1. T-Exercise 13

   $dB_t = rB_t dt$
   $dS_t = S_t \mu dt + \sigma S_t dW(t)$

   (a) $dS_t = S_t(\mu dt + \sigma dW(t))$ hence use (3.11) stochastic exponential:

   $S_t = S_0 \times exp\Big[(\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)\Big]$

   $X_t = log(S_t) = log(S_0) + (\mu - \frac{1}{2}\sigma^2)t + \sigma W(t) =$

   $= X_0 + (\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)$

   it is an Ito process interpreted as an integral equation, see (3.7)

   hence $dX_t = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW(t)$

   and then $d[X, X]_t = \sigma^2 dt$

   (b) $\varphi = (\varphi_t^0, \varphi_t^1)_{t \geq 0}$ with $V_0(\varphi) = 1$, investing strategy always $\varphi_t^1 = \frac{V_1(\varphi)}{2S_t}$

   $V_t(\varphi) = \varphi_t^0 S_t^0 + \varphi_t^1 S_t^1 = \varphi_t^0 B_t + \frac{V_t(\varphi)}{2S_t} S_t = \varphi_t^0 B_t + \frac{V_t(\varphi)}{2}$

   $\varphi_t^0 = \frac{V_t(\varphi)}{B_t}(1 - \frac{1}{2}) = \frac{V_t(\varphi)}{2B_t}$

   by (3.15) $dV_t(\varphi) = \varphi_t dS_t \Big(= \sum_{i=0}^{d} \varphi_i(t) dS_i(t) \Big)$

   $dV_t(\varphi) = \varphi_t^0 dB_t + \varphi_t^1 dS_t$

   substitute dS_t and dB_t

   $dV_t(\varphi) = \varphi_t^0 r B_t dt + \varphi_t^1(\mu S_t dt + \sigma S_t dW_t)$

   substitute φ_t^0 and φ_t^1

   $dV_t(\varphi) = \frac{V_t(\varphi)}{2B_t} r B_t dt + \frac{V_t(\varphi)}{2S_t}(\mu S_t dt + \sigma S_t dW_t) =$

   $= \frac{V_t(\varphi)}{2}((r + \mu)dt + \sigma dW_t)$

   this is the form of geometric Brownian Motion