Christian-Albrechts-Universität zu Kiel **WS 2017/18**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Dr. Giso Jahncke Sheet 07

Risk Management

Exercises for participants of the programme **Quantitative Finance**

C-Exercise 13

In this exercise we want to use a GARCH(1,1) model to estimate *value at risk* and *expected shortfall* of a portfolio loss.
For this purpose we assume that our one dimensional risk factor changes $(X_n)_{n\in\mathbb{N}}$ follow the GARCH(1,1) model:

$$\begin{aligned} X_n &= \sigma_n Y_n, \\ \sigma_n^2 &= \alpha_0 + \alpha_1 X_{n-1}^2 + \beta \sigma_{n-1}^2, \end{aligned}$$

with parameters $\vartheta = (\alpha_0, \alpha_1, \beta, \sigma_1) \in \Theta := (0,\infty)^4$ and iid standard normal random variables $(Y_n)_{n\in\mathbb{N}}$. We write

$$f_\vartheta^{(X_1,\dots,X_n)} : \mathbb{R}^n \to \mathbb{R}$$

for the common pdf of $(X_1,\dots,X_n)$ under the probability measure P_ϑ.

(a) Write a *scilab*-function

```
y= log_likelihood_GARCH_11(theta,x),
```

which returns the log likelihood function

$$L_n(\vartheta, x) = \log\left(f_\vartheta^{(X_1,\dots,X_n)}(x)\right)$$

evaluated at the parameter $\vartheta = (\alpha_0, \alpha_1, \beta, \sigma_1)$ and given historical risk factor changes $x = (x_1,\dots,x_n) \in \mathbb{R}^n$.

(b) Write a *scilab*-function

```
theta_hat=estimates_GARCH_11(x),
```

which computes the Maximum Likelihood estimates $\widehat{\vartheta}$ for given historical risk factor changes $x = (x_1,\dots,x_n) \in \mathbb{R}^n$ in the GARCH(1,1) model.

(c) Write a *scilab*-function

```
[VaR,ES]=VaR_ES_MC_GARCH_11(l,alpha,theta,x,k,m),
```

that computes the *value at risk* and *expected shortfall* estimates for the m-period loss operator $l : \mathbb{R} \to \mathbb{R}$, level $\alpha \in (0,1)$, parameters $\vartheta \in \Theta$ and given historical risk factor changes $x = (x_1,\dots,x_n) \in \mathbb{R}^n$ using the Monte-Carlo method with $k \in \mathbb{N}$ simulations.

(d) Compute the logarithmic returns $x_2,\dots,x_{6816}$ of the DAX time series, that we use as risk factor changes. Compute for the last trading day in the time series the 5 day ahead estimates of *value at risk* and *expected shortfall* at level $\alpha = 0.98$ using $k = 1000$ simulations.

Submit until: Thursday, 14.12.2017, 08:30 (before the lecture)