# MECH 309 Project Progress Report 1

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Handed in March 19, 2018

The work was split roughly equally among the team participants (33.3 % each). The scanned pages on the following pages are the accomplished work.

## 1 Exercise 1

Subquestions 1 and 2 were completed by Abtin and Vassili. The third one was started but not finished, as it is for now unclear how to treat the L-shape of the plate in the solution expression.

## 2 Exercise 2

Christopher did subsection 1,2,3,4 for this.

Q1

1. $-\nabla\cdot(K\nabla T) = -(K\nabla\cdot\nabla T + \nabla T\cdot\nabla K)$

$$(*) \quad = -\left[K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\partial T}{\partial x}\frac{\partial K}{\partial x}+\frac{\partial T}{\partial y}\frac{\partial K}{\partial y}\right]$$

2. Use midpoint 3 point difference for first partials.

e.g. $\frac{\partial T}{\partial x}(x_i,y_i) \approx \frac{1}{2h}\left[T(x_{i+1},y_j)-T(x_{i-1},y_j)\right]$; $h=\frac{2}{8}=\frac{1}{4}$

Three point midpoint diff. for second partials. With first partials - forward/backward 2-point differences

e.g. $\frac{\partial^2 T}{\partial x^2} \approx \frac{1}{2h}\left[\frac{\partial T}{\partial x}(x_{i+1},y_j)-\frac{\partial T}{\partial x}(x_{i-1},y_j)\right]$ | midpt diff.

$$\simeq \frac{1}{2h}\left[\frac{T_{i+1,j}-T_{i,j}}{h}-\frac{T_{ij}-T_{i-1,j}}{h}\right]$$

$$= \frac{1}{2h^2}\left[T_{i+1,j}-2T_{ij}+T_{i-1,j}\right] \quad \Big| \; h=1/4$$

$(*)$ yields: $\nabla\cdot(K\nabla T) = K_{ij}\left\{\frac{1}{2h^2}\left[T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}-4T_{ij}\right]\right\}+\frac{1}{4h^2}\left\{\left[(T_{i+1,j}-T_{i-1,j})\left(\partial K/\partial x\big|_{ij}\right)\right]+\right.$

$$\left.+\left[(T_{i,j+1}-T_{i,j-1})\left(\partial K/\partial y\big|_{ij}\right)\right]\right\}$$

(3)

$$\frac{\partial k}{\partial x} = -\sin(x+y) \qquad k = 2+\cos(x+y)$$
$$\qquad\qquad\qquad\qquad f = e^{-(x^2+y^2)}$$
$$\frac{\partial k}{\partial y} = -\sin(x+y)$$

$$\frac{k}{2h^2}\left[ T(x_{i+1}, y_j) + T(x_{i-1}, y_j) + T(x_i, y_{j+1}) + T(x_i, y_{j-1}) - 4T(x_i, y_j) \right]$$
$$+ \frac{1}{2h}\left(-\sin(x_i+y_j)\right)\left[ T(x_{i+1}, y_j) - T(x_{i-1}, y_j) + T(x_i, y_{j+1}) - T(x_i, y_{j-1}) \right] = -e^{-(x^2+y^2)}$$

$$\left[\frac{k}{2h^2} - \frac{\sin(x_i+y_j)}{2h}\right] T(x_{i+1}, y_j)$$
$$+ \left[\frac{k}{2h^2} + \frac{\sin(x_i+y_j)}{2h}\right] T(x_{i-1}, y_j)$$
$$+ \left[\frac{k}{2h^2} - \frac{\sin(x_i+y_j)}{2h}\right] T(x_i, y_{j+1})$$
$$+ \left[\frac{k}{2h^2} + \frac{\sin(x_i+y_j)}{2h}\right] T(x_i, y_{j-1})$$
$$+ \left[-\frac{4k}{2h^2}\right] T(x_i, y_j) = -e^{-(x_i^2+y_j^2)}$$

There are 6 boundaries and the temp on them is 0.

Left: $T(x_0, y_j) = 0 \quad j = [-\frac{n}{2}, \frac{n}{2}]$

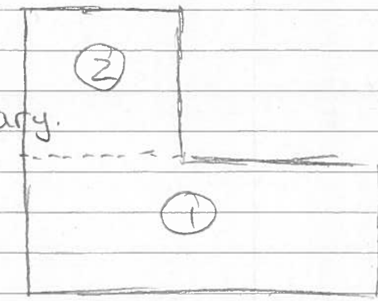
eqn becomes:

$$\left[\frac{k}{2h^2} - \frac{\sin(x_1+y_j)}{2h}\right] T(x_2, y_j)$$
$$+ \left[\frac{k}{2h^2} + \frac{\sin(x_1+y_j)}{2h}\right] 0$$
$$\left[\frac{k}{2h^2} - \frac{\sin(x_1+y_j)}{2h}\right] T(x_1, y_{j+1})$$

$T_{x_0, y_j} = 0 \quad j$ from $0$ to $\frac{n}{2}$.

$T_{x_i, 0} = 0$

③ cont'd




Split problem into parts:
Solve separately until boundary.
Impose continuity as BC for both problems ?

Unclear for now.

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2. Buckling of a tree
→ max possible height of tree before collapse under own weight
→ assume height bounded from above b/c of a buckling instability

2.1: Tree modelled as ~~massless~~ trunk
→ supports weight of foliage, mg, on top

→ Simple beam → Circular cross-sectional area $S$ and radius $R$

→ Bending Rigidity $EI$   $E$: Young's modulus   $I$: Second moment of Area

→ Find critical length $L_c$

Critical Load: $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$ OR $K^2L^2 = \frac{\pi^2 EI}{P_{cr}}$

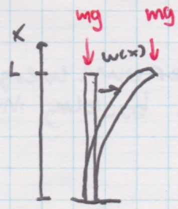
$\therefore L_c = \sqrt{\frac{\pi^2 EI}{P_{cr} K^2}}$   K value = 2.0 here

1. We would expect that $L_c$ would ~~have~~ be an increasing ~~relationship~~ function ~~with~~ of $EI$ as the $L_c = \sqrt{\frac{\pi^2 EI}{P_{cr} K^2}}$ Note that in our case, $K = 2.0$ and $P_{cr} = mg$

and therefore $L_c = \sqrt{\frac{\pi^2 EI}{4mg}}$. It would increase with increasing $EI$ and decrease with increasing $mg$.

- This makes sense as increasing the bending rigidity would make the tree stronger, and increasing the Load (weight) on the tree would make it be able to support less with an equal length.

2.




$$\forall x \in [0, L], \quad w^{(4)}(x) + k^2 w''(x) = 0$$

and $k^2 = \frac{mg}{EI}$

Vanishing shear force at $x = L$ is expressed by:

$$w^{(3)}(L) + k^2 w'(L) = 0$$

- 4 boundary conditions will be needed in order to solve this ODE. We would recommend to use:

At the fixed end: $w = 0$ and $\frac{dw}{dx} = 0$

Shear force

At the free end: $M = 0$ so $\frac{d^2w}{dx^2} = 0$ and $V = 0$, so $\frac{d^3w}{dx^3} + \lambda^2 \frac{dw}{dx} = 0$

moment

$\boxed{3,4}$ ~~treat~~ let $k^2 = \frac{mg}{EI}$ $\rightarrow$ $w^{(4)}(x) + k^2 w^{(II)}(x) = 0$

$\rightarrow \frac{d^4w}{dx^4} + k^2 \frac{d^2w}{dx^2} = 0$ let $w = e^{rx}$

$\therefore r^4(e^{rx}) + r^2k^2(e^{rx}) = 0$

$= e^{rx}\left[r^4 - r^2k^2\right] = 0$ Note $e^{rx}$ cannot be zero

$\therefore$ ~~$r^4 - r^2k^2$~~ $r^2\left[r^2 - k^2\right] = 0$ $\quad r^2 = k^2$ $\quad \therefore r =$ ~~$\pm\sqrt{k}$~~ $\pm K$

$\therefore r_1 = 0, \; r_2 = 0, \; r_3 = K, \; r_4 = -k$ $\quad \rightarrow r_3 = \sqrt{\frac{mg}{EI}} \quad r_4 = -\sqrt{\frac{mg}{EI}}$

Imaginary, as $\frac{mg}{EI}$ is negative

and we obtain the following solutions:

$e^{0x}, \; xe^{0x}, \; \cancel{e^{kx}}, \; \cancel{e^{-kx}}$

$\hookrightarrow e^{kx}\cos(kx), \; e^{kx}\sin(kx)$

by the principle of superposition, the general solution is:

$$A\sin(kx) + B\cos(kx) + Cx + D$$

For 3, a trivial solution to the ODE would be $w(x) = 0$. This would mean that the tree is not deflecting under the load, which would happen if there were no other factors such as wind to consider, but it would be very unstable. We still need to solve the ODE because in reality, this will not happen, and we will need to investigate the deflection of the tree.