QUANTUM MECHANICS IS NOT POSTULATED. IT IS DERIVED.

DESCRIPTIVE

- QUANTIZATION POSTULATE
- WAVEFUNCTION AXIOM
- BORN RULE
- MEASUREMENT POSTULATE

Standard quantum mechanics describes reality through a set of postulates—axioms taken as given. Its formalism is powerful, but it does not explain *why* quantization exists. **Relational Geometry (RG)** offers a generative alternative.

GENERATIVE

- GEOMETRIC CLOSURE
- TOPOLOGICAL NECESSITY
- ONTOLOGICAL UNITY

WE WILL DEMONSTRATE THAT THE ENTIRE STRUCTURE OF THE ATOM—QUANTIZED ENERGY LEVELS, AND EVEN THE FINE STRUCTURE CONSTANT—EMERGES AS A A NECESSARY CONSEQUENCE OF A SINGLE ONTOLOGICAL PRINCIPLE:

SPACETIME ≡ ENERGY.

Bad philosophy creates complexity. Good philosophy reveals geometry.

The Foundational Principle and Its Geometric Carriers

The RG framework begins by removing the false separation between structure (spacetime) and dynamics (energy).

SPACETIME ≡ ENERGY

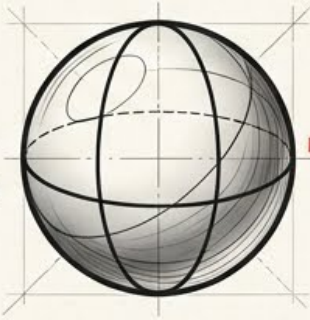
This is not an equation but an ontological identity. All physical phenomena are projections of this single, self-contained resource.

S1 (Circle)

The carrier for directional (kinematic) transformation, described by the projection

$\beta = v/c$

S2 (Sphere)

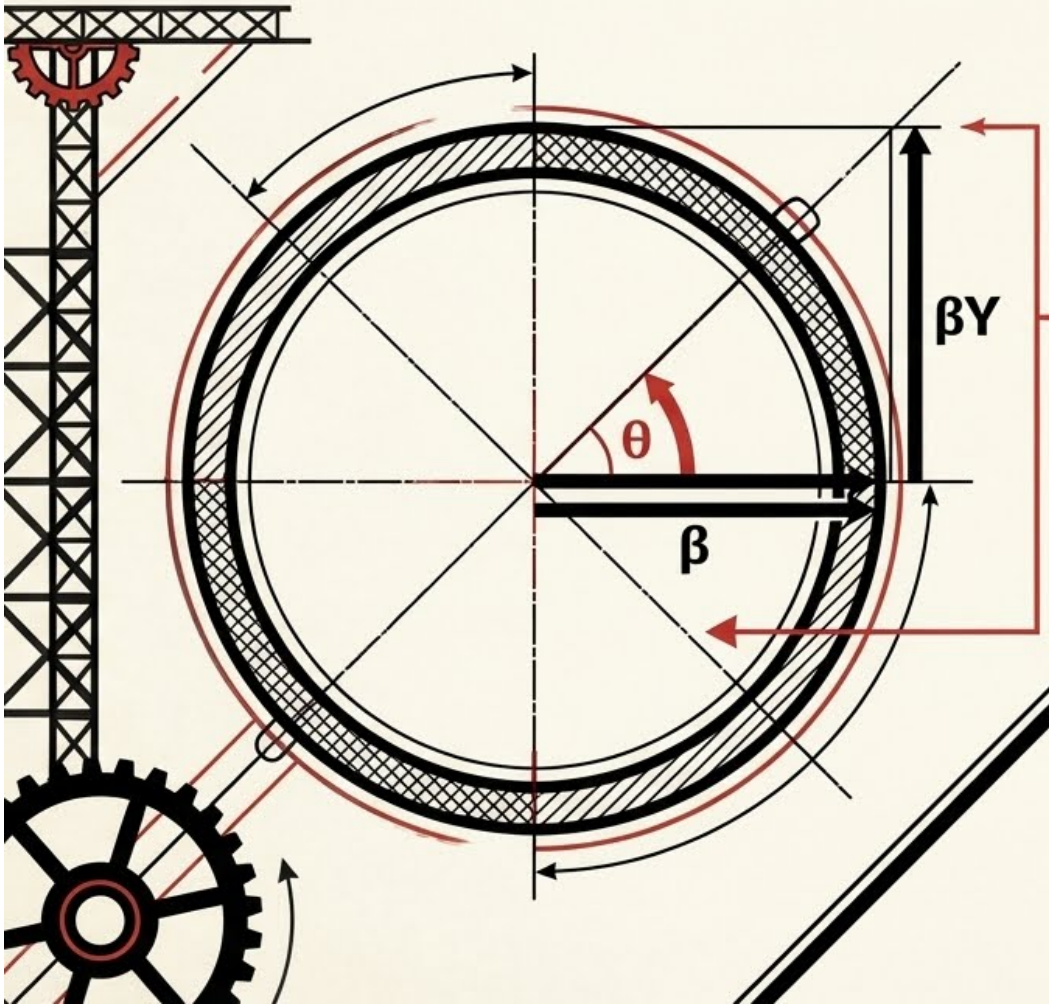


The carrier for omnidirectional (potential) transformation, described by the projection

$\kappa = v_e/c$

The de Broglie Relation is an Inevitable Consequence of S1 Geometry

The de Broglie relation is not a postulate, but a direct result of the geometry of the S1 carrier.



1. Momentum as a Projection:

$$p = \frac{E_0}{c}\left(\frac{\beta}{\beta_Y}\right)$$

2. Wavelength as Inverse Projection:

$$\lambda = \Lambda_0\left(\frac{\beta_Y}{\beta}\right)$$

3. The Invariant Product:

$$p\lambda = \frac{E_0}{c}\left(\frac{\beta}{\beta_Y}\right)\Lambda_0\left(\frac{\beta_Y}{\beta}\right) = \boxed{p\lambda = \frac{E_0}{c}\Lambda_0}$$

By calibrating the phase-length so that one full 2π winding corresponds to one quantum of action, we set $\frac{E_0}{c}\Lambda_0 = h$. The de Broglie relation $p\lambda = h$ is thus established as a geometric identity.

$$\boxed{p\lambda = h}$$

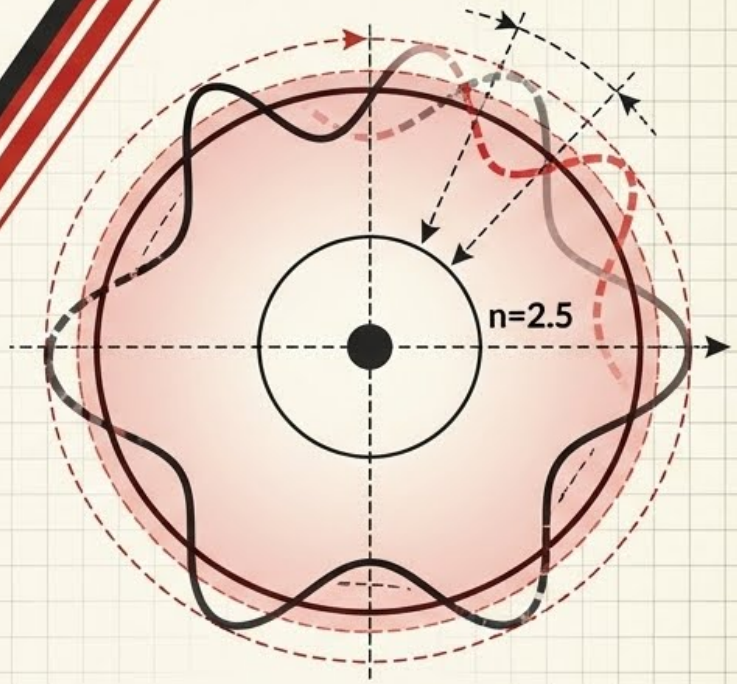
The First Engine of Quantization: Topological Closure

$$n\lambda = 2\pi r$$

This is not an ad-hoc rule, but a requirement for constructive interference.

Allowed State: n = 1, 2, 3...

Forbidden State:
Geometric Inconsistency



****The Origin of the Quantum Number*:** The principal quantum number *n* is not an arbitrary label but the **topological winding number** of the energy projection's phase. It counts the complete rotations required for the geometric configuration to close on itself.

The Second Engine: The Universal Scale Principle

$$\kappa^2 = \frac{r_{critical}}{r_{current}}$$

GRAVITY

$$\kappa_G^2 = \frac{R_s}{r}$$

The familiar Schwarzschild radius R_S defines the gravitational critical scale.

ELECTROMAGNETISM

$$\kappa_q^2 = \frac{Rq}{r}$$

We define an analogous electromagnetic critical radius Rq, the scale of energetic saturation.

Derivation of the Electromagnetic Critical Radius (Rq)

The saturation condition is $|U(Rq)| = (1/2)E_0$.

1. $\frac{e^2}{4\pi\varepsilon_0 Rq} = (1/2)m_e c^2$
2. Solving for Rq gives:

$$Rq = \frac{2e^2}{4\pi\varepsilon_0 m_e c^2}$$

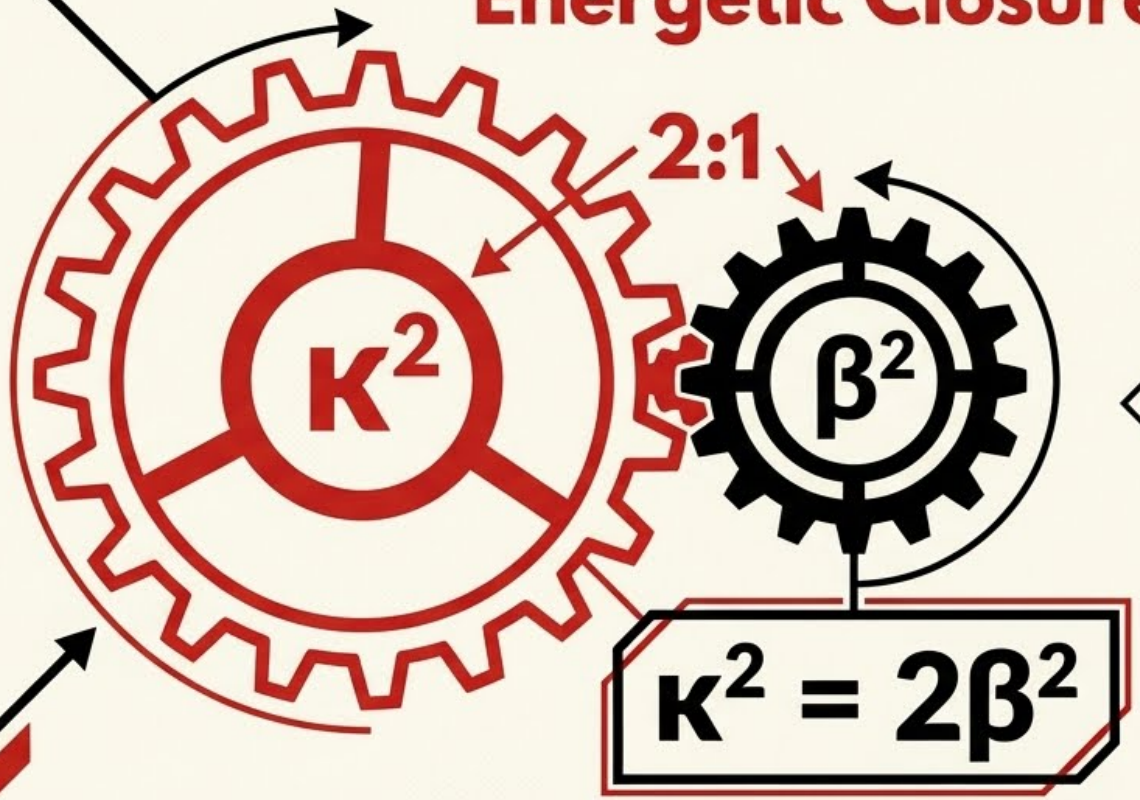
The Third Engine: Energetic Closure

The unified energy resource must be consistently partitioned between its potential (κ) and kinetic (β) modes.

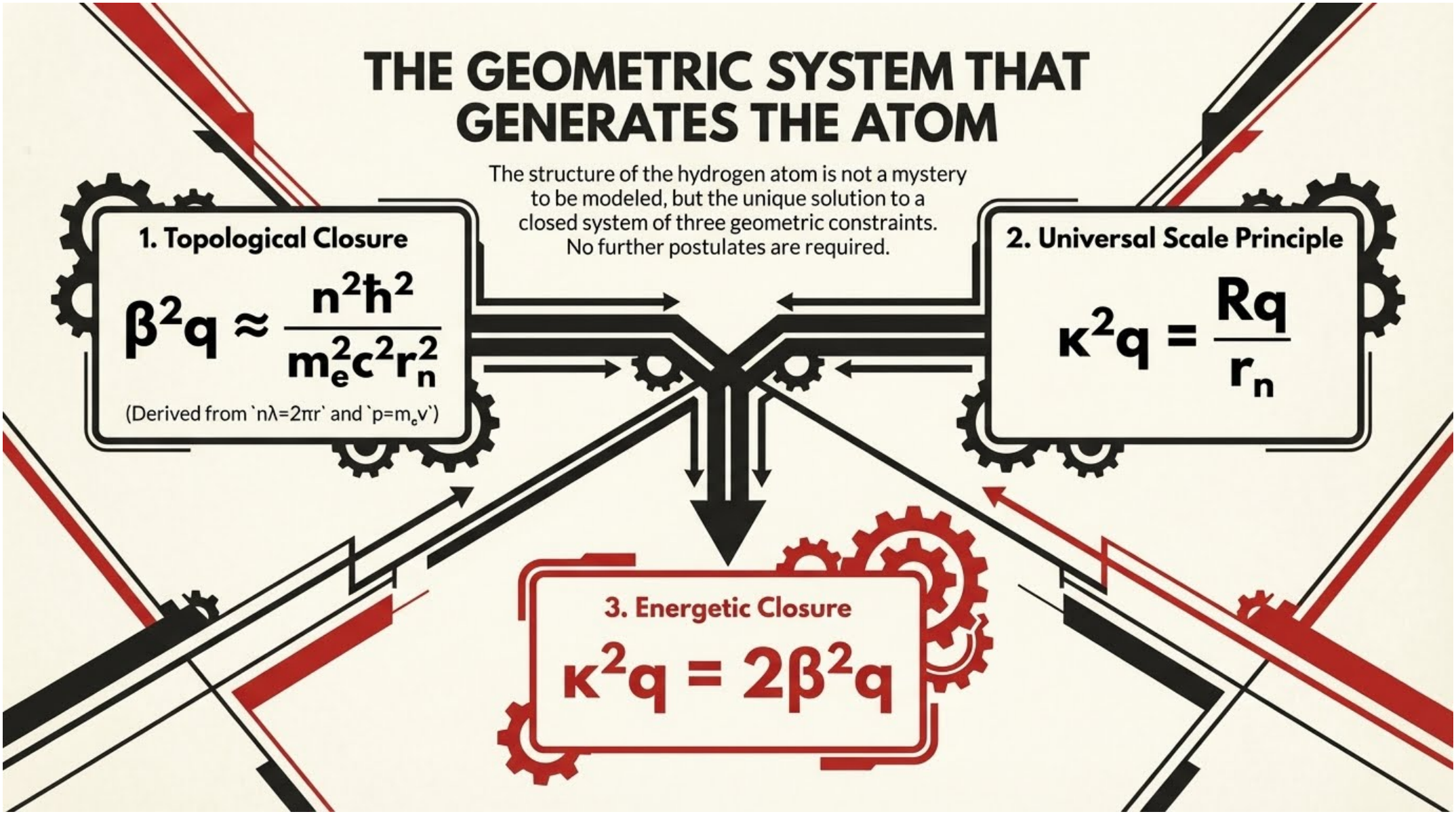
The "exchange rate" is fixed by the geometry of their respective carriers (S2 and S1).

The Rule of Balance

The ratio of the degrees of freedom of the carriers (**d.o.f.(S2)/d.o.f.(S1) = 2/1 = 2**) dictates the energetic balance.



This is the relational analogue of the **Virial Theorem.** It is a strict condition for any stable, closed system and the final constraint needed to solve for the structure of the atom.


THE GEOMETRIC SYSTEM THAT GENERATES THE ATOM
The structure of the hydrogen atom is not a mystery to be modeled, but the unique solution to a closed system of three geometric constraints. No further postulates are required.
1. Topological Closure
$\beta^2 q \approx \frac{n^2\hbar^2}{m_e^2 c^2 r_n^2}$
(Derived from `nλ=2πr` and `p=m_ev`)
2. Universal Scale Principle
$\kappa^2 q = \frac{Rq}{r_n}$
3. Energetic Closure
$\kappa^2 q = 2\beta^2 q$

Derivation of Quantized Atomic Radii

By solving the geometric system, the allowed orbital radii are strictly determined.

1. Substitute (1) into (3):

$$\kappa^2 q = 2\left[\frac{n^2\hbar^2}{m_e^2 c^2 r_n^2}\right]$$

$$\frac{Rq}{r_n} = \frac{2n^2\hbar^2}{m_e^2 c^2 r_n^2}$$

2. Equate with (2):

$$\frac{Rq}{r_n} = \frac{Rq}{r_n}$$

3. Solve for r_n:

$$r_n = \frac{2n^2\hbar^2}{m_e^2 c^2 Rq}$$

4. Substitute the value for Rq:

$$= \frac{2n^2\hbar^2}{m_e^2 c^2}\left[\frac{4\pi\varepsilon_0 m_e c^2}{2e^2}\right]$$

CANCEL

CANCEL

$$r_n = n^2 \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$$

The Bohr radius, a_0, emerges as the geometric constant for the ground state ($n = 1$):

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$$

The Fine Structure Constant is the Ground-State Velocity of the Electron

The fine structure constant α has historically been a purely empirical number (≈ 1/137). **RG reveals its ontological identity.**

Calculation:

We calculate the electron's kinetic projection β_1 in its ground state (n=1).

1. From Energetic Closure:
   $\beta_1^2 = (1/2)\kappa_1^2$
2. From Scale Principle:
   $\kappa_1^2 = R_q / a_0$
3. Combine:
   $$\beta_1^2 = \frac{1}{2} \cdot \frac{R_q}{a_0}$$

$\beta_1 = \alpha$

Substitution:

4. Substitute the derived expressions for R_q and a_0:
   $$\beta_1^2 = \frac{1}{2} \cdot \left[\frac{2e^2}{4\pi\varepsilon_0 m_e c^2}\right] \cdot \left[\frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2}\right]$$

Simplification:

Simplifying the expression yields:

$$\beta_1^2 = \frac{e^4}{[(4\pi\varepsilon_0)^2 \hbar^2 c^2} = \left[\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right]^2$$

We recognize the term in the brackets as the definition of the fine structure constant, α.

$\beta_1 = \alpha$. The fine structure constant is not fundamental. It is the geometric ratio defining the electron's kinetic state, fixed by the self-consistency of the atom.

Deriving the Complete Atomic Energy Spectrum

With $\beta_1 = \alpha$, the entire energy structure of the atom follows.

Derivation of Energy Levels

1. The kinetic projection for any level n scales as $\beta_n = \frac{\beta_1}{n} = \frac{\alpha}{n}$.
2. The **Energetic Closure** ($\kappa^2 = 2\beta^2$) is the relational virial theorem, which implies the total energy is the negative of the kinetic energy: $E_n = -K_n$.
3. The kinetic energy is $K_n = \frac{1}{2} m_e c^2 \beta_n^2$.
4. Substituting β_n yields the final formula:

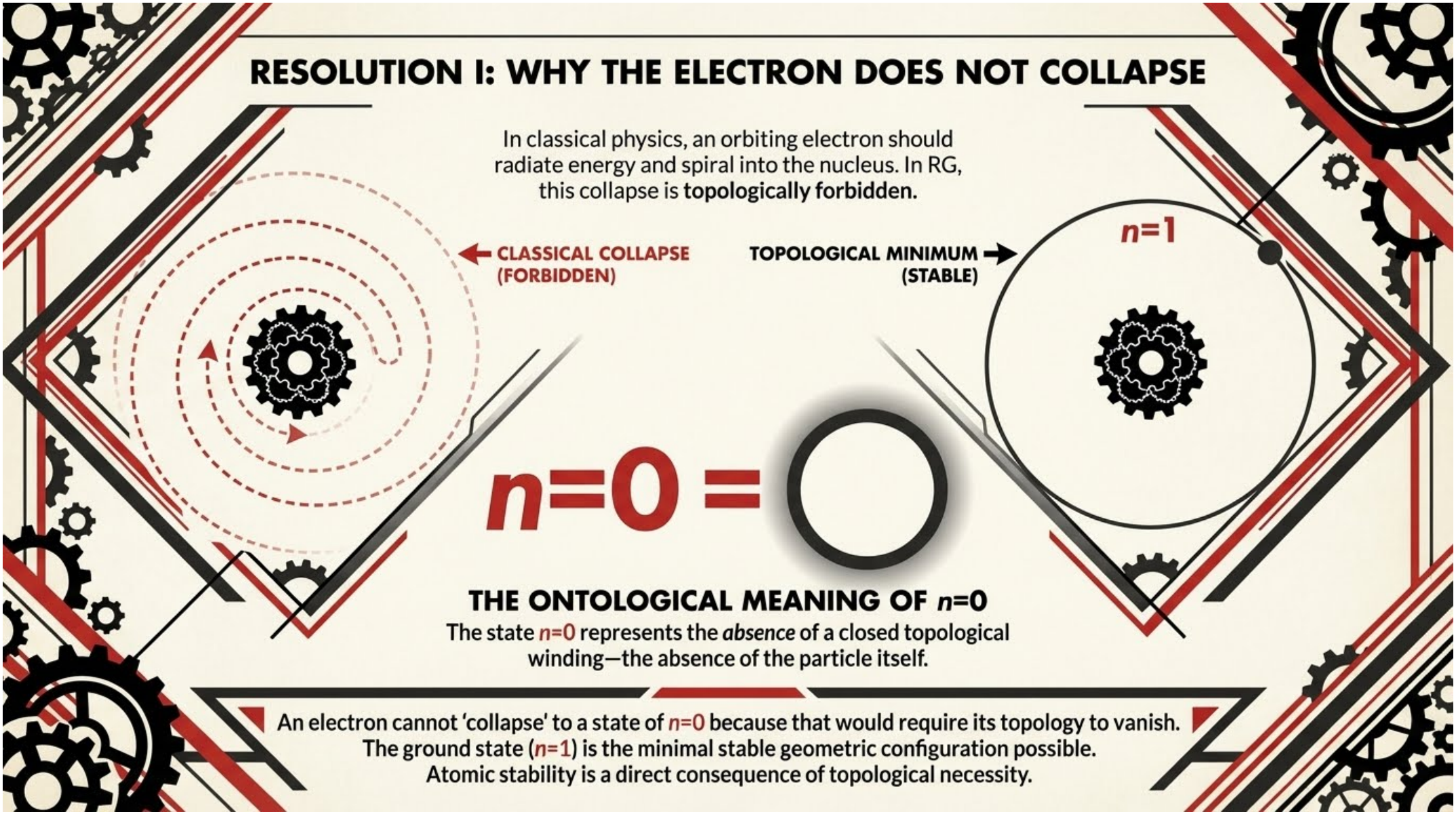
$$E_n = -\frac{1}{2} m_e c^2 \left(\frac{\alpha}{n}\right)^2 = -\frac{\alpha^2 m_e c^2}{2n^2}$$

Empirical Validation: Hydrogen Spectral Lines

Transition	Computed λ (nm)	Experimental λ (nm)
3 → 2	**656.34**	656.3
4 → 2	**486.17**	486.1
5 → 2	**434.08**	434.0
6 → 2	**410.21**	410.2

RESOLUTION I: WHY THE ELECTRON DOES NOT COLLAPSE

In classical physics, an orbiting electron should radiate energy and spiral into the nucleus. In RG, this collapse is **topologically forbidden.**



THE ONTOLOGICAL MEANING OF *n*=0

The state *n*=0 represents the *absence* of a closed topological winding—the absence of the particle itself.

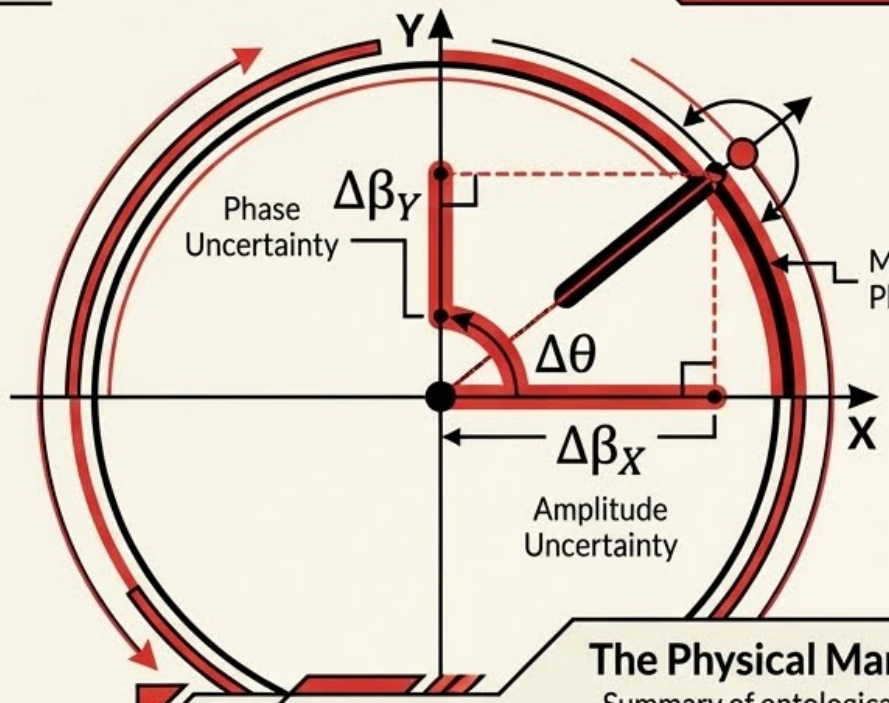
An electron cannot 'collapse' to a state of *n*=0 because that would require its topology to vanish.
The ground state (*n*=1) is the minimal stable geometric configuration possible.
Atomic stability is a direct consequence of topological necessity.

RESOLUTION II: THE GEOMETRIC ORIGIN OF UNCERTAINTY

The **Uncertainty Principle** is not a fundamental limit on measurement, but a necessary consequence of encoding energy on a closed manifold (S1).

$$\Delta\beta_X \Delta\beta_Y \geq |G| \left(\frac{2\pi}{n}\right)^2$$

The two orthogonal projections, Amplitude (**βX**) and Phase (**βY**), are not independent. Their uncertainties are geometrically linked.



The Role of Planck's Constant

$\hbar$ is not a fundamental constant of nature, but a **calibration factor**. It translates the dimensionless geometry of phase winding into the dimensional units of physical action (Joule·seconds).

The Physical Manifestation

Summary of ontological identification:

Uncertainty in position Δx ↔ de Broglie wavelength λ.

Uncertainty in momentum Δp ↔ scaled amplitude projection $m_e c \Delta\beta_X$.

The product $\Delta x \cdot \Delta p \approx \hbar$ is therefore an algebraic identity, the physical expression of underlying geometric closure.

A Shift in Ontology: From Description to Generation

Relational Geometry transforms the role of physical law. Laws are no longer external assumptions that model what is seen; they are generated as inevitable consequences of self-consistency.

Descriptive Physics (Standard)	Generative Physics (WILL RG)
Method: Phenomena observed first, then summarized into empirical laws.	**Method:** Laws emerge as inevitable consequences of relational geometry.
Nature of Laws: Assumptions introduced to model reality.	**Nature of Laws:** Identities enforced by geometric self-consistency.
Spacetime: Treated as an external background.	**Spacetime:** Projections of energy relations.
Dynamics: Evolution of states in time.	**Dynamics:** Ordered succession of balanced configurations; time is emergent.
Goal: Describe what is observed.	**Goal:** Show why nothing else is possible.

A SINGLE GEOMETRY FOR COSMOS AND QUANTUM

The same relational algebra that describes black hole orbits also generates the structure of the atom. The underlying geometry is identical; only the characteristic scale of the interaction changes.

Feature	Gravitational Case (Gravity)	Electromagnetic Case (Atom)
Critical Radius	$R_s = 2GM/c^2$	$R_q = 2e^2/(4\pi\varepsilon_0 m_e c^2)$
Potential (κ^2)	$\kappa^2 = R_s / r$	$\kappa^2 q = R_q / r_n$
Kinetic (β^2)	$\beta^2 = R_s / 2r$	$\beta^2{}_q = R_q / 2r_n$
Balance Law	$\kappa^2 = 2\beta^2$	$\kappa^2{}_q = 2\beta^2 q$

Both columns follow the same fundamental law: $\kappa^2 = 2\beta^2$. The apparent differences between the macro and micro worlds are matters of scale, not of principle.

THE END OF POSTULATES

By starting from a single principle—that spacetime and energy are two projections of a single entity—we have derived the complete structure of the hydrogen atom without postulates.

- **Quantization** arises from **Topological Closure.**
- The **Bohr Radius** is a consequence of **Geometric Consistency.**
- The **Fine Structure Constant** is a **Kinetic Ratio.**
- **Uncertainty** is a necessary feature of **Closed Projections.**

What physics calls 'quantum mechanics'is the algebraic expression of this geometric unity.

SPACETIME ≡ ENERGY