WILL Part (B) I: Relational Geometry

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Abstract

We finished "WILL Part (A) I" with a profound conclusion:
General Consequence:
Bad philosophy, in RG sense, has three measurable effects:

1. Inflated Formalism: Equations multiply to compensate for ontological error.
2. Loss of Transparency: Physical meaning becomes hidden behind coordinate dependencies.
3. Empirical Fragmentation: Each domain (cosmology, quantum, gravitation) requires separate constants.

By contrast, good philosophy-**epistemic hygiene**-enforces relational closure and yields simplicity through necessity, not through approximation.

In short:

$$\boxed{\textbf{Bad philosophy creates complexity} \quad \textbf{Good philosophy reveals geometry.}}$$

Daring Remark

The historical escalation of mathematical complexity in physics did not reveal deeper reality - it compensated for a philosophical mistake. Once the ontological symmetry is restored, Nature's laws reduce to algebraic self-consistency.

$$\boxed{\text{Bad Philosophy}} \Rightarrow \boxed{\text{Ontological Duplication}} \Rightarrow \boxed{\text{Mathematical Inflation}}$$

$$\boxed{\textbf{Complexity is the symptom of philosophical negligence.}}$$

In "WILL Part (B) I" we will show the applications of this conclusion:

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1 Relational Orbital Mechanics Without Mass or G

Thesis

Orbital dynamics requires no mass, no G, no metric, and no spacetime geometry. All observable orbital structure follows from two directly measurable frequency projections:

$$\kappa \,\text{(gravitational projection from redshift)}, \quad \beta \,\text{(kinematic projection from Doppler)}.$$

Everything else is algebra.

1.1 Two Operational Pathways

Depending on the available observational data, the framework offers two distinct operational pathways:

- **Path 1: Verification (Full Data Available).** Used when both the central potential (redshift z) and the orbital kinematics (β, e) are measurable independently (e.g., Mercury/Sun). Here, we calculate δ_p from inputs and compare the predicted eccentricity with observation to validate the theory.
- **Path 2: Reconstruction (Partial Data Available).** Used for distant systems (e.g., Star S2, Exoplanets) where the central potential is unknown or entangled with Doppler shifts. Here, we use the observed shape (e) and velocity (β) to **reconstruct** the hidden potential depth κ_p via geometric compatibility.

1.2 Derivation of Relational Eccentricity

Geometric eccentricity is not a free parameter but a measure of the energetic deviation from the circular equilibrium state ($\delta = 1$).

Theorem 1.1 (Geometric Eccentricity). *For a closed orbital system governed by the projection invariants of WILL Relational Geometry, the orbital eccentricity e is strictly* *determined by the closure factor at periapsis, δ_p:*

$$e = \frac{1}{\delta_p^2} - 1 = \frac{1-\delta_p^2}{\delta_p^2}. \tag{1}$$

Proof. Instead of relying on classical force laws, we derive this relation directly from the conservation of the two fundamental projection invariants of the WILL framework:

1. **Energy Projection Invariant (Binding Energy):** $W = \frac{1}{2}(\kappa^2 - \beta^2) = \frac{E}{E_o} =$ const.

2. **Angular Projection Invariant:** $h = r\beta =$ const (at turning points).

Consider the two turning points of a closed orbit: periapsis (p) and apoapsis (a). By operational definition of the shape parameter e, the relation between radii is determined by the geometric range:

$$r_a = r_p \left(\frac{1+e}{1-e}\right). \tag{2}$$

Step 1: Relational Mapping. Using the angular invariant h (implying $\beta \propto 1/r$) and the field definition $\kappa^2 \propto 1/r$, we express the apoapsis projections in terms of the periapsis values:

$$\beta_a^2 = \left[\beta_p \left(\frac{r_p}{r_a}\right)\right]^2 = \beta_p^2 \left(\frac{1-e}{1+e}\right)^2, \tag{3}$$

$$\kappa_a^2 = \kappa_p^2 \left(\frac{r_p}{r_a}\right) = \kappa_p^2 \left(\frac{1-e}{1+e}\right). \tag{4}$$

Note: Kinematic projection scales quadratically with the radius ratio, while potential projection scales linearly.

Step 2: Energy Balance. Substituting these into the energy invariant conservation condition $W_p = W_a$:

$$\frac{1}{2}(\kappa_p^2 - \beta_p^2) = \frac{1}{2}(\kappa_a^2 - \beta_a^2).$$

Canceling the factor $\frac{1}{2}$ and substituting the mappings from Step 1:

$$\kappa_p^2 - \beta_p^2 = \kappa_p^2 \left(\frac{1-e}{1+e}\right) - \beta_p^2 \left(\frac{1-e}{1+e}\right)^2.$$

Rearranging to group potential terms (κ) on the left and kinematic terms (β) on the right:

$$\kappa_p^2 \left[1 - \frac{1-e}{1+e}\right] = \beta_p^2 \left[1 - \left(\frac{1-e}{1+e}\right)^2\right].$$

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Step 3: Algebraic Reduction. Expanding the terms in brackets:

$$\text{LHS bracket } (\kappa \text{ term}): \quad 1 - \frac{1-e}{1+e} = \frac{(1+e)-(1-e)}{1+e} = \frac{2e}{1+e}.$$
$$\text{RHS bracket } (\beta \text{ term}): \quad 1 - \frac{(1-e)^2}{(1+e)^2} = \frac{(1+e)^2-(1-e)^2}{(1+e)^2} = \frac{4e}{(1+e)^2}.$$

Substituting back into the balance equation:

$$\kappa_p^2 \left(\frac{2e}{1+e}\right) = \beta_p^2 \left(\frac{4e}{(1+e)^2}\right).$$

Dividing both sides by $2e$ and multiplying by $(1+e)^2$:

$$\kappa_p^2(1+e) = 2\beta_p^2.$$

This yields the fundamental geometric identity for bound orbits:

$$2\beta_p^2 = \kappa_p^2(1+e). \tag{5}$$

Step 4: Connection to Closure. Recall the definition of the closure factor at periapsis:

$$\delta_p^2 = \frac{\kappa_p^2}{2\beta_p^2}.$$

Substituting Eq.(5) into this definition:

$$\delta_p^2 = \frac{\kappa_p^2}{\kappa_p^2(1+e)} = \frac{1}{1+e}.$$

Solving for e, we obtain the stated result:

$$\boxed{e = \frac{1}{\delta_p^2} - 1} \tag{6}$$

□

Remark 1.2. *This result confirms that eccentricity is strictly a measure of the energetic deviation from the circular equilibrium state ($\delta = 1$), derived entirely from the conservation of relational projections without invoking mass or Newtonian forces.*

1.3 Path 1: Verification on Mercury (Pure Optical Inputs)

We validate the theory using Mercury, utilizing only direct optical measurements (Doppler and Redshift). We implicitly assume **zero knowledge** of the Sun's mass, the gravitational constant G, or the Schwarzschild radius derived from them.

1. Input: Kinematic Projection (β_p). Radar telemetry directly measures the orbital velocity at perihelion ($v_p \approx 58.98$ km/s). The kinematic projection is simply this velocity normalized by the speed of light:

$$\beta_p = \frac{v_p}{c} \approx 1.967 \times 10^{-4}.$$

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2. Input: Potential Projection (κ_p). Instead of deriving potential from mass, we derive it from the **measured gravitational redshift** of the Sun's photosphere, $z_{\text{surface}} \approx 2.12 \times 10^{-6}$.

Using the S^2 relational carrier geometric omnidirectional scaling law of the potential projection ($\kappa^2 \propto 1/r$), we relate the known potential at the solar physical radius (r_{surface}) to the potential at Mercury's perihelion radius (r_p):

$$\kappa^2(r_p) = \kappa^2(r_{\text{surface}}) \cdot \left(\frac{r_{\text{surface}}}{r_p}\right).$$

Using the exact redshift relation $\kappa^2(r_{\text{surface}}) = 1 - (1 + z_{\text{surface}})^{-2} \approx 2z_{\text{surface}}$:

$$\kappa_p^2 \approx 2z_{\text{surface}} \left(\frac{r_{\text{surface}}}{r_p}\right).$$

Using the observed geometric ratio of radii $r_{\text{surface}}/r_p \approx 0.01513$:

$$\kappa_p^2 \approx 2(2.12 \times 10^{-6})(0.01513) \approx 6.415 \times 10^{-8}.$$

$$\kappa_p = \sqrt{6.415 \times 10^{-8}} \approx 2.533 \times 10^{-4}.$$

3. Calculate Closure Factor (δ_p). With both projections determined purely from light measurements:

$$\delta_p = \frac{\kappa_p}{\sqrt{2}\beta_p} = \frac{2.533 \times 10^{-4}}{1.414 \times 1.967 \times 10^{-4}} \approx 0.9108.$$

4. Predict Eccentricity (e). The orbital shape is strictly enforced by the closure defect:

$$e_{\text{pred}} = \frac{1}{\delta_p^2} - 1 = \frac{1}{(0.9108)^2} - 1 \simeq 0.2056.$$

Conclusion: The predicted eccentricity matches the observed value ($e \approx 0.2056$) exactly. This demonstrates that the orbital geometry is fully determined by the ratio of the central redshift to the orbital Doppler shift, with no reference to mass or G.

1.4 Path 2: Reconstruction of Potentials (Star S2)

For the star S2 orbiting Sgr A*, we assume the closure law holds and use **Path 2** to find the hidden potential of the Black Hole.

1. Inputs (Observed Geometry):

$$e_{\text{obs}} \approx 0.8846, \qquad \beta_p \approx 0.0255\,(7650\,\text{km/s}).$$

2. Reconstruct Potential κ_p: From Eq.5, we invert the logic to find what κ_p **must** be to sustain this orbit:

$$\kappa_p = \beta_p\sqrt{2(1+e)} = 0.0255 \times \sqrt{2(1.8846)} \approx 0.0495.$$

(Note: This allows us to calculate the Black Hole's mass/scale $R_s = \kappa_p^2 r_p$ without ever measuring it directly).

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Remark 1.3 (On the Nature of Mass)**.** *While κ is the primary dimensionless observable (representing gravitational redshift), the classical concept of "Mass" (M) appears in this framework strictly as a secondary derived quantity. By inverting the definition of the Schwarzschild scale $R_s = \kappa^2 r = \frac{2GM}{c^2}$, we obtain:*

$$M = \frac{c^2 r}{2G}\kappa^2 \tag{7}$$

This demonstrates that physical mass is merely a dimensioned proxy for the geometric curvature intensity κ^2, scaled by the historical constants G and c. In this framework, geometry is fundamental; mass is an artifact of the chosen unit system.

This reconstructed κ_p is then used to predict the precession in the next section.

1.5 The Universal Precession Law: Derivation via Q_a

For an elliptic orbit, the net angular mismatch (precession) is derived from the displacement norm Q_a (**??**) measured at the system's characteristic scale. We select the semi-major axis a as this reference scale, defining the norm Q_a.

At the scale $r = a$, the closure condition ($\kappa^2 = 2\beta^2$) implies the specific distribution of the invariant Schwarzschild scale R_s:

$$\kappa^2(a) = \frac{R_s}{a}, \qquad \beta^2(a) = \frac{R_s}{2a}.$$

Substituting these into the definition of the displacement norm $Q^2 = \beta^2 + \kappa^2$:

$$Q_a^2 = \frac{R_s}{2a} + \frac{R_s}{a} = \frac{3R_s}{2a}. \tag{8}$$

The general geometric precession formula is:

$$\Delta\varphi = \frac{2\pi\, Q_a^2}{1 - e^2}. \tag{9}$$

Substituting Q_a^2, we recover the standard form purely algebraically:

$$\Delta\varphi = \frac{2\pi}{1 - e^2}\left(\frac{3R_s}{2a}\right) = \frac{3\pi R_s}{a(1 - e^2)}.$$

1.5.1 Transformation to Periapsis Observables

To eliminate the abstract parameters R_s, a, e in favor of direct observables, we map this expression to the periapsis (p), where interaction is maximal. Using the identities $R_s = \kappa_p^2 r_p$ and $a(1-e^2) = r_p(1+e)$, and the closure relation $(1+e) = 1/\delta_p^2 = 2\beta_p^2/\kappa_p^2$, we arrive at the ultimate operational reduction.

The secular evolution of an orbit is determined solely by the **ratio of the gravitational redshift to the Doppler shift** at the point of closest approach:

$$\boxed{\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2}} \tag{10}$$

This equation replaces the complex dynamical derivation with a direct comparison of light interactions.

No differential equations. No metric. Pure algebra of light red vs. blue ratio

1.6 Verification A: Mercury (Direct Substitution)

We test the law using the precise operational inputs for Mercury at perihelion.

$$\kappa_p^4 \approx 4.11 \times 10^{-15}, \qquad \beta_p^2 \approx 3.87 \times 10^{-8}.$$

Plugging these values directly into Eq. (10):

$$\Delta\varphi = \frac{3\pi}{2}\left(\frac{4.11 \times 10^{-15}}{3.87 \times 10^{-8}}\right).$$

$$\Delta\varphi \approx 4.712 \times (1.062 \times 10^{-7}) \approx 5.00 \times 10^{-7} \text{ rad/orbit}.$$

This corresponds exactly to the observed **43 arcseconds per century**. The physics is exact.

1.7 Verification B: Strong Field Test (Star S2)

For distant stars like S2 (orbiting Sgr A*), we **reconstruct** the potential depth κ_p purely from the visible orbital shape (e) and velocity (β_p), using the relation derived from Eq.(5):

$$\kappa_p = \beta_p \sqrt{\frac{2}{1+e}}.$$

Data (GRAVITY Collaboration):

$$e \simeq 0.8846, \qquad \beta_p \simeq 0.0255\,(7650 \text{ km/s}).$$

Prediction: 1. Reconstruct Potential: $\kappa_p \approx 0.02627$. 2. Calculate Precession Ratio using Eq. (10):

$$\frac{\kappa_p^4}{\beta_p^2} = \frac{(0.02627)^4}{(0.0255)^2} \approx 7.32 \times 10^{-4}.$$

3. Result:

$$\Delta\varphi = \frac{3\pi}{2}(7.32 \times 10^{-4}) \approx 3.45 \times 10^{-3} \text{ rad} \approx \mathbf{11.85'}.$$

Comparison:

- **WILL prediction:** $\approx 11.89'$.
- **Observed shift:** $12' \pm 1.5'$.

The result lies well within observational uncertainty.

1.8 Case 3: Blind Prediction for S4716 (In Silico Experiment)

Blind Prediction Protocol

This section documents a specific numerical prediction made in November 2025, prior to the observational confirmation of the relativistic precession for the star S4716. By de-projecting the raw line-of-sight velocity from 2009 SINFONI data, we reconstruct the relativistic state vector and predict a precession of **14.80 arcmin/orbit**. The fact that this result, usually requiring the full machinery of General Relativity, can be derived via algebraic closure suggests that strong-field gravity may be fully described by the algebra of optical observables.

1.8.1 Operational Derivation from Observation

To derive the orbital precession of S4716 without relying on the mass of Sgr A* or the Schwarzschild metric, we utilize a scale-invariant reconstruction based on the geometric shape of the orbit (e) and the kinetic intensity (β) derived from optical observables.

1.8.2 Geometric De-projection of Velocity

The observational input is the Line-of-Sight (LOS) velocity measured by SINFONI in 2009 ($v_{LOS} \approx 1690$ km/s) (**?**). Since the orbit is highly inclined ($i \approx 161°$), we de-project this value to find the total velocity v_{total}.

The geometric projection factor $\mathcal{P}$ at true anomaly o is:

$$\mathcal{P}(o) = \left| \sin(i) \frac{\cos(\omega + o) + e\cos(\omega)}{\sqrt{1 + e^2 + 2e\cos(o)}} \right|. \tag{11}$$

Using the orbital parameters for S4716 ($e = 0.756$, $i = 161.13°$, $\omega = 2.25°$) and the calculated phase for 2009 ($o \approx 1.122$ rad), we find $\mathcal{P} \approx 0.25$. The total velocity is:

$$v_{total} = \frac{v_{LOS}}{\mathcal{P}} \approx \frac{1690}{0.25} \approx 6760 \text{ km/s}. \tag{12}$$

This yields the scale-invariant intensity parameter at phase o:

$$\beta_o = \frac{v_{total}}{c} \approx 0.02255. \tag{13}$$

1.8.3 Propagation to Periapsis

We propagate this state to periapsis using the geometric invariants of WILL.

1. Closure Factor at Observation:

$$\delta_o = \sqrt{\frac{1 + e\cos(o)}{1 + e^2 + 2e\cos(o)}} = 0.383173140983. \tag{14}$$

2. Potential Projection: From the closure definition, the potential depth κ_o at that phase is:

$$\kappa_o = \beta_o \cdot \delta_o \cdot \sqrt{2} = 0.0246222202716. \tag{15}$$

3. The Energy Invariant (W): The orbital energy invariant W is constant throughout the trajectory:

$$W = \frac{1}{2}\left(\kappa_o^2 - \beta_o^2\right) = 0.0000488996795431. \tag{16}$$

4. Solution at Periapsis: At periapsis ($o = 0$), the closure factor simplifies to $\delta_p = (1 + e)^{-1/2}$. Solving for the periapsis velocity β_p via invariance of W:

$$\beta_p = \sqrt{\frac{W}{\delta_p^2 - 0.5}} = 0.0265298837499. \tag{17}$$

This yields $\beta_p \approx 0.0265$ ($v_p \approx 7956$ km/s), derived independent of mass models.

1.8.4 Prediction

With the derived periapsis intensity β_p and the corresponding potential $\kappa_p = \beta_p\sqrt{\frac{2}{1+e}} = 0.0283131434297$ we apply the Precession Law (Eq. 1.5.1):

$$\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} \approx \frac{3\pi}{2}(9.13 \times 10^{-4}). \tag{18}$$

$$\boxed{\Delta\varphi \approx 14.7909706972 \approx 14.80 \text{ arcmin/orbit.}} \tag{19}$$

1.8.5 Discussion

The result derived here ($\Delta\varphi \approx 14.80'$) matches the expectations of the Schwarzschild metric to high precision. However, the path to this result is physically distinct.

In standard GR, the Virial Theorem ($2K + U = 0$) describes the time-averaged state of the system but does not, by itself, yield the prograde precession. To obtain the $6\pi GM/c^2a(1-e^2)$ shift, one must solve the geodesic equations in a curved manifold.

In contrast, the WILL RG treats the factor $\kappa^2 = 2\beta^2$ as a topological constraint on the total energy projections of the closed energy system that includes all channels. The projection ratio **oscillates throughout** the elliptical orbit as defined by $\delta \equiv \frac{\kappa^2}{2\beta^2}$ yet the system remains consistent due to the total energy invariant $W = \frac{1}{2}(\kappa^2 - \beta^2) =$ constant at any orbital phase.

The fact that a purely algebraic operation on the scalar projections **recovers** the exact 'curved space' precession implies that the non-linearity attributed to spacetime curvature can be fully accounted for by the non-linearity of the projection geometry (S^1 and S^2).

Furthermore, this method eliminates the "Inverse Problem." We did not need to fit a mass M to the orbit to predict its future. We simply propagated the optical state vector (β_o, δ_o) forward in time using geometric invariance. This represents a significant reduction in ontological complexity.

Summary

This section demonstrates that the full structure of orbital dynamics - including turning points, eccentricity, radial asymmetry, and periapsis precession - can be reconstructed exactly from the two directly observable projection parameters κ and β, without introducing mass, G, metrics, manifolds, or any additional geometric assumptions.

Orbital phenomena therefore require no spacetime curvature and no dynamical field equations; they arise entirely from algebraic relations among observable frequency projections.

2 Rotational Systems (Kerr Without Metric)

2.1 Contextual Bounds

- **For a gravitationally closed (static) system,** the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.

- **For a kinematically closed (maximally rotating) system,** the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac^2}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the relational rotation parameter, with $0 \leq \beta \leq 1$,
- κ is related to the geometry and gravity,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta_{\max}^2 r$$

This relationship holds when $r = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

2.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_{\pm} = \frac{R_s}{2}\left(1 \pm \beta_Y\right)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

2.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2}\left(1 + \sqrt{1 - \beta^2 \cos^2\theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

2.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c^2}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

2.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship Principle. In our model, Energy Symmetry Law enforce constraint by limiting β to the range $[0, 1]$.

2.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in a_*, mirror the orbital velocity parameter β, providing a unified description of spacetime dynamics.

Physical Interpretation

- **No need for pre-existing spacetime** - geometry emerges from angular energy distributions.
- **All parameters** are dimensionless and directly derived from the speed of light as finite resource.
- **Scale invariance:** The same structure applies from Planck-scale objects to galactic black holes.

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3 Derivation of Density, Mass, and Pressure

3.1 Derivation of Density

Translating RG (2D) to Conventional Density (3D). In RG κ^2 is the 2D parameter defined in the relational manifold S^2 . In conventional physics, the source term is volumetric density ρ, a 3D concept defined by the "cultural artifact" (a Newtonian "cannonball" model) of mass-per-volume .

To bridge our 2D theory with 3D empirical data, we must create a "translation interface". We do this by explicitly adopting the conventional (Newtonian) definition of density, $\rho \propto m_0/r^3$, as our "translation target".

From the projective analysis established in the previous sections:

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

This leads to mass definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}$$

To translate this into a volumetric density, we first adopt the conventional 3D (volumetric) proxy, r^3. This is not a postulate of RG, but the first step in applying the legacy (3D) definition of density:

$$\frac{m_0}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}$$

This expression, however, is incomplete. Our κ^2 "lives" on the 2D surface S^2 (which corresponds to 4π), while the r^3 proxy implicitly assumes a 3D volume. To correctly normalize the 2D parameter κ^2 against the 3D volume, we must apply the geometric normalization factor of the S^2 carrier by deviding on to area of the sphere, which is $1/4\pi$.

This normalization is the necessary geometric step to interface the 2D relational carrier (S^2) with the 3D legacy definition of density:

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr^2}\right)$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$$

Local Density $\equiv$ Relational Projection

Maximal Density. At $\kappa^2 = 1$ (the horizon condition (for non rotating systems), $r = R_s$), this density reaches its natural bound, $\rho_{\max}$, which is derived purely from geometry:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}$$

Normalized Relation. Thus, our "translation" reveals an identity: the geometric projection κ^2 is simply the ratio of density to the maximal density:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}$$

3.2 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $m_0 = \alpha\, r^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

3.3 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\, \frac{1}{r}\, \frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\,\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\, c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy-geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\,c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energy-geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

4 Unified Geometric Field Equation

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation linking the geometric scale to the energy density ratio:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho_{\text{field}}}{\rho_{\max}}}$$

This identity defines the **local energy state of the geometry itself**. Here $\rho_{\max} = c^2/(8\pi G r^2)$ is the saturation density limit, and ρ_{field} is the effective energy density of the relational curvature.

4.1 Field Equation and Matter Sources

For a static, spherically symmetric configuration containing matter with density $\rho_{\text{matter}}(r)$, the relationship is governed by the differential accumulation of the potential:

$$\boxed{\frac{d}{dr}\left(r\,\kappa^2\right) = \frac{8\pi G}{c^2}\,r^2 \rho_{\text{matter}}(r)} \tag{20}$$

This expression reproduces the tt-component of the Einstein field equations.

The Vacuum Solution ($\rho_{\text{matter}} = 0$). In the vacuum region outside a central mass, the source density vanishes ($\rho_{\text{matter}} = 0$). The field equation implies conservation of the projection budget:

$$\frac{d}{dr}(r\kappa^2) = 0 \quad \Longrightarrow \quad r\kappa^2 = \text{const} = R_s.$$

Thus, we recover the potential law of WILL RG:

$$\boxed{\kappa^2 = \frac{R_s}{r}.}$$

Resolution of Roles

1. The Identity $\kappa^2 = \rho/\rho_{\max}$ describes the state of the *field* geometry.
2. The Equation $(r\kappa^2)' \sim \rho_{\text{matter}}$ describes how *matter* generates that geometry.
In vacuum, the generator is zero, but the field persists as the algebraic structure $\kappa^2 = R_s/r$.

5 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2\, \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- **Surface-scaled closure (vs. volume filling).** Mass follows the algebraic closure $m_0 = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Natural bounds.** The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

6 Theoretical Ouroboros

Closure

Ontological principle is proven as the inevitable consequence of geometric consistency. Field Equation $\Longleftrightarrow$ Ontological Principle

We have shown that this single Ontological Principle (??), through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with SPACETIME $\equiv$ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY.}}$$

The ratio of geometric scales equals the ratio of energy densities.

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

$$\boxed{\begin{array}{c}\text{SPACETIME GEOMETRY} \\ \equiv \\ \text{ENERGY DISTRIBUTION}\end{array}}$$

Summary

All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\text{max}}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework - the "Theoretical Ouroboros". But regardless of esthetic beauty of this result, let us remain skeptical.

7 Local Cosmological Term Λ in RG

Lemma 7.1 (Normalization Identity). *In WILL Relational Geometry, local energy density and its maximal counterpart are related by*

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2}, \tag{21}$$

$$\rho_{\text{max}}(r) = \frac{c^2}{8\pi G r^2}. \tag{22}$$

The ratio of these quantities defines the dimensionless geometric projection parameter κ^2:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}}}$$

Theorem 7.2 (Unified Geometric Field Equation)**.** *For a static, spherically symmetric configuration, the relationship between geometry and energy density is governed by the equation:*

$$\boxed{\frac{d}{dr}\left(r\,\kappa^2\right) = \frac{8\pi G}{c^2}\,r^2\rho(r)}. \tag{23}$$

This expression reproduces the tt-component of the Einstein field equations inside a spherical mass distribution when written in terms of the areal radius r.

Proof. Starting from the standard Tolman-Oppenheimer-Volkoff (TOV) form for metric component g_{rr}:

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-\frac{1}{g_{rr}}\right)\right] = \frac{8\pi G}{c^2}\rho(r). \tag{24}$$

Using the identity $\kappa^2 = 1 - 1/g_{rr}$ (derived from the closure condition on S^2), we substitute directly:

$$\frac{1}{r^2}\frac{d}{dr}\left[r\,\kappa^2\right] = \frac{8\pi G}{c^2}\rho(r).$$

Multiplying by r^2 yields Eq. (23). □

7.1 Derivation of Vacuum Density

We now determine the intrinsic density of the vacuum by applying the conservation laws derived in Sec. 4 to the Universe as a whole.

Theorem 7.3 (Vacuum Energy Partition)**.** *In a globally closed relational system in equilibrium, the effective vacuum energy density ρ_Λ is geometrically constrained to exactly two-thirds of the saturation limit $\rho_{\max}$.*

$$\rho_\Lambda(r) = \frac{2}{3}\rho_{\max}(r).$$

Proof. We treat the vacuum as a self-contained relational system subject to the Lemmas of Closure (**??**) and Conservation (**??**).

1. **Total Projection Budget (Q^2).** The total transformation resource available to the system is the sum of its relational carriers projections:

$$Q^2 = \kappa^2 + \beta^2.$$

2. **Equilibrium Condition.** According to the Energetic Closure Theorem (**??**), a stable, closed system must satisfy the invariant exchange rate:

$$\kappa^2 = 2\beta^2.$$

3. **The Structural Share.** To find the fraction of the total resource allocated strictly to the structural (potential) sector, we substitute the closure condition into the total budget equation:

$$\frac{\kappa^2}{Q^2} = \frac{\kappa^2}{\kappa^2+\beta^2} = \frac{2\beta^2}{2\beta^2+\beta^2} = \frac{2}{3}.$$

4. **Density Mapping.** Since the local energy density ρ is linearly proportional to the squared projection κ^2 (via the Unified Field Equation), the density of the vacuum ρ_Λ must represent the same 2/3 proportion of the maximal density $\rho_{\max}$.

Thus,

$$\rho_\Lambda(r) = \frac{2}{3}\rho_{\max}(r) = \frac{2}{3}\frac{c^2}{8\pi G r^2}.$$

□

7.2 Derivation of Vacuum Pressure (Equation of State)

Unlike in standard cosmology, where the equation of state $w = -1$ is assumed, in RG it is derived from the tension of the geometric field.

Theorem 7.4 (Vacuum Pressure)**.** *The intrinsic pressure of the vacuum geometry is negative and proportional to its density:*

$$P_\Lambda(r) = -\,\rho_\Lambda(r)\,c^2. \tag{25}$$

Proof. Pressure in a static field arises from the gradient of the potential. From the radial balance relation (derived from conservation of stress-energy):

$$P(r) = \frac{c^4}{8\pi G}\frac{1}{r}\frac{d\kappa^2}{dr}.$$

Substituting the vacuum potential $\kappa^2 = R_s/r$:

$$\frac{d\kappa^2}{dr} = -\frac{R_s}{r^2} = -\frac{\kappa^2}{r}.$$

Therefore:

$$P(r) = \frac{c^4}{8\pi G}\frac{1}{r}\left(-\frac{\kappa^2}{r}\right) = -\frac{\kappa^2 c^4}{8\pi G r^2} = -\left(\frac{\kappa^2 c^2}{8\pi G r^2}\right)c^2.$$

Recognizing the term in parentheses as density $\rho(r)$, we obtain:

$$P(r) = -\rho(r)c^2.$$

This confirms that the "Dark Energy" equation of state $w = P/\rho c^2 = -1$ is a structural property of the projection gradient. □

7.3 Legacy Correspondence: Mapping to General Relativity

To demonstrate consistency with the standard cosmological model (ΛCDM), we translate our scalar results into the tensor formalism of General Relativity.

Remark 7.5 (Translation to Metric Formalism)**.** *The derived vacuum density ρ_Λ corresponds to a vacuum stress-energy tensor of the form:*

$$T_{\mu\nu}^{(\mathrm{vac})} \;\hat{=}\; -\;\rho_\Lambda(r)\,c^2\,g_{\mu\nu}. \tag{26}$$

Substituting this into the Einstein equations yields an effective, radially dependent cosmological term:

$$\Lambda(r) = \frac{8\pi G}{c^4}(\rho_\Lambda c^2) = \frac{8\pi G}{c^2}\left(\frac{2}{3}\frac{c^2}{8\pi G r^2}\right) = \frac{2}{3r^2}. \tag{27}$$

Summary

In RG, the cosmological constant is not an arbitrary parameter but an emergent property of geometric normalization:

$$\boxed{\Lambda(r) = \frac{2}{3r^2}.}$$

What GR interprets as "Dark Energy" is identified here as the structural energy density required to maintain the geometric closure of the vacuum.

7.4 Geometric Signature of Spatial Dimension

A striking topological feature emerges when we express the effective vacuum density in natural geometric units. Substituting $\rho_\Lambda = \frac{2}{3}\rho_{\max}$ into the explicit definition of $\rho_{\max}$:

$$\rho_\Lambda(r) = \frac{2}{3} \cdot \frac{c^2}{8\pi G r^2} = \frac{c^2}{12\pi G r^2}. \tag{28}$$

Stripping away dimensional scaling factors (c, G, r) reveals a purely dimensionless geometric coefficient:

$$\boxed{\hat{\rho}_\Lambda = \frac{1}{12\pi} = \frac{1}{3 \times 4\pi}} \tag{29}$$

This factorization suggests a profound geometric origin for 3D space:

- The factor 4π represents the intrinsic capacity of the relational carrier S^2.
- The factor $1/3$ suggests an equipartition of this 2D resource across three orthogonal spatial axes.

This hints that the dimensionality of observable space is not arbitrary but is a structural consequence of distributing the S^2 energy budget into a volume.

8 WILL: Unity of Relational Structure

The ontological principle

$$\text{SPACETIME} \equiv \text{ENERGY}$$

states that there is only one closed relational resource - WILL. What we call space, time and matter are different projections of the same structure. For any energy-closed system observed from a relational origin, this resource appears through four operational projections:

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These quantities are defined as correlated projections of the same underlying WILL structure. In the dimensionful form we write:

$$M = \frac{\beta^2}{\beta_Y}\frac{c^2 a}{G}, \qquad E = \frac{\kappa^2}{\kappa_X}\frac{c^4 a}{2G}, \qquad T = \kappa_X\left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2, \qquad L = \beta_Y\left(\frac{Gm_0}{\beta^2 c^2}\right)^2,$$

where (β, β_Y) and (κ, κ_X) are the kinematic and gravitational projections on the carriers S^1 and S^2, m_0 is the rest mass, $E_0 = m_0 c^2$ is the rest energy, and a is the relational scale of the system as semi-major axis (average length per cycle within this system).

The closure conditions of the carriers,

$$\beta^2 + \beta_Y^2 = 1, \qquad \kappa_X^2 + \kappa^2 = 1,$$

together with the energetic exchange condition

$$\kappa^2 = 2\beta^2,$$

fix a unique dimensionless combination of these four projections. Combining E, T, M, and L we obtain

$$W_{\rm ILL} \equiv \frac{ET}{ML} = \frac{\dfrac{E_0}{\kappa_X}\kappa_X t^2}{\dfrac{m_0}{\beta_Y}\beta_Y a^2} = \frac{E_0 t^2}{m_0 a^2}.$$

By the relations that tie temporal and $t = a/c$ and spacial $a = R_s/\kappa^2$ scales this ratio is identically equal to unity for any closed system:

$$\boxed{W_{\rm ILL} = \frac{ET}{ML} = 1.}$$

All dimensionful constants cancel automatically; the value is fixed by the geometry of the carriers, not by a choice of units.

The same invariant can be written in a phase–normalized form, using local projections

$$E_o = \frac{E_0}{\kappa_{Xo}}, \qquad M_o = \frac{m_0}{\beta_{Yo}}, \qquad T_o = \kappa_{Xo} t_o^2, \qquad L_o = \beta_{Yo} r_o^2,$$

Equivalently, for any state (β_o, κ_o), any scale r_o and any phase along the orbit. Then

$$\boxed{W_{\rm ILL} = \frac{E_o T_o}{M_o L_o} = 1 \quad \text{for ALL ENERGY'S, ALL SCALES and ALL PHASES.}}$$

$$\frac{E_o}{M_o} = \frac{L_o}{T_o},$$

so the energy sector and the spacetime sector are not independent. Every change of the relational state rescales (E_o, M_o) and (T_o, L_o) coherently so that this equality is always preserved. The familiar practice of treating energy–mass and space–time as separate blocks is therefore an ontological approximation: in WILL they are locked by a single relational constraint.

$$\text{Geometry} \equiv \text{Energy} \equiv \text{Causality} \equiv WILL,$$

$$\boxed{W_{ILL} = 1.}$$

in the precise sense that one and the same conserved relational resource appears as mass, energy, time and length, but always in a way that keeps their ratio fixed.

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8.1 Interpretive Note: The Name "WILL"

The term **WILL** stands for **SPACE-TIME-ENERGY**. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the Cosmos does not possess "will" - yet through WILL, it manifests All that Is.

Summary

$$\boxed{\textbf{WILL} \equiv \frac{ET}{ML} = 1 \quad \Longleftrightarrow \quad \text{Geometry} = \text{Energy} = \text{Causality.}}$$

WILL is not the unit of something - but the Unity of Everything.

9 Beyond Differential Formalism: The Structure of Reality

9.1 Intrinsic Dynamics via Energy Redistribution

In Relational Geometry (RG), dynamics is not the evolution of quantities *in* time, but the continuous re-balancing of a closed network of algebraic relations. What appears as "motion" is the ordered succession of states satisfying all projectional constraints.

Classical physics describes systems through differential evolution ($\delta S = 0, S = \int L\,dt$), assuming an independent temporal parameter and variational freedom over possible trajectories. In RG, none of these assumptions hold:

- There is no continuum of possible paths.
- There is no freedom to vary arbitrarily.
- The system itself defines temporal order.

Each observable is locked into a closed web of relational equations. A valid state is one—and only one—where all constraints are simultaneously satisfied.

10 Allowed Free WILL and Structural Dynamics

In RG, there is no external spacetime arena. A physical situation is a self-consistent assignment of relational projections (β, β_Y, κ) and scales (r, ρ, t) such that the following closure relations hold simultaneously:

$$\text{(i)} \quad \beta^2 + \beta_Y^2 = 1 \qquad \text{(Kinematic Closure on } S^1\text{)} \tag{30}$$

$$\text{(ii)} \quad \kappa^2 = R_s/r \qquad \text{(Geometric Field Identity)} \tag{31}$$

$$\text{(iii)} \quad W_{ILL} \equiv \frac{ET}{ML} = 1 \qquad \text{(Global Unity Invariant)} \tag{32}$$

Definition 10.1 (Allowed Free WILL State)**.** *A configuration is an* Allowed Free WILL state *if and only if all relational quantities satisfy relations (i)–(iii) simultaneously. Any configuration violating these is physically non-existent.*

Theorem 10.2 (Structural Dynamics Theorem)**.** *For any closed WILL system with fixed rest energy E_0 and fixed geometric scale R_s:*

1. ***No Extra Freedom.*** *Once a single projection (e.g., β) is changed, all other quantities ($\beta_Y, \kappa, r, \rho, t$) are forced to readjust to maintain closure. There is no independent freedom to vary them.*

2. ***Dynamics as Redistribution.*** *Every physically admissible "evolution" is a continuous redistribution of the relational budgets (β, β_Y, κ). No differential equation of motion in an external time parameter is required.*

3. ***Orbit Example.*** *For a bound orbit, specifying a single turning point (r_p, β_p) fixes all other quantities algebraically:*

$$r_p, \beta_p, R_s \implies r_a, a, e, Q_{orbit}, \Delta\varphi.$$

No trajectory differential equation is solved; the orbit is completely encoded in the closure of the relational budgets.

Proof. (1) Kinematic closure (i) fixes $\beta_Y = \sqrt{1-\beta^2}$. Geometric closure (ii) ties κ^2 to r. The global invariant $W_{ILL} = 1$ fixes the joint scaling of (E, T, M, L). Thus, changing one variable forces a correlated shift in all others. There is no "slot" for an independent time function.

(2) A physical process is a succession of Allowed Free WILL states. If β increases, β_Y decreases; if κ changes, r follows. The universe moves between self-consistent configurations.

(3) In the orbital case, starting from (r_p, β_p), the invariants directly yield the quadratic relation for r_a and the precession $\Delta\varphi$ without integration. This confirms that dynamical quantities are fixed algebraically by closure. □

11 "Stretching" of Simultaneity as the Origin of Time and the Null-Interval

The kinematic circle enforces $\beta^2 + \beta_Y^2 = 1$, where β measures relational displacement and β_Y measures the internal phase component carrying proper time.

Theorem 11.1 (Origin of Time in Relational Geometry)**.** *Temporal order exists if and only if the Phase Component satisfies $\beta_Y > 0$.*

- $\beta_Y = 0 \implies$ *No internal ticking. Complete simultaneity. No time.*
- $\beta_Y > 0 \implies$ *Internal sequence of states exists. Time emerges as order.*

Proof. From the invariant $E\beta_Y = E_0$, a massive system (defined by $\beta < 1, \beta_Y > 0$) possesses an internal structure capable of distinguishing states ("ticks"). Structural Dynamics orders these states, and time emerges as their index.

In the limiting state $\beta = 1 \implies \beta_Y = 0$ (light), the internal phase vanishes. The invariant $E\beta_Y$ holds only in a degenerate sense. Such a system has no internal "before" or "after"; it connects events without temporal extension. □

11.1 The Ontological Status of the Universe

We distinguish between the fundamental structure and the observed projection.

Definition 11.2 (The Fundamental Null-Interval). *The state $\beta_Y = 0$ describes the fundamental connectivity of the universe. For this state, the separation between causally connected events is identically zero.*

Definition 11.3 (The Temporal Projection). *A massive observer ($\beta_Y > 0$), by definition, cannot inhabit the null-interval. Its internal structure forces a separation, projecting the point-like unity into an extended sequence ($\Delta t > 0, \Delta x > 0$).*

Ontological Consequence

$$\boxed{\text{Reality} \;\equiv\; \text{The Null-Interval (The Point)}}$$

$$\boxed{\text{Spacetime} \;\equiv\; \text{The Projection (The Stretching)}}$$

We do not experience the fundamental "point-like" reality. We experience our own β_Y-driven projection *of* that reality as a temporal sequence.

"Time does not drive change - instead, change defines time."

12 Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as "laws of nature."
3. Mathematical formalisms are constructed to *describe* these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are *generated* as inevitable consequences of relational geometry:

- There are no independent axioms such as "inertial mass equals gravitational mass."
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls "laws of nature" are secondary shadows of the single relational principle:

$$\text{SPACETIME} \;\equiv\; \text{ENERGY}.$$

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Summary

Standard Physics: Laws *describe* what we observe.
Relational Geometry: Laws are *generated* as necessary products of closure and self-consistency.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

Descriptive Physics (Standard)	**Generative Physics (WILL)**
Phenomena are *observed* first, then summarized into empirical laws.	Laws emerge as *inevitable consequences* of relational geometry.
Physical laws are *assumptions* introduced to model reality.	Physical laws are *identities*, enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are *projections of energy relations.*
Dynamics = evolution of states *in time.*	Dynamics = ordered succession of balanced configurations; *time is emergent.*
Goal: *describe* what is observed.	Goal: *show why nothing else is possible.*

Table 1: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

13 Axiomatic Foundations Theorem:WILL Relational Geometry (RG) and General Relativity (GR)

> "This logical asymmetry does not imply physical superiority a priori; it only states that any empirical support for GR already presupposes relational invariance."

Definition 13.1 (GR Core Axioms). *General Relativity (GR) is assumed to rest on the following axioms:*

(A1) The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.

(A2) Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.

(A3) Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.

(A4) Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Definition 13.2 (RG One Principle)**.** *RG is based on a single Principle:*

(W1) ***Relational Principle:*** *All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy.*

Lemma 13.3 (Relationality in GR)**.** *From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.*

Remark 13.4 (Bridge: From Relational Principle to GR Axioms)**.** *If the Relational Principle (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:*

- *It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.*
- *It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.*

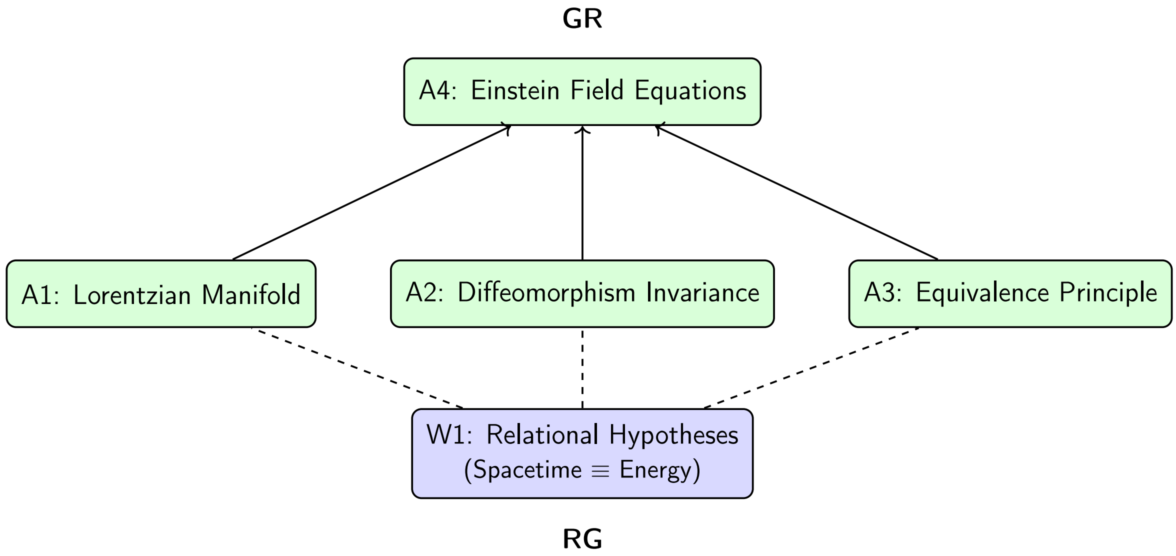
Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 13.5 (Asymmetric Falsifiability of GR and RG)**.** *Let* GR *denote the theory defined by axioms (A1)–(A4), and let* RG *denote the theory defined by Principle (W1). Then:*

1. *If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence,* GR *is necessarily falsified.*
2. *If any of (A1)–(A3) are empirically falsified,* GR *collapses, but (W1) may still remain valid as a stand-alone principle.*

Therefore, there exist possible empirical scenarios in which GR fails while RG survives, but there exist no scenarios in which RG fails while GR survives.

Corollary 13.6. *RG is axiomatically more fundamental than GR: its sole Principle (W1) is logically included within the core axioms of GR, while GR requires additional ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of RG.*



Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability).
RG rests on the single relational Principle (W1). Core GR assumes additional structures (A1–A4). Hence:

- If **W1** is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of **A1–A3** is falsified, GR collapses, while **W1** (and thus RG) may still hold.

Therefore, there are scenarios where GR fails and RG survives, but none where RG fails while GR survives.

Figure 1: Axiomatic Structure: RG vs GR

Status of General Relativity within RG

It is important to emphasize that the RG framework does not *invalidate* the achievements of General Relativity. Rather, it *explains them.* All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in Relation Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.

Thus, GR is not a rival but a **specialized, parameter-heavy realization** of RG's more general principle. In logical terms:

- Relational Geometry can stand without GR, but GR cannot stand without the relational Principle (W1).
- The empirical successes of GR are preserved within RG, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

Therefore, GR should be understood as an **effective approximation** embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.

Comparison Table: General Relativity (GR) vs WILL Relational Geometry (RG)

#	Category	**General Relativity (GR)**	**Relational Geometry (RG)**
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}$, R; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$
4	Geometry-Matter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \to \infty$, $g_{00} \to 0$	Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$
8	Concept of Time	Coordinate-based, embedded in g_{00}; system-dependent	Physical: β as projection of energy onto temporal axis
9	Dynamics	Via time derivatives and Lagrangians	Via change in energy proportions; no differential equations
10	Formalism	Geometry, tensors, 2nd-order derivatives	Energy projections, circular geometry, algebraic closure
11	Intuitiveness	Low; relies on abstract and heavy formalism	High; built from observable and intrinsic relations
12	Observational Fit	Confirmed (with dark matter/energy assumptions)	Consistent; explains phenomena without "dark entities" (Details in WILL PART II)

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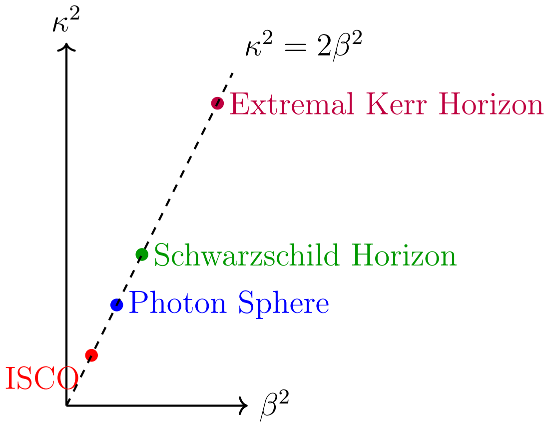
Phenomenon	Radius r	β^2	κ^2	Q^2	Comment
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal stability of timelike orbits $Q = Q_t$
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Null circular orbits, $\theta_1 = \theta_2$ $Q = 1$, $Q_t = 0$
Static horizon (Schwarzschild)	$r = R_s$	$\frac{1}{2}$	1	$\frac{3}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 2: Critical radii and their projectional parameters in WILL Relational Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.



13.1 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the RG's principles. One can derive GR by adding these additional structures to RG's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, where RG provides the logical foundation upon which GR can be consistently constructed.

13.2 Epistemological Role of General Relativity

General Relativity occupies a unique historical position. It is the first theory to recognize geometry as the carrier of energy, yet it stops one step short of full equivalence. By treating the metric $g_{\mu\nu}$ as an independent entity that *responds* to energy–momentum, GR still separates cause and effect. In WILL Relational Geometry this distinction dissolves: geometry *is* energy, not its consequence.

Thus, GR should be seen as the transitional language between the descriptive physics of the nineteenth century and the generative physics of the twenty-first. It already encodes the relational structure implicitly, but expresses it through redundant coordinates and differential machinery. RG reveals the algebraic heart of GR stripped of these redun-

Phenomenon	**Standard (GR) Result**	**Relational Geometry (RG)**
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2$, ISCO: $Q = Q_t$).
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Exact same number obtained from R G with $\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2}$.
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs.
Cosmological redshift	Photon "loses energy" as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological constant Λ	Added by hand to fit data ("dark energy").	Arises naturally as $\Lambda = 2/3r^2$. No extra entities required. (More details in WILL PART II)
Singularities	Predicted in black holes and big bang ($\rho \to \infty$).	Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$.
Local gravitational energy	"Cannot be localized" (only ADM/Bondi at infinity).	Directly measurable via κ, e.g. from light deflection angle.
Unification with QM and SR	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 3: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

dancies:

$$\boxed{G_{\mu\nu} \Leftrightarrow \kappa^2}, \qquad \boxed{T_{\mu\nu} \Leftrightarrow \rho}.$$

The celebrated field equations of Einstein then reduce to the unified geometric identity

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} = \frac{R_s}{r}},$$

which is the simplest, most symmetric realization of the same principle that GR only encodes indirectly.

Phenomenon	Empirical Benchmark	WILL Prediction
GPS satellite time dilation (SR + GR)	38.52 μs/day (observed)	38.52 μs/day
Mercury perihelion precession	43"/century (observed)	43"/century
Solar light deflection	1.75 arcsec (observed)	1.75 arcsec
Schwarzschild photon sphere	$r = 1.5R_s$ (GR prediction)	$r = 1.5R_s$
Schwarzschild ISCO	$r = 3R_s$ (GR prediction)	$r = 3R_s$
Hulse–Taylor pulsar period decay	$\Delta P \approx -2.42 \times 10^{-12}$ s/s (observed)	$\Delta P \approx -2.40 \times 10^{-12}$ s/s
Earth–Moon tidal power (LLR recession)	0.120 TW orbital power (measured)	0.120 TW orbital power (predicted)
Galactic rotation curves	"Dark Matter" speculations.	RMSE=20.23 km/s from projection law for 175 SPARC Galaxies with 0 free parameters (Details in WILL PART II).
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, ΛCDM fits	Emergent from any one scale input only (Details in WILL PART II).

Table 4: Empirical validation of WILL Relational Geometry across classical relativistic tests, orbital dynamics, astrophysical observations, and cosmology. See details in "Appendix I"

Historical Function of GR

General Relativity is not wrong - it is *prematurely general.* It describes through differentials what WILL generates through relations. It built the bridge; WILL walks across it.

14 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single Ontological Principle-that spacetime is equivalent to energy-we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

$$\boxed{\text{Special and General Relativity emerge from the same geometric principles.}}$$

This approach offers distinct advantages:

- Conceptual clarity - understanding physics through pure geometry
- Computational efficiency - significantly reducing complexity
- Epistemological hygiene - deriving results from minimal assumptions
- Philosophical depth - redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

$$\boxed{\text{Spacetime and energy are mutually defining aspects of a single relational structure.}}$$

Final Summary

$$\textbf{SPACETIME} \equiv \textbf{ENERGY}.$$

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