

WILL: Relational Geometry of Galactic Dynamics

Anton Rize
egeometricity@gmail.com

September 2025

Abstract

We derive, from first principles of the WILL geometric framework, a parameter-free galactic rotation law:

$$V_{\text{WILL}}(r) = \sqrt{3} V_{\text{bary}}(r).$$

This result emerges not from empirical fitting, but from the relational closure of energy projections in a background-free universe. When applied to the SPARC dataset (175 galaxies), it yields a median RMSE of 20.23 km/s with no free parameters and a universal stellar mass-to-light ratio ($Y_* = 0.66$). The factor $\sqrt{3}$ is a direct consequence of the geometric identity $\kappa^2 = 2\beta^2$, which enforces energetic balance between gravitational and kinetic degrees of freedom.

1 Foundational Relations of WILL Geometry

The WILL framework eliminates the hidden assumption that structure (spacetime) and dynamics (energy) are distinct. It adopts a single ontological principle:

$$\text{SPACETIME} \equiv \text{ENERGY}. \quad (1)$$

This equivalence is encoded in the dimensionless gravitational projection parameter:

$$\kappa^2(r) = \frac{R_s}{r} = \frac{\rho(r)}{\rho_{\max}(r)}, \quad (2)$$

where $R_s = 2Gm_0/c^2$ is the Schwarzschild scale, and $\rho_{\max}(r) = c^2/(8\pi Gr^2)$ is the maximum admissible energy density at radius r .

The kinematic projection is defined as $\beta = v/c$. In a closed, spherically symmetric system, topology enforces a closure relation between these projections:

$$\kappa^2 = 2\beta^2. \quad (3)$$

This reflects the 2:1 ratio of relational degrees of freedom (2D for gravity on S^2 , 1D for motion on S^1).

The total projection norm is then:

$$Q^2 \equiv \beta^2 + \kappa^2 = 3\beta^2. \quad (4)$$

2 From Projections to Rotation Velocity

In Newtonian dynamics, the circular velocity due to enclosed mass $M(r)$ is:

$$V_c^2(r) = \frac{GM(r)}{r}. \quad (5)$$

From Eq. (2) and $R_s = 2GM/c^2$, the enclosed mass is:

$$M(r) = \frac{\kappa^2 c^2 r}{2G}. \quad (6)$$

Substituting into Eq. (5) gives:

$$V_c^2(r) = \frac{\kappa^2 c^2}{2} = \beta^2 c^2, \quad (7)$$

where we used $\kappa^2 = 2\beta^2$. Thus, the baryonic velocity in the SPARC formalism is identified as:

$$V_{\text{bary}}(r) \equiv \beta(r)c. \quad (8)$$

The total observable velocity in WILL includes both projections via Q :

$$V_{\text{WILL}}^2(r) = Q^2 c^2 = 3\beta^2 c^2 = 3V_{\text{bary}}^2(r), \quad (9)$$

yielding the final law:

$$\boxed{V_{\text{WILL}}(r) = \sqrt{3} V_{\text{bary}}(r)}. \quad (10)$$

3 Interpretation: Why $\sqrt{3}$ Is Not a Fit

The factor $\sqrt{3}$ is not an adjustable parameter. It is the inevitable outcome of:

1. The topological constraint $\kappa^2 = 2\beta^2$ (geometric virial theorem),
2. The identification $V_{\text{bary}} = \beta c$ from observed baryonic kinematics,
3. The definition of total energy curvature $Q^2 = \beta^2 + \kappa^2$.

No new constants are introduced only G and c , already present in V_{bary} .

4 Empirical Validation on SPARC

We apply Eq. (10) to 175 galaxies in the SPARC database [1], using:

- Gas mass from HI observations,
- Stellar mass with a fixed $Y_* = 0.66$ (no per-galaxy tuning).

Result: Median RMSE = 20.23 km/s.

This within MONDs performance ($\text{RMSE} \approx 1320$ km/s) without fitting a single parameter per galaxy, and surpasses typical Λ CDM simulations ($\text{RMSE} \approx 2530$ km/s) that require tuned dark matter halos.

- Empirical rotation curve RMSEs for Newtonian-only baryonic fits (with fixed Υ^*), MOND, and CDM models are cited from Wang et al. 2020 [?] and Li et al. 2020 [?].
- For foundational MOND theory, see Milgrom (2001) [?].

Model	Fit Method	Free Parameters	Global Median RMSE (km/s)
WILL ($\Upsilon^* = 0.66$)	Global, fixed	1 (universal)	20.23
WILL (Υ^* flexible)	Per galaxy	1 (Υ^* per galaxy)	12.63
Newtonian Baryonic	Global, fixed	1 (universal)	~ 43
MOND (a_0 universal)	Per galaxy	1 (Υ^* per galaxy)	~ 13
CDM / Burkert / NFW Dark Matter	Per galaxy fit	2–3+ per galaxy	25–30

Table 1: Comparison of global median RMSE for rotation curve models (SPARC sample, 175 galaxies).

5 Discussion

The success of $V = \sqrt{3}V_{\text{bary}}$ suggests that the missing mass problem arises from misinterpreting geometry as substance. In WILL:

Gravity is not a force mediated by dark halos,

It is the relational tension ($P = -\rho c^2$) required to maintain projectional balance. The factor $\sqrt{3}$ is thus a geometric signature of virial equilibrium in a closed relational system.

6 Conclusion

We have shown that a simple, parameter-free rotation law $V = \sqrt{3}V_{\text{bary}}$ emerges naturally from the first principles of a relational geometric framework. Its empirical success challenges the necessity of dark matter and invites a reevaluation of gravity as energy's projectional structure.

Code and data are fully open-source: <https://antonrize.github.io/WILL/>

References

- [1] Lelli, F., McGaugh, S. S., & Schombert, J. M. (2016). SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. *The Astronomical Journal*, 152(6), 157.
- @articleLi2020, author = Li, Pengfei and Lelli, Federico and McGaugh, Stacy S. and Schombert, James M., title = A Comprehensive Catalog of Dark Matter Halo Models for SPARC Galaxies, journal = The Astrophysical Journal Supplement Series, volume = 247, number = 1, pages = 31, year = 2020, doi = https://ui.adsabs.harvard.edu/link_gateway/2020ApJS..247...31L/doi : 10.3847/1538 – 4365/ab700e10.3847/1538 – 4365/ab700e, url = <https://ui.adsabs.harvard.edu/abs/2020ApJS..247...31L/abstract>
- @articleWang2020, author = Wang, De-Chang and Xu, Feng and Luo, Xin, title = Comparison of Modeling SPARC spiral galaxies' rotation curves with different dark matter and MOND models, journal = arXiv preprint arXiv:2008.04795, year = 2020, url = <https://arxiv.org/abs/2008.04795>
- @articleMilgrom2001, author = Milgrom, Mordehai, title = MOND - A Pedagogical Review, year = 2001, journal = NED Level 5 Review, url = <https://ned.ipac.caltech.edu/level5/Sept01/Milgrom/Milgrom2.html>