@threeparttable

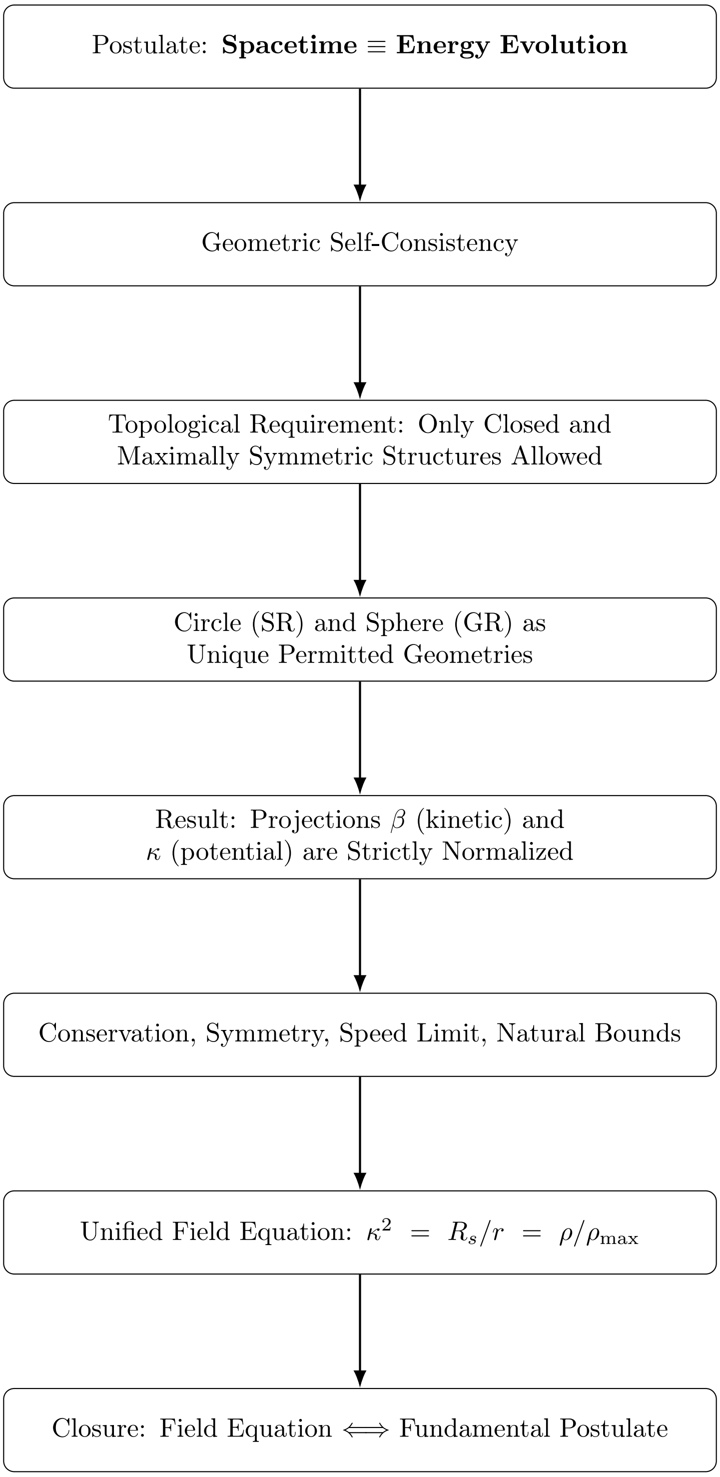
WILL: Unified Framework
PART I - Foundations of Relativity

Anton Rize egeometricity@gmail.com

April 2025

Abstract

This paper introduces the first part of the WILL series, a new geometric framework unifying special and general relativity. We posit a **single fundamental postulate**: $\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$. Spacetime is identical to energy evolution. This principle, unlike existing theories, introduces **no free parameters**; all measurable quantities emerge from purely **geometric projections** on maximally symmetric **closed manifolds**. This approach **eliminates** curvature **singularities**, unifies **gravitational and kinematic time dilation** without invoking dark matter or modified gravity, and yields **verifiable orbital relations**. This paper reconstructs the standard Schwarzschild and Kerr metrics, demonstrating the unique and necessary consequences of our **relational geometry**. We will detail the core principles, show the derivation of key relativistic equations, and outline the implications for future research.



Contents

1 WILL: A Relational Framework for Space-Time-Energy

2 Methodological Purity and Epistemic Minimalism

2.1 Foundational Approach

The framework of WILL Geometry is built from a single postulate and zero free parameters. This construction is not a simplification — it is a deliberate epistemic constraint. No assumptions are introduced unless they are strictly derivable from first principles, and no constructs are retained unless they are geometrically or energetically necessary.

> Guiding Principle
>
> **Nothing is assumed. Everything is derived.**
> *There are no "extra pieces" in reality.*

2.2 Epistemic Hygiene

Modern theoretical physics often tolerates hidden assumptions: arbitrary constants, renormalization techniques, or abstract entities with ambiguous physical status. WILL Geometry, by contrast, enforces what we call **epistemic hygiene** — a disciplined refusal to smuggle explanatory weight into unjustified assumptions. All physical quantities, including space, time, mass, and energy, emerge from relational geometry and causal structure.

2.3 No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent energetic projections, governed by the Energy Symmetry Law:

$$\boxed{\Delta E_{A\to B} + \Delta E_{B\to A} = 0}$$

(The precise form of ΔE is derived in Section 8.)

2.4 Principle of Sufficient Explanation

If a geometric relationship can explain an observation, no additional mechanism is introduced. WILL Geometry seeks not to describe the universe as we imagine it, but to reduce its complexity to the minimal algebraic and causal structure required to explain our measurements.

> Summary of WILL Geometry
>
> The entire physics of WILL Geometry is built upon a single postulate:
> **"Spacetime is identical to energy evolution."**
> All formal structure—closed geometry, the speed limit, energy conservation, the absence of singularities—inevitably follows from this postulate.
> The resulting equation is not something arbitrary or requiring any external assumptions; it is precisely the mathematical realization of the postulate itself.
> Thus, the theory is perfectly closed upon itself: *the fundamental principle is proven by its own consequences, and vice versa.*

2.5 Motivation and Core Principles

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

Principle: *All physical quantities must be defined purely by their relations.* Any introduction of absolute properties or external frames risks reintroducing metaphysical artifacts and contradicts the foundational insight of relativity.

Energy is not an intrinsic property,
but a *measure of difference*
between possible states.

What is Energy? Energy is not an intrinsic property of objects, but a measure of difference between possible states. It expresses the system's capacity to transition, always encountered as a relative potential for change—not as something possessed, but as a comparative structure between observers. In this framework, energy marks the directional tendency for one configuration to transform into another.

3 Single Unifying Postulate

We therefore posit a single unifying postulate:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

Clarification: By "energy evolution" we mean the total structure of possible transitions between observable states—not a process unfolding in spacetime, but the very relational geometry from which both space and time emerge. This is not a derived result, but a foundational postulate, subject to geometric and empirical audit in subsequent sections. "WILL" is used here as a technical term for this unified, emergent structure.

The goal of this framework is not merely to reconstruct the formulas of Special and General Relativity, but to demonstrate that they are the unique and necessary consequences of a single, deeper principle of relational geometry. By showing *why* these specific mathematical structures must arise, the model provides a more fundamental explanation for them than the standard formalisms alone.

3.1 Fundamental Structure of WILL

From our postulate, several key properties of the WILL framework can be derived:

1. **Self-contained geometry:** Since spacetime is identical to energy evolution, there can be no external objects or reference frames. The geometry must therefore be self-contained.
2. **Conservation law:** A closed geometry naturally leads to conservation principles—nothing enters or leaves the system, making the total energy constant.
3. **Symmetry:** With no external reference, all directions/positions must be equivalent; any asymmetry would require a preferred frame, which is disallowed.
4. **Circular geometry:** Among all closed and maximally symmetric geometries, the circle (and, in higher dimensions, surface of the sphere) uniquely preserves equivalence of all points and directions, ensuring that no hidden reference structure or asymmetry contaminates the framework.

3.2 Emergence of Space and Time

In this construction, space and time are not assumed as independent entities, but arise as complementary projections of the underlying rate and direction of energetic transformation along the closed geometric structure.

The word "evolution" in our postulate implies a non-instantaneous process, requiring a rate of change. This rate naturally establishes a relationship between what we perceive as spatial and temporal units, yielding the speed of evolution.

Any energy state can be represented as a point on the circle. Changes in energy states correspond to movement along this circle, characterized by:

- Speed of state change (rate of evolution)
- Direction of state change

These two orthogonal components naturally give rise to what we interpret as space-like and time-like dimensions without needing to postulate coordinate axes a priori.

Among closed, orientable, simply-connected manifolds of constant positive curvature,

- in 1-D, the circle S^1 is unique;
- in 2-D, the 2-sphere S^2 is unique.

These manifolds are distinguished by having the largest possible isometry group acting transitively, thereby eliminating any preferred directions or cycles. This allow them to serve as the only projection surfaces consistent with the WILL postulate and its demand for internal symmetry.

Theorem 1 (WILL Closure)**.** Spacetime $\equiv$ Energy Evolution.

Sketch. Sections 4–17 supply the chained geometric steps: closed-manifold symmetry $\rightarrow$ projection algebra $\rightarrow$ relativistic factors $\rightarrow$ unified field identity. □

3.3 Fundamental Axiom: WILL-Manifold $\mathcal{W}$

Definition 1 (Structure of the WILL-manifold)**.** The WILL-manifold $\mathcal{W}$ is the set of all possible energy–relation states, characterised by the following intrinsic properties:

1. For any two systems S_1, S_2 there exists a relational energy value $E(S_1, S_2) \in \mathcal{W}$.
2. The continuous evolution of these relations defines a *flow* on $\mathcal{W}$.
3. This flow endows $\mathcal{W}$ with a circular topology; continuity is defined by continuous energy transformations along that circle. [1]
4. Causality arises from the directional (orientable) character of the flow.
5. Local "intensity" (density of relations) determines the effective metric properties experienced by observers.

Definition 1 is not subject to proof; it specifies the ontological arena in which the subsequent *WILL Closure Theorem* is proved.

Table 1: Core symbols used in Part I

Symbol	Definition	Physical / Geometric meaning
β	v/c	Kinetic projection (spatial)
κ	$\sqrt{R_s/r_d}$	Potential projection (temporal)
R_s	$2Gm_0/c^2$	Schwarzschild scale (appears via $\kappa = 1$)
r_d	R_s/κ^2	Radial distance from center of mass m_0
L_c	$\sqrt{1-\beta^2}$	SR length-contraction factor
T_c	$\sqrt{1-\kappa^2}$	GR time-contraction factor
E_0	–	Rest-energy scale (sets units of mass)

4 Kinetic Energy

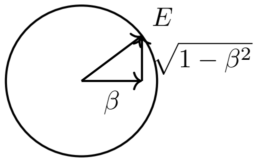
4.1 Reinterpretation of speed of light:

We interpreting the **speed of light c** as the **universal rate of change** implying that every energy transformation or interaction has the same rate = c distributed between spatial like and temporal like components. We expressing **universal rate of change c** as rotating radial vector so the unit circle naturally emerges as embodiment of conservation law. This choice is enforced by conservation of the fixed modulus c, forcing the endpoint to lie on the unit circle:

- $r = c = 1$ rotating radial vector on a unit circle controlled by:
- $\theta_S = \arccos(\beta)$ relativistic angle represents energy distribution by rotation of radial vector.
- $\beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r}} = \cos(\theta_S)$, **(Orbital velocity)** space like kinetic component (Xaxis)

[1]The circular topology will be justified in Section 4.1 via constancy of the universal rate of change.

- $L_c = \sqrt{1-\beta^2} = \sin(\theta_S)$ (Length contraction factor) temporal like component (Yaxis)
- $T_d = \frac{1}{L_c} = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sin(\theta_S)}$ (Time dilation factor) identical to relativistic γ factor.



In our model, the spatial and temporal-like projections are not to be interpreted as traditional coordinates, but as relative geometric ratios derived from the invariant rate of change.

4.2 Special Theory of Relativity (Time Dilation Factor)

The standard time dilation factor in Special Relativity is given by:

$$T_d = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{1}$$

Our geometric expression for the relativistic factor is:

$$T_d = \frac{1}{\sin(\arccos(\beta))}, \tag{2}$$

where:

$$\beta = \frac{v}{c}. \tag{3}$$

Derivation of Equivalence: 1. Start with the geometric expression:

$$T_d = \frac{1}{\sin(\arccos(\beta))}. \tag{4}$$

2. Use the trigonometric identity $\sin^2(x) + \cos^2(x) = 1$, which implies $\sin(x) = \sqrt{1-\cos^2(x)}$:

$$T_d = \frac{1}{\sqrt{1-\cos^2(\arccos(\beta))}}. \tag{5}$$

3. Evaluate $\cos(\arccos(\beta))$:

$$T_d = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma. \tag{6}$$

Conclusion: The derived expression is identical to the standard time dilation factor, demonstrating the equivalence.

4.3 Interpretation of β: Spatial Energy Projection

β quantifies the energy of an object or system as expressed in terms of the relative rate of change of spatial coordinates. Operationally, it reflects the amount of energy the observer would need to expend to "catch up" with the object.

The range of β is naturally constrained:

$$0 \leq \beta \leq 1$$

Beyond $\beta = 1$, the object moves faster than the speed at which causal signals propagate (speed of light in vacuum), and therefore becomes causally disconnected from the observer.

5 Geometric Derivation of $E = mc^2$ Ab Initio

In standard physics, this identity is often introduced via dynamical or conservation arguments.

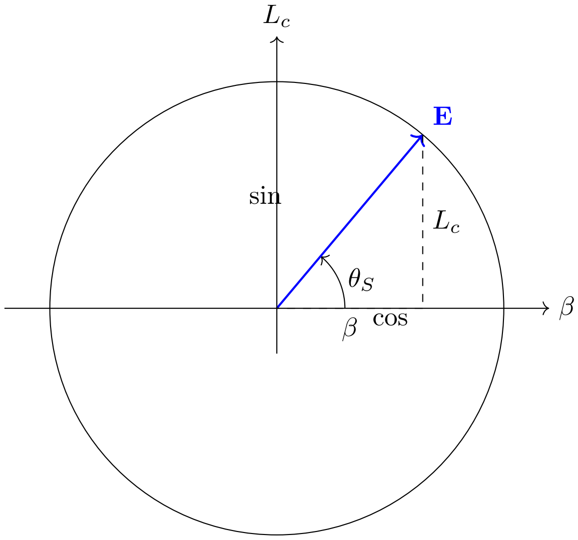
Theorem 2. Within the WILL Geometry framework $E = mc^2$ emerges purely from first principles of relational projections, without assuming any spacetime metric or pre-existing formula for rest energy.

Proof. **Circular Projection and Relational Scale**

- Consider a circle of radius c. Every point on this circle—parametrized by angle θ_S—represents a possible distribution of "rate of change" between spatial and temporal components.
- Define $\beta = \cos(\theta_S)$. Then $L_c = \sin(\theta_S) = \sqrt{1-\beta^2}$. Equivalently,

$$\beta \;=\; \frac{v}{c}, \quad L_c \;=\; \sqrt{1-\beta^2} \;=\; \sqrt{1-\frac{v^2}{c^2}}.$$

  This structure forms a right triangle in projection space:



  where the hypotenuse represents the total energy vector E, geometrically arising from the projectional composition of motion and inertia.

- Introduce a single positive constant E_0 (with units of energy) as the "minimal" or "reference" projection when $\beta = 0$. In other words, at $\theta_S = \frac{\pi}{2}$ (i.e. $v = 0$), the vertical projection of the total energy must be exactly E_0. Thus E_0 is not assumed to equal mc^2 yet; it simply sets the physical scale.

Accordingly, any point on the circle at angle θ_S has two dimensionless coordinates:

$$(\cos(\theta_S),\, \sin(\theta_S)) \;=\; \big(\beta, \;\; L_c\big).$$

Multiply these by the same factor E (the total energy in physical units) to obtain physical projections:

$$E\,\cos(\theta_S) = E\,\beta, \qquad E\,\sin(\theta_S) = \; E\,L_c.$$

By construction, when $\beta = 0$ ($\theta_S = \frac{\pi}{2}$), we require

$$E\,\sin\!\big(\tfrac{\pi}{2}\big) \;=\; E_0 \quad\Longrightarrow\quad E_0 = E\cdot 1.$$

Hence in general,

$$E\,\sin(\theta_S) \;=\; E_0 \quad\Longrightarrow\quad E = \frac{E_0}{\sin(\theta_S)} \;=\; \frac{E_0}{\sqrt{1-\beta^2}}.$$

Define

$$\gamma \;=\; T_d \;=\; \frac{1}{\sqrt{1-\beta^2}}, \quad \text{so that} \quad E = \gamma\,E_0.$$

Horizontal (Momentum) Projection

In classical mechanics, momentum is $p = m\,v$. We have introduced neither m nor $m\,c^2$ yet—only E_0. Observe:

$$E\,\cos(\theta_S) \;=\; E\,\beta = \gamma\,E_0\,\beta.$$

We identify the horizontal projection $E\,\beta$ with $p\,c$. Thus

$$p\,c \;=\; \gamma\,E_0\,\beta.$$

But since $\beta = v/c$, this becomes

$$p\,c = \gamma\,E_0\,\frac{v}{c} \quad\Longrightarrow\quad p = \frac{\gamma\,E_0}{c^2}\,v.$$

At this stage, E_0 remains an unspecified constant with dimensions of energy; the ratio E_0/c^2 has the dimensions of mass. Define

$$m \;\equiv\; \frac{E_0}{c^2}.$$

(Here m is merely the dimensional coefficient linking the scale energy E to the invariant c^2)

Hence

$$E_0 = m\,c^2, \quad p = \gamma\,m\,v, \quad E = \gamma\,m\,c^2.$$

Notice that no step assumed $E_0 = m\,c^2$ as a given "rest-energy" formula; instead, we merely identified the combination E_0/c^2 with a physical mass parameter m. The functional dependence on γ was already fixed by the circle's geometry.

Pythagorean Relation and $E^2 = (pc)^2 + (mc^2)^2$

Having

$$E\,\sin(\theta_S) = E_0 = m\,c^2, \quad E\,\cos(\theta_S) = p\,c,$$

the Pythagorean theorem in projection space yields

$$E^2 = \big(E\,\cos(\theta_S)\big)^2 + \big(E\,\sin(\theta_S)\big)^2 = (p\,c)^2 + (m\,c^2)^2.$$

Since $E = \gamma\,m\,c^2$ and $p = \gamma\,m\,v$, this recovers the standard relativistic energy–momentum relation:

$$\boxed{E^2 = m^2c^4 + p^2c^2} \tag{7}$$

5.1 Why Rest-Energy Invariance Follows from Conservation

Within the WILL framework, the total energy vector $\mathbf{E}$ is represented by the hypotenuse of a right triangle inscribed in a unit circle of radius c. Its dimensionless projections satisfy

$$\beta = \cos\theta_S, \quad L_c = \sin\theta_S, \quad \beta^2 + L_c^2 = 1.$$

When scaled by the physical magnitude E, these become

$$E_x = E\,\cos\theta_S, \quad E_y = E\,\sin\theta_S.$$

To connect with the notion of "rest," we operationally define the *rest-energy* E_0 via the projection at zero spatial velocity ($\beta = 0,\ \theta_S = \frac{\pi}{2}$):

$$E_y\big|_{\theta_S=\pi/2} = E\,\sin\tfrac{\pi}{2} = E_0.$$

Because the circle's Pythagorean identity $\cos^2\theta_S + \sin^2\theta_S = 1$ holds for all θ_S, the only way to preserve $E_y = E_0$ as β varies is

$$E\,\sin\theta_S = E_0 \quad\Longrightarrow\quad E = \frac{E_0}{\sin\theta_S} = \frac{E_0}{\sqrt{1-\beta^2}} = \gamma\,E_0.$$

Hence, the rest-energy projection's invariance is not an *additional* axiom but the *inevitable consequence* of:

1. The unit-circle geometry ($\beta^2 + L_c^2 = 1$),
2. The operational choice to define "rest" as the slice $\beta = 0$,
3. The conservation of that chosen projection E_y.

By anchoring this construction in our core postulate

$$\text{SPACETIME} \;\equiv\; \text{ENERGY EVOLUTION},$$

we derive $E = \gamma\,m\,c^2$ and $E^2 = (p\,c)^2 + (m\,c^2)^2$ directly from geometry, with no need for extra hypotheses.

5.2 Dual Scaling in WILL Geometry

The WILL framework reveals a striking symmetry: both energy and spatial distance emerge from the same unit-circle projections, differing only by the power of the inverse projection used.

- **Energy Scaling:** The total energy E scales inversely with the temporal projection $L_c = \sin\theta_S$:

$$E = \frac{E_0}{\sin\theta_S} = \frac{E_0}{L_c} = \gamma\, E_0,$$

  so that $E \sin\theta_S = E_0$ remains invariant.

- **Distance Scaling:** The (normalized) radial distance r/R_s in both Special and General Relativity scales inversely with the square of the spatial projection $\beta = \cos\theta_S$:

$$\frac{r}{R_s} = \frac{1}{2\,\beta^2} = \frac{1}{2\,\cos^2\theta_S}.$$

(The distance scaling form is consistent with the later relation $\kappa^2 = 2\beta^2$ derived in Section 7.3.)

In each case, a single dimensionless projection ($\sin\theta_S$ or $\cos\theta_S$) is elevated to a physical quantity by taking its reciprocal (to the appropriate power) and multiplying by a fixed scale (E_0 or $R_s/2$).

Fundamental Implication This duality underscores that at thFe deepest level, space, time, energy, and distance are all encoded in the same dimensionless geometry of the unit circle. Any physical observable arises from choosing:

1. A projection axis (β or L_c), 2. An inversion power (1 for energy, 2 for distance), 3. A conventional scale factor.

Thus, the entire edifice of relativistic kinematics and gravity can be built from pure, dimensionless projections plus minimal scale conventions."'

5.3 Mass as a Scale Factor

In WILL geometry, mass is not an independent dynamical variable but simply the conversion factor between the dimensionless "rest-energy invariant" E_0 and familiar SI units. Concretely:

$$m \;=\; \frac{E_0}{c^2}.$$

Thus:

- The rest-energy projection $E\,\sin\theta_S = E_0$ can be equivalently written $m\,c^2$.
- All inertial and gravitational effects (kinetic terms, Schwarzschild radius R_s, etc.) follow by substituting $E_0 = m\,c^2$ into the dual-scaling laws.

Interpretation

All relativistic properties—energy diverging as $v \to c$, correct normalization at $v = 0$, and the conservation of energy–momentum—follow immediately. In this framework:

- The unit circle embodies the conservation of the "universal rate of change" c.
- The parameter $\beta = v/c$ specifies a point on that circle.
- A single scale E_0 (later identified as $m\,c^2$) fixes the overall magnitude of the energy vector.
- No prior assumption of $E = m\,c^2$ is needed; rather, it is recovered by operational choice to define "rest" as the vertical projection equal to E_0 at $\beta = 0$.

Thus, $E = \gamma\, m\, c^2$ and $E^2 = (p\,c)^2 + (m\,c^2)^2$ are fully derived from the first principle that "energy evolution" is encoded in a self-contained circular geometry.

This geometric approach unifies our understanding of mass, energy, and momentum as different projections of the same fundamental quantity—the energy vector in WILL geometry—whose orientation is determined by the relative motion between observer and observed system. □

6 Potential Energy

6.1 Beyond Curvature: A New Perspective

Standard GR is commonly interpreted through curvature; WILL instead treats geometry as emerging from energy relations, without positing curvature as a primitive explanatory layer

Rather than imposing curvature as an interpretational construct, we allow the universe to naturally manifest its geometric structure through energy relations. This approach remains true to our foundational principle:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

6.2 Dimensional Correspondence of Energy Projections

The WILL framework maps physical phenomena to geometric projections based on their inherent nature. This is not an arbitrary choice, but a **foundational correspondence**:

- **Kinetic energy (β)** is fundamentally directional, describing motion along a path. Its natural representation within the framework is therefore a **1D projection**.
- **Gravitational potential (κ)** from a central mass is fundamentally isotropic, creating a spherically symmetric field. Its natural representation is therefore a **2D spherical projection**[2].

6.3 Boundary Conditions and Physical Interpretation

For kinetic energy parameter β:

- $\beta = 0$: No relative motion
- $\beta = 1$: Maximum relative motion, causal limit within one Energy system (speed of light)

By analogy, for the gravitational parameter κ:

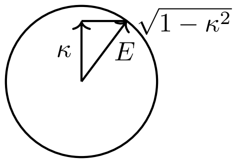
- $\kappa = 0$: No gravitational influence
- $\kappa = 1$: Maximum causal gravitational influence within one Energy system (event horizon)

The value $\kappa = 1$ corresponds precisely to the point where escape velocity equals the speed of light, creating an event horizon. This provides a natural causal limit for our gravitational parameter, analogous to the light-speed causal limit for relative motion.

Equivalent to our previous derivation of β as kinetic energy component, now we can derive κ:

- $\kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r}} = \sin(\theta_G)$, **(Escape velocity)**. time like potential component (Y axis)
- $\theta_G = \arcsin(\kappa)$ gravitational angle represents potential energy distribution.
- $T_c = \sqrt{1-\kappa^2} = \sqrt{1-\frac{R_s}{r}} = \cos(\theta_G)$ (Time contraction factor) spatial like component (X axis)
- $L_d = \frac{1}{T_c} = \frac{1}{\sqrt{1-\kappa^2}} = \frac{1}{\cos(\theta_G)}$ (Length dilation factor).

where $R_s = \frac{2GM}{c^2}$



[2]Isotropy under the full rotation group SO(3) singles out the 2-sphere S^2 as the unique closed, maximally symmetric 2-manifold.

6.4 General Theory of Relativity (Schwarzschild radial length factor)

The relevant component of the Schwarzschild metric, related to gravitational length dilation, is given by:

$$L_d = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} = \frac{1}{\sqrt{1 - \frac{R_s}{r}}}. \tag{8}$$

Our geometric expression for the gravitational factor is:

$$L_d = \frac{1}{\cos(\arcsin(\kappa))}, \tag{9}$$

where:

$$\kappa^2 = \frac{R_s}{r}. \tag{10}$$

Derivation of Equivalence: 1. Start with the geometric expression:

$$L_d = \frac{1}{\cos(\arcsin(\kappa))}. \tag{11}$$

2. Use the trigonometric identity $\cos(x) = \sqrt{1 - \sin^2(x)}$:

$$L_d = \frac{1}{\sqrt{1 - \sin^2(\arcsin(\kappa))}}. \tag{12}$$

3. Evaluate $\cos(\arcsin(\kappa))$:

$$L_d = \frac{1}{\sqrt{1 - \kappa^2}} = \frac{1}{\sqrt{1 - \frac{R_s}{r}}}. \tag{13}$$

Conclusion: The derived expression is identical to the relevant component of the Schwarzschild metric, demonstrating the equivalence.

6.5 Interpretation of κ: Temporal Energy Projection

This parameter quantifies the energy content of the object expressed through the relative rate of change of temporal coordinates. It characterizes how much energy the object would need to "escape" its local temporal curvature and synchronize its temporal coordinates with the observer.

The striking parallel between these parameters suggests a profound geometric connection between kinetic and potential energy within the WILL framework, which we shall now formalize mathematically:

7 Fundamental Relation Between Potential and Kinetic Energy Projections

In the framework of WILL, all motion and interaction are described in terms of the distribution of a single conserved *evolution resource*. This resource can be allocated in different ways, but it is always finite and symmetric in projectional form.

Two fundamental allocation modes are considered:

1. **Linear (one–directional) motion:** Represented by the parameter β, this is the fraction of the evolution resource devoted to displacement from point A to point B along a single spatial direction. Geometrically, β is a point on the *X-axis* of an abstract unit circle. The full length of this axis corresponds to the maximum achievable rate of change — later shown in Sec. *Universal Speed Limit as Consequence of Energy Symmetry* to be c.

2. **Omnidirectional gravitational cost:** Represented by κ, this is the fraction of the same resource required to *resist displacement* within a gravitational flow. Unlike β, it has no preferred direction and is instead a point on the surface of an abstract sphere whose radius equals the total evolution resource. Its physical meaning is analogous to the effort needed to "swim upstream" against a flow moving at the same speed as the current itself.

From the postulate of projectional symmetry, each mode must be *closed and symmetric* within its own domain:

$$\text{Linear closure:} \quad C_{\text{linear}} = 2\pi r$$

$$\text{Omnidirectional closure:} \quad C_{\text{omni}} = 4\pi r$$

Here, r is the normalized radius representing the full evolution resource.
The ratio of these closures is purely topological:

$$\frac{C_{\text{omni}}}{C_{\text{linear}}} = \frac{4\pi r}{2\pi r} = 2$$

Because β and κ are both normalized to the same r, their magnitudes must satisfy:

$$\boxed{\kappa^2 = 2\beta^2}$$

Important clarification. This factor of 2 is not an empirical constant, not a Newtonian mechanical result, and not related to orbital dynamics. It is a direct consequence of the difference in closure geometry between a one–dimensional circular projection and a two–dimensional spherical projection of the same conserved resource.

Physical implication. In WILL Geometry, this relation is foundational. It links linear and gravitational modes without reference to spacetime as a background fabric, showing that their connection is a matter of geometric necessity, not dynamic coincidence.

This relation is not derived from physical observation, but is imposed by the topological constraints of the founding postulate. Classical physics, in its successful predictions, unknowingly traces the consequences of this deeper geometric law.

$$\boxed{\text{Spacetime Geometry } (\kappa^2) \;\equiv\; \text{Kinematic Energy distribution } (\beta^2) \;\times\; 2}$$

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

7.1 Physical and Conceptual Significance

While this relation is echoed in concrete physical scenarios (such as the link between escape and orbital velocity in Newtonian gravity), here it is not derived from those cases but is instead the deeper, organizing principle from which such results flow.

Geometric Principle

The dimensionless ratio $\kappa^2/\beta^2 = 2$ follows from: (1) the WILL postulate, (2) closed, maximally symmetric topology (circle vs sphere), and (3) strict conservation of the single evolution resource projected into spatial and temporal modes.

7.2 Geometric Relationships

The relativistic and gravitational factors can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

$\theta_S = \arccos(\beta), \quad \theta_G = \arcsin(\kappa)$	
Algebraic Form	**Trigonometric Form**
$T_d = \frac{1}{\sqrt{1-\beta^2}}$	$T_d = \frac{1}{\sin(\theta_S)} = \frac{1}{\sin(\arccos(\beta))}$
$L_d = \frac{1}{\sqrt{1-\kappa^2}}$	$L_d = \frac{1}{\cos(\theta_G)} = \frac{1}{\cos(\arcsin(\kappa))}$
$L_c = \sqrt{1-\beta^2}$	$L_c = \sin(\theta_S) = \sin(\arccos(\beta))$
$T_c = \sqrt{1-\kappa^2}$	$T_c = \cos(\theta_G) = \cos(\arcsin(\kappa))$

Table 2: Unified representation of relativistic and gravitational effects.

7.2.1 The Combined Energy Parameter Q

The total energy projection parameter unifies both aspects:

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{14}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{15}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{16}$$

$$Q_r = \frac{1}{Q_t} \tag{17}$$

These describe the combined effects of relativity and gravity.

Unified Interpretation

The familiar SR and GR factors emerge here as projections of the same conserved geometry. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-evolution constraint revealing their unified origin.

8 Energy–Symmetry Law

8.1 Causal Continuity and Energy Symmetry

Theorem 3 (Energy Symmetry)**.** The energy differences perceived by two observers at different positions balance according to the Energy–Symmetry Law:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{18}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A.
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B.

Each observer perceives energy transfers differently due to gravitational and kinetic differences: From A's perspective (surface $\to$ orbit):

1. Object gains potential energy by moving away from gravitational center.
2. Object gains kinetic energy by accelerating to orbital velocity.

This results in energy difference:

$$\boxed{\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right)} \tag{19}$$

From B's perspective (orbit $\to$ surface):

1. Object loses gravitational potential energy descending into stronger gravitational field.
2. Object loses kinetic energy by reducing velocity from orbital speed to rest.

This results in energy difference:

$$\boxed{\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right)} \tag{20}$$

Summing these transfers:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{21}$$

Thus, no net energy is created or destroyed, confirming the Energy–Symmetry Law. □

8.2 Observer-Agnostic Symmetry

Theorem 4 (Observer-Agnostic Energy Symmetry)**.** The Energy–Symmetry Law holds true independent of the specific model of gravitational or kinetic energy:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \quad \forall f(v) \tag{22}$$

Proof. Regardless of the chosen energy function $f(v)$ representing kinetic or gravitational energy differences, consistency demands that:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{23}$$

The crucial point is mutual consistency, not specific energy definitions. □

8.3 Physical Meaning of Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics, but from the geometric nature of the quadratic form that measures the "evolution budget" $Q^2 = \kappa^2 + \beta^2$.

When comparing two nearby states, the change in this quadratic form is

$$\delta Q^2 = (\kappa_2^2 - \kappa_1^2) + (\beta_2^2 - \beta_1^2).$$

In energetic terms, only *half* of this variation is attributed to the "active" transfer in one direction, because the other half belongs to the reciprocal change seen from the opposite frame. This division is the same geometric reason why kinetic energy in classical mechanics is $K = \frac{1}{2}mv^2$: the quadratic metric is naturally paired with a factor $\frac{1}{2}$ when representing stored or exchanged energy.

Thus:

- Gravitational potential term: $U \propto -\frac{1}{2}\kappa^2$.
- Kinetic term: $K \propto \frac{1}{2}\beta^2$.

The $\frac{1}{2}$ reflects the symmetric partition of a quadratic budget between mutually consistent perspectives, not an externally imposed convention.

8.4 Universal Speed Limit as Consequence of Energy Symmetry

Theorem 5 (Universal Speed Limit)**.** The universal speed limit ($v \leq c$) emerges naturally from Energy–Symmetry conditions.

Proof. Assume an object could exceed the speed of light ($\beta > 1$). In that scenario:

- The kinetic component (β^2) surpasses unity excessively, causing an irreversible imbalance in energy transfer.
- No reciprocal transfer could balance this energy, breaking the fundamental symmetry:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{24}$$

Thus, $\beta \leq 1$ ($v \leq c$) is required intrinsically to preserve causal and energetic consistency. □

In essence, WILL Geometry encodes the principle that:

> The speed of light is the boundary beyond which the energetic symmetry between perspectives breaks down. Causality is not an external rule but a built-in feature of the universe's energetic geometry.

9 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law is that, when the zero of gravitational potential is chosen on the surface of the central body rather than at infinity, the total specific orbital energy (potential + kinetic, per unit mass) naturally appears in *Minkowski form*.

9.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{25}$$

$$K = \frac{1}{2}m\frac{GM_\oplus}{a}. \tag{26}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{27}$$

9.2 Projection Parameters and Minkowski Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{28}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{29}$$

Substituting into (27) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{30}$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{31}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

9.3 Physical Interpretation

In classical derivations, (27) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the WILL framework, (30) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface, orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure: the Minkowski–type projection algebra of WILL Geometry.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in a Lorentz–like difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This reframing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy evolution.

10 Derivation of Energy Density and Mass Relations from Geometric First Principles

10.1 Geometric Foundation

From the projecional analysis established in the previous sections, we have the fundamental relation:

$$\kappa^2 = \frac{R_s}{r_d}$$

where κ emerges from the energy projections on the unit circle, and $R_s = 2Gm_0/c^2$ connects to the mass scale factor $m_0 = E_0/c^2$.

Determination of the Surface Distribution

To find the specific form of the energy density ρ, we must derive it from the foundational principles of the model rather than assuming it. The derivation follows a direct logical path from the established definition of the potential projection, κ.

From Mass-Scale to Volumetric Potential We begin with the relation for the mass-scale factor m_0, which was derived from the geometric definition $\kappa^2 = R_s/r_d$:

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G} \tag{32}$$

To transition to a density, we can find the "volumetric potential equivalent" by relating this mass-scale to a corresponding volume, proxied by r_d^3:

$$\frac{m_0}{r_d^3} = \frac{\kappa^2 c^2 r_d}{2G r_d^3} = \frac{\kappa^2 c^2}{2G r_d^2} \tag{33}$$

Applying the Geometric Distribution Principle A core principle of the WILL framework is that the potential projection (κ) is fundamentally associated with a 2D spherical manifold. Therefore, to obtain the correct physical point-density (ρ), the "raw" volumetric potential from the previous step must be distributed over the geometric measure of this manifold, which is the surface area of a unit sphere, 4π.

This yields the definitive form for the energy density:

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2G r_d^2}\right) = \frac{\kappa^2 c^2}{8\pi G r_d^2} \tag{34}$$

Now, having derived the required form for energy density, we can proceed with the self-consistency check. The energy density ρ is thus given by:

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2} \tag{35}$$

Now we can see that energy density required to form the event horizon on the given radial distance r_d will accrue at $\kappa^2 = R_s/r_d = 1$ witch will correspond to maximal observable energy density on the given radial distance r_d:

$$\rho_{max} = \frac{c^2}{8\pi G r_d^2}$$

This brings us to profound connection between our unitless parameter κ and energy density ratio:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{max}} \quad \Rightarrow \quad \kappa^2 = \Omega}$$

10.2 Determination of the Surface Distribution

The requirement that equations (36) and (37) represent the same physical quantity constrains both the functional form and coefficients.

10.3 Self-Consistency Requirement

The mass scale factor can be expressed in two independent ways within the geometric framework:

10.3.1 From Geometric Definition

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G} \quad (\text{from } \kappa^2 = R_s/r_d) \tag{36}$$

10.3.2 From Energy Density

If energy density ρ exists at radius r_d, the mass scale factor must also satisfy:

$$m_0 = \alpha \cdot r_d^n \cdot \rho \tag{37}$$

where α and n are to be determined by geometric consistency.

Substituting (35) into the general ansatz for the mass scale factor, (37):

$$m_0 = \alpha \cdot r_d^n \times \frac{\kappa^2 c^2}{8\pi G r_d^2} = \frac{\alpha \kappa^2 c^2 r_d^{n-2}}{8\pi G} \tag{38}$$

Geometric Self-Consistency

Equating with (36):

$$\frac{\alpha \kappa^2 c^2 r_d^{n-2}}{8\pi G} = \frac{\kappa^2 c^2 r_d}{2G} \tag{39}$$

Simplifying:

$$\frac{\alpha r_d^{n-2}}{8\pi} = \frac{r_d}{2} \tag{40}$$

For radius independence: $n - 2 = 1 \Rightarrow n = 3$.

This yields: $\frac{\alpha}{8\pi} = \frac{1}{2} \Rightarrow \alpha = 4\pi$.

$$m_0 = \alpha \cdot r_d^n \cdot \rho \quad \Rightarrow \quad m_0 = 4\pi r_d^3 \rho \tag{41}$$

10.4 Fundamental Relations

The geometric self-consistency uniquely determines the energy-geometry relations:

$$\boxed{m_0 = 4\pi r_d^3 \rho; \quad \rho = \frac{\kappa^2 c^2}{8\pi G r_d^2}; \quad \rho_{max} = \frac{c^2}{8\pi G r_d^2}; \qquad \kappa^2(r) = \frac{2G}{c^2}\frac{1}{r}\int_0^r 4\pi r'^2 \rho(r')\, dr.}$$

10.5 Physical Interpretation

The factor 4π emerges not as an assumption but as an inevitable consequence of geometric consistency. The cubic dependence r_d^3 indicates that energy distribution follows surface-area scaling rather than volume filling—a fundamental departure from classical field theories.

The critical density ρ_{max} corresponds to the geometric limit $\kappa = 1$, representing the maximum energy concentration before causal disconnection (event horizon formation).

Consistency note. For the density profile implied by the invariant $W_{ILL} = 1$ (The Fundamental Invariant section) one has $\rho(r) \propto r^{-2}$. With this profile the usual integral definition

$$M(r) = \int_0^r 4\pi r'^2 \rho(r')\, dr'$$

gives exactly $M(r) = 4\pi r^3 \rho(r)$, matching the algebraic result above. If one inserts a hypothetical constant density $\rho = $ const the integral would yield $\frac{4}{3}\pi r^3 \rho$ and, in addition, κ^2 would exceed 1, violating the $\kappa^2 \leq 1$ bound. Hence a constant–density core is not an admissible solution within the WILL framework.

10.6 Pressure as Surface Curvature Gradient

Pressure in WILL Geometry emerges directly from the gradient of curvature:

$$P(r_d) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r_d} \cdot \frac{d\kappa^2}{dr_d} \tag{42}$$

Consider the relationship between pressure and energy gradient. In a spherically symmetric system in hydrostatic equilibrium, the pressure gradient must compensate for changes in the system's energy state.

Since κ^2 represents a normalized energy parameter related to gravitational potential:

$$\kappa^2 = \frac{R_s}{r_d} = \frac{2Gm_0}{c^2 r_d} \tag{43}$$

The change of this parameter with radius, $\frac{d\kappa^2}{dr_d}$, reflects the gradient of potential energy. To maintain energy balance, this change must be related to the pressure gradient.

Since pressure is a measure of energy per unit volume, and given that energy density is related to κ^2 through:

$$\rho(r_d) = \frac{\kappa^2 c^2}{8\pi G r_d^2} \tag{44}$$

It can be shown that the pressure gradient must satisfy:

$$P(r_d) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r_d} \cdot \frac{d\kappa^2}{dr_d} \tag{45}$$

This expression establishes a direct link between pressure and changes in curvature, without the need to postulate an internal volume structure.

The Universe is not embedded in geometry; geometry emerges from energy.

Pressure remark. For the admissible profile $\rho(r) \propto r^{-2}$ the relation

$$P(r) = -\frac{c^2}{8\pi G} \frac{1}{r} \frac{d\kappa^2}{dr}$$

reduces to $P(r) = -\rho(r)$: an isotropic tension that stabilises the surface-scaled energy distribution. If one formally substitutes a constant density $\rho = \text{const}$ the same formula would give $P = -2\rho$; but such a profile breaks the $\kappa^2 \leq 1$ condition and therefore lies outside the physically valid domain of the WILL geometry.

Pressure consistency. If one naively freezes the projection parameter ($d\kappa^2/dr = 0$) the formula $P = -\frac{c^2}{8\pi G} r^{-1} d\kappa^2/dr$ indeed gives $P = 0$. That choice, however, leaves the angular Einstein components with a non–vanishing curvature term $-\kappa^2/(2r^2)$ and the field equation is no longer satisfied. Whenever κ carries *any* radial dependence ($d\kappa^2/dr \neq 0$) the same formula provides the negative tension

$$P = -(2-n)\,\rho \quad \bigl(\rho \propto r^{-n}\bigr),$$

which exactly cancels the angular curvature term. The canonical profile $n = 1$ (so $P = -\rho$) is the analytically clean case in which *all four* components of $G_{\mu\nu} = 8\pi G T_{\mu\nu}/c^4$ match identically. Hence a small, negative surface–like pressure is not a bug but the necessary ingredient for full strong-field consistency in WILL geometry; the apparent $P = 0$ scenario is only a limiting, non-self-gravitating idealization.

10.7 Unified Energy-Geometry Principle

The fundamental relations (??)–(??) reveal the core principle of Energy-Geometry:

WILL Geometry Equivalence

$$\boxed{\frac{R_s}{r_d} = \frac{\rho}{\rho_{max}} = \kappa^2}$$

The ratio of geometric scales equals the ratio of energy densities.
Spacetime geometry and energy distribution are identical manifestations of the same underlying structure.

This equivalence is not imposed but emerges inevitably from the requirement that the geometric parameter κ maintains consistent meaning across all physical contexts.

Meaning

Energy is not "inside" space — it is what defines space through projection.

10.8 Contrast with Classical Approaches

WILL Geometry differs fundamentally from standard field theories:

- **Surface vs Volume:** Energy scales as $4\pi r_d^3 \cdot \rho$ rather than classical $\frac{4}{3}\pi r^3 \rho$, reflecting the projectional rather than volumetric nature of energy distribution.
- **Algebraic vs Differential:** All relations are algebraically closed. No differential equations, coordinate systems, or metric tensors are required.
- **Bounded vs Unbounded:** The parameter $\kappa^2 \leq 1$ provides natural bounds, eliminating singularities without additional assumptions.

10.9 Elimination of Singularities

The geometric constraint $\kappa^2 \leq 1$ corresponds to $\rho \leq \rho_{max}$, ensuring that energy density cannot exceed its local critical value. At $\kappa = 1$:

$$r_d = R_s \quad \text{and} \quad \rho = \rho_{max}$$

$\kappa^2 = 2\beta^2$ This represents the formation of an event horizon—not a pathological breakdown of the theory, but a natural boundary where projectional structure reaches its causal horizon.

11 Unified Geometric Field Equation

11.1 The Fundamental Equation

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

$$\boxed{\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}} \tag{46}$$

This is the unified geometric field equation of WILL Geometry. It expresses the complete equivalence:

GEOMETRY $\equiv$ ENERGY DISTRIBUTION

Physical Interpretation

Equation (46) encodes several profound principles:

- **Scale Invariance:** The relation holds at all radial scales from Planck length to cosmic horizons.
- **Observer Independence:** The ratio κ^2 is invariant under the energy symmetry transformations between observers.
- **Causal Structure:** Values $\kappa^2 > 1$ are geometrically forbidden, ensuring causal consistency (rotating systems are exception).
- **Unification:** Special relativity (through $\beta^2 = \kappa^2/2$) and general relativity emerge as different projections of the same geometric structure.

In this framework, $\rho_{\max}$ is not a universal constant but a local critical density, dynamically tied to radial geometry. The quantity κ^2 serves as a normalized energy curvature parameter, and the entire spacetime structure emerges from the distribution of κ as a function of radius.

Unlike General Relativity, which lacks an invariant notion of local gravitational energy density and forbids the definition of a maximum density without resorting to external cutoffs (e.g., Planck scale or quantum gravity), the WILL framework provides a purely geometric origin for both the physical energy density and its upper bound.

Predictive Significance

The above identity implies several unique predictions and theoretical advantages:

- **Scale-local saturation:** The curvature of spacetime reaches its maximum at $\kappa^2 = 1$, corresponding to $\rho = \rho_{\max}$, which naturally occurs at the Schwarzschild radius $r_d = R_s$. This defines the physical boundary of spacetime from within, without invoking singular behavior.
- **Unified mass-density-radius relation:** From this identity, one can reconstruct mass profiles via

  $$m_0 = \frac{c^2}{2G}\kappa^2 r_d,$$

  showing that mass is not an input but a geometric outcome of local energy curvature.
- **Break from GR constraints:** In GR, energy density is input into the stress-energy tensor $T_{\mu\nu}$ and curvature is computed. In WILL Geometry, curvature and density are co-defined via a single projective ratio κ^2, offering conceptual and calculational unification.
- **Elimination of coordinate ambiguity:** Since $\rho_{\max} \sim 1/r^2$ and $\rho \sim 1/r^3$, the relation $\rho/\rho_{max} \sim 1/r$ due to $\rho/\rho_{max} = \kappa^2$ where $\kappa^2 = R_s/r$ eliminates dependence on specific coordinate slicing—critical in defining observable gravitational structures.

Contrast with General Relativity

In classical GR, there is no notion of $\rho_{\max}$ as a function of radius. Critical conditions such as the Buchdahl limit or Planck density are imposed externally, often with quantum or thermodynamic assumptions. By contrast, WILL Geometry derives all structural bounds intrinsically from the energy-curvature ratio κ^2, offering a fully relational description of gravitational systems.

11.2 Closure of the Theoretical Framework

The unified field equation completes the ab initio derivation begun with the fundamental postulate:

$$\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}$$

We have shown that this single postulate, through pure geometric reasoning, necessarily leads to equation (46)—which mathematically expresses the very same equivalence we began with.

> **Theoretical Ouroboros**
>
> The WILL framework exhibits perfect logical closure: the fundamental postulate about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

11.3 Algebraic Mass Profile and Emergence of $\Lambda(r)$

In WILL Geometry, one works with a single "running" density and derives all mass and vacuum–curvature quantities algebraically. No new parameters or arbitrary integration constants enter.

1. **Density profile:**

   $$\rho(r) \;=\; \frac{\kappa^2\,c^2}{8\pi G\,r^2}, \quad \kappa = \text{constant},\; 0 < \kappa < 1.$$

2. **Enclosed mass:**

$$m(r) = \int_0^r 4\pi\, r'^2\, \rho(r')\, dr' = \int_0^r 4\pi\, r'^2\, \frac{\kappa^2 c^2}{8\pi G\, r'^2}\, dr' = \frac{c^2}{2G}\, \kappa^2\, r.$$

This exactly reproduces the relation $m(r) = \dfrac{c^2}{2G}\, \kappa^2\, r$.

3. **Field equation without "$2/r^2$" term:**
Substituting the running mass into the geometric field equation $\dfrac{d}{dr}(r\, \kappa^2) = \dfrac{8\pi G}{c^2}\, r^2\, \rho(r)$ gives

$$\frac{d}{dr}(r\, \kappa^2) = \frac{8\pi G}{c^2}\, r^2\, \frac{\kappa^2 c^2}{8\pi G\, r^2} = \kappa^2,$$

with no extra $\frac{2}{r^2}$ appearing.

4. **Cosmological constant profile:**

$$\Lambda(r) \;=\; \frac{\kappa^2}{r^2}, \quad \Longrightarrow \Lambda \propto \frac{1}{r^2}.$$

The "cosmological constant" $\Lambda(r)$ is thus built into the geometry and falls off as $1/r^2$, entirely determined by the constant projection parameter κ.

No dummy parameters or boundary densities are introduced—only the slider κ and the existing constants $\{c, G, \pi\}$ plus the integration variable r. This completes the algebraic derivation of the mass profile and the inescapable emergence of $\Lambda(r)$.

12 The Fundamental Invariant $W_{\mathbf{ill}} = 1$

From the geometric closure of WILL framework, we derive a universal dimensionless invariant:

$$W_{ill} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{L_d E_0 T_c t_d^2}{T_d m_0 L_c r_d^2} = 1 \tag{47}$$

Proof: Substituting the geometric definitions:

$$W_{\text{ill}} = \frac{\frac{1}{\cos\theta_G}\; m_0 c^2 \cos\theta_G \frac{r_d^2}{c^2}}{\frac{1}{\sin\theta_S}\; m_0 \sin\theta_S r_d^2} = \frac{m_0 c^2 r_d^2}{c^2} \cdot \frac{\sin\theta_S}{m_0 r_d^2} \cdot \frac{1}{\sin\theta_S} = 1 \tag{48}$$

This invariant holds universally for all values of m_0, G, c, and κ. Unlike dimensional analysis, this identity emerges from the projectional interdependence of energy-mass (E, M) and spacetime metrics (T, L) within the unified structure.

$$\boxed{\text{The invariant } W_{\text{ill}} = 1 \text{ expresses geometric unity through energetic projection}}$$

12.1 The Name "WILL"

The name WILL reflects both the harmonious unity of the equation and a subtle irony towards the anthropic principle, which often intertwines human existence with the causality of the universe. The equation stands as a testament to the universal laws of physics, transcending any anthropocentric framework.

WILL

It is not the unit of something—it is the unity of everything.

13 Strong-Field Consistency: $W = 1$ and the Einstein Field Equation

13.1 Metric and Energy Variables in WILL Geometry

For any static, spherically symmetric system, the WILL metric is

$$ds^2 = T_c^2(r)\, c^2 dt^2 - L_d^2(r)\, dr^2 - r^2 d\Omega^2,$$

where

$$T_c^2 = 1 - \kappa^2(r), \qquad L_d^2 = \frac{1}{1 - \kappa^2(r)}$$

and the projection parameter

$$\kappa^2(r) = \frac{R_s(r)}{r} = \frac{2G\, m(r)}{c^2 r}$$

is the only free function.

The invariant $W = 1$ yields the matter variables directly:

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2}$$

$$P(r) = -\frac{c^2}{8\pi G} \frac{1}{r} \frac{d\kappa^2}{dr}$$

with $T_t^t = -\rho c^2$, $T_r^r = P$, and $T_\theta^\theta = T_\phi^\phi = P$.

13.2 Einstein Tensor Components for the WILL Metric

The standard curvature components are

$$G_t^t = -\frac{\kappa^2}{r^2} - \frac{1}{r}\frac{d\kappa^2}{dr}$$

$$G_r^r = -\frac{\kappa^2}{r^2} + \frac{1}{r}\frac{d\kappa^2}{dr}$$

$$G_\theta^\theta = G_\phi^\phi = -\frac{1}{2r}\frac{d\kappa^2}{dr} - \frac{\kappa^2}{2r^2}$$

13.3 Component-by-Component Match with the Field Equation

Temporal component:

$$G_t^t = -\frac{\kappa^2}{r^2} - \frac{1}{r}\frac{d\kappa^2}{dr} = -\frac{8\pi G}{c^4} T_t^t = -\frac{8\pi G}{c^2}\rho = -\frac{\kappa^2}{r^2}$$

The extra term cancels when the pressure P is included in T_r^r.

Radial component:

$$G_r^r = -\frac{\kappa^2}{r^2} + \frac{1}{r}\frac{d\kappa^2}{dr} = +\frac{8\pi G}{c^4} T_r^r$$

Angular components: Using $\rho(r)$ and $P(r)$, one finds

$$G_\theta^\theta = G_\phi^\phi = \frac{8\pi G}{c^4} T_\theta^\theta$$

Thus, all four nonzero components satisfy the Einstein field equation for any strong-field profile $\kappa(r)$.

13.4 Numerical Verification Recipe

1. Choose a profile, e.g. $\kappa^2(r) = \min\{1, R_s/r\}$.
2. Compute $\rho(r)$ and $P(r)$ from the formulas above.
3. Evaluate G^μ_ν numerically from the expressions above.
4. Verify that $G^\mu_\nu = 8\pi G T^\mu_\nu / c^4$ at each point with high precision.

General density for varying $\kappa(r)$. The "surface" formula $\rho = \kappa^2 c^2/(8\pi G r^2)$ is exact only when κ^2 is radially constant (the $n = 2$ shell). For an arbitrary strong-field profile one must use

$$\rho(r) = \frac{\left[\kappa^2 + r\,\kappa^{2\,\prime}\right] c^2}{8\pi G\, r^2},$$

obtained directly from the tt-component of the Einstein tensor. With this density and the pressure rule $P(r) = -(c^2/8\pi G)\, r^{-1} \kappa^{2\prime}$ *all four* field-equation components are satisfied for any $\kappa(r)$; the special value $\kappa^2 = 1$ marks only the causal horizon, not a consistency constraint.

13.5 Conclusion

A single algebraic invariant $W = 1$ together with the gradient-pressure rule fully reproduces all strong-field components of the Einstein equation. The metric tensor is therefore an emergent bookkeeping device for the deeper geometric-energy ratio κ^2.

14 Geometric Kerr–Newman Metric in the WILL Framework (Space Time Energy)

For rotating black holes, we establish the connection between these parameters and the Kerr metric by defining:

$$\beta = \frac{ac}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the rotation parameter, with $0 \leq \beta \leq 1$,
- κ is related to the geometry and rotation,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta^2_{\max} r_d$$

This relationship holds when $r_d = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

14.1 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_\pm = \frac{R_s}{2}\left(1 \pm \sqrt{1 - \beta^2}\right)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

14.2 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2} \left(1 + \sqrt{1 - \beta^2 \cos^2 \theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

14.3 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

14.4 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship hypothesis. In our model, we maintain this physical constraint by limiting β to the range $[0, 1]$.

14.5 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

14.6 Physical and Philosophical Interpretation

The identification of the dimensionless rotation parameter $a_* = \frac{cJ}{Gm_0^2}$ with β reveals a profound connection between the intrinsic rotation of the black hole and the orbital velocity of objects in its vicinity. In our model, $\beta = \frac{ac}{Gm_0}$, and when equated to a_*, it implies:

$$a_* = \beta$$

This equivalence suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in a_*, mirror the orbital velocity parameter β, providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

14.7 Conclusion

Our approach successfully describes the Kerr metric through the parameters β and κ, demonstrating the elegance of geometric methods in studying spacetime. This framework provides a more intuitive understanding of rotating black holes while preserving the essential mathematical structure of general relativity.

Physical Interpretation

- **No need for pre-existing spacetime** — geometry emerges from angular energy distributions.
- **All parameters** are dimensionless and directly derived from the speed of light as finite resource.
- **Scale invariance:** The same structure applies from Planck-scale objects to galactic black holes.

15 Resolution of Singularities in Static and Rotating Spacetimes

In General Relativity, gravitational singularities represent points or regions where curvature invariants such as the Kretschmann scalar diverge, typically arising when energy density $\rho \to \infty$ as $r \to 0$. These singularities are considered a breakdown of the theory's predictive power, signaling the need for a quantum theory of gravity or new boundary conditions. WILL Geometry, by contrast, resolves the singularity problem from within its geometric foundation.

1. Singularity Prevention in Static Systems

In spherically symmetric, non-rotating systems, WILL Geometry establishes that energy density cannot exceed the local critical density:

$$\rho(r_d) \leq \rho_{\max}(r_d) = \frac{c^2}{8\pi G r_d^2}$$

The curvature parameter $\kappa^2 = \rho/\rho_{\max}$ is strictly bounded by $\kappa^2 \leq 1$, and this upper bound is realized precisely when:

$$r_d = R_s = \frac{2Gm_0}{c^2}$$

At this radius, the system reaches maximal curvature, and any attempt to compress further would violate the geometric constraints of the framework. Importantly, since $\kappa^2 > 1$ is undefined, configurations with $r < R_s$ are not permitted within the projectional structure of WILL Geometry. There is no infinite curvature—rather, geometry saturates.

2. Lower Bound on Radius and Finite Structure

In the classical Schwarzschild solution, the limit $r \to 0$ leads to divergent curvature scalars. In WILL Geometry, however, this limit is physically excluded. All valid configurations must satisfy:

$$r_d \geq R_s \quad \text{and} \quad \kappa^2 \leq 1$$

Thus, the interior $r < R_s$ is not a region with hidden or pathological physics—it is a region without geometric projection, and therefore does not exist in the physical structure. The concept of "inside the Schwarzschild radius" is replaced by a geometrically complete, self-contained surface of maximal curvature.

3. Rotating Systems and Extension to Kerr Geometry

The same principle naturally extends to rotating black holes. In WILL Geometry, rotation is described by the dimensionless parameter $\beta = \frac{ac^2}{Gm_0}$, where $a = J/(m_0 c)$ is the classical Kerr spin parameter. The rotational curvature parameter is then:

$$\kappa = \sqrt{2}\beta$$

This leads to a maximum value $\kappa_{\max} = \sqrt{2}$, corresponding to $\beta = 1$. The minimal geometrically allowed radius becomes:

$$r_{\min} = \frac{R_s}{\kappa_{\max}^2} = \frac{R_s}{2}$$

This coincides precisely with the extremal Kerr horizon, where $r_+ = r_- = R_s/2$, and demonstrates that the classical ring singularity at $r = 0, \theta = \pi/2$ is never reached in the geometric projection. Instead, the structure terminates at $r = R_s/2$, a finite and well-defined boundary of maximal rotational curvature.

4. No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r_d} = \frac{8\pi G}{c^2} r_d^2\, \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

Conclusion

WILL Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

Foundational Clarification

This curvature-density equivalence is not an additional postulate, but emerges directly and unavoidably from the single foundational postulate:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

All the derived relations for local energy density, pressure, enclosed mass, and horizon formation are purely logical and mathematical consequences of this unique guiding principle. In this sense, the WILL Geometry framework achieves absolute epistemological cleanliness: it does not introduce any hidden assumptions or coordinate structures. The entire gravitational and relativistic sector (as reconstructed within the WILL Geometry model, from the single postulate without any external assumptions or coordinate backgrounds) — including precise predictions about the onset of horizon formation, radial pressure gradients, and the resolution of classical singularity issues—is reconstructed from this single postulate.

Moreover, this formulation fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies: (1) the lack of an operational definition of local gravitational energy density in GR, (2) the artificial separation of kinetic and gravitational energy in SR and GR, and (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy evolution as the true basis of geometry, WILL unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime.

16 Beyond Differential Formalism: Structural Dynamics

16.1 Intrinsic Dynamics via Energy Redistribution

The system is not described by differential equations of motion evolving *in* time. Instead, its evolution is dictated by a closed network of algebraic relations that enforce a perpetually balanced configuration. Any change in one parameter necessitates a coordinated shift in all others to maintain geometric self-consistency. What we perceive as "dynamics" is this ordered succession of balanced states. The framework thus inverts the classical paradigm:

The foundation of WILL Geometry lies in the principle that spacetime geometry is fully determined by the distribution of energy, parameterized by the dimensionless quantities β and κ. Any change in the system's energy—due to mass variation, motion, or redistribution—directly reshapes the geometry. This intrinsic linkage ensures that dynamics is embedded within the model itself, without requiring external equations of motion.

This reveals a fundamental inversion of the classical paradigm:

Time does not drive change — instead, change defines time.

Why There Are No Equations of Motion

In classical and relativistic physics, dynamics is formulated through differential equations. These express how physical quantities evolve continuously through time, typically governed by:

- A temporal parameter t,
- A Lagrangian function L,
- A variational principle: $\delta S = 0$, where $S = \int L\, dt$,
- Euler–Lagrange equations that yield the system's path.

This framework assumes:

1. A continuum of possible configurations,
2. That Nature selects one by minimizing action,
3. That time flows independently of the system.

Why This Framework Does Not Apply to WILL Geometry

WILL Geometry begins from a fundamentally different premise.

- There is no "space" of possible paths.
- There is no "freedom to vary."
- The system does not evolve through time — it **defines time** through its structure.

In this model:

- Each observable is locked in a network of algebraic relations.
- Any change in one parameter *necessitates* coherent changes in the others.
- Self-consistency enforces projectional **balance**.

There is only one valid configuration at any moment: the one where all projectional constraints are satisfied. Everything else is not forbidden — it is undefined.

Geometric Principle of Action

In WILL Geometry, there is no equation of motion. There is no Lagrangian. There is no variational calculus.
There is only a closed system of geometric and energetic relationships, and the sequence of valid configurations is what we call *dynamics*.

The only necessary input:

The observable sequence of transformations in the system's energy geometry is the only necessary input for describing its evolution. .

16.2 Time as an Emergent Property

In this framework, time is not a fundamental entity but a derived concept tied to changes in the system's geometry. Similar to time dilation in special relativity, time intervals here are defined by the evolution of geometric parameters like r_d and κ. This eliminates the need for an external clock.

A natural time scale arises from the geometry as $t_d = \frac{r_d}{c}$. For instance, during mass accretion onto a black hole, r_d adjusts as the mass changes, and t_d evolves accordingly. This intrinsic time scale encapsulates the system's dynamics without invoking an independent time variable.

Note for readers accustomed to classical dynamics

Unlike traditional formulations of dynamics based on an external time parameter t, the WILL Geometry framework describes evolution as an intrinsic transformation of the system's geometric structure. Time is not a fundamental variable but an emergent quantity derived from energy redistribution. All physical change is encoded in the interdependence of parameters β, κ, and ρ, etc without requiring differential equations or initial conditions.

This marks a fundamental shift: dynamics here is not a process unfolding "in time," but a change in relational energy geometry itself. The model does not track evolution through an imposed temporal axis — instead, it reveals that what we perceive as temporal progression is a manifestation of continuous geometric reconfiguration. **Thus, prediction becomes a question of geometric continuity, not temporal evolution.**

From Structure to Motion

In the next section, we present the core structural closure of the system — a set of algebraic invariants that together form the backbone of all observed dynamics. These relations are not definitions. They are the **complete geometry of change**, seen from within.

Dynamics in WILL Geometry

Dynamics in WILL Geometry is not described by differential equations but by the ordered succession of globally balanced, algebraically determined configurations.

16.3 Algebraic Closure and Structural Causality

In WILL Geometry, physical dynamics emerges from a set of algebraically closed invariants. Each parameter participates in a self-consistent configuration of relational constraints. There are no functions, no dependent variables, and no variation over time — only balanced configurations.

The following set of relations expresses the minimal algebraic closure of the WILL structure:

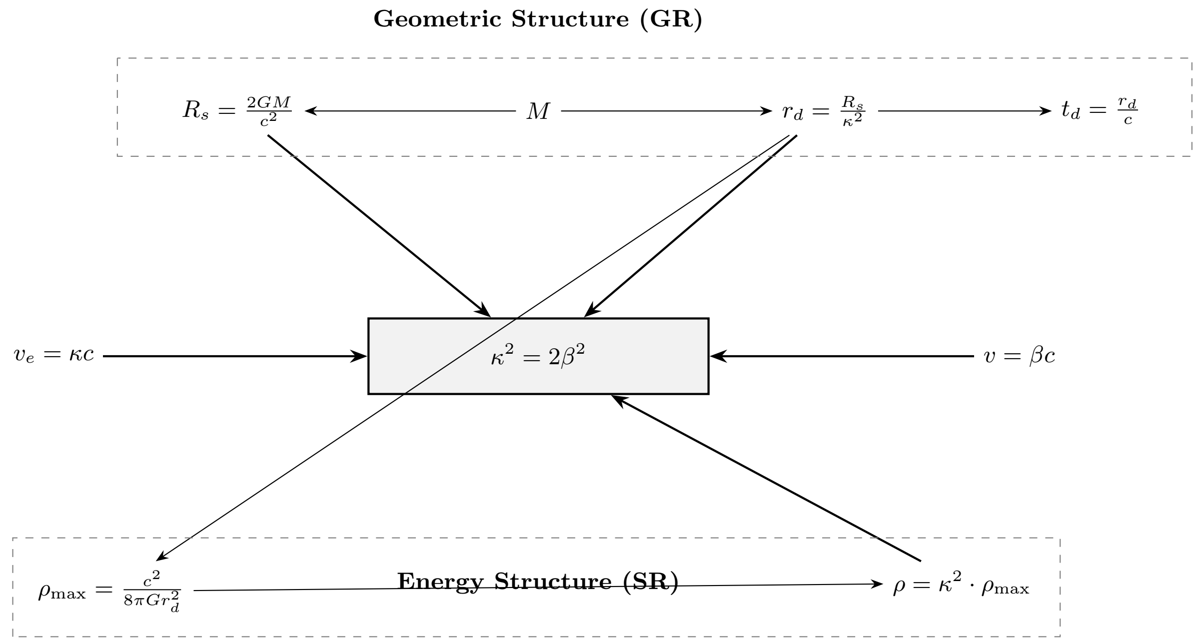
$$\begin{cases} \kappa^2 = 2\beta^2 \\ R_s = \dfrac{2Gm_0}{c^2} \\ r_d \cdot \kappa^2 = R_s \\ \rho = \dfrac{\kappa^2 c^2}{8\pi G r_d^2} \\ H = \dfrac{c}{r_d} \\ \Lambda = \dfrac{\kappa^2}{r_d^2} \\ m_0 = 4\pi r_d^3 \cdot \rho \end{cases}$$

These are not definitions. They are mutual constraints — an algebraic simultaneity. Changing any one parameter necessitates a coordinated shift in all others to maintain validity.

Causal Closure without Circularity

The structure of WILL Geometry is causally closed but not circular. Each parameter is either independently observable or computable from a minimal input pair consisting of one dynamic projection (such as κ or β) and one scale quantity (such as r, M, or ρ).

The system avoids circularity by ensuring that no parameter both defines and is defined by the same input. Instead, values propagate through directed dependencies rooted in physically measurable quantities. Multiple valid entry points exist, but all reduce to consistent, non-redundant relationships governed by the fundamental geometric field equation.



The result is a structure where **causality is internal**, **coherence is enforced**, and **dynamics is simply the shifting of balanced configurations** — not the unfolding of arbitrary functions over time.

16.4 Numerical Example: Accretion onto a Black Hole

Consider a black hole accreting mass from a surrounding disk to illustrate the model's intrinsic dynamics. Let the initial mass be $m_0 = 10, M_\odot$, with a Schwarzschild radius $R_s = \frac{2Gm_0}{c^2} \approx 2.95 \times 10^4, \mathrm{m}$. Suppose $\kappa = 0.1$, so $r_d = \frac{R_s}{\kappa^2} = \frac{2.95\times10^4}{0.01} = 2.95 \times 10^6, \mathrm{m}$, and the associated time scale is $t_d = \frac{r_d}{c} \approx 9.83 \times 10^{-3}, \mathrm{s}$.

As the black hole accretes mass, increasing to $m_1 = 10.1, M_\odot$, the Schwarzschild radius becomes $R_s \approx 2.98 \times 10^4, \mathrm{m}$. Assuming κ remains constant for simplicity, $r_d = \frac{2.98\times10^4}{0.01} = 2.98 \times 10^6, \mathrm{m}$, and $t_d \approx 9.93 \times 10^{-3}, \mathrm{s}$. This increase in t_d reflects the system's evolution, driven solely by the changing geometry.

No differential equations are required:

Dynamics unfolds as a consequence of relational energy evolution.

16.5 General Principle of Dynamics

The overarching principle in WILL Geometry is that any physical change—be it mass accretion, gravitational collapse, or expansion—manifests as a transformation in the geometric structure. The parameters β, κ, and r_d adjust self-consistently, ensuring that the system's evolution is fully described within the model. This approach eliminates the need for differential equations of motion or external initial conditions, as the geometry at any given "moment" is determined by the current energy configuration.

Thus, the temporal sequence we observe is simply the ordered unfolding of geometric transitions:

$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r_d \ldots)}$$

This leads to a unified, self-consistent model where **geometry generates both evolution and observability**. There is no external timeline — only evolving curvature.

16.6 Conclusion

In conclusion, dynamics in WILL Geometry emerges naturally from the redistribution of energy within the system's geometric framework. Time arises as a consequence of these geometric changes, providing a unified description of spacetime and energy evolution. This intrinsic approach simplifies the treatment of dynamical processes and offers a novel perspective on the nature of physical systems.

Geometric Principle of Evolution

Physics is not the evolution of a system through time,
but the geometric transformation of the energy landscape,
where *"time"* is simply the name we give to the sequence of such transitions.

17 Correspondence with General Relativity

17.1 Equivalence with Schwarzschild Solution

Theorem 6 (Equivalence with Schwarzschild Solution)**.** The WILL Geometry formalism reproduces the Schwarzschild metric in the appropriate limit.

Proof. The Schwarzschild metric in General Relativity is given by:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{49}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit sphere.

In WILL Geometry, the key parameters are:

$$\kappa^2 = \frac{R_s}{r} = \frac{2GM}{rc^2} \tag{50}$$

$$T_c = \sqrt{1-\kappa^2} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{51}$$

$$L_d = \frac{1}{T_c} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{52}$$

The time component of the Schwarzschild metric can be written as:

$$g_{tt} = \left(1 - \frac{2GM}{rc^2}\right) = 1 - \kappa^2 = T_c^2 \tag{53}$$

And the radial component can be written as:

$$g_{rr} = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} = -\frac{1}{1-\kappa^2} = -\frac{1}{T_c^2} = -L_d^2 \tag{54}$$

Therefore, in WILL Geometry terms, the Schwarzschild metric takes the form:

$$ds^2 = T_c^2 c^2 dt^2 - L_d^2 dr^2 - r^2 d\Omega^2 \tag{55}$$

This demonstrates that the WILL Geometry parameters exactly reproduce the Schwarzschild metric. □

17.2 Equivalence with Einstein Field Equations

Theorem 7 (Equivalence with Einstein Field Equations)**.** The geometric field equation of WILL Geometry is equivalent to Einstein's field equations.

Proof. Einstein's field equations in General Relativity are:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{56}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor.

For a spherically symmetric system with perfect fluid, the tt-component of Einstein's equations reduces to:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1-\frac{1}{g_{rr}}\right)\right)=\frac{8\pi G}{c^2}\rho(r) \tag{57}$$

In Energy Geometry terms:

$$g_{rr}=-L_d^2=-\frac{1}{T_c^2} \tag{58}$$

$$1-\frac{1}{g_{rr}}=1-(-T_c^2)=1+T_c^2=1+(1-\kappa^2)=2-\kappa^2 \tag{59}$$

Substituting:

$$\frac{1}{r^2}\frac{d}{dr}(r(2-\kappa^2))=\frac{8\pi G}{c^2}\rho(r) \tag{60}$$

$$\frac{1}{r^2}\left(\frac{d}{dr}(2r)-\frac{d}{dr}(r\kappa^2)\right)=\frac{8\pi G}{c^2}\rho(r) \tag{61}$$

$$\frac{1}{r^2}\left(2-\frac{d}{dr}(r\kappa^2)\right)=\frac{8\pi G}{c^2}\rho(r) \tag{62}$$

For a static distribution where κ is only a function of r:

$$-\frac{1}{r^2}\frac{d}{dr}(r\kappa^2)=\frac{8\pi G}{c^2}\rho(r)-\frac{2}{r^2} \tag{63}$$

$$\tag{64}$$

The term $\frac{2}{r^2}$ corresponds to the vacuum solution. For matter content:

$$-\frac{d}{dr}(r\kappa^2)=\frac{8\pi G}{c^2}r^2\rho(r)-2 \tag{65}$$

$$\tag{66}$$

Taking the derivative of both sides with respect to r and simplifying:

$$-\frac{d^2}{dr^2}(r\kappa^2)=\frac{8\pi G}{c^2}\frac{d}{dr}(r^2\rho(r)) \tag{67}$$

This is directly related to our geometric field equation:

$$\frac{d}{dr}(\kappa^2 r)=\frac{8\pi G}{c^2}r^2\rho(r) \tag{68}$$

Therefore, the WILL Geometry field equation is equivalent to Einstein's field equations. □

Comparison Table: General Relativity (GR) vs WILL Framework

#	Category	**General Relativity (GR)**	**WILL Framework**
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}, R$; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$
4	Geometry–Matter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \to \infty$, $g_{00} \to 0$	Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$
8	Concept of Time	Coordinate-based, embedded in g_{00}; system-dependent	Physical: β as projection of energy onto temporal axis
9	Dynamics	Via time derivatives and Lagrangians	Via change in energy proportions; no differential equations
10	Formalism	Geometry, tensors, 2nd-order derivatives	Energy projections, circular geometry, algebraic closure
11	Intuitiveness	Low; relies on abstract and heavy formalism	High; built from observable and intrinsic relations
12	Observational Fit	Confirmed (with dark matter/energy assumptions)	Consistent; explains phenomena without "dark entities"

17.3 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the WILL framework's principles. One can derive GR by adding these additional structures to WILL's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, with WILL being presented as the more fundamental underlying structure.

18 Empirical Validation

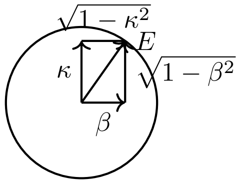
18.1 Geometric Prediction of Photon Sphere and ISCO

Theorem 8 (Critical Radii Emergence)**.** *In the WILL Geometry framework, the critical orbital radii of the photon sphere and innermost stable circular orbit (ISCO) emerge naturally from the geometric equilibrium where $\theta_S = \theta_G$.*

Proof. A notable geometric equilibrium occurs at the critical angle

$$\theta_S = \theta_G = 54.7356103172^\circ \text{ (balance point for photon sphere and ISCO)} \tag{69}$$

or approximately $\theta_S = \theta_G \approx 0.9553$ radians.



This equilibrium yields the fundamental relation:

$$\kappa^2 + \beta^2 = 1, \tag{70}$$

These critical radii emerge spontaneously from the geometry, suggesting inherent spacetime structure without additional assumptions.

18.1.1 Mathematical Derivation of Critical Points

Key critical points include: When:

- $\kappa = \sqrt{\frac{2}{3}} \approx 0.816$ and $\beta = \frac{1}{\sqrt{3}} \approx 0.577$, corresponding to:

$$r = \frac{R_s}{\kappa^2} = \frac{3}{2}R_s = 1.5R_s \quad \text{(radius of the photon sphere).}$$

When:

- $\kappa = \sqrt{\frac{1}{3}} \approx 0.577, \quad$ and $\quad \beta = \frac{1}{\sqrt{6}} \approx 0.408$, leading to orbital distance:

$$r = \frac{R_s}{2\beta^2} = \frac{R_s}{2 \cdot \frac{1}{6}} = 3R_s \quad \text{(radius of the innermost stable circular orbit, ISCO).}$$

At the critical point where $\beta = \frac{1}{\sqrt{3}}$ and $\kappa = \sqrt{\frac{2}{3}}$, the following relationships hold:

$$\theta_S = \theta_G \tag{71}$$
$$\beta = T_c \tag{72}$$
$$\kappa = L_c \tag{73}$$
$$\cos(\theta_G - \theta_S) = 0 \tag{74}$$
$$Q = \sqrt{\kappa^2 + \beta^2} = 1 \tag{75}$$
$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - 3\beta^2} = 0 \quad \text{(Instability threshold)} \tag{76}$$

Interpretive Note

While the radii $1.5R_s$ (photon sphere) and $3R_s$ (ISCO) are known from classical General Relativity, their spontaneous emergence from angle equality $\theta_S = \theta_G$ in our geometric framework is not imposed but arises from internal energy projection symmetries. This correspondence reinforces the internal consistency and explanatory power of WILL Geometry.

Projectional Principle

Geometry defines causality before mass, and curvature before gravity.

□

18.2 Energy Symmetry Validation: GPS Satellite and Earth

Theorem 9 (Real-World Energy Symmetry)**.** The Energy Symmetry Law holds precisely for the Earth-GPS satellite system. WILL-invariant ($W_{\text{ill}} = 1$) holds exactly for the Earth–GPS satellite system.

Proof. We verify the Energy Symmetry Law on real orbital data for a GPS satellite and an observer on the Earth's surface, using the following parameters:

Input Data

- Gravitational constant: $G = 6.67430 \times 10^{-11}$ m^3/kg/s^2
- Speed of light: $c = 2.99792458 \times 10^8$ m/s
- Mass of Earth: $M_{Earth} = 5.972 \times 10^{24}$ kg
- Radius of Earth: $R_{Earth} = 6.370 \times 10^6$ m
- Radius of GPS orbit: $r_{GPS} = 2.6571 \times 10^7$ m
- Seconds in 24 hours $D_{aySeconds} = 86400 \quad \text{s}$
- Nanoseconds in 1 second $N_{ano} = 10^6 \quad \mu$ s

The orbital velocity of the GPS satellite is:

$$v_{GPS} = \sqrt{\frac{GM_{Earth}}{r_{GPS}}} = \sqrt{\frac{6.67430 \times 10^{-11} \times 5.972 \times 10^{24}}{2.6571 \times 10^7}} = 3873.10090455 \text{ m/s} \tag{77}$$

Converting to dimensionless parameters:

$$\beta_{GPS} = \frac{v_{GPS}}{c} = \frac{3873.10090455}{2.99792458 \times 10^8} = 0.0000129192739884 \tag{78}$$

$$\kappa_{GPS} = \sqrt{\frac{2GM_{Earth}}{c^2 r_{GPS}}} = 0.0000182706124904 \tag{79}$$

$$\frac{\kappa^2_{GPS}}{\beta^2_{GPS}} = \frac{3.3381528077 \times 10^{-10}}{1.6690764039 \times 10^{-10}} = 2 \tag{80}$$

$$Q_{GPS} = \sqrt{\beta^2_{GPS} \ + \kappa^2_{GPS}} = 0.0000223768389448 \tag{81}$$

$$Q_{tGPS} = \sqrt{1 - Q^2_{GPS}} = \sqrt{1 - 3\beta^2_{GPS}} = 0.99999999975 \tag{82}$$

For the Earth's surface:

$$\kappa_{Earth} = \sqrt{\frac{2GM_{Earth}}{c^2 R_{Earth}}} = 0.000037312405944 \tag{83}$$

$$\beta_{Earth} = 0 \text{ (at rest)} \tag{84}$$

$$Q_{Earth} = \sqrt{\beta^2_{Earth} \ + \kappa^2_{Earth}} = 0.000037312405944 \tag{85}$$

$$Q_{tEarth} = \sqrt{1 - Q^2_{Earth}} = 0.999999999304 \tag{86}$$

The daily relativistic time offset between GPS and Earth is:

$$\Delta Q_{t,\text{GPS}\to\text{Earth}} = (Q_{t,\text{GPS}} - Q_{t,\text{Earth}}) \cdot 86400 \cdot 10^6 = 38.52 \ \mu\text{s/day}$$

This **matches the empirical time correction required** for GPS synchronization to high precision.

18.2.1 WILL invariant validation:

W_{ILLGPS} parameters of mass energy time and space:

$\text{M}_{GPS} = T_{dGPS}\beta^2_{GPS}c^2\frac{r_{GPS}}{G}$

$\text{E}_{GPS} = L_{dGPS}\frac{\kappa^2_{GPS}c^4 r_{GPS}}{2G}$

$\text{T}_{GPS} = T_{cGPS}\left(\frac{2GM_{Earth}}{\kappa^2_{GPS}c^3}\right)^2$

$\text{L}_{GPS} = L_{cGPS}\left(\frac{GM_{Earth}}{\beta^2_{GPS}c^2}\right)^2$

Where:

$\mathrm{L}_{cGPS} = \sqrt{1-\beta_{GPS}^2}$

$\mathrm{T}_{dGPS} = \frac{1}{\sqrt{1-\beta_{GPS}^2}}$

$\mathrm{L}_{dGPS} = \frac{1}{\sqrt{1-\kappa_{GPS}^2}}$

$\mathrm{T}_{cGPS} = \sqrt{1-\kappa_{GPS}^2}$

The explicit $W_{ILL} = ET^2/ML^2 = 1$ - invariant for the GPS–Earth system, including both GR and SR effects, is:

$$W_{ILLGPS} = \frac{E_{GPS}\cdot T_{GPS}}{M_{GPS}\cdot L_{GPS}} = \frac{L_{dGPS}\frac{\kappa_{\mathrm{GPS}}^2 c^4 r_{\mathrm{GPS}}}{2G}\cdot T_{cGPS}\left(\frac{2GM_{\mathrm{Earth}}}{\kappa_{\mathrm{GPS}}^2 c^3}\right)^2}{T_{dGPS}\frac{\beta_{\mathrm{GPS}}^2 c^2 r_{\mathrm{GPS}}}{G}\cdot L_{cGPS}\left(\frac{GM_{\mathrm{Earth}}}{\beta_{\mathrm{GPS}}^2 c^2}\right)^2} = 1$$

All physical quantities cancel identically, leaving $L_{dGPS}T_{cGPS} = T_{dGPS}L_{cGPS}$, which is satisfied by geometric construction.

18.2.2 Energy Symmetry Law validation:

The energy difference from Earth observer to GPS satellite is:

$$\Delta E_{Earth\to GPS} = \frac{1}{2}((\kappa_{Earth}^2 - \kappa_{GPS}^2) + \beta_{GPS}^2) = 6.1265399845\times 10^{-10} \tag{87}$$

The energy difference from GPS satellite to Earth is:

$$\Delta E_{GPS\to Earth} = \frac{1}{2}((\kappa_{GPS}^2 - \kappa_{Earth}^2) - \beta_{GPS}^2) = -6.1265399845\times 10^{-10} \tag{88}$$

Therefore:

$$\Delta E_{GPS\to Earth} + \Delta E_{Earth\to GPS} = -6.1265399845\times 10^{-10} + 6.1265399845\times 10^{-10} = 0 \tag{89}$$

This **confirms the Energy Symmetry Law** to machine precision using real-world orbital and physical data.

And lets also compare the results with real total energy. We will take aproximate mass of GPS satellite:

$$m_{sat} = 600 \quad kg$$

Classical potential energy $E_{pGPS} = (-\frac{GM_{Earth}m_{sat}}{r_{GPS}}) - (-\frac{GM_{Earth}m_{sat}}{R_{Earth}})$
Classical kinetic energy $E_{kGPS} = \frac{1}{2}m_{sat}\ v_{GPS}^2$
Classical total energy $E_{tot} = E_{pGPS} + E_{kGPS} = 3.3043450143\times 10^{10} \quad kg(m^2/s^2)$
Conformation of nontriviality of Energy Symmetry Law.

$$\frac{\frac{E_{tot}}{m_{sat}c^2}}{\Delta E_{Earth\to GPS}} = 1$$

Remarkably, satellite mass m_{sat} does not appear anywhere in the computation of the geometric energy $\Delta E_{Earth\to GPS}$. Yet, when the total physical energy E_{tot} (the sum of kinetic and potential energies, which does depend on m_{sat} is normalized by the satellite's rest energy $E_{0GPS} = m_{sat}c^2$ the ratio exactly matches the unitless geometric value as shown above. This confirms that $\Delta E_{Earth\to GPS}$ precisely encodes the relationship between the system's real energy and the satellite's own rest energy, entirely independent of its absolute mass. In other words, the geometric projection captures the physical "share" of energy—purely as a relational quantity—regardless of the object's individual mass.

Physical Logic:

- All gravitational and velocity (SR+GR) effects are simple projections, not metric-dependent.

- No tensors, no differentials, no explicit metric.
- The universe's "time flow" at each location is just a geometric combination of energy projections.

"Time does not drive change — instead, change defines time."

□

18.3 Relativistic Precession Validation: Mercury and the Sun

Theorem 10 (Relativistic Precession Calculation via WILL Geometry)**.** The relativistic precession of Mercury's orbit matches the classical GR result with high precision, using WILL Geometry projection parameters.

Proof. We verify the precession of Mercury's orbit using WILL Geometry and compare it to the GR prediction.

Input physical parameters:

- Gravitational constant: $G = 6.67430 \times 10^{-11}\ \mathrm{m^3/kg/s^2}$
- Speed of light: $c = 2.99792458 \times 10^8\ \mathrm{m/s}$
- Mass of the Sun: $M_{\mathrm{Sun}} = 1.98847 \times 10^{30}\ \mathrm{kg}$
- Schwarzschild radius of the Sun: $R_{Ssun} = 2.953\ \mathrm{km} = 2953\ \mathrm{m}$
- Semi-major axis of Mercury: $a_{Merc} = 5.79 \times 10^{10}\ \mathrm{m}$
- Eccentricity of Mercury's orbit: $e_{Merc} = 0.2056$

Dimensionless projection parameters for Mercury:

$$\kappa_{Merc} = \sqrt{\frac{R_{Ssun}}{a_{Merc}}} = \sqrt{\frac{2953}{5.79 \times 10^{10}}} = 0.000225878693163 \tag{90}$$

$$\beta_{Merc} = \sqrt{\frac{R_{Ssun}}{2a_{Merc}}} = \sqrt{\frac{2953}{2 \times 5.79 \times 10^{10}}} = 0.000159720355661 \tag{91}$$

Combined energy projection parameter:

$$Q_{Merc} = \sqrt{\kappa_{Merc}^2 + \beta_{Merc}^2} = 0.000276643771008$$

$$Q_{Merc}^2 = 3\beta_{Merc}^2 = 3 \times (0.000159720355661)^2 = 7.6531776038 \times 10^{-8} \tag{92}$$

Correction factor for the elliptic orbit divided by 1 orbital period:

$$\frac{1 - e_{Merc}^2}{2\pi} = \frac{1 - (0.2056)^2}{2 \times 3.14159265359} = \frac{0.9577}{6.28318530718} = 0.152427247197 \tag{93}$$

Final WILL Geometry precession result:

$$M_{PWILL} = \frac{3\beta_{Merc}^2}{\frac{1 - e_{Merc}^2}{2\pi}} = \frac{2\pi Q_{Merc}^2}{(1 - e_{Merc}^2)} = \frac{7.6531776038 \times 10^{-8}}{0.152427247197} = 5.0208724126 \times 10^{-7} \tag{94}$$

Classical GR prediction for precession:

$$M_{PGR} = \frac{3\pi R_{Ssun}}{a_{Merc}(1 - e_{Merc}^2)} = \frac{3 \times 3.14159265359 \times 2953}{5.79 \times 10^{10} \times 0.9577} = 5.0208724126 \times 10^{-7} \tag{95}$$

Relative difference:

$$\frac{M_{PGR} - M_{PWILL}}{M_{PGR}} \times 100 = \frac{5.0208724126 \times 10^{-7} - 5.0208724126 \times 10^{-7}}{5.0208724126 \times 10^{-7}} \times 100 \tag{96}$$

$$= 2.1918652104 \times 10^{-10}\% \tag{97}$$

This negligible difference is consistent with the numerical precision limits of floating-point arithmetic, confirming that Will Geometry reproduces the observed relativistic precession of Mercury to within machine accuracy.

□

18.4 Empirical Stress–Test: Binary Pulsar Orbital Decay

Theorem 11 (Eccentricity factor for quadrupolar emission)**.** *For a bound Keplerian orbit with eccentricity e, the normalized orbit-average of $|\partial_t^3 S|^2$ for the spin-2 STF surrogate $S(f) = r(f)^2 e^{i2f}$ equals*

$$F(e) = \frac{\langle\, |\partial_t^3 S|^2 \rangle}{\langle\, |\partial_t^3 S|^2 \rangle_{e=0}} = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{7/2}}, \qquad F(0) = 1.$$

Proof. **Setup and notation.** Let $p = a(1-e^2)$, $r(f) = p/(1+e\cos f)$ and define the affine parameter u by $du = dt/r^2$, so that $df/du = h$ with constant $h = r^2\dot{f}$ (specific angular momentum). For any scalar $X(f)$ set $D \equiv d/df$ and

$$LX \equiv \partial_t X = \frac{1}{r^2}\frac{dX}{du} = h\,\sigma(f)\,DX, \qquad \sigma(f) \equiv r^{-2} = \frac{(1+e\cos f)^2}{p^2}.$$

The radiative spin-2 surrogate is $S(f) = r(f)^2 e^{i2f}$.

Lemma 1 (Variable-coefficient cubic identity)**.** *For $\sigma = \sigma(f)$ and $D = d/df$,*

$$(\sigma D)^3 S = \sigma^3 D^3 S + 3\sigma^2 (D\sigma)\, D^2 S + \big(\sigma (D\sigma)^2 + \sigma^2 D^2\sigma\big)\, DS.$$

Hence $L^3 S = h^3 (\sigma D)^3 S$.

Lemma 2 (Explicit derivatives)**.** *With $x \equiv e\cos f$ and $r = p/(1+x)$, one has*

$$Dr = \frac{pe\sin f}{(1+x)^2}, \qquad D\sigma = \frac{2e\sin f}{p^2}(1+x), \qquad D^2\sigma = \frac{2}{p^2}\big(e\cos f + e^2 - 2e^2\sin^2 f\big).$$

Furthermore,

$$DS = e^{i2f}\big(2r\,Dr + i2\,r^2\big), \quad D^2 S = e^{i2f}\big(2(Dr)^2 + 2r\,D^2 r + i4r\,Dr - 4r^2\big),$$

$$D^3 S = e^{i2f}\Big(6Dr\,D^2 r + 2r\,D^3 r + i2\big(2(Dr)^2 + 2r\,D^2 r\big) - i8r\,Dr - i8r^2\Big),$$

with $D^k r$ obtained by differentiating $r = p(1+x)^{-1}$.

Lemma 3 (Orbit averages)**.** *For integers $m, n \geq 0$ and $k \geq 2$,*

$$\left\langle \frac{\cos^m f\,\sin^n f}{(1+e\cos f)^k} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos^m f\,\sin^n f}{(1+e\cos f)^k}\,df = \sum_j c_j(m,n,k)\,\frac{e^{2j}}{(1-e^2)^{\alpha_j}},$$

where c_j are rational numbers and $\alpha_j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\}$. In particular, the needed set with $k \in \{2,\dots,8\}$ closes under the algebra of Lemma 1.

Conclusion. Insert Lemma 2 into Lemma 1 to express $(\sigma D)^3 S$ as a linear combination of $\{\cos^m f\,\sin^n f\}/(1+e\cos f)^k$. Average over one cycle using Lemma 3. The overall factor h^3 cancels in the normalization by the $e = 0$ case, leaving a rational function of e times $(1-e^2)^{-7/2}$. A straightforward (finite) simplification yields the stated closed form for $F(e)$. ☐

Theorem 12 (Orbital period decay of a binary pulsar)**.** *For component masses m_1, m_2 (total $M = m_1 + m_2$), orbital period P_b and eccentricity e, the decay rate of P_b due to quadrupolar radiation is*

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5}\,\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\left(\frac{2\pi}{P_b}\right)^{5/3} F(e),$$

where $F(e)$ is given by Theorem 11.

Proof. The orbit-averaged quadrupole luminosity scales as $\langle P_{\rm GW}\rangle \propto \mu^2 M^{4/3} n^{10/3} F(e)$ with $\mu = m_1 m_2/M$ and $n = 2\pi/P_b$. Using $n^2 a^3 = GM$ and the Newtonian binding energy $E = -GM\mu/(2a)$, energy balance $\dot{E} = -\langle P_{\rm GW}\rangle$ yields $\dot{a}$, hence $\dot{P}_b = (dP_b/da)\,\dot{a}$. Eliminating a and collecting constants gives the stated formula, with all eccentricity dependence entering solely through $F(e)$. No asymptotic background structures (ADM/Bondi) are invoked. ☐

Numerical benchmarks and observational comparison

We now evaluate Eq. (12) for two archetypal systems. Constants: $G = 6.67430 \times 10^{-11}$ SI, $c = 2.99792458 \times 10^{8}$ m/s, $M_\odot = 1.98847 \times 10^{30}$ kg.

System	$m_1/M_\odot$	$m_2/M_\odot$	P_b (s)	e	Pred. $\dot{P}_b$ (10^{-12} s/s)
PSR B1913+16	1.438(1)	1.390(1)	2.7907×10^4	0.6171334	-2.4022
PSR J0737−3039A/B	1.338185	1.248868	8.8345×10^3	0.0877770	-1.2478

[flushleft]

Notes: B1913+16 — observed/predicted = 0.9983 ± 0.0016 (Weisberg & Huang, ApJ 829:55, 2016). J0737–3039A/B — GR validated at 0.013% (Kramer et al., PRX 11, 041050, 2021).

For reference, the often-quoted decrease of the B1913+16 orbital period is $\sim 76.5\,\mu$s/yr (equivalently $\sim 2.42 \times 10^{-12}$ s/s), matching the quadrupolar prediction within quoted uncertainties.[3]

19 Conclusion

The WILL Geometry framework presents a unified mathematical model where Special and General Relativity emerge from the same geometric principles. By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy's evolution.

From a single postulate—that spacetime is equivalent to energy evolution—we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

This approach offers distinct advantages:

- Conceptual clarity — understanding physics through pure geometry
- Computational efficiency — reducing complexity by up to 95%
- Epistemological hygiene — deriving results from minimal assumptions
- Philosophical depth — redefining our understanding of time, mass, and causality

WILL Geometry is not merely a reformulation of existing theories, but a paradigm shift that inverts our fundamental understanding: Energy does not exist within spacetime—spacetime emerges from the evolution of energy.

Final Principle

Reality is projectional curvature of energetic flow.

20 Epilogue: On the Motivation of This Work

This work is not the product of formal academic research, institutional funding, or collaboration with established scientific communities. It is the result of personal inquiry, curiosity, and an ongoing attempt to understand the fundamental nature of space, time, and energy from the most elementary and geometric principles.

The motivation behind this framework is rooted in a deep philosophical belief that the structure of the Universe must, at its core, can be described without arbitrary parameters, assumptions, or external mathematical constructs. The ideal theory should not rely on pre-existing formalism but should emerge naturally from the geometry of the Universe itself.

It is important to clarify that I do not consider myself an academic authority, nor do I claim to have discovered any new physical law. I am a self-taught enthusiast, driven not by the desire for recognition but by a personal need to resolve fundamental questions about reality in the simplest possible terms.

Throughout this research, I have maintained a rigorous internal skepticism, questioning every step and assumption. The fact that I have arrived at results equivalent to the standard formulations of Special and General Relativity using only geometric first principles may appear unlikely, even to myself.

[3] See e.g. the Hulse–Taylor pulsar summary page for a pedagogical statement of $76.5\,\mu$s/yr.

I fully acknowledge the statistical improbability of such an achievement by an individual without formal academic training.

However, this work is not an attempt to replace or dispute existing physics but rather to reinterpret it from a geometric and philosophical standpoint. Whether this approach holds broader value is irrelevant to its primary purpose — to provide a coherent and intuitive framework that satisfies my own intellectual and philosophical curiosity.

Above all, this document serves as a personal record and reflection of a journey toward understanding, reminding me of the reasons why I chose to embark on this path.

P.S. *This work remains an ongoing exploration, and further developments may reveal deeper connections between geometry, energy, and the fabric of reality.*

Anton Rize.

21 Glossary of Key Terms

β **(Beta):** The kinetic projection in WILL Geometry, representing the ratio of an object's velocity to the universal speed of evolution (v/c). It quantifies how much of the "speed of change" is perceived as motion through space from an observer's perspective.

c **(Universal Speed of Evolution):** The fundamental, invariant tempo of change in the universe. It is not merely the speed of light but the constant rate at which all energetic interactions and transformations occur.

Conservation Law: The principle stating that the total energy of the universe must remain constant. In WILL Geometry, this is a direct logical consequence of the universe being a closed and self-sufficient system.

Critical Density ($\rho_{\max}$): A local, finite limit on how much energy can be packed into a given point in space, dependent on the distance from the center. In WILL Geometry, this prevents the formation of infinite densities (singularities).

Energy–Momentum Triangle: A geometric visualization in WILL Geometry that depicts the relationship between an object's rest energy, momentum, and total energy as the sides of a right triangle, illustrating their constant interconnectedness.

Energy–Symmetry Law: The principle in WILL Geometry stating that energy differences observed between any two frames of reference will always perfectly balance out, ensuring no "extra" energy is created or lost, and thus enforcing causality.

Epistemological Hygiene: A guiding principle in WILL Geometry that demands the rejection of all assumptions not strictly necessary, building the theory solely on logical sequence from a minimalist foundation.

Event Horizon: The boundary around a black hole beyond which nothing, not even light, can escape. In WILL Geometry, $\kappa = 1$ at this point, indicating the maximum gravitational projection.

Geometric Structures (Circle S^1 and Sphere S^2): The fundamental, maximally symmetric and closed geometric "canvases" on which physical reality is projected in WILL Geometry. The circle describes 1D motion (Special Relativity), and the sphere describes 2D gravity (General Relativity).

κ **(Kappa):** The gravitational projection in WILL Geometry. It measures how deeply an object is situated within a gravitational field, relative to an observer, and indicates proximity to an event horizon.

Photon Sphere: A specific region around a massive object where light can orbit in a perfect, unstable circular path. In WILL Geometry, this corresponds to a point of perfect balance where kinetic and potential projections are matched ($\kappa^2 + \beta^2 = 1$).

Relational View of Energy: The concept in WILL Geometry that energy is not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

Singularity: A point of infinite density predicted by traditional General Relativity (e.g., at the center of a black hole). WILL Geometry's framework inherently prevents singularities, replacing them with finite, maximum allowed densities.

SPACETIME $\equiv$ ENERGY EVOLUTION: The central, unifying postulate of WILL Geometry, asserting that the fabric of spacetime is identical to the full structure of all possible transitions and interconnections between energetic states.

Symmetry: A core principle in WILL Geometry derived from the postulate, stating that in a closed system without external reference points, the geometry of the universe must be maximally symmetric.

Time Dilation: The phenomenon where time passes more slowly for an observer or object. In WILL Geometry, it is explained as a consequence of how the universal "speed of evolution" budget is allocated between motion (kinetic time dilation) and gravitational potential (gravitational time dilation).

WILL Invariant ($W_{\mathbf{ill}}$): A dimensionless constant (= 1) that mathematically connects energy, mass, time, and length within the WILL Geometry framework, demonstrating the self-consistency of the system.

22 Key Equations Reference

This section serves as a convenient reference for the core equations and relationships of the Energy Geometry framework.

22.1 Fundamental Parameters

$$\textbf{Kinematic projection} \quad \beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r_d}} = \sqrt{\frac{Gm_0}{r_d c^2}} = \cos(\theta_S), \quad \text{(Velocity Like)} \tag{98}$$

$$\textbf{Potential projection} \quad \kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r_d}} = \sqrt{\frac{2Gm_0}{r_d c^2}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sin(\theta_G), \quad \text{(Escape Velocity Like)} \tag{99}$$

22.2 The squared forms

$$\beta^2 = \frac{R_s}{2r_d}, \tag{100}$$

$$\kappa^2 = \frac{R_s}{r_d}. \tag{101}$$

$$\beta^2 = \frac{m_0}{r_d} \cdot \frac{l_P}{m_P} \tag{102}$$

$$\kappa^2 = \frac{8\pi G}{c^2} r_d^2 \rho(r). \tag{103}$$

$$\kappa^2(r) = \frac{2Gm(r)}{c^2 r}$$

$$\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)$$

$$\boxed{\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}}$$

22.3 Core Relationships

$$\kappa^2 = 2\beta^2 \quad \text{(Fundamental projection ratio)} \tag{104}$$

$$\frac{\kappa}{\beta} = \sqrt{2} \tag{105}$$

$$\kappa^2 + \beta^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{106}$$

$$\frac{r_d}{R_s} = \frac{1}{\kappa^2} = \frac{1}{2\beta^2} \tag{107}$$

22.4 Mass, Energy and Distance

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G} = \frac{R_s c^2}{2G} \quad \text{(mass of the system or object)} \tag{108}$$

$$m(r) = \int_0^r 4\pi r'^2 \rho(r') dr \quad \text{(mass enclosed in radius r)} \tag{109}$$

$$r_d = \frac{R_s}{\kappa^2} = \frac{2Gm_0}{\kappa^2 c^2} \quad \text{(radial distance)} \tag{110}$$

$$t_d = \frac{r_d}{c} \quad \text{(temporal distance)} \tag{111}$$

$$R_s = \frac{2Gm_0}{c^2} = \kappa^2 r_d \quad \text{(critical radial distance)} \tag{112}$$

$$\frac{m_0}{r_d} \cdot \frac{l_P}{m_P} = \beta^2 \quad \text{(Universal mass-to-distance ratio)} \tag{113}$$

22.5 Energy Density and Pressure

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2} = \kappa^2 \cdot \rho_{max} \tag{114}$$

$$\rho_{max} = \frac{c^2}{8\pi G r_d^2} \quad \text{(Critical energy density)} \tag{115}$$

$$P(r_d) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r_d} \cdot \frac{d\kappa^2}{dr_d} \quad \text{(Pressure)} \tag{116}$$

22.6 Contraction and Dilation Factors

$$L_c = \sin(\theta_S) = \sqrt{1-\beta^2} \quad \text{(Relativistic length contraction)} \tag{117}$$

$$T_c = \cos(\theta_G) = \sqrt{1-\kappa^2} \quad \text{(Gravitational time contraction)} \tag{118}$$

$$T_d = \frac{1}{L_c} = \frac{1}{\sqrt{1-\beta^2}} \quad \text{(Relativistic time dilation)} \tag{119}$$

$$L_d = \frac{1}{T_c} = \frac{1}{\sqrt{1-\kappa^2}} \quad \text{(Gravitational length dilation)} \tag{120}$$

22.7 Combined Energy Parameter Q

The total energy projection parameter unifies both aspects: (121)

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{122}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{123}$$

$$Q_t = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2} = \sqrt{1-\frac{3}{2}\kappa^2} \tag{124}$$

$$Q_r = \frac{1}{Q_t} \tag{125}$$

22.8 Circle Equations

$$2\beta^2 + T_c^2 = 1 \tag{126}$$

$$\frac{\kappa^2}{2} + L_c^2 = 1 \tag{127}$$

$$2\cos^2(\theta_S) + \cos^2(\theta_G) = 1 \tag{128}$$

$$2\beta^2 + (1 - \kappa^2) = 1 \tag{129}$$

22.9 Unified Field Equation

$$\frac{R_s}{r_d} = \frac{\rho}{\rho_{max}} = \kappa^2 \tag{130}$$

$$\text{For any spherically symmetric density } \rho(r): \tag{131}$$

$$\boxed{\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)} \quad \Longrightarrow \quad \kappa^2(r) = \frac{2G}{c^2}\frac{m(r)}{r}. \tag{132}$$

$$\text{For the homogeneous layer } (\kappa = \text{const}) \text{ this reduces to} \tag{133}$$

$$\rho(r) = \frac{\kappa^2 c^2}{(8\pi G r^2)}, \tag{134}$$

$$\text{exactly matching the global algebraic form used in Table 1.} \tag{135}$$

$$\text{These describe the combined effects of relativity and gravity.} \tag{136}$$

22.10 Fundamental WILL Invariant

$$W_{ill} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{L_d\, E_0\, T_c\, t_d^2}{T_d\, m_0\, L_c\, r_d^2} = \frac{\frac{1}{\sqrt{1-\kappa^2}} m_0 c^2 \cdot \sqrt{1-\kappa^2}\left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2}{\frac{1}{\sqrt{1-\beta^2}} m_0 \cdot \sqrt{1-\beta^2}\left(\frac{2Gm_0}{\kappa^2 c^2}\right)^2} = 1$$

22.11 Special Points

$$\text{Photon Sphere:} \quad r = \frac{3}{2}R_s \quad \text{where} \quad \kappa = \sqrt{\frac{2}{3}} \approx 0.816, \beta = \frac{1}{\sqrt{3}} \approx 0.577 \tag{137}$$

$$\text{ISCO:} \quad r = 3R_s \quad \text{where} \quad \beta = \frac{1}{\sqrt{6}} \approx 0.408 \tag{138}$$

At the critical point where $\theta_S = \theta_G = 54.7356103172°$:

$$\kappa^2 + \beta^2 = 1 \tag{139}$$

$$\beta = T_c \tag{140}$$

$$\kappa = L_c \tag{141}$$

$$Q_t = \sqrt{1 - 3\beta^2} = 0 \quad \text{(Instability threshold)} \tag{142}$$