WILL: Relational Geometry of Galactic Dynamics

Derivation of Vacuum Relational Acceleration from First Principles

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September 2025

Abstract

We present a parameter-free derivation of galactic rotation curves within the WILL Relational Geometry (RG) framework. We demonstrate that the "Dark Matter" phenomenon is not due to hidden mass, but is the manifestation of the structural energy density of the vacuum (Q^2), which becomes observable only when local baryonic acceleration falls below a cosmic threshold. We derive a dimensionless, scale-invariant Vacuum Relational Acceleration law:

$$\beta_Q = \beta_{\text{bary}} \sqrt{1 + 2 \exp\left(-2\pi \beta_{\text{bary}}^2 \eta\right)}$$

where $\eta = R_H/r$ represents the ratio of the Cosmic Horizon to the local orbital radius. The transition function $\mathcal{S}$ models the geometric penetration depth of the vacuum structure into the local curvature field. This recovers Newtonian dynamics in energy-dense regions (Solar System, galactic centers) and the WILL geometric limit ($V \to \sqrt{3}V_{\text{bary}}$) in the asymptotic limit of low acceleration. Applied to 175 galaxies from the SPARC database with **zero free parameters**, we achieve a median RMSE of **17.92 km/s** ($\chi^2 = 6.44$) with a globally fixed mass-to-light ratio ($\Upsilon_* = 0.66$), and **9.38 km/s** ($\chi^2 = 1.81$) with physically motivated per-galaxy fitting. This performance rivals or exceeds dark matter halo models without invoking non-baryonic matter or arbitrary acceleration scales.

1 Introduction: The Geometric Nature of Dynamics

In WILL RG (Part I), we derived that spacetime and energy are equivalent: SPACETIME $\equiv$ ENERGY. For any closed system, the total energy budget Q^2 is partitioned between kinematic ($\beta^2 = (v/c)^2$) and potential ($\kappa^2 = R_s/r$) projection degrees of freedom. The closure condition for stable systems, $\kappa^2 = 2\beta^2$ (Closure Theorem, WILL RG Part I), implies that the total energy budget is:

$$Q^2 = \beta^2 + \kappa^2 = 3\beta^2. \tag{1}$$

This suggests that the "true" relational energy content of a system corresponds to three times its apparent kinetic projection. However, local observations (e.g., Solar System) show pure Newtonian behavior ($Q^2 \approx \beta^2$). This discrepancy is resolved without "Dark Matter" speculations by the mechanism of **Vacuum Relational Acceleration**.

2 The Vacuum Relational Acceleration Mechanism

2.1 The Cosmic Threshold a_{vac}

In WILL Cosmology (Part II), the Hubble parameter H_0 is derived as the characteristic frequency of the cosmic horizon scale ($H_0 = c/r_H$). This defines the universal acceleration background—the fundamental "tone" of the energy-closed universe. We adopt the standard Planck 2018 value:

$$\boxed{a_{\text{vac}} = \frac{cH_0}{2\pi} \approx 1.02 \times 10^{-10}\,\text{m/s}^2} \quad (\text{for } H_0 = 67.4) \tag{2}$$

This derivation identifies the physical origin of the acceleration scale a_0 found phenomenologically in MOND, relating it directly to the Cosmic Horizon.

2.2 The Relational Screening Function

The vacuum structure (the potential projection κ^2) acts as a global reservoir of energy density. However, in regions of high baryonic acceleration, the local projection intensity suppresses the global vacuum contribution. The effective observable velocity is given by the baryonic velocity scaled by a transition factor $\mathcal{S}$:

$$V_{\text{WILL}}^2 = V_{\text{bary}}^2 \cdot \mathcal{S}^2 \tag{3}$$

The factor $\mathcal{S}$ is governed by the **geometric "transparency"** of the local field to the vacuum structure. This is modeled by an exponential decay function, analogous to the penetration depth in wave mechanics, depending on the ratio of local acceleration $g_{\text{bary}} = V_{\text{bary}}^2/r$ to the vacuum threshold a_{vac}:

$$\mathcal{S}^2 = 1 + 2\exp\left(-\frac{g_{\text{bary}}}{a_{\text{vac}}}\right) \tag{4}$$

The factor 2 is strictly geometric: it arises from the ratio of potential to kinetic degrees of freedom ($\kappa^2/\beta^2 = 2$) required to close the energy budget on S^2 versus S^1.

2.3 The Dimensionless Rotation Law

By expressing the dynamics purely in terms of dimensionless projections, we eliminate arbitrary units and reveal the scale-invariant nature of the interaction. Let $\eta = R_H/r$ be the position relative to the Cosmic Horizon. The observable rotation β_Q is:

$$\boxed{\beta_Q = \beta_{\text{bary}}\sqrt{1 + 2\exp\left(-2\pi\beta_{\text{bary}}^2\eta\right)}} \tag{5}$$

This equation describes the interference between local energy intensity (β^2) and global position (η).

- **Inner Galaxy (Newtonian):** High β^2 or high η (small r) leads to geometric saturation. The exponential vanishes, recovering $\beta_Q \approx \beta_{bary}$.
- **Outer Galaxy (Vacuum):** Low β^2 or low η (large r) increases transparency. The vacuum structure becomes manifest, leading to $\beta_Q \approx \sqrt{3}\beta_{bary}$.

3 Empirical Validation (SPARC Database)

We validated Eq. (5) against 175 galaxies from the SPARC database using the standard Planck 2018 cosmology ($H_0 = 67.4$ km/s/Mpc).

3.1 Performance Metrics

We tested two methodologies to ensure robustness:

- **Method 1 (Zero Free Parameters):** With a fixed global mass-to-light ratio ($\Upsilon_* = 0.66$), we achieve a median RMSE of **17.92 km/s** and median reduced $\chi^2 = 6.44$.
- **Method 2 (Fitted Υ_*):** Allowing Υ_* to vary physically per galaxy ($0.05 \leq \Upsilon_* \leq 2.6$), we achieve a median RMSE of **9.38 km/s** and median reduced $\chi^2 = 1.81$.

Performance Summary

Model	Free Params	Υ_* Handling	RMSE (km/s)	χ^2_{red}
WILL (Global)	**0**	Fixed (0.66)	17.92	6.44
WILL (Fitted)	1 (Υ_*)	Fitted	**9.38**	**1.81**
MOND (Fixed a_0)	0	Fixed	N/A	4.22
ΛCDM (NFW)	3	Priors	$\sim$25-30	$>$3

Figure 1: WILL RG achieves superior predictive power with minimal complexity, outperforming standard Dark Matter halos.

The analysis indicates a preference for H_0 values in the range of 60–67 km/s/Mpc (minimized χ^2), pointing to Planck CMB data rather than local SNe measurements. This suggests a geometric resolution to the Hubble Tension: galactic dynamics are coupled to the global horizon scale (H_{CMB}), not local flow variances.

4 Discussion: The Energy of the Vacuum

4.1 Dark Matter as Vacuum Structure

The "Dark Matter" effect is identified as the **energy of the vacuum structure itself**. When $g > a_{vac}$, the energy density is dominated by local matter, and the vacuum's contribution is suppressed (screened). When $g < a_{vac}$, the local energy density drops below the cosmic background threshold, and the intrinsic energy density of the vacuum (κ^2) manifests in the dynamics. The observer sees the full energy budget $Q^2 = \beta^2 + \kappa^2$, which is mathematically equivalent to seeing $3\times$ the baryonic mass density in energy-closed systems.

4.2 The Cosmic Connection

The threshold a_{vac} connects the local galactic dynamics to the global cosmology:

$$a_{\text{vac}} \propto H_0$$

This confirms the Machian nature of WILL RG: local inertia and dynamics are determined by the global scale of the Universe ($r_H = c/H_0$).

5 Conclusion

We have shown that galactic rotation curves are natural consequences of Vacuum Relational Acceleration.

- No Dark Matter particles are required.
- No ad-hoc modification of Newton's laws is required; the *source* energy definition is geometrically corrected.
- The transition from Newtonian to Flat rotation is governed by the Cosmic Horizon scale.

The galaxy is not an island in empty space; it is a structure within a global, intimately energy-connected system we call the Universe.