**Problem 729**

Range of Periodic Sequence

Analytical solution exists for :

Periodicity two, so

For period :

Becomes something like:

and cannot really be solved analytically. We have to resort to numerical root-finding methods. This is an 8th-order equation -> 8 complex roots (fundamental theorem of algebra), so there’s an *upper limit* of 8 solutions in the real plane.

Abel-Ruffini theorem states basically that we cannot analytically solve polynomial equations of degree 5 or higher. <https://en.wikipedia.org/wiki/Abel%E2%80%93Ruffini_theorem>

* We need very high precision floating point to store the numbers.
* Need to be very careful about Cycle Collisions: if is a root of both and , then it’s a period-2 point, not a new period-4 one. This is why my answer for S(5) was too high.
* Roots may lie extremely close together (< 10^-40). Roots are not degenerate in theory by the appear numerically degenerate.
* Optional idea: use interval arithmetic to rigorously enclose each root (slower but exact).

**Sequences identified**

1. Number of (real) roots for given period (***without*** accounting for cycle collisions)

0, 2, 6, 14, 30, 62, 126

<https://oeis.org/A000918>

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1. Number of (real) roots for given period (***with*** accounting for cycle collisions)

0, 2, 6, 12, 30, 54, 126

<https://oeis.org/A056267>

So if I’m correct, we need to find unique roots in order to solve this problem.

Computationally, computing the orbits and finding the range (min, max) is doable.

But how do we efficiently find so many roots?

**Next step, find a faster way to find/compute the roots than sampling over a range using Newton’s method.**