WILL Part I: Relational Geometry

Anton Rize
freewillrg@gmail.com *

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Abstract

This paper, the first in the *WILL* series. It is a relational rediscovery of GR and SR from the SPACETIME $\equiv$ ENERGY principle, yielding a singularity-free, algebraically simple formalism that matches empirical data without dark components. This principle derived by removing the hidden ontological assumption, implicit in modern physics, that structure (spacetime) and dynamics (energy) are separate phenomena (3.1).

Applying extreme methodological constraints it establishes *Relational Geometry (RG)*: a foundational framework where spacetime is an emergent property of relational energy transformations. This shift establishes an ontological transition from *descriptive* to *generative* physics: instead of introducing laws to model observations, it derives them as necessary consequences of RG itself - turning physics from a catalogue of phenomena into the logical unfolding of inevitable geometrical constrains on closed relational carriers S^1 (directional) and S^2 (omnidirectional).

Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energy-momentum relation, Schwarzschild and Einstein field equations via the dimensionless projections β (kinematic) and κ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$ (topologically derived, virial-like (Theorem: Closure). All results are empirically validated and listed in (Appendix I).

***WILL Part I* offers solutions to several long-standing problems, including:**

- Resolution of GR singularities (via naturally bounded $\rho_{\max} = \frac{c^2}{8\pi G r^2}$),
- Derivation of the equality of gravitational and inertial masses (from the common channel of rest-invariant scaling) 12.2,
- Removal of local energy ambiguity $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$
- Revelation of a clear relational symmetry between kinematic and potential projections,
- Developed closed system of equations for Relational Orbital Mechanics (ROM) without G, mass and differential formalism.
- Establishment of a computationally simpler and ontologically consistent foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

*This work is archived on Zenodo: ¸

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> *"There is no such thing as an empty space, i.e., a space without field. . . . Space-time does not claim existence on its own, but only as a structural quality of the field."*

*— Albert Einstein, **Relativity: The Special and the General Theory** (Appendix V: "Relativity and the Problem of Space"), 1952 edition, Methuen (London), p. 155; based on earlier 1920 additions.*

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This document must be read **literally**. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (*absolute energies, external backgrounds, hidden containers*) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

1 Foundational Principles

This Approach Does not Describe Physics; it Generates it.

Guiding Principle:

Nothing is assumed. Everything is derived.

Principle 1.1 (Epistemic Hygiene)**.** Epistemic Hygiene as Refusal to Import Unjustified Assumptions. *This line of reasoning derive physics by **removing hidden assumptions**, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced and no constructs are retained unless they are geometrically or energetically necessary.*

Principle 1.2 (Ontological Minimalism)**.** *Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities. This principle is not a statement about the nature of reality, but a rule of logical hygiene for constructing a theory.*

No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent relational projections.

Principle 1.3 (Relational Origin)**.** All physical quantities must be defined by their relations. *Any introduction of absolute properties risks reintroducing metaphysical artefacts and contradicts the foundational insight of relationalism.*

Principle 1.4 (Simplicity)**. Everything** must be expressed in the **simplest** form possible. *Any unjustified complexity risks reintroducing metaphysical artefacts and contradicts the foundational insight of Epistemic Hygiene.*

Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.

Principle 1.5 (Mathematical Transparency)**.**

1. *Every mathematical phrase, operational choice, or identity caries its on ontological statement.*
2. *Each mathematical object must correspond to explicitly identifiable relation between observers with transparent ontological origin.*
3. *Every symbol must be anchored to unique physical idea.*
4. *Introducing symbols without explicit necessity constitutes semantic inflation: the proliferation of symbols without corresponding physical meaning.*
5. *Number of symbols = Number of independent physical ideas.*

IMathematical hygiene

Mathematical hygiene is the geometry of reason

1.1 What is Energy in Relational Framework?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure. Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation. Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself. From these empirical and relational facts the definition follows unavoidably:

Energy :

Definition 1.6 (Energy)**.**

Energy is the relational measure of difference between possible states, conserved in any closed whole.
It is not an intrinsic property of an object, but ***comparative structure*** *between states (and observers), always manifesting as transformation.*

2 Ontological Blind Spot In Modern Physics

The standard formulation of General Relativity often relies on the concept of a pre-existing, container-like spacetime that then gets “filled” with fields and matter. This is in direct tension with the Relational Principle 1.3.

The standard derivation of the Einstein field equations begins with the Einstein-Hilbert action, which is built upon the metric as the fundamental variable. This metric is defined on a smooth manifold that is assumed to exist *a priori*. This manifold, even without a metric, carries topological and differential structure - an absolute scaffold. This is an ontological extra. It violates Principle 1.2 (Ontological Minimalism) because it introduces an entity (the manifold) that is not derived from relational observations.

In the standard formulation, the stress-energy tensor is derived from the variation of the matter Lagrangian with respect to the metric. This assumes that energy is a property of matter fields that can be localized in spacetime. However, this localization is frame-dependent (via the equivalence principle) and leads to well-known problems such as the non-uniqueness of the gravitational energy-momentum pseudotensor. This is a direct violation of the Relational Origin principle 1.3: energy is treated as an absolute property of matter rather than a relational measure.

We proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

2.1 Historical Pattern: breakthroughs delete, not add

- **Copernicus** eliminated the Earth/cosmos separation.
- **Newton** eliminated the terrestrial/celestial law separation.
- **Einstein** eliminated the space/time separation.
- **Maxwell** eliminated the electricity/magnetism separation.

Each step widened the relational circle and reduced the number of unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

2.2 The contemporary split: an unpaid ontological bill

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a *bi-variable* syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate *inside* the split. The split is therefore *not* an empirical discovery but an unpaid ontological debt.

2.3 Empirical bankruptcy of the separation

- **Local energy conservation** verified only *after* the metric is declared fixed; no experiment varies the *volume* of flat space and checks calorimetry.
- **Universality of free fall** tests $m_i = m_g$ numerically, not the claim that inertia resides *in* the object rather than in a geometric scaling relation.
- **Gravitational-wave polarisations** test spin content, not ontology; extra modes can still be called "matter on spacetime".

- **Casimir/Lamb shift** measure *differences* of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

In short, every "test" is an *internal consistency check* of a formalism that already presupposes two substances. None constitute *positive evidence* for the split.

2.4 Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an *un-evidenced metaphysical postulate*—the last geocentric epicycle in physics.

Summary

Any attempt to treat *"spacetime structure"* as separate from *"dynamics"* smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

Ontological Minimalism:

If no empirical or logical ground justifies the distinction between (structure) and (dynamics), the distinction must be dissolved.

SPACETIME ≡ ENERGY

This equivalence is not algebraic but ontological;
spacetime and energy are two descriptive projections of a single invariant entity we call:

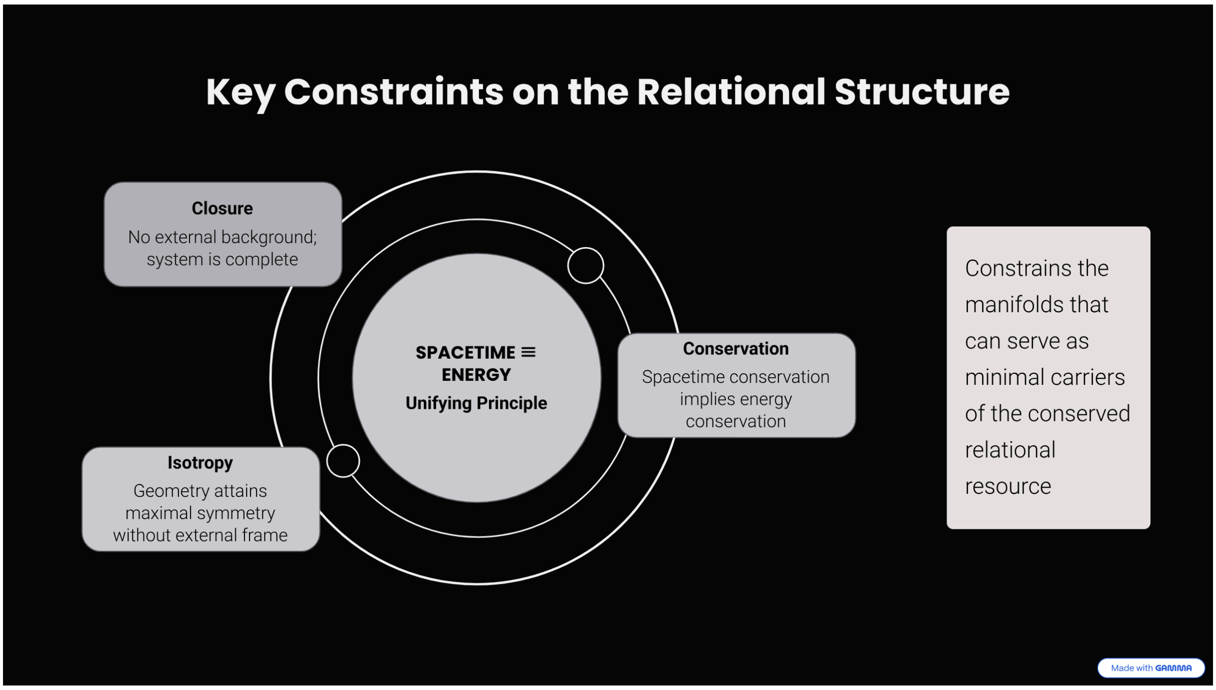
WILL

3 Unifying Principle Removing the Hidden Assumption

3.1 False Separation

Lemma 3.1 (False Separation)**.** *Any model that treats processes as unfolding* within *an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an* extraneous absolute.

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of interprocess data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. □



Corollary 3.2 (Structure–Dynamics Coincidence). *To avoid the artifact of Lemma 3.1, the structural arena and the dynamical content must be identified: geometry* is *energy, and energy* is *geometry.*

Principle 3.3 (Ontological Principle: Removing the Hidden Assumption).

$$\boxed{\textbf{\textit{SPACETIME}} \equiv \textbf{\textit{ENERGY}}}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight*: it removes the hidden unjustified separation between "structure" and "dynamics." Spacetime is not a "container" but emergent relational energy structure.*

Remark 3.4 (Auditability). *Principle 3.3 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).*

Definition 3.5 (WILL). **WILL ≡ SPACE-TIME-ENERGY** *is the technical term we use for unified relational structure determined by 3.3 . All physically meaningful quantities are relational features of WILL; no external container is permitted.*

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call - WILL.

4 Deriving the WILL Structure

Having established our Principle 3.3 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences.

We will demonstrate that this single principle is sufficient to enforce the closure, conservation, and isotropy of the relational structure, leading to a unique set of geometric carriers for energy.

Lemma 4.1 (Closure)**.** *Under 3.3, WILL is self-contained: there is no external reservoir into or from which the relational resource can flow.*

Proof. If WILL were not self-contained, there would exist an external structure mediating exchange. That external structure would then serve as a background distinct from the dynamics, contradicting Corollary 3.2. □

Lemma 4.2 (Conservation)**.** *Within WILL, the total relational "transformation resource" (energy) is conserved.*

Proof. By Lemma 4.1, no external fluxes exist. Any change in one part of WILL must be balanced by complementary change elsewhere. Hence a conserved global quantity is enforced at the relational level. □

Lemma 4.3 (Isotropy from Background–Free Relationality)**.** *If no external background is allowed (Cor. 3.2), then no direction can be* a priori *privileged. Thus the admissible relational geometry of WILL must be maximally symmetric (isotropic and homogeneous) at the level at which it encodes the conserved resource.*

Proof. Any alleged privilege that cannot be constructed from relations among participants is unobservable (pure gauge). Therefore the carrier used to encode the conserved WILL resource must have no intrinsic privileged direction or point. Therefore the encoding geometry must be maximally symmetric. □

4.1 Derivation of the Relational Carriers

Relational Carrier Conventions:

All references to "carriers" in the following section are to be read in the strict relational sense:

- **Degree of freedom (DOF):** A carrier with n DOF is an n-dimensional relational carrier used to encode the conserved transformation resource.
- **Direction:** A direction is an *oriented* relational ray. Opposite rays are physically distinct and are *not* identified. Any construction that merges opposite directions (e.g. antipodal identification) fails the relational requirement.
- **Closed carrier:** "Closed" means compact and without boundary: the transformation resource is finite and cannot leak into an external reservoir.
- **No background:** No external embedding space or privileged frame is allowed. All geometric structure must be reconstructible from relations between participants only.
- **Maximal symmetry:** The carrier is homogeneous and isotropic with respect to oriented directions: no point and no direction is *a priori* privileged.
- **Minimal relational carrier:** A carrier is "minimal" if it is closed, maximally symmetric, and uses exactly the required DOF to encode the resource. Under these constraints the classification theorems force S^1 (for 1DOF) and S^2 (for 2DOF) as the unique admissible carriers.

The lemmas of Closure, Conservation, and Isotropy (Lemmas 4.1–4.3) establish the **necessary properties** of any geometric carrier of the relational resource (energy). We must now identify these carriers.

To do this, we must first reject the *substantivalist* or "God's-eye view" common in physics, which illegitimately postulates an external 3D coordinate system ("State C") from which to describe the interaction between two states ("A" and "B"). Per Principle 1.3 (Relational Origin), such an external frame is an ontological speculation.

In a purely relational framework, only the participants (A and B) exist. All physics must be described **only** from mutual perspectives. This methodological constraint is not a simplification; it is an ontological necessity.

Theorem 4.4 (Minimal Relational Carriers of the Conserved Energy Resource)**.** *The minimal relational carriers satisfying the derived constraints of Closure, Conservation, and maximal Symmetry (Lemmas 4.1–4.3) are:*

(a) S^1 *for* directional *(Kinematic) relational transformation;*

(b) S^2 *for* omnidirectional *(Gravitational) relational transformation.*

Proof. The proof proceeds by classifying the minimal types of relations and applying the derived Lemmas:

(a) **Directional (Kinematic) Relation:** This is the simplest non-trivial 1 degree of freedom (1DOF) relation: transformation from State A to State B.

Per the Principle of Relational Origin, this interaction can only be described from the frame of A or B. From the perspective of B, any complex 3D motion of A (including transverse motion) is operationally perceived, within 1DOF relation, only as a change in the rate of approach or recession. Thus, the fundamental, operational description of a 1DOF two-state transformation is necessarily **one-dimensional (1D)**.

Applying the Lemmas: By Lemma 4.1 this 1D geometry must be **closed**. By Lemma 4.3 it must be **maximally symmetric**. The classification of connected, closed, 1-carrier yields S^1 (circle) as the **unique** (up to diffeomorphism) carrier satisfying these constraints.

(b) **Omnidirectional (Gravitational) Relation:** This is the other minimal relation 2 degree of freedom (2DOF) type: a central state (A) relating to the l**ocus** of all equidistant states (e.g., an orbit). This describes a "center-to-orbit" relationship. By Lemma 4.3 (Isotropy), the conserved WILL resource must be distributed uniformly across all possible orientations from this center. The minimal carrier required to describe all possible orientations from a center is a **two-dimensional (2D)** surface.

Applying the Lemmas: By Lemma 4.1, this 2D surface must be **closed**. By Lemma 4.3, it must be **maximally symmetric** (isotropic from the center). By the classification of constant-curvature surfaces, the unique closed, simply connected, maximally symmetric 2-carrier is the **2-sphere (S^2) (surface area of the sphere).**

The Principle of Ontological Minimalism (1.2), combined with the derived constraints, thus uniquely and necessarily selects S^1 and S^2 as the minimal relational carriers. □

Corollary 4.5 (Uniqueness)**.** *Under 3.3 with Closure, Conservation, and Isotropy (Lemmas 4.1–4.3), S^1 and S^2 are* necessary *relational carriers for, respectively, directional and omnidirectional modes of energy transformation.*

Remark 4.6 (Non-spatial Reading)**.** *Throughout, S^1 and S^2 are not to be interpreted as* spacetime *geometries. They are* relational carriers *that encode the closure, conservation, and isotropy of the transformational resource. Ordinary spatial and temporal notions are emergent descriptors of patterns within WILL.*

Summary:

From removing the hidden assumption 3.1 we inevitably arrive to 3.3 SPACETIME ≡ ENERGY from there we deduced: (i) closure, (ii) conservation, (iii) isotropy, and hence (iv) the unique selection of S^1 and S^2 as minimal relational carriers for directional and omnidirectional transformation. These objects are non-spatial encodings of conservation and symmetry; they are enforced by the3.3 rather than assumed independently.

5 The Amplitude-Phase Duality

The manifestation of any system is distributed between its internal (Phase) and external (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

Lemma 5.1 (Duality of Evolution). *The identification of spacetime with energy and its transformations necessitates two complementary relational measures:*

1. *the* ***amplitude*** *of transformation (external shift), and*
2. *the* ***phase*** *of transformation (internal order).*

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. Relational Carriers S^1 and S^2 both provides the minimal geometry enforcing such complementarity: there orthogonal projections furnish precisely two non-redundant coordinates. □

The orthogonal decomposition of the relational carriers S^1 and S^2 reveals a functional duality. Every physical state is a superposition of two projections:

Definition 5.2 (The Amplitude Projection (External Interaction)). *Denoted by β (kinematic) and κ (gravitational). This component measures the extent of external relation. It manifests as momentum or potential intensity. Physically, it represents the system's relational "shift" from the* ***Observer's relational Origin*** *(the rest frame).*

$$\textit{Amplitude} \to \textit{External Power (Kinetic/Potential)}$$

Definition 5.3 (The Phase Projection (Internal Evolution)). *Denoted by β_Y and κ_X. This component measures the internal ordering. It governs the intrinsic scale of proper time and proper length. A Phase of 1 represents maximal internal flow (rest/vacuum), while a Phase of 0 represents the cessation of internal causality (light-speed/horizon).*

$$\textit{Phase} \to \textit{Internal Order (Time/Structure)}$$

Theorem 5.4 (Universal Conservation of Relation). *For both kinematic (S^1) and gravitational (S^2) modes, the sum of the squared Amplitude and squared Phase is invariant:*

$$\underbrace{\textit{Amplitude}^2}_{\textit{External Interaction}} + \underbrace{\textit{Phase}^2}_{\textit{Internal Existence}} = 1 \tag{1}$$

Proof. By geometric nature of the relational carriers S^1 ($\beta^2 + \beta_Y^2 = 1$) and S^2 ($\kappa^2 + \kappa_X^2 = 1$), they satisfy [$\underbrace{\text{Amplitude}^2}_{\text{External Interaction}} + \underbrace{\text{Phase}^2}_{\text{Internal Existence}} = 1$] which encodes the finite relational budget. □

5.1 Consequence: Relativistic Effects

Proposition 5.5 (Physical Interpretation: Relativistic Effects). *The conservation law of Theorem (Universal Conservation of Relation) implies that any redistribution between the orthogonal components* $\underbrace{\textit{Amplitude}^2}_{\textit{External Interaction}} + \underbrace{\textit{Phase}^2}_{\textit{Internal Existence}} = 1$ *manifests physically as the relativistic effects of time dilation and length contraction.*

Proof. By Theorem (Universal Conservation of Relation) the components satisfy ($\beta^2 + \beta_Y^2 = 1$); ($\kappa^2 + \kappa_X^2 = 1$). An increase in Amplitude (β, κ) enforces a decrease in Phase measure (β_Y, κ_X). This reduction of β_Y, κ_X corresponds to dilation of proper time and contraction of proper length, while the growth of β, κ represents momentum/potential. Thus the relativistic and gravitational trade-off is the direct physical expression of the geometric closure of $S^1$and S^2 relational carriers. □

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Summary:
Geometry of spacetime is the shadow cast by the geometry of relations.

Remark 5.6. *This identifies Relativistic and Gravitational Time Dilation not as a mysterious "slowing down" of clocks, but as a strict geometric* ***phase rotation****. As a system invests more of its existence into external Amplitude (β or κ), it necessarily withdraws from its internal Phase (β_Y or κ_X), changing the rate of its proper time evolution.*

5.2 Ontological Status of the Relational Carriers S^1 and S^2

A natural question arises regarding the ontological status of the circle S^1 and the sphere S^2: What are they, and where do they "exist"?

The answer requires a shift in perspective. In WILL Relational Geometry, S^1 and S^2 are **not spatial entities** existing within a pre-defined container. They are the necessary **relational architectures** that implement the core identity SPACETIME $\equiv$ ENERGY.

Energy as Relational Transformation Capacity Recall that energy is defined as the *relational measure of difference between possible states.* It is not an intrinsic property but a relational potential for change. It is never observed directly, only through transformations.

The Carriers as Protocols of Interaction The Carriers S^1 and S^2 are the minimal, unique mathematical structures capable of hosting this relational "bookkeeping" for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, A and B:

- Observer A is the center of their own relational framework. Observer B is a point on A's S^1 (for kinematic relations) and S^2 (for gravitational relations).
- Simultaneously, observer B is the center of their own framework. Observer A is a point on B's S^1 and S^2.

There is no privileged "master" carrier. Each observable interaction is structured by these mutually-centered relational protocols. The parameters β and κ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta^2 + \beta_Y^2 = 1; \quad \kappa_X^2 + \kappa^2 = 1$) are the innate accounting rules of these protocols.

6 Emergence of Spacetime

In this construction, "space," "time," are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

Therefore, the question "Where are S^1 and S^2?" is a category error. They are not *in* space; they are the structures whose coordinated, multi-centered interactions **give rise to** the phenomenon we perceive as spacetime. Spacetime is the emergent, collective shadow cast by the dynamics of energy relations projected onto these architectures.

In essence, S^1 and S^2 are the ontological embodiment of the relational principle. They are derived as the only possible structures that can house the transformational resource (energy) in a closed, conserved, and isotropic system. Their status is that of a fundamental **relational geometry** from which physics is generated.

7 Energy as a Relation - What κ and β Actually Mean

Energy (1.6) is the measure of differences between states.

In relational framework:

- Physical parameters like energy, speed, and gravitational potential don't belong to objects.
- Instead, they represent how we, as observers, measure objects state differences from our own point of view.

In this view, your perspective is always the reference frame. You are always at the point of origin (0,0) on your (β,κ) plane.

Everything else is described by how it differs from your state:

- β is the measure of how much of the universal "speed of change" you see as motion through space, relative to yourself.
- κ is the measure of how deeply an object sits in a gravitational field, as seen from your position. It's your personal "ruler" for gravitational depth.

Think of κ and β as your own relational measuring tools:

- β is how far along your "motion ruler" you project another object's state.
- κ is how deep into your "gravity well" you see another object's state.

Thus relational understanding emerges naturally:

- Energy is the capacity to move between states.
- Saying "the object's energy" always implicitly means "the object's energy as measured from your perspective."

Here's a simple analogy:

> Imagine standing on a train platform. A train passes by rapidly: to you, it has significant kinetic energy. But if you jump onto the train, it instantly becomes stationary relative to you. Its kinetic energy is now zero - because your frame of reference shifted. The energy didn't vanish; your perspective changed.

Summary:

- The projections κ and β are your personal, relational measurements of energy difference.
- All physics boils down to describing how things differ in relation to you.

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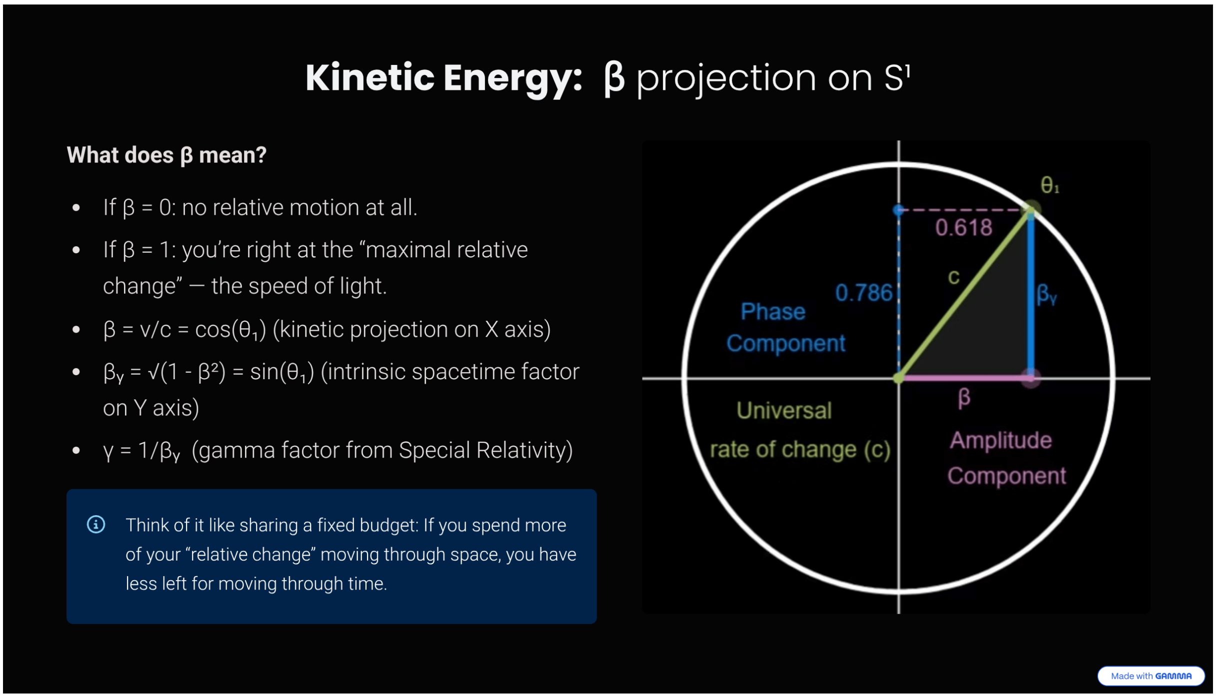

8 Kinetic Energy Projection on S^1

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Since S^1 encodes one-dimensional shift, the total energy E of the system must project consistently onto both axes:

$$E_X = E\beta, \qquad E_Y = E\beta_Y.$$

8.1 The Geometric Nature of Mass

To derive the energy transformation, we must first explicitly identify the physical meaning of the orthogonal projections.

Theorem 8.1 (Invariant Projection of Rest Energy)**.** *For any state (β, β_Y) on the relational circle S^1, the total energy E must scale such that its vertical projection remains constant and equal to the rest energy E_0.*

$$E\beta_Y = E_0.$$

Proof. Let the total energy E be distributed on the relational carrier S^1 with projections β (kinematic) and β_Y (internal).

$$E_Y = E \cdot \beta_Y$$

By Principle 1.3 (Relational Origin), the internal structure of an object (its rest energy E_0) is defined solely by relations internal to itself. Therefore, it must be invariant under changes in its relation to an external observer. In relation to itself kinematic projection is always ($\beta = 0 \Rightarrow \beta_Y = 1$). Therefore:

$$\boxed{E_Y \equiv E_0}$$

where E_0 is the energy measured in the frame where the system is at rest ($\beta = 0, \beta_Y = 1$). Thus, geometric consistency requires the vertical leg to be fixed:

$$E\beta_Y = E_0 \quad \Longrightarrow \quad E = \frac{E_0}{\beta_Y}.$$

The "hypotenuse" (Total Energy E) is therefore not a fixed-length vector that rotates (which would reduce E_Y), but a scalable relational magnitude that grows to preserve the invariant vertical leg E_0 against the closure constraint $\beta_Y = \sqrt{1-\beta^2}$. □

Summary:

The historical Lorentz factor γ is the reciprocal of β_Y. $\gamma = 1/\beta_Y$

8.2 Rest Energy and Mass Equivalence

Corollary 8.2 (Rest Energy and Mass Equivalence)**.** *Within the normalization $c = 1$, the invariant rest energy equals mass:*

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1, no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. □

Summary:

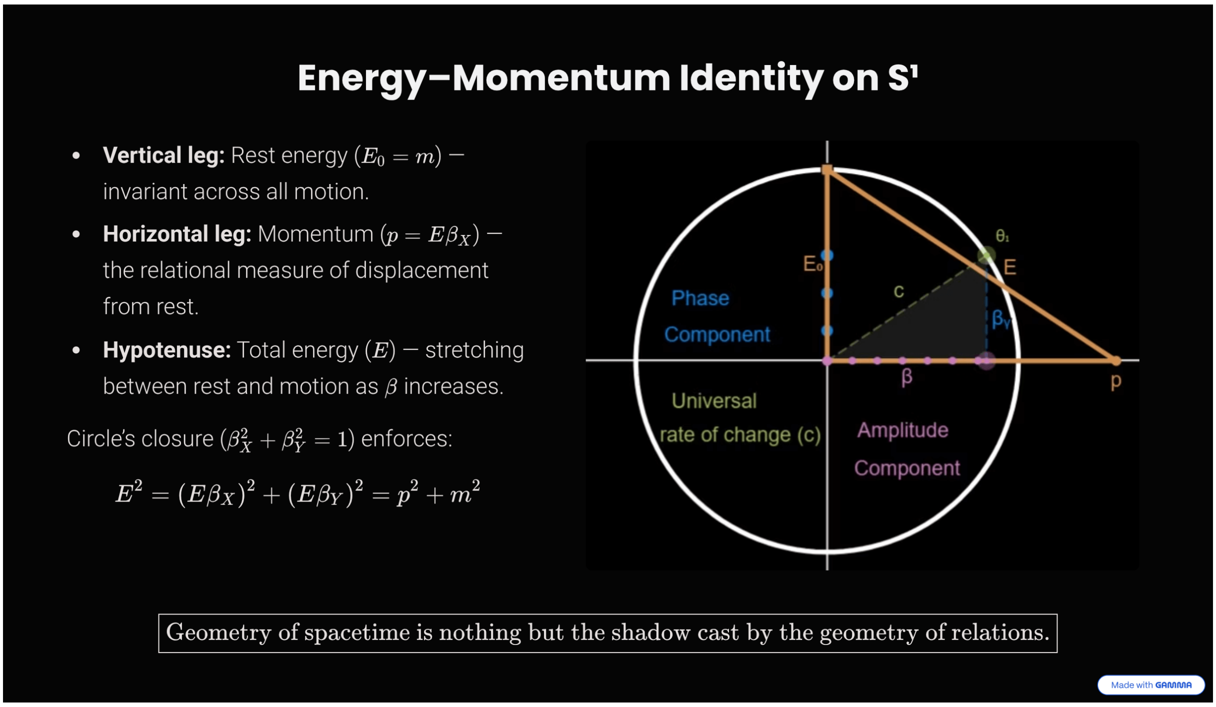
Mass is the invariant projection of total rest energy.

8.3 Energy–Momentum Relation

Proposition 8.3 (Horizontal Projection as Momentum)**.** *On the relational circle, the unique relational shift measure from rest is the horizontal projection $E\beta$; hence*

$$p \equiv E\beta \quad (c = 1).$$

Proof. The rest state is $(\beta, \beta_Y) = (0, 1)$. A shift measure must (i) vanish at rest, (ii) grow monotonically with $|\beta|$, and (iii) flip sign under $\beta \mapsto -\beta$. The only relational candidate satisfying (i)-(iii) is the horizontal projection $E\beta$. Thus the identification is necessary rather than conventional. □



Corollary 8.4 (Energy–Momentum Relation)**.** *With p identified by Proposition 8.3 and $m = E_0$, the closure identity yields*

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E, yielding the standard form. □

Remark 8.5 (Geometric Forms)**.** *The same identity may be expressed explicitly in terms of circle coordinates:*

$$E^2 = \left(\frac{\beta}{\beta_Y} E_0\right)^2 + E_0^2 = \big(\cot(\theta_1)\, E_0\big)^2 + E_0^2.$$

These are equivalent renderings of the same geometric necessity.

Remark 8.6 (Units sanity check - bookkeeping)**.** *Using $\beta = v/c$, the identification $p \equiv E\beta$ gives*

$$pc = E\,\frac{v}{c} \quad \Longrightarrow \quad p = \frac{E\,v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

Summary

The energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$ is geometric identity of S^1.

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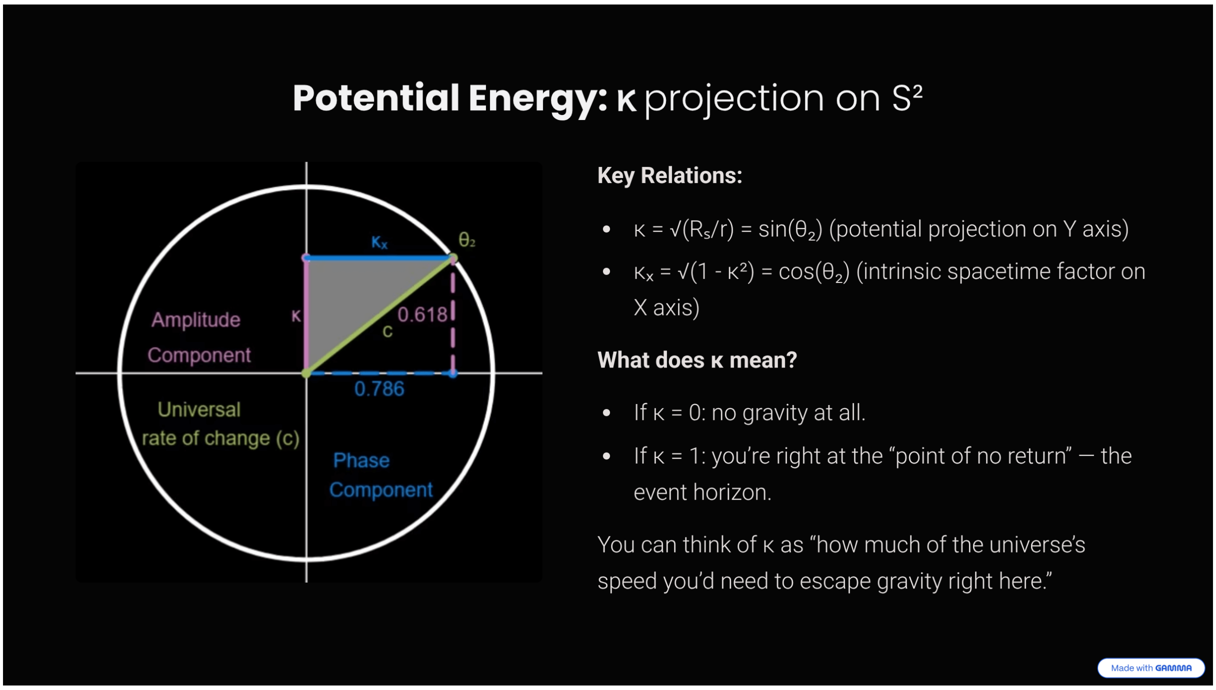
$\beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	**Trigonometric Form**
$\beta = v/c = \sqrt{1 - \beta_Y{}^2}$	$\beta = cos(\theta_1) = cos(\arccos(\beta))$
$\beta_Y = \sqrt{1 - (v/c)^2} = \sqrt{1 - \beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$

Table 1: Geometric representation of relativistic effects.



9 Potential Energy Projection on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational carriers encoding energy conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1 the relational geometry of the sphere, S^2, provides orthogonal projections, for two aspects of omnidirectional transformation. We define them as follows:

- **The Amplitude Component (κ):** This projection represents the *relational gravitational measure* between the object and the observer. It corresponds to the *extent* of transformation, which manifests physically as gravitation potential. A value of $\kappa = 1$ denotes *saturation*: the entire relational resource of the system has been allocated into the gravitational channel. No residual capacity remains for kinematic projection. This condition defines the relational horizon.

- **The Phase Component (κ_X):** This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the *sequence* of its transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of transformation":

$$\kappa_X^2 + \kappa^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

9.1 Gravitational Meridional Section of S^2

By isotropy the omnidirectional carrier is S^2, but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of S^2 with the parametrization $(\kappa_X, \kappa) = (\cos\theta_2, \sin\theta_2)$.

9.2 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure (κ, gravitation potential) necessarily requires a decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is the shadow cast by the geometry of relations.

9.3 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the escape velocity at radius r.

In the kinematic case, with $\beta = \cos\theta_1$, the energy relation can be written as

$$E^2 = \big(\cot\theta_1\, E_0\big)^2 + E_0^2, \tag{2}$$

so that the relativistic momentum is expressed as

$$p = E_0/c\,\cot\theta_1. \tag{3}$$

In full symmetry, the gravitational case follows from $\kappa = \sin\theta_2$. We define the gravitational energy as

$$E_g = \frac{E_0}{\kappa_X}, \qquad \kappa_X = \sqrt{1-\kappa^2}, \tag{4}$$

and introduce the gravitational analogue of momentum:

$$p_g \;=\; E_0/c\,\tan\theta_2. \tag{5}$$

This yields the gravitational energy relation

$$E_g^2 = (p_g c)^2 + (mc^2)^2. \tag{6}$$

Summary:

$$\beta = \cos\theta_1, \qquad \kappa = \sin\theta_2,$$

$$\beta \longleftrightarrow \kappa, \qquad \cot\theta_1 \longleftrightarrow \tan\theta_2.$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the same relational circle, expressed through complementary trigonometric forms.

10 Geometric composition of SR and GR factors

On the unit kinematic circle (S^1) we parametrize

$$(\beta, \beta_Y) = (\cos\theta_1,\ \sin\theta_1),$$

so that the invariant vertical projection reads

$$E\,\beta_Y = E_0 \quad \Rightarrow \quad \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin\theta_1}}, \qquad p = \frac{E}{c}\,\beta = \frac{E_0\,\beta}{\beta_Y} = E_0\cot\theta_1,$$

and therefore $E^2 = (pc)^2 + E_0^2$.

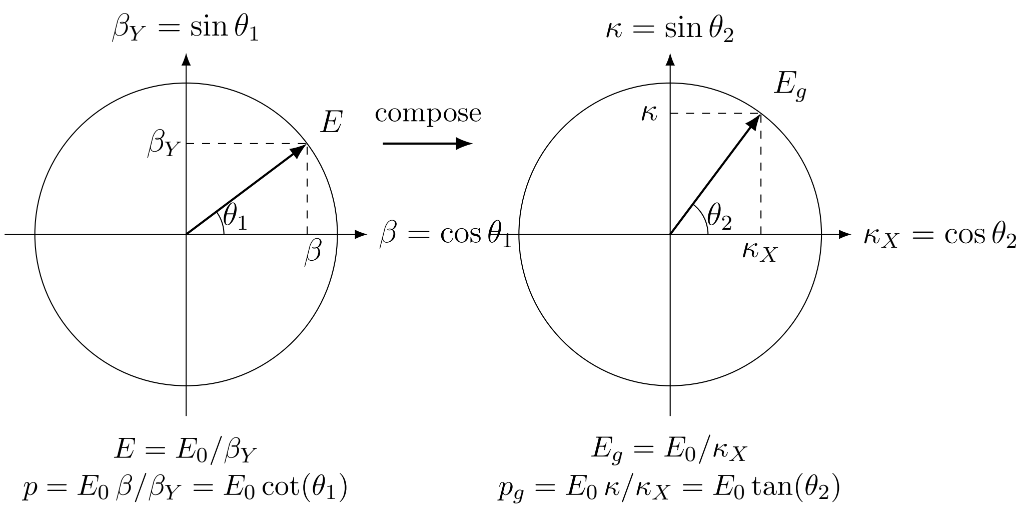
On the gravitational circle (S^2) we parametrize

$$(\kappa_X, \kappa) = (\cos\theta_2,\ \sin\theta_2),$$

so that the invariant horizontal projection reads

$$E_g\,\kappa_X = E_0 \quad \Rightarrow \quad \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos\theta_2}}, \qquad p_g = E_g\,\kappa = \frac{E_0\,\kappa}{\kappa_X} = E_0\tan\theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



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$\theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa),\ \kappa^2 = 2\beta^2$	
Algebraic Form	**Trigonometric Form**
$\beta = v/c$	$\beta = cos(\theta_1)$
$\kappa = \sqrt{R_s/r}$	$\kappa = sin(\theta_2)$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$
$p = E_0/c \cdot \beta/\beta_Y$	$p = E_0/c \cdot \cot(\theta_1)$
$p_g = E_0/c \cdot \kappa/\kappa_X$	$p_g = E_0/c \cdot \tan(\theta_2)$
$\tau = \beta_Y \kappa_X$	$\tau = \sin(\theta_1)\cos(\theta_2)$
$Q = \sqrt{\kappa^2+\beta^2} = \sqrt{3}\beta$	$Q = \sqrt{3}\cos(\theta_1)$
$Q_t = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2}$	$Q_t = \sqrt{1-3\cos^2(\theta_1)}$

Table 2: Unified representation of relativistic and gravitational effects for closed systems.

10.1 Clear Relational Symmetry Between Kinematic and Potential Projections

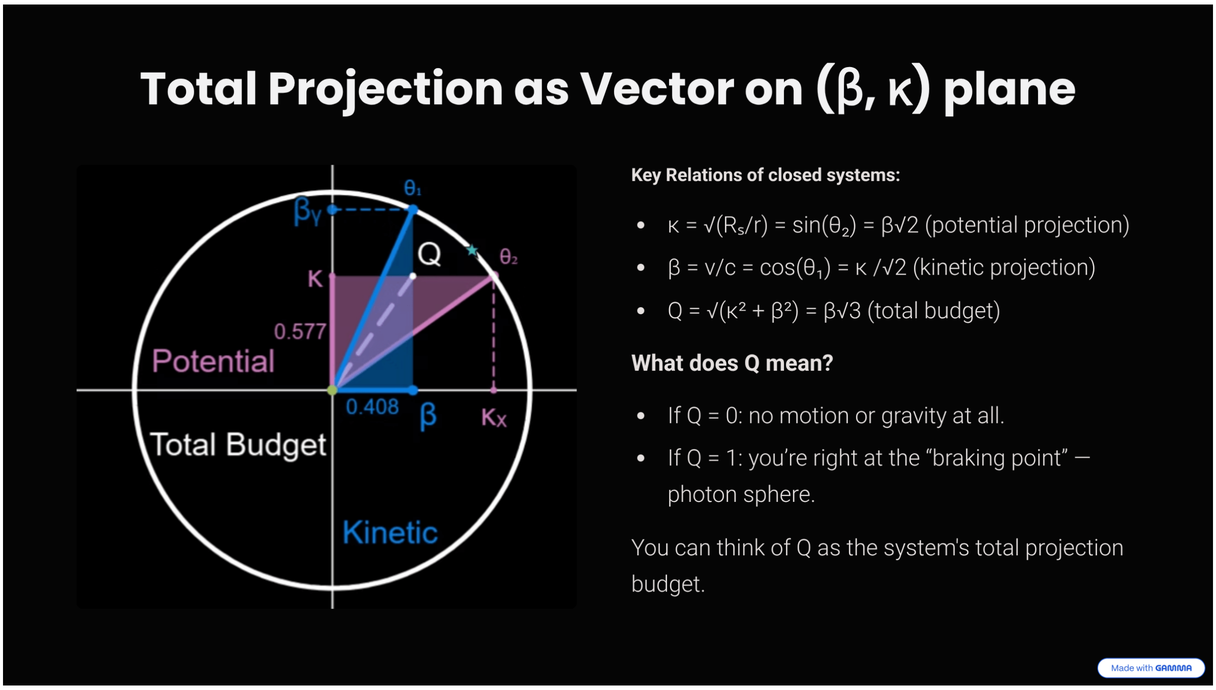
Now we can clearly see the underlying symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

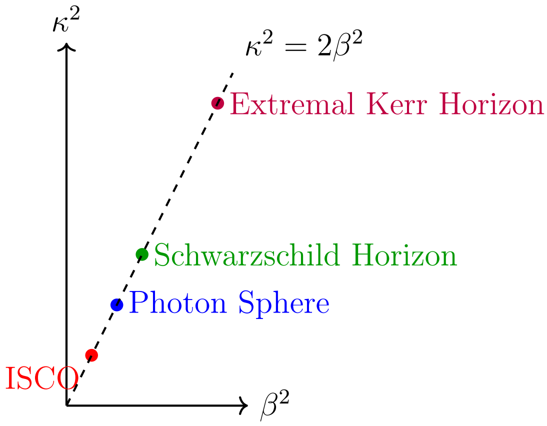
Summary

The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-transformation constraint revealing their unified origin.

Phenomenon	Radius r	β^2	κ^2	Q^2	Comment
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal stability of time-like orbits $Q = Q_t$
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Null circular orbits, $\theta_1 = \theta_2$ $Q = 1$, $Q_t = 0$
Static horizon (Schwarzschild)	$r = R_s$	$\frac{1}{2}$	1	$\frac{3}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 3: Critical radii and their projectional parameters in WILL Relational Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.





11 Total Relational Shift Q

When an observer observes another system, they assign to it a Total Relational Shift norm Q:

$$\boxed{Q^2 = \beta^2 + \kappa^2} \tag{7}$$

Relational reciprocity is the invariance of this norm under the self-centering operation of each observer.

Each observer places itself at the relational origin

$$(\beta, \kappa) = (0, 0).$$

If the other system now looks back, it again self–centres at $(0, 0)$ and applies the same rule. It measures the observer's (β, κ) and again obtains

$$Q^2 = \beta^2 + \kappa^2.$$

Thus Q is the *norm* of Total Relational Shift, not a spatial distance. Geometrically, the observer is always at the centre of its own S^1 (or S^2) carrier, and any external system is a point (β, κ) on that plane. The scalar Q measures the total deviation from the observer's relational origin.

Remark 11.1 (Closure-specific simplification). *Under energetic closure $\kappa^2 = 2\beta^2$ (circular/periodic systems), the norm reduces to $Q^2 = 3\beta^2$ (proved uniquely in Chapter C,* **??***).*

In general (open or elliptic) configurations, the full definition $Q^2 = \beta^2 + \kappa^2$ must be used.

11.1 Principle of Relational Reciprocity

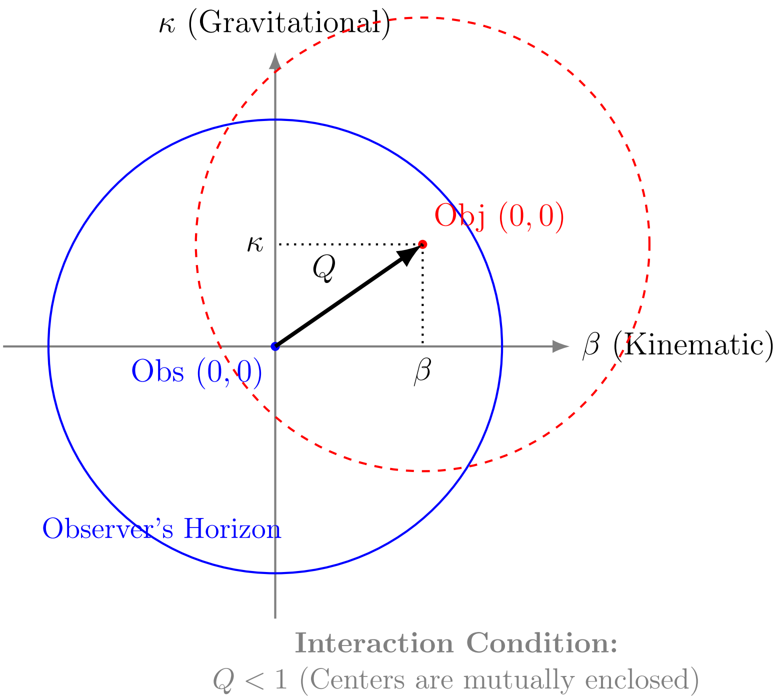


Figure 1: **Relational Self-Centering.** The total shift Q is defined by the orthogonal projections β and κ. Interaction is causal only when the center of the Object lies within the Observer's horizon ($Q < 1$), ensuring mutual coverage.

Self–centering reciprocity. Every observer performs self–centering:

$$(\beta, \kappa) = (0, 0).$$

When I observe another system, I assign to it (β, κ) and therefore a total shift

$$Q^2 = \beta^2 + \kappa^2.$$

When that system observes me, it again self–centres and obtains the same form. It assigns to me some (β, κ) and again computes $Q^2 = \beta^2 + \kappa^2$.

Reciprocity is therefore not a vector symmetry in a shared space. It is a symmetry of the *self–centering operation*: each observer applies the same rule and only the norm of shift is invariant.

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Summary

$$\boxed{\text{Relational reciprocity} \;=\; \text{invariance of the norm } Q \text{ under self–centering.}}$$

There is no common background arena. There are only mutual total shift magnitudes Q computed in each observer's own relational origin.

12 Equivalence Principle as Derived Identity

Lemma 12.1 (Unified Relational Scaling)**.** *Within the relational framework of WILL, both kinematic (S^1) and gravitational (S^2) transformations act as independent projections of the same invariant energy E_0. Each projection rescales the observable quantities by its respective geometric factor:*

$$E = \frac{E_0}{\beta_Y}, \qquad E_g = \frac{E_0}{\kappa_X}.$$

Proof. On the kinematic circle S^1, the invariant vertical projection corresponds to $\beta_Y = \sin\theta_1$. Preserving the same invariant leg E_0 forces the stretch $E/E_0 = 1/\beta_Y$. On the gravitational sphere S^2, the invariant horizontal projection is $\kappa_X = \cos\theta_2$, forcing $E_g/E_0 = 1/\kappa_X$. These transformations are independent and commute, each preserving the closure identity of its respective carrier. □

Theorem 12.2 (Equivalence of Inertial and Gravitational Response)**.** *Composing the independent relational stretches of Lemma 12.1 yields the total local energy scale*

$$E_{\rm loc} = \frac{E_0}{\tau} = \frac{E_0}{\beta_Y\,\kappa_X} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The corresponding inertial and gravitational projections share a single operational factor,

$$\tilde{p} = \frac{E_{\rm loc}}{c}\beta, \qquad \tilde{p}_g = \frac{E_{\rm loc}}{c}\kappa,$$

both governed by the same effective mass

$$m_{\rm eff} = \frac{E_0}{\beta_Y\,\kappa_X\,c^2} = \frac{E_0}{\tau c^2}.$$

Therefore,

$$\boxed{m_g \equiv m_i \equiv m_{\rm eff}},$$

and the Einstein equivalence principle follows as a necessary structural identity of WILL.

Corollary 12.3 (Mass Invariance under Relational Scaling)**.** *The invariant core E_0 denotes the complete internal equilibrium state ($\beta_Y = \kappa_X = 1$). Relational factors β_Y and κ_X rescale only external manifestations (energy, momentum, and rates), while E_0 remains unchanged. Hence,*

$$\boxed{m_g \equiv m_i \equiv m = E_0/c^2},$$

is not a dynamical statement but the definition of rest invariance itself.

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Remark 12.4 (Composition-Independence)**.** *Decomposing the invariant rest energy into internal channels,*

$$E_0 = \sum_a E_0^{(a)},$$

each term couples identically through the same geometric stretch:

$$E_{\text{loc}} = \sum_a \frac{E_0^{(a)}}{\tau}.$$

Since all channels scale by the same factor $1/\tau = 1/(\beta_Y \kappa_X)$*, ratios between channels cancel in all observables. Therefore, composition-independence of motion (Eotvos universality) follows identically, without requiring a postulate* $m_g = m_i$*.*

Remark 12.5 (Quantum Interface)**.** *The relational phase increment inherits the same scaling:*

$$\Delta\phi \propto E_{\text{loc}}\, \Delta\lambda,$$

where $\Delta\lambda$ *is the internal ordering parameter. Thus both kinematic and gravitational phase shifts share the same stretch* $1/\tau = 1/(\beta_Y \kappa_X)$*, yielding composition-independent matter-wave interference patterns.*

Summary:

In WILL, the equivalence of inertial and gravitational mass is not assumed but follows necessarily from the compositional closure of relational geometry. What General Relativity posits as a postulate, WILL reveals as a corollary.

13 Unification of Projections: The Geometric Exchange Rate

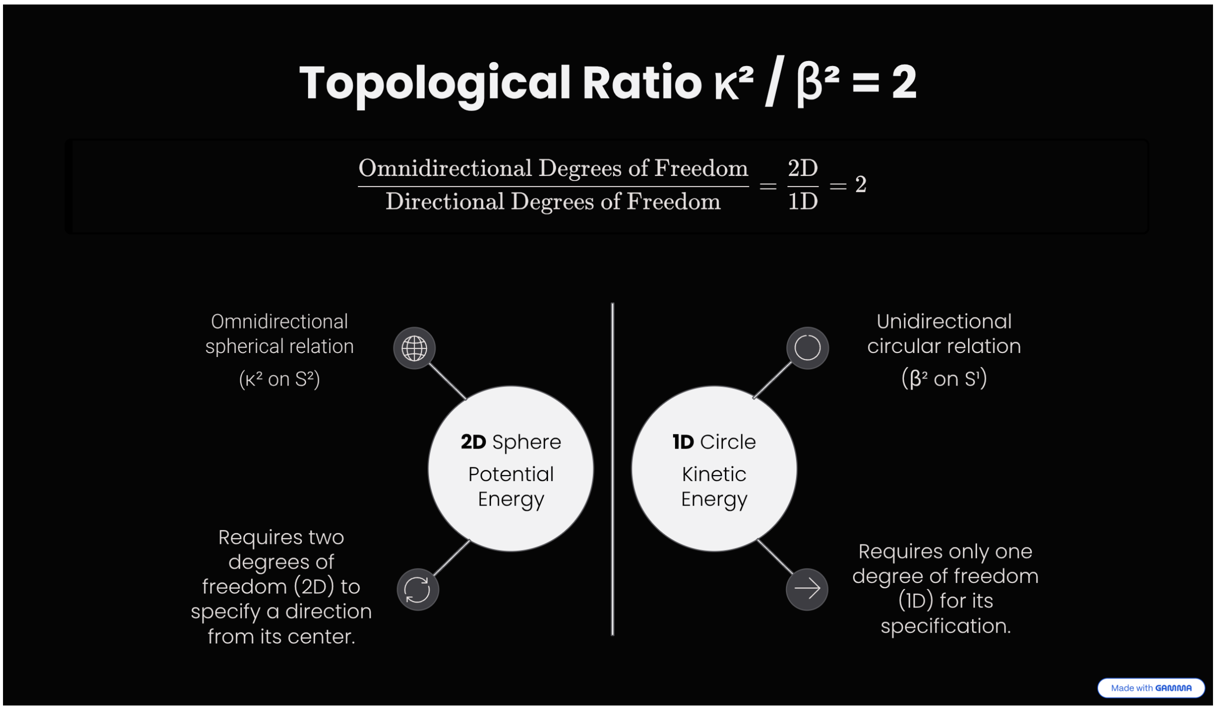
Having established that directional (kinematic) and omnidirectional (gravitational) relations are carried by the S^1 and S^2 respectively, we now derive the relationship that unifies them.

13.1 Derivation of the Energetic Closure Condition

Remark 13.1 (From slice to whole S^2)**.** *Although we parametrise a single meridional great circle* (κ_X, κ) *for algebraic convenience, the amplitude* κ^2 *denotes the* total omnidirectional budget *of the* S^2 *carrier. The exchange-rate factor* 2 *reflects that* S^2 *has two independent relational degrees of freedom even when calculations are carried out on a representative great-circle section.*

13.2 Uniqueness of the Exchange Rate (No Hidden Weighting)

Lemma 13.2 (DOF-Indifference)**.** *Under maximal symmetry (no privileged directions) and ontological minimalism (no hidden structure), any admissible conserved budget must assign* equal quadratic weight *to each independent relational degree of freedom.*



Proof. If two independent DOF contributed unequal weights to the conserved quadratic budget, then the theory would contain an implicit weighting structure that distinguishes DOF. This constitutes a privileged feature not derivable from relations, violating maximal symmetry and minimalism. □

Theorem 13.3 (Closure)**.** *Within WILL, the only exchange rate between the kinematic carrier S^1 (1 DOF) and the gravitational carrier S^2 (2 DOF) compatible with (i) closure in quadratic form, (ii) maximal symmetry, and (iii) ontological minimalism is:*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Proof. Let b denote the conserved quadratic budget associated with a single independent DOF. By Lemma 13.2, each DOF must contribute the same amount b.

The carrier S^1 has 1 DOF, so its total quadratic budget is $B_{S^1} = 1 \cdot b = b$. The carrier S^2 has 2 DOF, so its total quadratic budget is $B_{S^2} = 2 \cdot b$.

An exchange rate is precisely the statement that the omnidirectional budget is a fixed multiple of the directional budget:

$$B_{S^2} = \mathcal{R}\, B_{S^1}.$$

Substituting $B_{S^2} = 2b$ and $B_{S^1} = b$ gives $\mathcal{R} = 2$, hence

$$\kappa^2 = \mathcal{R}\,\beta^2 = 2\beta^2.$$

Any $\mathcal{R} \neq 2$ would require unequal DOF weighting (hidden structure) or an extra free parameter, violating the methodology. □

Remark 13.4 (Status)**.** *The factor 2 is not empirical calibration and not a coordinate choice. It is the unique consequence of (1 DOF vs 2 DOF) under symmetry and minimalism.*

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Definition 13.5 (Closure Factor).

$$\delta \equiv \frac{\kappa^2}{2\beta^2}$$

A subsystem is energetically closed *if* $\langle\delta\rangle_{cycle} = 1$. *For circular orbits,* $\delta \equiv 1$.

Corollary 13.6 (Energetic Closure Criterion). *Closed systems (momentary or periodic) satisfy* $\kappa^2 = 2\beta^2$ *identically. Open systems display* $\delta \neq 1$*, the magnitude of which quantifies the energy flow through unaccounted channels. When all channels are included, closure is restored.*

Remark 13.7 (Physical Interpretation). *The exchange rate between the kinematic and gravitational projections corresponds to the ratio of their relational dimensions. This purely geometric constant (2) replaces the empirical proportionalities of classical dynamics. It is the relational origin of the* ***virial theorem****: the kinetic and potential aspects of WILL maintain closure through the invariant ratio*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Illustrative Examples.

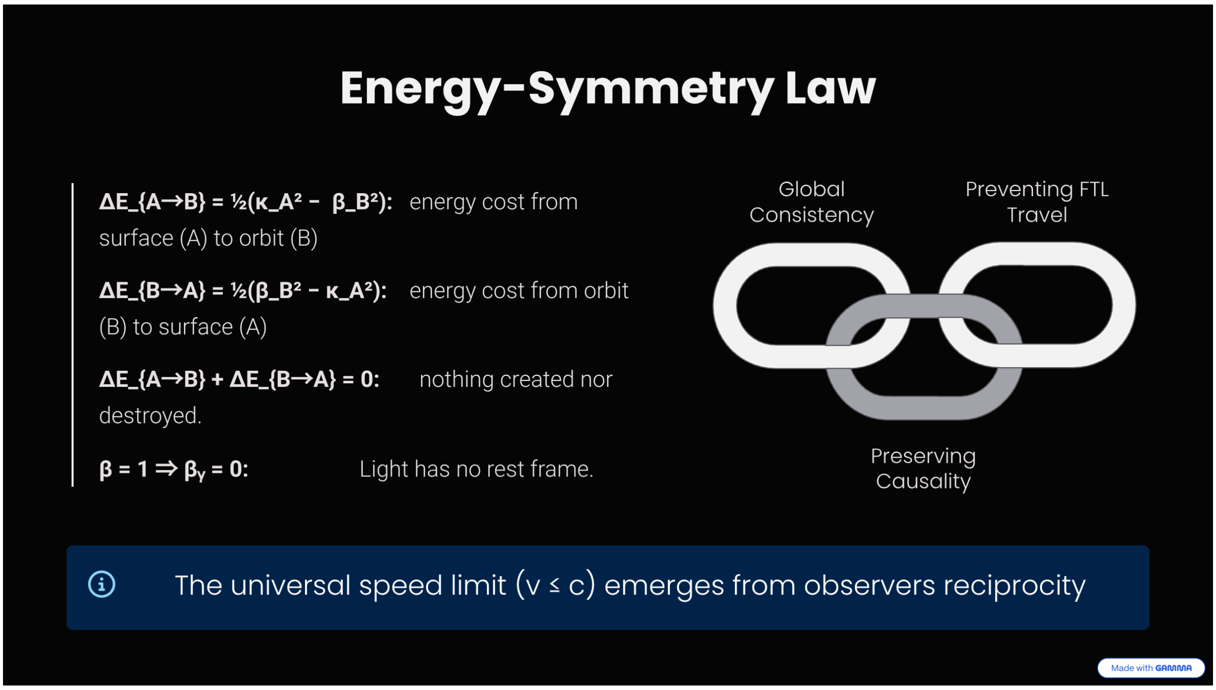
- **Circular Orbit (Closed).** A body at any orbital phase exactly satisfies $\kappa^2 = 2\beta^2$. The entire conserved resource is partitioned between kinetic and gravitational projections; no internal "breathing" and no external channel exists.
- **Elliptical Orbit (Closed).** A body satisfies $< \kappa^2 >= 2 < \beta^2 >$ exactly as an average per orbital cycle due to internal "breathing" of elliptical systems. Though this internal "breathing" is restricted by the Energy-Symmetry Law (14) so the difference $W = \frac{1}{2}(\kappa^2 - \beta^2) = \frac{E}{E_0} =$ constant at any orbital phase. No external channel exists.
- **Radiating Binary (Open).** An elliptical compact binary violates $< \kappa^2 >= 2 < \beta^2 >$ when only orbital degrees of freedom are counted, the closure defect δ quantifying energy lost to gravitational radiation. Including all channels restores closure.

Summary:

1. WILL is a closed relational structure, SPACETIME $\equiv$ ENERGY.
2. The simplest maximally symmetric carriers of these relations are S^1 and S^2.
3. The parameters $\beta = \cos\theta_1$ and $\kappa = \sin\theta_2$ are thus constrained to these carriers.
4. The geometric exchange rate between these modes equals the ratio of their relational dimensionalities: 2.

Remark 13.8 (Geometric Origin of Physical Law). *The relation between kinetic and potential energy is not an empirical coincidence but a geometric necessity of relational closure. Classical mechanics merely approximates this deeper invariant. Explicitly,*

$$\boxed{\textit{Geometric Distribution } (\kappa^2) \;\equiv\; 2 \times \textit{Kinetic Distribution } (\beta^2).}$$



14 Energy-Symmetry Law

In RG, every transformation is bidirectional: each change observed by A corresponds to an equal and opposite change observed by B. This reciprocity is the algebraic form of causal continuity, and its geometric expression is the Energy-Symmetry Law.

14.1 Causal Continuity and Energy Symmetry

Theorem 14.1 (Energy Symmetry). *The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the Energy-Symmetry Law:*

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{8}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A = 0$).
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A\to B} = \underbrace{\frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}\left(\beta_B^2 - \beta_A^2\right)}_{\text{Change in Kinetic}}} \tag{9}$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right) \tag{10}$$

From B's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)\right) = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right) \tag{11}$$

Summing these transfers gives:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{12}$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the Energy-Symmetry Law as a direct consequence of the closed geometry. □

14.2 The Specific Energy Transfer (ΔE):

This is the projectional energy difference between states, equivalent to the Total Relational Shift Q, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the **sum of the changes** in the potential and kinetic energy budgets:

$$\Delta E_{A\to B} = \Delta U_{A\to B} + \Delta K_{A\to B} = \frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right) + \frac{1}{2}\left(\beta_B^2 - \beta_A^2\right) \tag{13}$$

It is this quantity, ΔE, that is conserved and must balance to zero in any closed system.

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \tag{14}$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \tag{15}$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

14.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ originate from the quadratic nature of the energy budgets in RG. The energetic significance of a state is proportional to the **square** of its geometric projection. This is the unavoidable consequence of relational carriers closure condition (amplitude^2 + phase^2 = 1). By using only amplitudes (β^2 and κ^2) we operating with half's relational budgets of S^1 and S^2 carriers.

The individual energy budgets are:

- **Specific Potential Energy Budget:** $U/E_0 \propto -\frac{1}{2}\kappa^2$
- **Specific Kinetic Energy Budget:** $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the *changes* in these individual budgets.

14.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 14.2 (Universal Speed Limit)**.** *The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.*

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A\to B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B\to A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{16}$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. □

14.5 Single-Axis Energy Transfer and the Nature of Light

Theorem 14.3 (Single-Axis Transformation Principle)**.** *For light, the kinematic projection amplitude reaches its full extent:*

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

This means that all transformation of the relational resource occurs along a ***single X axis*** $\Rightarrow \beta = 1$*. The orthogonal Y axis is absent* $\Rightarrow \beta_Y = 0$*, and the total resource of transformation is entirely expressed on one geometric component.*

Proof. For massive bodies, the Energy–Symmetry Law (Section 14) partitions the transformation resource equally between orthogonal axes. The specific binding energy invariant for orbital motion is therefore:

$$W_{\text{mass}} = \frac{1}{2}\left(\kappa^2 - \beta^2\right), \tag{17}$$

where the factor $\frac{1}{2}$ arises from this dual-axis distribution (Theorem **??**). This invariant is conserved for closed systems and reduces to the classical Keplerian energy under the closure condition $\kappa^2 = 2\beta^2$ (Corollary 13.6).

Now consider light. By the Single–Axis Transformation Principle (Theorem **??**), its kinematic projection saturates the carrier:

$$\beta = 1 \quad \Rightarrow \quad \beta_Y = 0.$$

Geometrically, this collapses the relational architecture:

- The Y-axis (Phase component) vanishes entirely, as $\beta_Y = 0$ indicates no internal state evolution.
- No rest frame exists for self–centering (Section 11), eliminating the dual–framing that justifies the $\frac{1}{2}$ partitioning.
- The entire transformation resource concentrates on the single X-axis (Amplitude component).

Consequently, the energy invariant for photon interactions with a massive body (projection κ) must exclude the partitioning factor:

$$W_\gamma = \kappa^2 - \beta^2 = \kappa^2 - 1. \tag{18}$$

This is verified at asymptotic infinity ($\kappa = 0$):

$$\begin{aligned} W_{\text{mass}} &= \tfrac{1}{2}(0^2 - 0^2) = 0, \\ W_\gamma &= 0^2 - 1^2 = -1. \end{aligned}$$

The value $W_\gamma = -1$ corresponds to the full rest energy cost of creating a photon at infinity - whereas the $\frac{1}{2}$ factor would erroneously yield $-\frac{1}{2}$, violating energy conservation.

For a concrete test case, consider a photon interacting with a massive body where $\kappa_M = 0.4$:

$$\begin{aligned} W_{\text{mass}} &= \tfrac{1}{2}\left(0.4^2 - 1^2\right) = \tfrac{1}{2}\left(-0.84\right) = -0.42, \\ W_\gamma &= 0.4^2 - 1^2 = -0.84 = 2 \times W_{\text{mass}}. \end{aligned}$$

Thus, the gravitational effect on light is twice that on massive particles - matching the observed factor in light deflection.

The general energy potential follows directly: for light, the full geometric resource expresses unpartitioned along the single axis:

$$\Phi_\gamma = \kappa^2 c^2,$$

while massive bodies retain the partitioned form:

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This factor-of-2 is the geometric signature of axis count in relational space. No auxiliary approximations or background structures are introduced; the result emerges from the topological constraint $\beta_Y = 0$ applied to the closed carrier S^1. □

Summary

Energy Symmetry Law $\Delta E_{A\to B} + \Delta E_{B\to A} = 0$ - universal relational energy bookkeeping.
The Speed of Light is the boundary beyond which the energy symmetry law breaks down.
Causality is built-in feature of Relational Geometry.
Light has no rest frame. The disappearance of the Phase component (Y-axis $\beta_Y = 0$) concentrates the entire transformation resource on a single geometric component. This eliminates the $\frac{1}{2}$ partitioning factor, yielding $\boxed{\Phi_\gamma = \kappa^2 c^2}$ and explaining why light experiences exactly twice the geometric effect of massive bodies.
This explains the experimentally verified factor of 2 in gravitational lensing and Shapiro delay, traditionally requiring full General Relativity to derive.

IMPORTANT:

This document must be read **literally**. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (*absolute energies, external backgrounds, hidden containers*) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

15 Operational Independence and the Role of Constants

A frequent objection from conventional frameworks is that any model employing the Schwarzschild radius (R_s) must implicitly depend on the Newtonian constants G and m_0 (mass) as fundamental inputs. This objection assumes that mass is the primary ontological entity and geometry is a secondary derivative.

We demonstrate that this is incorrect. In the WILL RG framework, G and m_0 are not physical inputs but *output calibration tools*. The parameters κ and β are operationally measurable geometric intensities, and R_s is a system scale derived directly from the phase interactions of light.

15.1 Operational Measurability of Relational Projections

Theorem 15.1 (Operational Measurability)**.** *The relational projections are encoded directly in the combined phase interactions of light (spectroscopy) and are operationally independent of G, c, or m_0.*

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Proof. **Step 1: The Raw Observable (τ_W).** Spectroscopy does not measure "gravitational potential" or "kinematic velocity" as separate isolates; it measures the total accumulated phase difference between the source and the observer. We define the **Relational Spacetime Factor** τ_W as the inverse of the total measured redshift product:

$$\tau_W \equiv \frac{1}{Z_{\text{tot}}} = \frac{1}{(1 + z_{\text{obs}})}. \tag{19}$$

In the WILL framework, this single observable represents the product of the internal phase projections of the carriers S^2 and S^1:

$$\tau_W = \underbrace{\kappa_X}_{\text{Gravitational Phase}} \cdot \underbrace{\beta_Y}_{\text{Kinematic Phase}} = \sqrt{1-\kappa^2}\sqrt{1-\beta^2}. \tag{20}$$

Step 2: Exact Relational Identity. We reject weak-field approximations. The exact relationship between the signal τ_W and the structural norm Q is given by the algebraic expansion of the phase product:

$$\tau_W^2 = (1-\kappa^2)(1-\beta^2) = 1 - (\kappa^2 + \beta^2) + \kappa^2\beta^2. \tag{21}$$

Substituting $Q^2 = \kappa^2 + \beta^2$, we obtain the rigorous link between the optical observable and the geometric state:

$$\tau_W^2 = 1 - Q^2 + \kappa^2\beta^2. \tag{22}$$

This identity demonstrates that the optical signal contains the complete information about the system's structural state, measurable without prior knowledge of mass. □

15.2 Algebraic Determination of System Scale

We now derive algebraic formulas for the system scale R_s using three distinct observational methods. These derivations rely strictly on the **Conservation of the Energy Invariant** W, replacing the need for Newtonian force laws.

15.2.1 Method A: Differential (Two-Point Method)

This method is suitable when the orbital period is unknown, but the trajectory can be traced geometrically.

Theorem 15.2 (Two-Point Schwarzschild Scale). *Given measurements of geometric position (r) and kinematic intensity (β) at two arbitrary points along a trajectory:*

- *Radii: r_1, r_2 (from astrometry)*
- *Intensities: β_1, β_2 (from de-projected spectral line widths or proper motion)*

the Schwarzschild radius is:

$$\boxed{R_s = \frac{r_1 r_2}{r_2 - r_1}(\beta_1^2 - \beta_2^2).} \tag{23}$$

Proof. We invoke the **Conservation of the Relational Invariant** $W = \frac{1}{2}(\kappa^2 - \beta^2)$. This law states that for any closed system, the specific energy difference between the potential and kinematic projections is constant throughout the orbit.

Therefore, at any two points 1 and 2:

$$\frac{1}{2}(\kappa_1^2 - \beta_1^2) = \frac{1}{2}(\kappa_2^2 - \beta_2^2). \tag{24}$$

Rearranging to group the projections:

$$\kappa_1^2 - \kappa_2^2 = \beta_1^2 - \beta_2^2. \tag{25}$$

Substituting the field identity $\kappa^2 = R_s/r$:

$$\frac{R_s}{r_1} - \frac{R_s}{r_2} = \beta_1^2 - \beta_2^2. \tag{26}$$

Factoring out the scale parameter R_s:

$$R_s\left(\frac{r_2 - r_1}{r_1 r_2}\right) = \beta_1^2 - \beta_2^2. \tag{27}$$

Solving for R_s:

$$R_s = \frac{\beta_1^2 - \beta_2^2}{\frac{r_2 - r_1}{r_1 r_2}} = \frac{r_1 r_2}{r_2 - r_1}(\beta_1^2 - \beta_2^2). \tag{28}$$

This formula extracts the "mass" scale purely from geometric gradients. □

15.2.2 Method B: Geometric Resonance (Balance Point Method)

At the specific orbital phase $O_o = \arccos(-e)$, the system passes through its geometric balance point where the instantaneous radius r equals the semi-major axis a. At this unique phase, the closure condition $\kappa^2 = 2\beta^2$ is satisfied instantaneously.

Theorem 15.3 (Balance Point Formula)**.** *Given the geometric scale a and the total light signal τ_W measured at the balance point ($r = a$):*

$$\boxed{R_s = \frac{a}{2}\left(3 - \sqrt{1 + 8\tau_W^2(O_o)}\right).} \tag{29}$$

Proof. **Step 1: Spacetime Factor Expansion.** At the balance point ($r = a$), the closure condition implies $\kappa^2 = R_s/a$ and $\beta^2 = R_s/2a$. The observable τ_W is:

$$\tau_W^2 = (1 - \kappa^2)(1 - \beta^2) = \left(1 - \frac{R_s}{a}\right)\left(1 - \frac{R_s}{2a}\right). \tag{30}$$

Step 2: Exact Geometric Solution. Expanding the product:

$$\tau_W^2 = 1 - \frac{R_s}{2a} - \frac{R_s}{a} + \frac{R_s^2}{2a^2} = 1 - \frac{3R_s}{2a} + \frac{R_s^2}{2a^2}. \tag{31}$$

Multiplying by $2a^2$ to clear denominators and rearranging into standard quadratic form ($Ax^2 + Bx + C = 0$):

$$R_s^2 - 3aR_s + 2a^2(1 - \tau_W^2) = 0. \tag{32}$$

Solving for R_s using the quadratic formula ($x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a_{coef}}$):

$$R_s = \frac{3a \pm \sqrt{(3a)^2 - 4(1)(2a^2(1 - \tau_W^2))}}{2}. \tag{33}$$

Simplifying the term under the radical:

$$9a^2 - 8a^2(1 - \tau_W^2) = 9a^2 - 8a^2 + 8a^2\tau_W^2 = a^2(1 + 8\tau_W^2). \tag{34}$$

Thus:

$$R_s = \frac{3a \pm a\sqrt{1 + 8\tau_W^2}}{2} = \frac{a}{2}\left(3 \pm \sqrt{1 + 8\tau_W^2}\right). \tag{35}$$

For stable orbits, we must select the negative root. □

Remark 15.4 (Light Separation)**.** *This unit-less factor in brackets is:*

- ***Potential projection at semi-major axis:*** $\kappa^2 = \frac{1}{2}(3 - \sqrt{1 + 8\tau_W^2(O_o)})$

A unique way to separate gravitational part from the light signal.

15.2.3 Method C: Instantaneous (Arbitrary Phase Method)

Most generally, if the orbital geometry (a) is known, the scale R_s can be derived from a **single epoch** observation at any arbitrary radius r_o.

Theorem 15.5 (Arbitrary Phase Formula)**.** *Given the orbital geometry (a, r_o) and the single light observable τ_{Wo}:*

$$\boxed{R_s = \frac{r_o}{2(2a - r_o)}\left(4a - r_o - \sqrt{(4a - r_o)^2 - 8a(2a - r_o)(1 - \tau_{Wo}^2)}\right).} \tag{36}$$

Proof. **Step 1: Relational Invariant Form.** We start with the energy invariant relation $W = \frac{1}{2}(\kappa^2 - \beta^2)$. For a bound orbit, $W = R_s/4a$. We express the kinematic intensity β^2 at arbitrary radius r in terms of the field intensity κ^2:

$$\frac{1}{2}(\kappa^2 - \beta^2) = \frac{R_s}{4a} \implies \beta^2 = \kappa^2 - \frac{R_s}{2a}. \tag{37}$$

Substituting $\kappa^2 = R_s/r$:

$$\beta^2 = R_s\left(\frac{1}{r} - \frac{1}{2a}\right). \tag{38}$$

Step 2: Constraint via Observables. We substitute the expressions for κ^2 and β^2 into the exact observable constraint $\tau_{Wo}^2 = (1 - \kappa^2)(1 - \beta^2)$:

$$\tau_{Wo}^2 = \left(1 - \frac{R_s}{r}\right)\left(1 - R_s\left[\frac{1}{r} - \frac{1}{2a}\right]\right). \tag{39}$$

Let $A = \frac{1}{r}$ and $B = \frac{1}{r} - \frac{1}{2a}$. The equation becomes:

$$\tau_{Wo}^2 = (1 - AR_s)(1 - BR_s) = 1 - (A + B)R_s + ABR_s^2. \tag{40}$$

Rearranging into quadratic form:

$$(AB)R_s^2 - (A + B)R_s + (1 - \tau_{Wo}^2) = 0. \tag{41}$$

Step 3: Coefficient Expansion. We compute the coefficients explicitly:

$$AB = \frac{1}{r}\left(\frac{2a-r}{2ar}\right) = \frac{2a-r}{2ar^2} \tag{42}$$

$$A + B = \frac{1}{r} + \frac{2a-r}{2ar} = \frac{2a+2a-r}{2ar} = \frac{4a-r}{2ar} \tag{43}$$

Multiplying the entire quadratic equation by $2ar^2$ to clear denominators:

$$(2a-r)R_s^2 - r(4a-r)R_s + 2ar^2(1-\tau_{Wo}^2) = 0. \tag{44}$$

Step 4: Solution. Solving for R_s:

$$R_s = \frac{r(4a-r) \pm \sqrt{r^2(4a-r)^2 - 4(2a-r)(2ar^2)(1-\tau_{Wo}^2)}}{2(2a-r)}. \tag{45}$$

Factoring out r^2 from the radical term $\sqrt{r^2(\ldots)} = r\sqrt{(\ldots)}$:

$$R_s = \frac{r}{2(2a-r)}\left((4a-r) \pm \sqrt{(4a-r)^2 - 8a(2a-r)(1-\tau_{Wo}^2)}\right). \tag{46}$$

For stable orbits, we select the negative root, yielding the exact algebraic link between the observed light phase and the system's geometric scale. □

15.3 The Role of G as Translation Constant

The presence of R_s in these formulas does not imply a dependency on mass m_0 or the constant G. The objection that $R_s = 2Gm_0/c^2$ makes G fundamental rests on a categorical error: it mistakes a unit conversion factor for a physical source.

Theorem 15.6 (Constants as Converters)**.** *In WILL RG, G and m_0 are derived calibration tools used to translate geometric scales into legacy units.*

Proof. The operational procedure is strictly geometric:

1. **Measure:** Light phase τ_W (dimensionless) via spectroscopy.
2. **Measure:** Geometric scale r (meters/AU) via astrometry.
3. **Calculate:** System Scale $R_s = f(r, \tau_W)$ via Theorems 15.2, 15.3, or 15.4.

The physical calculation ends here. The system is fully defined. If, and only if, one wishes to interface with legacy catalogues, one employs the **unit converter**:

$$m_0 \equiv \frac{R_s c^2}{2G}. \tag{47}$$

The constant G describes the units we use (kilograms vs. meters), not the physics of the system. □

Remark 15.7 (Historical Artifact)**.** *The kilogram is a human convention. In WILL RG, the fundamental quantity is the dimensionless ratio $\kappa^2 = R_s/r$, which encodes the energy density ratio $\rho/\rho_{\max}$. The "mass" m_0 is a secondary bookkeeping device.*

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15.4 Summary

We have derived three algebraically distinct formulas for R_s, each optimized for different observational scenarios:

1. **Two-point** Requires two velocity measurements
2. **Balance point** Simplified form at special phase
3. **Arbitrary phase** Works for any single epoch

All formulas are:

- **Operationally independent** of G and m_0
- **Non-circular** (inputs are direct observables)
- **Algebraically exact** (no approximations beyond Keplerian closure)

The operational input is always $\tau_W = 1/[(1+z)(1+z_D)]$, a dimensionless quantity directly measurable from spectroscopy. This demonstrates that WILL RG formulas are empirically grounded and do not rely on hidden assumptions about gravitational constants.

16 Classical Keplerian Energy and Minkowski Interval as Ontologically Heavy Energy–Symmetry Approximation

A striking consequence of the Energy–Symmetry Law (Section 14) emerges when analysing the total specific orbital energy. Since energy in RG is defined *relationally, as the measure of difference between two states*, we naturally select these two states (e.g., the surface of the central body 'A' and the orbit 'B') as the reference points for the potential and kinetic energy budgets. Under this relational approach, the total specific orbital energy (potential + kinetic, per unit rest mass) naturally appears in a form **structurally identical to the Minkowski interval**.

16.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{48}$$

$$K = \frac{1}{2}m\frac{GM_\oplus}{a}. \tag{49}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{50}$$

16.2 Projection Parameters and Minkowski-like Form

Remark 16.1. *Here we explicitly define our projections using classical language to ease understanding for readers not yet familiar with WILL RG. For more explicit comparison and clearer derivations, we adopt standard Newtonian notation and define the projection parameters κ^2 and β^2 through their legacy equivalents involving G, M, and c. This is purely a pedagogical choice—the projections themselves remain dimensionless geometric quantities independent of these conventional constants.*

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{51}$$

$$\beta_{\rm orbit}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{52}$$

Substituting into (50) gives the exact identity:

$$\frac{E_{\rm tot}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\rm orbit}^2\big). \tag{53}$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\rm orbit}$, then

$$\frac{E_{\rm tot}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{54}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\kappa^2$ and $K/E_0 = \frac{1}{2}\beta^2$ as *budgets*. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer ΔE is the signed sum of budget *changes*.

16.3 Physical Interpretation

In classical derivations, (50) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG, (53) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface, orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While this framework refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

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Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in Energy–Symmetry difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This re framing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

17 Lagrangian and Hamiltonian as Ontologically Corrupted RG Approximations

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational Energy-Symmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms are not fundamental principles but ontologically "dirty" approximations of the relational WILL framework. By collapsing the **two-point relational structure** into a single-point description, classical mechanics **lose ontological clarity** and gain **mathematically inflated formalism** increasing the computational cost.

17.1 Definitions of Parameters

We consider a central mass M and a test mass m. The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2\dot{\phi}^2$ — total squared orbital speed at B.
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B.
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A.

17.2 The Relational Lagrangian

Instead of a relational energy, we define the *clean relational Lagrangian* L_{rel}, which represents the kinetic budget at point B relative to the potential budget at point A:

$$L_{\text{rel}} = T(B) + U(A) = \tfrac{1}{2}m\left(\dot{r}_B^2 + r_B^2\dot{\phi}^2\right) + \frac{GMm}{r_A}. \tag{55}$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \tfrac{1}{2}\left(\beta_B^2 + \kappa_A^2\right). \tag{56}$$

This two-point, relational form is the clean geometric statement.

17.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{57}$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

17.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \tag{58}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \tag{59}$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \tfrac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \frac{GMm}{r}. \tag{60}$$

17.4.1 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \tfrac{1}{2}mv^2 + \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 + \kappa^2\big), \tag{61}$$

$$H = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 - \kappa^2\big). \tag{62}$$

Here the "+" or "−" signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both are ontologically "dirty" approximations. The clean relational law, involving distinct points A and B, is collapsed into a local, one-point description. This shows that Hamiltonian and Lagrangian are just needlessly overcomplicated approximations that lose in ontological integrity.

Remark 17.1 (Mathematical Status: Groupoid vs. Group)**.** *From a category-theoretic perspective, the relational transitions $\Delta E_{A\to B}$ form a* ***Groupoid*** *structure, where operations are defined only between specific connected states. Standard field theories (Hamiltonian/Lagrangian) rely on the collapse of this structure into a* ***Group*** *acting on a global manifold (Quotienting). Thus, WILL operates at (Groupoid) level before the degeneration into global fields occurs.*

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2. For readers accustomed to standard frameworks, the following translation rules may help:

1. *General Relativity (metric form):*

$$\kappa_X \hat{=} \sqrt{-g_{tt}} \quad \text{(static spacetimes)}, \qquad \beta \hat{=} \frac{\|u^\mu_{\text{spatial}}\|}{u^t c}.$$

2. *Canonical mechanics (Lagrangian/Hamiltonian):* Quantities such as $p_i = \partial L/\partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are computational *shadows*, used only for legacy calculations and physically redundant.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

17.5 Third Ontological Collapse: Derivation of Newton's Third Law

We now demonstrate that Newton's Third Law, like the Lagrangian and Hamiltonian, is not a fundamental principle but another "degenerate shadow" of the WILL framework. It arises as a necessary mathematical consequence of the same ontological collapse — forcing a two-point relational law into a single-point, instantaneous formalism.

Theorem 17.2 (Newton's Third Law as a Degenerate Consequence)**.** *The Energy–Symmetry Law ($\Delta E_{A\to B} + \Delta E_{B\to A} = 0$) mathematically necessitates Newton's Third Law ($\vec{F}_{AB} = -\vec{F}_{BA}$) in the classical, non-relativistic limit where the two-point relational energy budget is collapsed into a single-point potential function $U(\vec{r})$.*

Proof. We begin with the foundational Energy–Symmetry Law (Section 14), the principle of causal balance for state transitions:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

In the classical, non-relativistic limit, this two-point relational law is "ontologically corrupted" into a single-point potential energy function, U. This function is assumed to depend only on the relative positions of the two interacting entities, A and B:

$$U = U(\vec{r}) \quad \text{where} \quad \vec{r} = \vec{r}_B - \vec{r}_A.$$

This $U(\vec{r})$ is the classical approximation of the system's relational energy budget. In this collapsed formalism, the force $\vec{F}$ is defined as the negative gradient of this potential.

(1) Force on B by A ($\vec{F}_{AB}$): This force is found by taking the gradient with respect to B's coordinates:

$$\begin{aligned}
\vec{F}_{AB} &= -\nabla_B U(\vec{r}_B - \vec{r}_A) && (63)\\
&= -\left(\frac{dU}{d\vec{r}}\right)\cdot\left(\frac{\partial\vec{r}}{\partial\vec{r}_B}\right) && (64)\\
&= -\nabla U(\vec{r})\cdot(\mathbf{I}) && (65)\\
&= -\nabla U(\vec{r}) && (66)
\end{aligned}$$

(2) Force on A by B ($\vec{F}_{BA}$): This force is found by taking the gradient with respect to A's coordinates:

$$\begin{aligned}
\vec{F}_{BA} &= -\nabla_A U(\vec{r}_B - \vec{r}_A) && (67)\\
&= -\left(\frac{dU}{d\vec{r}}\right)\cdot\left(\frac{\partial\vec{r}}{\partial\vec{r}_A}\right) && (68)\\
&= -\nabla U(\vec{r})\cdot(-\mathbf{I}) && (69)\\
&= +\nabla U(\vec{r}) && (70)
\end{aligned}$$

(3) Conclusion: By direct comparison of the results, we find:

$$\vec{F}_{AB} = -\nabla U(\vec{r}) \quad \text{and} \quad \vec{F}_{BA} = +\nabla U(\vec{r}).$$

Therefore, it is a mathematical tautology of the collapsed formalism that:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

This completes the proof. Newton's Third Law is not an independent physical axiom, but the built-in mathematical consequence of approximating the Energy Symmetry Law with a single potential function. The law of "equal and opposite forces" is revealed to be a degenerate approximation of the more fundamental, generative law of Relational Geometry. □

18 Substantialism vs. Relationalism

18.1 No-Go Theorem for Fundamental One-Point Dynamics

Foundational Assumptions

Definition 18.1 (Self–Centering)**.** *Each observer defines itself as the relational origin:*

$$(\beta, \kappa) = (0, 0).$$

This is an ontological definition of the observer's state, not a coordinate choice.

Definition 18.2 (Relational Reciprocity)**.** *The only invariant quantity between two observers is the norm of the Total Relational Shift:*

$$Q^2 = \beta^2 + \kappa^2.$$

Reciprocity is invariance of this norm under the self–centering operation performed independently by each observer.

Definition 18.3 (Absence of Background)**.** *There exists no shared background structure: no global state space, no external time parameter, and no common coordinate system simultaneously hosting the states of distinct observers.*

Definition 18.4 (Operationality)**.** *A physical quantity is admissible only if it is either:*

1. *directly measurable, or*
2. *algebraically derivable from measurable quantities,*

without invoking non-observable auxiliary structures.

Target Class

Definition 18.5 (One–Point Dynamics)**.** *By one-point dynamics we mean any formulation in which:*

- *a system is represented by a state x in a global space,*
- *physical law is given by a local evolution rule*

$$\dot{x} = F(x) \quad \text{or} \quad \delta x = \mathcal{L}(x),$$

- *temporal evolution is defined as transitions of the same point through neighboring states.*

This includes Lagrangian, Hamiltonian, field-theoretic, and metric-based formulations.

No-Go Result

Theorem 18.6 (No-Go for Fundamental One–Point Dynamics)**.** *Under Self–Centering, Relational Reciprocity, Absence of Background, and Operationality, no one-point dynamical formulation can be simultaneously:*

1. *relationally reciprocal,*
2. *operationally well-defined,*
3. *background-independent,*
4. *ontologically minimal.*

Therefore one-point dynamics cannot be fundamental.

Proof. A one-point evolution law requires identification of a system across multiple states, comparison of "earlier" and "later" states, and embedding of these states into a common structure in order to define $\dot{x}$ or δx.

By Self–Centering, an observer is always at $(\beta, \kappa) = (0, 0)$ in its own relational description. There is no operationally available notion of an observer's own worldline as a sequence of distinct states inside a shared arena. Any attempt to define such a sequence introduces an external state-labeling structure, contradicting Absence of Background.

By Relational Reciprocity, only the scalar norm Q is invariant under mutual self–centering. Directional quantities required by one-point dynamics (such as signed increments, tangent vectors, gradients, forces, or local generators of change) are not reciprocity-invariant objects. Hence they cannot be fundamental relational observables.

By Operationality, time derivatives and infinitesimal variations are inadmissible unless they are directly measurable or algebraically derivable from measurable quantities without additional structure. However, defining $\dot{x}$ or δx presupposes non-observable distinctions between arbitrarily close states and thus introduces auxiliary structure beyond the measurable invariants.

To restore a well-defined one-point dynamics one must add at least one of the following: a global time parameter, a shared state manifold, a persistent identity map between "the same" system at different moments, or a background metric/symplectic structure that defines local generators. Each of these additions violates Absence of Background and breaks the relational closure enforced by Self–Centering and Reciprocity.

Therefore one-point dynamics necessarily violates at least one of the stated foundational assumptions and cannot serve as a fundamental description. □

Corollary 18.7 (Constraint–Based Fundamental Law)**.** *Under the same assumptions, admissible fundamental laws must be algebraic and relational: they constrain mutual states through reciprocity-invariant quantities (such as Q and closure relations) rather than prescribing one-point evolution.*

18.2 Theorem of Minimality for Relational Constraint Laws

Purpose

We prove that even if one-point dynamics is permitted as a non-fundamental descriptive tool, it is strictly non-minimal. Relational constraint-based formulations are provably optimal with respect to ontological and operational economy.

Primitive Count and Ontological Cost

Definition 18.8 (Ontological Primitive)**.** *An ontological primitive is any irreducible structure that must be assumed in order to formulate a physical law and that cannot be eliminated by algebraic redefinition. Examples include background time, global state manifolds, local generators, metrics, or identity maps between states.*

Definition 18.9 (Ontological Cost)**.** *The ontological cost of a formulation is the minimal number of independent primitives required to state its laws in a closed and operationally meaningful form.*

Minimal Relational Formulation

Lemma 18.10 (Primitive Content of Relational Constraint Laws)**.** *A relational constraint-based formulation requires only:*

- *self–centering of observers,*
- *relational reciprocity,*
- *algebraic invariants between relational projections.*

No background structures, generators, or evolution parameters are required.

Proof. Relational constraints relate observable projections directly through algebraic identities such as

$$Q^2 = \beta^2 + \kappa^2, \qquad \kappa^2 = 2\beta^2.$$

These quantities are dimensionless, operationally measurable, and invariant under self–centering. No additional structure is needed to define or apply such relations. □

Primitive Content of One–Point Dynamics

Lemma 18.11 (Primitive Inflation in One–Point Dynamics)**.** *Any one-point dynamical formulation requires the introduction of at least one additional ontological primitive beyond those of relational constraints.*

Proof. To define a local evolution law $\dot{x} = F(x)$ or $\delta x = \mathcal{L}(x)$, one must introduce:

- a global space of states hosting x,
- a rule identifying the same system across multiple states,
- an ordering parameter distinguishing "before" and "after",
- a generator defining local change.

At least one of these structures is irreducible and cannot be derived from relational invariants alone. Hence one-point dynamics necessarily increases ontological cost. □

The Minimality Theorem

Theorem 18.12 (Strict Minimality of Relational Constraint Laws)**.** *Among all formulations capable of reproducing the same observable predictions, relational constraint-based laws have strictly lower ontological cost than any one-point dynamical formulation.*

Proof. By the first lemma, relational constraint laws achieve closure using only self–centering, reciprocity, and algebraic invariants. By the second lemma, any one-point dynamical formulation requires at least one additional primitive not present in the relational scheme. Therefore the ontological cost of one-point dynamics is strictly greater. Since both classes of formulations can reproduce the same empirical relations, the relational constraint formulation is minimal. □

Operational Consequence

Corollary 18.13 (Redundancy of Fundamental Dynamics)**.** *Any one-point dynamical law is either:*

- *empirically redundant with respect to an underlying relational constraint, or*
- *dependent on surplus ontological structure.*

In neither case can it be fundamental.

Summary

Minimality Result

$$\boxed{\text{Relational Constraints} \;<\; \text{One–Point Dynamics}}$$

The inequality denotes strict ontological and operational minimality. Dynamical formalisms persist only as descriptive shadows of a more economical relational structure.

19 General Consequence

Substantialism as Bad philosophy, in RG sense, has three measurable effects:

1. Inflated Formalism: Equations multiply to compensate for ontological error.
2. Loss of Transparency: Physical meaning becomes hidden behind coordinate dependencies.
3. Empirical Fragmentation: Each domain (cosmology, quantum, gravitation) requires separate constants.

By contrast, Relationalism as good philosophy-**epistemic hygiene**-enforces relational closure and yields simplicity through necessity, not through approximation.

In short:

$$\boxed{\textbf{Bad philosophy creates complexity} \quad \textbf{Good philosophy reveals geometry.}}$$

Daring Remark

The historical escalation of mathematical complexity in physics did not reveal deeper reality - it compensated for a philosophical mistake. Once the ontological symmetry is restored, Nature's laws reduce to algebraic self-consistency.

$$\boxed{\text{Bad Philosophy}} \Rightarrow \boxed{\text{Ontological Duplication}} \Rightarrow \boxed{\text{Mathematical Inflation}}$$

$$\boxed{\textbf{Mathematical complexity is the symptom of philosophical negligence.}}$$

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20 Relational Orbital Mechanics (R.O.M.) Without Mass or G

Thesis

Orbital dynamics requires no mass, no G, no metric, and no spacetime geometry. All observable orbital structure follows from two directly measurable frequency projections:

κ (gravitational projection from redshift), β (kinematic projection from Doppler).

Everything else is algebra.

20.1 Two Operational Pathways

Depending on the available observational data, the framework offers two distinct operational pathways:

- **Path 1: Verification (Full Data Available).** Used when both the central potential (redshift z) and the orbital kinematics (β, e) are measurable independently (e.g., Mercury/Sun). Here, we calculate δ_p from inputs and compare the predicted eccentricity with observation to validate the theory.
- **Path 2: Reconstruction (Partial Data Available).** Used for distant systems (e.g., Star S2, Exoplanets) where the central potential is unknown or entangled with Doppler shifts. Here, we use the observed shape (e) and velocity (β) to **reconstruct** the hidden potential depth κ_p via geometric compatibility.

20.2 Derivation of Relational Eccentricity

Geometric eccentricity is not a free parameter but a measure of the energetic deviation from the circular equilibrium state ($\delta = 1$).

Theorem 20.1 (Geometric Eccentricity)**.** *For a closed orbital system governed by the projection invariants of WILL Relational Geometry, the orbital eccentricity e is strictly determined by the closure factor at periapsis, δ_p:*

$$e = \frac{1}{\delta_p} - 1 = \frac{1 - \delta_p}{\delta_p}. \tag{71}$$

Proof. Instead of relying on classical force laws, we derive this relation directly from the conservation of the two fundamental projection invariants of the WILL framework:

1. **Energy Projection Invariant (Binding Energy):** $W = \frac{1}{2}(\kappa^2 - \beta^2) = \frac{E}{E_o} =$ const.
2. **Angular Projection Invariant:** $h = r\beta =$ const (at turning points).

Consider the two turning points of a closed orbit: periapsis (p) and apoapsis (a). By operational definition of the shape parameter e, the relation between radii is determined by the geometric range:

$$r_a = r_p \left(\frac{1+e}{1-e}\right). \tag{72}$$

Step 1: Relational Mapping. Using the angular invariant h (implying $\beta \propto 1/r$) and the field definition $\kappa^2 \propto 1/r$, we express the apoapsis projections in terms of the periapsis values:

$$\beta_a^2 = \left[\beta_p \left(\frac{r_p}{r_a}\right)\right]^2 = \beta_p^2 \left(\frac{1-e}{1+e}\right)^2, \tag{73}$$

$$\kappa_a^2 = \kappa_p^2 \left(\frac{r_p}{r_a}\right) = \kappa_p^2 \left(\frac{1-e}{1+e}\right). \tag{74}$$

Note: Kinematic projection scales quadratically with the radius ratio, while potential projection scales linearly.

Step 2: Energy Balance. Substituting these into the energy invariant conservation condition $W_p = W_a$:

$$\frac{1}{2}(\kappa_p^2 - \beta_p^2) = \frac{1}{2}(\kappa_a^2 - \beta_a^2).$$

Canceling the factor $\frac{1}{2}$ and substituting the mappings from Step 1:

$$\kappa_p^2 - \beta_p^2 = \kappa_p^2 \left(\frac{1-e}{1+e}\right) - \beta_p^2 \left(\frac{1-e}{1+e}\right)^2.$$

Rearranging to group potential terms (κ) on the left and kinematic terms (β) on the right:

$$\kappa_p^2 \left[1 - \frac{1-e}{1+e}\right] = \beta_p^2 \left[1 - \left(\frac{1-e}{1+e}\right)^2\right].$$

Step 3: Algebraic Reduction. Expanding the terms in brackets:

$$\text{LHS bracket (κ term):} \quad 1 - \frac{1-e}{1+e} = \frac{(1+e)-(1-e)}{1+e} = \frac{2e}{1+e}.$$

$$\text{RHS bracket (β term):} \quad 1 - \frac{(1-e)^2}{(1+e)^2} = \frac{(1+e)^2-(1-e)^2}{(1+e)^2} = \frac{4e}{(1+e)^2}.$$

Substituting back into the balance equation:

$$\kappa_p^2 \left(\frac{2e}{1+e}\right) = \beta_p^2 \left(\frac{4e}{(1+e)^2}\right).$$

Dividing both sides by $2e$ and multiplying by $(1+e)^2$:

$$\kappa_p^2(1+e) = 2\beta_p^2.$$

This yields geometric identity for bound orbits:

$$2\beta_p^2 = \kappa_p^2(1+e). \tag{75}$$

Step 4: Connection to Closure. Recall the definition of the closure factor at periapsis:

$$\delta_p = \frac{\kappa_p^2}{2\beta_p^2}.$$

Substituting Eq.(75) into this definition:

$$\delta_p = \frac{\kappa_p^2}{\kappa_p^2(1+e)} = \frac{1}{1+e}.$$

Solving for e, we obtain the stated result:

$$\boxed{e = \frac{1}{\delta_p} - 1 = \frac{2\beta_p^2}{\kappa_p^2} - 1 = 1 - \frac{2\beta_a^2}{\kappa_a^2}}$$

□

Remark 20.2. *This result confirms that eccentricity is strictly a measure of the energetic deviation from the circular equilibrium state ($\delta = 1$), derived entirely from the conservation of relational projections without invoking mass or Newtonian forces.*

SUMMARY

$$e \equiv \frac{1}{\delta_p} - 1 \equiv \frac{2\beta_p^2}{\kappa_p^2} - 1$$

ECCENTRICITY ≡ CLOSURE DEFECT

SPACETIME ≡ ENERGY

20.3 Path 1: Verification on Mercury (Pure Optical Inputs)

We validate the theory using Mercury, utilizing only direct optical measurements (Doppler and Redshift). We implicitly assume **zero knowledge** of the Sun's mass, the gravitational constant G, or the Schwarzschild radius derived from them.

Table 4: Used Inputs

Parameter	Symbol	Value	Source
Mercury Velocity (Perihelion)	v_p	$58,980$ m/s	(?)
Mercury Distance (Perihelion)	r_p	46.0012×10^9 m	(?)
Solar Radius (Nominal)	R_{sun}	6.957×10^8 m	(?)
Solar Gravitational Redshift	z_{sun}	2.1224×10^{-6}	(?)

1. Input: Kinematic Projection (β_p). Radar telemetry directly measures the orbital velocity at perihelion ($v_p \approx 58.98$ km/s). The kinematic projection is simply this velocity normalized by the speed of light:

$$\beta_p = \frac{v_p}{c} \approx 1.967 \times 10^{-4}.$$

$$\beta_p^2 \approx 3.8705094361 \times 10^{-8}$$

2. Input: Potential Projection (κ_p). Instead of deriving potential from mass, we derive it from the **measured gravitational redshift** of the Sun's photosphere, $z_{\text{sun}} \approx 2.1224 \times 10^{-6}$.

Using the S^2 relational carrier geometric omnidirectional scaling law of the potential projection ($\kappa^2 \propto 1/r$), we relate the known potential at the solar physical radius (R_{sun}) to the potential at Mercury's perihelion radius (r_p):

$$\kappa^2(r_p) = \kappa^2(R_{\text{sun}}) \cdot \left(\frac{R_{\text{sun}}}{r_p}\right).$$

Using the redshift relation $\kappa^2(R_{\text{sun}}) = 1 - (1 + z_{\text{sun}})^{-2} \approx 1 - (1 + 2.1224 \cdot 10^{-6})^{-2} \approx 4.2447864862 \cdot 10^{-6}$:

$$\kappa_p^2 \approx \kappa^2(R_{\text{sun}}) \left(\frac{R_{\text{sun}}}{r_p}\right).$$

Using the observed geometric ratio of radii $R_{\text{sun}}/r_p \approx 0.0151235185169$:

$$\kappa_p^2 \approx 4.2447864862 \cdot 10^{-6} \cdot 0.0151235185169 \approx 6.4196107024 \times 10^{-8}$$

3. Calculate Closure Fac tor (δ_p). With both projections determined purely from light measurements:

$$\delta_p = \frac{\kappa_p^2}{2\beta_p^2} = \frac{6.4196107024 \times 10^{-8}}{2 \times 3.8705094361 \times 10^{-8}} \approx 0.829297901025.$$

4. Predict Eccentricity (e). The orbital shape is strictly enforced by the closure defect:

$$e_{\text{pred}} = \frac{1}{\delta_p} - 1 = \frac{1}{0.829297901025} - 1 \simeq 0.205839299441.$$

Conclusion: The predicted eccentricity closely matches (within inputs uncertainty) the observed value ($e \approx 0.2056$). This demonstrates that the orbital geometry is fully determined by the ratio of the central redshift to the orbital Doppler shift, with no reference to mass or G.

20.4 Path 2: Reconstruction of Potentials (Star S2)

For the star S2 orbiting Sgr A*, we assume the closure law holds and use **Path 2** to find the hidden potential of the Black Hole.

1. Inputs (Observed Geometry):

$$e_{\text{obs}} \approx 0.8846, \qquad \beta_p \approx 0.0255\,(7650\,\text{km/s}).$$

2. Reconstruct Potential κ_p: From Eq.75, we invert the logic to find what κ_p **must** be to sustain this orbit:

$$\kappa_p = \beta_p \sqrt{\frac{2}{1 + e_{obs}}} \approx 0.0262691236567$$

(Note: This allows us to calculate the Black Hole's mass/scale $R_s = \kappa_p^2 r_p$ without ever measuring it directly).

Remark 20.3 (On the Nature of Mass)**.** *While κ is the primary dimensionless observable (representing gravitational redshift), the classical concept of "Mass" (M) appears in this framework strictly as a secondary derived quantity. By inverting the definition of the Schwarzschild scale $R_s = \kappa^2 r = \frac{2GM}{c^2}$, we obtain:*

$$M = \frac{c^2 r}{2G}\kappa^2 \tag{76}$$

This demonstrates that physical mass is merely a dimensioned proxy for the geometric curvature intensity κ^2, scaled by the historical constants G and c. In this framework, geometry is fundamental; mass is an artifact of the chosen unit system.

This reconstructed κ_p is then used to predict the precession in the next section.

20.5 The Universal Precession Law: Derivation via Q_a

Derivation of the Phase Shift. The precession is intrinsic to the relational shift. Since Q_a represents the norm of deviation from the Euclidean background, the system accumulates a phase mismatch over every closed cycle. The total angular shift is simply the full orbital phase (2π) scaled by the quadratic intensity of this shift (Q_a^2), normalized by the elliptical geometry factor ($1 - e^2$):

$$\Delta\varphi = \underbrace{2\pi}_{\text{Cycle}} \cdot \underbrace{Q_a^2}_{\text{Intensity}} \cdot \underbrace{\frac{1}{1-e^2}}_{\text{Shape Factor}} = \frac{2\pi\, Q_a^2}{1-e^2}.$$

This yields the general precession law strictly from geometric accumulation:

$$\Delta\varphi = \frac{2\pi\, Q_a^2}{1-e^2}. \tag{77}$$

Substituting Q_a^2, we recover the standard form purely algebraically: We select the semi-major axis a as this reference scale, defining the norm Q_a. At the scale $r = a$, the closure condition ($\kappa^2 = 2\beta^2$) implies the specific distribution of the invariant Schwarzschild scale R_s:

$$\kappa^2(a) = \frac{R_s}{a}, \qquad \beta^2(a) = \frac{R_s}{2a}.$$

Substituting these into the definition of the relational shift norm $Q^2 = \beta^2 + \kappa^2$:

$$Q_a^2 = \frac{R_s}{2a} + \frac{R_s}{a} = \frac{3R_s}{2a}. \tag{78}$$

$$\Delta\varphi = \frac{2\pi}{1-e^2}\left(\frac{3R_s}{2a}\right) = \frac{3\pi R_s}{a(1-e^2)}.$$

20.5.1 Transformation to Periapsis Observables

To eliminate the abstract parameters R_s, a, e in favor of direct observables, we map this expression to the periapsis (p), where interaction is maximal. Using the identities $R_s = \kappa_p^2 r_p$ and $a(1-e^2) = r_p(1+e)$, and the closure relation $(1+e) = 1/\delta_p = 2\beta_p^2/\kappa_p^2$, we arrive at the ultimate operational reduction.

The secular evolution of an orbit is determined solely by the **ratio of the gravitational redshift to the Doppler shift** (red vs. blue) at the point of closest approach:

$$\boxed{\Delta\varphi = \frac{3}{2}\pi\frac{\kappa_p^4}{\beta_p^2}} \tag{79}$$

This equation replaces the complex dynamical derivation with a direct comparison of light interactions.

No differential equations. No metric. Pure algebra of light red vs. blue ratio

This formula reveals that relativistic precession is fundamentally a fourth-order effect in the gravitational projection (κ^4) moderated by the kinematic projection. The factor $\frac{3}{2}\pi$ is purely geometric, emerging from the $S^1 \cdot S^2$ carrier structure.

20.6 Verification A: Mercury (Direct Substitution)

We test the law using the precise operational inputs for Mercury at perihelion.

$$\kappa_p^4 \approx 4.11 \times 10^{-15}, \qquad \beta_p^2 \approx 3.87 \times 10^{-8}.$$

Plugging these values directly into Eq. (79):

$$\Delta\varphi = \frac{3\pi}{2}\left(\frac{4.11 \times 10^{-15}}{3.87 \times 10^{-8}}\right).$$

$$\Delta\varphi \approx 4.712 \times (1.062 \times 10^{-7}) \approx 5.00 \times 10^{-7} \text{ rad/orbit}.$$

This matches the observed **43 arcseconds per century**. The physics is exact.

20.7 Verification B: Strong Field Test (Star S2)

For distant stars like S2 (orbiting Sgr A*), we **reconstruct** the potential depth κ_p purely from the visible orbital shape (e) and velocity (β_p), using the relation derived from Eq.(75):

$$\kappa_p = \beta_p\sqrt{\frac{2}{1+e}}.$$

Data (GRAVITY Collaboration):

$$e \simeq 0.8846, \qquad \beta_p \simeq 0.0255\,(7650 \text{ km/s}).$$

Prediction: 1. Reconstruct Potential: $\kappa_p \approx 0.02627$. 2. Calculate Precession Ratio using Eq. (79):

$$\frac{\kappa_p^4}{\beta_p^2} = \frac{(0.02627)^4}{(0.0255)^2} \approx 7.32 \times 10^{-4}.$$

3. Result:

$$\Delta\varphi = \frac{3\pi}{2}(7.32 \times 10^{-4}) \approx 3.45 \times 10^{-3} \text{ rad} \approx \mathbf{11.85'}.$$

Comparison:

- **WILL prediction:** $\approx 11.89'$.
- **Observed shift:** $12' \pm 1.5'$.

The result lies well within observational uncertainty.

Red vs. Blue

This demonstrates that orbital geometry is encoded in the ratio of gravitational redshift to Doppler shift, revealing that mass is not a fundamental input but a derived bookkeeping quantity

20.8 Case 3: Blind Prediction for S4716 (In Silico Experiment)

Blind Prediction Protocol

This section documents a specific numerical prediction made in November 2025, prior to the observational confirmation of the relativistic precession for the star S4716. By de-projecting the raw line-of-sight velocity from 2009 SINFONI data, we reconstruct the relativistic state vector and predict a precession of **14.80 arcmin/orbit**. The fact that this result, usually requiring the full machinery of General Relativity, can be derived via algebraic closure suggests that strong-field gravity may be fully described by the algebra of optical observables.

20.8.1 Operational Derivation from Observation

To derive the orbital precession of S4716 without relying on the mass of Sgr A* or the Schwarzschild metric, we utilize a scale-invariant reconstruction based on the geometric shape of the orbit (e) and the kinetic intensity (β) derived from optical observables.

20.8.2 Geometric De-projection of Velocity

The observational input is the Line-of-Sight (LOS) velocity measured by SINFONI in 2009 ($v_{LOS} \approx 1690$ km/s) (**?**). Since the orbit is highly inclined ($i \approx 161°$), we de-project this value to find the total velocity v_{total}.

The geometric projection factor $\mathcal{P}$ at true anomaly o is:

$$\mathcal{P}(o) = \left| \sin(i) \frac{\cos(\omega + o) + e\cos(\omega)}{\sqrt{1 + e^2 + 2e\cos(o)}} \right|. \tag{80}$$

Using the orbital parameters for S4716 ($e = 0.756$, $i = 161.13°$, $\omega = 2.25°$) and the calculated phase for 2009 ($o \approx 1.122$ rad), we find $\mathcal{P} \approx 0.25$. The total velocity is:

$$v_{total} = \frac{v_{LOS}}{\mathcal{P}} \approx \frac{1690}{0.25} \approx 6760 \text{ km/s}. \tag{81}$$

This yields the scale-invariant intensity parameter at phase o:

$$\beta_o = \frac{v_{total}}{c} \approx 0.02255. \tag{82}$$

20.8.3 Propagation to Periapsis

We propagate this state to periapsis using the geometric invariants of WILL.

1. Closure Factor at Observation:

$$\delta_o = \frac{1 + e\cos(o)}{1 + e^2 + 2e\cos(o)} = 0.383173140983. \tag{83}$$

2. Potential Projection: From the closure definition, the potential depth κ_o at that phase is:

$$\kappa_o = \beta_o \cdot \sqrt{\delta_o \cdot 2} = 0.0246222202716. \tag{84}$$

3. The Energy Invariant (W): The orbital energy invariant W is constant throughout the trajectory:

$$W = \frac{1}{2}\left(\kappa_o^2 - \beta_o^2\right) = 0.0000488996795431. \tag{85}$$

4. Solution at Periapsis: At periapsis ($o = 0$), the closure factor simplifies to $\delta_p = (1+e)^{-1}$. Solving for the periapsis velocity β_p via invariance of W:

$$\beta_p = \sqrt{\frac{W}{\delta_p - 0.5}} = 0.0265298837499. \tag{86}$$

This yields $\beta_p \approx 0.0265$ ($v_p \approx 7956$ km/s), derived independent of mass models.

20.8.4 Prediction

With the derived periapsis intensity β_p and the corresponding potential $\kappa_p = \beta_p\sqrt{\frac{2}{1+e}} =$ 0.0283131434297 we apply the Precession Law (Eq. 20.5.1):

$$\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} \approx \frac{3\pi}{2}(9.13 \times 10^{-4}). \tag{87}$$

$$\boxed{\Delta\varphi \approx 14.7909706972 \approx 14.80 \text{ arcmin/orbit.}} \tag{88}$$

20.9 Discussion

The result derived here ($\Delta\varphi \approx 14.80'$) matches the expectations of the Schwarzschild metric to high precision. However, the path to this result is physically distinct.

In standard GR, the Virial Theorem ($2K + U = 0$) describes the time-averaged state of the system but does not, by itself, yield the prograde precession. To obtain the $6\pi GM/c^2a(1-e^2)$ shift, one must solve the geodesic equations in a curved manifold.

In contrast, the WILL RG treats the factor $\kappa^2 = 2\beta^2$ as a topological constraint on the total energy projections of the closed energy system that includes all channels. The projection ratio **oscillates throughout** the elliptical orbit as defined by $\delta \equiv \frac{\kappa^2}{2\beta^2}$ yet the system remains consistent due to the total energy invariant $W = \frac{1}{2}(\kappa^2 - \beta^2) =$ constant at any orbital phase.

The fact that a purely algebraic operation on the scalar projections **recovers** the same 'curved space' precession implies that the non-linearity attributed to spacetime curvature can be fully accounted for by the non-linearity of the projection geometry (S^1 and S^2).

Furthermore, this method eliminates the "Inverse Problem." We did not need to fit a mass M to the orbit to predict its future. We simply propagated the optical state vector

(β_o, δ_o) forward in time using geometric invariance. This represents a significant reduction in ontological complexity.

Summary

This section demonstrates that the full structure of orbital dynamics - including turning points, eccentricity, radial asymmetry, and periapsis precession - can be reconstructed from the two directly observable projection parameters κ and β, without introducing mass, G, metrics, manifolds, or any additional geometric assumptions. Orbital phenomena therefore require no spacetime curvature and no dynamical field equations; they arise entirely from algebraic relations among observable frequency projections.

21 Derivation of Density, Mass, and Pressure

IMPORTANT:

This document must be read **literally**. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (*absolute energies, external backgrounds, hidden containers*) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

21.1 Derivation of Density

Translating RG (2D) to Conventional Density (3D). In RG κ^2 is the 2D parameter defined in the relational carrier S^2 . In conventional physics, the source term is volumetric density ρ, a 3D concept defined by the "cultural artifact" (a Newtonian "cannonball" model) of mass-per-volume .

To bridge our 2D theory with 3D empirical data, we must create a "translation interface". We do this by explicitly adopting the conventional (Newtonian) definition of density, $\rho \propto m_0/r^3$, as our "translation target".

From the projective analysis established in the previous sections:

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

This leads to mass definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}$$

To translate this into a volumetric density, we first adopt the conventional 3D (volumetric) proxy, r^3. This is not a postulate of RG, but the first step in applying the legacy (3D) definition of density:

$$\frac{m_0}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}$$

This expression, however, is incomplete. Our κ^2 "lives" on the 2D surface S^2 (which corresponds to 4π), while the r^3 proxy implicitly assumes a 3D volume. To correctly normalize the 2D parameter κ^2 against the 3D volume, we must apply the geometric normalization factor of the S^2 carrier by deviding on to area of the sphere, which is $1/4\pi$.

This normalization is the necessary geometric step to interface the 2D relational carrier (S^2) with the 3D legacy definition of density:

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr^2}\right)$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$$

Local Density $\equiv$ Relational Projection

Maximal Density. At $\kappa^2 = 1$ (the horizon condition (for non rotating systems), $r = R_s$), this density reaches its natural bound, $\rho_{\max}$, which is derived purely from geometry:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}$$

Normalized Relation. Thus, our "translation" reveals an identity: the geometric projection κ^2 is simply the ratio of density to the maximal density:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}$$

21.2 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $m_0 = \alpha\, r^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

For the mass m_0 to remain a constant independent of the measurement scale r, the exponent must be $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

21.3 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\,\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\,c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy-geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\,c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energy-geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

Physical meaning

The negative pressure is not an exotic substance but the geometric tension required to maintain self-consistency when κ^2 varies radially. It's the relational analogue of surface tension in a soap bubble.

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22 Unified Geometric Field Equation

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation linking the geometric scale to the energy density ratio:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho_{\text{field}}}{\rho_{\max}}}$$

This identity defines the **local energy state of the geometry itself**. Here $\rho_{\max} = c^2/(8\pi G r^2)$ is the saturation density limit, and ρ_{field} is the effective energy density of the relational curvature.

22.1 Field Equation and Matter Sources

For a static, spherically symmetric configuration containing matter with density $\rho_{\text{matter}}(r)$, the relationship is governed by the differential accumulation of the potential:

$$\boxed{\frac{d}{dr}\left(r\,\kappa^2\right) = \frac{8\pi G}{c^2}\,r^2 \rho_{\text{matter}}(r)} \tag{89}$$

This expression reproduces the tt-component of the Einstein field equations.

The Vacuum Solution ($\rho_{\text{matter}} = 0$). In the vacuum region outside a central mass, the source density vanishes ($\rho_{\text{matter}} = 0$). The field equation implies conservation of the projection budget:

$$\frac{d}{dr}(r\kappa^2) = 0 \quad \Longrightarrow \quad r\kappa^2 = \text{const} = R_s.$$

Thus, we recover the potential law of WILL RG:

$$\boxed{\kappa^2 = \frac{R_s}{r}.}$$

Resolution of Roles

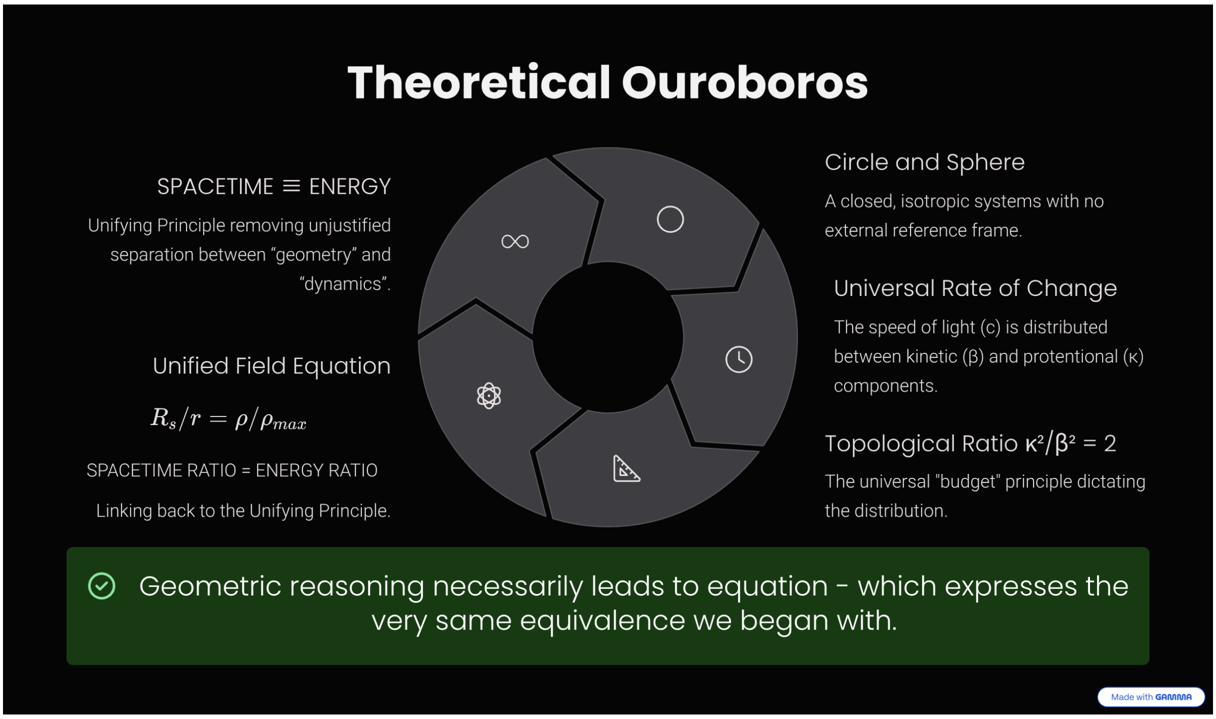
1. The Identity $\kappa^2 = \rho/\rho_{\max}$ describes the state of the *field* geometry.
2. The Equation $(r\kappa^2)' \sim \rho_{\text{matter}}$ describes how *matter* generates that geometry.
In vacuum, the generator is zero, but the field persists as the algebraic structure $\kappa^2 = R_s/r$.

23 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2\, \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.



WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- **Surface-scaled closure (vs. volume filling).** Mass follows the algebraic closure $m_0 = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Natural bounds.** The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

24 Theoretical Ouroboros

Closure

Ontological principle is proven as the inevitable consequence of geometric consistency. Field Equation $\iff$ Ontological Principle

We have shown that this single Ontological Principle (3.3), through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with SPACETIME $\equiv$ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY.}}$$

The ratio of geometric scales equals the ratio of energy densities.

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

$$\boxed{\begin{array}{c}\text{SPACETIME GEOMETRY}\\ \equiv \\ \text{ENERGY DISTRIBUTION}\end{array}}$$

Summary

All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework - the "Theoretical Ouroboros". But regardless of esthetic beauty of this result, let us remain skeptical.

25 W_{ILL}: Unity of Relational Structure

The ontological principle

$$\text{SPACETIME} \equiv \text{ENERGY}$$

states that there is only one closed relational resource - WILL. What we call space, time and matter are different projections of the same structure. For any energy-closed system observed from a relational origin, this resource appears through four operational projections:

These quantities are defined as correlated projections of the same underlying WILL structure. In the dimensionful form we write:

$$M = \frac{\beta^2}{\beta_Y}\frac{c^2 a}{G}, \qquad E = \frac{\kappa^2}{\kappa_X}\frac{c^4 a}{2G}, \qquad T = \kappa_X\left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2, \qquad L = \beta_Y\left(\frac{Gm_0}{\beta^2 c^2}\right)^2,$$

where (β, β_Y) and (κ, κ_X) are the kinematic and gravitational projections on the carriers S^1 and S^2, m_0 is the rest mass, $E_0 = m_0c^2$ is the rest energy, and a is the relational scale of the system as semi-major axis (average length per cycle within this system).

The closure conditions of the carriers,

$$\beta^2 + \beta_Y^2 = 1, \qquad \kappa_X^2 + \kappa^2 = 1,$$

together with the energetic exchange condition

$$\kappa^2 = 2\beta^2,$$

fix a unique dimensionless combination of these four projections. Combining E, T, M, and L we obtain

$$W_{\text{ILL}} \equiv \frac{ET}{ML} = \frac{\dfrac{E_0}{\kappa_X}\kappa_X t^2}{\dfrac{m_0}{\beta_Y}\beta_Y a^2} = \frac{E_0 t^2}{m_0 a^2}.$$

By the relations that tie temporal and $t = a/c$ and spacial $a = R_s/\kappa^2$ scales this ratio is identically equal to unity for any closed system:

$$\boxed{W_{\text{ILL}} = \frac{ET}{ML} = 1.}$$

All dimensionful constants cancel automatically; the value is fixed by the geometry of the carriers, not by a choice of units.

The same invariant can be written in a phase–normalized form, using local projections

$$E_o = \frac{E_0}{\kappa_{Xo}}, \qquad M_o = \frac{m_0}{\beta_{Yo}}, \qquad T_o = \kappa_{Xo} t_o^2, \qquad L_o = \beta_{Yo} r_o^2,$$

Equivalently, for any state (β_o, κ_o), any scale r_o and any phase along the orbit. Then

$$\boxed{W_{\text{ILL}} = \frac{E_o T_o}{M_o L_o} = 1 \quad \text{for ALL ENERGY'S, ALL SCALES and ALL PHASES.}}$$

$$\frac{E_o}{M_o} = \frac{L_o}{T_o},$$

so the energy sector and the spacetime sector are not independent. Every change of the relational state rescales (E_o, M_o) and (T_o, L_o) coherently so that this equality is always preserved. The familiar practice of treating energy–mass and space–time as separate blocks is therefore an ontological approximation: in WILL they are locked by a single relational constraint.

$$\text{Geometry} \equiv \text{Energy} \equiv \text{Causality} \equiv WILL,$$

$$\boxed{W_{ILL} = 1.}$$

in the precise sense that one and the same conserved relational resource appears as mass, energy, time and length, but always in a way that keeps their ratio fixed.

25.1 Interpretive Note: The Name "WILL"

The term **WILL** stands for **SPACE-TIME-ENERGY**. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the Cosmos does not possess "will" - yet through WILL, it manifests All that Is.

Summary

$$\boxed{\textbf{WILL} \equiv \frac{ET}{ML} = 1 \quad \Longleftrightarrow \quad \text{Geometry} = \text{Energy} = \text{Causality.}}$$

WILL is not the unit of something - but the Unity of Everything.

26 Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as "laws of nature."
3. Mathematical formalisms are constructed to *describe* these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are *generated* as inevitable consequences of relational geometry:

- There are no independent axioms such as "inertial mass equals gravitational mass."
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls "laws of nature" are secondary shadows of the single relational principle:

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

Summary

Standard Physics: Laws *describe* what we observe.
Relational Geometry: Laws are *generated* as necessary products of closure and self-consistency.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

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Descriptive Physics (Standard)	**Generative Physics (WILL)**
Phenomena are *observed* first, then summarized into empirical laws.	Laws emerge as *inevitable consequences* of relational geometry.
Physical laws are *assumptions* introduced to model reality.	Physical laws are *identities*, enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are *projections of energy relations*.
Dynamics = evolution of states *in time*.	Dynamics = ordered succession of balanced configurations; *time is emergent*.
Goal: *describe* what is observed.	Goal: *show why nothing else is possible*.

Table 5: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

27 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single Ontological Principle-that spacetime is equivalent to energy - we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time and space are merely different projections of the same underlying structure.

$$\boxed{\text{Special and General Relativity emerge from the same geometric principles.}}$$

This approach offers distinct advantages:

- Conceptual clarity - understanding physics through pure geometry
- Computational efficiency - significantly reducing complexity

Phenomenon	**Standard (GR) Result**	**Relational Geometry (RG)**
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ 38.52 μs/day verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2 = 54.73°$ ("magic angle") $Q^2 = \kappa^2 + \beta^2 = 1$. ISCO: $Q = Q_t$,
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Same number obtained from RG with $\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} = 43''$/century. Using simle algebra.
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs. $\Delta P \approx -2.42 \times 10^{-12}$ s/s (predicted)
Cosmological redshift	Photon "loses energy" as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, ΛCDM fits	$H_0 \equiv \sqrt{8\pi G\rho_\gamma/(3\alpha^2)} = 68.15$ Derived from CMB temperature and α connecting micro and macro scales (Details in WILL PART II-III).
Cosmological constant Λ	Added by hand to fit data ("dark energy").	Arises naturally as $\Lambda = 2/3r^2$. No extra entities required. (More details in WILL PART I and II)
Singularities	Predicted in black holes and big bang ($\rho \to \infty$).	Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$.
Local gravitational energy	"Cannot be localized" (only ADM/Bondi at infinity).	Directly measurable via κ, e.g. from light deflection angle or red shift.
Unification with QM	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 6: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

- Epistemological hygiene - deriving results from minimal assumptions

- Philosophical depth - redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

$$\boxed{\text{Spacetime and energy are mutually defining aspects of a single relational structure.}}$$

Final Summary

SPACETIME ≡ ENERGY.

28 Closed Algebraic System of Relational Orbital Mechanics (R.O.M.)

R.O.M. does not describe how a body moves under forces; it classifies the algebraically allowed relational states of a bound two-body system.

$$\boxed{\kappa^2 = 1 - \frac{1}{(1+z)^2}} \quad (z = \text{gravitational redshift})$$

$$\boxed{\beta^2 = 1 - \frac{1}{(1+z_D)^2}} \quad (z_D = \text{transverse Doppler shift})$$

Observational Z Inputs

$Z_{tot}\left(o\right) = \left(1 + z_o\left(o\right)\right)\left(1 + z_{Do}\left(o\right)\right)$ (product of gravitational red shift and transverse Doppler shift)
$\tau_{Wo}\left(o\right) = \kappa_{Xo}\left(o\right) \cdot \beta_{Yo}\left(o\right) = \left(Z_{tot}\left(o\right)\right)^{-1}$ (product of projectional phase factors on S^1 and S^2 carriers)

Global System Parameters

$R_s = \kappa^2 a = \frac{2Gm_0}{c^2} = \frac{2}{3}Q_o\left(O_o\right)^2 a = \frac{r_1 r_2}{r_2 - r_1}(\beta_1^2 - \beta_2^2) = \frac{a}{2}(3 - \sqrt{1 + 8\tau_{Wo}(O_o)^2}) = \frac{r_o(o)}{2(2a - r_o(o))}\left(4a - r_o\left(o\right) - \sqrt{\left(4a - r_o\left(o\right)\right)^2 - 8a\left(2a - r_o\left(o\right)\right)\left(1 - \tau_{Wo}\left(o\right)^2\right)}\right)$ (Schwarzschild radius - system scale)
$\kappa = \sqrt{\frac{R_s}{a}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sqrt{\kappa_p^2(1-e)} = \sqrt{4W} = \sqrt{\frac{1}{2}(3 - \sqrt{1 + 8\tau_{Wo}(O_o)^2})}$ (potential projection at semi-major axis)
$a = \frac{R_s}{\kappa^2} = \frac{R_s}{4W} = \frac{\beta_o c}{\omega} \cdot \frac{\sqrt{1-e^2}}{\sqrt{1+e^2+2e\cos(o)}}$ (semi-major axis)
$\beta = \frac{\kappa}{\sqrt{2}} = \beta_p\sqrt{\frac{1-e}{1+e}} = \sqrt{2W} = \sqrt{\kappa_o^2 - \frac{\kappa_o^2}{2} \cdot \left(1 + \left(\frac{1}{\delta_o(o)} - 1\right)\right)} = \beta_o\left(o\right)\frac{\sqrt{1-e^2}}{\sqrt{1+e^2+2\cdot e\cdot\cos(o)}}$ (kinetic projection on semi major axis)
$m_0 = \frac{\kappa^2 c^2 a}{2G} = 4\pi\rho a^3$ (mass parameter)
$o =$ orbital phase in radians
$\delta = \frac{\kappa_p^2}{2\beta_p^2} = \frac{1}{1+e}$ (closure factor, measured at r_p)

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Eccentricity Relations

$e = \frac{1}{\delta} - 1 = 1 - \frac{2\beta_a^2}{\kappa_a^2} = \frac{2\beta_p^2}{\kappa_p^2} - 1$ (eccentricity derived from closure)
$e_Y = \sqrt{1 - e^2}$ (eccentricity's orthogonal value)
$e_X = \frac{1+e}{1-e} = \frac{\delta_a}{\delta_p} = \frac{\kappa_a^2 \beta_p^2}{\kappa_p^2 2\beta_a^2}$ (shape factor)

Constants (Fixed for the Orbit)

$r_p = a(1-e)$ (radius at perihelion)
$\kappa_p = \kappa\sqrt{\frac{1}{1-e}} = Q_p\sqrt{\frac{2}{3+e}}$ (potential at perihelion)
$\beta_p = \frac{V_p}{c} = \frac{\kappa_p}{\delta\sqrt{2}} = \sqrt{\kappa_p^2 \cdot \frac{1+e}{2}}$ (kinetic at perihelion)
$W = \frac{\beta^2}{2} = \frac{1}{2}(\kappa_o^2 - \beta_o^2) = \frac{1}{4}\kappa_p^2\ (1-e) = \frac{1}{2}\left(\kappa_o^2 - \frac{\kappa_o^2}{2} \cdot \left(1 + \left(\frac{1}{\delta_o(o)} - 1\right)\right)\right)$ (energy invariant - binding energy)
$\Delta\phi_{\mathrm{WILL}} = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} = \frac{2\pi Q^2}{1-e^2}$ (precession of perihelion per orbit)
$h_W = a \cdot \beta c \cdot e_Y$ (angular momentum)
$\omega = \frac{\beta c}{a}$ (angular frequency)
$T = \frac{2\pi}{\omega}$ (orbital period)
$Q_p = \sqrt{\kappa_p^2 + \beta_p^2}$ (relational shift at perihelion)

Time Integration

$\omega_o\,(o) = \frac{\beta \cdot c}{a} \cdot \frac{(1+e\cdot\cos(o))^2}{(1-e^2)^{\frac{3}{2}}}$ (angular frequency for time integration)
$\Delta_{t1} = \int_0^o \frac{1}{\omega_\theta(\theta)}\,d\theta$ (time duration of given phase interval)

Apocenter Relations

$\beta_a = \sqrt{\beta_p^2 e_X^2} = \beta\sqrt{e_X}$ (kinetic projection at apocenter)
$\kappa_a = \sqrt{2W + \beta_a^2}$ (potential projection at apocenter)
$\delta_{\mathrm{a}} = \frac{1}{1-e} = \frac{\kappa_a^2}{2\beta_a^2}$ (closure factor at apocenter)

Phase Variables (Depend on o)

$r = r_o(o) = a\frac{1-e^2}{1+e\cos o} = \frac{R_s}{\kappa_o^2}$ (radial distance at phase o)
$\kappa_o = \sqrt{\frac{R_s}{r}} = \kappa_p\sqrt{\frac{1+e\cos o}{1+e}}$ (local potential at phase o)
$\kappa_{Xo} = \sqrt{1-\kappa_o^2}$ (gravitational phase factor at phase o)
$\beta_o = \sqrt{\kappa_o^2 - 2W}$ (local kinetic from energy invariant)
$\beta_{Yo} = \sqrt{1-\beta_o^2}$ (relativistic phase factor at phase o)
$\delta_o = \frac{1+e\cos o}{1+e^2+2e\cos o} = \frac{\kappa_o^2}{2\beta_o^2}$ (local closure factor at phase o)
$Q_o = \sqrt{\kappa_o^2 + \beta_o^2}$ (local relational shift vector at phase o)
$\omega_o = a\beta c\frac{e_Y}{r_o^2} = \frac{\beta c}{a}\frac{(1+e\cos o)^2}{(1-e^2)^{3/2}}$ (angular speed)

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$\eta_o = \frac{r}{a} = 2 - \frac{2\beta_o(o)^2}{\kappa_o(o)^2}$ (phase scale amplitude)
$t_o = \frac{r}{c}$ (temporal scale at phase o)
$z_o = \frac{1}{\kappa_{Xo}} - 1$ (redshift at phase o)
$z_{Do} = \frac{1}{\beta_{Yo}} - 1$ (transverse Doppler shift at phase o)
$\tau_{Wo}(o) = \kappa_{Xo}(o) \cdot \beta_{Yo}(o) = Z_{tot}(o)^{-1}$ (phase spacetime factor)

Relational Geometry (WILL)

$\theta_1 = \arccos(\beta)$ (distribution angle on S^1, non-physical)
$\theta_2 = \arcsin(\kappa)$ (distribution angle on S^2, non-physical)
$\beta_Y = \sin(\theta_1) = \sqrt{1-\beta^2}$ (relativistic phase factor)
$\kappa_X = \cos(\theta_2) = \sqrt{1-\kappa^2}$ (gravitational phase factor)
$\tau_W = \kappa_X \beta_Y$ (relational spacetime factor)

$\Delta_Q = Q_o^2 - Q^2$ (phase relational shift amplitude)
$O_o = \arccos(1-\delta^{-1}) = \arccos(-e) = \arccos\left(\frac{2\beta_a^2}{\kappa_a^2} - 1\right)$ (orbital balance point where $\kappa_o^2 = 2\beta_o^2$ is true)
$t = \frac{a}{c}$ (temporal scale as phase period)
$z = \frac{1}{\kappa_X} - 1$ (redshift)
$z_D = \frac{1}{\beta_Y} - 1$ (transverse Doppler shift)

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