Foundations of Relativity

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1 Geometric Derivation of $E = mc^2$ Ab Initio

In standard physics, this identity is often introduced via dynamical or conservation arguments.

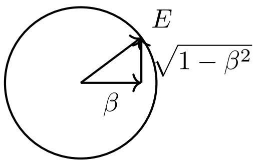
Theorem 1. Within the WILL Geometry framework $E = mc^2$ emerges purely from first principles of relational projections, without assuming any spacetime metric or pre-existing formula for rest energy.

Proof. **Circular Projection and Relational Scale**

- Consider a circle of radius c. Every point on this circle—parametrized by angle θ_S—represents a possible distribution of "rate of change" between spatial and temporal components.
- Define $\beta = \cos(\theta_S)$. Then $L_c = \sin(\theta_S) = \sqrt{1-\beta^2}$. Equivalently,

  $$\beta \;=\; \frac{v}{c}, \quad L_c \;=\; \sqrt{1-\beta^2} \;=\; \sqrt{1-\frac{v^2}{c^2}}.$$

  This structure forms a right triangle in projection space:

  where the hypotenuse represents the total energy vector E, geometrically arising from the projectional composition of motion and inertia.
- Introduce a single positive constant E_0 (with units of energy) as the "minimal" or "reference" projection when $\beta = 0$. In other words, at $\theta_S = \frac{\pi}{2}$ (i.e. $v = 0$), the vertical projection of the total energy must be exactly E_0. Thus E_0 is not assumed to equal mc^2 yet; it simply sets the physical scale.

Accordingly, any point on the circle at angle θ_S has two dimensionless coordinates:

$$(\cos(\theta_S),\ \sin(\theta_S)) \;=\; \big(\beta,\ \ L_c\big).$$

Multiply these by the same factor E (the total energy in physical units) to obtain physical projections:

$$E\,\cos(\theta_S) = E\,\beta, \qquad E\,\sin(\theta_S) = \,E\,L_c.$$

By construction, when $\beta = 0$ ($\theta_S = \frac{\pi}{2}$), we require

$$E\,\sin\!\big(\tfrac{\pi}{2}\big) \;=\; E_0 \quad\Longrightarrow\quad E_0 = E \cdot 1.$$

Hence in general,

$$E\,\sin(\theta_S) \;=\; E_0 \quad\Longrightarrow\quad E = \frac{E_0}{\sin(\theta_S)} \;=\; \frac{E_0}{\sqrt{1-\beta^2}}.$$

Define

$$\gamma \;=\; T_d \;=\; \frac{1}{\sqrt{1-\beta^2}}, \quad \text{so that} \quad E = \gamma\,E_0.$$

Horizontal (Momentum) Projection

In classical mechanics, momentum is $p = m\,v$. We have introduced neither m nor $m\,c^2$ yet—only E_0. Observe:

$$E\,\cos(\theta_S) \;=\; E\,\beta = \gamma\,E_0\,\beta.$$

We identify the horizontal projection $E\,\beta$ with $p\,c$. Thus

$$p\,c \;=\; \gamma\,E_0\,\beta.$$

But since $\beta = v/c$, this becomes

$$p\,c = \gamma\,E_0\,\frac{v}{c} \quad\Longrightarrow\quad p = \frac{\gamma\,E_0}{c^2}\,v.$$

At this stage, E_0 remains an unspecified constant with dimensions of energy; the ratio E_0/c^2 has the dimensions of mass. Define

$$m \;\equiv\; \frac{E_0}{c^2}.$$

(Here m is merely the dimensional coefficient linking the scale energy E to the invariant c^2)

Hence

$$E_0 = m\,c^2, \quad p = \gamma\,m\,v, \quad E = \gamma\,m\,c^2.$$

Notice that no step assumed $E_0 = m\,c^2$ as a given "rest-energy" formula; instead, we merely identified the combination E_0/c^2 with a physical mass parameter m. The functional dependence on γ was already fixed by the circle's geometry.

Pythagorean Relation and $E^2 = (pc)^2 + (mc^2)^2$

Having

$$E\,\sin(\theta_S) = E_0 = m\,c^2, \quad E\,\cos(\theta_S) = p\,c,$$

the Pythagorean theorem in projection space yields

$$E^2 = \big(E\,\cos(\theta_S)\big)^2 + \big(E\,\sin(\theta_S)\big)^2 = (p\,c)^2 + (m\,c^2)^2.$$

Since $E = \gamma\,m\,c^2$ and $p = \gamma\,m\,v$, this recovers the standard relativistic energy–momentum relation:

$$\boxed{E^2 = m^2c^4 + p^2c^2} \tag{1}$$

1.1 Why Rest-Energy Invariance Follows from Conservation

Within the WILL framework, the total energy vector $\mathbf{E}$ is represented by the hypotenuse of a right triangle inscribed in a unit circle of radius c. Its dimensionless projections satisfy

$$\beta = \cos\theta_S, \quad L_c = \sin\theta_S, \quad \beta^2 + L_c^2 = 1.$$

When scaled by the physical magnitude E, these become

$$E_x = E\,\cos\theta_S, \quad E_y = E\,\sin\theta_S.$$

To connect with the notion of "rest," we operationally define the *rest-energy* E_0 via the projection at zero spatial velocity ($\beta = 0,\ \theta_S = \frac{\pi}{2}$):

$$E_y\big|_{\theta_S=\pi/2} = E\,\sin\tfrac{\pi}{2} = E_0.$$

Because the circle's Pythagorean identity $\cos^2\theta_S + \sin^2\theta_S = 1$ holds for all θ_S, the only way to preserve $E_y = E_0$ as β varies is

$$E\,\sin\theta_S = E_0 \quad\Longrightarrow\quad E = \frac{E_0}{\sin\theta_S} = \frac{E_0}{\sqrt{1-\beta^2}} = \gamma\,E_0.$$

Hence, the rest-energy projection's invariance is not an *additional* axiom but the *inevitable consequence* of:

1. The unit-circle geometry ($\beta^2 + L_c^2 = 1$),
2. The operational choice to define "rest" as the slice $\beta = 0$,
3. The conservation of that chosen projection E_y.

By anchoring this construction in our core postulate

$$\text{SPACETIME} \;\equiv\; \text{ENERGY EVOLUTION},$$

we derive $E = \gamma\, m\, c^2$ and $E^2 = (p\, c)^2 + (m\, c^2)^2$ directly from geometry, with no need for extra hypotheses.

1.2 Dual Scaling in WILL Geometry

The WILL framework reveals a striking symmetry: both energy and spatial distance emerge from the same unit-circle projections, differing only by the power of the inverse projection used.

- **Energy Scaling:** The total energy E scales inversely with the temporal projection $L_c = \sin\theta_S$:

  $$E = \frac{E_0}{\sin\theta_S} = \frac{E_0}{L_c} = \gamma\, E_0,$$

  so that $E \sin\theta_S = E_0$ remains invariant.

- **Distance Scaling:** The (normalized) radial distance r/R_s in both Special and General Relativity scales inversely with the square of the spatial projection $\beta = \cos\theta_S$:

  $$\frac{r}{R_s} = \frac{1}{2\,\beta^2} = \frac{1}{2\,\cos^2\theta_S}.$$

(The distance scaling form is consistent with the later relation $\kappa^2 = 2\beta^2$ derived in Section 7.3.)

In each case, a single dimensionless projection ($\sin\theta_S$ or $\cos\theta_S$) is elevated to a physical quantity by taking its reciprocal (to the appropriate power) and multiplying by a fixed scale (E_0 or $R_s/2$).

Fundamental Implication This duality underscores that at thFe deepest level, space, time, energy, and distance are all encoded in the same dimensionless geometry of the unit circle. Any physical observable arises from choosing:

1. A projection axis (β or L_c), 2. An inversion power (1 for energy, 2 for distance), 3. A conventional scale factor.

Thus, the entire edifice of relativistic kinematics and gravity can be built from pure, dimensionless projections plus minimal scale conventions."'

1.3 Mass as a Scale Factor

In WILL geometry, mass is not an independent dynamical variable but simply the conversion factor between the dimensionless "rest-energy invariant" E_0 and familiar SI units. Concretely:

$$m \;=\; \frac{E_0}{c^2}.$$

Thus:

- The rest-energy projection $E\,\sin\theta_S = E_0$ can be equivalently written $m\,c^2$.
- All inertial and gravitational effects (kinetic terms, Schwarzschild radius R_s, etc.) follow by substituting $E_0 = m\,c^2$ into the dual-scaling laws.

Interpretation

All relativistic properties—energy diverging as $v \to c$, correct normalization at $v = 0$, and the conservation of energy–momentum—follow immediately. In this framework:

- The unit circle embodies the conservation of the "universal rate of change" c.
- The parameter $\beta = v/c$ specifies a point on that circle.
- A single scale E_0 (later identified as $m\,c^2$) fixes the overall magnitude of the energy vector.
- No prior assumption of $E = m\,c^2$ is needed; rather, it is recovered by operational choice to define "rest" as the vertical projection equal to E_0 at $\beta = 0$.

Thus, $E = \gamma\,m\,c^2$ and $E^2 = (p\,c)^2 + (m\,c^2)^2$ are fully derived from the first principle that "energy evolution" is encoded in a self-contained circular geometry.

This geometric approach unifies our understanding of mass, energy, and momentum as different projections of the same fundamental quantity—the energy vector in WILL geometry—whose orientation is determined by the relative motion between observer and observed system. $\square$