Martin Luther University Halle-Wittenberg
Chair of Economics, especially Macroeconomics
Professor Dr Oliver Holtemöller

Advanced Macroeconomics

Exam Winter 2024/25

Problem Set No. 1 n:10

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************ Student Data ************
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Contents

Overview of Code Files

File Name	File Description	Reference Files
`equilibrium.m` `twoperiodequilibrium_tax.m`	Solves Task 2: New equilibrium with τ	`Example_5_1.m` `twoperiodequilibrium.m`
`sensitivity.m`	Solves Task 3: Sensitivity analysis of τ on g_1	`Example_5_2.m`

Table 1: Table of code files.

Octave version 9.2.0 was used for this assignment

1 Task 1

Budget Constraints for Period 1 and 2:
The budget constraint for period 1 can be described as:

$$c_1 + k_2 = k_1 + (1-\tau)r_1k_1 + w_1n_1$$

Where consumption in period $1(c_1)$ and the capital that will be saved for period $2(k_2)$ is limited by the capital in period $1(k_1)$, taxed revenue on capital gains($(1-\tau)r_1k_1$), as well as wage(w_1) multiplied by hours worked(n_1).
The period 2 constraint is similar, with the only difference there not being any k_3 needing to be saved for any future period.

$$c_2 = k_2 + (1-\tau)r_2k_2 + w_2n_2$$

Intertemporal Budget Constraint:
It is achieved by combining the budget constraint of the first two periods. While keeping in mind to discount the value of the second period's budget constraint by the potential return for capital saved for this second period, with taxes accounted for.

$$c_1 + \frac{c_2}{1+(1-\tau)r_2} = k_1 + (1-\tau)r_1k_1 + w_1n_1 + \frac{k_2 + (1-\tau)r_2k_2 + w_2n_2}{1+(1-\tau)r_2}$$

Lagrangian:
The Lagrangian is constructed by combining the utility function with the intertemporal budget constraint using a Lagrange multiplier λ

$$\mathcal{L} = \ln c_1 + \frac{1}{1+\rho}\ln c_2 +$$
$$\lambda\left[k_1 + (1-\tau)r_1k_1 + w_1n_1 + \frac{k_2 + (1-\tau)r_2k_2 + w_2n_2}{1+(1-\tau)r_2} - \left(c_1 + \frac{c_2}{1+(1-\tau)r_2}\right)\right]$$

FOC 1:

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} - \lambda = 0$$

$$\implies \lambda = \frac{1}{c_1}.$$

FOC 2:

$$\frac{\partial \mathcal{L}}{\partial c_2} = \frac{1}{(1+\rho)c_2} - \lambda \cdot \frac{1}{1+(1-\tau)r_2} = 0$$

$$\implies \lambda = \frac{1+(1-\tau)r_2}{(1+\rho)c_2}.$$

Euler Equation:
To get the Euler equation, we set the 2 FOCs equal to each other:

$$\frac{1}{c_1} = \frac{1+(1-\tau)r_2}{(1+\rho)c_2}$$

$$\implies \frac{c_2}{c_1} = \frac{1+(1-\tau)r_2}{1+\rho}$$

This equation describes the relationship between c_1 and c_2 in optimum. We see that households having to pay taxes has an effect on this relationship, seeing as it is a variable within the equation.

2 Task 2

After the addition of the tax from Task 1, the consumption budget constraints need to be updated. These are changed to the new equations gotten in Task 1. These newly updated equations are crucial for the equilibrium, since they define the amount taxed, which then is relevant for the government spending, which appears in the other newly added equations.

```
c1 + k2 - ( 1 - tax ) * r1 * k1 - w1 * n1 - k1
c2 - k2 - ( 1 - tax ) * r2 * k2 - w2 * n2;
```

Equations for g1 and g2 are added to the model so that these two variables have an output one the equilibrium is processed. These are given in the Task 2 instructions as:

$$g_1 = \tau r_1 k_1, \quad g_2 = \tau r_2 k_2$$

Lastly the Euler equation derived in Task 1 is added.

```
c2 - (( 1 + ( 1 - tax ) * r2 ) / ( 1 + rho )) * c1
```

Importantly, the following equations remain unchanged in the model, as they continue to describe the relationships between variables without being affected by addition of tax. These are the production functions, wage equations, and rental rate equations:

```
y1 - A1 * k1^alpha * n1^(1 - alpha)
y2 - A2 * k2^alpha * n2^(1 - alpha)

w1 - (1 - alpha) * A1 * k1^alpha * n1^(-alpha)
w2 - (1 - alpha) * A2 * k2^alpha * n2^(-alpha)

r1 - alpha * A1 * k1^(alpha - 1) * n1^(1 - alpha)
r2 - alpha * A2 * k2^(alpha - 1) * n2^(1 - alpha)
```

When computing the equilibrium of the two period model, the tax variable is added to the code and given the assigned value:

$$\tau = 0.38$$

Running the code, the equilibrium is computed as:

Variable	**Value**
y_1	1
y_2	0.88504
c_1	1.2204
c_2	1.4497
w_1	0.7
w_2	0.61953
r_1	0.3
r_2	0.39891
k_2	0.6656
g_1	0.114
g_2	0.10089

3 Task 3

Below is a figure of the sensitivity of the effect of tax (τ) on government spending in period 1 (g_1):

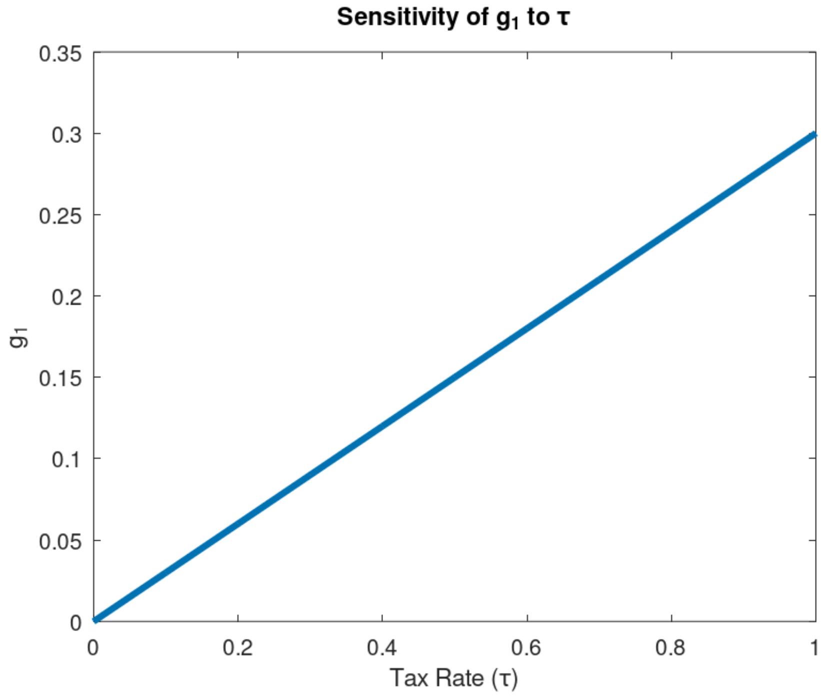

Figure 1: Sensitivity analysis results for the model.

It is observed that the plot is linear with a positive upwards trend. This is in accordance with the fact that the higher the tax, the more the government is able to spend of public consumption.

More specifically, government spending in period 1 is defined in question 2 as:

$$g_1 = \tau r_1 k_1$$

This equation indicated a positive linear relationship between τ and g_1, as observed in figure 1 above. Additionally we know that k_1 is fixed as an exogenous variable, and the equation defining r_1 is not affected by τ.

4 Sources

Software

`https://octave.org/index`

Textbook

Alogoskoufis, G. (2003). Dynamic macroeconomics. The MIT Press.

Online Sources

`https://www.rug.nl/ggdc/productivity/pwt/?lang=en`

Lecture Material

Holtemöller, O., 2024. Advanced Macroeconomics. Chapter 5. Lecture Slides. p. 4 - 9

```
twoperiodequilibrium.m
Example_5_1.m
Example_5_2.m
```

AI Tools

ChatGPT was used to clarify concepts, as well as explain mathematic reasoning, in order to guide and understand steps taken when deriving mathematical equations. Such math derivations were subsequently produced by hand, and any information was double checked with lecture slides and the textbook.

Declaration of Independence

I hereby declare that I have worked independently on this problem set.
If I have received help, I have explicitly referred to it.

28/11/24, Anton Cronet