**T-Exercise 4**

we have **one** share,
risk factor is *yield to maturity* $Z_n = -\frac{1}{T-t_n} log(P_n)$

(a) $V_n = f(t_n, Z_n)$ which is $P_n$ since there is only one share
$V_n = exp(Z_n(T-t_n)) = exp(Z_n(t_n - T))$

(b) risk factor change: $X_{n+1} = Z_{n+1} - Z_n = -\frac{log(P_{n+1})}{T-t_{n+1}} + \frac{log(P_n)}{T-t_n} = \frac{(T-t_{n+1})log(P_n) - (T-t_n)log(P_{n+1})}{(T-t_{n+1})(T-t_n)}$

(c) $l_{[n]} = -f(t_{n+1}, Z_n + X_{n+1}) + f(t_n, Z_n) =$
$$- exp\Big( (Z_n + X_{n+1})(t_{n+1} - T) \Big) + exp\Big( Z_n(t_n - T) \Big) =$$
$$- exp\Big( - (Z_n + X_{n+1})(T - t_{n+1}) \Big) + exp\Big( - Z_n(T - t_n) \Big) =$$
$$- exp\Big( - Z_n(T - t_n - \Delta t) - X_{n+1}(T - t_{n+1}) \Big) + exp\Big( - Z_n(T - t_n) \Big) =$$
$$- exp\Big( - Z_n(T - t_n) + Z_n\Delta t - X_{n+1}(T - t_{n+1}) \Big) + exp\Big( - Z_n(T - t_n) \Big) =$$
$$\underbrace{exp\Big( - Z_n(T - t_n) \Big)}_{P_n} \Big( 1 - exp\Big( - X_{n+1}(T - t_{n+1}) + Z_n\Delta t \Big) \Big)$$

(d) $l^{\Delta}_{[n]}(x) = \underbrace{-\Big( exp(-Z_n(T-t_n))Z_n \Big)\Delta t}_{-D_1 f(t,Z_n)\Delta t = \frac{\partial f(t,Z_n)}{\partial t}\Delta t} \underbrace{- \Big( exp(-Z_n(T-t_n))(t_n - T)x \Big)}_{D_2 f(t,Z_n)x = \frac{\partial f(t,Z_n)}{\partial x}x} =$
$$- exp\Big( - Z_n(T - t_n) \Big) \Big( Z_n\Delta t + (t_n - T)x \Big) = exp\Big( - Z_n(T - t_n) \Big) \Big( (T - t_n)x - Z_n\Delta t \Big)$$