1. T-Exercise 8 (American put option in the CRR model)

   $X(t)$ and $Y(t)$ are functions of both t and W(t), so use (3.9) for that:

   $df(X(t)) = \Big(\partial_1 f(t, X(t)) + \partial_2 f(t, X(t))\mu(t) + \frac{1}{2}\partial_{22} f(t, X(t))\sigma^2\Big)dt + \partial_2 f(t, X(t))\sigma(t)dW(t)$

   (a) $d(X(t)) = d(tW(t)) = W(t)dt + tdW(t)$ (remember that $\mu(t) = 0$ as W(t) is std Brownian motion)

   (b) $\partial_2 Y(t) = -\frac{W(t)}{(1+t)^2}$

   $dY(t) = (1+t)^{-1}dW(t) - \frac{W(t)}{(1+t)^2}dtY(t) = \frac{W(t)}{1+t}$

2. T-Exercise 9 (Exchange rates)

   $dD(t) = D(t)\mu dt + D(t)\sigma dW(t)$
   $E(t) := 1/D(t)$

   (a) $E(t) = \frac{1}{D(t)} = f(D(t))$ hence use (3.8) for that:

   $df(X(t)) = [f'(X(t))\mu + \frac{1}{2}f''(X(t))\sigma^2]dt + f'(X(t))\sigma dW(t)$

   $dE(t) = (-D^{-2}(t)\mu(t) + D^{-3}(t)\sigma(t)^2)dt - D^{-2}(t)\sigma(t)dW(t)$

   where $\mu(t) = D(t)\mu$ and $\sigma(t) = D(t)\sigma$ from $dD(t)$ process

   substitute and get $dE(t) = E(t)\Big((\sigma^2 - \mu)dt - \sigma dW(t)\Big)$

   (b) Use the stochastic exponential for the Ito process

   (3.11) $\mathcal{E}(X)(t) = exp\Big(\int_0^t(\mu(s) - \frac{1}{2}\sigma^2)ds + \int_0^t \sigma(s)dW(s)\Big)$ it follows

   that for linear SDE $dX(t) = \mu X(t)dt + \sigma X(t)dW(t)$ with $X(0) = x$ and constant μ and σ can be rephrased as $dX(t) = X(t)dY(t)$ with $Y(t) = \mu t + \sigma W(t)$

   using (3.11) we get $X(t) = x\mathcal{E}(Y)(t) = xexp((\frac{1}{2}\sigma^2 - \mu)t - \sigma W(t))$

   $dD(t) = D(t)\mu dt + D(t)\sigma dW(t) = D(t)(\mu dt + \sigma dW(t)) = D(t)dY(t)$ where

   $dY(t) = \mu dt + \sigma dW(t)$

   $D(t) = D(0)\mathcal{E}(Y)(t) = D(0)exp\Big[(\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)\Big]$

   $E(t) = E(0)\mathcal{E}(Y)(t) = E(0)exp\Big[(\frac{1}{2}\sigma^2 - \mu)t - \sigma W(t)\Big]$

   where $dY(t) = (\sigma^2 - \mu)dt - \sigma dW(t)$

   (c) Plugin $\mu = \frac{1}{2}\sigma^2$ and we get:

   $D(t) = D(0)exp(\sigma W(t))$ and $E(t) = E(0)exp(-\sigma W(t))$

   Drift part vanishes, diffusion is symmetric, hence E_t and D_t are symmetric

   EV's are the same, so no currency has an advantage