

1. (a) For $i = 1, 2, 3$ represent the process $f_i(X(t))$ as an Ito process, i.e. in the form

Use (3.3): $df(X(t)) = f'(X(t))dX(t) + \frac{1}{2}f''(X(t))d[X, X](t)$

$$df(X(t)) = \sum_{i=1}^d \partial_i f(X(t))dX(t) + \frac{1}{2} \sum_{i,j=1}^d \partial_{ij} f(X(t))d[X_i, X_j](t)$$

Or (3.8): $df(X(t)) = [f'(X(t))\mu + \frac{1}{2}f''(X(t))\sigma^2]dt + f'(X(t))\sigma dW(t)$

Function depends on both t and X

i. $f_1(x) = x^3$

$$d(f_1(x)) = (3x_t^2\mu(t) + \frac{3 \cdot 2}{2}\sigma^2(t))dt + 3x_t^2\sigma(t)dW(t) = (3x_t^2\mu(t) + 3\sigma^2(t))dt + 3x_t^2\sigma(t)dW(t)$$

ii. $f_2(x) = \exp(x)$

$$d(f_2(x)) = (\exp(x_t)\mu(t) + \frac{1}{2}\exp(x_t)\sigma^2(t))dt + \exp(x_t)\sigma(t)dW(t)$$

iii. $f_3(x) = 6x + 2$

$$d(f_3(x)) = (6\mu(t) + 0)dt + 6\sigma(t)dW(t) = 6\mu(t)dt + 6\sigma(t)dW(t)$$

- (b) By $d[X, X](t) = \sigma^2(t)dt$ we just consider diffusion coefficient of previously computed processes

i. $d[f_1(x), f_1(x)](t) = 9x_t^4\sigma^4(t)dt$

ii. $d[f_2(x), f_2(x)](t) = e^{2x}\sigma^2(t)dt$

iii. $d[f_3(x), f_3(x)](t) = 36\sigma^2(t)dt$

- (c) By (3.2) $dX(t)dY(t) = d[X, Y](t)$, and since the quadratic variation of standard Brownian motion W is $[W, W](t) = t$ it yields to $d[X, Y](t) = \sigma_x(t)\sigma_y(t)dt$

$$d[f_1(x), f_3(x)](t) = 3x_t^2\sigma(t)6\sigma(t)dt = 18x_t^2\sigma^2(t)dt$$