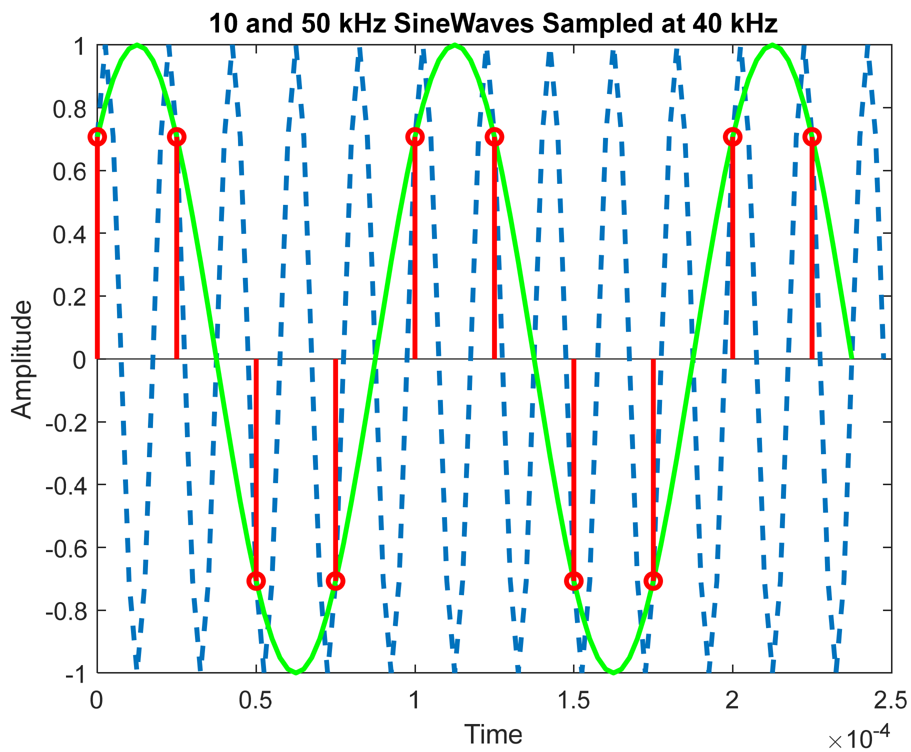
- The 50 kHz sine wave is sampled at 40 kHz




Time Domain Representation

# Compare – The Reconstructed Signal

- What do you notice about the samples for each sine wave?



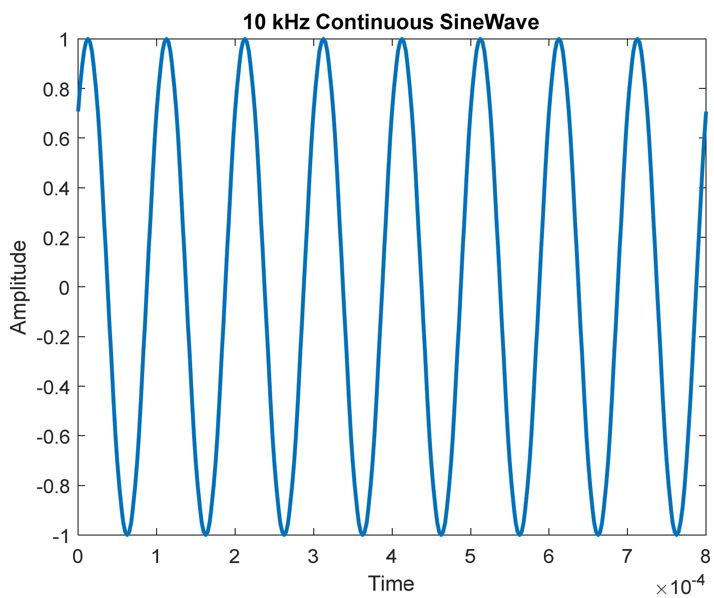

The samples for the 10 kHz and the 50 kHz sinewaves are the same when sampled at 40 kHz

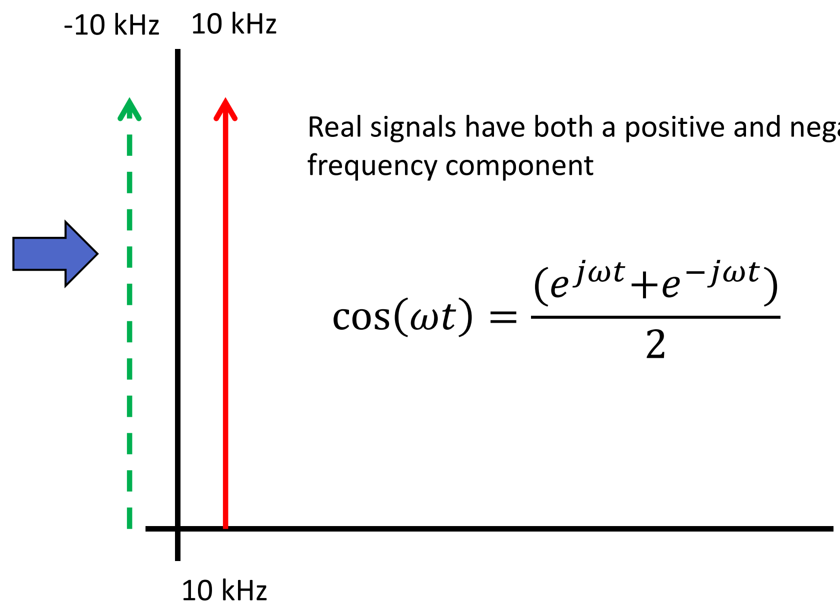
We've lost some information about the 50 kHz sine wave when sampled incorrectly

<u>This is ALIASING – The 50kHz sampled sinusoid appears as a 10kHz sinusoid after sampling – also referred to as folding.</u>

# Consider the Frequency Domain

- Sampling creates copies of the signal at intervals of the sample rate



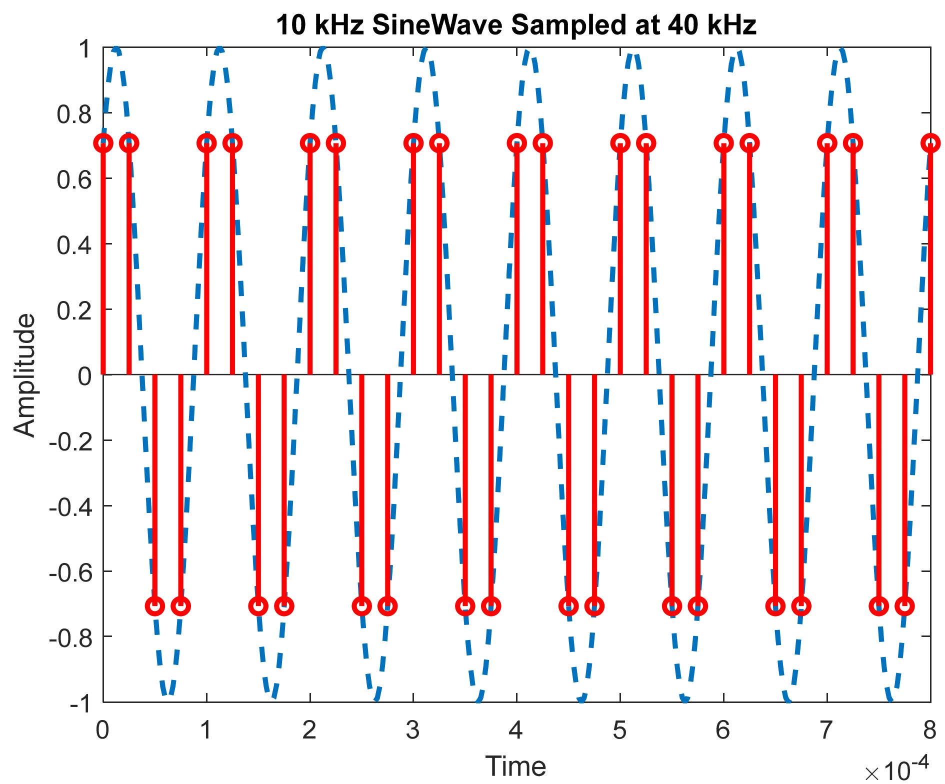

Real signals have both a positive and negative frequency component

$$\cos(\omega t) = \frac{(e^{j\omega t} + e^{-j\omega t})}{2}$$

Spectrum of continuous Signal

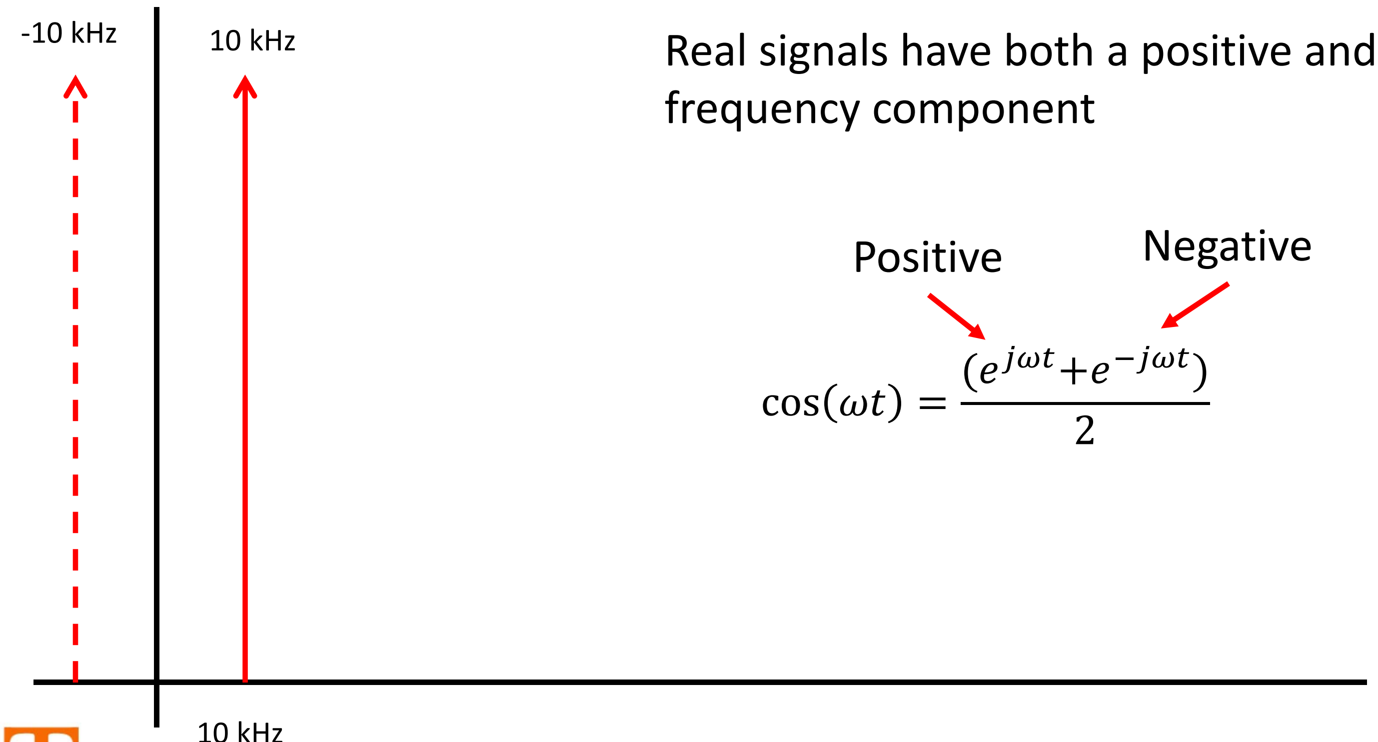
# Consider the Frequency Domain

- Sampling creates copies of the signal at intervals of the sample rate



# Where do the copies exist?

- A continuous signal at 10 kHz.
- A negative frequency component exists at -10 kHz
- Sometimes referred to as the Upper and Lower sideband



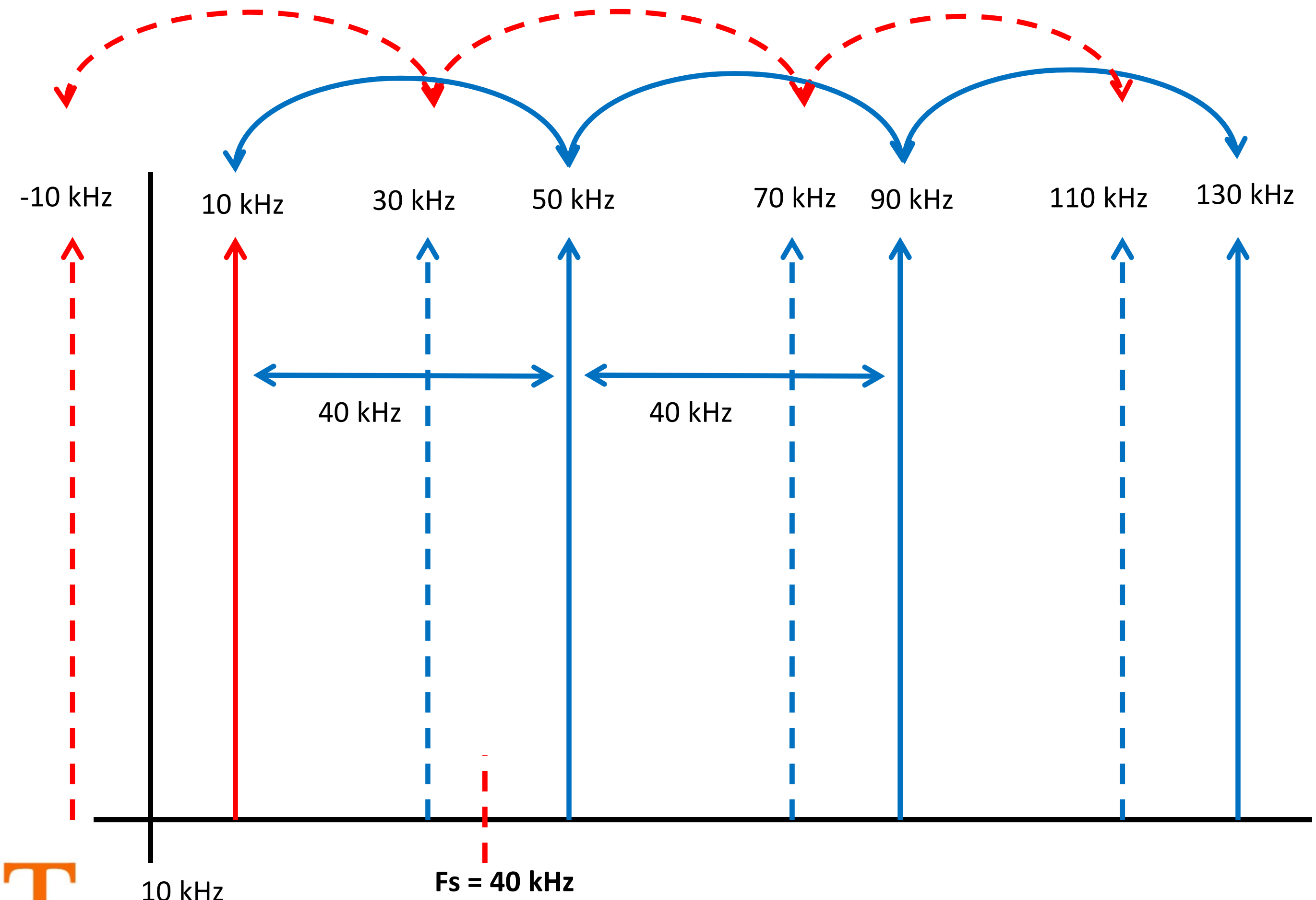

Real signals have both a positive and negative frequency component

Positive Negative

$$\cos(\omega t) = \frac{(e^{j\omega t} + e^{-j\omega t})}{2}$$

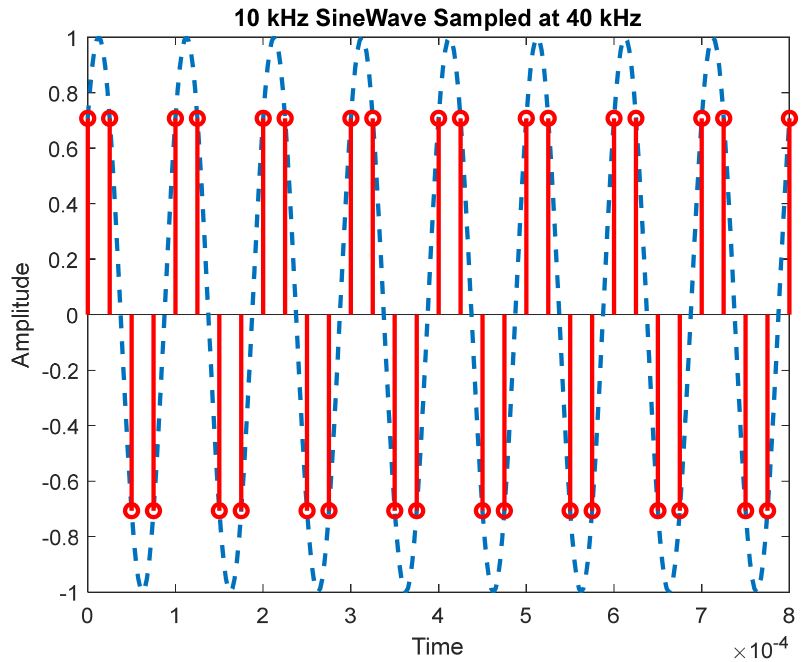
# Where do the copies exist when sampled at $F_s = 40\ kHz$?

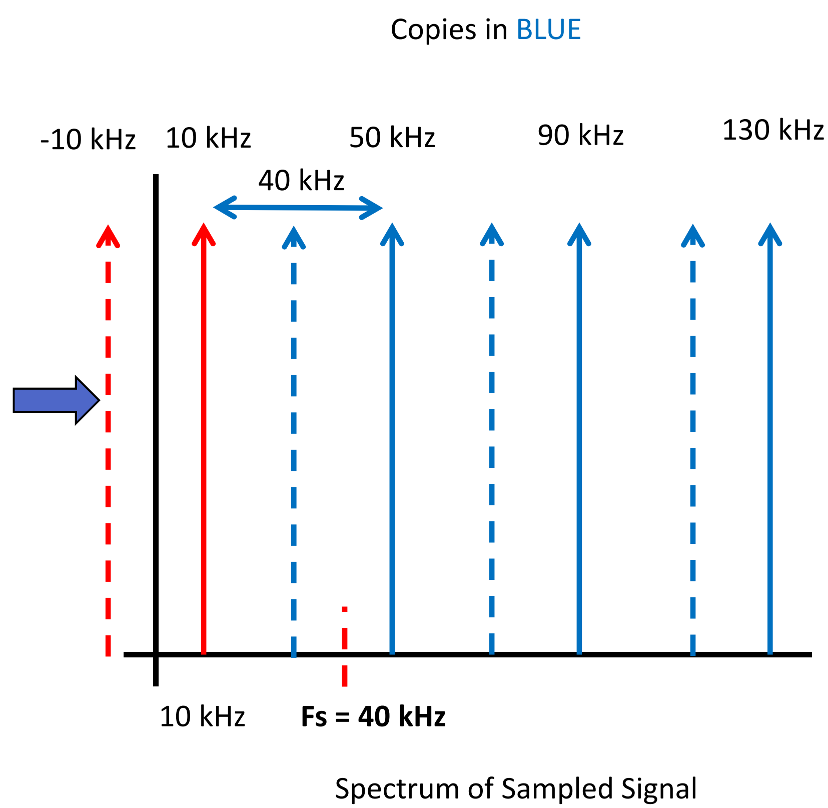
- Signals are duplicated <u>every $F_s = 40\ kHz$</u>



# Consider the Frequency Domain

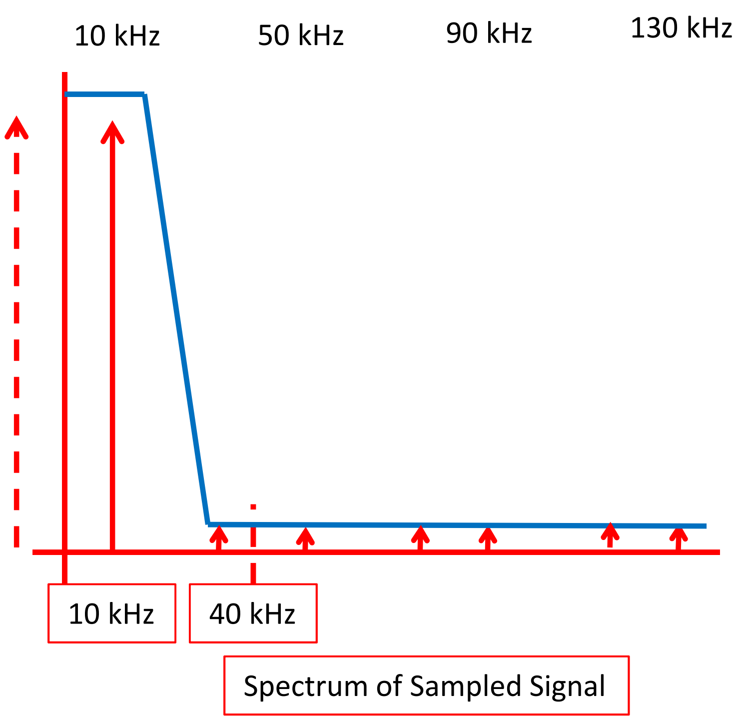
- Spectrum of the sampled signal

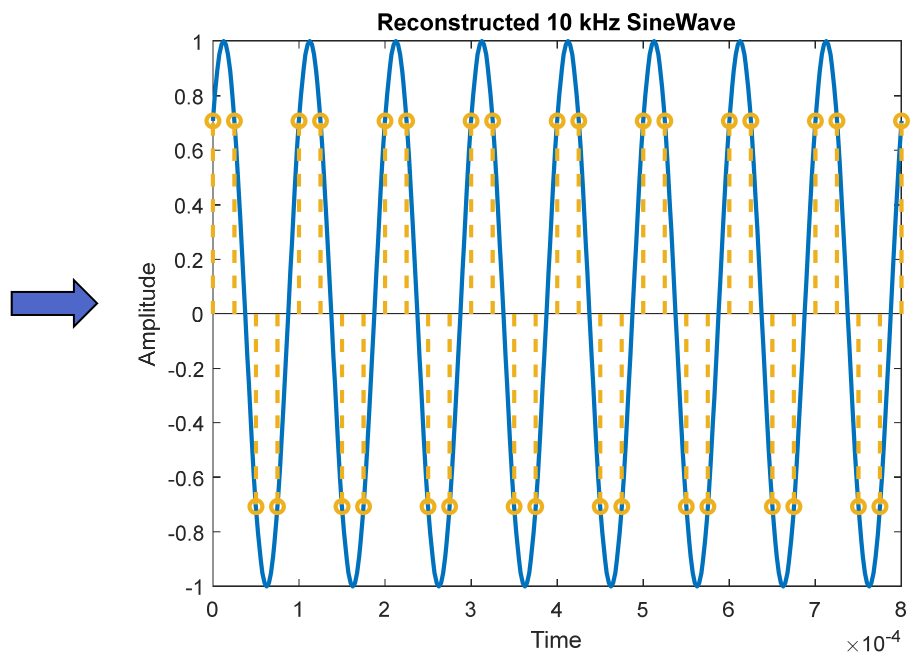


# Reconstructing the Signal

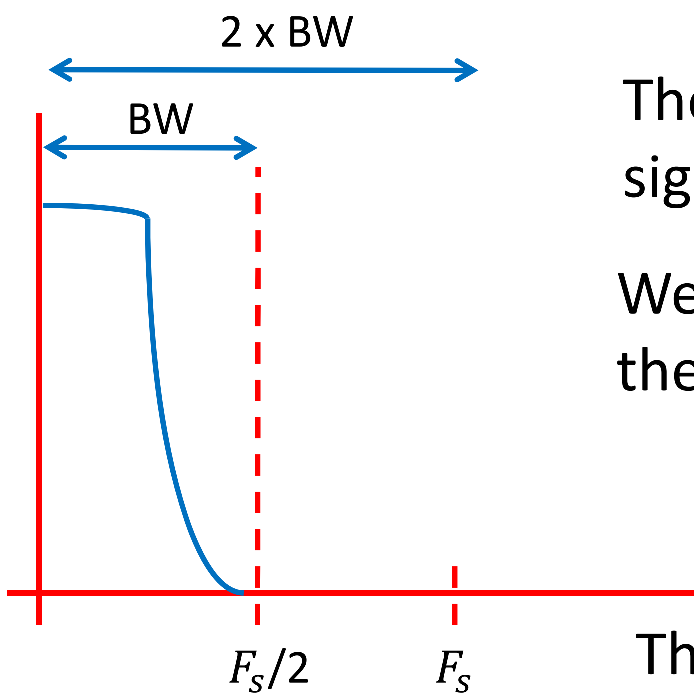
- Placing a low pass filter after the signal removes the copies and reproduces the original sine wave.
- No loss of information



# The Nyquist Rate

- If we sample at a rate that is at least twice the highest frequency of the signal, then we can reconstruct the signal with no loss.




The highest frequency in the signal determines its bandwidth

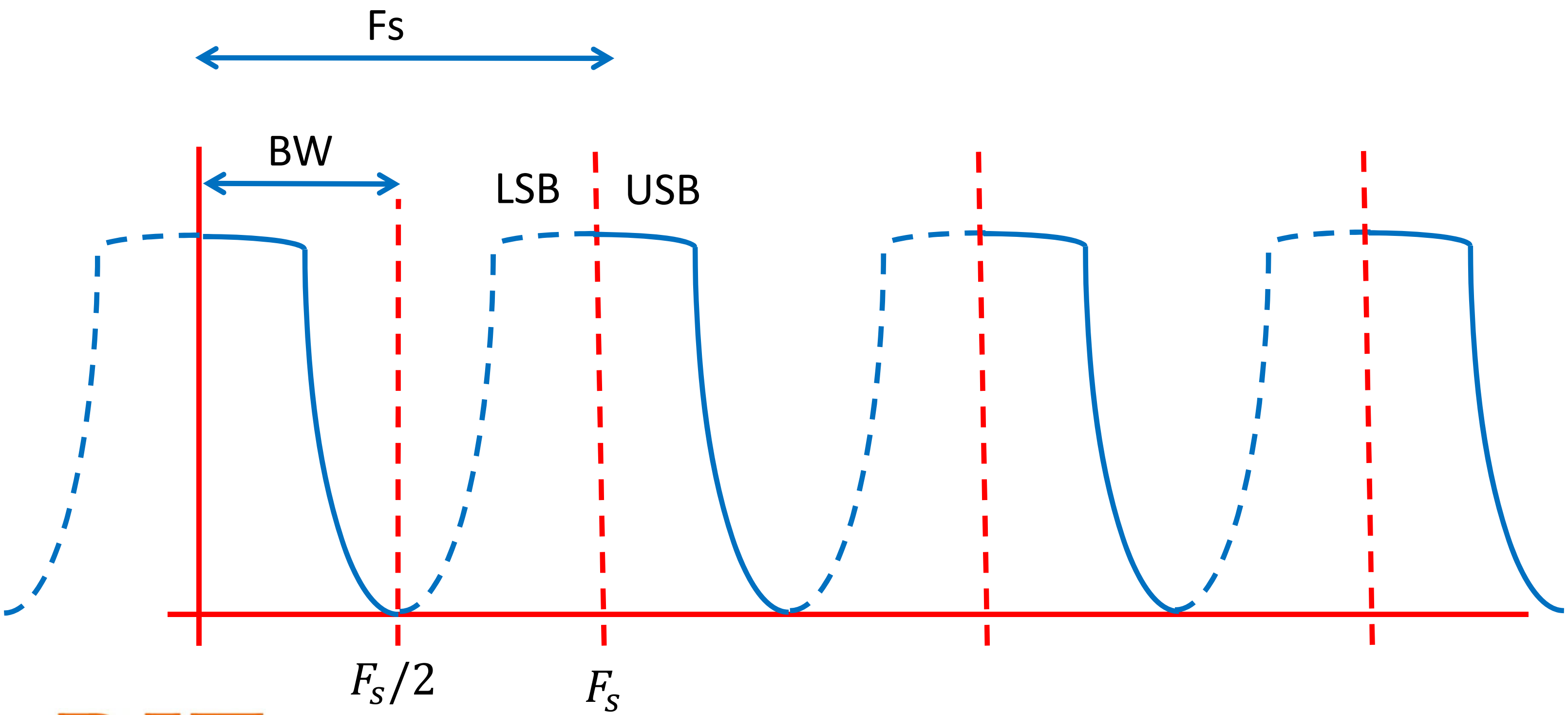
We must sample at least twice the BW to not lose information!

$$F_s \geq 2 \times BW$$

The Nyquist Rate is ½ sample rate
- Also called the folding frequency

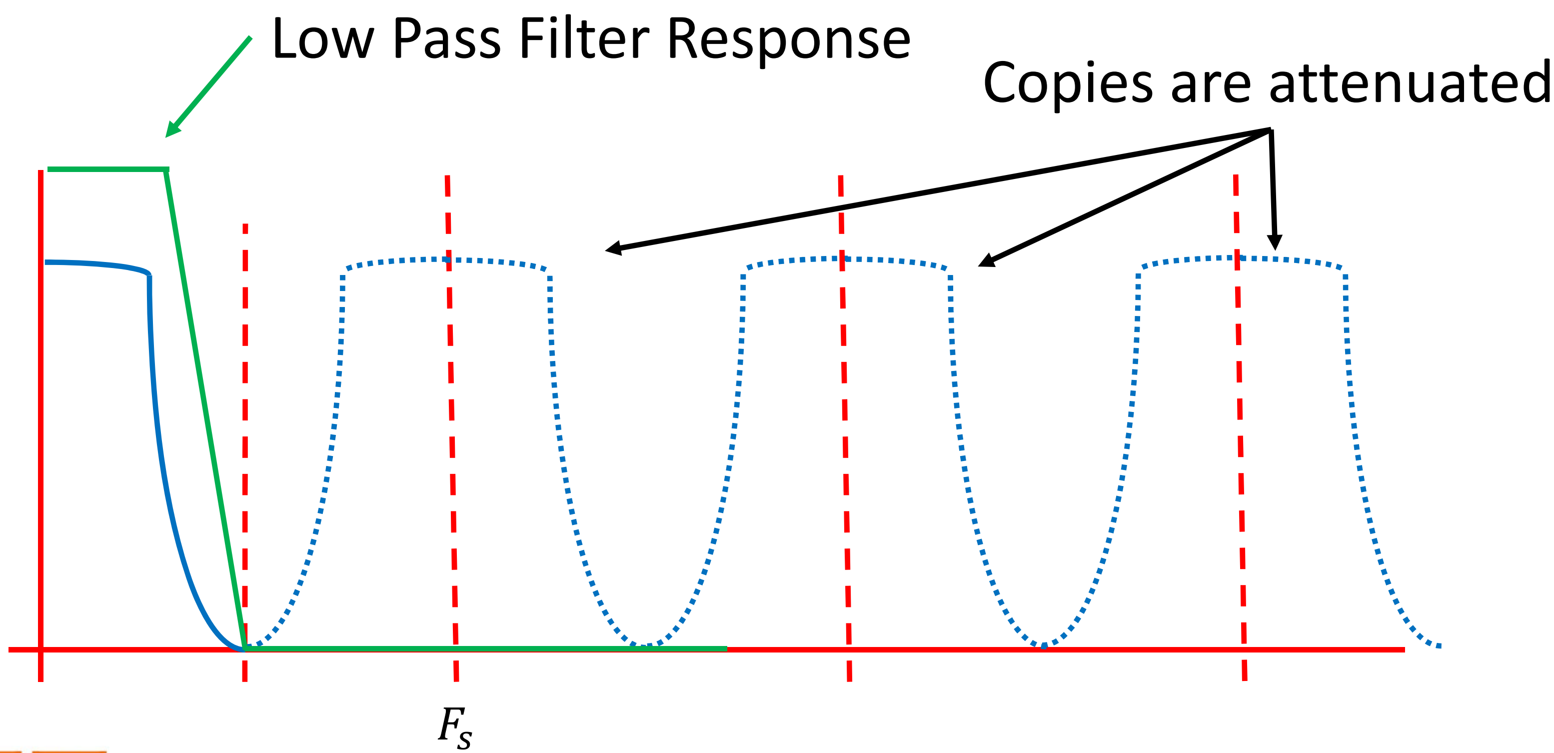
# Sampling at 2X the Nyquist Rate

- When the signal is sampled, the spectrum is duplicated at <u>intervals of the sample rate</u>



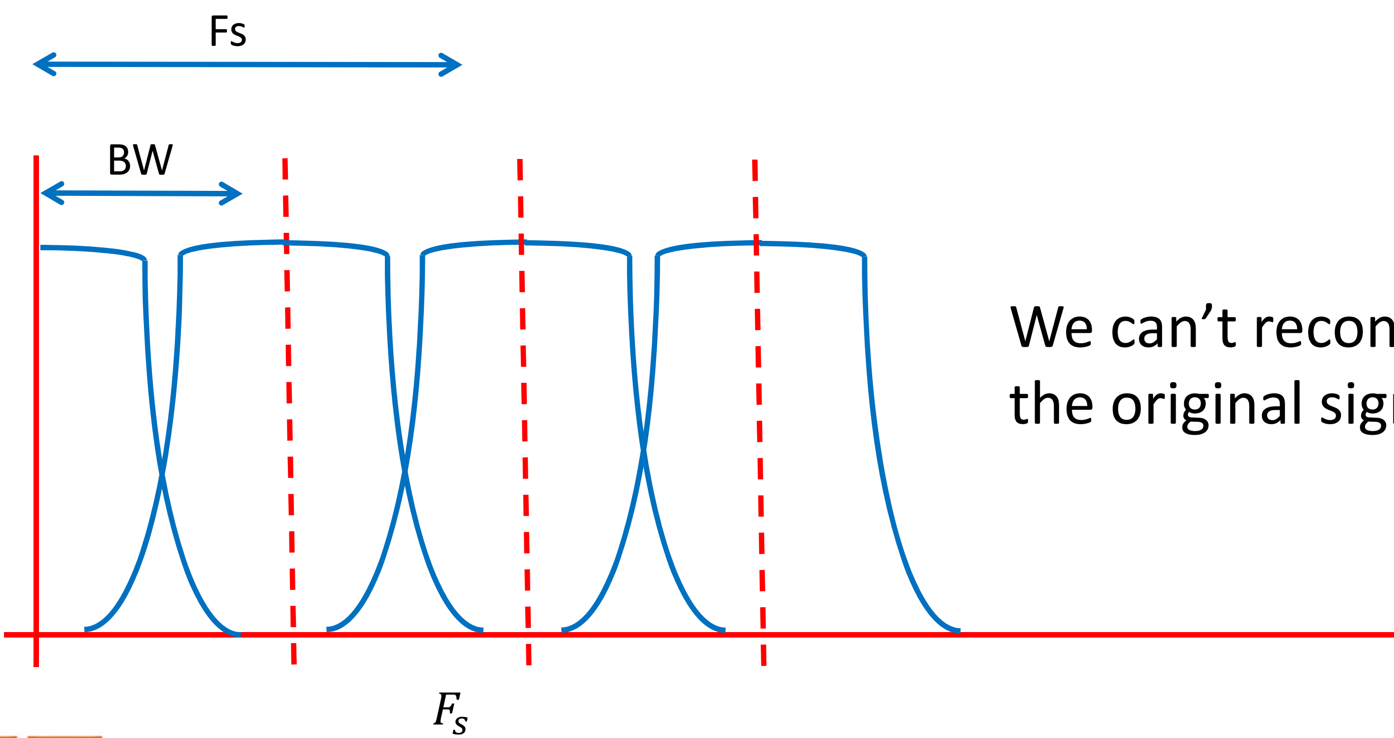
# Sampling at 2X the Nyquist Rate

- Filtering the sampled signal reconstructs the original signal without loss of information



# Sampling at less than 2X the Nyquist Rate

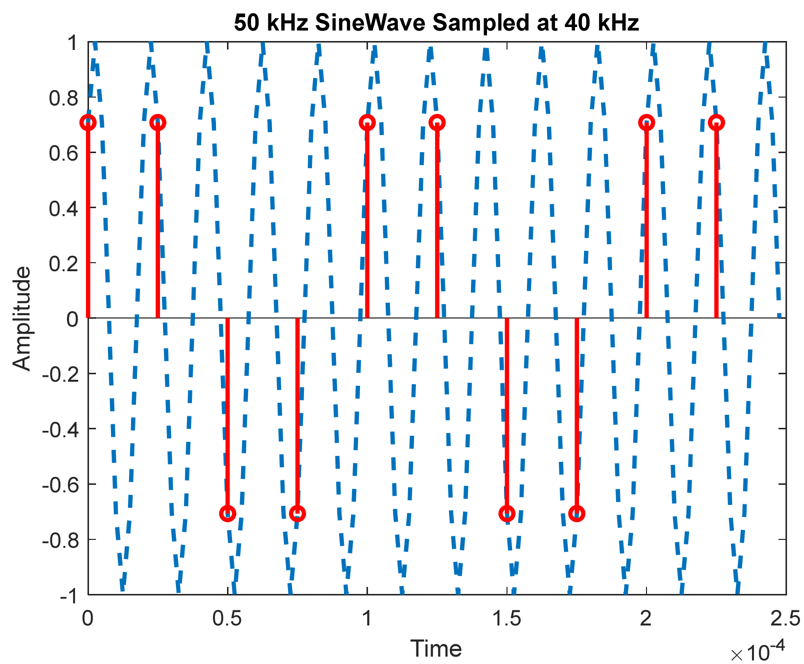
- If the sample rate is too low ( $F_s < 2 \times BW$ ) then information is lost. This is called aliasing.




We can't reconstruct the original signal

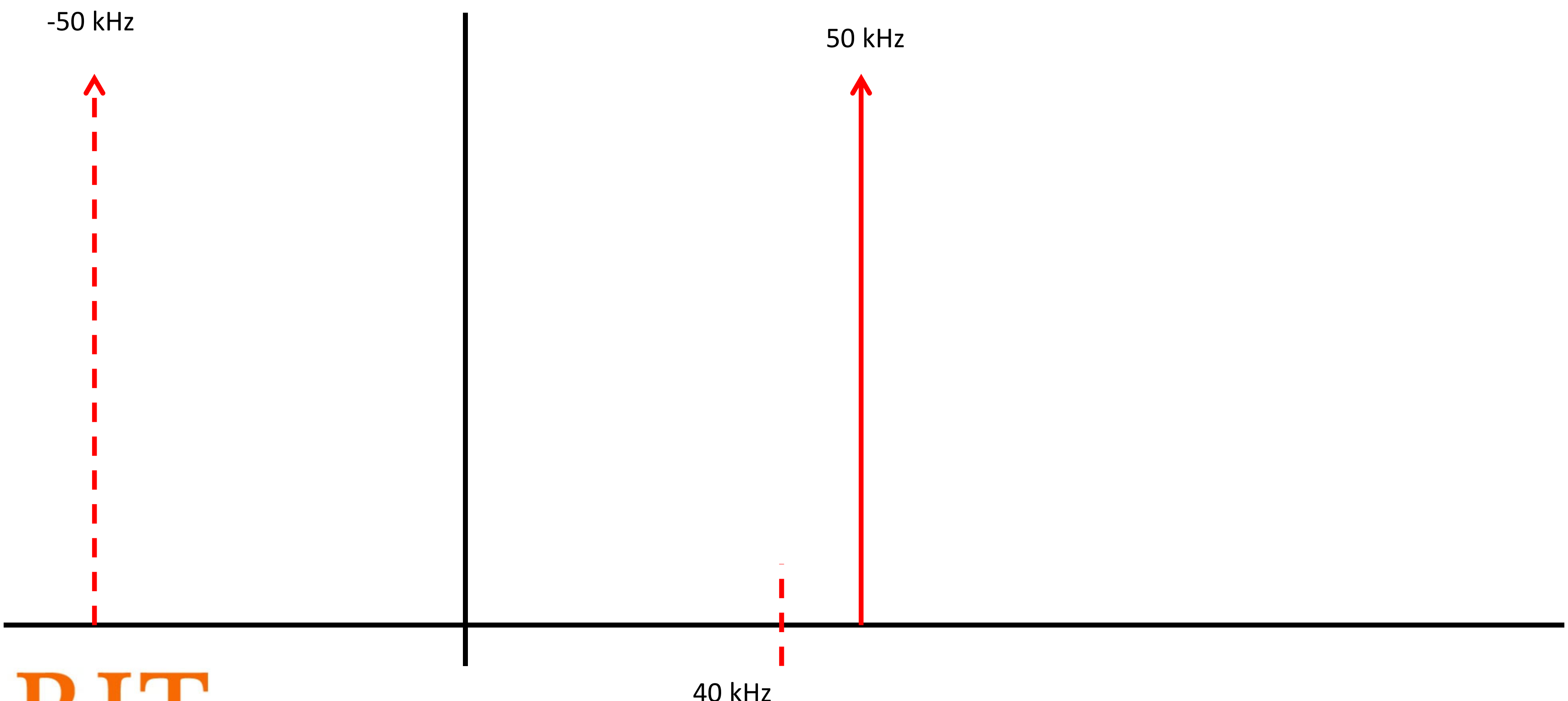
# Back to our previous example

- Assume that we are sampling a 50 kHz sinewave at fs = 40 kHz
  - Does this meet the criteria for proper sampling?



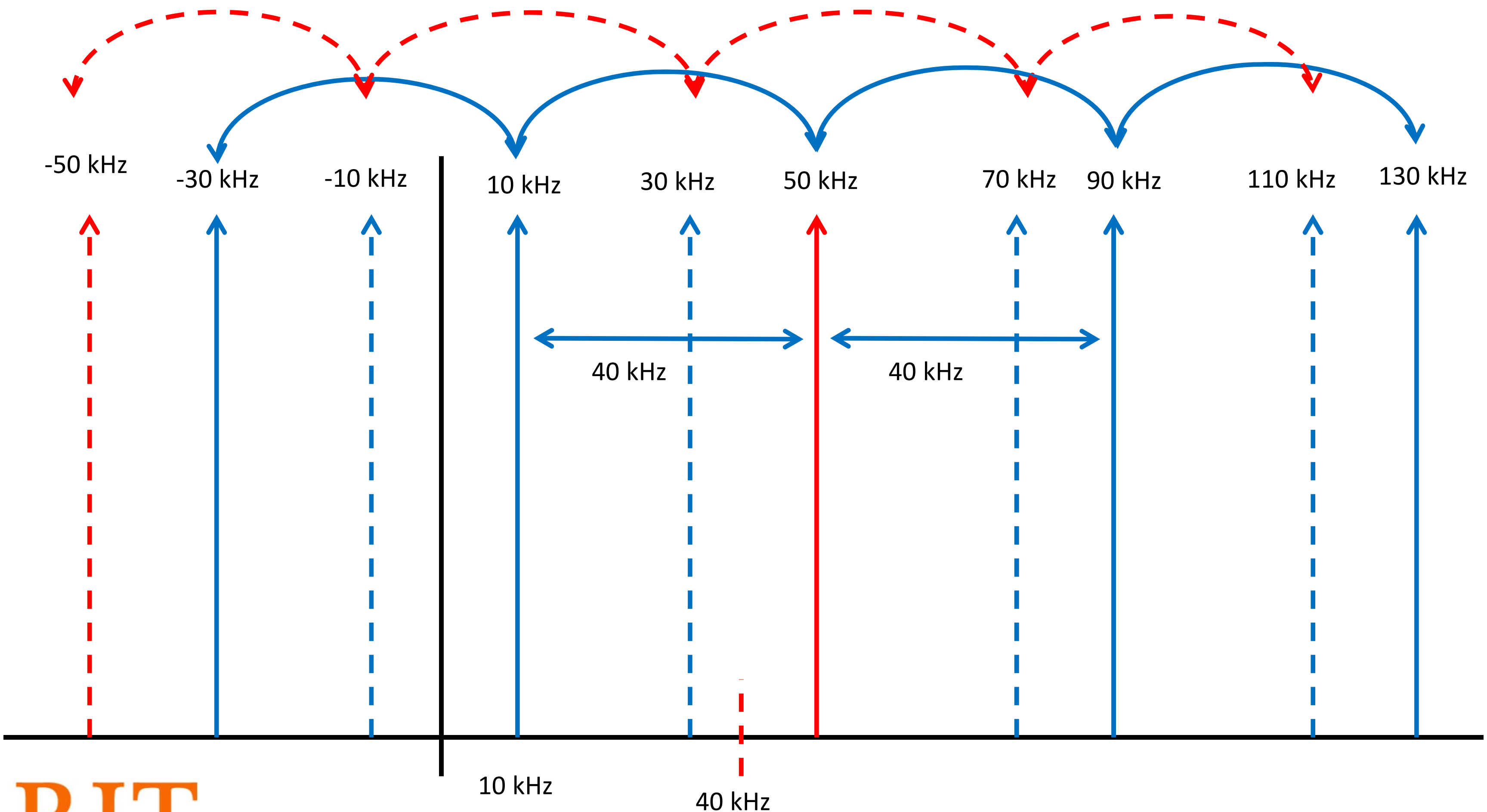
# Where do the copies exist?

- Starting with the continuous signal at 50 kHz.
  - There is a negative frequency component at -50 kHz



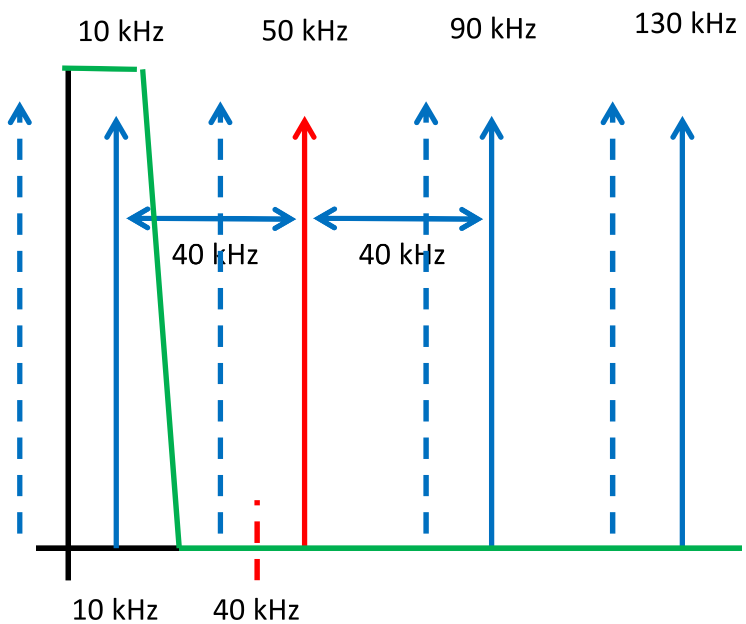
# Where do the copies exist?

- Signals are duplicated every $F_s = 40\ kHz$

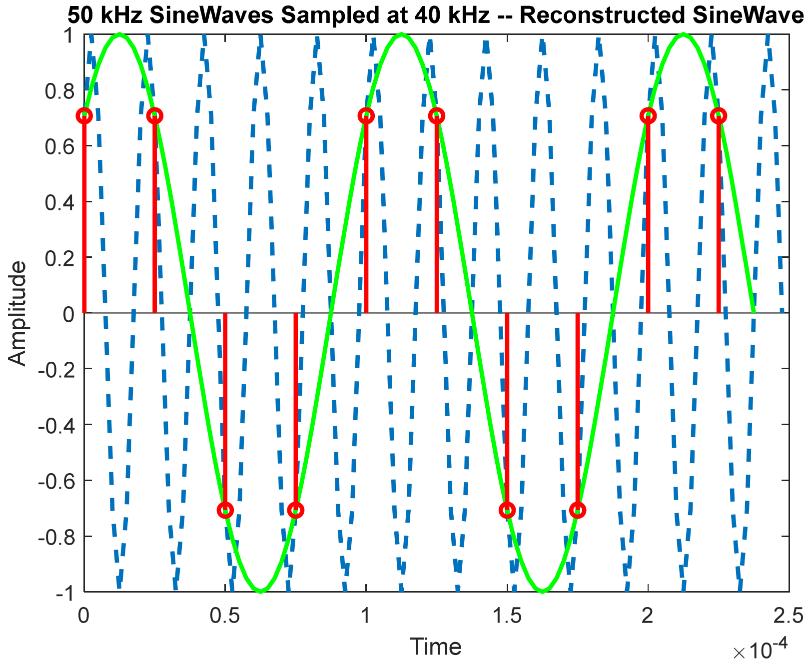


# Back to our previous example

- Filtering the signal with a low pass filter would result in a 10 kHz sinusoid
  - The 50 kHz sinusoid has aliased into the baseband




Spectrum of Sampled Signal



# ICP Sampling Sinusoids

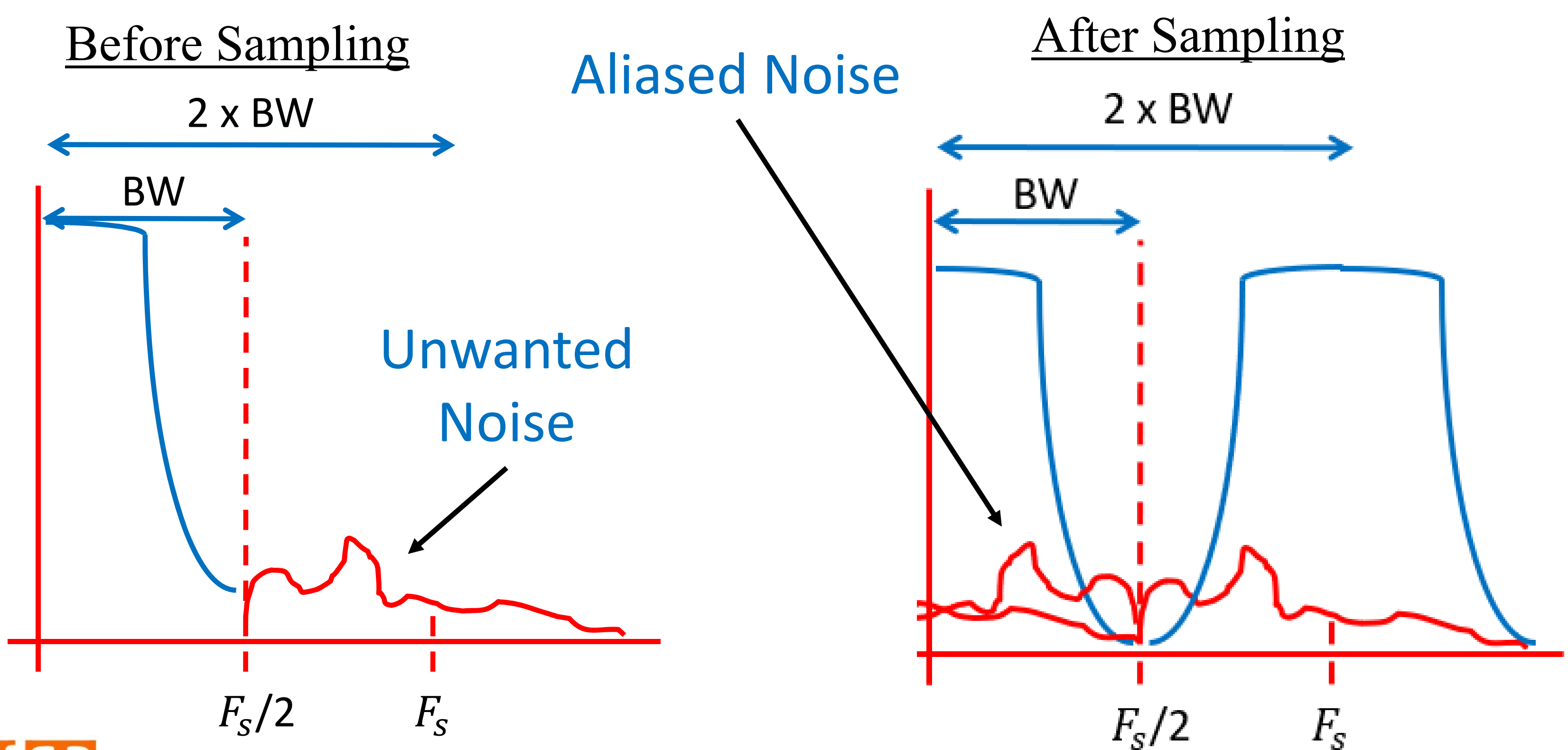
- Consider a single sinusoid:

$$x(t) = 10\sin(2\pi \times 150\mathrm{t})$$

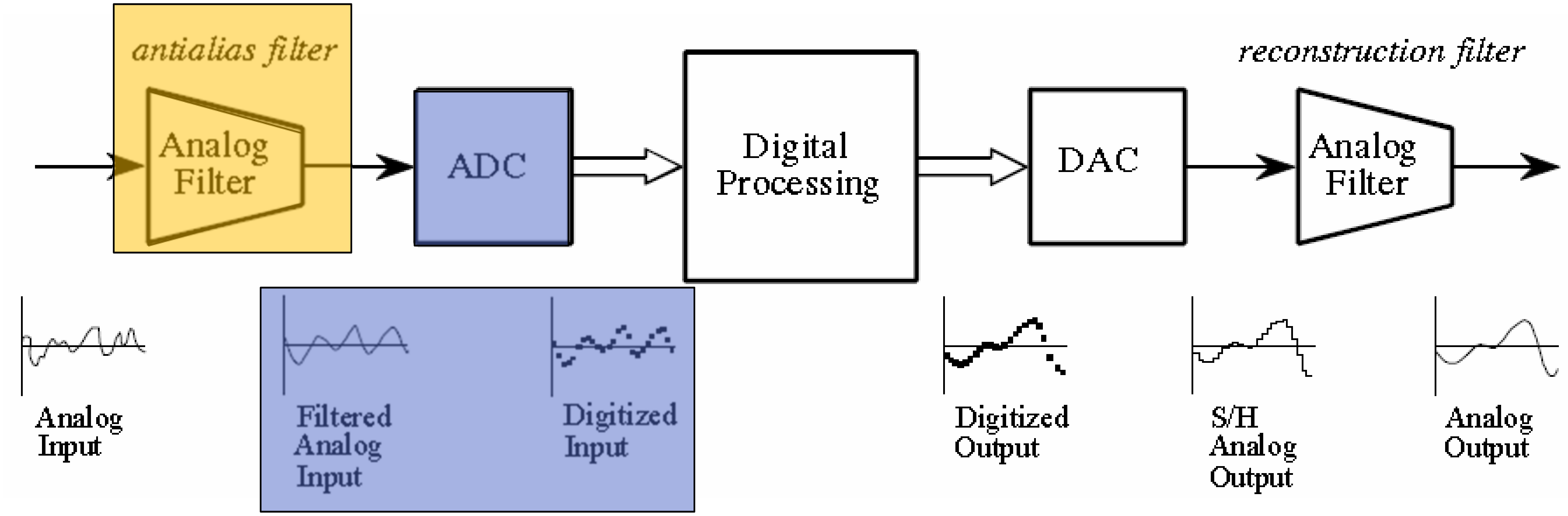
- What is the minimum sampling frequency for this signal?
- Sketch the frequency spectrum from -600Hz to 1kHz if the signal is sampled at 400 Hz

# Aliasing Noise

- If there is unwanted information or noise above the Nyquist rate, that energy will alias into the desired signal after sampling (the baseband region)



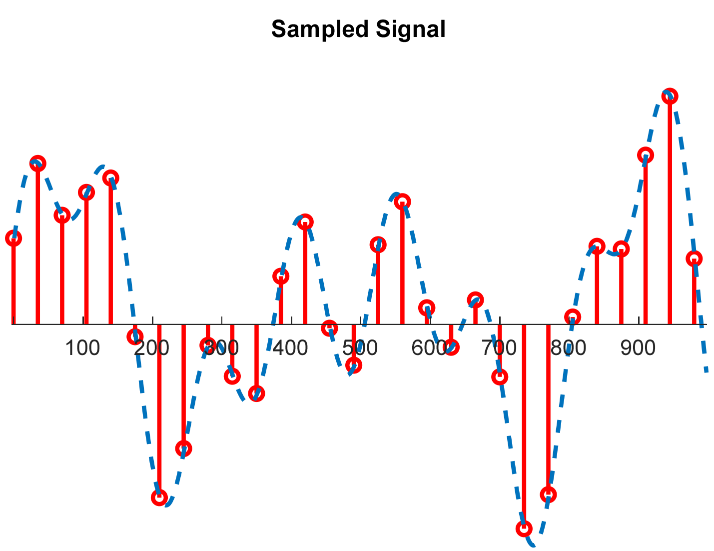
# Preventing Aliasing




- To prevent aliasing due to sampling from occurring, an analog filter is placed in front of the ADC
- This filter, generally a low pass, attenuates energy that is outside of the Nyquist region (above fs/2)
- This is called an anti-alias(ing) filter

# Signal Reconstruction

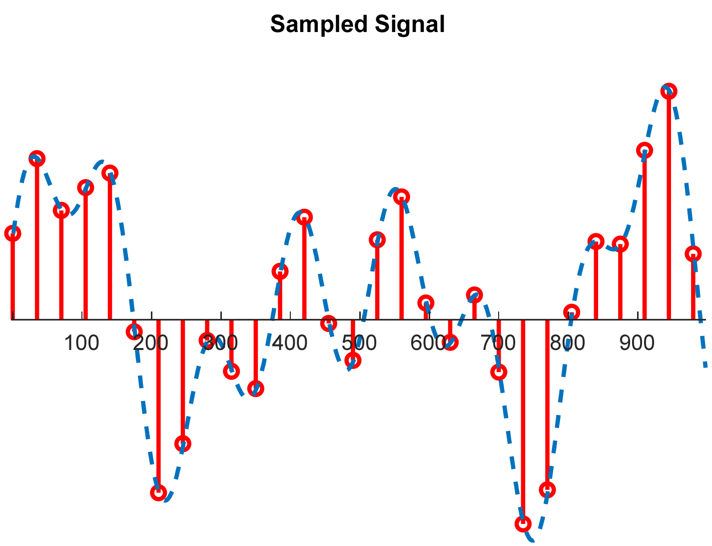
- How do we convert the samples back to a continuous signal?

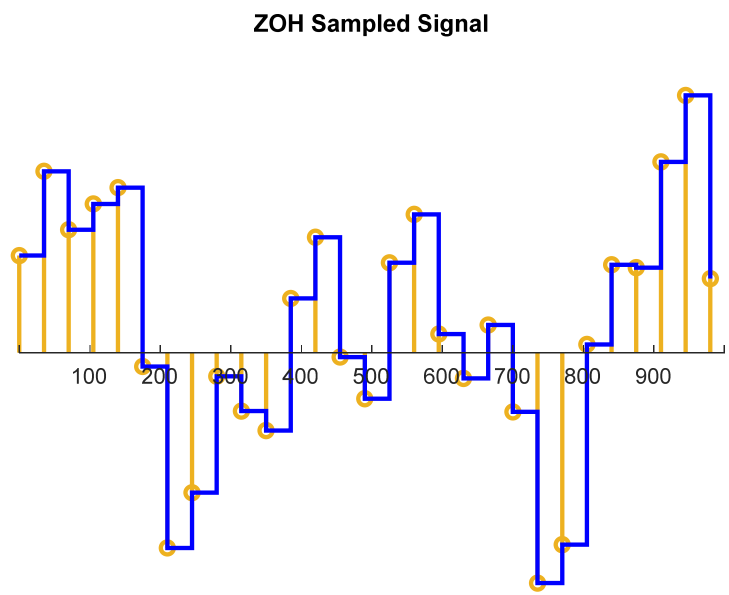



- The numerical values need to be converted into a voltage. This is done by a Digital to Analog Converter (DAC)
- Somehow we need to "connect the dots" between each sample point.

# Signal Reconstruction

- We could hold the value of the sample over 1 sample time – Called a Zero Order Hold (ZOH)

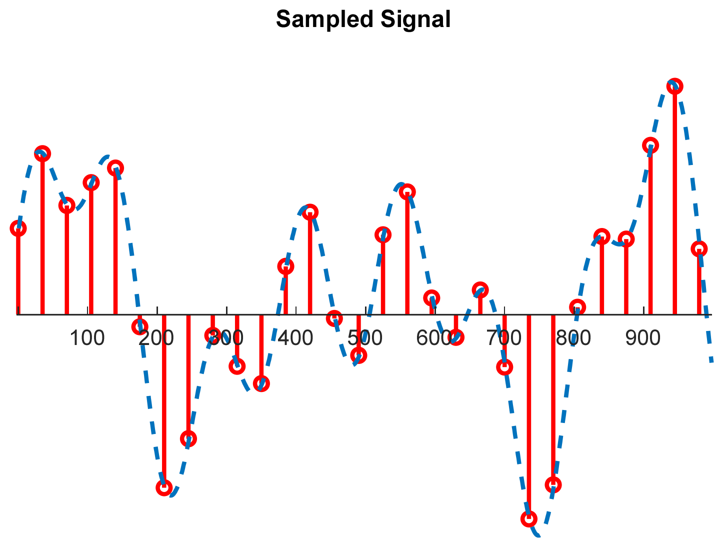


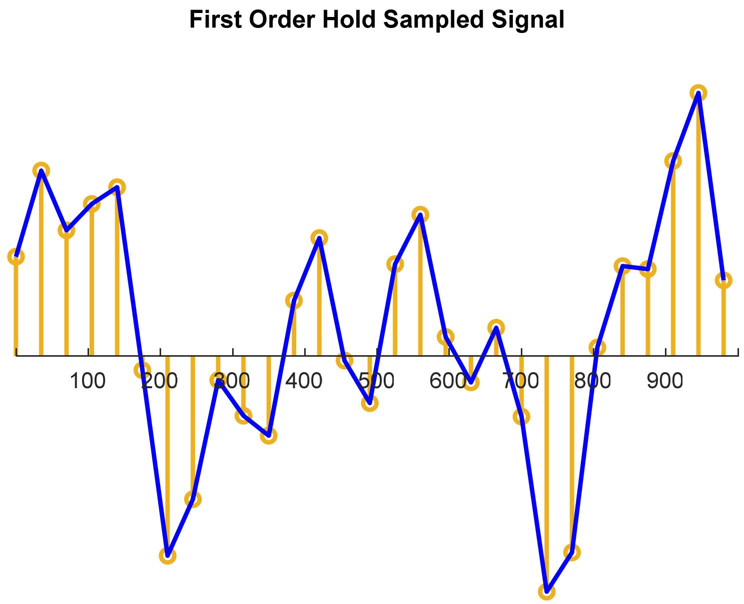




- This is what a DAC does at its output

# Signal Reconstruction

- We could run a straight line between the points (First Order Hold)

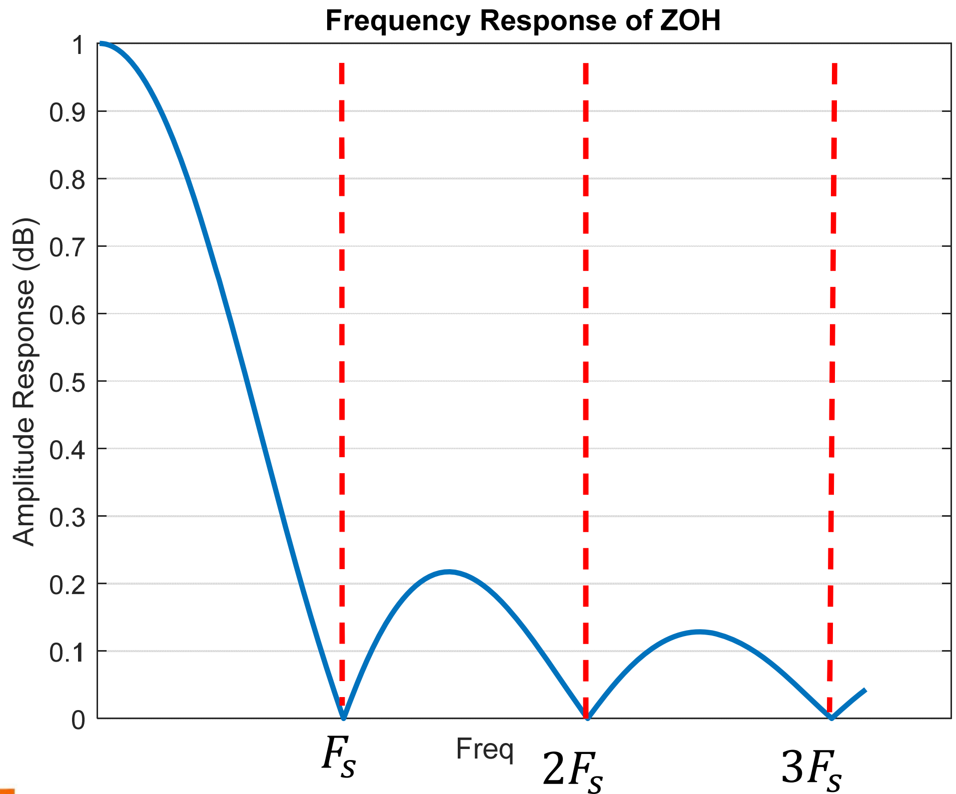



- Not as practical to achieve with DAC output

# What is the impact of the ZOH?

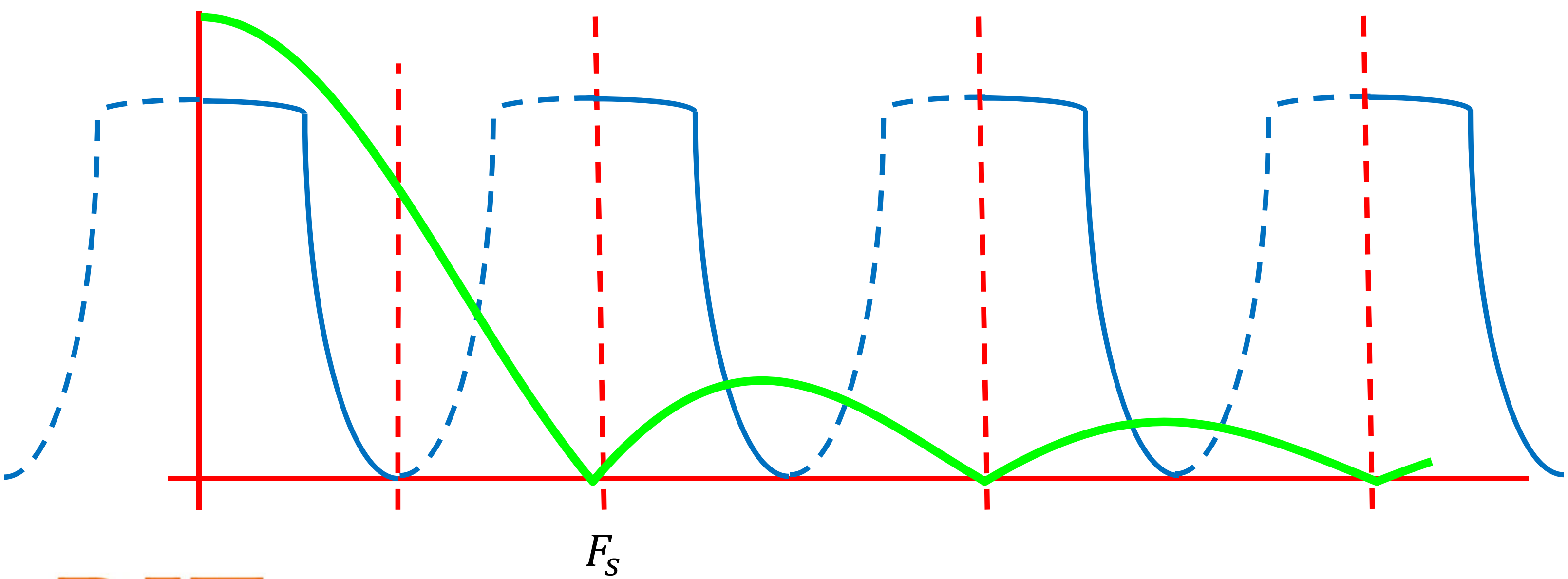
- The frequency response of the Zero Order Hold is a SINC function in the frequency domain




$$Sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

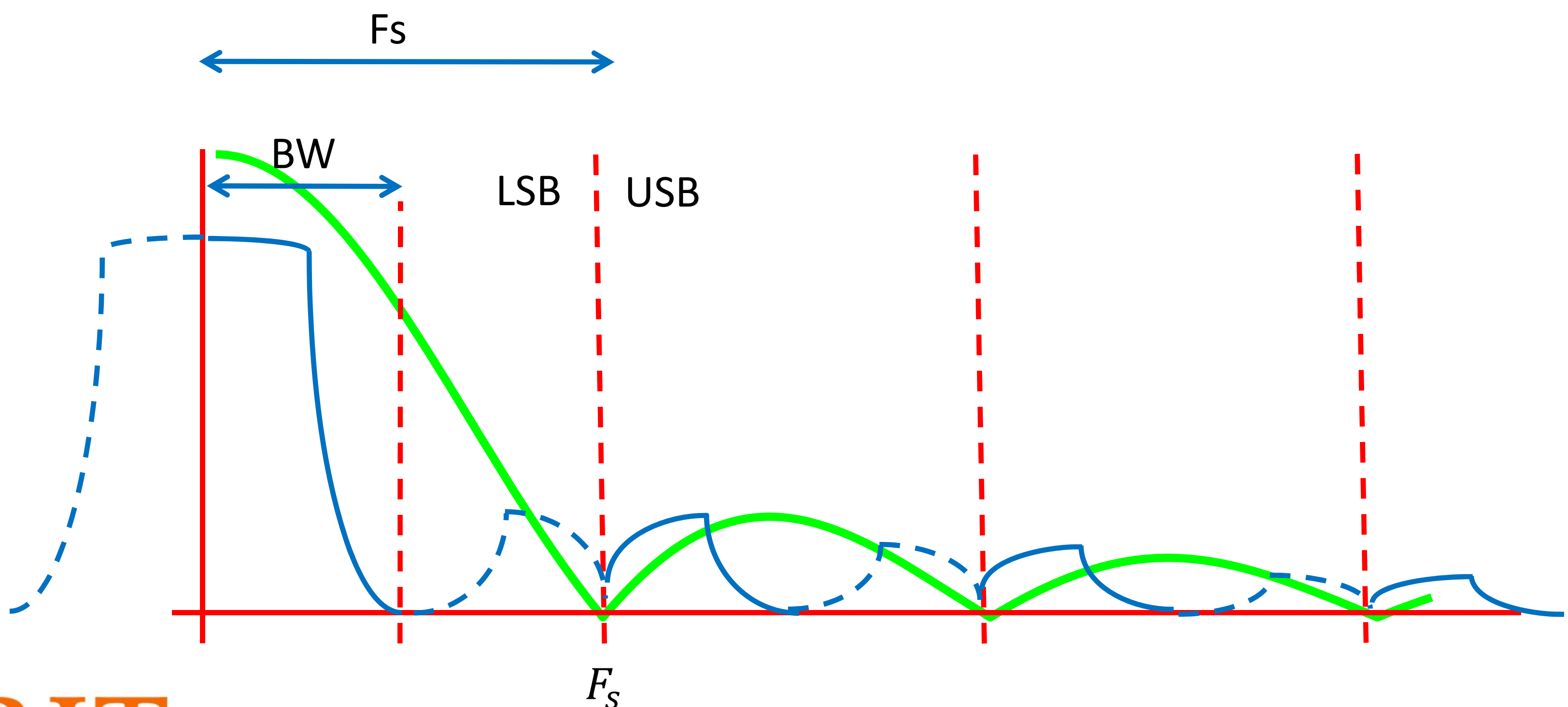
# Sampling at 2X the Nyquist Rate

- The ZOH frequency response is not “flat” across the range from DC to $F_s/2$ so it creates distortion in the signal.



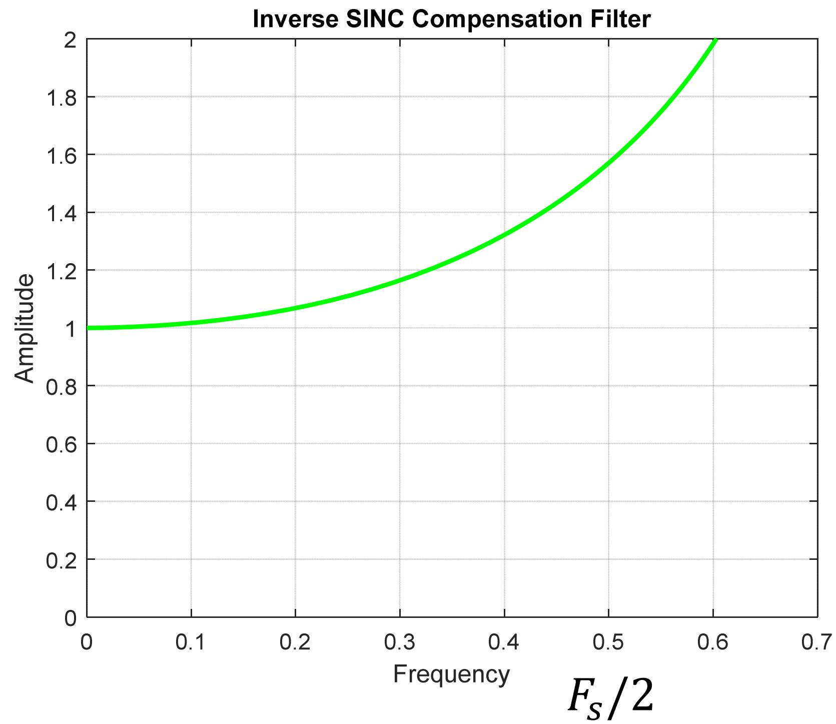
# Reconstruction Filter

- After the ZOH, there is still energy left as sampling artifacts



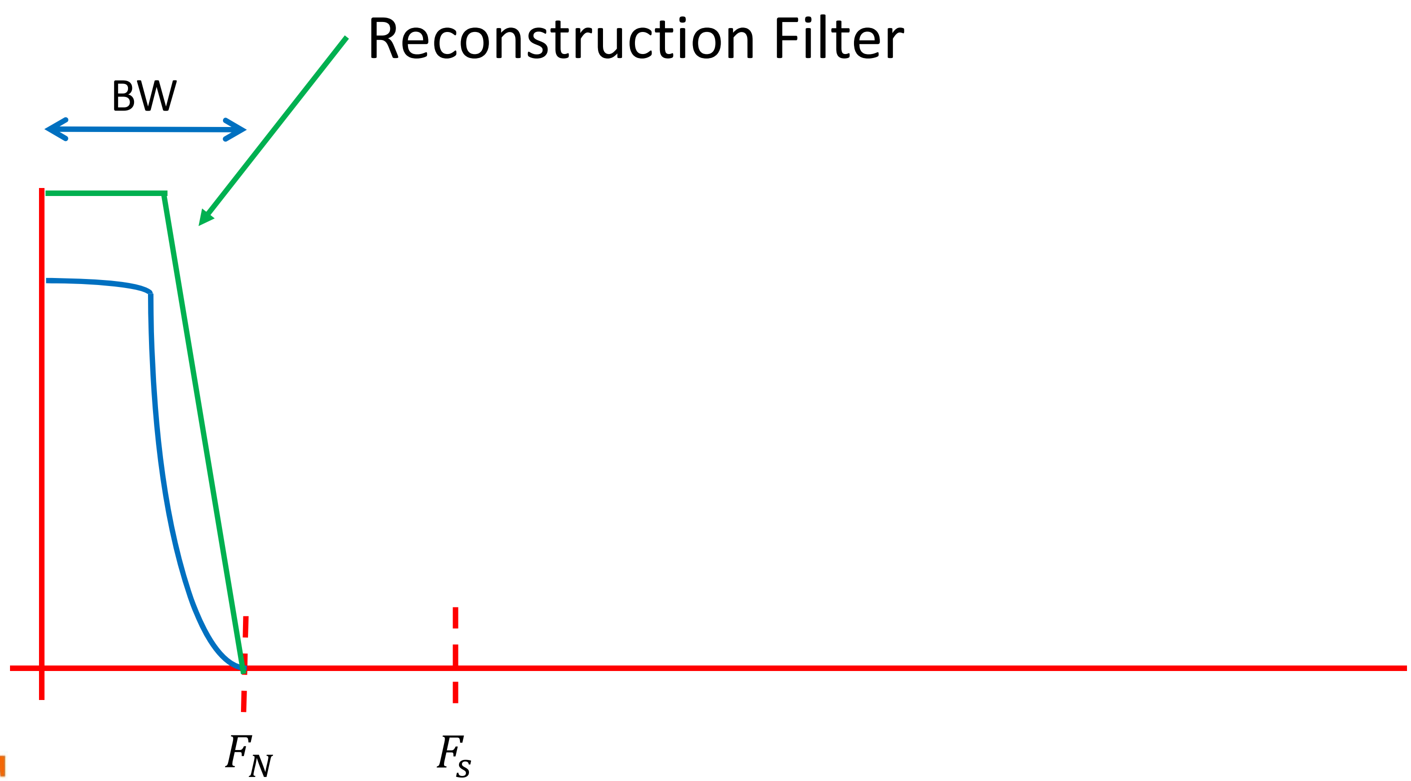
# Sampling at 2X the Nyquist Rate

- To compensate for the distortion, a reconstruction filter with a 1/Sinc(x) characteristic can be used to cancel out the distortion.



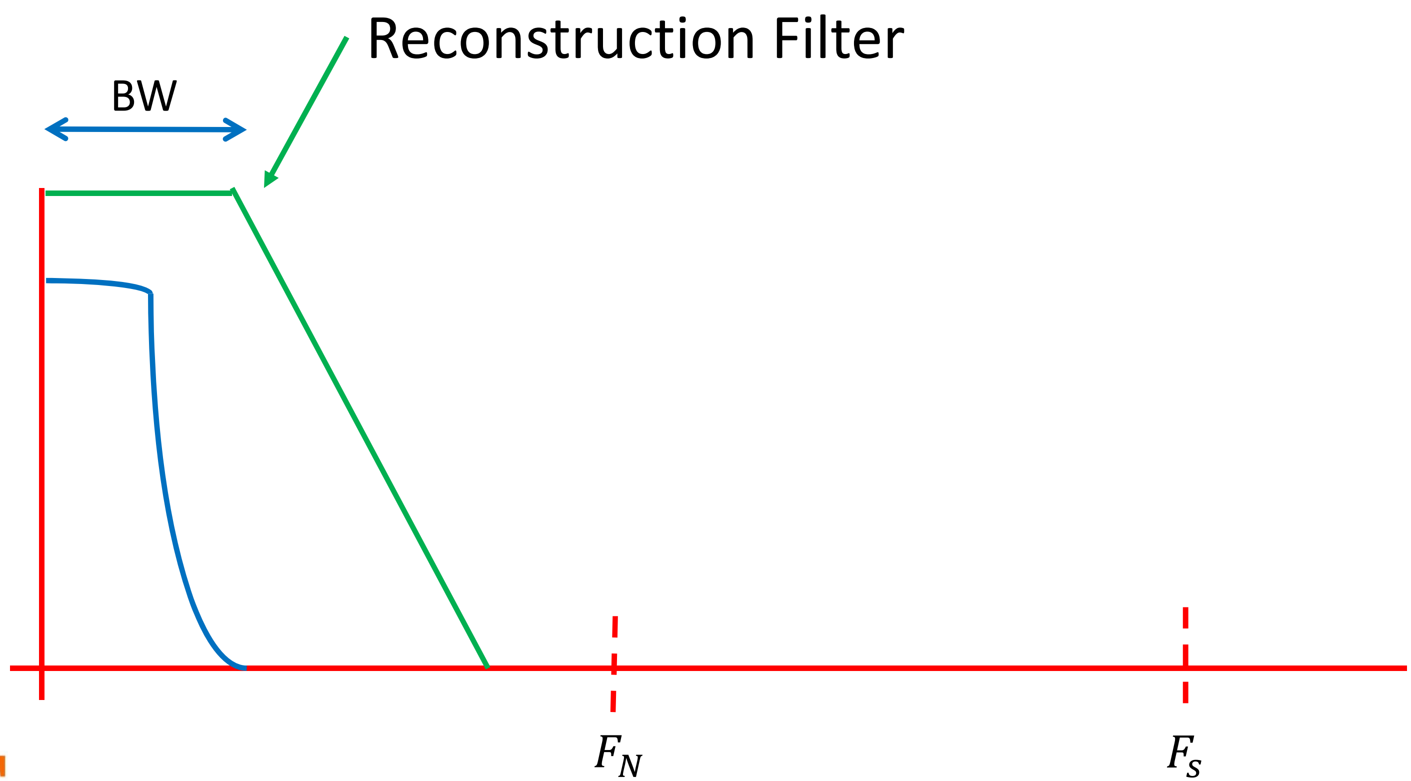
# Reconstruction Filter

- Apply an additional reconstruction filter to reduce the artifacts

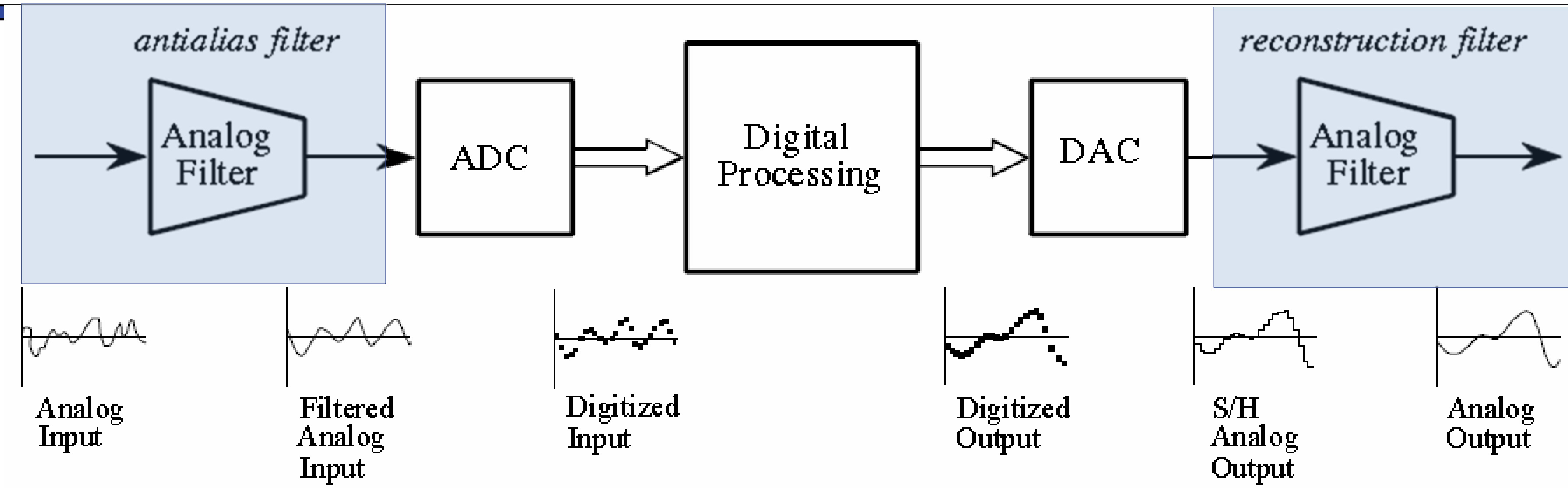


# Reconstruction Filter

- Sampling at a rate higher than 2X Nyquist eases the requirements on the analog reconstruction filter



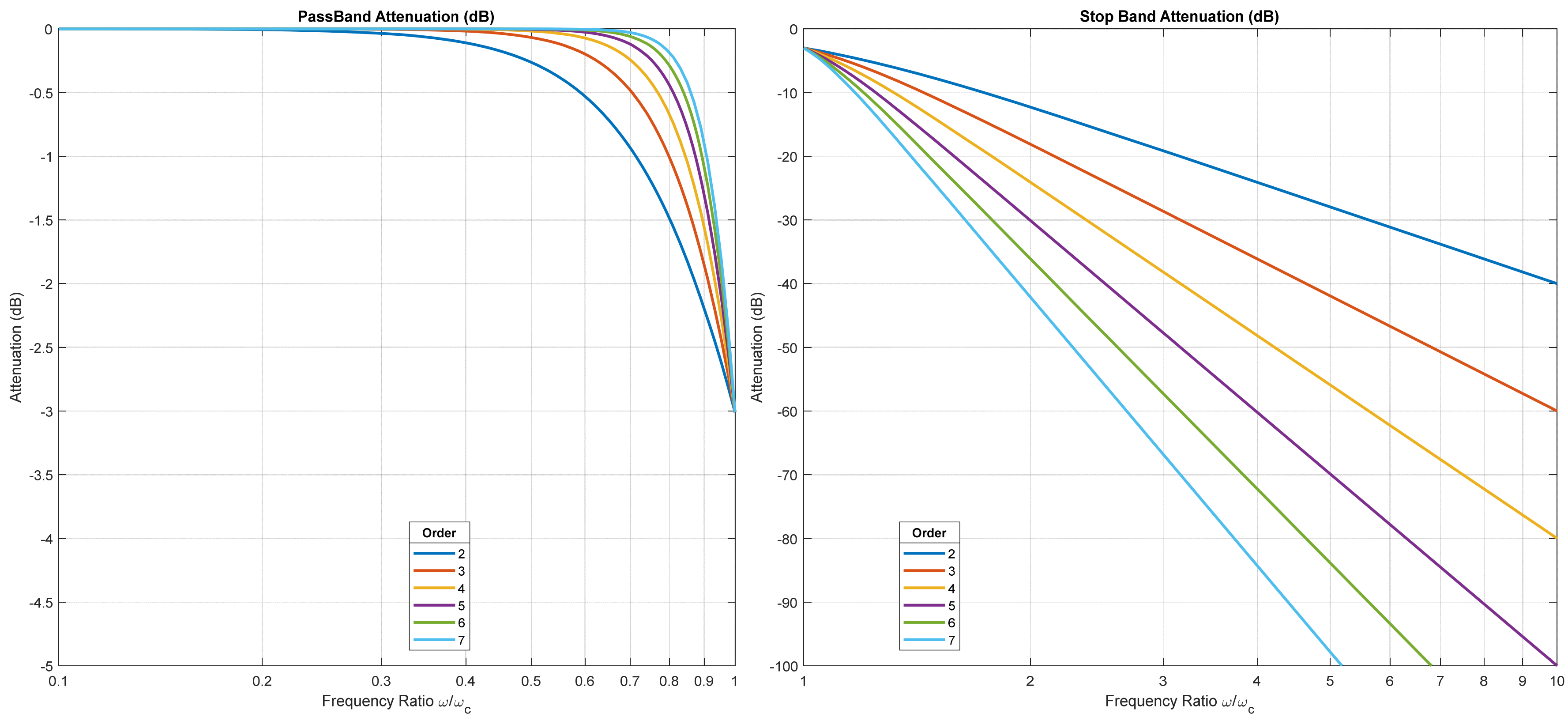
# Anti-Alias and Reconstruction Filters




- Three basic types of filters:
  - Chebyshev
    - fastest roll-off in frequency, frequency domain
  - Butterworth
    - maximally flat, frequency domain
  - Bessel
    - Best step response, time domain

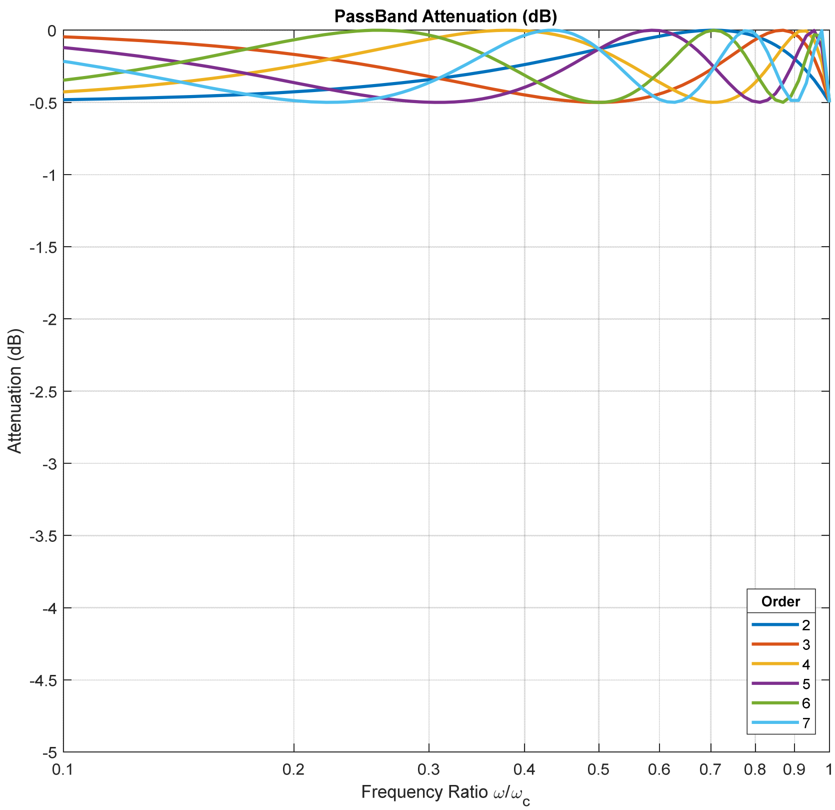
# Butterworth Filters

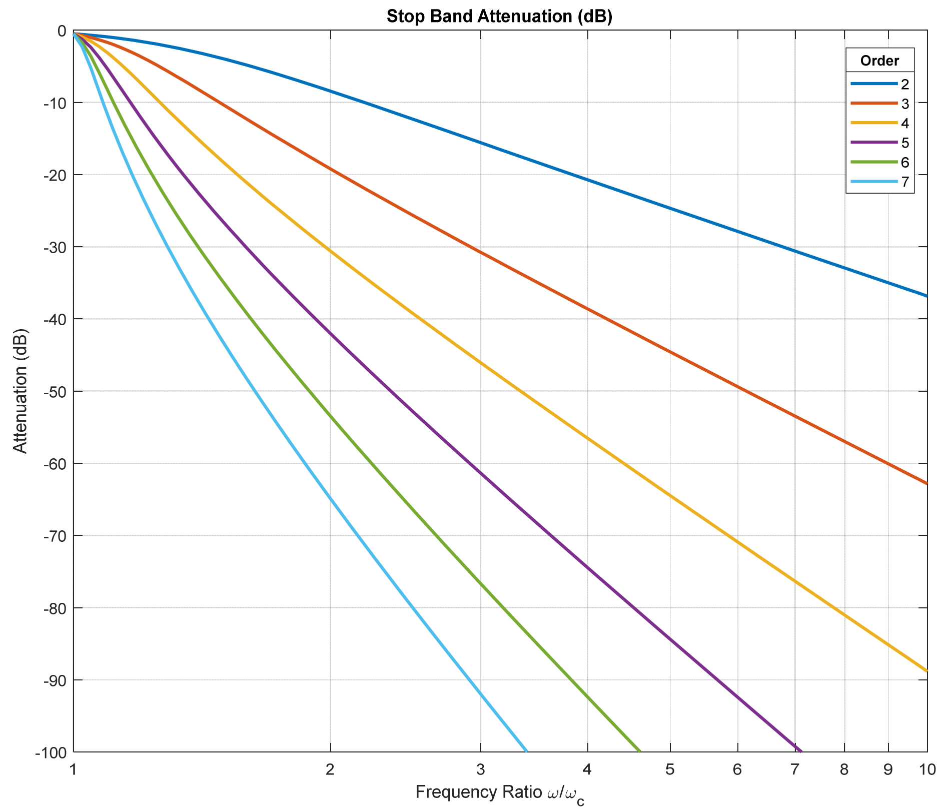
- Passband and Stopband Curves



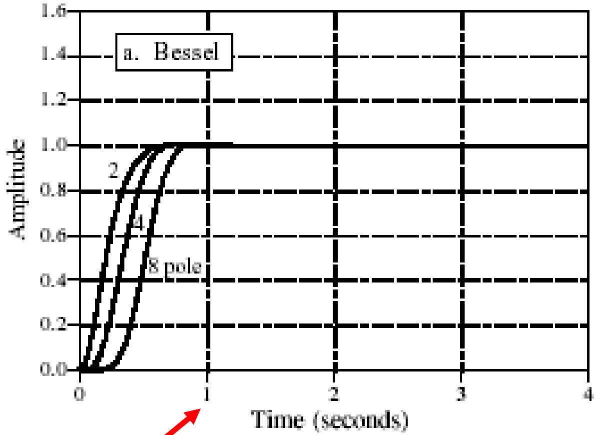
# Chebyshev Filters

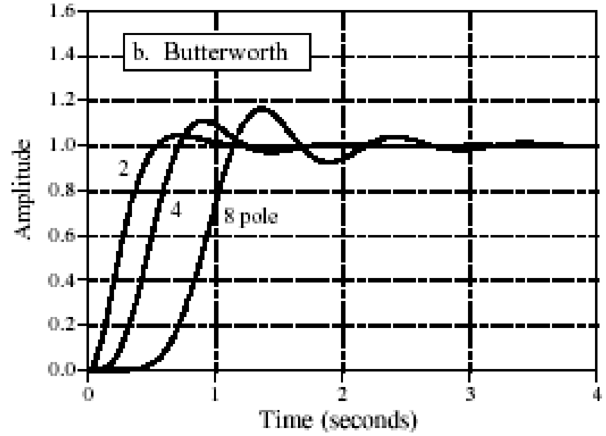
- Passband and Stopband Curves

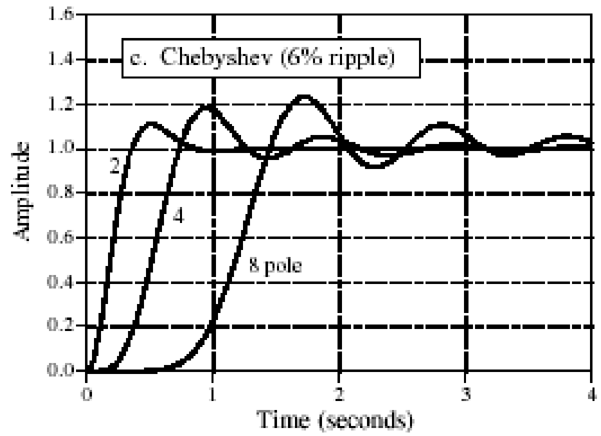


# Comparison of Filter Step Response



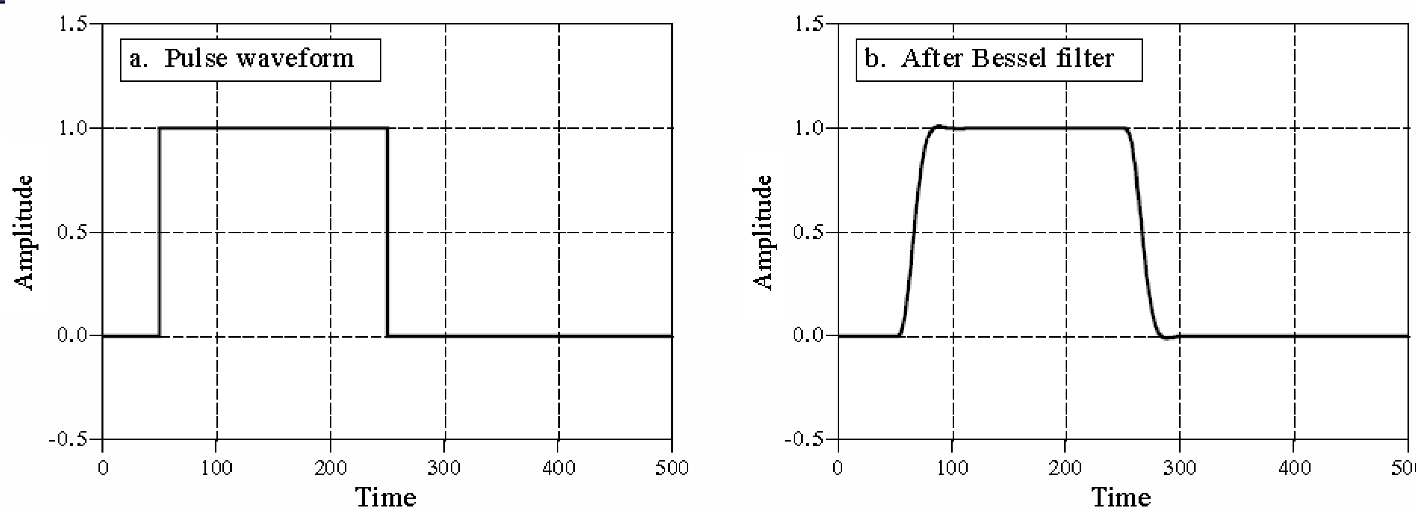




Bessel has good step response –

No significant ringing or overshoot!

# Pulse Response




- Depending on the type of signal being sampled or reconstructed, you may choose one type of filter over the other
  - Choose Bessel if you are concerned about overshoot
  - Choose Chebyshev if you need more attenuation