Machine Learning - Laboratory 4

```
import numpy as np
import pickle
import pandas as pd
from pdffuns import *
import matplotlib.pyplot as plt
```

Problem 1

Section a)

```
# Load the data
pfile = 'lab4.p'

with open(pfile, "rb") as fp:
    X, Y = pickle.load(fp)

for x in X:
    npX = np.array(x)
    print(f"X.shape = {npX.shape}")

for y in Y:
    npY = np.array(y)
    print(f"Y.shape = {npY.shape}")


X.shape = (2, 120)
X.shape = (2, 80)
Y.shape = (2, 120)
Y.shape = (2, 80)
```

Section b)

The classifier is already designed in a parametrized way, so we will be able to use it for both problems.

```
def classify(training_data, input_data, classifier="ML", param = 1,
verbose=False):
    """
    Train the model based on the training data

    Parameters
    ----------
    training_data : list
        List of training data points
    input_data : list
```

```
        List of input data points
    classifier : str
        Classifier to use. Options are:
            - "ML" : Maximum likelihood
            - "PZ" : Parzen window
            - "knn" : k-nearest neighbor
    param : int
        Parameter for the classifier. For knn this is the number of
neighbors
    verbose : bool
        If True, print some information of the dataset that is being
used

    """

    # LEARN THE PARAMETERS FROM THE TRAINING DATA
    # Get the number of classes and the number of dimensions
    M = len(training_data)
    l = len(training_data[0])

    # Append both number of samples in the same array
    N = []
    for i in range(M):
        N.append(len(training_data[i][0]))

    if verbose:
        print(f"M = {M}")
        print(f"l = {l}")
        for i in range(M):
            print(f"samples_w{i+1} = {N[i]}")

    # Determine prior probabilities
    Pw = []

    for i in range(M):
        Pw.append(N[i]/sum(N))

    # Initialize the parameters
    my = np.zeros(shape=(M, l), dtype=float)
    Sgm = np.zeros(shape=(M, l, l), dtype=float)

    # Initialize the probability density function and the discriminant
function
    pxw = np.zeros(shape=(M, np.shape(input_data)[1]))
    g = np.zeros(shape=pxw.shape)                          # Discriminant
function values for both classes
    g_max = np.zeros(shape=(np.shape(input_data)[1]))   # Array of
maximun g value for each sample
    C = np.zeros(shape=(np.shape(input_data)[1]))       # Array of
classification labels
```

```
    p = 0

    for i in range(M):
        if classifier == "ML":
            # Estimate parameters my[k], Sgm[k]
            for j in range(l):
                my[i][j] = np.mean(training_data[i][j])

            Sgm[i] = np.cov(training_data[i])

            # Compute the probability density function of the input
data using the learnt parameters from the training data
            pxw[i] = norm2D_samples(my[i].reshape(-1, 1), Sgm[i],
input_data)

        if classifier == "PZ":
            h1 = param
            hn = h1/np.sqrt(N[i])
            hnI = hn**2 * np.eye(l)

            # iterate over all feature vectors in class i
            for j in range(0, N[i]):
                # feature vector j of class i
                xk = training_data[i][:,j].reshape(l,1)
                # sum up the probabilities of each of the N[i]
distributions
                # Note that there is one distribution for each
datapoint!
                pxw[i] = pxw[i] + norm2D_samples(xk, hnI, input_data)
                # divide by number of feature vectors in class i
            pxw[i] = 1 / N[i] * pxw[i]

        if classifier == "knn":
            pxw[i] = knn2D(training_data[i], input_data, param)

    # - update p
    p += Pw[i] * pxw[i]

    # Determine discriminant functions, g[k], k = 0,...,M-1
    for i in range(M):
        g[i] = (Pw[i] * pxw[i])

    # Divide by the total probability to get the posterior
probabilities
    g = g/p

    # Once computed the discriminant function values, we need to check
what class has te maximun value on each
    for i in range(np.shape(g)[1]):
        C[i] = np.argmax(g[:, i])
```

```
    g_max = np.max(g, axis=0)

    return g_max, C

def computeMetrics(C, Y):
    """
    Compute the metrics given the classification labels and the true
labels

    Parameters
    ----------
    C : array-like
        Classification labels
    Y : array-like
        True labels
    """

    M = len(Y)
    n_samples = []

    for i in range(M):
        n_samples.append(len(Y[i]))

    # Initialization of the metrics
    accuracy = []
    error = 0

    # Compute for each class
    for i in range(M):
        # Compute the accuracy
        acc_value = (np.sum(C[i] == Y[i])/n_samples[i])
        accuracy.append(acc_value)

        # Compute the number of misclassified samples
        error += np.sum(C[i] != Y[i])

    # Divide the number of missclassified samples by the total number
of samples to get the error
    error /= sum(n_samples)

    return accuracy, error

M = len(Y)

# Create the ground truth labels
ground_truth_w1 = np.zeros(shape=(120), dtype=int)
ground_truth_w2 = np.ones(shape=(80), dtype=int)
ground_truth = [ground_truth_w1, ground_truth_w2]
```

```
gx = np.full(shape=(M), fill_value=np.array)
Cx = np.full(shape=(M), fill_value=np.array)

gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y, X[k], verbose=False)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y, Y[k], verbose=False)

accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y, X[k], verbose=False, classifier="PZ",
param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y, Y[k], verbose=False, classifier="PZ",
param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y, X[k], verbose=False, classifier="PZ",
param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y, Y[k], verbose=False, classifier="PZ",
param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y, X[k], verbose=False, classifier="knn",
param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y, Y[k], verbose=False, classifier="knn",
param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
```

```
    gx[k], Cx[k] = classify(Y, X[k], verbose=False, classifier="knn",
param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y, Y[k], verbose=False, classifier="knn",
param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: invalid value encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: overflow encountered in divide
  g = g/p
```

Data Frame of the metrics

```
row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Parzen h1 = 5 testing',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
df['P(correct|w1)'] = [accuracy_recl[0], accuracy_test[0],
accuracy_recl_pz[0], accuracy_test_pz[0], accuracy_recl_pz_5[0],
accuracy_test_pz_5[0], accuracy_recl_knn[0], accuracy_test_knn[0],
accuracy_recl_knn_5[0], accuracy_test_knn_5[0]]
df['P(correct|w2)'] = [accuracy_recl[1], accuracy_test[1],
accuracy_recl_pz[1], accuracy_test_pz[1], accuracy_recl_pz_5[1],
accuracy_test_pz_5[1], accuracy_recl_knn[1], accuracy_test_knn[1],
accuracy_recl_knn_5[1], accuracy_test_knn_5[1]]
display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification     0.100       0.933333

1                                ML testing     0.045       0.966667

2          Parzen h1 = 0.1 reclassification     0.395       0.341667

3                    Parzen h1 = 0.1 testing     0.565       0.266667

4             Parzen h1 = 5 reclassification     0.080       0.925000

5                      Parzen h1 = 5 testing     0.060       0.950000

6  Nearest neighbor kn = 1 reclassification     0.600       0.000000

7           Nearest neighbor kn = 1 testing     0.085       0.950000

8  Nearest neighbor kn = 5 reclassification     0.090       0.925000

9           Nearest neighbor kn = 5 testing     0.075       0.933333


   P(correct|w2)
0         0.8500
1         0.9375
2         1.0000
3         0.6875
4         0.9125
5         0.9250
6         1.0000
7         0.8625
8         0.8875
9         0.9125
```

Problem 2

```python
pfile = 'lab4_2.p'
with open(pfile, "rb") as fp:
    X_2D3cl, X_2D4cl, X_2D4cl_ms, X_2D4cl_hs, \
    X_3D3cl_ms, Y_2D3cl, Y_2D4cl, Y_2D4cl_ms, \
    Y_2D4cl_hs, Y_3D3cl_ms = pickle.load(fp)

for x in X_2D3cl:
    npX = np.array(x)
    print(f"X.shape = {npX.shape}")

for y in Y_2D3cl:
    npY = np.array(y)
```

```
    print(f"Y.shape = {npY.shape}")


X.shape = (2, 25)
X.shape = (2, 50)
X.shape = (2, 30)
Y.shape = (2, 25)
Y.shape = (2, 50)
Y.shape = (2, 30)
```

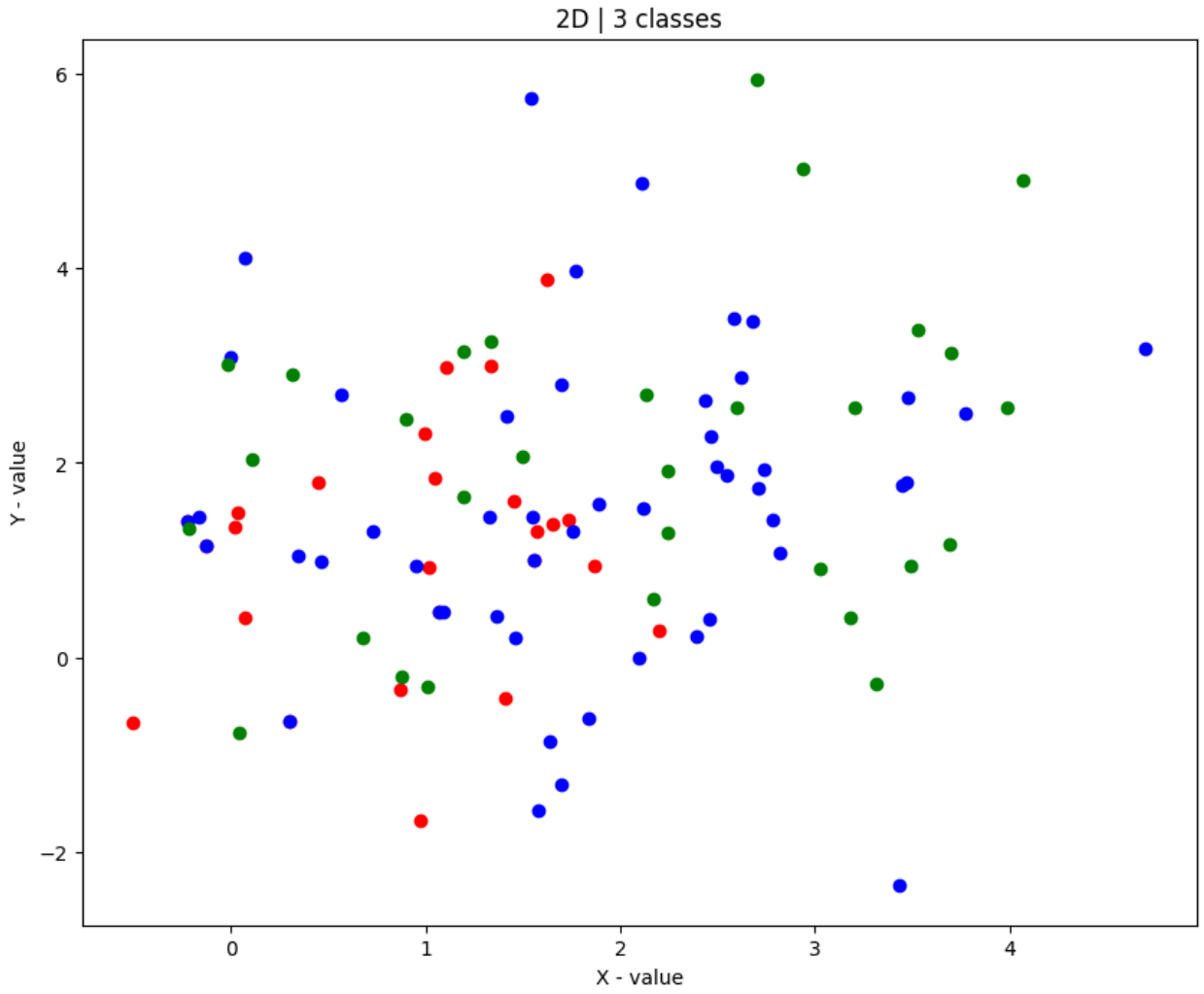
Section a)

```
# PLOT THE DATA

M = len(X_2D3cl)
colors = ['red', 'blue', 'green']

for i in range(M):
    plt.scatter(X_2D3cl[i][0], X_2D3cl[i][1], label=f'Class {i+1}',
color=colors[i])

plt.rcParams.update({'figure.figsize':(10,8), 'figure.dpi':100})
plt.title('2D | 3 classes')
plt.xlabel('X - value')
plt.ylabel('Y - value')
plt.show()
```




```
# CLASSIFY THE DATA

# Create the ground truth labels
ground_truth = []
for i in range(M):
    class_ground_truth = np.zeros(shape=(len(X_2D3cl[i][0])),
dtype=int)
    class_ground_truth = class_ground_truth + i
    ground_truth.append(class_ground_truth)

gx = np.full(shape=(M), fill_value=np.array)
Cx = np.full(shape=(M), fill_value=np.array)
gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D3cl, X_2D3cl[k], verbose=False)
    gy[k], Cy[k] = classify(Y_2D3cl, Y_2D3cl[k], verbose=False)
```

```
accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D3cl, X_2D3cl[k], verbose=False,
classifier="PZ", param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D3cl, Y_2D3cl[k], verbose=False,
classifier="PZ", param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D3cl, X_2D3cl[k], verbose=False,
classifier="PZ", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D3cl, Y_2D3cl[k], verbose=False,
classifier="PZ", param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D3cl, X_2D3cl[k], verbose=False,
classifier="knn", param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D3cl, Y_2D3cl[k], verbose=False,
classifier="knn", param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D3cl, X_2D3cl[k], verbose=False,
classifier="knn", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D3cl, Y_2D3cl[k], verbose=False,
classifier="knn", param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: invalid value encountered in divide
  g = g/p
```

```
# DATAFRAME WITH THE RESULTS

row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Parzen h1 = 5 testing',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
for i in range(M):
    df[f'P(correct|w{i+1})'] = [accuracy_recl[i], accuracy_test[i],
accuracy_recl_pz[i], accuracy_test_pz[i], accuracy_recl_pz_5[i],
accuracy_test_pz_5[i], accuracy_recl_knn[i], accuracy_test_knn[i],
accuracy_recl_knn_5[i], accuracy_test_knn_5[i]]

display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification  0.466667           0.32

1                                ML testing  0.533333           0.24

2          Parzen h1 = 0.1 reclassification  0.209524           0.48

3                   Parzen h1 = 0.1 testing  0.638095           0.44

4            Parzen h1 = 5 reclassification  0.457143           0.16

5                     Parzen h1 = 5 testing  0.533333           0.16

6  Nearest neighbor kn = 1 reclassification  0.714286           0.00

7           Nearest neighbor kn = 1 testing  0.657143           0.32
```

```
8  Nearest neighbor kn = 5 reclassification  0.523810           0.36

9           Nearest neighbor kn = 5 testing  0.552381           0.36


   P(correct|w2)  P(correct|w3)
0           0.86       0.166667
1           0.78       0.133333
2           0.82       1.000000
3           0.22       0.533333
4           0.96       0.166667
5           0.82       0.133333
6           0.00       1.000000
7           0.38       0.300000
8           0.74       0.133333
9           0.64       0.200000
```

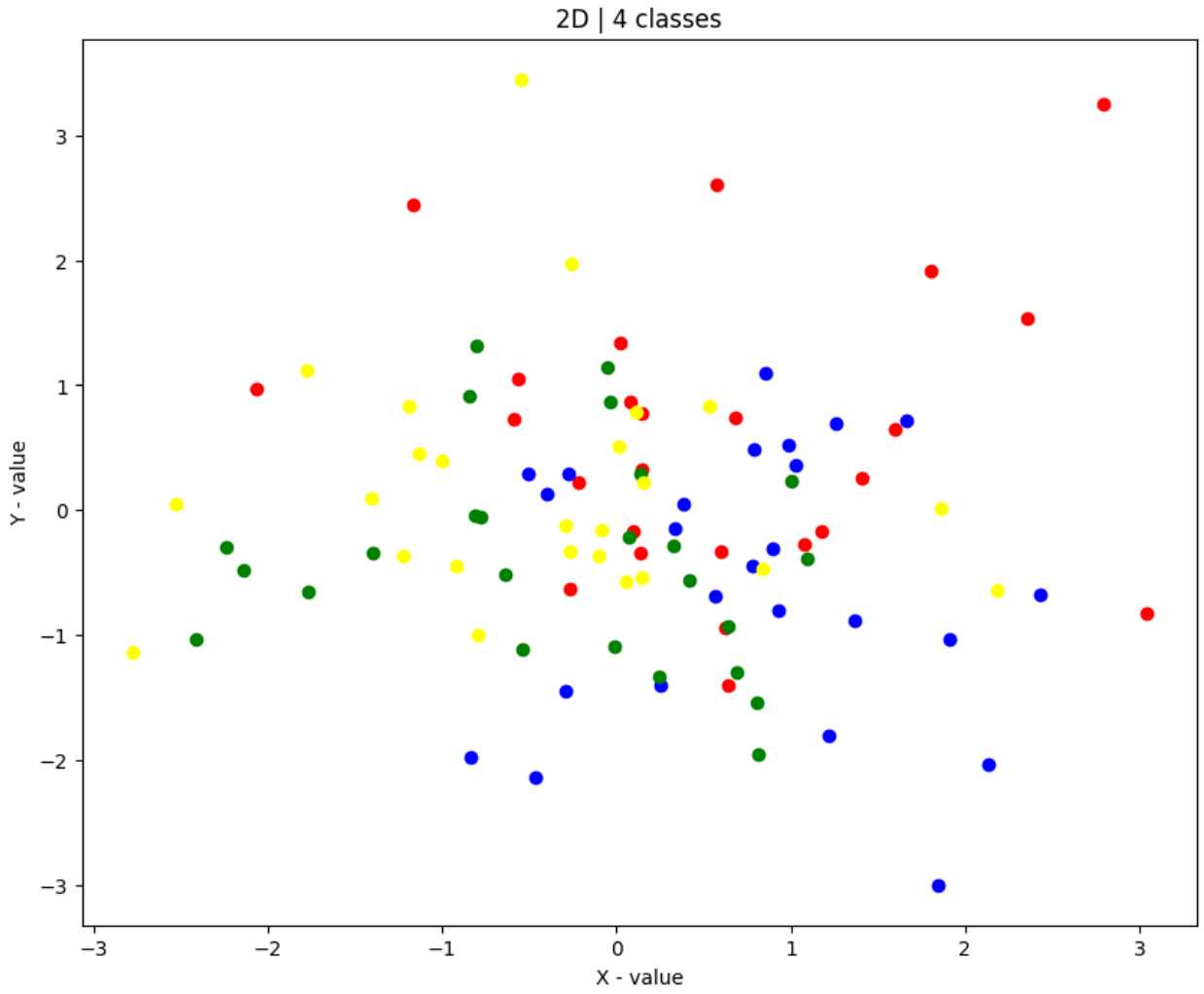
Section b)

```
# PLOT THE DATA

M = len(X_2D4cl)
colors = ['red', 'blue', 'green', 'yellow']

for i in range(M):
    plt.scatter(X_2D4cl[i][0], X_2D4cl[i][1], label=f'Class {i+1}',
color=colors[i])

plt.rcParams.update({'figure.figsize':(10,8), 'figure.dpi':100})
plt.title('2D | 4 classes')
plt.xlabel('X - value')
plt.ylabel('Y - value')
plt.show()
```




```
# CLASSIFY THE DATA

# Create the ground truth labels
ground_truth = []
for i in range(M):
    class_ground_truth = np.zeros(shape=(len(X_2D4cl[i][0])),
dtype=int)
    class_ground_truth = class_ground_truth + i
    ground_truth.append(class_ground_truth)

gx = np.full(shape=(M), fill_value=np.array)
Cx = np.full(shape=(M), fill_value=np.array)
gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl, X_2D4cl[k], verbose=False)
    gy[k], Cy[k] = classify(Y_2D4cl, Y_2D4cl[k], verbose=False)
```

```
accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl, X_2D4cl[k], verbose=False,
classifier="PZ", param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl, Y_2D4cl[k], verbose=False,
classifier="PZ", param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl, X_2D4cl[k], verbose=False,
classifier="PZ", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl, Y_2D4cl[k], verbose=False,
classifier="PZ", param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl, X_2D4cl[k], verbose=False,
classifier="knn", param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl, Y_2D4cl[k], verbose=False,
classifier="knn", param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl, X_2D4cl[k], verbose=False,
classifier="knn", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl, Y_2D4cl[k], verbose=False,
classifier="knn", param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
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RuntimeWarning: overflow encountered in divide
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```
# DATAFRAME WITH THE RESULTS

row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Parzen h1 = 5 testing',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
for i in range(M):
    df[f'P(correct|w{i+1})'] = [accuracy_recl[i], accuracy_test[i],
accuracy_recl_pz[i], accuracy_test_pz[i], accuracy_recl_pz_5[i],
accuracy_test_pz_5[i], accuracy_recl_knn[i], accuracy_test_knn[i],
accuracy_recl_knn_5[i], accuracy_test_knn_5[i]]

display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification      0.48           0.52

1                                ML testing      0.63           0.28

2          Parzen h1 = 0.1 reclassification      0.16           0.80

3                    Parzen h1 = 0.1 testing      0.71           0.32

4            Parzen h1 = 5 reclassification      0.46           0.52

5                      Parzen h1 = 5 testing      0.62           0.24
```

```
6  Nearest neighbor kn = 1 reclassification      0.75           0.00

7           Nearest neighbor kn = 1 testing      0.69           0.32

8  Nearest neighbor kn = 5 reclassification      0.53           0.44

9           Nearest neighbor kn = 5 testing      0.61           0.24

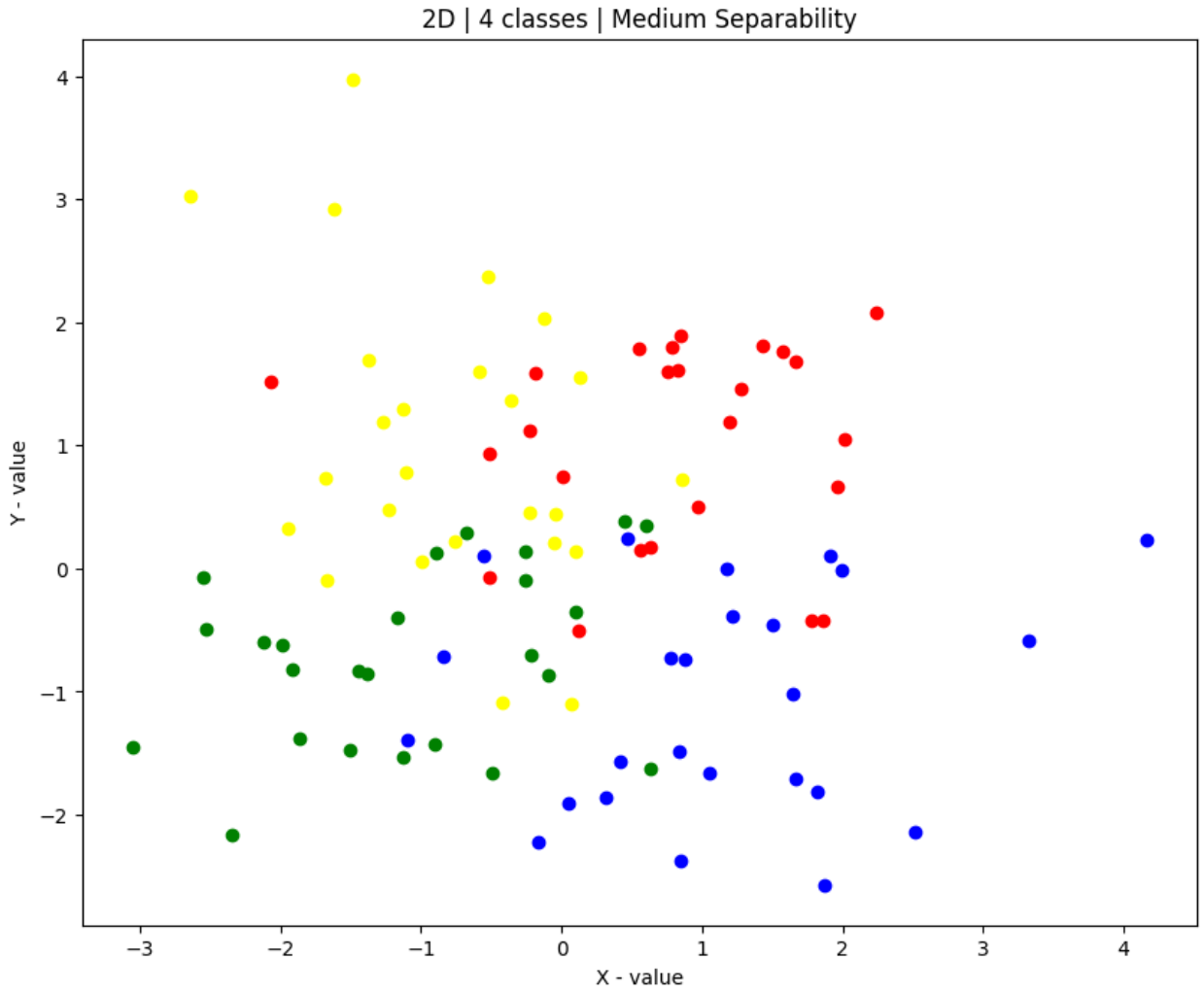

   P(correct|w2)  P(correct|w3)  P(correct|w4)
0           0.56           0.44           0.56
1           0.52           0.28           0.40
2           0.80           0.76           1.00
3           0.32           0.12           0.40
4           0.64           0.48           0.52
5           0.52           0.36           0.40
6           0.00           0.00           1.00
7           0.48           0.12           0.32
8           0.48           0.52           0.44
9           0.52           0.40           0.40
```

Section c)

```
M = len(X_2D4cl_ms)
colors = ['red', 'blue', 'green', 'yellow']

for i in range(M):
    plt.scatter(X_2D4cl_ms[i][0], X_2D4cl_ms[i][1], label=f'Class
{i+1}', color=colors[i])

plt.rcParams.update({'figure.figsize':(10,8), 'figure.dpi':100})
plt.title('2D | 4 classes | Medium Separability')
plt.xlabel('X - value')
plt.ylabel('Y - value')
plt.show()
```




```
# CLASSIFY THE DATA

# Create the ground truth labels
ground_truth = []
for i in range(M):
    class_ground_truth = np.zeros(shape=(len(X_2D4cl_ms[i][0])),
dtype=int)
    class_ground_truth = class_ground_truth + i
    ground_truth.append(class_ground_truth)

gx = np.full(shape=(M), fill_value=np.array)
Cx = np.full(shape=(M), fill_value=np.array)
gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_ms, X_2D4cl_ms[k], verbose=False)
    gy[k], Cy[k] = classify(Y_2D4cl_ms, Y_2D4cl_ms[k], verbose=False)
```

```
accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_ms, X_2D4cl_ms[k], verbose=False,
classifier="PZ", param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_ms, Y_2D4cl_ms[k], verbose=False,
classifier="PZ", param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_ms, X_2D4cl_ms[k], verbose=False,
classifier="PZ", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_ms, Y_2D4cl_ms[k], verbose=False,
classifier="PZ", param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_ms, X_2D4cl_ms[k], verbose=False,
classifier="knn", param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_ms, Y_2D4cl_ms[k], verbose=False,
classifier="knn", param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_ms, X_2D4cl_ms[k], verbose=False,
classifier="knn", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_ms, Y_2D4cl_ms[k], verbose=False,
classifier="knn", param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: invalid value encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: overflow encountered in divide
  g = g/p
```

```
# DATAFRAME WITH THE RESULTS

row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
for i in range(M):
    df[f'P(correct|w{i+1})'] = [accuracy_recl[i], accuracy_test[i],
accuracy_recl_pz[i], accuracy_test_pz[i], accuracy_recl_pz_5[i],
accuracy_test_pz_5[i], accuracy_recl_knn[i], accuracy_test_knn[i],
accuracy_recl_knn_5[i], accuracy_test_knn_5[i]]

display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification      0.29           0.72

1                                ML testing      0.29           0.64

2          Parzen h1 = 0.1 reclassification      0.40           0.68

3                    Parzen h1 = 0.1 testing      0.59           0.56

4            Parzen h1 = 5 reclassification      0.28           0.80

5                      Parzen h1 = 5 testing      0.28           0.76
```

```
6  Nearest neighbor kn = 1 reclassification      0.75           0.00

7           Nearest neighbor kn = 1 testing      0.42           0.60

8  Nearest neighbor kn = 5 reclassification      0.31           0.76

9           Nearest neighbor kn = 5 testing      0.30           0.72

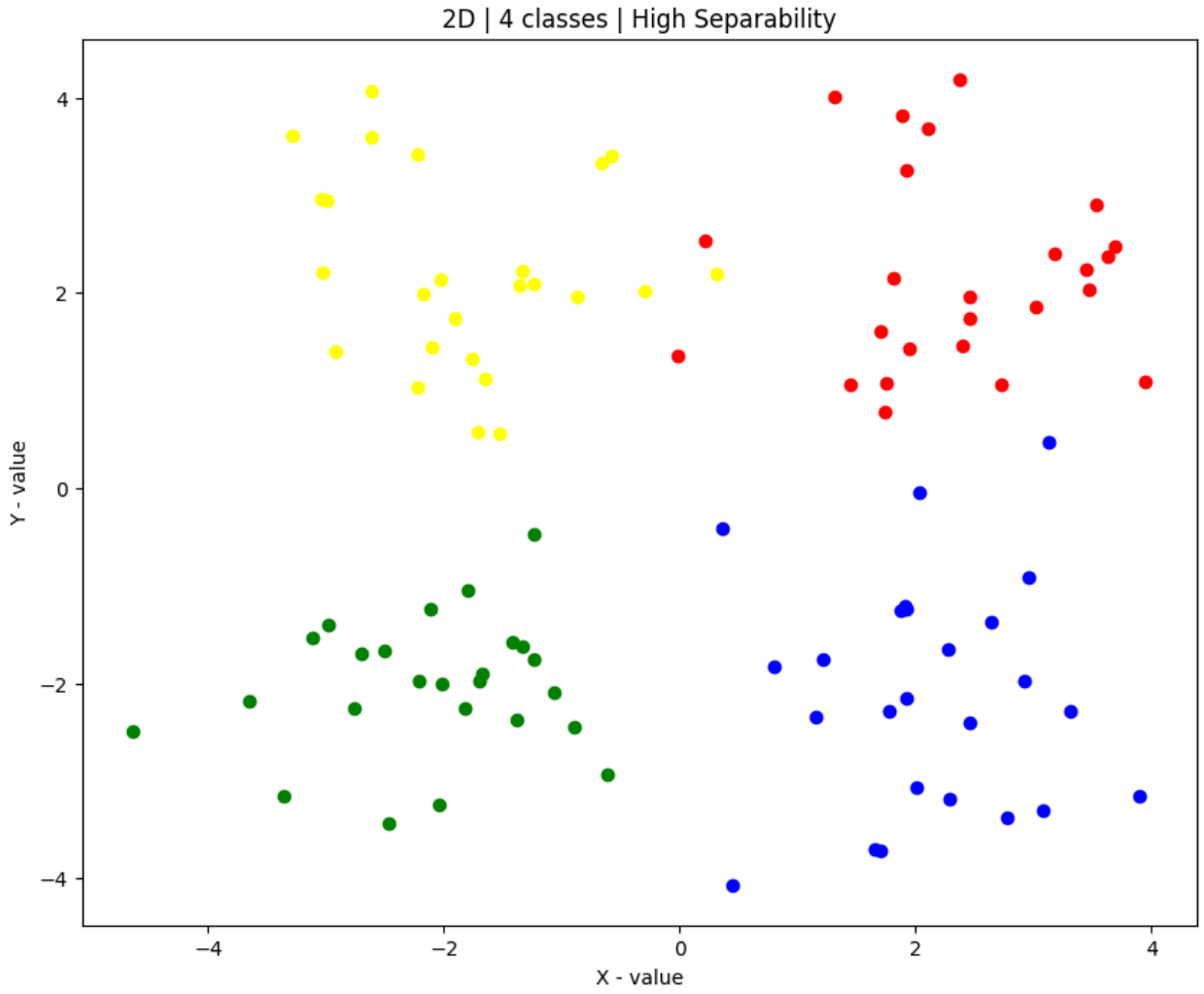

   P(correct|w2)  P(correct|w3)  P(correct|w4)
0           0.72           0.68           0.72
1           0.64           0.76           0.80
2           0.28           0.44           1.00
3           0.04           0.28           0.76
4           0.68           0.68           0.72
5           0.64           0.72           0.76
6           0.00           0.00           1.00
7           0.48           0.52           0.72
8           0.68           0.60           0.72
9           0.60           0.68           0.80
```

Section d)

```
M = len(X_2D4cl_hs)
colors = ['red', 'blue', 'green', 'yellow']

for i in range(M):
    plt.scatter(X_2D4cl_hs[i][0], X_2D4cl_hs[i][1], label=f'Class
{i+1}', color=colors[i])

plt.rcParams.update({'figure.figsize':(10,8), 'figure.dpi':100})
plt.title('2D | 4 classes | High Separability')
plt.xlabel('X - value')
plt.ylabel('Y - value')
plt.show()
```




```
# CLASSIFY THE DATA

# Create the ground truth labels
ground_truth = []
for i in range(M):
    class_ground_truth = np.zeros(shape=(len(X_2D4cl_hs[i][0])),
dtype=int)
    class_ground_truth = class_ground_truth + i
    ground_truth.append(class_ground_truth)

gx = np.full(shape=(M), fill_value=np.array)
Cx = np.full(shape=(M), fill_value=np.array)
gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_hs, X_2D4cl_hs[k], verbose=False)
    gy[k], Cy[k] = classify(Y_2D4cl_hs, Y_2D4cl_hs[k], verbose=False)
```

```
accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_hs, X_2D4cl_hs[k], verbose=False,
classifier="PZ", param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_hs, Y_2D4cl_hs[k], verbose=False,
classifier="PZ", param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_hs, X_2D4cl_hs[k], verbose=False,
classifier="PZ", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_hs, Y_2D4cl_hs[k], verbose=False,
classifier="PZ", param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_hs, X_2D4cl_hs[k], verbose=False,
classifier="knn", param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_hs, Y_2D4cl_hs[k], verbose=False,
classifier="knn", param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_2D4cl_hs, X_2D4cl_hs[k], verbose=False,
classifier="knn", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_2D4cl_hs, Y_2D4cl_hs[k], verbose=False,
classifier="knn", param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: invalid value encountered in divide
  g = g/p
```

```
# DATAFRAME WITH THE RESULTS

row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Parzen h1 = 5 testing',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
for i in range(M):
    df[f'P(correct|w{i+1})'] = [accuracy_recl[i], accuracy_test[i],
accuracy_recl_pz[i], accuracy_test_pz[i], accuracy_recl_pz_5[i],
accuracy_test_pz_5[i], accuracy_recl_knn[i], accuracy_test_knn[i],
accuracy_recl_knn_5[i], accuracy_test_knn_5[i]]

display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification      0.07           0.92

1                                ML testing      0.05           0.96

2          Parzen h1 = 0.1 reclassification      0.72           0.04

3                   Parzen h1 = 0.1 testing      0.76           0.00

4            Parzen h1 = 5 reclassification      0.05           0.96

5                     Parzen h1 = 5 testing      0.05           0.96

6  Nearest neighbor kn = 1 reclassification      0.75           0.00

7           Nearest neighbor kn = 1 testing      0.08           0.88
```

```
8  Nearest neighbor kn = 5 reclassification      0.08           0.88

9           Nearest neighbor kn = 5 testing      0.04           0.92


   P(correct|w2)  P(correct|w3)  P(correct|w4)
0           0.88           0.96           0.96
1           0.88           1.00           0.96
2           0.00           0.08           1.00
3           0.04           0.00           0.92
4           0.92           0.96           0.96
5           0.88           1.00           0.96
6           0.00           0.00           1.00
7           0.88           0.96           0.96
8           0.92           0.96           0.92
9           0.92           1.00           1.00
```

We can easily notice that the more splitted is the data, the easier is for the classifier to classify it.

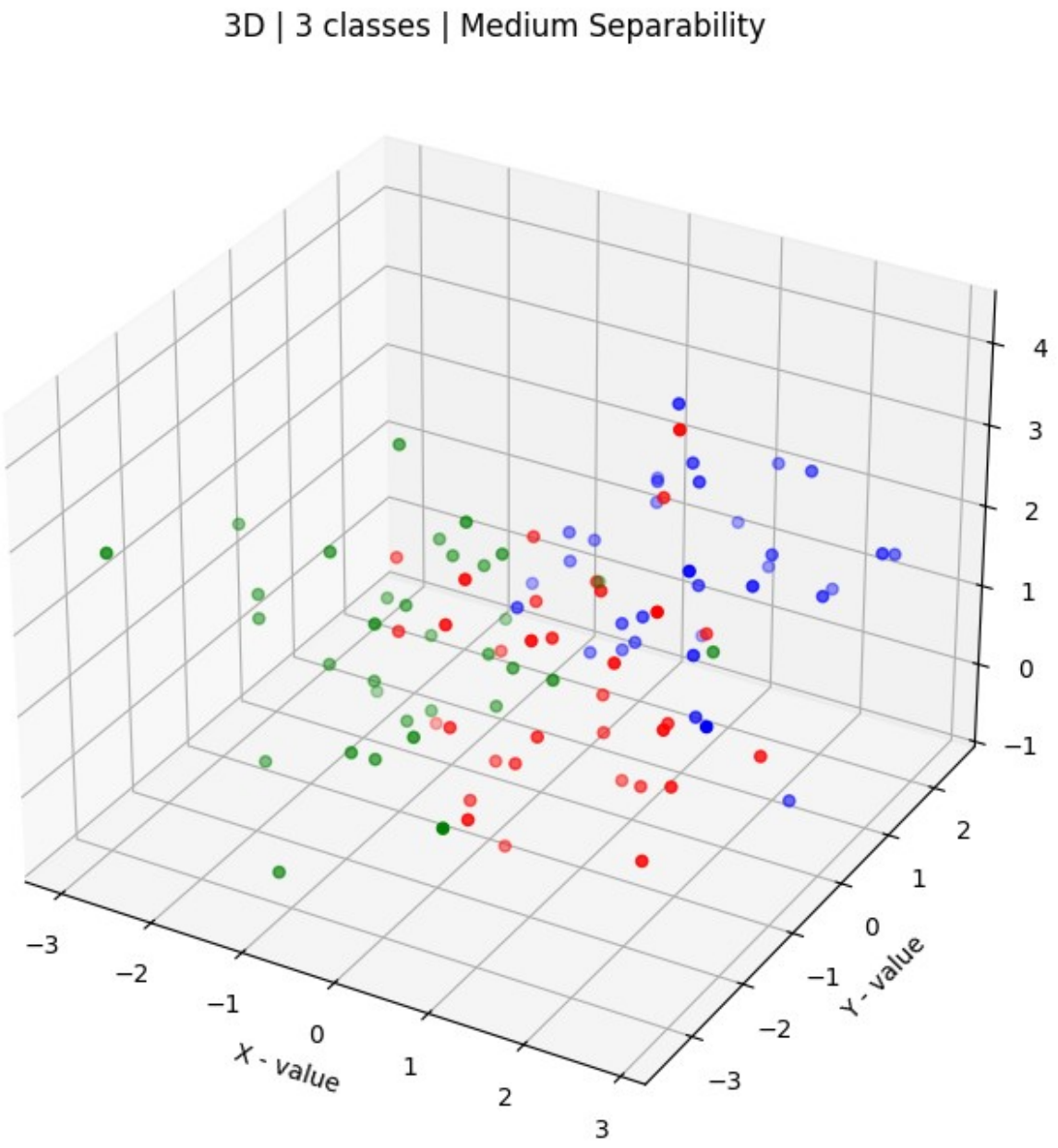
Section e)

```
M = len(X_3D3cl_ms)
colors = ['red', 'blue', 'green']

fig = plt.figure()
ax = fig.add_subplot(projection='3d')

for i in range(M):
    ax.scatter(X_3D3cl_ms[i][0], X_3D3cl_ms[i][1], X_3D3cl_ms[i][2],
label=f'Class {i+1}', color=colors[i])

plt.rcParams.update({'figure.figsize':(10,8), 'figure.dpi':100})
plt.title('3D | 3 classes | Medium Separability')
ax.set_xlabel('X - value')
ax.set_ylabel('Y - value')
ax.set_zlabel('Z - value')
plt.show()
```




```
# CLASSIFY THE DATA

# Create the ground truth labels
ground_truth = []
for i in range(M):
    class_ground_truth = np.zeros(shape=(len(X_3D3cl_ms[i][0])),
dtype=int)
    class_ground_truth = class_ground_truth + i
    ground_truth.append(class_ground_truth)

gx = np.full(shape=(M), fill_value=np.array)
```

```
Cx = np.full(shape=(M), fill_value=np.array)
gy = np.full(shape=(M), fill_value=np.array)
Cy = np.full(shape=(M), fill_value=np.array)

#ML
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_3D3cl_ms, X_3D3cl_ms[k], verbose=False)
    gy[k], Cy[k] = classify(Y_3D3cl_ms, Y_3D3cl_ms[k], verbose=False)

accuracy_test, error_test = computeMetrics(Cx, ground_truth)
accuracy_recl, error_recl = computeMetrics(Cy, ground_truth)

#Parzen = 0.1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_3D3cl_ms, X_3D3cl_ms[k], verbose=False,
classifier="PZ", param=0.1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_3D3cl_ms, Y_3D3cl_ms[k], verbose=False,
classifier="PZ", param=0.1)

accuracy_test_pz, error_test_pz = computeMetrics(Cx, ground_truth)
accuracy_recl_pz, error_recl_pz = computeMetrics(Cy, ground_truth)

#Parzen = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_3D3cl_ms, X_3D3cl_ms[k], verbose=False,
classifier="PZ", param=5)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_3D3cl_ms, Y_3D3cl_ms[k], verbose=False,
classifier="PZ", param=5)

accuracy_test_pz_5, error_test_pz_5 = computeMetrics(Cx, ground_truth)
accuracy_recl_pz_5, error_recl_pz_5 = computeMetrics(Cy, ground_truth)

#kNN = 1
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_3D3cl_ms, X_3D3cl_ms[k], verbose=False,
classifier="knn", param=1)
for k in range(0, M):
    gy[k], Cy[k] = classify(Y_3D3cl_ms, Y_3D3cl_ms[k], verbose=False,
classifier="knn", param=1)

accuracy_test_knn, error_test_knn = computeMetrics(Cx, ground_truth)
accuracy_recl_knn, error_recl_knn = computeMetrics(Cy, ground_truth)

#kNN = 5
for k in range(0, M):
    gx[k], Cx[k] = classify(Y_3D3cl_ms, X_3D3cl_ms[k], verbose=False,
classifier="knn", param=5)
for k in range(0, M):
```

```
    gy[k], Cy[k] = classify(Y_3D3cl_ms, Y_3D3cl_ms[k], verbose=False,
classifier="knn", param=5)

accuracy_test_knn_5, error_test_knn_5 = computeMetrics(Cx,
ground_truth)
accuracy_recl_knn_5, error_recl_knn_5 = computeMetrics(Cy,
ground_truth)
```

```
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: divide by zero encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: invalid value encountered in divide
  g = g/p
C:\Users\danie\AppData\Local\Temp\ipykernel_2912\3549508331.py:93:
RuntimeWarning: overflow encountered in divide
  g = g/p
```

```
# DATAFRAME WITH THE RESULTS

row_names = [
    'ML reclassification',
    'ML testing',
    'Parzen h1 = 0.1 reclassification',
    'Parzen h1 = 0.1 testing',
    'Parzen h1 = 5 reclassification',
    'Parzen h1 = 5 testing',
    'Nearest neighbor kn = 1 reclassification',
    'Nearest neighbor kn = 1 testing',
    'Nearest neighbor kn = 5 reclassification',
    'Nearest neighbor kn = 5 testing'
]

df = pd.DataFrame(row_names)
df['P(error)'] = [error_recl, error_test, error_recl_pz,
error_test_pz, error_recl_pz_5, error_test_pz_5, error_recl_knn,
error_test_knn, error_recl_knn_5, error_test_knn_5]
for i in range(M):
    df[f'P(correct|w{i+1})'] = [accuracy_recl[i], accuracy_test[i],
accuracy_recl_pz[i], accuracy_test_pz[i], accuracy_recl_pz_5[i],
accuracy_test_pz_5[i], accuracy_recl_knn[i], accuracy_test_knn[i],
accuracy_recl_knn_5[i], accuracy_test_knn_5[i]]

display(df)
```

```
                                          0  P(error)  P(correct|
w1)  \
0                       ML reclassification  0.212121       0.666667

1                                ML testing  0.171717       0.909091
```

```
2          Parzen h1 = 0.1 reclassification  0.545455       0.181818

3                   Parzen h1 = 0.1 testing  0.666667       0.363636

4            Parzen h1 = 5 reclassification  0.171717       0.757576

5                     Parzen h1 = 5 testing  0.202020       0.848485

6  Nearest neighbor kn = 1 reclassification  0.666667       0.000000

7           Nearest neighbor kn = 1 testing  0.333333       0.818182

8  Nearest neighbor kn = 5 reclassification  0.212121       0.666667

9           Nearest neighbor kn = 5 testing  0.212121       0.848485


   P(correct|w2)  P(correct|w3)
0       0.848485       0.848485
1       0.848485       0.727273
2       0.181818       1.000000
3       0.212121       0.424242
4       0.818182       0.909091
5       0.818182       0.727273
6       0.000000       1.000000
7       0.545455       0.636364
8       0.787879       0.909091
9       0.818182       0.696970
```

Authors

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