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# Internal Notes and Outtakes

## 1 Neutrino Mass and Mixing

### 1.1 Neutrino Mass Terms

With the SM neutrinos $\nu_\mathrm{L} = (\nu_e, \nu_\mu, \nu_\tau)^T$ and an arbitrary number of right-handed neutrino fields $\nu_{\mathrm{R}i}$ that are singlets under the SM gauge group, there are two possibilities for constructing mass terms in the Lagrangian.

1. The usual Dirac mass term reads
$$\mathcal{L}_\mathrm{Dirac} = -\overline{\nu_\mathrm{L}} m_\mathrm{D} \nu_\mathrm{R} + \text{h.c.} \tag{1}$$
in the "LR convention". In the literature one also encounters the "RL convention" where the Dirac mass term is $\overline{\nu_\mathrm{R}} m_\mathrm{D} \nu_\mathrm{L}$.

2. In addition, there can be the Majorana mass terms
$$\mathcal{L}_\mathrm{Majorana} = -\frac{1}{2}\overline{\nu_\mathrm{L}} m_\mathrm{L} \nu_\mathrm{L}^c - \frac{1}{2}\overline{\nu_\mathrm{R}^c} m_\mathrm{R} \nu_\mathrm{R} + \text{h.c.} \tag{2}$$

The conclusion that Eqs. (1) and (2) indeed contain mass terms follows from the general form of a fermion mass term, $\overline{L} m R + \text{h.c.}$ (i.e., a term containing one left-handed field $L$, one right-handed field $R$, and a constant $m$), and Eq. (5) below.

In general, $\nu_\mathrm{R}$ is a vector in flavor space, and $m_\mathrm{D}$, $m_\mathrm{L}$ as well as $m_\mathrm{R}$ are complex matrices. The Majorana mass matrices $m_\mathrm{L}$ and $m_\mathrm{R}$ are symmetric. All Lagrangians are understood to describe the low-energy effective theory after spontaneous breaking of $SU(2)_\mathrm{L} \times U(1)_Y$ and $U(1)_X$. Depending on the symmetry breaking pattern, some entries of $m_\mathrm{L}$, $m_\mathrm{D}$ or $m_\mathrm{R}$ may be zero. The superscript $c$ denotes charge conjugation,
$$\psi^c = \mathcal{C}\,\overline{\psi}^T \quad , \quad \overline{\psi^c} = -\psi^T \mathcal{C}^{-1} \tag{3}$$
for any fermion (4-component spinor) $\psi$, where $\mathcal{C}$ is the charge conjugation matrix, which satisfies
$$\mathcal{C}^\dagger = \mathcal{C}^{-1}\,, \tag{4a}$$
$$\mathcal{C}^T = -\mathcal{C}\,, \tag{4b}$$
$$\mathcal{C}\gamma_\mu^T \mathcal{C}^{-1} = -\gamma_\mu\,. \tag{4c}$$

In the standard representation, $\mathcal{C} = i\gamma_2\gamma_0$. From Eq. (4c), it follows that

$$\mathcal{C}\gamma_5\mathcal{C}^{-1} = \gamma_5 \quad \Leftrightarrow \quad \mathcal{C}\gamma_5 = \gamma_5\mathcal{C}\,. \tag{4d}$$

Note that

$$(\psi_\mathrm{L})^c = \mathcal{C}\overline{\psi_\mathrm{L}}^T = \mathcal{C}(\overline{\psi_\mathrm{L}}P_\mathrm{R})^T = \mathcal{C}P_\mathrm{R}\overline{\psi_\mathrm{L}}^T = P_\mathrm{R}\mathcal{C}\overline{\psi_\mathrm{L}}^T = P_\mathrm{R}(\psi_\mathrm{L})^c\,,$$

i.e., the charge conjugate of a left-handed spinor is right-handed and vice versa,

$$(\psi_\mathrm{L})^c = (\psi^c)_\mathrm{R} \quad \text{and} \quad (\psi_\mathrm{R})^c = (\psi^c)_\mathrm{L}\,. \tag{5}$$

In order to streamline notation, we use the abbreviations $\psi_\mathrm{L}^c \equiv (\psi_\mathrm{L})^c$ and $\psi_\mathrm{R}^c \equiv (\psi_\mathrm{R})^c$. A field that satisfies

$$\psi^c = \psi \tag{6}$$

(neglecting a possible phase factor), is called a Majorana field.

Combining Eqs. (1) and (2), we arrive at the most general neutrino mass term

$$\mathcal{L}^\nu_\mathrm{Mass} = -\frac{1}{2}\overline{\nu_\mathrm{L}}m_\mathrm{L}\nu_\mathrm{L}^c - \overline{\nu_\mathrm{L}}m_\mathrm{D}\nu_\mathrm{R} - \frac{1}{2}\overline{\nu_\mathrm{R}^c}m_\mathrm{R}\nu_\mathrm{R} + \text{h.c.} \tag{7}$$

It can be rewritten in the compact form

$$\mathcal{L}^\nu_\mathrm{Mass} = -\frac{1}{2}\overline{\Psi}M\Psi^c + \text{h.c.}\,, \tag{8}$$

where

$$\Psi = \begin{pmatrix} \nu_\mathrm{L} \\ \nu_\mathrm{R}^c \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} m_\mathrm{L} & m_\mathrm{D} \\ m_\mathrm{D}^T & m_\mathrm{R} \end{pmatrix}.$$

As $M$ is symmetric, it can be block-diagonalized by substituting

$$\Psi = U^*\chi = U^*\begin{pmatrix} \chi_\mathrm{l} \\ \chi_\mathrm{h} \end{pmatrix} \quad , \quad M' = \begin{pmatrix} m_\mathrm{l} & 0 \\ 0 & m_\mathrm{h} \end{pmatrix} = U^T M\, U\,, \tag{9}$$

where $U$ is a unitary matrix. Then the Lagrangian becomes

$$\mathcal{L}_\mathrm{Mass} = -\frac{1}{2}\overline{\chi_\mathrm{l}}m_\mathrm{l}\chi_\mathrm{l}^c - \frac{1}{2}\overline{\chi_\mathrm{h}}m_\mathrm{h}\chi_\mathrm{h}^c + \text{h.c.}\,, \tag{10}$$

i.e., we have obtained two fields with pure Majorana mass terms. In the special case $m_\mathrm{L} = m_\mathrm{R} = 0$, $m_\mathrm{D} \neq 0$, these can be combined to form one field with a Dirac mass term.

## 1.2 Classical See-Saw Scenario

Assuming $m_\mathrm{L} \ll m_\mathrm{D} \ll m_\mathrm{R}$,[1] approximate block-diagonalization is achieved by

$$U = \begin{pmatrix} \mathbb{1} & m_\mathrm{D}^* m_\mathrm{R}^{*\,-1} \\ -m_\mathrm{R}^{-1}m_\mathrm{D}^T & \mathbb{1} \end{pmatrix}, \tag{11}$$

[1]By this we mean that all eigenvalues of $m_\mathrm{L}$ are much smaller than those of $m_\mathrm{D}$ etc.

yielding the fields

$$\chi_{\mathrm{l}} \simeq \nu_{\mathrm{L}} \quad \text{and} \quad \chi_{\mathrm{h}} \simeq \nu_{\mathrm{R}}^c \tag{12a}$$

with mass matrices

$$m_{\mathrm{l}} \simeq m_{\mathrm{L}} - m_{\mathrm{D}} m_{\mathrm{R}}^{-1} m_{\mathrm{D}}^T \quad \text{and} \quad m_{\mathrm{h}} \simeq m_{\mathrm{R}}\,. \tag{12b}$$

The corrections are suppressed by powers of $m_{\mathrm{D}} m_{\mathrm{R}}^{-1}$. Thus, for $m_{\mathrm{R}} \gg m_{\mathrm{D}}$ the light neutrinos are predominantly composed of the left-handed states that participate in the weak interactions, and the heavy neutrinos are singlets under the SM gauge group to a good approximation. According to Eqs. (9) and (11), the mixing between light and heavy neutrinos is governed by the matrix $m_{\mathrm{D}} m_{\mathrm{R}}^{-1}$,

$$\nu_{\mathrm{L}} \simeq \chi_{\mathrm{l}} + m_{\mathrm{D}} m_{\mathrm{R}}^{-1}\, \chi_{\mathrm{h}}\,, \tag{13a}$$

$$\nu_{\mathrm{R}} \simeq \chi_{\mathrm{h}}^c - m_{\mathrm{R}}^{-1} m_{\mathrm{D}}^T\, \chi_{\mathrm{l}}^c\,. \tag{13b}$$

Note that we have only block-diagonalized $M$. Hence, $m_{\mathrm{l}}$ and $m_{\mathrm{h}}$ are non-diagonal matrices in general, and $\chi$ does not contain the mass eigenstates.

### 1.3 Light and Heavy Sterile Neutrinos

In scenarios with one eV-scale singlet $\nu_{\mathrm{R}1}$[2] and a number of much heavier singlets, block-diagonalizing like in Eq. (11) is not very useful, since we would like to separate light and heavy degrees of freedom, where the light ones include $\nu_{\mathrm{R}1}$. Instead, we put the light singlet into

$$\chi_{\mathrm{l}} \simeq \begin{pmatrix} \nu_{\mathrm{L}} \\ \nu_{\mathrm{R}1}^c \end{pmatrix} \tag{14}$$

and block-diagonalize such that we obtain a matrix of the form

$$M' = \begin{pmatrix} m_\nu & m_{\mathrm{LR}} & 0 \\ m_{\mathrm{LR}}^T & M_1 & 0 \\ 0 & 0 & m_{\mathrm{h}} \end{pmatrix}, \tag{15}$$

where $m_\nu$ is a $3\times 3$ matrix, $m_{\mathrm{LR}}$ is a $3\times 1$ "matrix" and the zeros are matrices of suitable dimensions.[3]

### 1.4 Active-Sterile Neutrino Mixing

Let us now diagonalize the light neutrino mass matrix

$$m_{\mathrm{l}} = \begin{pmatrix} m_\nu & m_{\mathrm{LR}} \\ m_{\mathrm{LR}}^T & M_1 \end{pmatrix} \tag{16}$$

[2]By this we mean that the mass eigenstate consisting mainly of $\nu_{\mathrm{R}1}$ has a mass of order 1 eV, anticipating small mixing between $\nu_{\mathrm{R}1}$ and the other states.

[3]When comparing to our paper [1], note that here we have included the MeV-scale singlet in the heavy degrees of freedom and removed $M_{\mathrm{RR}}$ by the block-diagonalization. Thus, the eV-scale neutrino is not an exact $U(1)_X$ eigenstate, but we can safely ignore this complication due to the assumed smallness of $M_{\mathrm{RR}}$.


(a) MES (b) Mess (c) FES (d) Fez

Figure 1: Confusingly similar terms

by changing to the basis of mass eigenstates (MES, cf. Fig. 1(a)) $\nu$,

$$\chi_1 = V^* \nu = V^* \left(\nu_1, \nu_2, \nu_3, N_1\right)^T . \tag{17}$$

The unitary matrix $V$ is uniquely determined from

$$V^\dagger \, m_1^\dagger m_1 \, V = \mathrm{diag}(m_1^2, m_2^2, m_3^2, m_{N_1}^2) \tag{18}$$

and the requirement that the r.h.s. of

$$V^T m_1 \, V = \mathrm{diag}(m_1, m_2, m_3, m_{N_1}) \tag{19}$$

be real and positive. Note that the MES are Dirac spinors describing Majorana particles. The corresponding Majorana spinors are $\nu_{\mathrm{Majorana}} = \nu + \nu^c$, but we will not use them.

It is often convenient to consider just one SM neutrino. Then $m_\nu$ and $m_{\mathrm{LR}}$ are simply numbers, and $V$ can be parameterized by an angle $\theta$,[4]

$$\begin{pmatrix} \nu_{\mathrm{L}} \\ \nu_{\mathrm{R}1}^c \end{pmatrix} \simeq \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ N_1 \end{pmatrix} . \tag{20}$$

### 1.5 Interactions and Definition of Mixing Parameters

In order to define the observable lepton mixing parameters, we have to take into account the gauge interactions. In a basis where the mass matrix of the charged leptons $e_{\mathrm{L}}$ is diagonal, the Lagrangian of the charged current weak interactions

$$\mathcal{L}_{\mathrm{cc}} = -\frac{g_2}{\sqrt{2}} \overline{e_{\mathrm{L}}} \, \not{W}^- \nu_{\mathrm{L}} + \mathrm{h.c.} \tag{21}$$

defines the SM neutrino flavor eigenstates (FES, cf. Fig. 1(c)) $\nu_{\mathrm{L}}$.[5] Likewise, the Lagrangian of the $U(1)_X$ neutrino interactions

$$\mathcal{L}_{V\nu} = -g_X X_{\nu_{\mathrm{R}}} \overline{\nu_{\mathrm{R}1}} \, \not{V} \nu_{\mathrm{R}1} \tag{22}$$

defines the sterile neutrino FES $\nu_{\mathrm{R}1}$.

[4] Strictly speaking, $V$ could also contain one Majorana phase.

[5] If the charged lepton mass matrix is not diagonal, we can diagonalize it by the transformations $e_{\mathrm{R}} \to U_{\mathrm{R}}^e e_{\mathrm{R}}$, $e_{\mathrm{L}} \to U_{\mathrm{L}}^e e_{\mathrm{L}}$, $\nu_{\mathrm{L}} \to U_{\mathrm{L}}^e \nu_{\mathrm{L}}$. Note that left-handed charged leptons and neutrinos are transformed by the same matrix and that the neutrino mass matrices change accordingly.

Note also that many works in the literature, including [2], claim that the charged lepton mass matrix is non-diagonal in the FES basis in general. This is incorrect, because the charged lepton FES and MES are identical *by definition*.

| | Vector/Matrix | Elements |
|---|---|---|
| Flavor eigenstates | $\chi_l \simeq (\nu_\mathrm{L}, \nu_{\mathrm{R}1}^c)^T = (\nu_e, \nu_\mu, \nu_\tau, \nu_{\mathrm{R}1}^c)^T$ | $\chi_{l\alpha}$, $\alpha \in \{e, \mu, \tau, s\}$ |
| Mass eigenstates | $\nu = (\nu_1, \nu_2, \nu_3, N_1)^T$ | $\nu_i$, $i = 1, \ldots, 4$ |
| Mixing matrix | $V^*$ | $V_{\alpha i}^*$ |
| | $\chi_l = V^* \nu$ | |

Table 1: Overview of notation

When we change to the MES basis according to Eq. (17), the charged current Lagrangian becomes

$$\mathcal{L}_\mathrm{cc} \simeq -\frac{g_2}{\sqrt{2}} \sum_{\alpha \in \{e,\mu,\tau\}} \sum_{i=1}^{4} \overline{\alpha_\mathrm{L}}\, V_{\alpha i}^* \not{W}^- \nu_i + \mathrm{h.c.}\,, \tag{23}$$

where we do not have an exact equality due to the approximation in Eq. (14), i.e., due to neglecting the admixture of heavy sterile neutrinos in $\nu_\mathrm{L}$. The smallness of this admixture justifies referring to both $\nu_\mathrm{L}$ (and $\nu_{\mathrm{R}1}$) and $\chi_l$ as flavor eigenstates. The matrix $V^*$ is the analogue of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in the SM.[6] Its elements $V_{\alpha i}$ ($\alpha \in \{e, \mu, \tau, s\}$, $i = 1, \ldots, 4$) are the experimentally accessible lepton mixing parameters. In the $U(1)_X$ sector, we obtain

$$\mathcal{L}_{V\nu} \simeq -g_X X_{\nu_\mathrm{R}} \sum_{i,j=1}^{4} \overline{\nu_i^c}\, V_{si}^* V_{sj} \not{V} \nu_j^c\,, \tag{24}$$

noticing that we denote two different quantities by $V$ due to the shortage of letters available for unitary matrices. If we consider just one SM neutrino, Eq. (20) yields

$$\mathcal{L}_{V\nu} \simeq -g_X X_{\nu_\mathrm{R}} \left[\sin^2\theta\, \overline{\nu_1^c} \not{V} \nu_1^c + \cos^2\theta\, \overline{N_1^c} \not{V} N_1^c - \frac{1}{2}\sin 2\theta \left(\overline{N_1^c} \not{V} \nu_1^c + \mathrm{h.c.}\right)\right]. \tag{25}$$

The appearance of charge conjugate fields in Eqs. (24) and (25) is unfortunate. Most likely it is an unnecessary complication that can be removed by a little Dirac algebra exploiting $\mathcal{L}^T = \mathcal{L}^\dagger = \mathcal{L}$. For the moment, this is left as an exercise to the interested reader. Alternatively, we could write the Lagrangian in terms of Majorana fields and use the formalism of [3] to derive the Feynman rules.

### 1.6 Parameterization of Mixing Matrices

Being a unitary $3 \times 3$ matrix, $V_\mathrm{PMNS}$ of the SM can be parameterized by 3 angles $\theta_{12}, \theta_{13}, \theta_{23}$ and 6 phases $\delta, \varphi_1, \varphi_2, \delta_e, \delta_\mu, \delta_\tau$. We use the parameterization

$$V_\mathrm{PMNS} = \mathrm{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}) \cdot U_\mathrm{PMNS} \cdot \mathrm{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1) \tag{26a}$$

[6] The somewhat unusual complex conjugate appearing here could be avoided by changing $V^* \to V$ in Eq. (17) at the expense of changes in Eqs. (18) and (19). As we will not consider complex parameters, we can largely ignore this detail.

with

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{26b}$$

where $s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$ etc. The phases $\delta_e, \delta_\mu$ and $\delta_\tau$ can be removed by a global phase transformation of the charged lepton fields and are thus unphysical. Unlike in the quark sector, this is not possible for $\varphi_1$ and $\varphi_2$, since the Majorana mass term (10) does not allow to change the phases of the neutrino fields, as otherwise the mass eigenvalues would become complex. Hence, these phases are observable in principle and called Majorana phases. However, only the Dirac phase $\delta$ is observable in neutrino oscillations.

We use the convention where $0 \leq \theta_{12} \leq \frac{\pi}{4}$ and $0 \leq \theta_{13}, \theta_{23} \leq \frac{\pi}{2}$. This implies that the neutrino mass eigenvalues are not ordered in general, i.e., $m_1 < m_2 < m_3$ does *not* hold. All phases are chosen to lie between 0 and $2\pi$.

For the matrix $V^*$ describing the mixing in our scenario with a light sterile neutrino, no standard parameterization exists. The mixing of the SM neutrinos among themselves is determined by $V^*_{\alpha i}$ ($\alpha \in \{e, \mu, \tau\}$ and $i = 1, 2, 3$), which we can still approximately parameterize as in Eqs. (26). To describe the mixing with the sterile neutrino, we can simply use the elements $V^*_{si}$ and $V^*_{\alpha 4}$, or effective mixing angles like the $\vartheta_{\alpha\beta}$ of [4].

### 1.7 Open Questions

Some people (typically of the kind that prefer Weyl over Dirac spinors) like to criticize our scenario with the statement "Majorana particles cannot have vector couplings". See, for example, the email discussion with subject "U(1) for Majorana" in early April 2017. The confusion arises primarily from not distinguishing between the terms "Majorana particle" and "Majorana spinor". However, there are two questions arising from the discussion that are not entirely settled and may deserve a closer look.

1. "Da man einen Massenterm hat, der $\nu_\text{L}$ und $\nu_\text{L}^c$ koppelt, erfuellt $\nu_\text{L}$ nicht die Diracgleichung. D.h. man kann mit so einem Lagrangian eigentlich nichts ausrechnen, weil die Diracgleichung natuerlich andauernd implizit oder explizit verwendet wird." [Felix Kahlhöfer]

   It is true that we can write down a propagator only for mass eigenstates (eigenstates of the Hamiltonian), i.e., for the Majorana spinor $\nu = \nu_\text{L} + \nu_\text{L}^c$ in our case. The reason why $\nu_\text{L}$ is nevertheless useful is most likely that for highly relativistic particles $\nu_\text{L}$ *approximately* satisfies the Dirac equation (since the mass is negligible). To give some physical intuition: the propagator for $\nu$ describes the probability for a particle to travel from $x$ to $y$. If it is left-handed at $x$, the full propagator includes two possibilities: either the particle is still left-handed at $y$, or it has become right-handed. The probability for the latter is suppressed by some power of $m/E$ in the relativistic case. Hence, we can approximate the full propagator by the propagator for $\nu_\text{L}$.

Questions remaining: does all this mean that for consistency we should use the *massless* propagator for $\nu_{\mathrm{L}}$ in calculations? How does this affect the QFT treatment of neutrino oscillations?

2. "Meine Bücher sagen, die schwache Wechselwirkung habe eine $(V-A)$-Kopplung, woran man sieht, dass Parität maximal verletzt ist. Wenn das für die Neutrinos in Wirklichkeit aber eine reine $A$-Kopplung ist [weil der Vektorstrom für einen Majoranaspinor verschwindet], ist dann P nicht mehr verletzt?" [Jörn]

# 2 Sterile Neutrino Scenarios

This section is to contain a complete list of all possible neutrino sectors that can be used to realize our scenario with interaction between dark matter and neutrinos by exchange of a light vector. All scenarios contain 3 left-handed neutrino states $\nu_{\mathrm{L}}$, which belong to $SU(2)_{\mathrm{L}}$ doublets $\ell_{\mathrm{L}}$. The scalar breaking $U(1)_X$ and giving a mass to the vector is denoted by $\Theta$.

## 2.1 Preliminary: Charges of Sterile Neutrinos

Let $X_{\nu_{\mathrm{R}}}$ be the $U(1)_X$ charge of $\nu_{\mathrm{R}}$. By definition, this means that the effect of a $U(1)_X$ transformation on $\nu_{\mathrm{R}}$ is

$$\nu_{\mathrm{R}} \to \exp(-i\alpha X_{\nu_{\mathrm{R}}})\, \nu_{\mathrm{R}}\,, \tag{27}$$

where $\alpha \in \mathbb{R}$ is the transformation parameter and depends on $x$ for a gauge symmetry. Complex conjugation of (27) yields

$$\nu_{\mathrm{R}}^* \to \exp(i\alpha X_{\nu_{\mathrm{R}}})\, \nu_{\mathrm{R}}^*\,. \tag{28}$$

On the other hand, by definition

$$\nu_{\mathrm{R}}^* \to \exp(-i\alpha X_{\nu_{\mathrm{R}}^*})\, \nu_{\mathrm{R}}^*\,. \tag{29}$$

Comparing (28) and (29), we find that $\nu_{\mathrm{R}}$ and $\nu_{\mathrm{R}}^*$ have opposite charges, $X_{\nu_{\mathrm{R}}^*} = -X_{\nu_{\mathrm{R}}}$. By analogous reasoning we can easily convince ourselves that transposition does not affect the charge, i.e., $X_{\nu_{\mathrm{R}}^T} = X_{\nu_{\mathrm{R}}}$. Combining everything with $\overline{\nu_{\mathrm{R}}} = \nu_{\mathrm{R}}^\dagger \gamma^0$ and Eq. (3), we conclude

$$X_{\overline{\nu_{\mathrm{R}}}} = X_{\nu_{\mathrm{R}}^c} = -X_{\nu_{\mathrm{R}}}\,. \tag{30}$$

## 2.2 Majorana Neutrinos

### 2.2.1 Scenario M1: 3 Sterile Neutrinos with $U(1)_X$ Charge

The new states in the neutrino sector are 3 SM singlets $\nu_{\mathrm{R}}$, all with the same $U(1)_X$ charge. (*How can we avoid trouble with anomalies? Cf. [5] and refs. [8–11] therein, and even better 1310.6582*) The Lagrangian for the neutrino sector contains

$$\mathcal{L}_\nu \supset -\frac{1}{\Lambda}\overline{\ell_{\mathrm{L}}}\phi\Theta Y_\nu \nu_{\mathrm{R}} - \overline{\nu_{\mathrm{R}}^c}\Theta' \frac{Y_M}{2}\nu_{\mathrm{R}} + \text{h.c.}\,, \tag{31}$$

which contains another scalar $\Theta'$ with suitable $U(1)_X$ charge and $\langle\Theta'\rangle \sim \langle\Theta\rangle$. The gauge symmetries of our scenario forbid the renormalizable Yukawa terms $\overline{\ell_{\rm L}}\phi\nu_{\rm R}$ and $\overline{\ell_{\rm L}}\Theta\nu_{\rm R}$. Therefore, we can obtain neutrino Yukawa couplings only from a dimension-5 operator involving a high-energy cutoff scale $\Lambda$. At least this generically leads to small Dirac neutrino masses $m_D \sim Y_\nu v_\phi v_\Theta/\Lambda$. It may also be possible to generate some couplings via loops [6].

After $U(1)_X$ breaking, $\mathcal{L}_\nu$ leads to Dirac and Majorana masses. The mass eigenstates are

- 3 light active neutrinos $\nu_{1,2,3}$,
- 1 relatively light sterile neutrino $\nu_s$ with mass $\sim 1\,\text{eV}$ relevant for oscillations and neutrino-DM scattering,
- 2 heavy sterile neutrinos $\nu_{h_{1,2}}$ with masses $\sim 1\,\text{MeV}$ (around the scale of $U(1)_X$ breaking).

Of course, we could exchange $\Theta$ and $\Theta'$ in the Lagrangian if we also exchange their $U(1)_X$ charges.

#### 2.2.2 Scenario M2: 1 Sterile Neutrino with $U(1)_X$ Charge

The new states in the neutrino sector are 3 SM singlets $\nu_{\rm R}$. Only one of them, let's say $\nu_{\rm R}^1$ without loss of generality, has a non-zero $U(1)_X$ charge. This allows us to write down Yukawa couplings and a Majorana mass term for the uncharged singlets (denoted by $\nu_{\rm R}^{2,3}$) without involving $\Theta$ and $\Theta'$. Thus,

$$\mathcal{L}_\nu \supset -\frac{1}{\Lambda}\overline{\ell_{\rm L}}\phi\Theta Y_\nu \nu_{\rm R}^1 - \overline{\ell_{\rm L}}\phi Y'_\nu \nu_{\rm R}^{2,3} - \overline{\nu_{\rm R}^{1\,c}}\Theta'\frac{Y_M}{2}\nu_{\rm R}^1 - \overline{\nu_{\rm R}^{2,3c}}\frac{M_{\rm R}}{2}\nu_{\rm R}^{2,3} - \overline{\nu_{\rm R}^{2,3c}}\Theta\frac{Y'_M}{2}\nu_{\rm R}^1 + \text{h.c.}\,. \tag{32}$$

As there is no reason for $M_{\rm R}$ to be close to the electroweak scale, we expect it to be very large, like in the usual see-saw mechanism. Then the mass eigenstates are

- 3 light active neutrinos $\nu_{1,2,3}$,
- 1 relatively light sterile neutrino $\nu_s$ with mass $\sim 1\,\text{eV}$ relevant for oscillations and neutrino-DM scattering,
- 2 heavy sterile neutrinos $\nu_{h_{1,2}}$ with masses $\sim 10^{14}\,\text{GeV}$.

### 2.3 Dirac Neutrinos

#### 2.3.1 Scenario D1: 3 Right-Handed Neutrinos with $U(1)_X$ Charge

The new states in the neutrino sector are 3 SM singlets $\nu_{\rm R}$, all with the same $U(1)_X$ charge. We do not introduce a Majorana mass term, which can be motivated by lepton number conservation. The Lagrangian for the neutrino sector contains

$$\mathcal{L}_\nu \supset -\frac{1}{\Lambda}\overline{\ell_{\rm L}}\phi\Theta Y_\nu \nu_{\rm R} + \text{h.c.}\,, \tag{33}$$

which leads to 3 Dirac neutrino mass eigenstates $\nu_{1,2,3}$, all of which couple to the $U(1)_X$ vector via their right-handed components, and that's it. This scenario seems to have been discussed in [7], where also a UV completion with heavy messengers is specified that yields the non-renormalizable Yukawa coupling. (*Unfortunately, [7] does not contain the word "anomaly".*)

At first sight, this scenario has trouble with $K$ and $W$ decays. However, it may survive these constraints for the following reasons.

- Beacom et al. [8] assume $V - A$ couplings between neutrinos and the $U(1)_X$ gauge boson. However, in our case only $\nu_\mathrm{R}$ couple to this vector, which implies a $V + A$ coupling. The resulting branching ratios will be suppressed (at least) by the small neutrino mass.

- The coupling of the vector to $\nu_\mathrm{R}$ does not break SM gauge invariance. Consequently, the $E/m_\nu$ enhancement of the branching ratios should not be present. (Maybe this is equivalent to the first point: if we multiply $E/m_\nu$ by $m_\nu$, the enhancement disappears.)

#### 2.3.2 Scenario D2: 3 Uncharged Right-Handed Neutrinos and an Extra Generation with $U(1)_X$ Charge

The new states in the neutrino sector are 3 SM singlets $\nu_\mathrm{R}$ without $U(1)_X$ charge. In addition, we introduce a fourth neutrino generation with the SM singlet fields $N_\mathrm{L}$ and $N_\mathrm{R}$. They have identical $U(1)_X$ charges. (Thus, we could combine them to a vector-like fermion, which indicates that they do not contribute to anomalies. *I think.*) We do not introduce Majorana mass terms. The Lagrangian for the neutrino sector contains

$$\mathcal{L}_\nu \supset -\overline{\ell_\mathrm{L}}\phi Y_\nu \nu_\mathrm{R} - \overline{N_\mathrm{L}} M_N N_\mathrm{R} - \overline{N_\mathrm{L}} \Theta Y'_\nu \nu_\mathrm{R} + \text{h.c.}\,. \tag{34}$$

The mass eigenstates are

- 3 light active Dirac neutrinos $\nu_{1,2,3}$,

- 1 sterile Dirac neutrino $\nu_s$ with mass $\sim M_N$ relevant for oscillations and neutrino-DM scattering.

Advantages of this scenario are its renormalizability[7] and the presumed absence of trouble with $K$ and $W$ decays (all neutrinos couple *either* to the $W$ *or* to the light vector). An obvious disadvantage is the absence of a motivation for the mass scales. Most importantly, $M_N$ has no reason to be around an eV. But no reason not to be either.

One could think of extending the scenario to a Dirac see-saw [9], introducing several generations of $N_\mathrm{L,R}$ fields. Then the overall scale of $M_N$ could be very large, like in the normal see-saw, and we would obtain the same mass spectrum as in scenario M2. The most challenging part would be to justify the existence of an eV-scale sterile neutrino, but this problem does not seem worse than in the normal see-saw. [9] also discusses some approaches.

[7] A non-renormalizable term $\frac{1}{\Lambda}\overline{\ell_\mathrm{L}}\phi\Theta Y''_\nu N_\mathrm{R}$ could be introduced in analogy to the previous scenarios, but it is not necessary.

### 2.4 Scenario X: Decoupled Particles

Mainly for completeness: we could introduce a particle $N$ with $U(1)_X$ charge and a suitable (Dirac or Majorana) mass but without mixing with the active neutrinos (no Yukawa couplings $\overline{\ell_\mathrm{L}}\phi\Theta N$ etc.). This can be considered the $\Lambda \to \infty$ limit of scenarios M1 and M2. The cosmological aspects of the scenario should be ok, there is certainly no problem with $K$ and $W$ decays, and neutrino oscillation anomalies are simply not addressed. As $N$ does not couple to the SM neutrinos at all, it would be somewhat far-fetched to call it "sterile neutrino".

# 3 Sterile Neutrinos from Left-Right Symmetry

The standard left-right symmetric gauge group is $G_\mathrm{LR} = SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times \mathrm{U}(1)_{B-L}$. In this case the generator of $U(1)_Y$ arises after spontaneous symmetry breaking to the Standard Model as

$$\frac{1}{2}Y = T_3^\mathrm{R} + \frac{B-L}{2}\,, \tag{35}$$

where $T_a^\mathrm{R}$ are the generators of $SU(2)_\mathrm{R}$. The sterile neutrinos are members of the $SU(2)_\mathrm{R}$ doublet $\ell_\mathrm{R} = (N_\mathrm{R}, e_\mathrm{R})$. If we simply extend $G_\mathrm{LR}$ to $G_\mathrm{LR} \times U(1)_X$, this implies that sterile neutrinos and right-handed charged leptons have the same $U(1)_X$ charge, which would be a disaster because the charged leptons would couple to the light vector.

The hope is to break the LR symmetry in a more intelligent way that distinguishes between sterile neutrinos and charged leptons.[8] So we change the high-energy gauge group to $G'_\mathrm{LR} = SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{A} \times \mathrm{U}(1)_\mathrm{B}$. It is broken to $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_Y \times U(1)_X$ by the vev

$$\langle\chi\rangle = \begin{pmatrix} v_\mathrm{R} \\ 0 \end{pmatrix} \tag{36}$$

of a scalar $\chi$, which is a $SU(2)_\mathrm{R}$ doublet and has charges $a$ and $b$ under $\mathrm{U}(1)_\mathrm{A}$ and $\mathrm{U}(1)_\mathrm{B}$, respectively. (*Change to $a_\chi$, $b_\chi$?*)

In order to determine the charges of the leptons under $U(1)_Y$ and $U(1)_X$, we have to find the linear combinations of $G_\mathrm{LR}$ generators that are not broken by $\langle\chi\rangle$. Let us denote them by

$$G_n = \tau_3 + \rho_n \mathrm{A}\mathbb{1} + \sigma_n \mathrm{B}\mathbb{1} \quad (n = 1, 2)\,, \tag{37}$$

(*Change $n = 1, 2$ to $n = X, Y$?*) where $\tau_3$ and $\mathbb{1}$ are $2 \times 2$ matrices in $SU(2)_\mathrm{R}$ space and $\rho_n, \sigma_n$ are so far arbitrary numbers. By definition, group elements generated by $G_n$ leave the vev invariant,

$$e^{i\alpha_n G_n} \langle\chi\rangle = \langle\chi\rangle\ . \tag{38}$$

Consequently, $G_n$ annihilate the vev,

$$G_n \langle\chi\rangle = 0\,. \tag{39}$$

[8] This is not unheard of. For instance, it is possible to keep $N_\mathrm{R}$ uncharged under $U(1)_Y$ while $e_\mathrm{R}$ are charged.

Combining Eqs. (36,37,39) leads to

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \rho_n a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \sigma_n b \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & * \end{pmatrix} , \tag{40}$$

where $*$ is an arbitrary number.

Now it seems that a possible choice is $a = b = 1$, $\rho_1 = \sigma_2 = 0$, $\sigma_1 = \rho_2 = -1$. Then $U(1)_Y$ is generated by $\tau_3 - A$ and $U(1)_X$ by $\tau_3 - B$. Assigning charges 1 and $-1$ under $\mathrm{U(1)_A}$ and $\mathrm{U(1)_B}$ to $\ell_\mathrm{R}$, we obtain $e_\mathrm{R}$ with $Y = -2$ and $U(1)_X$ charge 0 as well as $N_\mathrm{R}$ with $Y = 0$ and $U(1)_X$ charge 2.

*So it seems that the earlier negative conclusion I explained to Jasper was wrong – I didn't take into account the possibility of linear combinations of the three original generators, i.e., of arbitrary $\rho_n, \sigma_n$. The LR model may work after all. A necessary condition is that we can assign charges such that all quarks and left-handed leptons remain uncharged under $U(1)_X$. Pre-weekend update: $\rho_n, \sigma_n$ might be restricted by some orthogonality conditions. Otherwise the various gauge bosons ($U(1)_Y$ boson, $Z'$, $U(1)_X$ vector) may mix. Work in progress...*

# 4 Classification of Scenarios

Rather than considering the frustrating details of model building, it may be more fun to classify the possible choices of particles that realize Torsten's proposal.

1205.5809 considered $t$-channel exchange of a vector between two fermions. We need an interaction that yields a matrix element $\propto E^2$ in the limit $t \to 0$. We also want Sommerfeld enhancement of DM self-interactions, in the original scenario due to many exchanges of the light vector.

$s$-channel exchange cannot give the necessary enhancement because the only way to enhance an $s$-channel interaction is a resonance. However, in the expanding universe the resonance condition can only be satisfied for too short a time.

$u$-channel exhange could work even in a minimal setup with only a DM fermion and a DR scalar (fermion in the $u$-channel). We have

$$s + t + u \simeq s + u \simeq 2m_\mathrm{DM}^2 \tag{41}$$

and $s \simeq m_\mathrm{DM}^2$, thus $u \simeq m_\mathrm{DM}^2$, so the intermediate propagator $(u - m_\mathrm{DM}^2)^{-1}$ could blow up as desired. Sommerfeld enhancement would now work by exchanging the scalar.

If we add a vector, we could have both $t$- and $u$-channel diagrams, but probably it is not possible for both of them to be enhanced simultaneously.

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