WILL: Relational Geometry of Galactic Dynamics

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Abstract

We derive, from first principles of the WILL Relational Geometry (RG) framework, a parameter-free galactic rotation law:

$$V_{\text{WILL}}(r) = \sqrt{3}\, V_{\text{bary}}(r).$$

This result emerges not from empirical fitting, but from the relational closure of energy projections in a background-free universe. When applied to the SPARC dataset (175 galaxies), it yields a median RMSE of **20.23 km/s** with **no free parameters** and a **universal stellar mass-to-light ratio** ($Y_* = 0.66$). The factor $\sqrt{3}$ is a direct consequence of the closure condition $\kappa^2 = 2\beta^2$, which enforces energetic balance between gravitational and kinetic degrees of freedom derived from first principles in WILL Part I: Relational Geometry.

1 Foundational Relations of WILL RG

The WILL framework eliminates the hidden assumption that structure (spacetime) and dynamics (energy) are distinct. It adopts a single ontological principle:

$$\text{SPACETIME} \equiv \text{ENERGY}. \tag{1}$$

This equivalence is encoded in the dimensionless gravitational projection parameter:

$$\kappa^2(r) = \frac{R_s}{r} = \frac{\rho(r)}{\rho_{\max}(r)}, \tag{2}$$

where $R_s = 2Gm_0/c^2$ is the Schwarzschild scale, and $\rho_{\max}(r) = c^2/(8\pi G r^2)$ is the maximum admissible density at radius r.

The kinematic projection is defined as $\beta = v/c$. In a closed, spherically symmetric system, topology enforces a closure relation between these projections:

$$\kappa^2 = 2\beta^2. \tag{3}$$

This reflects the 2:1 ratio of relational degrees of freedom (2D for gravity on S^2, 1D for motion on S^1).

The total projection norm is then:

$$Q^2 \equiv \beta^2 + \kappa^2 = 3\beta^2. \tag{4}$$

2 From Projections to Rotation Velocity

In Newtonian dynamics, the circular velocity due to enclosed mass $M(r)$ is:

$$V_c^2(r) = \frac{GM(r)}{r}. \tag{5}$$

From Eq. (2) and $R_s = 2GM/c^2$, the enclosed mass is:

$$M(r) = \frac{\kappa^2 c^2 r}{2G}. \tag{6}$$

Substituting into Eq. (5) gives:

$$V_c^2(r) = \frac{\kappa^2 c^2}{2} = \beta^2 c^2, \tag{7}$$

where we used $\kappa^2 = 2\beta^2$. Thus, the baryonic velocity in the SPARC formalism is identified as:

$$V_{\text{bary}}(r) \equiv \beta(r)c. \tag{8}$$

The total observable velocity in WILL includes both projections via Q:

$$V_{\text{WILL}}^2(r) = Q^2 c^2 = 3\beta^2 c^2 = 3V_{\text{bary}}^2(r), \tag{9}$$

yielding the final law:

$$\boxed{V_{\text{WILL}}(r) = \sqrt{3}\, V_{\text{bary}}(r)}. \tag{10}$$

3 Interpretation: Why $\sqrt{3}$ Is Not a Fit

The factor $\sqrt{3}$ is **not an adjustable parameter**. It is the inevitable outcome of:

1. The topological constraint $\kappa^2 = 2\beta^2$ (geometric virial theorem),
2. The identification $V_{\text{bary}} = \beta c$ from observed baryonic kinematics,
3. The definition of total energy projection $Q^2 = \beta^2 + \kappa^2$.

No new constants are introduced - only G and c, already present in V_{bary}.

4 Empirical Validation on SPARC

We apply Eq. (10) to 175 galaxies in the SPARC database [1], using:

- Gas mass from HI observations,
- Stellar mass with a **fixed** $\Upsilon^* = 0.66$ (no per-galaxy tuning).

Result: Median RMSE = **20.23 km/s**.

This within MOND's performance (RMSE $\approx$ 13–20 km/s) *without fitting a single parameter per galaxy*, and surpasses typical ΛCDM simulations (RMSE $\approx$ 25–30 km/s) that require tuned dark matter halos.

- Empirical rotation curve RMSEs for Newtonian-only baryonic fits (with fixed Υ^*), MOND, and CDM models are cited from Wang et al. 2020 [**?**] and Li et al. 2020 [**?**].
- For foundational MOND theory, see Milgrom (2001) [**?**].

Model	Fit Method	Free Parameters	Global Median RMSE (km/s)
WILL ($\Upsilon^* = 0.66$)	Global, fixed	1 (universal)	20.23
WILL (Υ^* flexible)	Per galaxy	1 (Υ^* per galaxy)	12.63
Newtonian Baryonic	Global, fixed	1 (universal)	∼43
MOND (a_0 universal)	Per galaxy	1 (Υ^* per galaxy)	∼13
CDM / Burkert / NFW Dark Matter	Per galaxy fit	2–3+ per galaxy	25–30

Table 1: Comparison of global median RMSE for rotation curve models (SPARC sample, 175 galaxies).

5 Discussion

5.1 Hypothesis: Internal vs. External Observation (The "Carousel" Effect)

A fundamental question arises: if the universal rotation law is $V = \sqrt{3}V_{\text{bary}}$, why does the Solar System follow pure Newtonian dynamics ($V = V_{\text{bary}}$)?

The answer lies in the relational nature of observation. We must distinguish between two modes of measurement:

- **Inter-system Observation (External View):** When we observe a distant galaxy, we are external to its gravitational binding energy. We are not part of its "system." Therefore, we observe the **total energy budget** required to maintain that galaxy's structure against the vacuum. We see both the kinetic motion (β^2) and the structural tension (κ^2) required for closure.

$$Q_{\text{ext}}^2 = \beta^2 + \kappa^2 = 3\beta^2 \quad \Longrightarrow \quad V = \sqrt{3}V_{\text{bary}}$$

- **Intra-system Observation (Internal View):** When we observe the Solar System or Milky Way, we are embedded within the same gravitational potential well (κ_{local}) as the planets or stars. We are, effectively, "riding the same carousel." The background potential κ^2 is a shared baseline for both the observer (Earth) and the target (Jupiter).

Potential Screening Principle

Local Potential Screening: For an observer embedded within the system, the binding potential κ^2 acts as a common background frame, not as an observable kinematic difference. The relative measurement cancels out the structural tension, leaving only the kinetic differential:

$$Q_{\text{int}}^2 \approx \beta^2 \quad \Longrightarrow \quad V \approx V_{\text{bary}}$$

Thus, the factor $\sqrt{3}$ is the signature of a **holistic** observation of a closed system from the outside (Galactic Scale), while Newtonian dynamics represents the **differential** observation from the inside (Local Scale).

6 Vector Analysis of Observation Modes

To resolve the apparent discrepancy between galactic dynamics (where $V \approx \sqrt{3}V_{\text{bary}}$) and local solar system dynamics (where $V \approx V_{\text{bary}}$), we must treat the relational displacement Q strictly as a vector quantity in the (β, κ) plane.

6.1 Definition of Relational Vector

Any physical state is characterized by a relational displacement vector $\mathbf{Q}$ relative to the observer's origin:

$$\mathbf{Q} = \begin{pmatrix} \beta \\ \kappa \end{pmatrix} \tag{11}$$

The magnitude of this vector determines the total observable energy budget (and thus the effective orbital velocity):

$$V_{\text{obs}}^2 = c^2 |\mathbf{Q}|^2 = c^2(\beta^2 + \kappa^2) \tag{12}$$

6.2 Case 1: Inter-system Observation (External View)

Consider an observer located far outside the target system (e.g., measuring a distant galaxy). The observer resides in the asymptotic vacuum relative to the target's potential well.

- **Observer State:** The observer defines the relational zero: $\mathbf{Q}_{\text{obs}} = (0, 0)$.
- **Target State:** The target system (galaxy) exhibits both kinematic motion and structural potential binding: $\mathbf{Q}_{\text{sys}} = (\beta, \kappa)$.

The measured displacement is the absolute vector:

$$\mathbf{Q}_{\text{ext}} = \mathbf{Q}_{\text{sys}} - \mathbf{Q}_{\text{obs}} = \begin{pmatrix} \beta \\ \kappa \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \beta \\ \kappa \end{pmatrix} \tag{13}$$

Applying the closure condition for stable systems ($\kappa^2 = 2\beta^2$):

$$|\mathbf{Q}_{\text{ext}}|^2 = \beta^2 + 2\beta^2 = 3\beta^2 \quad \Longrightarrow \quad V_{\text{ext}} = \sqrt{3}V_{\text{bary}} \tag{14}$$

This explains the "Dark Matter" effect as the observation of the full vector magnitude, including the orthogonal potential component κ.

6.3 Case 2: Intra-system Observation (Internal View)

Consider an observer embedded within the same system as the target (e.g., Earth observing Jupiter). Both the observer and the target share the same background gravitational potential scale defined by the central mass (Sun/Galaxy).

- **Common Potential:** $\kappa_{\text{background}} \approx \text{const}$ locally.
- **Observer State:** $\mathbf{Q}_{\text{obs}} = (\beta_{\text{obs}}, \kappa_{\text{background}})$.
- **Target State:** $\mathbf{Q}_{\text{target}} = (\beta_{\text{target}}, \kappa_{\text{background}})$.

The observable is the *relative* displacement vector between the two bodies:

$$\mathbf{Q}_{\text{int}} = \mathbf{Q}_{\text{target}} - \mathbf{Q}_{\text{obs}} = \begin{pmatrix} \beta_{\text{target}} - \beta_{\text{obs}} \\ \kappa_{\text{background}} - \kappa_{\text{background}} \end{pmatrix} = \begin{pmatrix} \Delta\beta \\ 0 \end{pmatrix} \tag{15}$$

The common structural potential component κ subtracts out. The observer perceives only the differential kinetic projection:

$$|\mathbf{Q}_{\text{int}}|^2 = (\Delta\beta)^2 \quad \Longrightarrow \quad V_{\text{int}} \approx V_{\text{bary}} \tag{16}$$

Thus, internal observation naturally recovers Newtonian dynamics without requiring screening mechanisms or adjustable parameters. The "Dark" component (κ) exists but is geometrically invisible to an internal observer, just as voltage difference is zero between two points at the same high potential.

6.4 Remark

The remaining scatter (RMSE 20.23 km/s) is expected due to the assumption of a universal Υ^* and perfect geometric virial equilibrium. The fact that a parameter-free geometric law performs comparably to tuned Dark Matter models suggests that the $\sqrt{3}$ factor captures the fundamental driver of galactic dynamics, while astrophysical variations account for the residuals.

7 Conclusion

We have shown that a simple, parameter-free rotation law - $V = \sqrt{3}V_{\text{bary}}$ - emerges naturally from the first principles of a relational geometric framework. Its empirical success challenges the necessity of dark matter and invites a reevaluation of gravity as energy's projectional structure.

Code and data are fully open-source at: https://antonrize.github.io/WILL/

References

[1] Lelli, F., McGaugh, S. S., & Schombert, J. M. (2016). SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. *The Astronomical Journal*, 152(6), 157.

@articleLi2020, author = Li, Pengfei and Lelli, Federico and McGaugh, Stacy S. and Schombert, James M., title = A Comprehensive Catalog of Dark Matter Halo Models for SPARC Galaxies, journal = The Astrophysical Journal Supplement Series, volume = 247, number = 1, pages = 31, year = 2020, doi = https://ui.adsabs.harvard.edu/link$_{g}ateway/2020ApJS$..247...31L/doi : 10.3847/1538 − 4365/ab700e10.3847/1538 − 4365/ab700e, url = $https$: //$ui.adsabs.harvard.edu/abs/2020ApJS$..247...31$L/abstract$

@articleWang2020, author = Wang, De-Chang and Xu, Feng and Luo, Xin, title = Comparison of Modeling SPARC spiral galaxies' rotation curves with different dark matter and MOND models, journal = arXiv preprint arXiv:2008.04795, year = 2020, url = https://arxiv.org/abs/2008.04795

@articleMilgrom2001, author = Milgrom, Mordehai, title = MOND - A Pedagogical Review, year = 2001, journal = NED Level 5 Review, url = https://ned.ipac.caltech.edu/level5/Sept01/Milgrom/Milgrom$2_2.html$