Болотников Антон БФЗ-19-1

К. М. Задание Домашнее 1

№1) $\psi(x) = \frac{A}{x^2+a^2}$

$\langle\psi|\psi\rangle = 1 = A^2\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} = * = A^2\cdot\frac{\pi}{2a^3} \Leftrightarrow A = \sqrt{\frac{2a^3}{\pi}}$ ✓

$\left|\text{Ответ: } A = \sqrt{\frac{2a^3}{\pi}}\right|^{(1)}$

№2) $\psi(x) = \frac{B}{x+ib}$

$(x+ib)(x-ib) = x^2+b^2$

$\langle\psi|\psi\rangle = 1 = B^2\int_{-\infty}^{\infty}\frac{dx}{(x+ib)(x-ib)} = B^2\int_{-\infty}^{\infty}\frac{dx}{x^2+b^2} = \frac{B^2}{b}\operatorname{arctg}\frac{x}{b}\Big|_{-\infty}^{\infty} =$

$= 1 = \frac{B^2\pi}{b} \Leftrightarrow \sqrt{\frac{b}{\pi}} = B$ ✓

$\left|\text{Ответ: } B = \sqrt{\frac{b}{\pi}}\right|^{(2)}$

$* = A^2\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^2} = \left|\begin{array}{ll} U = \frac{1}{x} & dU = -\frac{1}{x^2} \\ dV = \frac{x}{(x^2+a^2)^2} & V = -\frac{1}{2(x^2+a^2)}\end{array}\right| = \underbrace{-\frac{A^2}{2x(x^2+a^2)^2}\Big|_{-\infty}^{\infty}}_{=0} -$

$A^2\int_{-\infty}^{\infty}\frac{dx}{2x^2(x^2+a^2)} = 0{,}5\int_{-\infty}^{\infty}\frac{-dx\cdot A^2}{x^2(x^2+a^2)} = 0{,}5\,\frac{A^2}{a^2}\cdot\underbrace{\frac{1}{x}\Big|_{-\infty}^{\infty}}_{=0} + \frac{A^2\cdot 0{,}5}{a^3}\cdot\operatorname{arctg}\left(\frac{x}{a}\right)\Big|_{-\infty}^{\infty} = *$

где, $\frac{1}{x^2(x^2+a^2)} = \frac{A'}{x^2} + \frac{B'x+D'}{(x^2+a^2)} = \frac{A'(x^2+a^2)+B'x^3+D'x^2}{x^2(x^2+a^2)} = \frac{1}{a^2x^2} + \left(-\frac{1}{a^2(x^2+a^2)}\right),$

$\Rightarrow \begin{array}{ll} x^3: & B' = 0 \\ x^2: & A' + D' = 0 \\ x: & \\ 1: & A' = \frac{1}{a^2}\end{array} \Rightarrow \begin{array}{l} A' = \frac{1}{a^2} \\ D' = -\frac{1}{a^2} \\ B' = 0\end{array}$

№4

$\delta(ax) = \frac{1}{|a|}\delta(x)$

$\delta(f(x)) = \delta\Big(\overbrace{\sum f'(x_i)}^{=C}\cdot(x-x_i)\Big) = |\text{по свойству } \delta\text{-ф-ии}| = \sum\frac{\delta(x-x_i)}{|f'(x_i)|}$ ✓

$f(x) \approx \underbrace{f(x_i)}_{=0} + \sum\underbrace{f'(x_i)}_{=C}\cdot(x-x_i)$

доказано

№5 1) $\langle f_1|f_2\rangle = \int_0^a \alpha_1\alpha_2 e^{-\frac{2i\pi x}{a}}dx = \alpha_1\alpha_2\cdot\frac{a}{(-2i\pi)}\exp\left(-2i\pi\frac{x}{a}\right)\Big|_0^a =$

$\alpha_1\alpha_2\cdot\frac{a}{(-2i\pi)} - \alpha_1\alpha_2\cdot\frac{a}{(-2i\pi)} = 0$ ✓

доказано

2) $\langle f_1 | f_1 \rangle = 1 = \int_0^a \alpha_1^2 \exp(-i\pi x/a + i\pi x/a) = \alpha_1^2 x \Big|_0^a \iff \alpha_1 = \sqrt{\frac{1}{a}}$

$\langle f_2 | f_2 \rangle =$ по сути абсолютно аналогично $\iff \alpha_2 = \sqrt{\frac{1}{a}}$ Ответ

(На самом деле $\alpha_1 = \alpha_2$ логичный результат при заданных представлениях f_1 и f_2).

3) $\langle f_1 | \psi \rangle = \alpha_1 \sqrt{\frac{2}{a}} \int_0^a \exp(-i\pi x/a) \sin\frac{\pi x}{a} dx =$

$= \alpha_1 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \int_0^a e^{\wedge}\left(-\frac{i\pi x}{a}\right) \cdot \left(e^{\wedge}\left(\frac{i\pi x}{2}\right) - e^{\wedge}\left(-\frac{i\pi x}{2}\right)\right) dx$

$= \alpha_1 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \int_0^a \left(1 - e^{\wedge}\left(\frac{-2i\pi x}{a}\right)\right) dx = \alpha_1 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \cdot \left(x + \frac{a}{2i\pi} \cdot e^{-\frac{2i\pi x}{a}}\right)\Big|_0^a$

$= \alpha_1 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \left(\left(a + \frac{a}{2i\pi}\right) - \frac{a}{2i\pi}\right) = \sqrt{2} \cdot \sqrt{a} \cdot \frac{\alpha_1}{2i} = \frac{\alpha_1 \sqrt{a}}{\sqrt{2}\, i} = \frac{-i}{\sqrt{2}}$

$\langle f_2 | \psi \rangle = \alpha_2 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \int_0^a \left(e^{\wedge}\left(\frac{2i\pi x}{a}\right) - 1\right) dx = \alpha_2 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \left(\frac{a}{2i\pi} e^{\frac{2i\pi x}{a}} - x\right)\Big|_0^a$

$= \alpha_2 \sqrt{\frac{2}{a}} \cdot \frac{1}{2i} \left(\left(\frac{a}{2i\pi} - a\right) - \frac{a}{2i\pi}\right) = \frac{i}{\sqrt{2}}$

$\Rightarrow |\psi\rangle = -\frac{i}{\sqrt{2}} |f_1\rangle + \frac{i}{\sqrt{2}} |f_2\rangle$ ← Ответ

интегралы нечётной ф-ии

1/3 $\langle \varphi | \psi \rangle = AB \int_{-\infty}^{\infty} \frac{dx}{(x - ib)(x^2 + a^2)} = AB \left(\frac{1}{a^2 - b^2} \int_{-\infty}^{\infty} \frac{dx}{(x - ib)} - \frac{1}{a^2 - b^2} \int_{-\infty}^{\infty} \underbrace{\frac{x\,dx}{(x^2 + a^2)}}_{=0} + \ldots \right.$

$\frac{1}{(x - ib)(x^2 + a^2)} = \frac{C}{(x - ib)} + \frac{Dx + E}{(x^2 + a^2)} = \frac{C(x^2 + a^2) + Dx(x - ib) + E(x - ib)}{(x - ib)(x^2 + a^2)}$

$x^2: C + D = 0$
$x^1: -ibD + E = 0$
$x^0: a^2 C - ibE = 1$
$\Rightarrow \begin{matrix} Cib + E = 0 \\ a^2 C - ibE = 1 \end{matrix} \Rightarrow \begin{matrix} C = \frac{1}{a^2 - b^2} \\ D = -\frac{1}{a^2 - b^2} \\ E = \frac{ib}{b^2 - a^2} \end{matrix}$

0 (нечётн. ф-ия)

$\ldots + \frac{ib}{b^2 - a^2} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)} \Bigg) = \frac{AB}{b^2 - a^2} \left(\frac{ib}{a} \cdot \operatorname{arctg}\left(\frac{x}{a}\right)\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{\infty} \left(\frac{x\,dx}{x^2 + b^2} + ib \frac{dx}{x^2 + b^2} \right) \right)$

$= AB/(b^2 - a^2) \cdot \left(\frac{ib}{a} \pi - i\pi \right) = \frac{AB i\pi}{b^2 - a^2} \left(\frac{b}{a} - \frac{a}{a} \right) = \frac{i\sqrt{2ab}}{b + a}$

| Ответ: $\frac{i\sqrt{2ab}}{b + a}$ | (3)