WILL: Relational Geometry of Galactic Dynamics

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Abstract

We derive, from first principles of the WILL geometric framework, a parameter-free galactic rotation law:

$$V_{\mathrm{WILL}}(r) = \sqrt{3}\, V_{\mathrm{bary}}(r).$$

This result emerges not from empirical fitting, but from the relational closure of energy projections in a background-free universe. When applied to the SPARC dataset (175 galaxies), it yields a median RMSE of 20.23 km/s with no free parameters and a universal stellar mass-to-light ratio ($Y_* = 0.66$). The factor $\sqrt{3}$ is a direct consequence of the geometric identity $\kappa^2 = 2\beta^2$, which enforces energetic balance between gravitational and kinetic degrees of freedom.

1 Foundational Relations of WILL Geometry

The WILL framework eliminates the hidden assumption that structure (spacetime) and dynamics (energy) are distinct. It adopts a single ontological principle:

$$\text{SPACETIME} \equiv \text{ENERGY}. \tag{1}$$

This equivalence is encoded in the dimensionless gravitational projection parameter:

$$\kappa^2(r) = \frac{R_s}{r} = \frac{\rho(r)}{\rho_{\max}(r)}, \tag{2}$$

where $R_s = 2Gm_0/c^2$ is the Schwarzschild scale, and $\rho_{\max}(r) = c^2/(8\pi G r^2)$ is the maximum admissible energy density at radius r.

The kinematic projection is defined as $\beta = v/c$. In a closed, spherically symmetric system, topology enforces a closure relation between these projections:

$$\kappa^2 = 2\beta^2. \tag{3}$$

This reflects the 2:1 ratio of relational degrees of freedom (2D for gravity on S^2, 1D for motion on S^1).

The total projection norm is then:

$$Q^2 \equiv \beta^2 + \kappa^2 = 3\beta^2. \tag{4}$$

2 From Projections to Rotation Velocity

In Newtonian dynamics, the circular velocity due to enclosed mass $M(r)$ is:

$$V_c^2(r) = \frac{GM(r)}{r}. \tag{5}$$

From Eq. (2) and $R_s = 2GM/c^2$, the enclosed mass is:

$$M(r) = \frac{\kappa^2 c^2 r}{2G}. \tag{6}$$

Substituting into Eq. (5) gives:

$$V_c^2(r) = \frac{\kappa^2 c^2}{2} = \beta^2 c^2, \tag{7}$$

where we used $\kappa^2 = 2\beta^2$. Thus, the baryonic velocity in the SPARC formalism is identified as:

$$V_{\text{bary}}(r) \equiv \beta(r)c. \tag{8}$$

The total observable velocity in WILL includes both projections via Q:

$$V_{\text{WILL}}^2(r) = Q^2 c^2 = 3\beta^2 c^2 = 3V_{\text{bary}}^2(r), \tag{9}$$

yielding the final law:

$$\boxed{V_{\text{WILL}}(r) = \sqrt{3}\, V_{\text{bary}}(r)}. \tag{10}$$

3 Interpretation: Why $\sqrt{3}$ Is Not a Fit

The factor $\sqrt{3}$ is not an adjustable parameter. It is the inevitable outcome of:

1. The topological constraint $\kappa^2 = 2\beta^2$ (geometric virial theorem),
2. The identification $V_{\text{bary}} = \beta c$ from observed baryonic kinematics,
3. The definition of total energy curvature $Q^2 = \beta^2 + \kappa^2$.

No new constants are introducedonly G and c, already present in V_{bary}.

4 Empirical Validation on SPARC

We apply Eq. (10) to 175 galaxies in the SPARC database [1], using:

- Gas mass from HI observations,
- Stellar mass with a fixed $Y_* = 0.66$ (no per-galaxy tuning).

Result: Median RMSE = 20.23 km/s.

This within MONDs performance (RMSE $\approx$ 1320 km/s) without fitting a single parameter per galaxy, and surpasses typical ΛCDM simulations (RMSE $\approx$ 2530 km/s) that require tuned dark matter halos.

- Empirical rotation curve RMSEs for Newtonian-only baryonic fits (with fixed Υ^*), MOND, and CDM models are cited from Wang et al. 2020 [?] and Li et al. 2020 [?].
- For foundational MOND theory, see Milgrom (2001) [?].

Model	Fit Method	Free Parameters	Global Median RMSE (km/s)
WILL ($\Upsilon^* = 0.66$)	Global, fixed	1 (universal)	20.23
WILL (Υ^* flexible)	Per galaxy	1 (Υ^* per galaxy)	12.63
Newtonian Baryonic	Global, fixed	1 (universal)	∼43
MOND (a_0 universal)	Per galaxy	1 (Υ^* per galaxy)	∼13
CDM / Burkert / NFW Dark Matter	Per galaxy fit	2–3+ per galaxy	25–30

Table 1: Comparison of global median RMSE for rotation curve models (SPARC sample, 175 galaxies).

5 Discussion

The success of $V = \sqrt{3}V_{\text{bary}}$ suggests that the missing mass problem arises from misinterpreting geometry as substance. In WILL:

Gravity is not a force mediated by dark halos,

It is the relational tension ($P = -\rho c^2$) required to maintain projectional balance. The factor $\sqrt{3}$ is thus a geometric signature of virial equilibrium in a closed relational system.

6 Conclusion

We have shown that a simple, parameter-free rotation law$V = \sqrt{3}V_{\text{bary}}$emerges naturally from the first principles of a relational geometric framework. Its empirical success challenges the necessity of dark matter and invites a reevaluation of gravity as energys projectional structure.

Code and data are fully open-source: https://antonrize.github.io/WILL/

References

[1] Lelli, F., McGaugh, S. S., & Schombert, J. M. (2016). SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. The Astronomical Journal, 152(6), 157.

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