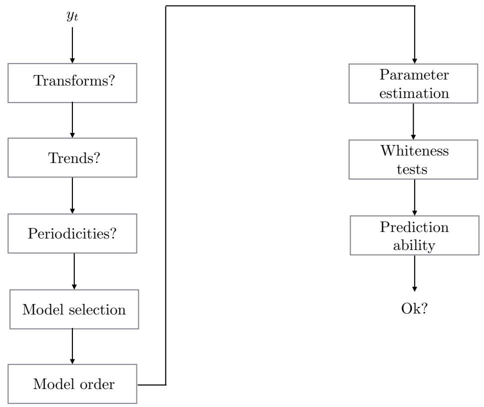
Partial Autocorrelation Function

Andreas Jakobsson

The partial autocorrelation function



The partial autocorrelation function

We begin by examining the model selection step. How can we select a suitable model? This may be done in several ways; for now, we focus on how to select between AR, MA, and ARMA models.

As discussed earlier, we may use the ACF to determine is something is well modelled using an MA process as

$$r_y(k) = \begin{cases} \sigma_e^2 \left(c_k + c_1 c_{k+1} + \ldots + c_{q-k} c_q \right) & \text{if } |k| \leq q \\ 0 & \text{if } |k| > q \end{cases}$$

Can we do something similar for AR or ARMA processes?

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Can we do something similar for AR or ARMA processes?

The partial autocorrelation function (PACF) aims to have the same role for AR processes; recall the definition of an AR(k) process

$$y_t = \phi_{k,1} y_{t-1} + \ldots + \phi_{k,k} y_{t-k} + e_t$$

where we now use $\phi_{k,\ell}$ to denote the ℓth (negative) AR coefficient of the kth order AR model.

The PACF is the sequence $\phi_{k,k}$, computed for different model orders of the process. If the model data is an AR(p) process, this means that

$$\phi_{k,k} = \begin{cases} \text{non-zero} & k \leq p \\ 0 & k > p \end{cases}$$

The partial autocorrelation function

For large N, it holds that

$$E\{\hat{\phi}_{k,k}\} = 0$$
$$V\{\hat{\phi}_{k,k}\} = \frac{1}{N}$$

for $k > p$. Furthermore, $\hat{\phi}_{k,k}$ is asymptotically Normal distributed (for $k > p$).

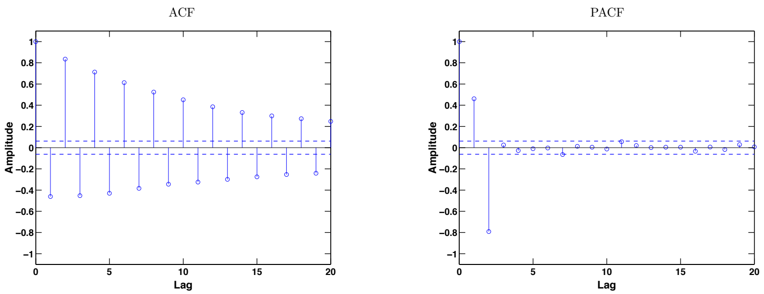
This means that the (approximative) 95% confidence interval for an AR(p) process can be expressed as

$$\hat{\phi}_{k,k} \approx 0 \pm \frac{2}{\sqrt{N}}, \quad \text{for } k > p$$

ACF and PACF

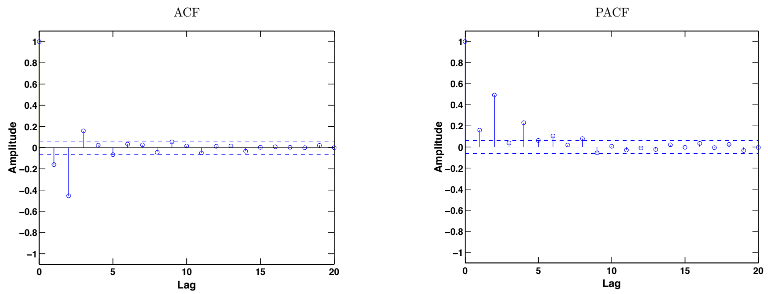
	ACF	PACF	IACF
AR(p)			
	Damped exponential and/or sine functions	$\phi_{k,k} = 0$ for $k > p$	$\rho^i(k) = 0$ for $k > p$
MA(q)			
	$\rho(k) = 0$ for $k > q$	Damped exponential and/or sine functions	Damped exponential and/or sine functions
ARMA(p, q)			
	Damped exponential and/or sine functions after lag $\|p - q\|$	Damped exponential and/or sine functions after lag $\|p - q\|$	Damped exponential and/or sine functions after lag $\|p - q\|$

ACF and PACF



What would be a reasonable model?

ACF and PACF



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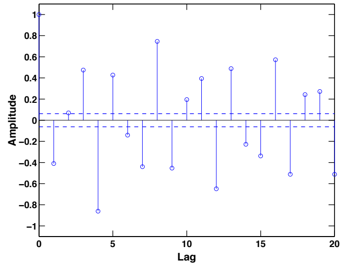
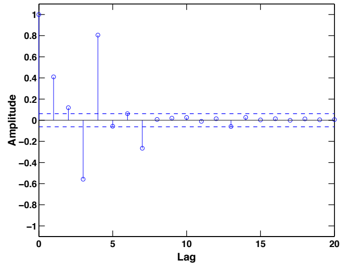


PACF



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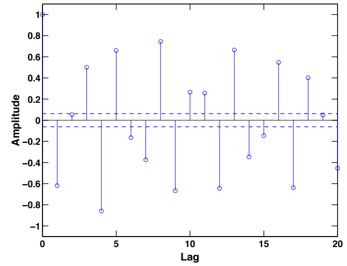

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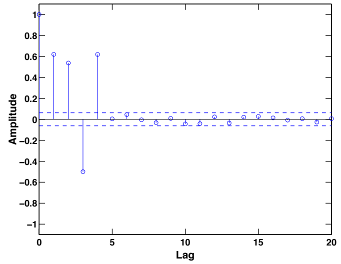

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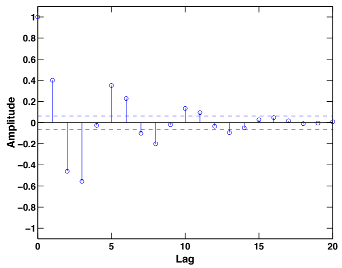

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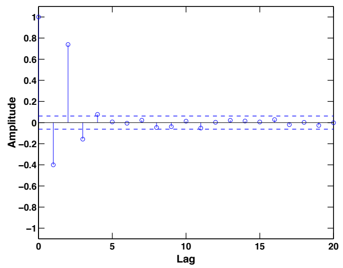

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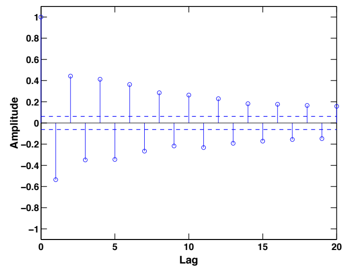

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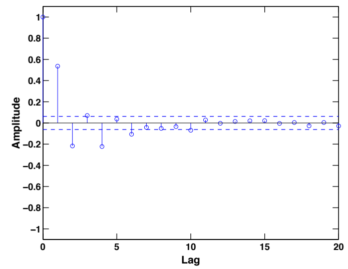

What would be a reasonable model?

How can one model the sunspots?