Christian-Albrechts-Universität zu Kiel **WS 2017/18**
Mathematisches Seminar
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Sheet 02

Risk Management

Exercises for participants of the programme **Quantitative Finance**

C-Exercise 3

Denote by S_n the price of a stock at day $t_n, n \in \mathbb{N}$ and by $X_n := \log\left(\frac{S_n}{S_{n-1}}\right), n \geq 2$ the log return of the stock. Assume that the conditional distribution of X_{n+1}, given the stock prices up to time t_n, is a $N\left(\mu_{n+1}, \sigma_{n+1}^2\right)$-distribution with

$$\mu_{n+1} := \frac{1}{251}\sum_{k=n-250}^{n} X_k \quad \text{and} \quad \sigma_{n+1}^2 := \frac{1}{250}\sum_{k=n-250}^{n}(X_k - \mu_{n+1})^2 \quad \text{for } n \geq 252,$$

i.e. the conditional distribution of the log return at time t_{n+1} is normally distributed with empirical mean and empirical variance of the log returns from the past trading year. (We ignore the days of the first trading year.)

(a) Write a scilab function

```
VaR_log_normal (s, alpha),
```

that computes for given stock prices $s = (s_{n-251}, s_{n-250}, \ldots, s_n)$ of the past trading year the Value at Risk at level α of the next trading day's loss L_{n+1} for this stock.

(b) Assume that the DAX time series data follow this model. Compute for each day after the first 252 days the $\text{VaR}_{98\%}$ of the DAX time series and visualize the violations, i.e. the days when the actual loss lies above the computed VaR. How much violations do you expect theoretically, how much do you observe?

Hint: The VaR of the loss L_{n+1} at time t_{n+1} is given by

$$\text{VaR}_\alpha(L_{n+1}) = S_n\left(1 - \exp(\mu_{n+1} + \sigma_{n+1} q_{1-\alpha})\right) ,$$

where $q_{1-\alpha}$ denotes the $(1-\alpha)$-quantile of the standard normal distribution.

Please give a description of your *scilab* operations in the `sce`-file.

Useful *scilab* commands: `read_csv`, `evstr`, `mean`, `variance`, `cdfnor`

Please turn over.

T-Exercise 4

For time points $t_n = n\Delta t, n \in \{0, \ldots, N\}$, let P_n denote the price of a zero coupon bond with maturity $T > t_N$ at time t_n. A Bank buys a portfolio consisting of one share of the zero coupon bond. For the purpose of risk management the bank chooses the so called *yield to maturity* $Z_n := -\frac{1}{T-t_n}\log(P_n)$ as a risk factor.

(a) Derive the function that computes the portfolio value from the risk factor.

(b) Derive the risk factor change X_{n+1} at time t_{n+1}.

(c) Derive the loss operator $l_{[n]}$, i.e. the function that computes the loss at time t_{n+1} from the risk factor change.

(d) Derive the linearized loss operator $l_{[n]}^{\Delta}$, i.e. the function that computes the linearized loss at time t_{n+1} from the risk factor change.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.
Submit until: Thursday, 09.11.2017, 08:30 (before the lecture)