Model order estimation

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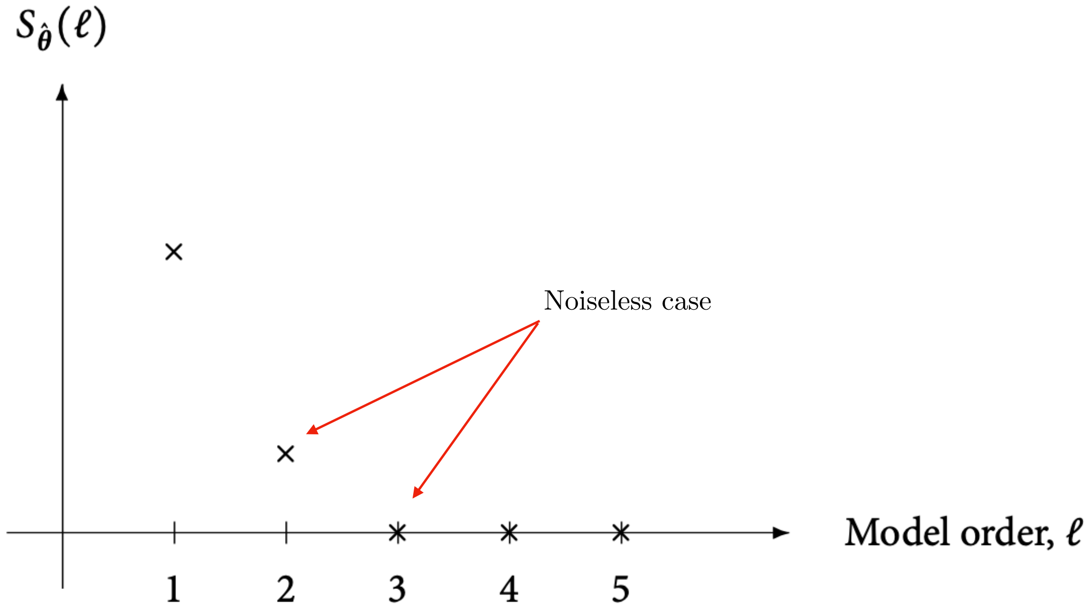
Model order estimation



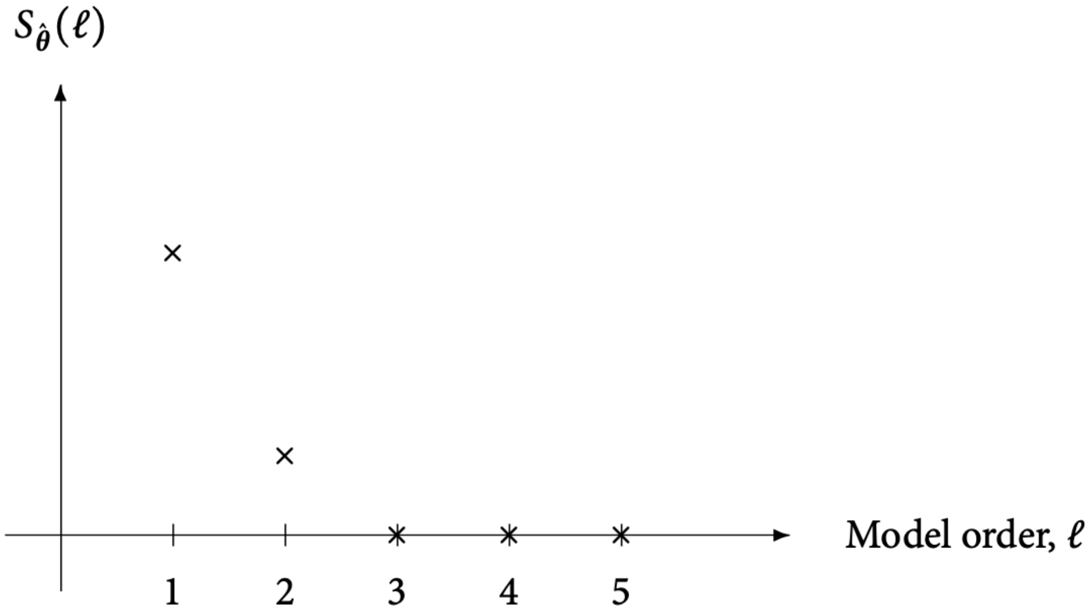
Form an estimate of the residual variance

$$S_{\boldsymbol{\theta}}(\ell) = \sum_t \left| y_t - \hat{y}_{t|t-1}(\hat{\boldsymbol{\theta}}_\ell) \right|^2$$

where ℓ denotes the assumed model order.

ML parameter estimate

Model order estimation



For the *generalized information criteria* (GIC), one adds $\alpha_{\ell,N}$ to $S_{\boldsymbol{\theta}}(\ell)$, where

$$\begin{aligned} \alpha_{\ell,N} &= 2\ell && \text{for AIC} \\ \alpha_{\ell,N} &= \ell \ln N && \text{for BIC} \\ \alpha_{\ell,N} &= 3\ell && \text{for KIC} \end{aligned}$$

yielding

$$\hat{\ell} = \arg \min_{\ell \in [1, \ell_{max}]} \text{GIC}(\ell)$$

where

$$\text{GIC}(\ell) = N \ln \hat{\sigma}^2_{e,\ell} + \alpha_{\ell,N}$$

A commonly used alternative is the *final prediction error* (FPE) defined as

$$\text{FPE}(\ell) = \hat{\sigma}^2_{e,\ell} \frac{1 + \ell/N}{1 - \ell/N} \approx \hat{\sigma}^2_{e,\ell} \left(1 + \frac{2\ell}{N} \right)$$

This is used, for instance, in Matlab.

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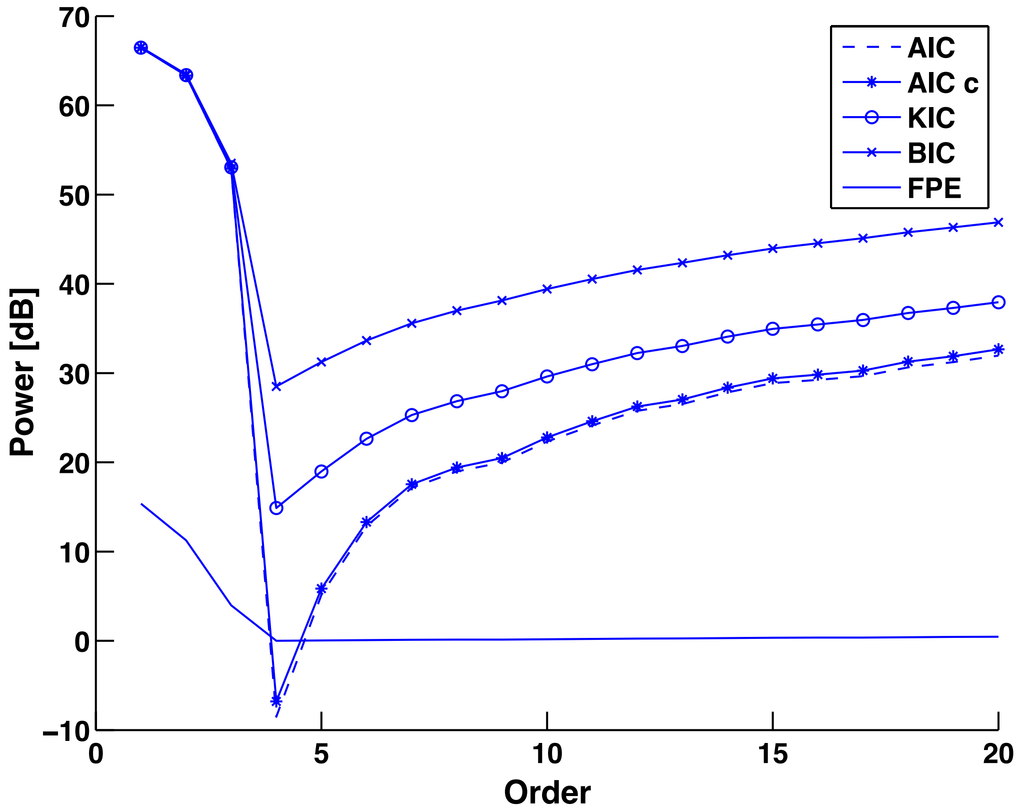

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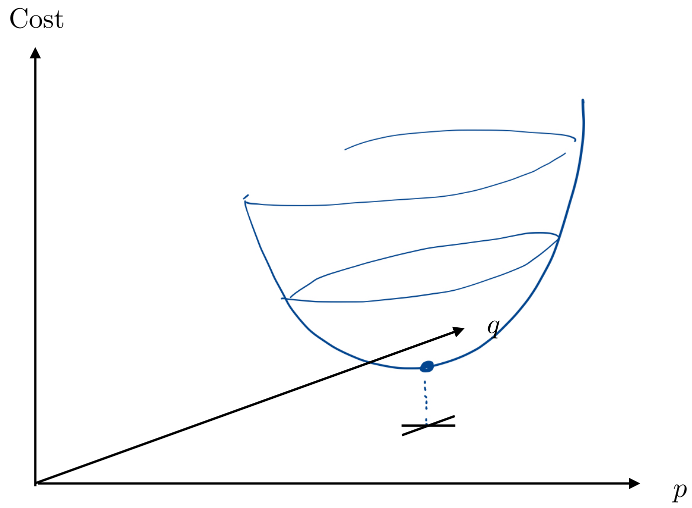
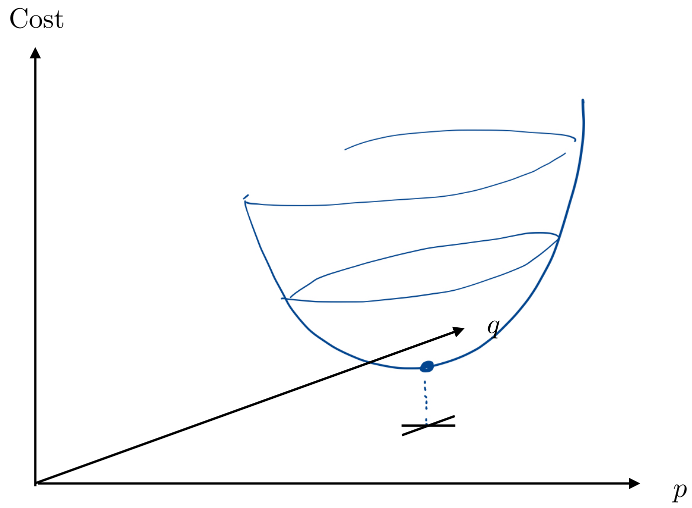


In the case of an ARMA(p, q) process, the number of unknowns is $\ell = p + q$. For instance, AIC will then be

$$\mathrm{AIC}(p, q) = N \ln \hat{\sigma}^2_{e,p,q} + 2(p + q)$$

Model order estimation



Important to note:

- Model order estimation is difficult, especially when N is small.
- There is no reliable algorithm. Treat all estimates with scepticism.
- At best, these algorithm can give you *a feel for an appropriate order*, at worst, they may indicate something *completely wrong*.
- Do not rely on your model order estimate!

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