1 Monte-Carlo Approximator

If $f : \Omega \to \mathbb{R}$ then

$$\hat{I} := \int_\Omega \mathrm{d}x \, \langle x|$$

can be approximated via

$$\hat{I}_n := \frac{|\Omega|}{n} \sum_{i=1}^{n} \langle X_i| \,, \quad X_i \in U(\Omega)$$

then $\lim_{n\to\infty} \hat{I}_n \, |f\rangle = \hat{I} \, |f\rangle$, with an error which scales as$\mathcal{O}(1/\sqrt{n})$. The approximator is also unbiased meaning $\mathbb{E}[\hat{I}_n \, |f\rangle] = \hat{I} \, |f\rangle$, for any n.