WILL Part I: Relational Geometry

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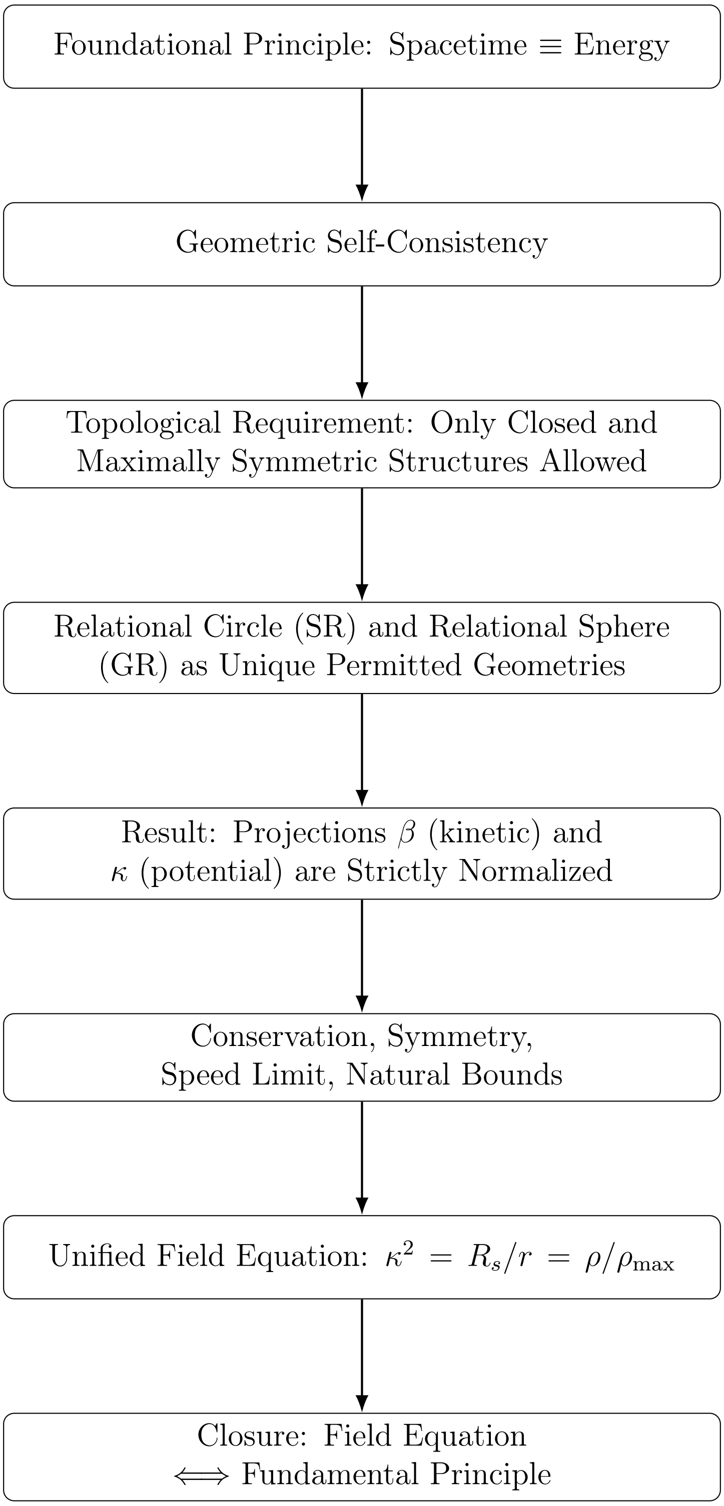
September 2025

*This work is archived on Zenodo: https://doi.org/10.5281/zenodo.17115271


Foundational Principle: Spacetime $\equiv$ Energy
Geometric Self-Consistency
Topological Requirement: Only Closed and Maximally Symmetric Structures Allowed
Relational Circle (SR) and Relational Sphere (GR) as Unique Permitted Geometries
Result: Projections β (kinetic) and κ (potential) are Strictly Normalized
Conservation, Symmetry, Speed Limit, Natural Bounds
Unified Field Equation: $\kappa^2 = R_s/r = \rho/\rho_{\max}$
Closure: Field Equation $\Longleftrightarrow$ Fundamental Principle

Abstract

This paper, the first in the WILL series, applies extreme methodological constraints in constructing a theory and establishes Relational Geometry (RG): a foundational framework where spacetime is not a background arena but an emergent property of energy transformations.

From a single principle, SPACETIME ≡ ENERGY, it aims to derive the complete geometric structure of physics across physical domains. This equivalence is not postulated but derived by removing the hidden ontological assumption, implicit in modern physics, that structure (spacetime) and dynamics (energy) are separate phenomena.

This shift establishes an ontological transition from descriptive to generative physics: instead of introducing laws to model observations, it derives them as necessary consequences of RG itself turning physics from a catalogue of phenomena into the logical unfolding of a single principle.

The result is a singularity-free, ontologically clean formalism that reproduces the core equations of Special Relativity (SR) and General Relativity (GR) as geometric projections on closed relational manifolds S^1 (directional) and S^2 (omnidirectional). Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energymomentum relation, Schwarzschild and Kerr solutions, and Einstein field equations via the dimensionless projections β (kinematic) and κ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$ (geometrically derived, virial-like). All results are empirically validated (e.g., GPS time shift 38.52 μs/day, Mercury precession to 10^{-10}%, and others listed in Appendix I).

WILL Part I offers solutions to several long-standing problems, including:

- Resolution of GR singularities (via naturally bounded $\rho_{\max}$),
- Derivation of the equality of gravitational and inertial masses (from the common channel of rest-invariant scaling),
- Removal of local energy ambiguity $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$,
- Revelation of a clear relational symmetry between kinematic and potential projections,
- Establishment of a computationally simpler and ontologically consistent foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

Contents

IMPORTANT:

This document must be read literally. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (absolute energies, external backgrounds, hidden containers) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

1 Foundational Approach

This Approach Does not Describe Physics; it Generates it. 19.1

Guiding Principle:

Nothing is assumed. Everything is derived.

1.1 Methodological Purity

This framework is constructed under a single epistemic constraint: to derive all of physics by removing one hidden assumption, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced unless they follow strictly from first principles, and no constructs are retained unless they are geometrically or energetically necessary.

Principle 1.1 (Methodological Minimalism). Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities. This principle is not a statement about the nature of reality, but a rule of logical hygiene for constructing a theory.

No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent relational projections.

1.2 Epistemic Hygiene

Modern physics often tolerates hidden assumptions: arbitrary constants, external backgrounds, or abstract entities with ambiguous physical status. Here we enforce epistemic hygiene: a refusal to import unjustified assumptions.

> Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.

Principle 1.2 (Relational Origin). All physical quantities must be defined by their relations. Any introduction of absolute properties or external frames risks reintroducing metaphysical artifacts and contradicts the foundational insight of relativity.

1.3 Mathematical Transparency

1. Each mathematical object must correspond to explicitly identifiable relation between observers with transparent ontological origin.
2. Every symbol must be anchored to unique physical idea.
3. Introducing symbols without explicit necessity constitutes semantic inflation: the proliferation of symbols without corresponding physical meaning.
4. Number of symbols = Number of independent physical ideas.

IMathematical hygiene

$$\boxed{\text{Mathematical hygiene is the geometry of reason}}$$

2 Ontological Blind Spot In Modern Physics

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

2.1 Historical Pattern: breakthroughs delete, not add

- Copernicus eliminated the Earth/cosmos separation.
- Newton eliminated the terrestrial/celestial law separation.
- Einstein eliminated the space/time separation.
- Maxwell eliminated the electricity/magnetism separation.

Each step widened the relational circle and reduced the number of unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

2.2 The contemporary split: an unpaid ontological bill

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a bivariable syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate inside the split. The split is therefore not an empirical discovery but an unpaid ontological debt.

2.3 Empirical bankruptcy of the separation

- Local energy conservation verified only after the metric is declared fixed; no experiment varies the volume of flat space and checks calorimetry.
- Universality of free fall tests $m_i = m_g$ numerically, not the claim that inertia resides in the object rather than in a geometric scaling relation.
- Gravitational-wave polarisations test spin content, not ontology; extra modes can still be called "matter on spacetime".
- Casimir/Lamb shift measure differences of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

In short, every "test" is an internal consistency check of a formalism that already presupposes two substances. None constitute positive evidence for the split.

2.4 Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an un-evidenced metaphysical postulate—the last geocentric epicycle in physics.

Summary

Any attempt to treat "spacetime structure" as separate from "dynamics" smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

3 Unifying Principle Removing the Hidden Assumption

3.1 False Separation

Lemma 3.1 (False Separation). Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of interprocess data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. □

Corollary 3.2 (Structure–Dynamics Coincidence). To avoid the artifact of Lemma 3.1, the structural arena and the dynamical content must be identified: geometry is energy, and energy is geometry.

Principle 3.3 (Working Principle: Removing the Hidden Assumption).

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight: it removes the hidden unjustified separation between geometry and dynamics. Space and time are not containers but emergent descriptors of relational energy.

Remark 3.4 (Auditability). Principle 3.3 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call - WILL.

3.2 What is Energy in Relational Framework?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure. Crucially, this quantity is never observed directly, but only through differences between states: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation. Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself. From these empirical and relational facts the definition follows unavoidably:

Energy :

Energy is the relational measure of difference between possible states, conserved in any closed whole.
It is not an intrinsic property of an object, but comparative structure between states (and observers), always manifesting as transformation.

4 Deriving the WILL Structure

Having established our Principle 3.3 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences. We will demonstrate that this single principle is sufficient to enforce the closure, conservation, and isotropy of the relational structure, leading to a unique set of geometric carriers for energy.

Definition 4.1 (WILL). WILL $\equiv$ SPACE-TIME-ENERGY is the technical term we use for unified relational structure determined by 3.3 . All physically meaningful quantities are relational features of WILL; no external container is permitted.

Lemma 4.2 (Closure). Under 3.3, WILL is self-contained: there is no external reservoir into or from which the relational resource can flow.

Proof. If WILL were not self-contained, there would exist an external structure mediating exchange. That external structure would then serve as a background distinct from the dynamics, contradicting Corollary 3.2. □

Lemma 4.3 (Conservation). Within WILL, the total relational "transformation resource" (energy) is conserved.

Proof. By Lemma 4.2, no external fluxes exist. Any change in one part of WILL must be balanced by complementary change elsewhere. Hence a conserved global quantity is enforced at the relational level. □

Lemma 4.4 (Isotropy from Background–Free Relationality). If no external background is allowed (Cor. 3.2), then no direction can be a priori privileged. Thus the admissible relational geometry of WILL must be maximally symmetric (isotropic and homogeneous) at the level at which it encodes the conserved resource.

Proof. A privileged direction requires an extrinsic reference to distinguish it. In a purely relational setting, distinctions must be constructible from relations internal to WILL. If a direction were privileged in the geometry that encodes the conserved resource, such privilege would not be derivable from purely internal comparisons (which are symmetric by construction), and would reintroduce an external orienting structure. Therefore the encoding geometry must be maximally symmetric. □

4.1 Classification of Minimal Relational Transformations

The lemmas of Closure, Conservation, and Isotropy establish the necessary properties of any geometric carrier of the relational resource. To identify these carriers, we must first classify the simplest possible types of physical relations that can exist in a background-free framework, according to the Principle of Methodological Minimalism (1.1).

(a) Directional (Kinematic) Relation: The simplest possible non-trivial relation is between two distinct states (A and B). The minimal description of this directed relation requires only a single degree of freedom (the axis connecting A and B). Per Principle 1.1, introducing additional dimensions would be an unjustified complication. For this description to be self-contained (Lemma 4.2), the 1D geometry must be closed. This uniquely specifies the circle (S^1) as the necessary and sufficient carrier for minimal directional relations.

(b) Omnidirectional (Gravitational) Relation: The simplest possible isotropic relation is between a central state (A) and the locus of all states equidistant to it. The minimal description for all directions of this relation requires a surface representing all possible orientations from the center. Such a surface has two degrees of freedom (corresponding to the two angles needed to specify a direction). For this description to be closed and maximally symmetric (Lemmas 4.2, 4.4), the geometry must be that of a 2-sphere (S^2). It is the necessary and sufficient carrier for minimal omnidirectional relations.

The Principle of Methodological Minimalism thus restricts the admissible relational geometries to these two minimal classes. The following theorem provides their formal identification.

Theorem 4.5 (Minimal Relational Carriers of the Conserved Resource). The only closed, maximally symmetric manifolds that can serve as minimal carriers of the conserved relational resource are:

(a) S^1 for directional (one-degree-of-freedom) relational transformation;

(b) S^2 for omnidirectional (central, all-directions-equivalent) relational transformation.

Proof. By Lemma 4.4, we require closed, maximally symmetric manifolds.

(a) In one relational degree of freedom, the classification of connected closed 1-manifolds yields S^1 as the unique (up to diffeomorphism) option. Its isometry group acts transitively with isotropy at each point, providing maximal symmetry.

(b) For omnidirectional relational transformation from a distinguished center, the encoding manifold must be a closed, simply connected, constant positive curvature 2-manifold with full isotropy at every point. By the uniformization/classification of constant-curvature surfaces, the maximally symmetric representative is S^2. Quotients of S^2 by nontrivial finite groups introduce global identifications that spoil global isotropy; these are excluded by Lemma 4.4. Hence S^2 is uniquely selected. □

Corollary 4.6 (Uniqueness). Under 3.3 with Closure, Conservation, and Isotropy (Lemmas 4.2–4.4), S^1 and S^2 are necessary relational carriers for, respectively, directional and omnidirectional modes of energy transformation.

Remark 4.7 (Non-spatial Reading). Throughout, S^1 and S^2 are not to be interpreted as spacetime geometries. They are relational manifolds that encode the closure, conservation, and isotropy of the transformational resource. Ordinary spatial and temporal notions are emergent descriptors of patterns within WILL.

Summary:

From removing the hidden assumption 3.1 we inevitably arrive to 3.3 SPACETIME ≡ ENERGY from there we deduced: (i) closure, (ii) conservation, (iii) isotropy, and hence (iv) the unique selection of S^1 and S^2 as minimal relational carriers for directional and omnidirectional transformation. These objects are non-spatial encodings of conservation and symmetry; they are enforced by the3.3 rather than assumed independently.

4.2 Ontological Status of the Relational Manifolds S^1 and S^2

A natural question arises regarding the ontological status of the circle S^1 and the sphere S^2: What are they, and where do they "exist"?

The answer requires a fundamental shift in perspective. In WILL Relational Geometry, S^1 and S^2 are not spatial entities existing within a pre-defined container. They are the necessary relational architectures that implement the core identity SPACETIME ≡ ENERGY.

Energy as Relational Bookkeeping Recall that energy is defined as the relational measure of difference between possible states. It is not an intrinsic property but a comparative structure that guarantees causal continuity. It is never observed directly, only through transformations.

The Manifolds as Protocols of Interaction The manifolds S^1 and S^2 are the minimal, unique mathematical structures capable of hosting this relational "bookkeeping" for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, A and B:

- Observer A is the center of their own relational framework. Observer B is a point on A's S^1 (for kinematic relations) and S^2 (for gravitational relations).
- Simultaneously, observer B is the center of their own framework. Observer A is a point on B's S^1 and S^2.

There is no privileged "master" manifold. Each observable interaction is structured by these mutually-centered relational protocols. The parameters β and κ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta_X^2 + \beta_Y^2 = 1; \quad \kappa_X^2 + \kappa_Y^2 = 1$) are the innate accounting rules of these protocols.

Emergence of Spacetime Therefore, the question "Where are S^1 and S^2?" is a category error. They are not in space; they are the structures whose coordinated, multi-centered interactions give rise to the phenomenon we perceive as spacetime. Spacetime is the emergent, collective shadow cast by the dynamics of energy relations projected onto these architectures.

In essence, S^1 and S^2 are the ontological embodiment of the relational principle. They are derived as the only possible structures that can house the transformational resource (energy) in a closed, conserved, and isotropic system. Their status is that of a fundamental relational geometry from which physics is generated.

5 Emergence of Spacetime

In this construction, "space," "time," are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

5.1 The Duality of Transformation

Lemma 5.1 (Duality of Evolution). The identification of spacetime with energy and its transformations necessitates two complementary relational measures:

1. the extent of transformation (external displacement), and
2. the sequence of transformation (internal order).

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The circle S^1 provides the minimal geometry enforcing such complementarity: its orthogonal projections furnish precisely two non-redundant coordinates. □

We define this orthogonal projections as follows:

- The Amplitude Component (β_X): This projection represents the relational measure between the system and the observer. It corresponds to the extent of transformation, which manifests physically as momentum (as shown in next section).
- The Phase Component (β_Y): This projection represents the internal structure of a system. It governs the intrinsic scale of its proper space and proper time units, corresponding to the sequence of its transformation. A value of $\beta_Y = 1$ represents a complete and undisturbed manifestation of this internal structure, a state we identify as rest.

5.2 Conservation Law of Relational Transformation

Theorem 5.2 (Conservation Law of Relational transformation). The orthogonal components of transformation (β_X, β_Y) are bound by the closure relation

$$\beta_X^2 + \beta_Y^2 = 1.$$

Proof. Since S^1 is closed, every point on the circle is constrained by the Pythagorean identity of its projections. Thus no state can exceed or fall short of the finite relational "budget." This closure enforces conservation across all processes. □

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

5.3 Consequence: Relativistic Effects

Proposition 5.3 (Physical Interpretation: Relativistic Effects). The conservation law of Theorem 5.2 implies that any redistribution between the orthogonal components (β_X, β_Y) manifests physically as the relativistic effects of time dilation and length contraction.

Proof. By Theorem 5.2, the components satisfy $\beta_X^2 + \beta_Y^2 = 1$. An increase in the relational displacement β_X enforces a decrease in the internal measure β_Y. This reduction of β_Y corresponds to dilation of proper time and contraction of proper length, while the growth of β_X represents momentum. Thus the relativistic trade-off is the direct physical expression of the geometric closure of S^1. □

Summary:

Geometry of spacetime is the shadow cast by the geometry of relations.

6 Kinetic Energy Projection on S^1

Since S^1 encodes one-dimensional displacement, the total energy E of the system must project consistently onto both axes:

$$E_X = E\beta_X, \qquad E_Y = E\beta_Y.$$

Theorem 6.1 (Invariant Projection of Rest Energy). For any state (β_X, β_Y) on the relational circle, the vertical projection of the total energy is invariant:

$$E\beta_Y = E_0.$$

Proof. When $\beta_X = 0$, closure enforces $\beta_Y = 1$, yielding $E = E_0$. Since closure applies for all θ_1, the vertical projection $E\beta_Y$ remains equal to this rest value in every state. □

Corollary 6.2 (Total Energy Relation). From Theorem 6.1 it follows that

$$E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sqrt{1-\beta_X^2}}.$$

Remark 6.3 (Lorentz Factor). The historical Lorentz factor γ is nothing more than the reciprocal of β_Y. No additional structure is introduced: all content is already present in $E\beta_Y = E_0$.

Summary:

The historical Lorentz factor γ is the reciprocal of β_Y. $\gamma = 1/\beta_Y$

6.1 Rest Energy and Mass Equivalence

Corollary 6.4 (Rest Energy and Mass Equivalence). Within the normalization $c = 1$, the invariant rest energy equals mass:

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1, no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. □

Summary:

Mass is the invariant projection of total rest energy.

6.2 Energy–Momentum Relation

Proposition 6.5 (Horizontal Projection as Momentum). On the relational circle, the unique relational measure of displacement from rest is the horizontal projection $E\beta_X$; hence

$$p \equiv E\beta_X \quad (c = 1).$$

Proof. The rest state is $(\beta_X, \beta_Y) = (0, 1)$. A displacement measure must (i) vanish at rest, (ii) grow monotonically with $|\beta_X|$, and (iii) flip sign under $\beta_X \mapsto -\beta_X$. The only relational candidate satisfying (i)(iii) is the horizontal projection $E\beta_X$. Thus the identification is necessary rather than conventional. □

Corollary 6.6 (Energy–Momentum Relation). With p identified by Proposition 6.5 and $m = E_0$, the closure identity yields

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta_X)^2+(E\beta_Y)^2 = E^2$. Substituting $p = E\beta_X$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E, yielding the standard form. □

Remark 6.7 (Geometric Forms). The same identity may be expressed explicitly in terms of circle coordinates:

$$E^2 = \left(\frac{\beta_X}{\beta_Y}E_0\right)^2 + E_0^2 = \left(\cot(\theta_1)\,E_0\right)^2 + E_0^2.$$

These are equivalent renderings of the same geometric necessity.

Remark 6.8 (Units sanity check bookkeeping). Using $\beta_X = v/c$, the identification $p \equiv E\beta_X$ gives

$$pc = E\,\frac{v}{c} \quad\Longrightarrow\quad p = \frac{E\,v}{c^2}.$$

With $E = \frac{1}{\beta_Y}mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta_X}{\beta_Y}mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

$\beta_X = \beta, \quad \beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	Trigonometric Form
$1/\beta_Y = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(v/c)^2}$	$1/\beta_Y = 1/\sin(\theta_1) = 1/\sin(\arccos(\beta))$
$\beta_Y = \sqrt{1-\beta^2} = \sqrt{1-(v/c)^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$

Table 1: Geometric representation of relativistic effects.

Summary

The energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$ is geometric identity of S^1.

7 Potential Energy Projection on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational manifolds encoding conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1 the relational geometry of the sphere, S^2, provides orthogonal projections, for two aspects of omnidirectional transformation. We define them as follows:

- The Amplitude Component (κ_Y): This projection represents the relational gravitational measure between the object and the observer. It corresponds to the extent of transformation, which manifests physically as gravitation potential. A value of $\kappa_Y = 1$ corresponds precisely to the point where escape velocity equals the speed of light, creating an event horizon. This provides a natural causal limit for our gravitational parameter, analogous to the relative motion it determinant by the same radial normalization.

- The Phase Component (κ_X): This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the sequence of its transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of transformation":

$$\kappa_X^2 + \kappa_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

7.1 Gravitational Meridional Section of S^2

By isotropy the omnidirectional carrier is S^2, but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of S^2 with the parametrization $(\kappa_X, \kappa) = (\cos\theta_2, \sin\theta_2)$.

7.2 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure (κ_Y, gravitation potential) necessarily requires a decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is the shadow cast by the geometry of relations.

Notation simplicity:

From here on we will write $\beta = \beta_X, \quad \beta_Y = \sqrt{1-\beta^2}, \quad \kappa = \kappa_Y, \qquad \kappa_X = \sqrt{1-\kappa^2}$ for notation simplicity.

7.3 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the escape velocity at radius r.

In the kinematic case, with $\beta = \cos\theta_1$, the energy relation can be written as

$$E^2 = \big(\cot\theta_1\, E_0\big)^2 + E_0^2, \tag{1}$$

so that the relativistic momentum is expressed as

$$p = E_0/c\, \cot\theta_1. \tag{2}$$

In full symmetry, the gravitational case follows from $\kappa = \sin\theta_2$. We define the gravitational energy as

$$E_g = \frac{E_0}{\kappa_X}, \qquad \kappa_X = \sqrt{1-\kappa^2}, \tag{3}$$

and introduce the gravitational analogue of momentum:

$$p_g \;=\; E_0/c\,\tan\theta_2. \tag{4}$$

This yields the gravitational energy relation

$$E_g^2 = (p_g c)^2 + (mc^2)^2. \tag{5}$$

Summary:

$$\beta = \cos\theta_1, \qquad \kappa = \sin\theta_2,$$

$$\beta \;\longleftrightarrow\; \kappa, \qquad \cot\theta_1 \;\longleftrightarrow\; \tan\theta_2.$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the same relational circle, expressed through complementary trigonometric forms.

7.4 Geometric composition of SR and GR factors

On the unit kinematic circle (S^1) we parametrize

$$(\beta, \beta_Y) = (\cos\theta_1,\ \sin\theta_1),$$

so that the invariant vertical projection reads

$$E\,\beta_Y = E_0 \quad\Rightarrow\quad \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin\theta_1}}, \qquad p = \frac{E}{c}\,\beta = \frac{E_0\,\beta}{\beta_Y} = E_0\cot\theta_1,$$

and therefore $E^2 = (pc)^2 + E_0^2$.

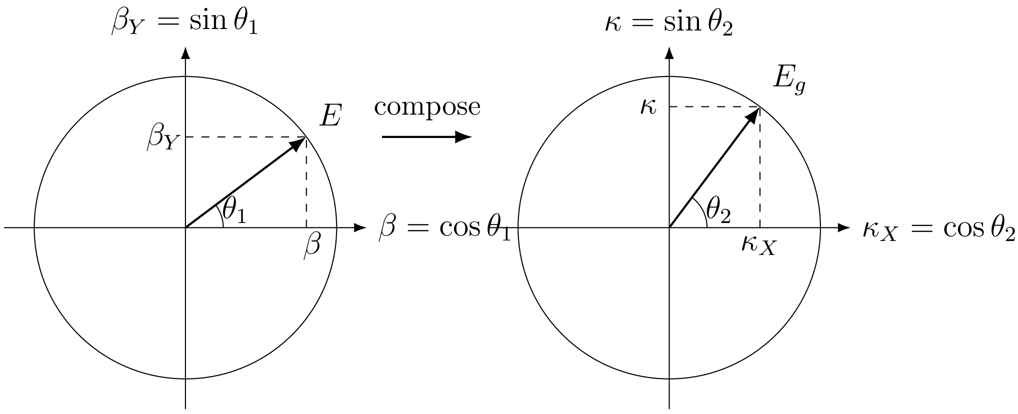
On the gravitational circle (S^2) we parametrize

$$(\kappa_X, \kappa) = (\cos\theta_2,\ \sin\theta_2),$$

so that the invariant horizontal projection reads

$$E_g\,\kappa_X = E_0 \quad\Rightarrow\quad \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos\theta_2}}, \qquad p_g = E_g\,\kappa = \frac{E_0\,\kappa}{\kappa_X} = E_0\tan\theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



$E = E_0/\beta_Y, \quad p = E_0\,\beta/\beta_Y = E_0\cot\theta_1$ $E_g = E_0/\kappa_X, \quad p_g = E_0\,\kappa/\kappa_X = E_0\tan\theta_2$

7.5 Clear Relational Symmetry Between Kinematic and Potential Projections

Now we can clearly see the underling symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

$\theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa),\ \kappa^2 = 2\beta^2$	
Algebraic Form	Trigonometric Form
$\beta = v/c$	$\beta = cos(\theta_1)$
$\kappa = \sqrt{R_s/r}$	$\kappa = sin(\theta_2)$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$
$p = E_0/c \cdot \beta/\beta_Y$	$p = E_0/c \cdot \cot(\theta_1)$
$p_g = E_0/c \cdot \kappa/\kappa_X$	$p_g = E_0/c \cdot \tan(\theta_2)$
$\tau = \beta_Y \kappa_X$	$\tau = \sin(\theta_1)\cos(\theta_2)$
$Q = \sqrt{\kappa^2 + \beta^2} = \sqrt{3}\beta$	$Q = \sqrt{3}\cos(\theta_1)$
$Q_t = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2}$	$Q_t = \sqrt{1-3\cos^2(\theta_1)}$

Table 2: Unified representation of relativistic and gravitational effects for closed systems.

Summary

The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic (β) and gravitational (κ) modes are not separate effects but dual aspects of one energy-transformation constraint revealing their unified origin.

8 Equivalence Principle as Derived Identity

Lemma 8.1 (Unified Relational Scaling). Within the relational framework of WILL, both kinematic (S^1) and gravitational (S^2) transformations act as independent projections of the same invariant energy E_0. Each projection rescales the observable quantities by its respective geometric factor:

$$E = \frac{E_0}{\beta_Y}, \qquad E_g = \frac{E_0}{\kappa_X}.$$

Proof. On the kinematic circle S^1, the invariant vertical projection corresponds to $\beta_Y = \sin\theta_1$. Preserving the same invariant leg E_0 forces the stretch $E/E_0 = 1/\beta_Y$. On the gravitational sphere S^2, the invariant horizontal projection is $\kappa_X = \cos\theta_2$, forcing $E_g/E_0 = 1/\kappa_X$. These transformations are independent and commute, each preserving the closure identity of its respective manifold. □

Theorem 8.2 (Equivalence of Inertial and Gravitational Response). Composing the independent relational stretches of Lemma 8.1 yields the total local energy scale

$$E_{\mathrm{loc}} = \frac{E_0}{\tau} = \frac{E_0}{\beta_Y\,\kappa_X} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The corresponding inertial and gravitational projections share a single operational factor,

$$\tilde{p} = \frac{E_{\mathrm{loc}}}{c}\beta, \qquad \tilde{p}_g = \frac{E_{\mathrm{loc}}}{c}\kappa,$$

both governed by the same effective mass

$$m_{\mathrm{eff}} = \frac{E_0}{\beta_Y\, \kappa_X\, c^2} = \frac{E_0}{\tau c^2}.$$

Therefore,

$$\boxed{m_g \equiv m_i \equiv m_{\mathrm{eff}}},$$

and the Einstein equivalence principle follows as a necessary structural identity of WILL.

Corollary 8.3 (Mass Invariance under Relational Scaling). The invariant core E_0 denotes the complete internal equilibrium state ($\beta_Y = \kappa_X = 1$). Relational factors β_Y and κ_X rescale only external manifestations (energy, momentum, and rates), while E_0 remains unchanged. Hence,

$$\boxed{m_g \equiv m_i \equiv m = E_0/c^2},$$

is not a dynamical statement but the definition of rest invariance itself.

Remark 8.4 (Composition-Independence). Decomposing the invariant rest energy into internal channels,

$$E_0 = \sum_a E_0^{(a)},$$

each term couples identically through the same geometric stretch:

$$E_{\mathrm{loc}} = \sum_a \frac{E_0^{(a)}}{\tau}.$$

Since all channels scale by the same factor $1/\tau = 1/(\beta_Y \kappa_X)$, ratios between channels cancel in all observables. Therefore, composition-independence of motion (Eцtvцs universality) follows identically, without requiring a postulate $m_g = m_i$.

Remark 8.5 (Quantum Interface). The relational phase increment inherits the same scaling:

$$\Delta\phi \propto E_{\mathrm{loc}}\, \Delta\lambda,$$

where $\Delta\lambda$ is the internal ordering parameter. Thus both kinematic and gravitational phase shifts share the same stretch $1/\tau = 1/(\beta_Y \kappa_X)$, yielding composition-independent matter-wave interference patterns.

Summary:

In WILL, the equivalence of inertial and gravitational mass is not assumed but follows necessarily from the compositional closure of relational geometry. What General Relativity posits as a postulate, WILL reveals as a corollary.

9 Unification of Projections: The Geometric Exchange Rate

Having established that directional (kinematic) and omnidirectional (gravitational) relations are carried by the unique manifolds S^1 and S^2 respectively, we now derive the relationship that unifies them.

9.1 Derivation of the Energetic Closure Condition

The Principle of a unified relational resource (Energy) requires a self-consistent "exchange rate" between its different modes of expression. This rate is not an arbitrary parameter but is dictated by the intrinsic geometry of the relations themselves:

Lemma 9.1 (Dimensional Ratio of Relational Modes). Directional (kinematic) and omnidirectional (gravitational) modes of WILL are carried by S^1 and S^2, requiring one and two degrees of freedom respectively. The intrinsic ratio of their relational dimensionalities is therefore

$$\frac{\deg(S^2)}{\deg(S^1)} = \frac{2}{1} = 2,$$

formally realized as the ratio of their total solid angles:

$$\frac{\Omega(S^2)}{\Omega(S^1)} = \frac{4\pi}{2\pi} = 2.$$

Theorem 9.2 (Energetic Closure Condition). Since energy is quadratic in its relational amplitudes, the unique quadratic balance between directional (S^1) and omnidirectional (S^2) resource distributions compatible with closure is

$$\boxed{\kappa^2 = 2\beta^2.}$$

Proof. The relational resource of WILL is conserved under redistribution between its kinematic and gravitational modes. Given the dimensional ratio of Lemma 9.1, an omnidirectional transformation consumes twice the relational degrees of freedom of a directional one. The energetic significance of a state is proportional not to the amplitude of a projection (β, κ), but to its square (β^2, κ^2). This principle is ubiquitous, reflected in the Pythagorean (sum-of-squares) structure of the relational manifolds themselves. There for the corresponding energetic relation is quadratic: $\kappa^2 = 2\beta^2$. □

Definition 9.3 (Closure Defect). $\delta \equiv \kappa^2 - 2\beta^2$. A subsystem is energetically closed iff $\langle\delta\rangle_{\text{cycle}} = 0$. For circular orbits, $\delta \equiv 0$.

Corollary 9.4 (Energetic Closure Criterion). Closed systems (momentary or periodic) satisfy $\kappa^2 = 2\beta^2$ identically. Open systems display $\delta \neq 0$, the magnitude of which quantifies the energy flow through unaccounted channels. When all channels are included, closure is restored.

Diagnostic Invariant:

The condition $\kappa^2 = 2\beta^2$ defines energetic closure. In closed systems (circular or periodic), it holds exactly; in open systems, its deviation δ measures interaction with external channels.

Remark 9.5 (Physical Interpretation). The exchange rate between the kinematic and gravitational projections corresponds to the ratio of their relational dimensions. This purely geometric constant (2) replaces the empirical proportionalities of classical dynamics. It is the relational analogue of the virial theorem: the kinetic and potential aspects of WILL maintain closure through the invariant ratio

$$\boxed{\kappa^2 = 2\beta^2.}$$

Illustrative Examples.

- Circular Orbit (Closed). A body in circular orbit exactly satisfies $\kappa^2 = 2\beta^2$. The entire conserved resource is partitioned between kinetic and gravitational projections; no external channel exists.

- Radiating Binary (Open). An elliptical compact binary violates $\kappa^2 = 2\beta^2$ when only orbital degrees of freedom are counted, the closure defect δ quantifying energy lost to gravitational radiation. Including all channels restores closure.

Summary:

1. WILL defines the universe as a closed relational structure, SPACETIME $\equiv$ ENERGY.

2. The simplest maximally symmetric carriers of these relations are S^1 and S^2.

3. The parameters $\beta = \cos\theta_1$ and $\kappa = \sin\theta_2$ are thus constrained to these manifolds.

4. The geometric exchange rate between these modes equals the ratio of their relational dimensionalities: 2.

Remark 9.6 (Geometric Origin of Physical Law). The relation between kinetic and potential energy is not an empirical coincidence but a geometric necessity of relational closure. Classical mechanics merely approximates this deeper invariant. Explicitly,

$$\boxed{\text{Geometric Distribution } (\kappa^2) \;\equiv\; 2 \times \text{Kinetic Distribution } (\beta^2).}$$

10 EnergySymmetry Law

In RG, every transformation is bidirectional: each change observed by A corresponds to an equal and opposite change observed by B. This reciprocity is the algebraic form of causal continuity, and its geometric expression is the EnergySymmetry Law.

10.1 Causal Continuity and Energy Symmetry

Theorem 10.1 (Energy Symmetry). The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the EnergySymmetry Law:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{6}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A = 0$).

- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A\to B} = \underbrace{\frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}\left(\beta_B^2 - \beta_A^2\right)}_{\text{Change in Kinetic}}} \tag{7}$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right) \tag{8}$$

From B's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)\right) = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right) \tag{9}$$

Summing these transfers gives:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{10}$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the EnergySymmetry Law as a direct consequence of the closed geometry. □

10.2 The Specific Energy Transfer (ΔE):

This is the physical quantity representing the actual work done and change in motion, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the sum of the changes in the potential and kinetic energy budgets:

$$\Delta E_{A\to B} = \Delta U_{A\to B} + \Delta K_{A\to B} = \frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right) + \frac{1}{2}\left(\beta_B^2 - \beta_A^2\right) \tag{11}$$

It is this quantity, ΔE, that is conserved and must balance to zero in any closed cycle.

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \tag{12}$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \tag{13}$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

10.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics but from the fundamental quadratic nature of the energy budgets in RG.

The energetic significance of a state is proportional to the square of its geometric projection. This is analogous to how kinetic energy is proportional to velocity squared (v^2) or how the energy in a wave is proportional to its amplitude squared (A^2). The individual energy budgets are defined as:

- Specific Potential Energy Budget: $U/E_0 \propto -\frac{1}{2}\kappa^2$
- Specific Kinetic Energy Budget: $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the changes in these individual budgets.

10.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 10.2 (Universal Speed Limit). The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A\to B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B\to A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{14}$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. □

10.5 SingleAxis Energy Transfer and the Nature of Light

Theorem 10.3 (SingleAxis Transformation Principle). For light, the kinematic projection reaches its full extent:

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

This means that all transformation of the relational energy occurs along a single orthogonal axis. The complementary branch of the bidirectional energy exchange is absent, and the total resource of transformation is entirely expressed on one geometric component.

Proof. For massive systems, the EnergySymmetry Law distributes the total energy exchange evenly between two orthogonal projections:

$$U/E_0 = -\tfrac{1}{2}\kappa^2, \qquad K/E_0 = +\tfrac{1}{2}\beta^2.$$

The symmetry of exchange arises because both branches — (κ, κ_X) and (β, β_Y) — coexist and compensate each other. Each side carries one half of the total transformation resource, ensuring

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

For light, however, $\beta = 1$ implies $\beta_Y = 0$. The complementary projection disappears; there is no dual observer-frame available for symmetric partition. As a result, the transformation cannot be divided between two orthogonal branches. The full relational resource of the interaction is realised on a single axis.

Therefore, the specific energy potential for light is not halved but complete:

$$\boxed{\Phi_\gamma = \kappa^2 c^2,}$$

while for a massive body the potential remains partitioned,

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This explains why light experiences a total geometric effect exactly twice that of a massive particle in the same field, without introducing any auxiliary approximations. □

> Interpretive Note Light occupies the boundary state where relational reciprocity collapses into self-reference. It is not a massless limit but a distinct single-axis state of the energy geometry. A photon is simultaneously its own counter-frame and its own anti-state. The factor of two that appears in gravitational deflection and frequency shift is a direct signature of this one-axis transformation.

Summary

Light has no rest frame. The Speed of Light is the boundary beyond which the energy symmetry law breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

11 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law (Section 10) emerges when analysing the total specific orbital energy. Since energy in RG is defined relationally, as the measure of difference between two states, we naturally select these two states (e.g., the surface of the central body 'A' and the orbit 'B') as the reference points for the potential and kinetic energy budgets. Under this relational approach, the total specific orbital energy (potential + kinetic, per unit rest mass) naturally appears in a form structurally identical to the Minkowski interval.

11.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{15}$$

$$K = \frac{1}{2} m \frac{GM_\oplus}{a}. \tag{16}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{17}$$

11.2 Projection Parameters and Minkowski-like Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{18}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{19}$$

Substituting into (17) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{20}$$

This is already in the form of a hyperbolic difference of squares: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{21}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\kappa^2$ and $K/E_0 = \frac{1}{2}\beta^2$ as budgets. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer ΔE is the signed sum of budget changes.

11.3 Physical Interpretation

In classical derivations, (17) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG, (20) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface, orbit})$, and is invariantly expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While this framework refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in a Minkowski–like difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This re framing answers why the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

12 Lagrangian and Hamiltonian as Ontologically Corrupted RG Approximations

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational EnergySymmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms are not fundamental principles but ontologically "dirty" approximations of the relational WILL framework. By collapsing the two-point relational structure into a single-point description, classical mechanics gains computational convenience at the cost of ontological clarity.

12.1 Definitions of Parameters

We consider a central mass M and a test mass m. The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2\dot{\phi}^2$ — total squared orbital speed at B.
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B.
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A.

12.2 The Relational Lagrangian

Instead of a relational energy, we define the clean relational Lagrangian L_{rel}, which represents the kinetic budget at point B relative to the potential budget at point A:

$$L_{\text{rel}} = T(B) - U(A) = \tfrac{1}{2}m\left(\dot{r}_B^2 + r_B^2\dot{\phi}^2\right) - \frac{GMm}{r_A}. \tag{22}$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \tfrac{1}{2}\big(\beta_B^2 + \kappa_A^2\big). \tag{23}$$

This two-point, relational form is the clean geometric statement.

12.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{24}$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

12.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \tag{25}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \tag{26}$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \tfrac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \frac{GMm}{r}. \tag{27}$$

12.5 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \tfrac{1}{2}mv^2 + \frac{GMm}{r} \quad\longleftrightarrow\quad \tfrac{1}{2}mc^2\big(\beta^2 + \kappa^2\big), \tag{28}$$

$$H = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad\longleftrightarrow\quad \tfrac{1}{2}mc^2\big(\beta^2 - \kappa^2\big). \tag{29}$$

Here the "+" or "−" signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both are ontologically "dirty" approximations. The clean relational law, involving distinct points A and B, is collapsed into a local, one-point description. This shows that Hamiltonian and Lagrangian are just needlessly overcomplicated approximations that lose in ontological integrity.

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2. For readers accustomed to standard frameworks, the following translation rules may help:

1. General Relativity (metric form):

$$\kappa_X \hat{=} \sqrt{-g_{tt}} \quad \text{(static spacetimes)}, \qquad \beta \hat{=} \frac{\|u^\mu_{\text{spatial}}\|}{u^t c}.$$

2. Canonical mechanics (Lagrangian/Hamiltonian): Quantities such as $p_i = \partial L/\partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are computational shadows, useful for legacy calculations but physically redundant.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

12.5.1 Third Ontological Collapse: Derivation of Newton's Third Law

We now demonstrate that Newton's Third Law, like the Lagrangian and Hamiltonian, is not a fundamental principle but another "degenerate shadow" of the WILL framework. It arises as a necessary mathematical consequence of the same ontological collapse — forcing a two-point relational law into a single-point, instantaneous formalism.

Theorem 12.1 (Newton's Third Law as a Degenerate Consequence). The Energy–Symmetry Law ($\Delta E_{A\to B} + \Delta E_{B\to A} = 0$) mathematically necessitates Newton's Third Law ($\vec{F}_{AB} = -\vec{F}_{BA}$) in the classical, non-relativistic limit where the two-point relational energy budget is collapsed into a single-point potential function $U(\vec{r})$.

Proof. We begin with the foundational Energy–Symmetry Law (Section 10), the principle of causal balance for state transitions:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

In the classical, non-relativistic limit, this two-point relational law is "ontologically corrupted" into a single-point potential energy function, U. This function is assumed to depend only on the relative positions of the two interacting entities, A and B:

$$U = U(\vec{r}) \quad \text{where} \quad \vec{r} = \vec{r}_B - \vec{r}_A.$$

This $U(\vec{r})$ is the classical approximation of the system's relational energy budget. In this collapsed formalism, the force $\vec{F}$ is defined as the negative gradient of this potential.

(1) Force on B by A ($\vec{F}_{AB}$): This force is found by taking the gradient with respect to B's coordinates:

$$\begin{align}
\vec{F}_{AB} &= -\nabla_B U(\vec{r}_B - \vec{r}_A) \tag{30}\\
&= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_B}\right) \tag{31}\\
&= -\nabla U(\vec{r}) \cdot (\mathbf{I}) \tag{32}\\
&= -\nabla U(\vec{r}) \tag{33}
\end{align}$$

(2) Force on A by B ($\vec{F}_{BA}$): This force is found by taking the gradient with respect to A's coordinates:

$$\begin{align}
\vec{F}_{BA} &= -\nabla_A U(\vec{r}_B - \vec{r}_A) \tag{34}\\
&= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_A}\right) \tag{35}\\
&= -\nabla U(\vec{r}) \cdot (-\mathbf{I}) \tag{36}\\
&= +\nabla U(\vec{r}) \tag{37}
\end{align}$$

(3) Conclusion: By direct comparison of the results, we find:

$$\vec{F}_{AB} = -\nabla U(\vec{r}) \quad \text{and} \quad \vec{F}_{BA} = +\nabla U(\vec{r}).$$

Therefore, it is a mathematical tautology of the collapsed formalism that:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

This completes the proof. Newton's Third Law is not an independent physical axiom, but the built-in mathematical consequence of approximating the Energy Symmetry Law with a single potential function. The law of "equal and opposite forces" is revealed to be a degenerate approximation of the more fundamental, generative law of Relational Geometry. □

12.6 General Consequence

Bad philosophy, in RG sense, has three measurable effects:

1. Inflated Formalism: Equations multiply to compensate for ontological error.
2. Loss of Transparency: Physical meaning becomes hidden behind coordinate dependencies.
3. Empirical Fragmentation: Each domain (cosmology, quantum, gravitation) requires separate constants.

By contrast, good philosophyepistemic hygieneenforces relational closure and yields simplicity through necessity, not through approximation.

In short:

$\boxed{\text{Badŕphilosophyŕcreatesŕcomplexity} \quad \text{Goodŕphilosophyŕrevealsŕgeometry.}}$

Daring Remark

The historical escalation of mathematical complexity in physics did not reveal deeper reality - it compensated for a philosophical mistake. Once the ontological symmetry is restored, Natures laws reduce to algebraic self-consistency.

$$\boxed{\text{Bad Philosophy}} \Rightarrow \boxed{\text{Ontological Duplication}} \Rightarrow \boxed{\text{Mathematical Inflation}}$$

$$\boxed{\text{Complex mathematics is the consequence of bad philosophy.}}$$

13 Rotational Systems (Kerr Without Metric)

13.1 Contextual Bounds

- For a gravitationally closed (static) system, the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.
- For a kinematically closed (maximally rotating) system, the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac^2}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the relational rotation parameter, with $0 \leq \beta \leq 1$,
- κ is related to the geometry and gravity,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta^2_{\max} r$$

This relationship holds when $r = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

13.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_{\pm} = \frac{R_s}{2}\left(1 \pm \beta_Y\right)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

13.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2}\left(1 + \sqrt{1 - \beta^2 \cos^2\theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

13.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c^2}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

13.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship Principle. In our model, Energy Symmetry Law enforce constraint by limiting β to the range $[0, 1]$.

13.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in a_*, mirror the orbital velocity parameter β, providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

Physical Interpretation

- No need for pre-existing spacetime geometry emerges from angular energy distributions.
- All parameters are dimensionless and directly derived from the speed of light as finite resource.
- Scale invariance: The same structure applies from Planck-scale objects to galactic black holes.

14 W_{ILL} Unity of Relational Structure

From the geometric closure of WILL we derive the universal invariant connecting mass, energy, time, and length. Each is a projection of the same relational whole:

$$M = \frac{\beta^2}{\beta_Y} c^2 \frac{r}{G}, \tag{38}$$

$$E = \frac{\kappa^2}{\kappa_X} \frac{c^4 r}{2G}, \tag{39}$$

$$T = \kappa_X \left(\frac{2Gm_0}{\kappa^2 c^3} \right)^2, \tag{40}$$

$$L = \beta_Y \left(\frac{Gm_0}{\beta^2 c^2} \right)^2. \tag{41}$$

Combining these projections yields the universal dimensionless invariant:

$$\boxed{W_{\mathrm{ILL}} = \frac{E\,T}{M\,L} = \frac{\frac{E_0}{\kappa_X}\,\kappa_X t^2}{\frac{m_0}{\beta_Y}\,\beta_Y r^2} = 1.}$$

All dimensional constants and parameters cancel identically. This equality is not a result of unit choice, but the manifestation of structural closure:

$$\boxed{\mathrm{WILL} = 1.}$$

14.1 Interpretive Note: The Name "WILL"

The term WILL stands for SPACETIMEENERGY. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the cosmos does not possess will - yet through WILL, it manifests all that is.

Summary

$$\boxed{\text{WILL} \equiv \frac{ET}{ML} = 1 \quad \Longleftrightarrow \quad \text{Geometry} = \text{Energy} = \text{Causality.}}$$

$\boxed{\text{It is not the unit of something - it is the unity of everything.}}$

15 Beyond Differential Formalism: Structural Dynamics

15.1 Intrinsic Dynamics via Energy Redistribution

In Relational Geometry (RG), dynamics is not the evolution of quantities in time, but the continuous re-balancing of a closed network of algebraic relations. What appears as "motion" or "change" is the ordered succession of states satisfying all projectional constraints.

Summary

Time does not drive change change defines time.

15.2 Why There Are No Equations of Motion

Classical physics describes systems through differential evolution:

$$\delta S = 0, \qquad S = \int L\,dt,$$

assuming an independent temporal parameter and a variational freedom over possible trajectories. In RG, none of these assumptions hold:

- There is no continuum of possible paths.
- There is no freedom to vary.
- The system itself defines temporal order.

Each observable is locked into a closed web of relational equations. A valid state is oneand only onewhere all constraints are simultaneously satisfied.

One valid configuration

There is only one physically meaningful configuration at any moment: the one where all relational projections close exactly. Everything else is not forbiddenit simply does not exist.

15.3 Why the Metric Is Physically Meaningless

The metric tensor $g_{\mu\nu}$ presupposes an external backdrop on which events are placed. In RG there is no such background: distance and duration are nothing but relational measures between energy states. The relations (β, κ) already encode all observable structure. Introducing a metric would merely restate these relations redundantly in coordinate form.

Summary

In RG there is no metric, no Lagrangian, no equations of motion. There is only a closed algebraic structure of energetic relations. Its self-consistency defines both geometry and causality.

16 Derivation of Density, Mass, and Pressure

16.1 Geometric Foundation

From the projective analysis established in the previous sections, the fundamental invariant is

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

16.2 Derivation of Energy Density

From Mass Scale to Volumetric Potential. Starting from the geometric relation,

$$m_0 = \frac{\kappa^2 c^2 r}{2G},$$

we associate m_0 with a volumetric proxy r^3, obtaining a raw volumetric potential,

$$\frac{m_0}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}.$$

Applying the Geometric Distribution Principle. Because the potential projection κ is distributed over S2 - a 2D spherical manifold, the volumetric expression must be normalized over the unit-sphere area 4π. This yields the physical energy density,

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr^2}\right).$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}.$$

Local Energy Density $\equiv$ Relational Projection

Maximal Density. At $\kappa^2 = R_s/r = 1$, the horizon condition is reached, corresponding to the maximal observable energy density at radius $r = R_s$:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}.$$

Normalized Relation. Thus the fundamental identification is

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}.$$

16.3 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $m_0 = \alpha\, r^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

16.4 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\,\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\, c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energygeometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\,c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energygeometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

17 Local Cosmological Term in RG

Lemma 17.1 (Normalization Identity). In WILL Relational Geometry, local energy density and its maximal counterpart are related by

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2}, \tag{42}$$

$$\rho_{\max}(r) = \frac{c^2}{8\pi G r^2}, \tag{43}$$

$$\Rightarrow \quad \boxed{\kappa^2 = \frac{\rho}{\rho_{\max}}}. \tag{44}$$

This ratio defines the dimensionless geometric projection parameter κ^2 as a normalized energy measure.

Theorem 17.2 (Geometric Field Equation). For a static, spherically symmetric configuration, the unified field equation of WILL RG reads

$$\boxed{\frac{d}{dr}\big(r\,\kappa^2\big) = \frac{8\pi G}{c^2}\,r^2\rho(r)}. \tag{45}$$

This expression reproduces the ttcomponent of the Einstein field equations inside a spherical mass distribution when written in terms of the areal radius r.

Proof. Starting from the standard TolmanOppenheimerVolkoff (TOV) form,

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1 - \frac{1}{g_{rr}}\right)\right] = \frac{8\pi G}{c^2}\rho(r), \tag{46}$$

and defining $\kappa^2(r) \equiv 1 - \frac{1}{g_{rr}} = \frac{2Gm(r)}{rc^2}$, one obtains Eq. (45) directly. □

Definition 17.3 (Vacuum Stress Tensor). Let the intrinsic geometric normalization $\rho_{\max}(r)$ act as a vacuum background of isotropic tension:

$$T^{(\mathrm{vac})}_{\mu\nu} = -\rho_{\max}(r)\, c^2\, g_{\mu\nu}. \tag{47}$$

Corollary 17.4 (Emergent Cosmological Term). Substituting $T^{(\mathrm{vac})}_{\mu\nu}$ into Einsteins equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

yields an effective local cosmological term

$$\Lambda(r) = \frac{8\pi G}{c^4}\rho_{\max}(r)c^2 = \frac{8\pi G}{c^2}\rho_{\max}(r) = \frac{1}{r^2}. \tag{48}$$

Hence the cosmological contribution is not postulated but arises algebraically from the normalization of relational energy.

Theorem 17.5 (Equation of State and Vacuum Equivalence). From the radial curvature balance,

$$P(r) = \frac{c^4}{8\pi G}\frac{1}{r}\frac{d\kappa^2}{dr}, \tag{49}$$

and using $\kappa^2 = R_s/r$, one finds

$$P(r) = -\rho(r)\, c^2. \tag{50}$$

Therefore the built-in equation of state of Relational Geometry exactly matches that of the cosmological vacuum, confirming the identification

$$P_\Lambda = -\rho_\Lambda c^2, \qquad \rho_\Lambda \equiv \rho_{\max}, \qquad \Lambda(r) = \frac{8\pi G}{c^2}\rho_\Lambda(r) = \frac{1}{r^2}.$$

Summary

In RG, the cosmological constant emerges naturally from the geometric normalization of energy density:

$$\boxed{\Lambda(r) = \frac{1}{r^2}.}$$

Thus the vacuum energy of GR corresponds to the intrinsic curvature limit of relational geometry itself.

18 Unified Geometric Field Equation

18.1 The Theoretical Ouroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}} \tag{51}$$

The ratio of geometric scales equals the ratio of energy densities.

This is the unified geometric field equation of WILL Relational Geometry. It expresses the complete equivalence:

$$\text{SPACETIME GEOMETRY} \equiv \text{ENERGY DISTRIBUTION}$$

We have shown that this single foundational Principle, through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with a single foundational Principle SPACETIME $\equiv$ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

IMAGES/1_WILL-Relational-Geometry.png

The foundational Principle SPACETIME $\equiv$ ENERGY closes into the unified field equation

$$\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}},$$

Theoretical Ouroboros

The RG framework exhibits perfect logical closure: the fundamental Principle about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework the Theoretical Ouroboros. But regardless of aesthetic beauty of this result lets remain sceptical.

18.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2 \ \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- Surfacescaled closure (vs. volume filling). Mass follows the algebraic closure $m_0 = 4\pi r^3\rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.

- Natural bounds. The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

Summary

The WILL framework postulates no external laws or assumptions. All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

19 Algebraic Closure and Structural Dynamics

Physical dynamics in WILL Relational Geometry (RG) emerges not from temporal evolution equations but from a set of algebraically closed invariants. Each parameter participates in a selfconsistent configuration of relational constraints. Changing any one parameter necessitates a coordinated shift in all others to maintain validity.

Definition 19.1 (Algebraic Closure of the WILL Structure). The minimal algebraic closure of the RG system is expressed by the set:

$$\begin{cases} \kappa^2 = 2\beta^2, \\ R_s = \dfrac{2Gm_0}{c^2}, \\ r = \dfrac{R_s}{\kappa^2}, \\ \rho = \dfrac{\kappa^2 c^2}{8\pi G r^2}, \\ \rho_{\max} = \dfrac{c^2}{8\pi G r^2}, \\ t = \dfrac{r}{c}, \\ m_0 = 4\pi r^3 \rho. \end{cases}$$

These relations are not independent definitions but mutual constraints. Each variable acquires meaning only within the self-consistent closure of the entire system.

No differential equations are required:

Dynamics unfolds as a consequence of relational energy transformations.

Lemma 19.2 (Causal Closure without Circularity). Every observable quantity in RG can be determined from a minimal input pair composed of one dynamic projection (e.g. κ or β) and one scale parameter (e.g. r, M, or ρ). No parameter both defines and is defined by the same input. Thus, the system avoids circularity while remaining causally closed.

Theorem 19.3 (Structural Dynamics). Within the closed algebraic configuration $(\kappa, \beta, \rho, r, t)$, any local variation of one parameter entails a coherent transformation of all others, such that the algebraic closure is preserved:

$$\delta\Phi(\kappa, \beta, \rho, r, t) = 0, \qquad \Phi \text{ representing the constraint manifold.}$$

Therefore, dynamics is the propagation of constraint consistency rather than temporal evolution.

Corollary 19.4 (Emergent Time). Since every admissible configuration is defined by its energygeometry relations, the parameter conventionally called time is nothing more than an ordered index of transitions between self-consistent configurations:

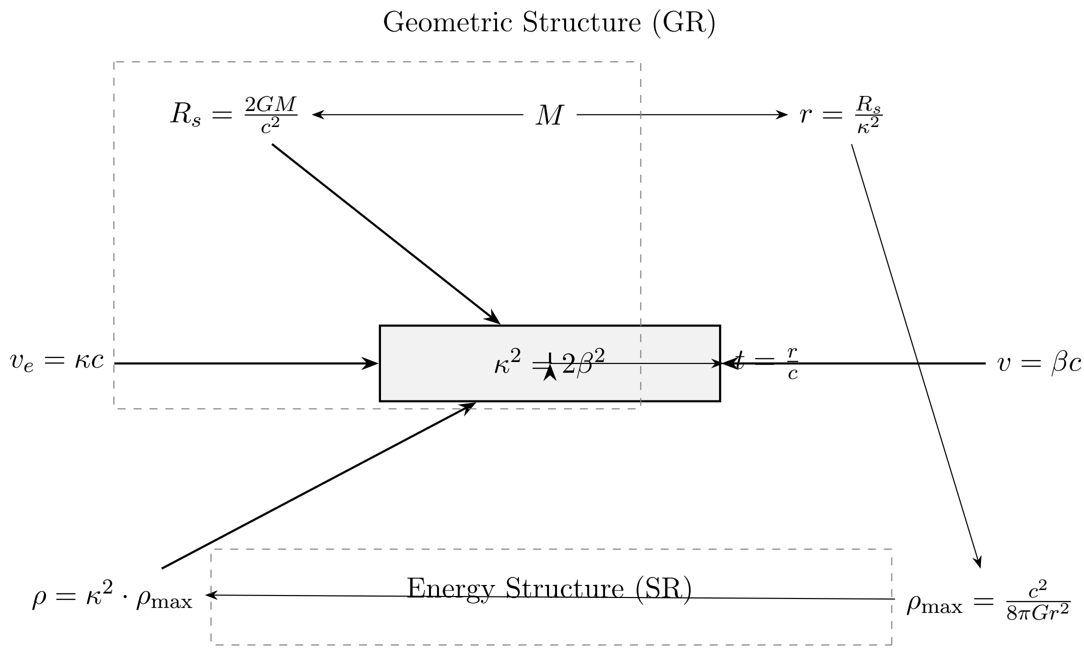
$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r, \ldots)}.$$

Thus, causality in RG is structural, not temporal; it refers to coherence of relations rather than to sequences of events.

Interpretation

Dynamics without Differentials: In RG, no differential equations of motion are required. Physical change is expressed through algebraic redistribution of geometric and energetic quantities. Time is emergent, coherence is enforced, and the universe evolves as a continuous sequence of balanced configurations.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$



The result is a structure where causality is internal, coherence is enforced, and dynamics is simply the shifting of balanced configurations — not the unfolding of arbitrary functions over time.

Philosophical Closure

The entire relational structure can be read as a self-consistent equation of existence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY} \iff W_{\text{ILL}} = 1.}$$

Geometry, density, motion, and causality are not separate domains but inter-translations within one algebraic identity. The universe is not governed by differential laws it is their solution.

19.1 Numerical Example: Accretion onto a Black Hole

Consider a black hole accreting mass from a surrounding disk to illustrate the model's intrinsic dynamics. Let the initial mass be $m_0 = 10, M_\odot$, with a Schwarzschild radius $R_s = \frac{2Gm_0}{c^2} \approx 2.95 \times 10^4, \text{m}$. Suppose $\kappa = 0.1$, so $r = \frac{R_s}{\kappa^2} = \frac{2.95\times10^4}{0.01} = 2.95 \times 10^6, \text{m}$, and the associated time scale is $t = \frac{r}{c} \approx 9.83 \times 10^{-3}, \text{s}$.

As the black hole accretes mass, increasing to $m_1 = 10.1, M_\odot$, the Schwarzschild radius becomes $R_s \approx 2.98 \times 10^4, \text{m}$. Assuming κ remains constant for simplicity, $r = \frac{2.98\times10^4}{0.01} = 2.98 \times 10^6, \text{m}$, and $t \approx 9.93 \times 10^{-3}, \text{s}$. This increase in t reflects the system's evolution, driven solely by the changing geometry.

Summary

In Relational Geometry, causality is not temporal but structural. Each variable $(\kappa, \beta, \rho, r, t)$ participates in an algebraically closed configuration that defines a self-consistent physical state. Any local variation of one variable entails a coherent transformation of all others, preserving the closure of the system. Hence, dynamics is the propagation of constraint consistency, and time is the ordered index of these transformations.

$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r, ...)}$$

Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as "laws of nature."
3. Mathematical formalisms are constructed to describe these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are generated as inevitable consequences of relational geometry:

- There are no independent axioms such as "inertial mass equals gravitational mass."
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls "laws of nature" are secondary shadows of the single relational principle:

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

Key Distinction

Standard Physics: Laws describe what we observe.
RG Framework: Laws are generated as necessary products of relational geometry.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

20 Axiomatic Foundations Theorem:WILL Relational Geometry (RG) and General Relativity (GR)

This logical asymmetry does not imply physical superiority a priori; it only states that any empirical support for GR already presupposes relational invariance.

Descriptive Physics (Standard)	Generative Physics (WILL)
Phenomena are observed first, then summarized into empirical laws.	Laws emerge as inevitable consequences of relational geometry.
Physical laws are assumptions introduced to model reality.	Physical laws are identities, enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are projections of energy relations.
Dynamics = evolution of states in time.	Dynamics = ordered succession of balanced configurations; time is emergent.
Goal: describe what is observed.	Goal: show why nothing else is possible.

Table 3: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

Definition 20.1 (GR Core Axioms). General Relativity (GR) is assumed to rest on the following axioms:

(A1) The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.

(A2) Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.

(A3) Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.

(A4) Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Definition 20.2 (RG One Principle). RG is based on a single Principle:

(W1) Relational Principle: All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy.

Lemma 20.3 (Relationality in GR). From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.

Remark 20.4 (Bridge: From Relational Principle to GR Axioms). If the Relational Principle (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:

- It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.
- It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.

Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 20.5 (Asymmetric Falsifiability of GR and RG). Let GR denote the theory defined by axioms (A1)–(A4), and let RG denote the theory defined by Principle (W1). Then:

1. If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence, GR is necessarily falsified.
2. If any of (A1)–(A3) are empirically falsified, GR collapses, but (W1) may still remain valid as a stand-alone principle.

Therefore, there exist possible empirical scenarios in which GR fails while RG survives, but there exist no scenarios in which RG fails while GR survives.

Corollary 20.6. RG is axiomatically more fundamental than GR: its sole Principle (W1) is logically included within the core axioms of GR, while GR requires additional ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of RG.

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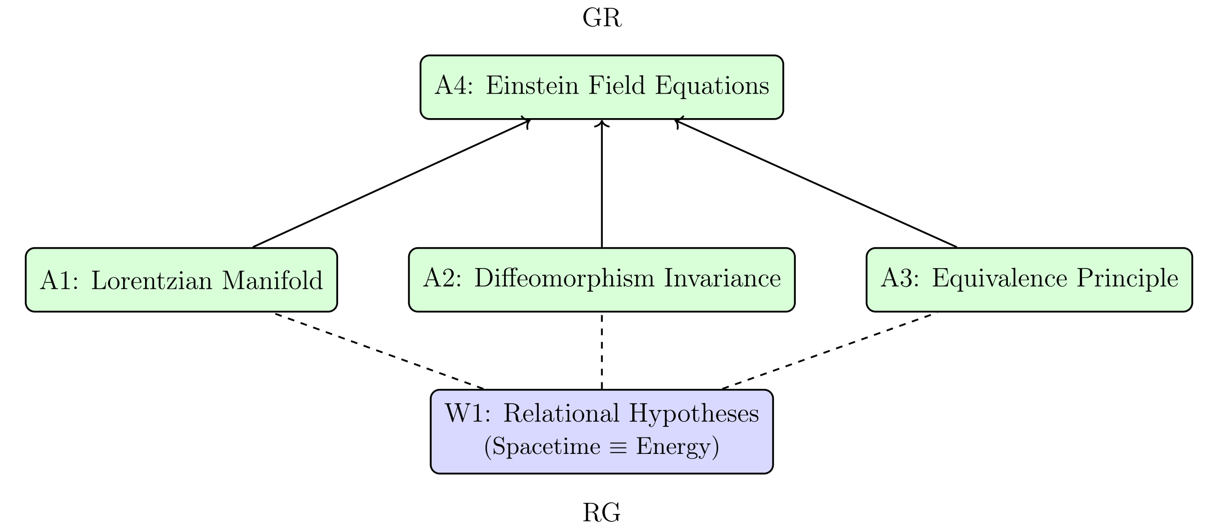
Status of General Relativity within RG

It is important to emphasize that the RG framework does not invalidate the achievements of General Relativity. Rather, it explains them. All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in Relation Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.
Thus, GR is not a rival but a specialized, parameter-heavy realization of RG's more general principle. In logical terms:

- Relational Geometry can stand without GR, but GR cannot stand without the relational Principle (W1).
- The empirical successes of GR are preserved within RG, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

Therefore, GR should be understood as an effective approximation embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.



Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability).
RG rests on the single relational Principle (W1). Core GR assumes additional structures (A1–A4). Hence:

- If W1 is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of A1–A3 is falsified, GR collapses, while W1 (and thus RG) may still hold.

Therefore, there are scenarios where GR fails and RG survives, but none where RG fails while GR survives.

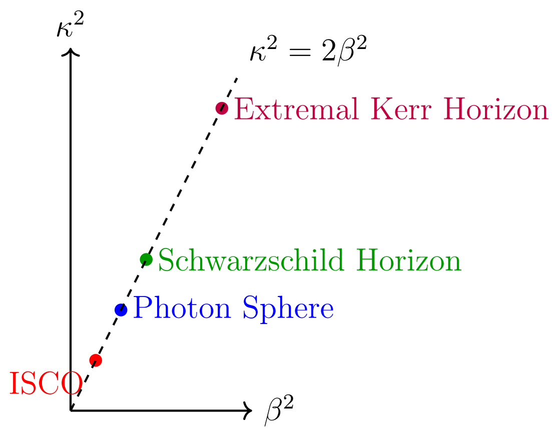
Figure 1: Axiomatic Structure: RG vs GR

Comparison Table: General Relativity (GR) vs WILL Relational Geometry (RG)

#	Category	General Relativity (GR)	Relational Geometry (RG)
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}, R$; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$
4	GeometryMatter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \to \infty$, $g_{00} \to 0$	Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$

Phenomenon	Radius r	β^2	κ^2	Q^2	Comment
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal stability of timelike orbits $Q = Q_t$
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Null circular orbits, $\theta_1 = \theta_2$ $Q = 1$, $Q_t = 0$
Static horizon (Schwarzschild)	$r = R_s$	$\frac{1}{2}$	1	$\frac{3}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 4: Critical radii and their projectional parameters in WILL Relational Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.



20.1 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the RG's principles. One can derive GR by adding these additional structures to RG's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, where RG provides the logical foundation upon which GR can be consistently constructed.

20.2 Epistemological Role of General Relativity

General Relativity occupies a unique historical position. It is the first theory to recognize geometry as the carrier of energy, yet it stops one step short of full equivalence. By treating the metric $g_{\mu\nu}$ as an independent entity that responds to energy–momentum, GR still separates cause and effect. In WILL Relational Geometry this distinction dissolves: geometry is energy, not its consequence.

Thus, GR should be seen as the transitional language between the descriptive physics of the nineteenth century and the generative physics of the twenty-first. It already encodes the relational structure implicitly, but expresses it through redundant coordinates and differential machinery. RG reveals the algebraic heart of GR stripped of these redun-

Phenomenon	Standard (GR) Result	Relational Geometry (RG)
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2$, ISCO: $Q = Q_t$).
Mercurys perihelion precession	Complex expansion of Einstein field equations.	Exact same number obtained from Relational Geometry with $\Delta_\phi = \frac{2\pi Q^2_{Merc}}{(1-e^2_{Merc})}$.
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs.
Cosmological redshift	Photon loses energy as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological constant Λ	Added by hand to fit data (dark energy).	Arises naturally as $\Lambda = \kappa^2/r^2$. No extra entities required. (Details in WILL PART II)
Singularities	Predicted in black holes and big bang ($\rho \to \infty$).	Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$.
Local gravitational energy	Cannot be localized (only ADM/Bondi at infinity).	Directly measurable via κ, e.g. from light deflection angle.
Unification with QM and SR	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 5: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

dancies:

$$\boxed{G_{\mu\nu} \;\Leftrightarrow\; \kappa^2}, \qquad \boxed{T_{\mu\nu} \;\Leftrightarrow\; \rho}.$$

The celebrated field equations of Einstein then reduce to the unified geometric identity

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} = \frac{R_s}{r}},$$

which is the simplest, most symmetric realization of the same principle that GR only encodes indirectly.

Phenomenon	Empirical Benchmark	WILL Prediction
GPS satellite time dilation (SR + GR)	38.52 μs/day (observed)	38.52 μs/day
Mercury perihelion precession	43"/century (observed)	43"/century
Solar light deflection	1.75 arcsec (observed)	1.75 arcsec
Schwarzschild photon sphere	$r = 1.5R_s$ (GR prediction)	$r = 1.5R_s$
Schwarzschild ISCO	$r = 3R_s$ (GR prediction)	$r = 3R_s$
Hulse–Taylor pulsar period decay	$\Delta P \approx -2.42 \times 10^{-12}$ s/s (observed)	$\Delta P \approx -2.40 \times 10^{-12}$ s/s
Earth–Moon tidal power (LLR recession)	0.120 TW orbital power (measured)	0.120 TW
Galaxy rotation curves (Milky Way example)	Flat curves beyond ∼10 kpc	Flat curves from projection law
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, ΛCDM fits	Emergent absolute scale from κ, β closure

Table 6: Empirical validation of WILL Relational Geometry across classical relativistic tests, orbital dynamics, astrophysical observations, and cosmology. See details in "Appendix I"

Historical Function of GR

General Relativity is not wrong it is prematurely general. It describes through differentials what WILL generates through relations. It built the bridge; WILL walks across it.

21 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single foundational Principlethat spacetime is equivalent to energywe derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

$$\boxed{\text{Special and General Relativity emerge from the same geometric principles.}}$$

This approach offers distinct advantages:

- Conceptual clarity understanding physics through pure geometry
- Computational efficiency significantly reducing complexity
- Epistemological hygiene deriving results from minimal assumptions

- Philosophical depth redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

$$\boxed{\text{Spacetime and energy are mutually defining aspects of a single relational structure.}}$$

Final Summary

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

21.1 References:

- Schwarzschild, K. (1916). Sitzungsberichte der Kцniglich Preussischen Akademie der Wissenschaften.
- Tolman, R.C. (1939). Phys. Rev. 55, 364.

  Oppenheimer, J.R., Volkoff, G.M. (1939). Phys. Rev. 55, 374.
- Dyson, F.W.; Eddington, A.S.; Davidson, C. (1920). Phil. Trans. R. Soc. A 220, 291333.

  Lebach, D.E. et al. (1995). Phys. Rev. Lett. 75, 1439.

  Lambert, S.B., Le Poncin-Lafitte, C. (2009). A&A 499, 331.
- Ashby, N. (2003). Living Rev. Relativity 6:1;
- Will, C.M. (2014). Living Rev. Relativity 17:4.
- Williams, J.G.; Turyshev, S.G.; Boggs, D.H. (2006/2014/2016). LLR analyses (Adv. Space Res. 37, 6771; JPL).

  Dickey, J.O. et al. (1994). Science 265, 482.

  Egbert, G.D.; Ray, R.D. (2000). Nature 405, 775778.
- Weisberg, J.M.; Huang, Y. (2016). ApJ 829, 55.

  Kramer, M. et al. (2021). Phys. Rev. X 11, 041050.
- Bardeen, J.M., Press, W.H., Teukolsky, S.A. (1972). ApJ 178, 347.
- Misner, Thorne, Wheeler (1973). Gravitation.

22 Appendix I

22.1 Correspondence with General Relativity

To facilitate comparison with standard General Relativity, we recast the relational parameters (κ, β) in metric form. Under the identification $\kappa^2 = 2GM/(rc^2)$, the WILL RG reproduces the Schwarzschild line element and the corresponding Einstein field equations in algebraic form.

22.2 Equivalence with Schwarzschild Solution

Theorem 22.1 (Equivalence with Schwarzschild Solution). The WILL RG formalism reproduces the Schwarzschild metric in the appropriate limit.

Proof. The Schwarzschild metric in General Relativity is given by:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{52}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit sphere.

In WILL Geometry, the key parameters are:

$$\kappa^2 = \frac{R_s}{r} = \frac{2GM}{rc^2} \tag{53}$$

$$\kappa_X = \sqrt{1-\kappa^2} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{54}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{55}$$

The time component of the Schwarzschild metric can be written as:

$$g_{tt} = \left(1 - \frac{2GM}{rc^2}\right) = 1 - \kappa^2 = \kappa_X^2 \tag{56}$$

And the radial component can be written as:

$$g_{rr} = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} = -\frac{1}{1-\kappa^2} = -\frac{1}{\kappa_X^2} \tag{57}$$

Therefore, in WILL Geometry terms, the Schwarzschild metric takes the form:

$$ds^2 = \kappa_X^2 c^2 dt^2 - \frac{1}{\kappa_X^2} dr^2 - r^2 d\Omega^2 \tag{58}$$

This demonstrates that the WILL Geometry parameters exactly reproduce the Schwarzschild metric. □

22.3 Equivalence with Einstein Field Equations

Theorem 22.2 (Equivalence with Einstein Field Equations). The geometric field equation of WILL RG is equivalent to the corresponding component of Einstein's field equations for a static, spherically symmetric mass distribution.

Proof. The standard form for the tt-component of Einstein's field equations inside a spherically symmetric perfect fluid is given by one of the Tolman–Oppenheimer–Volkoff (TOV) equations:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1 - \frac{1}{g_{rr}}\right)\right) = \frac{8\pi G}{c^2}\rho(r), \tag{59}$$

where $\rho(r)$ is the energy density at radius r.

For a static spherical system, the metric component g_{rr} is related to the mass enclosed within radius r, denoted $m(r)$, by

$$1-\frac{1}{g_{rr}}=\frac{2Gm(r)}{rc^2}. \tag{60}$$

We define the interior WILL parameter $\kappa^2(r)$ precisely as this quantity:

$$\kappa^2(r)\equiv\frac{2Gm(r)}{rc^2}. \tag{61}$$

This ensures a smooth transition to the exterior vacuum solution ($\kappa^2=R_s/r$ with $R_s=2GM/c^2$ constant) at the object's surface.

With this definition,

$$r\left(1-\frac{1}{g_{rr}}\right)=r\kappa^2(r), \tag{62}$$

and substitution into the field equation yields

$$\frac{1}{r^2}\frac{d}{dr}\left(r\kappa^2(r)\right)=\frac{8\pi G}{c^2}\rho(r). \tag{63}$$

Multiplying both sides by r^2 gives the differential form of the WILL field equation:

$$\frac{d}{dr}\left(r\kappa^2\right)=\frac{8\pi G}{c^2}r^2\rho(r). \tag{64}$$

From the geometric definition of energy density in WILL,

$$\rho(r)=\frac{\kappa^2(r)c^2}{8\pi Gr^2},\qquad \rho_{\max}(r)=\frac{c^2}{8\pi Gr^2},$$

it follows immediately that

$$\frac{\rho(r)}{\rho_{\max}(r)}=\kappa^2(r).$$

Moreover, since $r\kappa^2(r)=2Gm(r)/c^2$, differentiation gives

$$\frac{d}{dr}\left(r\kappa^2\right)=\frac{2G}{c^2}\frac{dm}{dr}=\frac{2G}{c^2}\cdot 4\pi r^2\rho(r)=\frac{8\pi G}{c^2}r^2\rho(r),$$

confirming consistency with mass conservation.

Thus, the Einstein field equation is mathematically equivalent to the identity

$$\kappa^2(r)=\frac{\rho(r)}{\rho_{\max}(r)},$$

which is itself the direct expression of the foundational principle:

$$\text{SPACETIME}\equiv\text{ENERGY}.$$

Profound Simplicity:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1-\frac{1}{g_{rr}}\right)\right)=\frac{8\pi G}{c^2}\rho(r)\iff\frac{\rho(r)}{\rho_{\max}(r)}=\kappa^2(r)\iff\kappa^2=\kappa^2$$

$$\text{SPACETIME}\equiv\text{ENERGY}$$

This demonstrates that the apparent complexity of the Einstein field equations arises from representational choices, not physical necessity. The underlying reality is a simple, self-consistent relational identity. Complex Mathematics is the Consequence of Bad Philosophy. The exact equivalence between the two formulations - under the geometrically motivated definition $\kappa^2(r) = 2Gm(r)/(rc^2)$ - completes the proof. □

Corollary 22.3 (Backtranslation). Given the Schwarzschild metric in standard form, the substitutions $\kappa^2 = 2GM/(rc^2)$, $\kappa_X^2 = 1 - \kappa^2$ map every tensorial component of $g_{\mu\nu}$ onto algebraic relations among κ, ρ, r in RG. Thus GR is a differential realization of the same algebraic closure.

22.4 Empirical Validation

22.4.1 Geometric Prediction of Photon Sphere and ISCO

The critical orbital radii in WILL Relational Geometry are not simply solved for, but emerge as direct consequences of geometric symmetries within the system's projection budget, $Q = \sqrt{\beta^2 + \kappa^2}$.

22.4.2 Photon Sphere from Geometric Equilibrium ($\theta_1 = \theta_2$)

Theorem 22.4. The Photon Sphere is generated from the principle of perfect equilibrium between the kinematic and potential projection angles. This occurs when they become equal, a condition corresponding to the "magic angle".

Proof. We begin with the symmetry condition for this state:

$$\theta_1 = \theta_2 \tag{65}$$

From the definitions $\beta = \cos\theta_1$ and $\kappa_X = \cos\theta_2$, this directly implies $\beta = \kappa_X$. We unpack the definition of $\kappa_X = \sqrt{1-\kappa^2}$:

$$\begin{aligned} \beta &= \sqrt{1-\kappa^2} \\ \beta^2 &= 1-\kappa^2 \\ \implies \kappa^2 + \beta^2 &= 1 \quad \text{(which is the condition } Q^2 = 1\text{)} \end{aligned}$$

We now solve this by applying the system's closure condition , $\kappa^2 = 2\beta^2$:

$$\begin{aligned} (2\beta^2) + \beta^2 &= 1 \\ 3\beta^2 = 1 &\implies \beta^2 = \frac{1}{3} \end{aligned}$$

From this, we find the corresponding κ^2:

$$\kappa^2 = 2\beta^2 = \frac{2}{3}$$

Solving for angles will give us:

$$\theta_1 = \theta_2 = 54.7356103172^\circ \quad \text{("magic angle")}$$

(66)

(the same numerical value as the so-called magic angle, now reinterpreted as the geometric balance defining the photon sphere.)

Finally, we derive the physical radius from the definition $r = R_s/\kappa^2$:

$$r_{ps} = \frac{R_s}{2/3} = \frac{3}{2}R_s = 1.5R_s \tag{67}$$

Thus, the symmetry of angle equality inevitably generates the radius of the photon sphere.
□

22.4.3 ISCO from Budgetary Equilibrium ($Q = Q_t$)

Theorem 22.5. The Innermost Stable Circular Orbit (ISCO) is generated from the principle of perfect equilibrium between the total projection budget ($Q = \sqrt{\beta^2 + \kappa^2}$) and its orthogonal complement ($Q_t = \sqrt{1 - Q^2}$). This represents a state of marginal stability.

Proof. We begin with the symmetry condition for this state:

$$Q = Q_t \tag{68}$$

By squaring both sides and using the definition $Q_t^2 = 1 - Q^2$, we find the value of the total budget Q^2:

$$\begin{aligned} Q^2 &= Q_t^2 \\ Q^2 &= 1 - Q^2 \\ 2Q^2 &= 1 \implies Q^2 = \frac{1}{2} \end{aligned}$$

We now have a new condition $\kappa^2 + \beta^2 = 1/2$. by applying the closure condition, $\kappa^2 = 2\beta^2$:

$$\begin{aligned} (2\beta^2) + \beta^2 &= \frac{1}{2} \\ 3\beta^2 &= \frac{1}{2} \implies \beta^2 = \frac{1}{6} \end{aligned}$$

From this, we find the corresponding κ^2:

$$\kappa^2 = 2\beta^2 = \frac{2}{6} = \frac{1}{3}$$

Finally, we derive the physical radius from $r = R_s/\kappa^2$:

$$r_{isco} = \frac{R_s}{1/3} = 3R_s \tag{69}$$

Thus, the symmetry of budgetary equilibrium inevitably generates the radius of the ISCO.
□

> Interpretive Note While the radii $1.5R_s$ and $3R_s$ are known from General Relativity, their emergence here from two distinct and fundamental geometric symmetries ($\theta_1 = \theta_2$ and $Q = Q_t$) is not imposed but arises from the internal consistency of the WILL framework. This reinforces the explanatory power of Relational Geometry.

Geometry before locality

Relational Geometry defines causality before mass, and curvature before gravity.

22.5 Empirical Check: Circular Earth Orbits (SR×GR Factorization and L/H Collapse)

Setup and Definitions. We test two identities of the WILL framework on wellmeasured circular orbits around Earth:

1. the closure diagnostic $\kappa^2 = 2\,\beta^2$ for circular motion;
2. the collapsed (legacy) energy forms

$$\frac{L}{E_0} = \tfrac{1}{2}\big(\beta^2 + \kappa^2\big), \qquad \frac{H}{E_0} = \tfrac{1}{2}\big(\beta^2 - \kappa^2\big),$$

   where $E_0 \equiv mc^2$ is the rest energy of the test mass m.

All quantities are dimensionless when divided by E_0. We use standard (legacy) constants and Earth parameters in SI units:

$$\mu_\oplus \equiv GM_\oplus = 3.986004418 \times 10^{14}\ \mathrm{m^3\,s^{-2}}, \quad c = 299{,}792{,}458\ \mathrm{m\,s^{-1}}, \quad R_\oplus = 6{,}371{,}000\ \mathrm{m}.$$

Theorem 22.6. Projection parameters. For a circular orbit of radius r with orbital speed v,

$$\beta^2 \equiv \frac{v^2}{c^2}$$

,

$$\kappa^2 \equiv \frac{2GM_\oplus}{rc^2} = \frac{R_s}{r}$$

,

$$R_s \equiv \frac{2GM_\oplus}{c^2}$$

.

For circular motion the empirical relation $v^2 = \mu_\oplus/r$ holds to high accuracy, hence

$$\beta^2 = \frac{\mu_\oplus}{rc^2} \quad\Longrightarrow\quad \boxed{\kappa^2 = 2\,\beta^2},$$

i.e. the closure condition is an exact analytic identity for ideal circular orbits.

Legacy energies from projection budgets. WILL assigns quadratic budgets

$$\frac{T}{E_0} = \tfrac{1}{2}\beta^2, \qquad \frac{U}{E_0} = -\tfrac{1}{2}\kappa^2,$$

so that the legacy Lagrangian and Hamiltonian (after the onepoint "ontological collapse") read

$$\boxed{\frac{L}{E_0} = \tfrac{1}{2}\big(\beta^2 + \kappa^2\big)}, \qquad \boxed{\frac{H}{E_0} = \tfrac{1}{2}\big(\beta^2 - \kappa^2\big)}.$$

These are direct rewritings of the relational budgets in terms of β, κ.

Proof. Numerical Evaluation (SI units)

We now evaluate the above identities for two standard circular orbits. Numerical differences from zero in the $\kappa^2 - 2\beta^2$ check reflect only rounding; the analytic identity guarantees exact cancellation.

(A) LEO at $\sim$400 km altitude.

$$r = R_\oplus + 400{,}000 \text{ m} = 6{,}771{,}000 \text{ m}, \qquad v = \sqrt{\mu_\oplus / r} \approx 7{,}672.60 \text{ m s}^{-1}.$$

Hence

$$\beta^2 = \frac{v^2}{c^2} \approx 6.5500340 \times 10^{-10}, \qquad \kappa^2 = \frac{2\mu_\oplus}{rc^2} \approx 1.3100068 \times 10^{-9},$$

$$\kappa^2 - 2\,\beta^2 \approx -2.1 \times 10^{-25} \quad (\approx 0),$$

$$\frac{L}{E_0} = \tfrac{1}{2}\big(\beta^2 + \kappa^2\big) \approx 9.8250510 \times 10^{-10}, \qquad \frac{H}{E_0} = \tfrac{1}{2}\big(\beta^2 - \kappa^2\big) \approx -3.2750170 \times 10^{-10}.$$

(B) GPS orbit at $\sim$20,200 km altitude.

$$r = R_\oplus + 20{,}200{,}000 \text{ m} = 26{,}571{,}000 \text{ m}, \qquad v = \sqrt{\mu_\oplus / r} \approx 3{,}873.16 \text{ m s}^{-1}.$$

Hence

$$\beta^2 \approx 1.6691235 \times 10^{-10}, \qquad \kappa^2 \approx 3.3382470 \times 10^{-10},$$

$$\kappa^2 - 2\,\beta^2 \approx 0 \quad \text{(within rounding)},$$

$$\frac{L}{E_0} \approx 2.5036852 \times 10^{-10}, \qquad \frac{H}{E_0} \approx -8.3456175 \times 10^{-11}.$$

Conclusion. Both circularorbit cases confirm:

$$\boxed{\kappa^2 = 2\,\beta^2}, \qquad \boxed{\frac{L}{E_0} = \tfrac{1}{2}(\beta^2 + \kappa^2), \quad \frac{H}{E_0} = \tfrac{1}{2}(\beta^2 - \kappa^2)}.$$

In the WILL reading, the subsystem is energetically closed, and the "legacy" L, H are just ontologically corrupted approximations of the underlying Energy Symmetry law. For nonclosed subsystems (e.g., radiating binaries), $\kappa^2 - 2\beta^2 \neq 0$ until all energy channels (such as gravitational radiation) are included.

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

□

22.6 Relational Self-Reference of Light (Gravitational Lensing)

Theorem 22.7 (SingleAxis Transformation Principle). For light, the kinematic projection reaches its full extent:

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

$$\alpha = 2\kappa^2 = \frac{4Gm_0}{bc^2} \tag{70}$$

This means that all transformation of the relational energy occurs along a single orthogonal axis. The complementary branch of the bidirectional energy exchange is absent, and the total resource of transformation is entirely expressed on one geometric component.

Proof. For massive systems, the EnergySymmetry Law distributes the total energy exchange evenly between two orthogonal projections:

$$U/E_0 = -\tfrac{1}{2}\kappa^2, \qquad K/E_0 = +\tfrac{1}{2}\beta^2.$$

The symmetry of exchange arises because both branches — (κ, κ_X) and (β, β_Y) — coexist and compensate each other. Each side carries one half of the total transformation resource, ensuring

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

For light, however, $\beta = 1$ implies $\beta_Y = 0$. The complementary projection disappears; there is no dual observer-frame available for symmetric partition. As a result, the transformation cannot be divided between two orthogonal branches. The full relational resource of the interaction is realized on a single projection.

Therefore, the specific energy potential for light is not halved but complete:

$$\boxed{\Phi_\gamma = \kappa^2 c^2,}$$

while for a massive body the potential remains partitioned,

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This explains why light experiences a total geometric effect exactly twice that of a massive particle in the same field, without introducing any auxiliary approximations.

$$\alpha = -\frac{1}{v^2}\int_{-\infty}^{\infty} \partial_\perp \Phi \, dz,$$

and for light ($v = c$) with $\kappa^2(r) = 2GM/(c^2 r)$ where $r = b$ one finds

$$\alpha = \frac{1}{c^2}\int c^2\, \partial_\perp \kappa^2 \, dz = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\left(-\frac{b}{(b^2+z^2)^{3/2}}\right) dz = \frac{4GM}{bc^2} = 2\kappa^2.$$

Thus the GR value is recovered without invoking metric or geodesics. □

Interpretive Note Light occupies the boundary state where relational reciprocity collapses into self-reference. It is not a massless limit but a distinct single-axis state of the energy geometry. A photon is simultaneously its own counter-frame and its own anti-state. The factor of two that appears in gravitational deflection and frequency shift is a direct signature of this one-axis transformation.

Summary

Light has no rest frame. The Speed of Light is the boundary beyond which the energy symmetry law breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

22.7 GPS Satellite and Earth

Theorem 22.8 (Real-World Energy Symmetry). WILL prediction matches the empirical time correction required for GPS synchronization to high precision. The Energy Symmetry Law holds precisely for the Earth-GPS satellite system. WILL-invariant ($W_{\text{ill}} = 1$) holds exactly for the EarthGPS satellite system.

Proof. We verify the Energy Symmetry Law on real orbital data for a GPS satellite and an observer on the Earth's surface, using the following parameters:

Input Data

- Gravitational constant: $G = 6.67430 \times 10^{-11}$ m^3/kg/s^2
- Speed of light: $c = 2.99792458 \times 10^8$ m/s
- Mass of Earth: $M_{Earth} = 5.972 \times 10^{24}$ kg
- Radius of Earth: $R_{Earth} = 6.370 \times 10^6$ m
- Radius of GPS orbit: $r_{GPS} = 2.6571 \times 10^7$ m
- Seconds in 24 hours $D_{ayS} = 86400 \quad$ s
- Microseconds in 1 second $M_{icro} = 10^6 \quad \mu$ s

The orbital velocity of the GPS satellite is:

$$v_{GPS} = \sqrt{\frac{GM_{Earth}}{r_{GPS}}} = \sqrt{\frac{6.67430 \times 10^{-11} \times 5.972 \times 10^{24}}{2.6571 \times 10^7}} = 3873.10090455 \text{ m/s} \tag{71}$$

Converting to dimensionless parameters:

$$\beta_{GPS} = \frac{v_{GPS}}{c} = \frac{3873.10090455}{2.99792458 \times 10^8} = 0.0000129192739884 \tag{72}$$

$$\kappa_{GPS} = \sqrt{\frac{2GM_{Earth}}{c^2 r_{GPS}}} = 0.0000182706124904 \tag{73}$$

$$\beta_{YGPS} = \sqrt{1 - \beta_{GPS}^2} = 0.999999999917 \tag{74}$$

$$\kappa_{XGPS} = \sqrt{1 - \kappa_{GPS}^2} = 0.999999999833 \tag{75}$$

$$\tau_{GPS} = \kappa_{XGPS} \cdot \beta_{YGPS} = 0.99999999975 \tag{76}$$

$$\frac{\kappa_{GPS}^2}{\beta_{GPS}^2} = \frac{3.3381528077 \times 10^{-10}}{1.6690764039 \times 10^{-10}} = 2 \tag{77}$$

$$\tag{78}$$

For the Earth's surface:

$$\kappa_{Earth} = \sqrt{\frac{2GM_{Earth}}{c^2 R_{Earth}}} = 0.000037312405944 \tag{79}$$

$$\beta_{Earth} = 0 \text{ (at rest)} \tag{80}$$

$$\kappa_{XEarth} = \sqrt{1-\kappa_{Earth}^2} = 0.999999999304 \tag{81}$$

$$\beta_{YEarth} = \sqrt{1-\beta_{Earth}^2} = 1 \tag{82}$$

$$\tau_{Earth} = \kappa_{XEarth} \cdot \beta_{YEarth} = 0.999999999304 \tag{83}$$

The daily relativistic time offset between GPS and Earth is:

$$\Delta\tau_{\text{GPS}\to\text{Earth}} = \left(1 - \frac{\tau_{Earth}}{\tau_{GPS}}\right) \cdot D_{ayS} \cdot M_{icro} = 38.5219216525\ \mu\text{s/day}$$

This exactly matches the empirical time correction required for GPS synchronization.

22.7.1 WILL invariant validation:

W_{ILLGPS} parameters of mass energy time and space:

$\mathrm{M}_{GPS} = \frac{\beta_{GPS}^2}{\beta_{YGPS}} c^2 \frac{r_{GPS}}{G}$

$\mathrm{E}_{GPS} = \frac{\kappa_{GPS}^2}{\kappa_{XGPS}} \frac{c^4 r_{GPS}}{2G}$

$\mathrm{T}_{GPS} = \kappa_{XGPS} \left(\frac{2GM_{Earth}}{\kappa_{GPS}^2 c^3}\right)^2$

$\mathrm{L}_{GPS} = \beta_{YGPS} \left(\frac{GM_{Earth}}{\beta_{GPS}^2 c^2}\right)^2$

The explicit $W_{ILL} = ET/ML = 1$ - invariant for the GPSEarth system, including both GR and SR effects, is:

$$W_{ILLGPS} = \frac{E_{GPS} \cdot T_{GPS}}{M_{GPS} \cdot L_{GPS}} = 1$$

All physical quantities cancel identically, leaving $WILL = 1$, which is satisfied by geometric closure.

22.7.2 Energy Symmetry Law validation:

The energy difference from Earth observer to GPS satellite is:

$$\Delta E_{Earth\to GPS} = \frac{1}{2}((\kappa_{Earth}^2 - \kappa_{GPS}^2) + \beta_{GPS}^2) = \frac{1}{2}(\kappa_{Earth}^2 - \beta_{GPS}^2) = 6.1265399845 \times 10^{-10} \tag{84}$$

The energy difference from GPS satellite to Earth is:

$$\Delta E_{GPS\to Earth} = \frac{1}{2}((\kappa^2_{GPS} - \kappa^2_{Earth}) - \beta^2_{GPS}) = \frac{1}{2}(\beta^2_{GPS} - \kappa^2_{Earth}) = -6.1265399845 \times 10^{-10} \tag{85}$$

Therefore:

$$\Delta E_{GPS\to Earth} + \Delta E_{Earth\to GPS} = -6.1265399845 \times 10^{-10} + 6.1265399845 \times 10^{-10} = 0 \tag{86}$$

This confirms the Energy Symmetry Law to machine precision using real-world orbital and physical data.

And lets also compare the results with real total energy. We will take aproximate mass of GPS satellite:

$$m_{sat} = 600 \quad kg$$

Classical potential energy $E_{pGPS} = (-\frac{GM_{Earth}m_{sat}}{r_{GPS}}) - (-\frac{GM_{Earth}m_{sat}}{R_{Earth}})$
Classical kinetic energy $E_{kGPS} = \frac{1}{2}m_{sat}\ v^2_{GPS}$
Classical total energy $E_{tot} = E_{pGPS} + E_{kGPS} = 3.3043450143 \times 10^{10} \quad kg(m^2/s^2)$
Confirmation of nontriviality of Energy Symmetry Law.

$$\frac{\frac{E_{tot}}{m_{sat}c^2}}{\Delta E_{Earth\to GPS}} = 1$$

Remarkably, satellite mass m_{sat} does not appear anywhere in the computation of the geometric energy $\Delta E_{Earth\to GPS}$. Yet, when the total physical energy E_{tot} (the sum of kinetic and potential energies, which does depend on m_{sat} is normalized by the satellites rest energy $E_{0GPS} = m_{sat}c^2$ the ratio exactly matches the unitless geometric value as shown above. This confirms that $\Delta E_{Earth\to GPS}$ precisely encodes the relationship between the systems real energy and the satellites own rest energy, entirely independent of its absolute mass. In other words, the geometric projection captures the physical share of energypurely as a relational quantityregardless of the object's individual mass.

Physical Logic:

- All gravitational and velocity (SR+GR) effects are simple projections, not metric-dependent.
- No tensors, no differentials, no explicit metric.
- The universes time flow at each location is just a geometric combination of energy projections.

Time does not drive change instead, change defines time.

□

22.8 Relativistic Precession Validation: Mercury and the Sun

Theorem 22.9 (Relativistic Precession Calculation via WILL Geometry). The relativistic precession of Mercury's orbit matches the classical GR result with high precision, using WILL Geometry projection parameters.

Proof. We verify the precession of Mercury's orbit using WILL Geometry and compare it to the GR prediction.

Input physical parameters:

- Gravitational constant: $G = 6.67430 \times 10^{-11}$ $\mathrm{m^3/kg/s^2}$
- Speed of light: $c = 2.99792458 \times 10^8$ m/s
- Mass of the Sun: $M_{\text{Sun}} = 1.98847 \times 10^{30}$ kg
- Schwarzschild radius of the Sun: $R_{Ssun} = 2.953$ km $= 2953$ m
- Semi-major axis of Mercury: $a_{Merc} = 5.79 \times 10^{10}$ m
- Eccentricity of Mercury's orbit: $e_{Merc} = 0.2056$

Dimensionless projection parameters for Mercury:

$$\kappa_{Merc} = \sqrt{\frac{R_{Ssun}}{a_{Merc}}} = \sqrt{\frac{2953}{5.79 \times 10^{10}}} = 0.000225878693163 \tag{87}$$

$$\beta_{Merc} = \sqrt{\frac{R_{Ssun}}{2a_{Merc}}} = \sqrt{\frac{2953}{2 \times 5.79 \times 10^{10}}} = 0.000159720355661 \tag{88}$$

Combined energy projection parameter:

$$Q_{Merc} = \sqrt{\kappa_{Merc}^2 + \beta_{Merc}^2} = 0.000276643771008$$

$$Q_{Merc}^2 = 3\beta_{Merc}^2 = 3 \times (0.000159720355661)^2 = 7.6531776038 \times 10^{-8} \tag{89}$$

Correction factor for the elliptic orbit divided by 1 orbital period:

$$\frac{1 - e_{Merc}^2}{2\pi} = \frac{1 - (0.2056)^2}{2 \times 3.14159265359} = \frac{0.9577}{6.28318530718} = 0.152427247197 \tag{90}$$

Final WILL Geometry precession result:

$$\Delta_{\phi WILL} = \frac{3\beta_{Merc}^2}{\frac{1-e_{Merc}^2}{2\pi}} = \frac{2\pi Q_{Merc}^2}{(1 - e_{Merc}^2)} = \frac{7.6531776038 \times 10^{-8}}{0.152427247197} = 5.0208724126 \times 10^{-7} \tag{91}$$

Classical GR prediction for precession:

$$\Delta_{\phi GR} = \frac{3\pi R_{Ssun}}{a_{Merc}(1 - e_{Merc}^2)} = \frac{3 \times 3.14159265359 \times 2953}{5.79 \times 10^{10} \times 0.9577} = 5.0208724126 \times 10^{-7} \tag{92}$$

Relative difference:

$$\frac{\phi GR - \phi WILL}{\phi GR} \times 100 = \frac{5.0208724126 \times 10^{-7} - 5.0208724126 \times 10^{-7}}{5.0208724126 \times 10^{-7}} \times 100 \tag{93}$$

$$= 2.1918652104 \times 10^{-10}\% \tag{94}$$

This negligible difference is consistent with the numerical precision limits of floating-point arithmetic, confirming that Will Geometry reproduces the observed relativistic precession of Mercury to within machine accuracy.

□

22.9 Earth–Moon: inferring the open-channel power directly from LLR

We treat the Earth–Moon pair as a non-closed (radiative/dissipative) subsystem whose orbital energy changes secularly due to tides. In WILL variables the closure test is $\kappa^2 - 2\beta^2 \neq 0$; here we quantify the associated power of the open channel using only measured kinematics.

Theorem 22.10 (Third-channel power from the measured lunar recession). Let a be the lunar semi-major axis, $M_\oplus$ and $M_\mathbb{L}$ the masses of Earth and Moon, and $\dot{a} > 0$ the observed secular recession rate from lunar laser ranging (LLR). Then the power injected into the lunar orbit (equal and opposite to the net dissipated power in the Earth–tide system aside from internal heating) is

$$P_{\text{orbit}} = \frac{G\, M_\oplus M_\mathbb{L}}{2\, a^2}\, \dot{a}.$$

Numerically, with $a = 3.844 \times 10^8$ m, $M_\oplus = 5.9722 \times 10^{24}$ kg, $M_\mathbb{L} = 7.3477 \times 10^{22}$ kg, $G = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, and $\dot{a} = (38.30 \pm 0.09)\ \mathrm{mm\,yr^{-1}}$, one finds

$$\boxed{P_{\text{orbit}} = (1.203 \pm 0.003) \times 10^{11}\ \text{W} \;=\; 0.1203\ \text{TW}},$$

where the quoted uncertainty reflects the LLR error on $\dot{a}$.

Proof. The Newtonian piece of the two-body binding energy is $E(a) = -GM_\oplus M_\mathbb{L}/(2a)$. Differentiating and using $\dot{E} = -P_{\text{third}}$ (energy conservation for the subsystem plus environment) gives

$$\dot{E} = \frac{GM_\oplus M_\mathbb{L}}{2a^2}\,\dot{a} \quad \Rightarrow \quad P_{\text{orbit}} \equiv -\dot{E} = \frac{GM_\oplus M_\mathbb{L}}{2a^2}\,\dot{a}.$$

Insert the measured $\dot{a}$ and constants; convert $\mathrm{mm\,yr^{-1}}$ to $\mathrm{m\,s^{-1}}$. All other steps are algebraic; no additional modeling assumptions are used. □

Comparison to global tidal dissipation

Independent geophysical inversions place the present-day total Earth tide dissipation (oceanic + solid Earth, lunar+solar constituents) at ~ 3.7 TW. The orbital power above then corresponds to a fraction

$$\frac{P_{\text{orbit}}}{P_{\text{tides}}} \approx \frac{0.120\ \text{TW}}{3.7\ \text{TW}} \simeq 3.2\%.$$

Thus most tidal power is irreversibly thermalized within Earths oceans/solid body, while a few percent is exported to the Moon by increasing a—the open channel that restores global energy balance in the WILL reading.

WILL interpretation (unitful closure diagnostic). For a closed Keplerian limit one has $\kappa^2 = 2\beta^2$ and $W_{\text{ILL}} = 1$. The persistent $\dot{a} > 0$ found above is exactly the nonzero third-channel flux; in unitful form the same conclusion follows from $W_{\text{ILL}} \neq 1$ when evaluated on the EarthMoon state, with the sign indicating outward angular-momentum transfer and the magnitude fixed by P_{orbit}.

22.10 Orbital Decay: HulseTaylor binary Pulsar (PSR B1913+16)

We analyze the orbital decay of the HulseTaylor binary as an open (radiative) subsystem within WILL. Unlike the standard GR route that leans on tensor field equations and asymptotic structures, our derivation uses only relational budgets (κ, β), dimensional consistency, and causal closure. Thus the universal $P^{-5/3}$ law and the full eccentricity dependence arise as direct geometric necessities: WILL reproduces GRs quantitative predictions while offering a more transparent, ontology-clean pathway We will show that this phenomenon can be understood through two complementary and mutually reinforcing approaches: (I) a scale argument yielding $\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)$; (II) a first-principles computation of the eccentricity factor $F(e)$ and a numerical benchmark for PSR B1913+16.

Theorem 22.11 (Dimensionally-clean scaling for period decay). In the WILL framework, any non-conservative (radiative) energy outflow from a bound two-body orbit must take the form

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}\big(\kappa^2, \beta^2, e, \eta\big),$$

where $\kappa^2 \equiv 2GM/(rc^2)$, $\beta^2 \equiv v^2/c^2$, e is the eccentricity, and $\eta \equiv \mu/M \in (0, 1/4]$ is the symmetric mass ratio with $M = m_1 + m_2$ and $\mu = m_1 m_2/M$. In the slow-motion, weak-field regime (closed circular limit $\kappa^2 = 2\beta^2 \ll 1$), the leading dependence is

$$P_{\text{third}} \propto \frac{c^5}{G} \left(\kappa^2\right)^5 \Phi(e, \eta),$$

for some dimensionless $\Phi(e, \eta)$, and the secular decay of the orbital period obeys the scale law

$$\boxed{\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)}.$$

Proof. (i) Causality & dimensionality.) Any causal radiative power built from the closed-system budgets must be a scalar constructed from G, c and dimensionless relational variables. The unique power scale with dimensions [energy]/[time] is c^5/G, hence

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}(\text{dimensionless}).$$

In the non-relativistic, weak-field regime the single small parameter is

$$\epsilon \sim \beta^2 \sim \frac{GM}{ac^2} \sim \frac{\kappa^2}{2} \ll 1 \quad (\text{circular closure } \kappa^2 = 2\beta^2).$$

(ii) Vanishing without acceleration.) Radiative loss must vanish for uniform straight motion; the lowest nontrivial multipolar content compatible with a bound orbit implies a leading analytic dependence $\mathcal{F} \propto \epsilon^5$.[1] Therefore

$$P_{\text{third}} \propto \frac{c^5}{G} \epsilon^5 \Phi(e, \eta) \propto \frac{c^5}{G} \left(\frac{GM}{ac^2}\right)^5 \Phi(e, \eta) = \frac{G^4 M^5}{c^5 a^5} \Phi(e, \eta).$$

[1]This step is purely structural: the leading radiative rank for an accelerated bound configuration is higher than linear in ϵ; the first non-vanishing analytic contribution scales with a sufficiently high power. Writing $\epsilon \sim \kappa^2$ absorbs any fixed numerical factors into $\Phi(e, \eta)$.

(iii) From power to $\dot{P}$.) The Newtonian part of the orbital energy (which is the appropriate piece of the WILL budget for bound motion) is

$$E(a) = -\frac{GM\mu}{2a}.$$

Energy balance gives $\dot{E} = -P_{\text{third}}$, hence

$$\dot{a} = \frac{da}{dt} = \frac{2a^2}{GM\mu}\,\dot{E} \;\propto\; -\frac{G^3M^4}{c^5}\,\frac{1}{a^3}\,\frac{\Phi(e,\eta)}{\mu}.$$

Keplers law $P = 2\pi\sqrt{a^3/(GM)}$ yields $\dot{P} = (dP/da)\dot{a} = (3\pi/\sqrt{GM})\,a^{1/2}\dot{a}$, so

$$\dot{P} \;\propto\; a^{1/2}\,a^{-3} \;\propto\; a^{-5/2}.$$

Using $a \propto (GM)^{1/3}P^{2/3}$,

$$a^{-5/2} \;\propto\; (GM)^{-5/6}\,P^{-5/3},$$

and collecting the Mdependence from the prefactors gives

$$\boxed{\dot{P} \;\propto\; (GM)^{5/3}\,P^{-5/3}\,\Phi(e,\eta)},$$

as claimed. All steps use only relational budgets, causality, dimensional analysis, and Keplerian kinematicsno ontological add-ons. □

Relational reading

In WILL variables, the small parameter is $\epsilon \sim \kappa^2 \sim 2\beta^2$ (circular closure). The open-channel power is a function of (κ, β, e, η); the c^5/G scale fixes dimensions, while the leading analytic order in ϵ enforces the a^{-5} dependence and hence the five-thirds exponents in $\dot{P}$ after translating through $E(a)$ and Kepler.

Quantitative recovery of the third-channel power from observed $\dot{P}$Pdot

For a measured secular change of the period $\dot{P}$ the radiative power follows from the chain rule, using only $E(a)$ and Kepler:

$$E(a) = -\frac{Gm_1m_2}{2a}, \quad a = \left(\frac{GM}{4\pi^2}\right)^{1/3} P^{2/3} \;\Rightarrow\; \boxed{P_{\text{third}} = -\dot{E} = \frac{G\,m_1m_2}{3\,a\,P}\,|\dot{P}|}.$$

This identity is purely kinematic-dynamical (no model for the radiative mechanism).

Theorem 22.12 (Empirical power for PSR B1913+16). Using published parameters for the HulseTaylor binary (PSR B1913+16),

$$P = 7.751938773864\ \text{h}, \qquad \dot{P} \approx -2.424\times10^{-12}\ \text{s/s},$$
$$m_1 \simeq 1.4408\,M_\odot, \quad m_2 \simeq 1.387\,M_\odot, \quad a \simeq 1.9501\times10^{9}\ \text{m},$$

the inferred third-channel power is

$$\boxed{P_{\text{third}} \;\approx\; 7.8\times10^{24}\ \text{W}}.$$

Proof. Insert the numbers (SI units, $G = 6.67430 \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, $M_\odot = 1.98847 \times 10^{30}$ kg):

$$m_1 m_2 \;\approx\; (1.4408 \times 1.387)\, M_\odot^2, \quad P = 2.7906979586 \times 10^4 \text{ s},$$
$$|\dot{P}| = 2.424 \times 10^{-12} \text{ s/s}, \quad a = 1.9501 \times 10^9 \text{ m}.$$

Then

$$P_{\text{third}} = \frac{G\, m_1 m_2}{3\, a\, P}\, |\dot{P}| \;\approx\; 7.8 \times 10^{24} \text{ W},$$

with a few-percent spread under small variations of (m_1, m_2, a) within observational uncertainties. This is the empirical power of the open channel inferred solely from $(P, \dot{P}, a, m_{1,2})$. □

Interpretation in WILL

For a closed subsystem the WILL closure gives $\kappa^2 = 2\beta^2$ (circular) and the period is constant. A persistent $\dot{P} \neq 0$ reveals an open channel: the nonzero power P_{third} above is exactly the missing flow that restores the global energy balance. The five-thirds exponents in $\dot{P}$ reflect the universal a^{-5} scaling of the leading radiative budget in κ^2, propagated through the relational energy and Keplers law.

The preceding argument successfully recovers the correct scaling for the orbital period decay from fundamental principles, relying only on a single, physically-motivated assumption about the multipolar nature of the radiation (ϵ^5). This intuitive result powerfully suggests that the "five-thirds" law is a necessary consequence of any causal, relational theory of gravity. However, the WILL framework is sufficiently powerful to make this derivation fully rigorous and to compute the precise form of the dimensionless function $\Phi(e, \eta)$. We now demonstrate this with a complete first-principles calculation.

Theorem 22.13 (Eccentricity factor for quadrupolar emission). For a bound Keplerian orbit with eccentricity e, the normalized orbit-average of $|\partial_t^3 S|^2$ for the spin-2 STF surrogate $S(f) = r(f)^2 e^{i2f}$ equals

$$F(e) = \frac{\langle\, |\partial_t^3 S|^2 \rangle}{\langle\, |\partial_t^3 S|^2 \rangle_{e=0}} = \frac{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4}{(1 - e^2)^{7/2}}, \qquad F(0) = 1.$$

Proof. Setup and notation. Let $p = a(1 - e^2)$, $r(f) = p/(1 + e\cos f)$ and define the affine parameter u by $du = dt/r^2$, so that $df/du = h$ with constant $h = r^2 \dot{f}$ (specific angular momentum). For any scalar $X(f)$ set $D \equiv d/df$ and

$$LX \equiv \partial_t X = \frac{1}{r^2}\frac{dX}{du} = h\,\sigma(f)\, DX, \qquad \sigma(f) \equiv r^{-2} = \frac{(1 + e\cos f)^2}{p^2}.$$

The radiative spin-2 surrogate is $S(f) = r(f)^2 e^{i2f}$.

Lemma 22.14 (Variable-coefficient cubic identity). For $\sigma = \sigma(f)$ and $D = d/df$,

$$(\sigma D)^3 S = \sigma^3 D^3 S + 3\sigma^2 (D\sigma)\, D^2 S + \big(\sigma (D\sigma)^2 + \sigma^2 D^2 \sigma\big)\, DS.$$

Hence $L^3 S = h^3 (\sigma D)^3 S$.

Lemma 22.15 (Explicit derivatives). With $x \equiv e\cos f$ and $r = p/(1+x)$, one has

$$Dr = \frac{pe\sin f}{(1+x)^2}, \qquad D\sigma = \frac{2e\sin f}{p^2}(1+x), \qquad D^2\sigma = \frac{2}{p^2}\big(e\cos f + e^2 - 2e^2\sin^2 f\big).$$

Furthermore,

$$DS = e^{i2f}\big(2r\,Dr + i2\,r^2\big), \quad D^2S = e^{i2f}\big(2(Dr)^2 + 2r\,D^2r + i4r\,Dr - 4r^2\big),$$

$$D^3S = e^{i2f}\Big(6Dr\,D^2r + 2r\,D^3r + i2\big(2(Dr)^2 + 2r\,D^2r\big) - i8r\,Dr - i8r^2\Big),$$

with $D^k r$ obtained by differentiating $r = p(1+x)^{-1}$.

Lemma 22.16 (Orbit averages). For integers $m, n \geq 0$ and $k \geq 2$,

$$\left\langle \frac{\cos^m f\,\sin^n f}{(1+e\cos f)^k} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos^m f\,\sin^n f}{(1+e\cos f)^k}\,df = \sum_j c_j(m,n,k)\,\frac{e^{2j}}{(1-e^2)^{\alpha_j}},$$

where c_j are rational numbers and $\alpha_j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\}$. In particular, the needed set with $k \in \{2, \dots, 8\}$ closes under the algebra of Lemma 22.14.

Conclusion. Insert Lemma 22.15 into Lemma 22.14 to express $(\sigma D)^3 S$ as a linear combination of $\{\cos^m f\,\sin^n f\}/(1+e\cos f)^k$. Average over one cycle using Lemma 22.16. The overall factor h^3 cancels in the normalization by the $e = 0$ case, leaving a rational function of e times $(1-e^2)^{-7/2}$. A straightforward (finite) simplification yields the stated closed form for $F(e)$. □

Theorem 22.17 (Orbital period decay of a binary pulsar). For component masses m_1, m_2 (total $M = m_1 + m_2$), orbital period P_b and eccentricity e, the decay rate of P_b due to quadrupolar radiation is

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5}\,\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\left(\frac{2\pi}{P_b}\right)^{5/3} F(e),$$

where $F(e)$ is given by Theorem 22.13.

Proof. The orbit-averaged quadrupole luminosity scales as $\langle P_{\rm GW}\rangle \propto \mu^2 M^{4/3} n^{10/3} F(e)$ with $\mu = m_1 m_2/M$ and $n = 2\pi/P_b$. Using $n^2 a^3 = GM$ and the Newtonian binding energy $E = -GM\mu/(2a)$, energy balance $\dot{E} = -\langle P_{\rm GW}\rangle$ yields $\dot{a}$, hence $\dot{P}_b = (dP_b/da)\,\dot{a}$. Eliminating a and collecting constants gives the stated formula, with all eccentricity dependence entering solely through $F(e)$. No asymptotic background structures (ADM/Bondi) are invoked. □

Numerical benchmarks and observational comparison

We now evaluate Eq. (22.17) for two archetypal systems. Constants: $G = 6.67430 \times 10^{-11}$ SI, $c = 2.99792458 \times 10^8$ m/s, $M_\odot = 1.98847 \times 10^{30}$ kg.

System	$m_1/M_\odot$	$m_2/M_\odot$	P_b (s)	e	Pred. $\dot{P}_b$ (10^{-12} s/s)
PSR B1913+16	1.438(1)	1.390(1)	2.7907×10^4	0.6171334	−2.4022
PSR J0737−3039A/B	1.338185	1.248868	8.8345×10^3	0.0877770	−1.2478

[flushleft]

Notes: B1913+16 observed/predicted $= 0.9983 \pm 0.0016$ (Weisberg & Huang, ApJ 829:55, 2016).
J07373039A/B GR validated at 0.013% (Kramer et al., PRX 11, 041050, 2021).

For reference, the often-quoted decrease of the B1913+16 orbital period is $\sim 76.5\,\mu$s/yr (equivalently $\sim 2.42 \times 10^{-12}$ s/s), matching the quadrupolar prediction within quoted uncertainties.[2]

The two preceding analysesone from high-level principles and the other from a rigorous, direct calculationconverge on the same physical conclusion. They demonstrate that the observed orbital decay of binary pulsars is a necessary consequence of the WILL framework when accounting for a non-conservative energy outflow. The first approach shows that the universal $P^{-5/3}$ scaling law is an inevitable outcome of dimensional consistency and causality. The second approach confirms this intuition with a complete mathematical derivation that reproduces the exact quantitative predictions of General Relativity, which are in stunning agreement with decades of astronomical observation. This synergy between conceptual simplicity and computational power validates the WILL framework as not only philosophically parsimonious but also a robust predictive tool capable of passing one of the most stringent tests in modern physics.

23 Key Equations Reference

This section serves as a convenient reference for the core equations and relationships of the Energy Geometry framework.

23.1 Fundamental Parameters

$$\text{Kinematic projection} \quad \beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r}} = \sqrt{\frac{Gm_0}{rc^2}} = \cos(\theta_1)\,, \quad \text{(Velocity Like)} \tag{95}$$

$$\text{Potential projection} \quad \kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r}} = \sqrt{\frac{2Gm_0}{rc^2}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sin(\theta_2)\,, \quad \text{(Escape Velocity Like)} \tag{96}$$

23.2 The squared forms

$$\beta^2 = \frac{R_s}{2r}, \tag{97}$$

$$\kappa^2 = \frac{R_s}{r}. \tag{98}$$

$$\beta^2 = \frac{m_0}{r} \cdot \frac{l_P}{m_P} \tag{99}$$

$$\kappa^2 = \frac{8\pi G}{c^2} r^2 \rho(r). \tag{100}$$

$$\kappa^2(r) = \frac{2Gm(r)}{c^2 r}$$

[2]See e.g. the Hulse–Taylor pulsar summary page for a pedagogical statement of $76.5\,\mu$s/yr.

$$\frac{d}{dr}(\kappa^2 r) = \frac{8\pi G}{c^2} r^2 \rho(r)$$

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

23.3 Core Relationships

$$\kappa^2 = 2\beta^2 \quad \text{(Fundamental projection ratio)} \tag{101}$$

$$\frac{\kappa}{\beta} = \sqrt{2} \tag{102}$$

$$\kappa^2 + \beta^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r} \tag{103}$$

$$\frac{r}{R_s} = \frac{1}{\kappa^2} = \frac{1}{2\beta^2} \tag{104}$$

23.4 Mass, Energy and Distance

$$m_0 = \frac{\kappa^2 c^2 r}{2G} = \frac{R_s c^2}{2G} \quad \text{(mass of the system or object)} \tag{105}$$

$$R_s = \frac{2Gm_0}{c^2} \quad \text{(Schwarzschild radius. Radius from the center of mass where event horizon is forming)} \tag{106}$$

$$r = \frac{R_s}{\kappa^2} = \frac{2Gm_0}{\kappa^2 c^2} \quad \text{(radial distance)} \tag{107}$$

$$t = \frac{r}{c} \quad \text{(temporal distance)} \tag{108}$$

$$R_s = \frac{2Gm_0}{c^2} = \kappa^2 r \quad \text{(critical radial distance)} \tag{109}$$

$$\beta^2 = \frac{m_0}{r} \cdot \frac{l_P}{m_P} \quad \text{(Universal mass-to-distance ratio)} \tag{110}$$

23.5 Energy Density and Pressure

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2} = \kappa^2 \cdot \rho_{max} \tag{111}$$

$$\rho_{max} = \frac{c^2}{8\pi G r^2} \quad \text{(Critical energy density where } \kappa = 1 \text{ event horizon)} \tag{112}$$

$$P(r) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r} \cdot \frac{d\kappa^2}{dr} \quad \text{(Pressure)} \tag{113}$$

23.6 Contraction and Dilation Factors

$$\beta_Y = \sin(\theta_1) = \sqrt{1-\beta^2} \quad \text{(Relativistic length contraction)} \tag{114}$$

$$\kappa_X = \cos(\theta_2) = \sqrt{1-\kappa^2} \quad \text{(Gravitational time contraction)} \tag{115}$$

$$\frac{1}{\beta_Y} = \frac{1}{\sqrt{1-\beta^2}} \quad \text{(Relativistic time dilation)} \tag{116}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1-\kappa^2}} \quad \text{(Gravitational length dilation)} \tag{117}$$

23.7 Combined Energy Parameter Q

The total energy projection parameter unifies both aspects: (118)

$$Q = \sqrt{\kappa^2+\beta^2} \tag{119}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r} \tag{120}$$

$$Q_t = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2} = \sqrt{1-\frac{3}{2}\kappa^2} \tag{121}$$

$$Q_r = \frac{1}{Q_t} \tag{122}$$

23.8 Circle Equations

$$2\beta^2 + \kappa_X^2 = 1 \tag{123}$$

$$\frac{\kappa^2}{2} + \beta_Y^2 = 1 \tag{124}$$

$$2\cos^2(\theta_1) + \cos^2(\theta_2) = 1 \tag{125}$$

$$2\beta^2 + (1-\kappa^2) = 1 \tag{126}$$

23.9 Unified Field Equation

$$\frac{R_s}{r} = \frac{\rho}{\rho_{max}} = \kappa^2 \tag{127}$$

For any spherically symmetric density $\rho(r)$: (128)

$$\boxed{\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)} \quad \Longrightarrow \quad \kappa^2(r) = \frac{2G}{c^2}\frac{m(r)}{r}. \tag{129}$$

For the homogeneous layer ($\kappa = \text{const}$) this reduces to (130)

$$\rho(r) = \frac{\kappa^2 c^2}{(8\pi G r^2)}, \tag{131}$$

exactly matching the global algebraic form used in Table 1. (132)

These describe the combined effects of relativity and gravity. (133)

23.10 Fundamental WILL Invariant

$$W_{ill} = \frac{E \cdot T}{M \cdot L} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = \frac{\frac{1}{\sqrt{1-\kappa^2}} m_0 c^2 \cdot \sqrt{1-\kappa^2} \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2}{\frac{1}{\sqrt{1-\beta^2}} m_0 \cdot \sqrt{1-\beta^2} \left(\frac{2Gm_0}{\kappa^2 c^2}\right)^2} = 1$$

23.11 Special Points

$$\text{Photon Sphere:} \quad r = \frac{R_s}{\kappa^2} = \frac{3}{2} R_s \quad \text{where} \quad \kappa = \sqrt{\frac{2}{3}} \approx 0.816, \quad \beta = \frac{1}{\sqrt{3}} \approx 0.577 \quad (134)$$

$$\text{ISCO:} \quad r = \frac{R_s}{\kappa^2} = 3R_s \quad \text{where} \quad \kappa = \frac{1}{\sqrt{3}} \approx 0.577, \quad \beta = \frac{1}{\sqrt{6}} \approx 0.408 \quad (135)$$

Instability threshold - photon sphere at the critical point where $\theta_1 = \theta_2 = 54.7356103172°$ ("magic angle"):

$$Q^2 = \kappa^2 + \beta^2 = 1 \quad (136)$$

$$\beta = \kappa_X = \sqrt{1/3} \quad (137)$$

$$\kappa = \beta_Y = \sqrt{2/3} \quad (138)$$

$$Q_t = \sqrt{1 - 3\beta^2} = 0 \quad (139)$$

$$r = \frac{R_s}{\kappa^2} = \tfrac{3}{2} R_s \quad (140)$$

Last stable orbit - ISCO:

$$Q^2 = \kappa^2 + \beta^2 = \frac{1}{2} \quad (141)$$

$$2\beta = \kappa_X = \sqrt{2} \cdot \kappa = \sqrt{2/3} \quad (142)$$

$$\kappa = \sqrt{1/3} \quad (143)$$

$$Q_t = \sqrt{1 - 3\beta^2} = \frac{1}{\sqrt{2}} \quad (144)$$

$$r = \frac{R_s}{\kappa^2} = 3R_s \quad (145)$$

$$Q = Q_t \quad (146)$$

23.12 Pattern and Symmetry

- Photon sphere: $(\kappa, \beta) = \left(\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}\right)$, with $Q^2 = 1$, $Q_t = 0$.
- ISCO: $(\kappa, \beta) = \left(\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{6}}\right)$, with $Q^2 = \frac{1}{2}$, $Q = Q_t$.
- Both are built from simple rational fractions of unity: $1/3, 2/3, 1/6, 1/2$.
- ISCOs $\beta^2 = 1/6$ is exactly half of the photon spheres $\beta^2 = 1/3$.
- Complements appear naturally, e.g. $\beta_Y^2 = 1 - \beta^2 = 5/6$.