Christian-Albrechts-Universität zu Kiel **SS 2017**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Ole Martin, Adrian Theopold Sheet 07

Computational Finance

Exercises for participants of **mathematical programmes**

Remark: As you have 2 weeks time to work on this exercise sheet, there are 4 exercises on this exercise sheet which yield a total of **16 points**!

C-Exercise 20 (Model calibration to market prices)
Write a scilab function

```
sigma = BS_EuCall_Calibrate (S0, r, T, K, V, sigma0)
```

that calibrates the Black-Scholes model to given prices of European call options. I.e., for the initial stock price $S(0)$, interest rate r, a vector of maturities T, a vector of strikes K and a vector of corresponding option prices V, the routine shall determine the volatility parameter σ that "fits as well as possible", where "fitting well" is to be understood in the sense of Formula (4.17). The parameter σ_0 is the starting value for the optimization. Use the closed formula (3.23) to compute the prices of call options in the Black-Scholes model.

Test the function for $S_0 = 12658$, $r = 0$, $\sigma_0 = 0.3$ and real price data of European call options on the german DAX index from 01.06.2017 provided in the file

```
Dax_CallPrices_Eurex20170601.csv
```

which is available on the OLAT.

Hint: Have a look at section 4.4 of the course and make yourself familiar with the Scilab command `leastsq`.

C-Exercise 21 (Valuation of European options in the B-S model using Monte-Carlo)
Write a scilab function

```
[V0, c1, c2] = EuOption_BS_MC (S0, r, sigma, T, K, M, g)
```

that computes the initial price of a European option with payoff $g(S_T)$ at maturity T in the Black-Scholes model and the asymptotic 95%-confidence interval $[c_1, c_2]$ assuming finite variance of $g(S_T)$ via the Monte-Carlo approach using $M \in \mathbb{N}$ simulations. As an example consider the european put $g(x) = (K-x)^+$ with strike price $K = 100$, $S_0 = 95$, $r = 0.05$, $\sigma = 0.2$ and $T = 1$.

Hint: The initial option price is of the form $V(0) = E_Q(f(Z))$, where $Z \sim N(0,1)$ under Q. How does the function f look like?

Useful scilab command: `grand`

C-Exercise 22 (Sampling from the Beta distribution by the acceptance/rejection method)
The density of the Beta distribution with parameters $\alpha_1 > 1$ and $\alpha_2 > 1$ is given by

$$f(x) = \frac{1}{B(\alpha_1, \alpha_2)} x^{\alpha_1 - 1}(1-x)^{\alpha_2 - 1} 1_{[0,1]}(x),$$

where $B(\alpha_1, \alpha_2) = \int_0^1 x^{\alpha_1 - 1}(1-x)^{\alpha_2 - 1}\, \mathrm{d}x$ is the Beta function. Write a Scilab function

```
X = Sample_Beta_AR (alpha1, alpha2, N)
```

that generates and returns $N \in \mathbb{N}$ independent samples from the Beta distribution with parameters $\alpha_1 > 1$ and $\alpha_2 > 1$ by means of the acceptance/rejection method. In your algorithm, you may sample only from the uniform distribution on $[0,1]$ using the function `rand`.

For $\alpha_1 = 2$, $\alpha_2 = 3$, generate $N = 2000$ samples, and plot them in a histogram.

Useful Scilab commands: `beta`, `rand`, `histplot`

T-Exercise 23 (Simulation of mixed distributions)
A distribution with cdf F is called a mixture if for $i = 1, \dots, m$ there exist cdfs F_i on $\mathbb{R}$ and $\omega_i > 0$ real numbers with

$$\sum_{i=1}^{m} \omega_i = 1.$$

such that

$$F(x) = \sum_{i=1}^{m} \omega_i F_i(x).$$

These distributions might for example be used to model demand behaviour in financial markets.

a) For $i = 1, \dots, m$ let X_j be random variables distributed according to F_j and Z an independent random variable with $\mathbb{P}(Z = j) = \omega_j$. Show that the cdf of

$$X = \sum_{i=1}^{m} \mathbb{1}_{\{Z=i\}} X_i$$

is F.

b) Show that for the j-th centered moment of X it holds that

$$\mathbb{E}\left[(X-\mu)^j\right] = \sum_{i=1}^{m} \sum_{k=0}^{j} \binom{j}{k} \omega_i (\mu_i - \mu)^{j-k} \mathbb{E}\left[(X_i - \mu_i)^k\right]$$

where $\mu = \mathbb{E}[X]$ and $\mu_i = \mathbb{E}[X_i]$.

Hint: Binomial formula

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Fri, 16.06.2017, 10:00
Discussion: 19/21.06.2017