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БФЗ-19-1

Переоформленное Д/з 1 по КМ

Лист 1

$$N1: \quad \psi(x) = \frac{A}{x^2+a^2}, \quad A?$$

$$\langle\psi|\psi\rangle = 1 \Rightarrow A^2\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} = 1, \text{ рассмотрим } \int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2}:$$

$$\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} = \left| \begin{matrix} u = \frac{1}{x} & du = -\frac{1}{x^2} \\ dV = \frac{x\,dx}{(x^2+a^2)^2} & V = -\frac{1}{2(x^2+a^2)} \end{matrix} \right| = \underbrace{-\frac{1}{x}\cdot\frac{1}{2(x^2+a^2)}\Big|_{-\infty}^{\infty}}_{=0} + \int_{-\infty}^{\infty}-\frac{1}{2}\frac{dx}{x^2(x^2+a^2)^2} = (1)$$

$$\frac{1}{x^2(x^2+a^2)} = \frac{C}{x^2} + \frac{Dx+E}{(x^2+a^2)} = \frac{C(x^2+a^2)+Dx^3+Ex^2}{x^2(x^2+a^2)}$$

$$\Rightarrow \begin{matrix} x^3: 0 = D \\ x^2: C+E=0 \\ x^1: - \\ 1: a^2\cdot C = 1 \end{matrix} \Rightarrow \begin{matrix} C = \frac{1}{a^2} \\ E = -\frac{1}{a^2} \end{matrix} \Rightarrow \frac{1}{x^2(x^2+a^2)} = \frac{1}{a^2x^2} - \frac{1}{a^2(x^2+a^2)}$$

$$(1) \Rightarrow \int_{-\infty}^{\infty}-\frac{1}{2}\frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{2}\int_{-\infty}^{\infty}\left(\frac{1}{a^2x^2} - \frac{1}{a^2(x^2+a^2)}\right)dx = -\frac{1}{2}\cdot\left(\underbrace{-\frac{1}{a^2x}\Big|_{-\infty}^{+\infty}}_{=0} - \frac{1}{a^3}\operatorname{arctg}\left(\frac{x}{a}\right)\Big|_{-\infty}^{\infty}\right)$$

$$= -\frac{1}{2}\cdot -\frac{1}{a^3}\cdot\pi = \frac{\pi}{2a^3} = \int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} \Rightarrow$$

$$\cdot A^2\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} = \frac{A^2\pi}{2a^3} = 1 \Leftrightarrow A = \sqrt{\frac{2a^3}{\pi}} \quad ✓$$

$$N2 \quad \varphi(x) = \frac{B}{x+ib} \Rightarrow \varphi^*(x) = \frac{B}{x-ib}, \quad B?$$

$$\langle\varphi|\varphi\rangle = 1 = B^2\int_{-\infty}^{\infty}\frac{1}{x^2+b^2}dx = \frac{B^2}{b}\operatorname{arctg}\frac{x}{b}\Big|_{-\infty}^{\infty} = 1 = \frac{B^2}{b}\pi \Leftrightarrow B = \sqrt{\frac{b}{\pi}} \quad ✓$$

N4

$$\delta(f(x)) = \delta\left(\sum_i C_i(x-x_i)\right) = \left|\delta(ax) = \frac{1}{|a|}\delta(x)\right| = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|}$$

где $f(x) \approx \underset{\substack{\uparrow \\ \text{нули} \\ \text{ф-ии} \\ (\text{т.е}=0)}}{f(x_i)} + \sum_i \underbrace{f'(x_i)}_{=C}\cdot(x-x_i)$

См. след. страницу:

№5, пункт 1

$$\langle f_1|f_2\rangle = \int_0^a \alpha_1\alpha_2 \exp(-2i\pi x/a)\,dx = \alpha_1\alpha_2\cdot\left(\frac{a}{-2i\pi}\right)\exp\left(-\frac{2i\pi x}{a}\right)\Big|_0^a$$

$$= \alpha_1\alpha_2\left(\frac{a}{-2i\pi}\right) - \alpha_1\alpha_2\left(\frac{a}{-2i\pi}\right) = 0 \checkmark \text{ док.}$$

№5 пункт 2

$$\langle f_1|f_1\rangle = 1 = \alpha_1^2\int_{-0}^{a}\exp(-i\pi x/a + i\pi x/a)\,dx = \alpha_1^2 x\Big|_0^a = 1 \Leftrightarrow \alpha = \sqrt{\frac{1}{a}}$$

$\langle f_2|f_2\rangle = 1$ = аналогично $\Rightarrow \alpha_2 = \sqrt{\frac{1}{a}}$

№5 пункт 3

$\sin\left(\frac{\pi x}{a}\right)$ через экспон.

$$c_1 = \langle f_1|\psi\rangle = \sqrt{\frac{1}{a}}\cdot\sqrt{\frac{2}{a}}\int_0^a e^{-\frac{i\pi x}{a}}\cdot\left(e^{\frac{i\pi x}{a}} - e^{-\frac{i\pi x}{a}}\right)\cdot\frac{1}{2i}\,dx$$

$$= \frac{1}{\sqrt{2}\cdot i\cdot a}\cdot\int_0^a\left(1 - e^{-\frac{2i\pi x}{a}}\right)dx = \frac{1}{\sqrt{2}\cdot i\cdot a}\left(x\Big|_0^a - \frac{a}{-2i\pi}e^{-\frac{2i\pi x}{a}}\Big|_0^a\right)$$

$$= \frac{1}{\sqrt{2}\cdot i\cdot a}\cdot\left(a - \left(\frac{a}{-2i\pi x} - \frac{a}{-2i\pi x}\right)\right) = \frac{1}{\sqrt{2}\,i} = -\frac{i}{\sqrt{2}} = c_1$$

$$c_2 = \langle f_2|\psi\rangle = \frac{1}{\sqrt{2}\cdot i\cdot a}\int_0^a\left(e^{\frac{2i\pi x}{a}} - 1\right)dx = -\frac{1}{\sqrt{2}\cdot i} = \frac{i}{\sqrt{2}} = c_2$$

$$\Rightarrow |\psi\rangle = \frac{-i}{\sqrt{2}}|f_1\rangle + \frac{i}{\sqrt{2}}|f_2\rangle \checkmark$$ (проверка: $\left|\frac{i}{\sqrt{2}}\right|^2 + \left|-\frac{i}{\sqrt{2}}\right|^2 = 1$, да!)

№3:

$$\langle\varphi|\psi\rangle = AB\int_{-\infty}^{\infty}\frac{dx}{(x-ib)(x^2+a^2)}$$, подумаем о подинтегр. выражении:

$$\frac{1}{(x-ib)(x^2+a^2)} = \frac{C}{(x-ib)} + \frac{Dx+E}{(x^2+a^2)} = \frac{C(x^2+a^2) + Dx(x-ib) + E(x-ib)}{(x-ib)(x^2+a^2)}$$

x^2: $C + D = 0$

x^1: $-ibD + E = 0$ $\Rightarrow a^2C + b^2D = 1$

1: $a^2C - ibE = 1$ $\Rightarrow a^2C - b^2C = 1 \Rightarrow$ $C = \frac{1}{a^2-b^2}$, $D = -\frac{1}{a^2-b^2}$, $E = \frac{ib}{b^2-a^2}$ $\Rightarrow$

$$\frac{1}{(x-ib)(x^2+a^2)} = \frac{1}{(a^2-b^2)}\frac{1}{(x-ib)} - \frac{1}{(a^2-b^2)}\frac{x}{(x^2+a^2)} - \frac{ib}{(a^2-b^2)(x^2+a^2)}$$

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№3, продолжение:

0 ← интегр. от нечётн. ф-ии

$$\Rightarrow \int_{-\infty}^{\infty} \frac{AB\,dx}{(x-ib)(x^2+a^2)} = \frac{AB}{(a^2-b^2)} \int_{-\infty}^{\infty} \left(\frac{1}{(x-ib)} - \underbrace{\frac{x}{(x^2+a^2)}}_{=0} - \frac{ib}{(x^2+a^2)} \right) dx$$

$= 0 \leftarrow$ интегр от нечётн. ф-ии

$$= \frac{AB}{(a^2-b^2)} \int_{-\infty}^{\infty} \left(\frac{x+ib}{\underbrace{(x-ib)(x+ib)}_{x^2+b^2}} - \frac{ib}{(x^2+a^2)} \right) dx = \frac{AB}{(a^2-b^2)} \cdot \int_{-\infty}^{\infty} \left(\frac{x}{x^2+b^2} + \frac{ib}{x^2+b^2} - \frac{ib}{x^2+a^2} \right) dx$$

$\uparrow \frac{1}{b} \operatorname{arctg} \frac{x}{b}$

$$= \frac{AB}{a^2-b^2} \cdot \left(ib \cdot \frac{1}{b} \cdot \pi - ib \cdot \frac{1}{a} \cdot \pi \right) = \frac{AB\,\pi i b}{a^2-b^2} \left(\frac{1}{b} - \frac{1}{a} \right) =$$

$$\frac{AB}{a^2-b^2} \cdot \pi \cdot ib \cdot \frac{a-b}{ba} = AB\pi \cdot i \cdot b \cdot \frac{1}{ba(b+a)} = \sqrt{\frac{2a^3b}{\pi^2}} \cdot \frac{\pi \cdot i \cdot b}{b a (b+a)}$$

$$= \frac{i\sqrt{2ab}}{b+a} \checkmark$$