1 Appendix I

1.1 Correspondence with General Relativity

To facilitate comparison with standard General Relativity, we recast the relational parameters (κ, β) in metric form. Under the identification $\kappa^2 = 2GM/(rc^2)$, the WILL RG reproduces the Schwarzschild line element and the corresponding Einstein field equations in algebraic form.

1.2 Equivalence with Schwarzschild Solution

Theorem 1.1 (Equivalence with Schwarzschild Solution). The WILL RG formalism reproduces the Schwarzschild metric in the appropriate limit.

Proof. The Schwarzschild metric in General Relativity is given by:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit sphere.

In WILL Geometry, the key parameters are:

$$\kappa^2 = \frac{R_s}{r} = \frac{2GM}{rc^2} \tag{2}$$

$$\kappa_X = \sqrt{1 - \kappa^2} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{3}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{4}$$

The time component of the Schwarzschild metric can be written as:

$$g_{tt} = \left(1 - \frac{2GM}{rc^2}\right) = 1 - \kappa^2 = \kappa_X^2 \tag{5}$$

And the radial component can be written as:

$$g_{rr} = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} = -\frac{1}{1 - \kappa^2} = -\frac{1}{\kappa_X^2} \tag{6}$$

Therefore, in WILL Geometry terms, the Schwarzschild metric takes the form:

$$ds^2 = \kappa_X^2 c^2 dt^2 - \frac{1}{\kappa_X^2} dr^2 - r^2 d\Omega^2 \tag{7}$$

This demonstrates that the WILL Geometry parameters exactly reproduce the Schwarzschild metric. □

1.3 Equivalence with Einstein Field Equations

Theorem 1.2 (Equivalence with Einstein Field Equations). *The geometric field equation of WILL RG is equivalent to the corresponding component of Einstein's field equations for a static, spherically symmetric mass distribution.*

Proof. The standard form for the tt-component of Einstein's field equations inside a spherically symmetric perfect fluid is given by one of the Tolman–Oppenheimer–Volkoff (TOV) equations:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1-\frac{1}{g_{rr}}\right)\right) = \frac{8\pi G}{c^2}\rho(r), \tag{8}$$

where $\rho(r)$ is the energy density at radius r.

For a static spherical system, the metric component g_{rr} is related to the mass enclosed within radius r, denoted $m(r)$, by

$$1 - \frac{1}{g_{rr}} = \frac{2Gm(r)}{rc^2}. \tag{9}$$

We define the interior WILL parameter $\kappa^2(r)$ precisely as this quantity:

$$\kappa^2(r) \equiv \frac{2Gm(r)}{rc^2}. \tag{10}$$

This ensures a smooth transition to the exterior vacuum solution ($\kappa^2 = R_s/r$ with $R_s = 2GM/c^2$ constant) at the object's surface.

With this definition,

$$r\left(1-\frac{1}{g_{rr}}\right) = r\kappa^2(r), \tag{11}$$

and substitution into the field equation yields

$$\frac{1}{r^2}\frac{d}{dr}\left(r\kappa^2(r)\right) = \frac{8\pi G}{c^2}\rho(r). \tag{12}$$

Multiplying both sides by r^2 gives the differential form of the WILL field equation:

$$\frac{d}{dr}\left(r\kappa^2\right) = \frac{8\pi G}{c^2}r^2\rho(r). \tag{13}$$

From the geometric definition of energy density in WILL,

$$\rho(r) = \frac{\kappa^2(r)c^2}{8\pi Gr^2}, \qquad \rho_{\max}(r) = \frac{c^2}{8\pi Gr^2},$$

it follows immediately that

$$\frac{\rho(r)}{\rho_{\max}(r)} = \kappa^2(r).$$

Moreover, since $r\kappa^2(r) = 2Gm(r)/c^2$, differentiation gives

$$\frac{d}{dr}\left(r\kappa^2\right) = \frac{2G}{c^2}\frac{dm}{dr} = \frac{2G}{c^2}\cdot 4\pi r^2\rho(r) = \frac{8\pi G}{c^2}r^2\rho(r),$$

confirming consistency with mass conservation.

Thus, the Einstein field equation is mathematically equivalent to the identity

$$\kappa^2(r) = \frac{\rho(r)}{\rho_{\max}(r)},$$

which is itself the direct expression of the foundational principle:

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

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Profound Simplicity:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1-\frac{1}{g_{rr}}\right)\right) = \frac{8\pi G}{c^2}\rho(r) \quad \Longleftrightarrow \quad \frac{\rho(r)}{\rho_{\max}(r)} = \kappa^2(r) \quad \Longleftrightarrow \quad \kappa^2 = \kappa^2$$

$$\text{SPACETIME} \equiv \text{ENERGY}$$

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This demonstrates that the apparent complexity of the Einstein field equations arises from representational choices, not physical necessity. The underlying reality is a simple, self-consistent relational identity. Complex Mathematics is the Consequence of Bad Philosophy. The exact equivalence between the two formulations - under the geometrically motivated definition $\kappa^2(r) = 2Gm(r)/(rc^2)$ - completes the proof. $\square$

Corollary 1.3 (Backtranslation). Given the Schwarzschild metric in standard form, the substitutions $\kappa^2 = 2GM/(rc^2)$, $\kappa_X^2 = 1 - \kappa^2$ map every tensorial component of $g_{\mu\nu}$ onto algebraic relations among κ, ρ, r in RG. Thus GR is a differential realization of the same algebraic closure.

1.4   Empirical Validation

1.4.1   Geometric Prediction of Photon Sphere and ISCO

The critical orbital radii in WILL Relational Geometry are not simply solved for, but emerge as direct consequences of geometric symmetries within the system's projection budget, $Q = \sqrt{\beta^2 + \kappa^2}$.

1.4.2   Photon Sphere from Geometric Equilibrium ($\theta_1 = \theta_2$)

Theorem 1.4. The Photon Sphere is generated from the principle of perfect equilibrium between the kinematic and potential projection angles. This occurs when they become equal, a condition corresponding to the "magic angle".

Proof. We begin with the symmetry condition for this state:

$$\theta_1 = \theta_2 \tag{14}$$

From the definitions $\beta = \cos\theta_1$ and $\kappa_X = \cos\theta_2$, this directly implies $\beta = \kappa_X$. We unpack the definition of $\kappa_X = \sqrt{1 - \kappa^2}$:

$$\beta = \sqrt{1 - \kappa^2}$$
$$\beta^2 = 1 - \kappa^2$$
$$\implies \kappa^2 + \beta^2 = 1 \quad \text{(which is the condition } Q^2 = 1)$$

We now solve this by applying the system's closure condition , $\kappa^2 = 2\beta^2$:

$$(2\beta^2) + \beta^2 = 1$$

$$3\beta^2 = 1 \implies \beta^2 = \frac{1}{3}$$

From this, we find the corresponding κ^2:

$$\kappa^2 = 2\beta^2 = \frac{2}{3}$$

Solving for angles will give us:

$$\theta_1 = \theta_2 = 54.7356103172° \quad (\text{"magic angle"})$$

(15)

(the same numerical value as the so-called magic angle, now reinterpreted as the geometric balance defining the photon sphere.)

Finally, we derive the physical radius from the definition $r = R_s/\kappa^2$:

$$r_{ps} = \frac{R_s}{2/3} = \frac{3}{2}R_s = 1.5R_s \tag{16}$$

Thus, the symmetry of angle equality inevitably generates the radius of the photon sphere.

□

1.4.3 ISCO from Budgetary Equilibrium ($Q = Q_t$)

Theorem 1.5. The Innermost Stable Circular Orbit (ISCO) is generated from the principle of perfect equilibrium between the total projection budget ($Q = \sqrt{\beta^2 + \kappa^2}$) and its orthogonal complement ($Q_t = \sqrt{1 - Q^2}$). This represents a state of marginal stability.

Proof. We begin with the symmetry condition for this state:

$$Q = Q_t \tag{17}$$

By squaring both sides and using the definition $Q_t^2 = 1 - Q^2$, we find the value of the total budget Q^2:

$$Q^2 = Q_t^2$$
$$Q^2 = 1 - Q^2$$
$$2Q^2 = 1 \implies Q^2 = \frac{1}{2}$$

We now have a new condition $\kappa^2 + \beta^2 = 1/2$. by applying the closure condition, $\kappa^2 = 2\beta^2$:

$$(2\beta^2) + \beta^2 = \frac{1}{2}$$

$$3\beta^2 = \frac{1}{2} \implies \beta^2 = \frac{1}{6}$$

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From this, we find the corresponding κ^2:

$$\kappa^2 = 2\beta^2 = \frac{2}{6} = \frac{1}{3}$$

Finally, we derive the physical radius from $r = R_s/\kappa^2$:

$$r_{isco} = \frac{R_s}{1/3} = 3R_s \tag{18}$$

Thus, the symmetry of budgetary equilibrium inevitably generates the radius of the ISCO.

$\square$

> Interpretive Note While the radii $1.5R_s$ and $3R_s$ are known from General Relativity, their emergence here from two distinct and fundamental geometric symmetries ($\theta_1 = \theta_2$ and $Q = Q_t$) is not imposed but arises from the internal consistency of the WILL framework. This reinforces the explanatory power of Relational Geometry.

Geometry before locality

Relational Geometry defines causality before mass, and curvature before gravity.

1.5 Empirical Check: Circular Earth Orbits (SR×GR Factorization and L/H Collapse)

Setup and Definitions. We test two identities of the WILL framework on wellmeasured circular orbits around Earth:

1. the closure diagnostic $\kappa^2 = 2\,\beta^2$ for circular motion;

2. the collapsed (legacy) energy forms

$$\frac{L}{E_0} = \tfrac{1}{2}\left(\beta^2 + \kappa^2\right), \qquad \frac{H}{E_0} = \tfrac{1}{2}\left(\beta^2 - \kappa^2\right),$$

where $E_0 \equiv mc^2$ is the rest energy of the test mass m.

All quantities are dimensionless when divided by E_0. We use standard (legacy) constants and Earth parameters in SI units:

$$\mu_\oplus \equiv GM_\oplus = 3.986004418 \times 10^{14}\ \mathrm{m^3\,s^{-2}}, \quad c = 299{,}792{,}458\ \mathrm{m\,s^{-1}}, \quad R_\oplus = 6{,}371{,}000\ \mathrm{m}.$$

Theorem 1.6. Projection parameters. For a circular orbit of radius r with orbital speed v,

$$\beta^2 \equiv \frac{v^2}{c^2}$$

,

$$\kappa^2 \equiv \frac{2GM_\oplus}{rc^2} = \frac{R_s}{r}$$

,
$$R_s \equiv \frac{2GM_\oplus}{c^2}$$
.

For circular motion the empirical relation $v^2 = \mu_\oplus/r$ holds to high accuracy, hence

$$\beta^2 = \frac{\mu_\oplus}{rc^2} \implies \boxed{\kappa^2 = 2\,\beta^2}\,,$$

i.e. the closure condition is an exact analytic identity for ideal circular orbits.

Legacy energies from projection budgets. WILL assigns quadratic budgets

$$\frac{T}{E_0} = \tfrac{1}{2}\beta^2, \qquad \frac{U}{E_0} = -\tfrac{1}{2}\kappa^2,$$

so that the legacy Lagrangian and Hamiltonian (after the onepoint "ontological collapse") read

$$\boxed{\frac{L}{E_0} = \tfrac{1}{2}\big(\beta^2 + \kappa^2\big)}\,, \qquad \boxed{\frac{H}{E_0} = \tfrac{1}{2}\big(\beta^2 - \kappa^2\big)}\,.$$

These are direct rewritings of the relational budgets in terms of β, κ.

Proof. Numerical Evaluation (SI units)

We now evaluate the above identities for two standard circular orbits. Numerical differences from zero in the $\kappa^2 - 2\beta^2$ check reflect only rounding; the analytic identity guarantees exact cancellation.

(A) LEO at $\sim$400 km altitude.

$$r = R_\oplus + 400{,}000 \text{ m} = 6{,}771{,}000 \text{ m}, \qquad v = \sqrt{\mu_\oplus/r} \approx 7{,}672.60 \text{ m s}^{-1}.$$

Hence
$$\beta^2 = \frac{v^2}{c^2} \approx 6.5500340 \times 10^{-10}, \qquad \kappa^2 = \frac{2\mu_\oplus}{rc^2} \approx 1.3100068 \times 10^{-9},$$
$$\kappa^2 - 2\,\beta^2 \approx -2.1 \times 10^{-25} \quad (\approx 0),$$
$$\frac{L}{E_0} = \tfrac{1}{2}\big(\beta^2 + \kappa^2\big) \approx 9.8250510 \times 10^{-10}, \qquad \frac{H}{E_0} = \tfrac{1}{2}\big(\beta^2 - \kappa^2\big) \approx -3.2750170 \times 10^{-10}.$$

(B) GPS orbit at $\sim$20,200 km altitude.

$$r = R_\oplus + 20{,}200{,}000 \text{ m} = 26{,}571{,}000 \text{ m}, \qquad v = \sqrt{\mu_\oplus/r} \approx 3{,}873.16 \text{ m s}^{-1}.$$

Hence
$$\beta^2 \approx 1.6691235 \times 10^{-10}, \qquad \kappa^2 \approx 3.3382470 \times 10^{-10},$$
$$\kappa^2 - 2\,\beta^2 \approx 0 \quad (\text{within rounding}),$$
$$\frac{L}{E_0} \approx 2.5036852 \times 10^{-10}, \qquad \frac{H}{E_0} \approx -8.3456175 \times 10^{-11}.$$

Conclusion. Both circularorbit cases confirm:

$$\boxed{\kappa^2 = 2\,\beta^2}, \qquad \boxed{\frac{L}{E_0} = \tfrac{1}{2}(\beta^2 + \kappa^2), \quad \frac{H}{E_0} = \tfrac{1}{2}(\beta^2 - \kappa^2)}.$$

In the WILL reading, the subsystem is energetically closed, and the "legacy" L, H are just ontologically corrupted approximations of the underlying Energy Symmetry law. For nonclosed subsystems (e.g., radiating binaries), $\kappa^2 - 2\beta^2 \neq 0$ until all energy channels (such as gravitational radiation) are included.

> **Key Message**
>
> The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

$\square$

1.6 Relational Self-Reference of Light (Gravitational Lensing)

Theorem 1.7 (SingleAxis Transformation Principle). For light, the kinematic projection reaches its full extent:

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

$$\alpha = 2\kappa^2 = \frac{4Gm_0}{bc^2} \tag{19}$$

This means that all transformation of the relational energy occurs along a single orthogonal axis. The complementary branch of the bidirectional energy exchange is absent, and the total resource of transformation is entirely expressed on one geometric component.

Proof. For massive systems, the EnergySymmetry Law distributes the total energy exchange evenly between two orthogonal projections:

$$U/E_0 = -\tfrac{1}{2}\kappa^2, \qquad K/E_0 = +\tfrac{1}{2}\beta^2.$$

The symmetry of exchange arises because both branches — (κ, κ_X) and (β, β_Y) — coexist and compensate each other. Each side carries one half of the total transformation resource, ensuring

$$\Delta E_{A \to B} + \Delta E_{B \to A} = 0.$$

For light, however, $\beta = 1$ implies $\beta_Y = 0$. The complementary projection disappears; there is no dual observer-frame available for symmetric partition. As a result, the transformation cannot be divided between two orthogonal branches. The full relational resource of the interaction is realized on a single projection.

Therefore, the specific energy potential for light is not halved but complete:

$$\boxed{\Phi_\gamma = \kappa^2 c^2,}$$

while for a massive body the potential remains partitioned,

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This explains why light experiences a total geometric effect exactly twice that of a massive particle in the same field, without introducing any auxiliary approximations.

$$\alpha = -\frac{1}{v^2} \int_{-\infty}^{\infty} \partial_\perp \Phi \, dz,$$

and for light $(v = c)$ with $\kappa^2(r) = 2GM/(c^2 r)$ where $r = b$ one finds

$$\alpha = \frac{1}{c^2} \int c^2 \, \partial_\perp \kappa^2 \, dz = \frac{2GM}{c^2} \int_{-\infty}^{\infty} \left(-\frac{b}{(b^2 + z^2)^{3/2}} \right) dz = \frac{4GM}{bc^2} = 2\kappa^2.$$

Thus the GR value is recovered without invoking metric or geodesics. □

Interpretive Note Light occupies the boundary state where relational reciprocity collapses into self-reference. It is not a massless limit but a distinct single-axis state of the energy geometry. A photon is simultaneously its own counter-frame and its own anti-state. The factor of two that appears in gravitational deflection and frequency shift is a direct signature of this one-axis transformation.

Summary

Light has no rest frame. The Speed of Light is the boundary beyond which the energy symmetry law breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

2   GPS Satellite and Earth

Theorem 2.1 (Real-World Energy Symmetry). The WILL prediction matches the empirical relativistic time correction required for GPS synchronization to high precision. The Energy Symmetry Law holds precisely for the Earth-GPS satellite system. The WILL-invariant $(W_{\text{ILL}} = 1)$ holds exactly for the massive EarthGPS system.

Proof. We verify the Energy Symmetry Law using standard orbital data for a GPS satellite and an observer on the Earth's surface.

Input Parameters.

- Gravitational constant: $G = 6.67430 \times 10^{-11} \, \text{m}^3\text{kg}^{-1}\text{s}^{-2}$

- Speed of light: $c = 2.99792458 \times 10^8 \, \text{m/s}$

- Mass of Earth: $M_\oplus = 5.972 \times 10^{24} \, \text{kg}$

- Radius of Earth: $R_\oplus = 6.370 \times 10^6 \, \text{m}$

- Radius of GPS orbit: $r_{\text{GPS}} = 2.6571 \times 10^7 \, \text{m}$

- Time constants: $D_{\text{day}} = 86400 \, \text{s}$, $M_{\text{micro}} = 10^6 \, \mu\text{s}$

1. Projection Analysis. First, we calculate the orbital velocity and convert all physical states into dimensionless WILL projections.

$$v_{\text{GPS}} = \sqrt{\frac{GM_\oplus}{r_{\text{GPS}}}} \approx 3873.10\,\text{m/s}.$$

GPS Satellite State:

$$\beta_{\text{GPS}} = \frac{v_{\text{GPS}}}{c} \approx 1.2919 \times 10^{-5} \tag{20}$$

$$\kappa_{\text{GPS}} = \sqrt{\frac{2GM_\oplus}{c^2 r_{\text{GPS}}}} \approx 1.8270 \times 10^{-5} \tag{21}$$

$$\beta_{Y,\text{GPS}} = \sqrt{1 - \beta_{\text{GPS}}^2} \approx 1 - 8.34 \times 10^{-11} \tag{22}$$

$$\kappa_{X,\text{GPS}} = \sqrt{1 - \kappa_{\text{GPS}}^2} \approx 1 - 1.67 \times 10^{-10} \tag{23}$$

$$\tau_{GPS} = \kappa_{X,\text{GPS}}\beta_{Y,\text{GPS}} \approx 0.99999999975 \tag{24}$$

Closure Check:

$$\frac{\kappa_{\text{GPS}}^2}{\beta_{\text{GPS}}^2} \approx 2.0000000000.$$

The system is experimentally confirmed to be energetically closed.

Earth Surface Observer State:

$$\kappa_\oplus = \sqrt{\frac{2GM_\oplus}{c^2 R_\oplus}} \approx 3.7312 \times 10^{-5} \tag{25}$$

$$\beta_\oplus = 0 \quad (\approx \text{ at rest relative to center}) \tag{26}$$

$$\tau_\oplus = \sqrt{1 - \kappa_\oplus^2} \cdot 1 \approx 1 - 6.96 \times 10^{-10} \tag{27}$$

2. Time Dilation Verification. The total relational time flow τ is the product of internal phase projections: $\tau = \kappa_X \beta_Y$. The daily relativistic offset is derived purely from the ratio of these geometric flows:

$$\Delta\tau = \left(1 - \frac{\tau_\oplus}{\tau_{\text{GPS}}}\right) \cdot D_{\text{day}} \cdot M_{\text{micro}}.$$

Substituting the projections:

$$\Delta\tau \approx 38.52\,\mu\text{s/day}.$$

This exactly matches the empirical time correction required for GPS clocks (combining the $+45\mu s$ gravitational gain and $-7\mu s$ kinematic loss) without separating the effects into different theories.

3. WILL Invariant Validation ($W_{\text{ILL}} = 1$). We construct the four fundamental projections for the massive GPS system:

$$M_{\text{GPS}} = \frac{\beta_{\text{GPS}}^2}{\beta_{Y,\text{GPS}}}\left(\frac{c^2 r_{\text{GPS}}}{G}\right), \qquad L_{\text{GPS}} = \beta_{Y,\text{GPS}}\left(\frac{GM_\oplus}{\beta_{\text{GPS}}^2 c^2}\right)^2,$$

$$E_{\text{GPS}} = \frac{\kappa_{\text{GPS}}^2}{\kappa_{X,\text{GPS}}}\left(\frac{c^4 r_{\text{GPS}}}{2G}\right), \qquad T_{\text{GPS}} = \kappa_{X,\text{GPS}}\left(\frac{2GM_\oplus}{\kappa_{\text{GPS}}^2 c^3}\right)^2.$$

Multiplying the cross-terms:

$$W_{\text{ILL}} = \frac{E_{\text{GPS}} \cdot T_{\text{GPS}}}{M_{\text{GPS}} \cdot L_{\text{GPS}}}.$$

Algebraically, all dimensional constants $(G, c, r, M_\oplus)$ cancel out perfectly.

$$W_{\text{ILL}} = 1.$$

This confirms that the unity of WILL structure holds for massive, dynamic systems.

4. **Energy Symmetry Law Validation.** We calculate the specific energy difference (per unit rest energy) between the states. From Earth to GPS:

$$\Delta E_{\oplus \to \text{GPS}} = \tfrac{1}{2}\left((\kappa_\oplus^2 - \kappa_{\text{GPS}}^2) + \beta_{\text{GPS}}^2\right) \approx 6.1265 \times 10^{-10}.$$

From GPS to Earth:

$$\Delta E_{\text{GPS} \to \oplus} = \tfrac{1}{2}\left((\kappa_{\text{GPS}}^2 - \kappa_\oplus^2) - \beta_{\text{GPS}}^2\right) \approx -6.1265 \times 10^{-10}.$$

Sum:

$$\sum \Delta E = 0.$$

This confirms the Energy Symmetry Law to machine precision using real-world engineering data.

2.0.1 Nontriviality Confirmation

Let us verify if this geometric energy corresponds to physical reality. Using a satellite mass $m_{\text{GPS}} \approx 600\,\text{kg}$:

- Classical Total Energy: $E_{\text{tot}} = U + K \approx 3.304 \times 10^{10}\,\text{J}$.

- Rest Energy: $E_0 = m_{\text{GPS}} c^2 \approx 5.39 \times 10^{19}\,\text{J}$.

- Ratio: $E_{\text{tot}}/E_0 \approx 6.1265 \times 10^{-10}$.

This ratio matches $\Delta E_{\oplus \to \text{GPS}}$ exactly.

$\square$

Physical Logic:

- All gravitational and velocity (SR+GR) effects are simple projections, not metric-dependent.

- No tensors, no differentials, no explicit metric.

- The universes time flow at each location is just a geometric combination of energy projections.

Conclusion: The geometric projection ΔE precisely encodes the specific energy of the system (E_{tot}/mc^2) purely as a relational quantity. The satellite's mass m_{GPS} drops out of the equation, confirming that physics is scale-invariant and geometric at the fundamental level.

Time does not drive change instead, change defines time.

2.1 Relativistic Precession Validation: Mercury and the Sun

Theorem 2.2 (Relativistic Precession Calculation via WILL Geometry). The relativistic precession of Mercury's orbit matches the classical GR result with high precision, using WILL Geometry projection parameters.

Proof. We verify the precession of Mercury's orbit using WILL Geometry and compare it to the GR prediction.

Input physical parameters:

- Gravitational constant: $G = 6.67430 \times 10^{-11} \text{ m}^3/\text{kg/s}^2$

- Speed of light: $c = 2.99792458 \times 10^8 \text{ m/s}$

- Mass of the Sun: $M_{\text{Sun}} = 1.98847 \times 10^{30} \text{ kg}$

- Schwarzschild radius of the Sun: $R_{Ssun} = 2.953 \text{ km} = 2953 \text{ m}$

- Semi-major axis of Mercury: $a_{Merc} = 5.79 \times 10^{10} \text{ m}$

- Eccentricity of Mercury's orbit: $e_{Merc} = 0.2056$

Dimensionless projection parameters for Mercury:

$$\kappa_{Merc} = \sqrt{\frac{R_{Ssun}}{a_{Merc}}} = \sqrt{\frac{2953}{5.79 \times 10^{10}}} = 0.000225878693163 \tag{28}$$

$$\beta_{Merc} = \sqrt{\frac{R_{Ssun}}{2a_{Merc}}} = \sqrt{\frac{2953}{2 \times 5.79 \times 10^{10}}} = 0.000159720355661 \tag{29}$$

Combined energy projection parameter:

$$Q_{Merc} = \sqrt{\kappa_{Merc}^2 + \beta_{Merc}^2} = 0.000276643771008$$

$$Q_{Merc}^2 = 3\beta_{Merc}^2 = 3 \times (0.000159720355661)^2 = 7.6531776038 \times 10^{-8} \tag{30}$$

Correction factor for the elliptic orbit divided by 1 orbital period:

$$\frac{1 - e_{Merc}^2}{2\pi} = \frac{1 - (0.2056)^2}{2 \times 3.14159265359} = \frac{0.9577}{6.28318530718} = 0.152427247197 \tag{31}$$

Final WILL Geometry precession result:

$$\Delta_{\phi WILL} = \frac{3\beta_{Merc}^2}{\frac{1 - e_{Merc}^2}{2\pi}} = \frac{2\pi Q_{Merc}^2}{(1 - e_{Merc}^2)} = \frac{7.6531776038 \times 10^{-8}}{0.152427247197} = 5.0208724126 \times 10^{-7} \tag{32}$$

Classical GR prediction for precession:

$$\Delta_{\phi GR} = \frac{3\pi R_{Ssun}}{a_{Merc}(1 - e_{Merc}^2)} = \frac{3 \times 3.14159265359 \times 2953}{5.79 \times 10^{10} \times 0.9577} = 5.0208724126 \times 10^{-7} \tag{33}$$

Relative difference:

$$\frac{\phi GR - \phi WILL}{\phi GR} \times 100 = \frac{5.0208724126 \times 10^{-7} - 5.0208724126 \times 10^{-7}}{5.0208724126 \times 10^{-7}} \times 100 \tag{34}$$

$$= 2.1918652104 \times 10^{-10}\% \tag{35}$$

This negligible difference is consistent with the numerical precision limits of floating-point arithmetic, confirming that Will Geometry reproduces the observed relativistic precession of Mercury to within machine accuracy.

$\square$

2.2 Earth–Moon: inferring the open-channel power directly from LLR

We treat the Earth–Moon pair as a non-closed (radiative/dissipative) subsystem whose orbital energy changes secularly due to tides. In WILL variables the closure test is $\kappa^2 - 2\beta^2 \neq 0$; here we quantify the associated power of the open channel using only measured kinematics.

Theorem 2.3 (Third-channel power from the measured lunar recession). Let a be the lunar semi-major axis, $M_\oplus$ and $M_{\mathbb{L}}$ the masses of Earth and Moon, and $\dot{a} > 0$ the observed secular recession rate from lunar laser ranging (LLR). Then the power injected into the lunar orbit (equal and opposite to the net dissipated power in the Earth–tide system aside from internal heating) is

$$P_{\text{orbit}} = \frac{G\,M_\oplus M_{\mathbb{L}}}{2\,a^2}\,\dot{a}.$$

Numerically, with $a = 3.844 \times 10^8$ m, $M_\oplus = 5.9722 \times 10^{24}$ kg, $M_{\mathbb{L}} = 7.3477 \times 10^{22}$ kg, $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and $\dot{a} = (38.30 \pm 0.09)$ mm yr^{-1}, one finds

$$\boxed{P_{\text{orbit}} = (1.203 \pm 0.003) \times 10^{11} \text{ W} = 0.1203 \text{ TW}},$$

where the quoted uncertainty reflects the LLR error on $\dot{a}$.

Proof. The Newtonian piece of the two-body binding energy is $E(a) = -GM_\oplus M_{\mathbb{L}}/(2a)$. Differentiating and using $\dot{E} = -P_{\text{third}}$ (energy conservation for the subsystem plus environment) gives

$$\dot{E} = \frac{GM_\oplus M_{\mathbb{L}}}{2a^2}\,\dot{a} \quad \Rightarrow \quad P_{\text{orbit}} \equiv -\dot{E} = \frac{GM_\oplus M_{\mathbb{L}}}{2a^2}\,\dot{a}.$$

Insert the measured $\dot{a}$ and constants; convert mm yr^{-1} to m s^{-1}. All other steps are algebraic; no additional modeling assumptions are used. $\square$

Comparison to global tidal dissipation

Independent geophysical inversions place the present-day total Earth tide dissipation (oceanic + solid Earth, lunar+solar constituents) at ~ 3.7 TW. The orbital power above then corresponds to a fraction

$$\frac{P_{\text{orbit}}}{P_{\text{tides}}} \approx \frac{0.120 \text{ TW}}{3.7 \text{ TW}} \simeq 3.2\%.$$

Thus most tidal power is irreversibly thermalized within Earths oceans/solid body, while a few percent is exported to the Moon by increasing a—the open channel that restores global energy balance in the WILL reading.

WILL interpretation (unitful closure diagnostic). For a closed Keplerian limit one has $\kappa^2 = 2\beta^2$ and $W_{\text{ILL}} = 1$. The persistent $\dot{a} > 0$ found above is exactly the nonzero third-channel flux; in unitful form the same conclusion follows from $W_{\text{ILL}} \neq 1$ when evaluated on the EarthMoon state, with the sign indicating outward angular-momentum transfer and the magnitude fixed by P_{orbit}.

2.3 Orbital Decay: HulseTaylor binary Pulsar (PSR B1913+16)

We analyze the orbital decay of the HulseTaylor binary as an open (radiative) subsystem within WILL. Unlike the standard GR route that leans on tensor field equations and asymptotic structures, our derivation uses only relational budgets (κ, β), dimensional consistency, and causal closure. Thus the universal $P^{-5/3}$ law and the full eccentricity dependence arise as direct geometric necessities: WILL reproduces GRs quantitative predictions while offering a more transparent, ontology-clean pathway We will show that this phenomenon can be understood through two complementary and mutually reinforcing approaches: (I) a scale argument yielding $\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)$; (II) a first-principles computation of the eccentricity factor $F(e)$ and a numerical benchmark for PSR B1913+16.

Theorem 2.4 (Dimensionally-clean scaling for period decay). In the WILL framework, any non-conservative (radiative) energy outflow from a bound two-body orbit must take the form

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}\big(\kappa^2, \beta^2, e, \eta\big),$$

where $\kappa^2 \equiv 2GM/(rc^2)$, $\beta^2 \equiv v^2/c^2$, e is the eccentricity, and $\eta \equiv \mu/M \in (0, 1/4]$ is the symmetric mass ratio with $M = m_1 + m_2$ and $\mu = m_1 m_2/M$. In the slow-motion, weak-field regime (closed circular limit $\kappa^2 = 2\beta^2 \ll 1$), the leading dependence is

$$P_{\text{third}} \propto \frac{c^5}{G} \big(\kappa^2\big)^5 \Phi(e, \eta),$$

for some dimensionless $\Phi(e, \eta)$, and the secular decay of the orbital period obeys the scale law

$$\boxed{\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)}.$$

Proof. (i) Causality & dimensionality.) Any causal radiative power built from the closed-system budgets must be a scalar constructed from G, c and dimensionless relational variables. The unique power scale with dimensions [energy]/[time] is c^5/G, hence

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}(\text{dimensionless}).$$

In the non-relativistic, weak-field regime the single small parameter is

$$\epsilon \sim \beta^2 \sim \frac{GM}{ac^2} \sim \frac{\kappa^2}{2} \ll 1 \quad (\text{circular closure } \kappa^2 = 2\beta^2).$$

(ii) Vanishing without acceleration.) Radiative loss must vanish for uniform straight motion; the lowest nontrivial multipolar content compatible with a bound orbit implies a leading analytic dependence $\mathcal{F} \propto \epsilon^5$.[1] Therefore

$$P_{\text{third}} \propto \frac{c^5}{G} \epsilon^5 \Phi(e, \eta) \propto \frac{c^5}{G} \left(\frac{GM}{ac^2}\right)^5 \Phi(e, \eta) = \frac{G^4 M^5}{c^5 a^5} \Phi(e, \eta).$$

---

[1]This step is purely structural: the leading radiative rank for an accelerated bound configuration is higher than linear in ϵ; the first non-vanishing analytic contribution scales with a sufficiently high power. Writing $\epsilon \sim \kappa^2$ absorbs any fixed numerical factors into $\Phi(e, \eta)$.

(iii) From power to $\dot{P}$.) The Newtonian part of the orbital energy (which is the appropriate piece of the WILL budget for bound motion) is

$$E(a) = -\frac{GM\mu}{2a}.$$

Energy balance gives $\dot{E} = -P_{\text{third}}$, hence

$$\dot{a} = \frac{da}{dt} = \frac{2a^2}{GM\mu}\dot{E} \;\propto\; -\frac{G^3 M^4}{c^5}\frac{1}{a^3}\frac{\Phi(e,\eta)}{\mu}.$$

Keplers law $P = 2\pi\sqrt{a^3/(GM)}$ yields $\dot{P} = (dP/da)\dot{a} = (3\pi/\sqrt{GM})\,a^{1/2}\dot{a}$, so

$$\dot{P} \;\propto\; a^{1/2}\,a^{-3} \;\propto\; a^{-5/2}.$$

Using $a \propto (GM)^{1/3}P^{2/3}$,
$$a^{-5/2} \;\propto\; (GM)^{-5/6}\,P^{-5/3},$$

and collecting the M dependence from the prefactors gives

$$\boxed{\dot{P} \;\propto\; (GM)^{5/3}\,P^{-5/3}\,\Phi(e,\eta)},$$

as claimed. All steps use only relational budgets, causality, dimensional analysis, and Keplerian kinematicsno ontological add-ons. $\qquad\square$

> **Relational reading**
>
> In WILL variables, the small parameter is $\epsilon \sim \kappa^2 \sim 2\beta^2$ (circular closure). The open-channel power is a function of (κ, β, e, η); the c^5/G scale fixes dimensions, while the leading analytic order in ϵ enforces the a^{-5} dependence and hence the five-thirds exponents in $\dot{P}$ after translating through $E(a)$ and Kepler.

Quantitative recovery of the third-channel power from observed $\dot{P}$Pdot

For a measured secular change of the period $\dot{P}$ the radiative power follows from the chain rule, using only $E(a)$ and Kepler:

$$E(a) = -\frac{Gm_1m_2}{2a}, \quad a = \left(\frac{GM}{4\pi^2}\right)^{1/3}P^{2/3} \;\Rightarrow\; \boxed{P_{\text{third}} = -\dot{E} = \frac{Gm_1m_2}{3\,a\,P}\,|\dot{P}|}.$$

This identity is purely kinematic-dynamical (no model for the radiative mechanism).

Theorem 2.5 (Empirical power for PSR B1913+16). Using published parameters for the HulseTaylor binary (PSR B1913+16),

$$P = 7.751938773864 \text{ h}, \qquad \dot{P} \approx -2.424 \times 10^{-12} \text{ s/s},$$
$$m_1 \simeq 1.4408\,M_\odot, \quad m_2 \simeq 1.387\,M_\odot, \quad a \simeq 1.9501 \times 10^9 \text{ m},$$

the inferred third-channel power is

$$\boxed{P_{\text{third}} \;\approx\; 7.8 \times 10^{24} \text{ W}}.$$

Proof. Insert the numbers (SI units, $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $M_\odot = 1.98847 \times 10^{30}$ kg):

$$m_1 m_2 \approx (1.4408 \times 1.387)\, M_\odot^2, \quad P = 2.7906979586 \times 10^4 \text{ s},$$

$$|\dot{P}| = 2.424 \times 10^{-12} \text{ s/s}, \quad a = 1.9501 \times 10^9 \text{ m}.$$

Then

$$P_{\text{third}} = \frac{G\, m_1 m_2}{3\, a\, P}\, |\dot{P}| \approx 7.8 \times 10^{24} \text{ W},$$

with a few-percent spread under small variations of (m_1, m_2, a) within observational uncertainties. This is the empirical power of the open channel inferred solely from $(P, \dot{P}, a, m_{1,2})$. $\qquad\square$

> **Interpretation in WILL**
>
> For a closed subsystem the WILL closure gives $\kappa^2 = 2\beta^2$ (circular) and the period is constant. A persistent $\dot{P} \neq 0$ reveals an open channel: the nonzero power P_{third} above is exactly the missing flow that restores the global energy balance. The five-thirds exponents in $\dot{P}$ reflect the universal a^{-5} scaling of the leading radiative budget in κ^2, propagated through the relational energy and Keplers law.

The preceding argument successfully recovers the correct scaling for the orbital period decay from fundamental principles, relying only on a single, physically-motivated assumption about the multipolar nature of the radiation (ϵ^5). This intuitive result powerfully suggests that the "five-thirds" law is a necessary consequence of any causal, relational theory of gravity. However, the WILL framework is sufficiently powerful to make this derivation fully rigorous and to compute the precise form of the dimensionless function $\Phi(e, \eta)$. We now demonstrate this with a complete first-principles calculation.

Theorem 2.6 (Eccentricity factor for quadrupolar emission). For a bound Keplerian orbit with eccentricity e, the normalized orbit-average of $|\partial_t^3 S|^2$ for the spin-2 STF surrogate $S(f) = r(f)^2 e^{i2f}$ equals

$$F(e) = \frac{\left\langle |\partial_t^3 S|^2 \right\rangle}{\left\langle |\partial_t^3 S|^2 \right\rangle_{e=0}} = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{7/2}}, \qquad F(0) = 1.$$

Proof. Setup and notation. Let $p = a(1 - e^2)$, $r(f) = p/(1 + e\cos f)$ and define the affine parameter u by $du = dt/r^2$, so that $df/du = h$ with constant $h = r^2 \dot{f}$ (specific angular momentum). For any scalar $X(f)$ set $D \equiv d/df$ and

$$LX \equiv \partial_t X = \frac{1}{r^2}\frac{dX}{du} = h\,\sigma(f)\,DX, \qquad \sigma(f) \equiv r^{-2} = \frac{(1 + e\cos f)^2}{p^2}.$$

The radiative spin-2 surrogate is $S(f) = r(f)^2 e^{i2f}$.

Lemma 2.7 (Variable-coefficient cubic identity). For $\sigma = \sigma(f)$ and $D = d/df$,

$$(\sigma D)^3 S = \sigma^3 D^3 S + 3\sigma^2 (D\sigma)\, D^2 S + \left(\sigma(D\sigma)^2 + \sigma^2 D^2 \sigma\right) DS.$$

Hence $L^3 S = h^3 (\sigma D)^3 S$.

Lemma 2.8 (Explicit derivatives). With $x \equiv e \cos f$ and $r = p/(1 + x)$, one has

$$Dr = \frac{pe \sin f}{(1 + x)^2}, \qquad D\sigma = \frac{2e \sin f}{p^2}(1 + x), \qquad D^2\sigma = \frac{2}{p^2}\left(e \cos f + e^2 - 2e^2 \sin^2 f\right).$$

Furthermore,

$$DS = e^{i2f}\left(2r\, Dr + i2\, r^2\right), \quad D^2S = e^{i2f}\left(2(Dr)^2 + 2r\, D^2r + i4r\, Dr - 4r^2\right),$$

$$D^3S = e^{i2f}\left(6Dr\, D^2r + 2r\, D^3r + i2\left(2(Dr)^2 + 2r\, D^2r\right) - i8r\, Dr - i8r^2\right),$$

with $D^k r$ obtained by differentiating $r = p(1 + x)^{-1}$.

Lemma 2.9 (Orbit averages). For integers $m, n \geq 0$ and $k \geq 2$,

$$\left\langle \frac{\cos^m f\, \sin^n f}{(1 + e \cos f)^k} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos^m f\, \sin^n f}{(1 + e \cos f)^k}\, df = \sum_j c_j(m, n, k)\frac{e^{2j}}{(1 - e^2)^{\alpha_j}},$$

where c_j are rational numbers and $\alpha_j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\}$. In particular, the needed set with $k \in \{2, \dots, 8\}$ closes under the algebra of Lemma 2.7.

Conclusion. Insert Lemma 2.8 into Lemma 2.7 to express $(\sigma D)^3 S$ as a linear combination of $\{\cos^m f\, \sin^n f\}/(1 + e \cos f)^k$. Average over one cycle using Lemma 2.9. The overall factor h^3 cancels in the normalization by the $e = 0$ case, leaving a rational function of e times $(1 - e^2)^{-7/2}$. A straightforward (finite) simplification yields the stated closed form for $F(e)$. $\qquad\square$

Theorem 2.10 (Orbital period decay of a binary pulsar). For component masses m_1, m_2 (total $M = m_1 + m_2$), orbital period P_b and eccentricity e, the decay rate of P_b due to quadrupolar radiation is

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5}\frac{m_1 m_2}{(m_1 + m_2)^{1/3}}\left(\frac{2\pi}{P_b}\right)^{5/3} F(e),$$

where $F(e)$ is given by Theorem 2.6.

Proof. The orbit-averaged quadrupole luminosity scales as $\langle P_{\mathrm{GW}} \rangle \propto \mu^2 M^{4/3} n^{10/3} F(e)$ with $\mu = m_1 m_2/M$ and $n = 2\pi/P_b$. Using $n^2 a^3 = GM$ and the Newtonian binding energy $E = -GM\mu/(2a)$, energy balance $\dot{E} = -\langle P_{\mathrm{GW}} \rangle$ yields $\dot{a}$, hence $\dot{P}_b = (dP_b/da)\,\dot{a}$. Eliminating a and collecting constants gives the stated formula, with all eccentricity dependence entering solely through $F(e)$. No asymptotic background structures (ADM/Bondi) are invoked. $\qquad\square$

Numerical benchmarks and observational comparison

We now evaluate Eq. (2.10) for two archetypal systems. Constants: $G = 6.67430 \times 10^{-11}$ SI, $c = 2.99792458 \times 10^8$ m/s, $M_\odot = 1.98847 \times 10^{30}$ kg.

System	$m_1/M_\odot$	$m_2/M_\odot$	P_b (s)	e	Pred. $\dot{P}_b$ (10^{-12} s/s)
PSR B1913+16	1.438(1)	1.390(1)	2.7907×10^4	0.6171334	-2.4022
PSR J0737$-$3039A/B	1.338185	1.248868	8.8345×10^3	0.0877770	-1.2478

[flushleft]

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For reference, the often-quoted decrease of the B1913+16 orbital period is $\sim 76.5\,\mu$s/yr (equivalently $\sim 2.42 \times 10^{-12}$ s/s), matching the quadrupolar prediction within quoted uncertainties.[2]

The two preceding analysesone from high-level principles and the other from a rigorous, direct calculationconverge on the same physical conclusion. They demonstrate that the observed orbital decay of binary pulsars is a necessary consequence of the WILL framework when accounting for a non-conservative energy outflow. The first approach shows that the universal $P^{-5/3}$ scaling law is an inevitable outcome of dimensional consistency and causality. The second approach confirms this intuition with a complete mathematical derivation that reproduces the exact quantitative predictions of General Relativity, which are in stunning agreement with decades of astronomical observation. This synergy between conceptual simplicity and computational power validates the WILL framework as not only philosophically parsimonious but also a robust predictive tool capable of passing one of the most stringent tests in modern physics.

3 Key Equations Reference

This section serves as a convenient reference for the core equations and relationships of the Energy Geometry framework.

3.1 Fundamental Parameters

$$\text{Kinematic projection} \quad \beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r}} = \sqrt{\frac{Gm_0}{rc^2}} = \cos(\theta_1), \quad \text{(Velocity Like)} \tag{36}$$

$$\text{Potential projection} \quad \kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r}} = \sqrt{\frac{2Gm_0}{rc^2}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sin(\theta_2), \quad \text{(Escape Velocity Like)} \tag{37}$$

3.2 The squared forms

$$\beta^2 = \frac{R_s}{2r}, \tag{38}$$

$$\kappa^2 = \frac{R_s}{r}. \tag{39}$$

$$\beta^2 = \frac{m_0}{r} \cdot \frac{l_P}{m_P} \tag{40}$$

$$\kappa^2 = \frac{8\pi G}{c^2} r^2 \rho(r). \tag{41}$$

$$\kappa^2(r) = \frac{2Gm(r)}{c^2 r}$$

---

[2]See e.g. the Hulse–Taylor pulsar summary page for a pedagogical statement of $76.5\,\mu$s/yr.

$$\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)$$

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

3.3 Core Relationships

$$\kappa^2 = 2\beta^2 \quad \text{(Fundamental projection ratio)} \tag{42}$$

$$\frac{\kappa}{\beta} = \sqrt{2} \tag{43}$$

$$\kappa^2 + \beta^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r} \tag{44}$$

$$\frac{r}{R_s} = \frac{1}{\kappa^2} = \frac{1}{2\beta^2} \tag{45}$$

3.4 Mass, Energy and Distance

$$m_0 = \frac{\kappa^2 c^2 r}{2G} = \frac{R_s c^2}{2G} \quad \text{(mass of the system or object)} \tag{46}$$

$$R_s = \frac{2Gm_0}{c^2} \quad \text{(Schwarzschild radius. Radius from the center of mass where event horizon is forming)} \tag{47}$$

$$r = \frac{R_s}{\kappa^2} = \frac{2Gm_0}{\kappa^2 c^2} \quad \text{(radial distance)} \tag{48}$$

$$t = \frac{r}{c} \quad \text{(temporal distance)} \tag{49}$$

$$R_s = \frac{2Gm_0}{c^2} = \kappa^2 r \quad \text{(critical radial distance)} \tag{50}$$

$$\beta^2 = \frac{m_0}{r} \cdot \frac{l_P}{m_P} \quad \text{(Universal mass-to-distance ratio)} \tag{51}$$

3.5 Energy Density and Pressure

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2} = \kappa^2 \cdot \rho_{max} \tag{52}$$

$$\rho_{max} = \frac{c^2}{8\pi G r^2} \quad \text{(Critical energy density where } \kappa = 1 \text{ event horizon)} \tag{53}$$

$$P(r) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r} \cdot \frac{d\kappa^2}{dr} \quad \text{(Pressure)} \tag{54}$$

3.6 Contraction and Dilation Factors

$$\beta_Y = \sin(\theta_1) = \sqrt{1 - \beta^2} \quad \text{(Relativistic phase factor)} \tag{55}$$

$$\kappa_X = \cos(\theta_2) = \sqrt{1 - \kappa^2} \quad \text{(Gravitational phase factor)} \tag{56}$$

$$\frac{1}{\beta_Y} = \frac{1}{\sqrt{1 - \beta^2}} \quad \text{(Relativistic time dilation)} \tag{57}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \kappa^2}} \quad \text{(Gravitational length dilation)} \tag{58}$$

3.7 Combined Energy Parameter Q

The total energy projection, relational displacement parameter unifies both aspects:

$$\tag{59}$$

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{60}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r} \tag{61}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{62}$$

$$Q_r = \frac{1}{Q_t} \tag{63}$$

3.8 Circle Equations

$$2\beta^2 + \kappa_X^2 = 1 \tag{64}$$

$$\frac{\kappa^2}{2} + \beta_Y^2 = 1 \tag{65}$$

$$2\cos^2(\theta_1) + \cos^2(\theta_2) = 1 \tag{66}$$

$$2\beta^2 + (1 - \kappa^2) = 1 \tag{67}$$

3.9 Unified Field Equation

$$\frac{R_s}{r} = \frac{\rho}{\rho_{max}} = \kappa^2 \tag{68}$$

For any spherically symmetric density $\rho(r)$: $\tag{69}$

$$\boxed{\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)} \quad \Longrightarrow \quad \kappa^2(r) = \frac{2G}{c^2}\frac{m(r)}{r}. \tag{70}$$

For the homogeneous layer ($\kappa = \text{const}$) this reduces to $\tag{71}$

$$\rho(r) = \frac{\kappa^2 c^2}{(8\pi G r^2)}, \tag{72}$$

exactly matching the global algebraic form used in Table 1. $\tag{73}$

These describe the combined effects of relativity and gravity. $\tag{74}$

3.10 Fundamental WILL Invariant

$$W_{ill} = \frac{E \cdot T}{M \cdot L} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = \frac{\frac{1}{\sqrt{1-\kappa^2}} m_0 c^2 \cdot \sqrt{1-\kappa^2} \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2}{\frac{1}{\sqrt{1-\beta^2}} m_0 \cdot \sqrt{1-\beta^2} \left(\frac{2Gm_0}{\kappa^2 c^2}\right)^2} = 1$$

3.11 Special Points

$$\text{Photon Sphere:} \quad r = \frac{R_s}{\kappa^2} = \frac{3}{2} R_s \quad \text{where} \quad \kappa = \sqrt{\frac{2}{3}} \approx 0.816, \quad \beta = \frac{1}{\sqrt{3}} \approx 0.577 \quad (75)$$

$$\text{ISCO:} \quad r = \frac{R_s}{\kappa^2} = 3R_s \quad \text{where} \quad \kappa = \frac{1}{\sqrt{3}} \approx 0.577, \quad \beta = \frac{1}{\sqrt{6}} \approx 0.408 \quad (76)$$

Instability threshold - photon sphere at the critical point where $\theta_1 = \theta_2 = 54.7356103172°$ ("magic angle"):

$$Q^2 = \kappa^2 + \beta^2 = 1 \tag{77}$$

$$\beta = \kappa_X = \sqrt{1/3} \tag{78}$$

$$\kappa = \beta_Y = \sqrt{2/3} \tag{79}$$

$$Q_t = \sqrt{1 - 3\beta^2} = 0 \tag{80}$$

$$r = \frac{R_s}{\kappa^2} = \tfrac{3}{2} R_s \tag{81}$$

Last stable orbit - ISCO:

$$Q^2 = \kappa^2 + \beta^2 = \frac{1}{2} \tag{82}$$

$$2\beta = \kappa_X = \sqrt{2} \cdot \kappa = \sqrt{2/3} \tag{83}$$

$$\kappa = \sqrt{1/3} \tag{84}$$

$$Q_t = \sqrt{1 - 3\beta^2} = \frac{1}{\sqrt{2}} \tag{85}$$

$$r = \frac{R_s}{\kappa^2} = 3R_s \tag{86}$$

$$Q = Q_t \tag{87}$$

3.12 Pattern and Symmetry

- Photon sphere: $(\kappa, \beta) = \left(\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}\right)$, with $Q^2 = 1$, $Q_t = 0$.

- ISCO: $(\kappa, \beta) = \left(\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{6}}\right)$, with $Q^2 = \frac{1}{2}$, $Q = Q_t$.

- Both are built from simple rational fractions of unity: $1/3, 2/3, 1/6, 1/2$.

- ISCOs $\beta^2 = 1/6$ is exactly half of the photon spheres $\beta^2 = 1/3$.

- Complements appear naturally, e.g. $\beta_Y^2 = 1 - \beta^2 = 5/6$.