

WILL Part III: Geometric Quantization and the Origin of the Fine Structure Constant

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Abstract

This paper, the third in the WILL series, applies the principles of Relational Geometry to the quantum realm. From the foundational principle **SPACETIME** \equiv **ENERGY**, we derive the complete structure of the hydrogen atom. By enforcing geometric and topological closure, we demonstrate that quantization is not a separate postulate but an inevitable consequence of a self-consistent relational system. This framework derives the Bohr radius, the quantized energy levels, and reveals the fine structure constant, α , to be nothing more than the kinetic projection parameter, β , of the electron in its ground state. The derivation requires no classical force analogues, probabilistic wavefunctions, or differential equations, establishing atomic structure as a direct manifestation of relational geometry.

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1 The Principle of Geometric Closure in Quantum Systems

We begin with the foundational principle established in WILL Part I: any stable, self-contained system must be described by a closed and self-consistent relational geometry.

Principle 1.1 (Geometric Closure). *A system in a stable, stationary state must be represented by a geometry that is topologically closed. All relational projections describing the system must return to their initial values after a complete cycle, ensuring the continuity and integrity of the configuration.*

For a bound electron in an atom, this principle has a direct and profound consequence.

Proposition 1.2 (Topological Quantization Condition). *The principle of Geometric Closure, when applied to a bound electron, requires its relational wave to form a standing wave around the orbital circumference. This is expressed by the condition:*

$$n\lambda_n = 2\pi r_n, \quad n = 1, 2, 3, \dots$$

where λ_n is the electron's de Broglie wavelength on the n -th orbital of radius r_n .

Proof. If the wave did not close on itself (i.e., if n were not an integer), destructive interference would prevent the formation of a stable, time-independent configuration. Such a state would be geometrically and energetically inconsistent, and thus is not a permissible solution in WILL Geometry. The integer n is therefore a necessary condition for a stable state's existence. \square

Definition 1.3 (The Quantum Number as a Topological Invariant). *The principal quantum number, n , is not an externally imposed parameter but is the **topological winding number** of the energy projection's phase. It counts the number of complete phase rotations required for the geometric configuration to close upon itself.*

Corollary 1.4 (Geometric Quantization of Angular Momentum). *The topological closure condition, combined with the de Broglie relation $\lambda_n = h/p_n$, directly yields the quantization of angular momentum as a geometric necessity, not a separate physical postulate.*

$$n \frac{h}{p_n} = 2\pi r_n \quad \implies \quad \boxed{p_n r_n = n\hbar}$$

2 The Universal Scale Principle and Relational Projections

As established in Part I, the state of any system is fully described by its potential projection (κ) and kinetic projection (β). The potential projection is governed by a universal principle connecting the system's geometry to its intrinsic limits.

Principle 2.1 (Universal Scale Principle). *The potential projection κ^2 is the dimensionless ratio of a system's intrinsic critical scale to its current observational scale.*

$$\kappa^2 = \frac{r_{critical}}{r_{current}}$$

Remark 2.2 (Structural Symmetry of Interactions). *The identical inverse-square form of the gravitational and electromagnetic interactions ($F \propto 1/r^2$) is not a coincidence. It reflects that both are manifestations of the same underlying geometric principle. This compels us to define a critical scale for electromagnetism that is directly analogous to gravity's Schwarzschild radius.*

Definition 2.3 (Electromagnetic Critical Radius). *The electromagnetic critical radius, R_e , is the intrinsic scale at which a particle's electrostatic potential energy budget becomes saturated. In the WILL framework, this corresponds to the radius where the potential energy magnitude equals half the particle's rest energy, consistent with the limit $\kappa = 1$ in our energy budget formalism $|U| = \frac{1}{2}E_0\kappa^2$.*

Theorem 2.4 (Derivation of the Electromagnetic Critical Radius). *For an electron of mass m_e and charge e , the electromagnetic critical radius is given by:*

$$R_e = \frac{2e^2}{4\pi\epsilon_0 m_e c^2}$$

Proof. We apply the condition from the definition, setting the potential energy magnitude equal to half the rest energy:

$$|U(R_e)| = \frac{1}{2}E_0$$

Substituting the physical expressions for the electron:

$$\frac{e^2}{4\pi\epsilon_0 R_e} = \frac{1}{2}m_e c^2$$

Solving for R_e directly yields the result. This scale is twice the "classical electron radius" and represents the true point of energetic saturation for the electron's potential field in WILL Geometry. \square

3 Derivation of Atomic Structure from First Principles

The complete structure of the hydrogen atom is now constrained by a closed system of three fundamental geometric principles, with no further assumptions required.

Lemma 3.1 (The Geometric System of Equations). *The stable states of the hydrogen atom are the solutions to the following system:*

1. **Topological Closure:** $\beta_n^2 \approx \frac{n^2 \hbar^2}{m_e^2 c^2 r_n^2}$

2. **Scale Principle:** $\kappa_n^2 = \frac{R_e}{r_n}$

3. **Geometric Closure:** $\kappa_n^2 = 2\beta_n^2$

The non-relativistic approximation for β_n is used, consistent with the low energy scale of the hydrogen atom.

Theorem 3.2 (Derivation of Quantized Atomic Radii). *The allowed orbital radii r_n are derived solely from the simultaneous solution of the geometric system.*

$$r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{m_e e^2}$$

Proof. We begin by substituting the expression for β_n^2 from (1) into the geometric closure condition (3):

$$\kappa_n^2 = 2 \left(\frac{n^2 \hbar^2}{m_e^2 c^2 r_n^2} \right)$$

Next, we equate this with the expression for κ_n^2 from the scale principle (2):

$$\frac{R_e}{r_n} = \frac{2n^2 \hbar^2}{m_e^2 c^2 r_n^2}$$

We solve for r_n . Canceling one factor of r_n from both sides gives:

$$r_n = \frac{2n^2 \hbar^2}{m_e^2 c^2 R_e}$$

Finally, we substitute the derived expression for the critical radius, $R_e = \frac{2e^2}{4\pi\epsilon_0 m_e c^2}$:

$$r_n = \frac{2n^2 \hbar^2}{m_e^2 c^2} \left(\frac{4\pi\epsilon_0 m_e c^2}{2e^2} \right)$$

All dependencies on c and one factor of m_e cancel, yielding the final expression for the quantized radii. \square

Corollary 3.3 (The Bohr Radius as a Geometric Consequence). *The fundamental length scale of the atom, the Bohr radius a_0 , emerges not from a force balance, but from the intersection of the three geometric principles. For the ground state ($n = 1$):*

$$a_0 = r_1 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$$

4 The Geometric Origin of the Fine Structure Constant

We now arrive at the central result of this paper. With the atomic length scale a_0 derived from pure geometry, we can calculate the electron's relational projections and uncover the identity of a fundamental constant of nature.

Theorem 4.1 (The Fine Structure Constant as the Ground-State Kinetic Projection). *The fine structure constant, α , is identical to the kinetic projection, β_1 , of the electron in the ground state of the hydrogen atom.*

$$\alpha = \beta_1$$

Proof. We use our geometric system to find the value of β_1 . From the geometric closure condition, $\beta_1^2 = \kappa_1^2/2$. From the scale principle, $\kappa_1^2 = R_e/r_1 = R_e/a_0$. Therefore:

$$\beta_1^2 = \frac{1}{2} \frac{R_e}{a_0}$$

Substituting the expressions for R_e and a_0 :

$$\beta_1^2 = \frac{1}{2} \left(\frac{2e^2}{4\pi\epsilon_0 m_e c^2} \right) \left(\frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} \right)$$

The terms simplify to:

$$\beta_1^2 = \frac{e^4}{(4\pi\epsilon_0)^2 \hbar^2 c^2} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)^2$$

We recognize the term in parentheses as the standard definition of the fine structure constant, α .

$$\beta_1^2 = \alpha^2 \implies \beta_1 = \alpha$$

The theorem is proven. □

Remark: A Derived Constant

The fine structure constant has historically been regarded as a fundamental, dimensionless constant of nature whose value ($\approx 1/137$) is determined only by experiment. This derivation reveals its true identity: it is a **derived ratio of geometric projections**, fixed by the self-consistency requirements of the electron's relational geometry.

5 Emergence of Quantized Energy Levels

The final piece of the atomic puzzle, the energy spectrum, now follows as a direct consequence of the derived geometry.

Theorem 5.1 (Quantized Energy Levels). *The energy levels of the hydrogen atom are given by:*

$$E_n = -\frac{\alpha^2 m_e c^2}{2n^2}$$

Proof. The total energy E_n of a stable bound state is given by the virial theorem, which in our framework is a consequence of the geometric closure condition $\kappa_n^2 = 2\beta_n^2$. The theorem states $E_n = -K_n$. The kinetic energy budget is defined as $K_n = \frac{1}{2}m_e c^2 \beta_n^2$. Therefore:

$$E_n = -\frac{1}{2}m_e c^2 \beta_n^2$$

From the result $\beta_1 = \alpha$ and the quantization condition $p_n \propto 1/r_n \propto 1/n^2$, it follows that $p_n \propto 1/n$ and thus $\beta_n = \beta_1/n = \alpha/n$. Substituting this into the energy expression gives the final result. □

6 Why the Electron Does Not Collapse into the Nucleus: Topological and Ontological Resolution

6.1 Statement of the Problem

A longstanding paradox in both classical and early quantum theory is the apparent instability of the hydrogen atom: classically, an orbiting electron should continuously radiate energy and spiral into the nucleus, resulting in atomic collapse. However, atoms are empirically stable, and the electron remains at a finite distance from the nucleus in its ground state. Standard quantum mechanics resolves this paradox via the uncertainty principle and the existence of a lowest-energy stationary state. Here, we show that in the WILL geometric framework, atomic stability arises as a purely topological and ontological necessity, with no need for additional “quantum” postulates.

6.2 Geometric Condition for Stable Projection

Recall the fundamental geometric quantization condition:

$$n\lambda_n = 2\pi r_n, \quad n = 1, 2, 3, \dots \quad (1)$$

where n is the winding (topological) number, λ_n is the de Broglie wavelength, and r_n is the orbital radius for the n th energy level.

6.3 Ontological Meaning of $n = 0$

Within this framework, n has a strict ontological interpretation: it counts the number of complete phase rotations (windings) of the energy projection around the nucleus. The $n = 0$ case would correspond to zero phase winding—a state with no closed projection and thus no physical electron:

A system with $n = 0$ does not correspond to a physical electron bound to the nucleus; it represents the absence of any closed energetic projection. There is simply no object to “collapse.”

6.4 Mathematical Exclusion of Collapse

Suppose we attempt to reduce the orbital radius r_n to zero (i.e., electron “falling into the nucleus”). By the quantization condition:

$$r_n \rightarrow 0 \implies \lambda_n \rightarrow 0 \quad (2)$$

But from the de Broglie relation,

$$\lambda_n = \frac{h}{p_n} \implies p_n \rightarrow \infty \quad (3)$$

That is, the required electron momentum and energy diverge as $r_n \rightarrow 0$, making such a state energetically forbidden. Furthermore, the topological winding number n can only take integer values $n \geq 1$; $n = 0$ does not correspond to any physically realizable projection.

6.5 Minimal Stable State

Thus, the ground state ($n = 1$) is not just the lowest allowed energy configuration, but the minimal topologically permissible projection of energy around the nucleus:

$$n = 1 \iff \text{single closed winding of phase projection} \quad (4)$$

There is no valid state with $n < 1$; the electron cannot “collapse” further because the geometric structure of the system no longer exists.

6.6 Conclusion

The stability of the hydrogen atom, and the impossibility of the electron collapsing into the nucleus, arises naturally in the WILL framework as a consequence of topological closure. The electron’s existence as a bound system is identical to the existence of a nonzero winding number. There is no need to invoke additional quantum mechanical principles; the geometric ontology of energy projection alone guarantees atomic stability.

7 Geometric Origin of the Uncertainty Principle

In the WILL framework, the uncertainty principle is not a postulate of quantum theory, nor a consequence of non-commuting operators. It is a direct geometric necessity arising from the closure of energy projection on the unit circle S^1 . This section derives the principle from first principles, without invoking wavefunctions, probabilities, or external measurement axioms.

7.1 Phase Winding as the Ontological Basis of Quantization

The quantum number n is not an externally imposed label but the **topological winding number** of the electron's energy projection around its orbital path. This follows directly from the foundational principle:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

For a closed orbit of radius r_n , the projection must return to its initial phase after one full circuit. This enforces the geometric closure condition:

$$n\lambda_n = 2\pi r_n, \quad n = 1, 2, 3, \dots \quad (5)$$

where λ_n is the de Broglie wavelength. The integer n counts the number of complete phase rotations - i.e., the number of times the projection wraps around S^1 before closing. No state with $n = 0$ exists; such a configuration would correspond to the absence of a closed projection, and thus to no physical electron.

7.2 Minimal Phase Grain and Observational Granularity

Because the phase must close after n windings, the smallest physically distinguishable phase increment is bounded by:

$$\Delta\theta \geq \frac{2\pi}{n}. \quad (6)$$

This is not a limitation of instrumentation but a **limit of definability**: any phase difference smaller than $2\pi/n$ cannot be distinguished from a full winding and therefore lacks relational identity. The quantity $2\pi/n$ is the *minimal phase grain* - a topological invariant of the closed geometry.

7.3 Orthogonal Projections on S^1 and Their Uncertainties

The unit circle S^1 supports two orthogonal relational projections:

- **Amplitude Component** (β_X): $\beta_X = \cos\theta$, representing the relational (kinetic) aspect of the projection. This manifests physically as momentum.
- **Phase Component** (β_Y): $\beta_Y = \sin\theta$, representing the internal (temporal) aspect. This governs proper time and frequency scales.

For small variations, the uncertainties in these projections are:

$$\Delta\beta_X = |\sin \theta| \Delta\theta, \quad (7)$$

$$\Delta\beta_Y = |\cos \theta| \Delta\theta. \quad (8)$$

Their product is therefore:

$$\Delta\beta_X \Delta\beta_Y = |\sin \theta \cos \theta| (\Delta\theta)^2 = \frac{1}{2} |\sin 2\theta| (\Delta\theta)^2. \quad (9)$$

Substituting the minimal phase grain $\Delta\theta \geq 2\pi/n$, we obtain the **Geometric Uncertainty Relation**:

$$\Delta\beta_X \Delta\beta_Y \geq \frac{1}{2} |\sin 2\theta| \left(\frac{2\pi}{n} \right)^2 \quad (10)$$

This relation is purely topological and algebraic. It requires no probabilistic interpretation, no Hilbert space, and no external observer. It is the inevitable consequence of demanding closure on a finite, self-contained projection.

Epistemic Disclaimer

Equation (10) is not an approximation or empirical rule. It is a necessary condition for the existence of a well-defined, closed energy projection. Any attempt to interpret β_X and β_Y as independent classical variables will violate this bound and produce unphysical states.

7.4 Calibration to Physical Observables: Emergence of Planck's Constant

The geometric uncertainty relation (Eq. (10)) is expressed in dimensionless projection space. To connect it with empirical physics, we must map these projections to measurable quantities. This mapping arises directly from the ontological nature of a particle in WILL Geometry.

The Ontological Origin of Spatial Uncertainty (Δx). In the WILL framework, a particle is not a point object existing in spacetime; it is a quantized unit of spacetime itself, whose minimal spatial extent is its de Broglie wavelength λ_n . This is a direct consequence of the principle $\text{SPACETIME} \equiv \text{ENERGY}$. The "position" of such a wave-like quantum is therefore inherently delocalized.

Consequently, the fundamental, irreducible uncertainty in its position, Δx , is not a limit of measurement but a **definition of the particle's own geometric scale**:

$$\Delta x \approx \lambda_n. \quad (11)$$

Momentum and the Emergence of the Heisenberg Relation. The uncertainty in momentum, Δp , remains the scaled amplitude projection as previously defined:

$$\Delta p = m_e c \Delta\beta_X. \quad (12)$$

The de Broglie relation, $p_n = h/\lambda_n$, can be rewritten as $p_n \lambda_n = h$. By combining this with our geometric definition of Δx from Eq. (11), the Heisenberg Uncertainty Principle emerges not as a postulate, but as an algebraic identity:

$$\boxed{\Delta x \cdot \Delta p \approx \hbar}. \quad (13)$$

This relation is thus shown to be a direct consequence of the particle's ontological status as a self-contained quantum of spacetime. This physical uncertainty is the manifest expression of the underlying Geometric Uncertainty Relation (Eq. (10)), with the two being linked via the mappings $\Delta p \propto \Delta \beta_X$ and $\Delta x \propto \lambda \propto \Delta \beta_Y$.

Cycle-averaged geometric bound. Over one closed winding on S^1 , the average of $|\sin 2\theta|$ equals $2/\pi$. Hence the cycle-averaged product obeys

$$\left\langle \Delta \beta_X \Delta \beta_Y \right\rangle_{cycle} \geq \frac{1}{\pi} (\Delta \theta)^2 \geq \frac{1}{\pi} \left(\frac{2\pi}{n} \right)^2 = \frac{4\pi}{n^2}.$$

This bound is purely geometric (closure on S^1) and refers to a full winding, not to a pointwise angle.

The Nature of Planck's Constant. Critically, \hbar is not a fundamental constant of nature but a **conversion factor** between topological winding and physical scale. It is the calibration constant that translates the dimensionless geometry of phase into the dimensional world of meters, kilograms, and seconds.

Epistemic Note

In WILL Geometry, \hbar is epiphenomenal. The fundamental invariant is the winding number n ; \hbar is a human artifact for translating topological closure into SI units.

7.5 Superposition and Collapse as Phase Coherence and Locking

In standard quantum mechanics, superposition is a linear combination of states, and collapse is an ill-defined measurement-induced transition. In WILL Geometry, both are reinterpreted geometrically.

Superposition = Unresolved Winding. A system exhibits interference when its phase projection remains **coherent across multiple paths**. For example, in the double-slit experiment, the electron's phase is not yet locked to any external reference; its winding number is defined only relative to the global experimental setup. Multiple geometric trajectories remain compatible, and their phase differences produce interference.

Collapse = Phase Locking via Energy Symmetry. When the system interacts with an environment (detector, air molecule, thermal photon), the Energy Symmetry Law enforces:

$$\Delta E_{\text{system} \rightarrow \text{env}} + \Delta E_{\text{env} \rightarrow \text{system}} = 0. \quad (14)$$

This mutual closure requires the system to adopt a **definite phase configuration** relative to the environment. The winding network locks, eliminating compatibility between

multiple paths. Interference vanishes - not because of “observation,” but because of **energetic accountability**.

No wavefunction collapse postulate is needed. No observer consciousness. Only geometric closure.

Key Insight

Decoherence is not probabilistic - it is the deterministic resolution of phase ambiguity under the constraint of global energy symmetry.

7.6 Path Integral as Sum Over Winding Configurations

Feynman’s path integral formalism posits that a particle “samples” all possible trajectories, weighted by $e^{iS/\hbar}$. In WILL Geometry, this is reinterpreted without integrals or complex amplitudes.

The physical process is a **sum over all phase winding configurations** that satisfy global closure. Each configuration corresponds to a distinct way the projection can wrap around its geometric manifold while maintaining consistency with boundary conditions.

- The “classical path” is the configuration with **stationary phase** - i.e., minimal phase shear across the network. - Constructive interference occurs when phase differences between paths equal $2\pi m$ (integer winding match). - Destructive interference occurs for half-integer mismatches.

No complex numbers are required. The “amplitude” is simply the **geometric compatibility** of a winding configuration with the global closure constraint. The path integral is thus reduced to a combinatorial count of topologically allowed states.

This reframing eliminates the ontological baggage of infinite-dimensional configuration spaces and restores the path integral to its geometric essence: a catalog of self-consistent phase windings.

Summary of Steps 4–6

- \hbar emerges from phase granularity, not as a fundamental constant.
- Superposition = unresolved winding; collapse = phase locking via energy symmetry.
- Path integral = sum over topologically allowed winding configurations.

All quantum phenomena arise from the geometry of closed projections - no additional postulates required.

7.7 Entanglement as Shared Phase Origin

In standard quantum mechanics, entanglement is a non-local correlation with no classical analogue. In WILL Geometry, it arises naturally from the **topological unity of phase winding**.

Consider two electrons created in a common process (e.g., atomic decay or pair production). Their energy projections originate from a **single closed phase network** on a

shared manifold. This means their winding numbers are not independent but **co-defined** by a global closure condition.

Let the total phase of the composite system be:

$$\Phi_{\text{total}} = \phi_A + \phi_B = 2\pi n_{\text{total}},$$

where n_{total} is a fixed integer. Any local change in ϕ_A must be compensated by ϕ_B to preserve closure:

$$\delta\phi_A + \delta\phi_B = 0.$$

This is not “spooky action at a distance.” It is **relational bookkeeping**: the two subsystems are parts of a single geometric whole. Measuring one fixes its phase relative to the environment, which **locks the other** to maintain global energy symmetry:

$$\Delta E_{A \rightarrow \text{env}} + \Delta E_{\text{env} \rightarrow A} = 0 \quad \Rightarrow \quad \text{phase of } B \text{ adjusts accordingly.}$$

Bell’s theorem is not violated because the correlations were **never separable** - they were born from a common phase origin. No hidden variables are needed; only **topological unity**.

Key Insight

Entanglement is not a property of particles - it is a property of the **phase network** that defines them.

7.8 Quantum Field Theory as a Network of Interlocked Circles

In WILL Geometry, a quantum field is not a fundamental entity but a **dense network of interlocked S^1 projections**, each representing a local degree of freedom of WILL=spacetime-energy.

Particles as Localized Winding Excitations. A particle corresponds to a **localized defect** in the phase network - a region where the winding number differs from the ground state. Creation and annihilation are not operator actions but **topological transitions**: adding or removing a winding unit.

Vacuum as Ground-State Winding. The vacuum is not “empty.” It is the **lowest-energy winding configuration** consistent with global closure. Virtual particles are temporary **phase fluctuations** that do not violate energy symmetry because they are confined within the uncertainty bound:

$$\Delta E \Delta t \geq \hbar.$$

Gauge Symmetries as Winding Conservation Laws.

- - **U(1) symmetry (electromagnetism)**: conservation of total phase winding on S^1 .
- - **SU(2) symmetry (weak force)**: conservation of winding on a 3-sphere S^3 (double cover of rotation group).

- - **SU(3) symmetry (strong force)**: conservation of winding on a higher-dimensional manifold with 8 topological generators.

Renormalization is not a fix for infinities but a **rescaling of the winding granularity** to match observational resolution.

No Lagrangian density is needed. The dynamics of the field are encoded in the **algebraic closure** of the network:

$$\sum_{\text{nodes}} \Delta n_i = 0,$$

ensuring global phase consistency.

Epistemic Note

Quantum field theory in WILL is not a theory of fields in spacetime - it is a **theory of spacetime as a field of phase relations**.

7.9 Final Synthesis: The WILL Uncertainty Principle

We now unify all quantum phenomena under a single geometric principle.

Definition 7.1 (WILL Uncertainty Principle). *On any closed manifold encoding a physical observable, the product of uncertainties between two non-parallel projections is bounded below by the square of the minimal phase grain, set by the topological winding number:*

$$\Delta A \Delta B \geq |G(A, B)| \left(\frac{2\pi}{n} \right)^2$$

where $G(A, B)$ is the geometric coupling (e.g., $\frac{1}{2}|\sin 2\theta|$ for S^1), and n is the winding number of the closed projection.

This principle explains:

- **Position-momentum uncertainty**: $A = x, B = p \rightarrow$ projections on S^1 .
- **Energy-time uncertainty**: $A = E, B = t \rightarrow$ conjugate phase and frequency.
- **Angular momentum uncertainty**: $A = L_x, B = L_y \rightarrow$ non-orthogonal projections on S^2 .

The “commutator” $[A, B] = i\hbar$ is not fundamental - it is the **Poisson bracket of the underlying geometric projections**, elevated to operator form in the legacy formalism.

Probability as Epistemic Ignorance. The Born rule ($P = |\psi|^2$) emerges not as a law of nature but as a **measure of phase coherence** in an incomplete network. If the global winding state were known, outcomes would be deterministic. Probability reflects **relational incompleteness**, not ontological randomness.

The End of the Measurement Problem. There is no “collapse.” There is only **phase locking** to satisfy energy symmetry with the environment. The observer is not special - they are part of the same phase network.

Theoretical Uroboros of Quantum Mechanics

Quantum mechanics is not a theory of particles or waves. It is the geometry of closed energy projections. Uncertainty, superposition, entanglement, and quantization are all necessary consequences of:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

8 Hypothesis: Energy Symmetry as the Origin of Decoherence

Statement: Interference phenomena and coherent superpositions are only permitted in systems that are not energetically bound via interaction with **ANY** external object or system (including detector or measuring environment). The act of measurement corresponds to a physical interaction that invokes the principle of mutual energy conservation between the system and the detector:

$$\boxed{\Delta E_{A \rightarrow C} + \Delta E_{C \rightarrow A} = 0}$$

Interpretation: Before any interaction, the system’s internal energy projection is unconstrained and may evolve or propagate through multiple coherent geometric trajectories simultaneously. Upon interaction, the requirement of energy symmetry forces the system to project into a single, well-defined energetic configuration. This projection eliminates the compatibility of multiple phase paths and thereby collapses the interference pattern.

Implications:

- Collapse is not epistemic (observer-dependent), but a geometric-energetic necessity arising from reciprocal closure.
- Decoherence is the energetic resolution of potential superposition into a single pathway dictated by energy-matching boundary conditions.
- Observation corresponds to a physical event, not a metaphysical concept.

Next Steps: To validate the hypothesis, we must:

- Test it against canonical quantum experiments (double slit, EPR, delayed choice).
- Model systems near the decoherence threshold (partial interaction).
- Explore energetic asymmetries between detector and system.

8.1 Empirical Validation: Decoherence from Pre-Interaction Events

Objective: To test the hypothesis that coherence and interference in quantum systems are disrupted not by epistemic acts of observation, but by physical energy exchange that enforces mutual energetic closure between the system and the environment.

Core Statement: A particle arriving at an interferometric structure (e.g., a double slit) with prior energetic entanglement (via scattering, emission, or thermal exchange) enters the system already constrained by symmetry:

$$\Delta E_{A \rightarrow C} + \Delta E_{C \rightarrow A} = 0$$

Therefore, its internal projection must resolve to a definite energetic configuration, preventing coherent interference.

8.2 Test Cases and Results

- **Case 1: Electrons in high vacuum** – Path length: 1 m; – Mean free path: 10–100 km; – Result: *Clear interference observed.* – **Conclusion:** No prior interaction \Rightarrow full coherence preserved.
- **Case 2: C₆₀ molecules in ultra-high vacuum** – Coherent interference visible at low temperature; – When heated ($T > 3000\text{ K}$), thermal IR emission occurs; – *Interference pattern disappears.* – **Conclusion:** Internal energy leakage \Rightarrow decoherence without measurement.
- **Case 3: Electrons in atmospheric air** – Mean free path $\sim 4\mu\text{m} \ll$ system size (0.1–1 m); – *No interference observed.* – **Conclusion:** High probability of pre-interaction \Rightarrow loss of coherence.
- **Case 4: Photons with partial phase scattering before slit** – Partially diffusive medium inserted; – *Interference visibility decreases;* – **Conclusion:** Partial energy leakage \Rightarrow partial loss of phase integrity.

8.3 Summery:

Across all tested regimes, the hypothesis holds:

- **Coherence exists** when no energetic connection exists between system and environment.
- **Interference disappears** when mutual energy conservation applies due to prior or current interactions.
- This behavior scales smoothly: partial interaction leads to partial decoherence, consistent with phase degradation models.

Implication: This supports a non-probabilistic, physically grounded account of quantum decoherence based purely on energy geometry, in full alignment with the foundational principle of WILL Geometry.

9 Conclusion

By a rigorous application of the principles of WILL Relational Geometry, we have derived the complete structure of the hydrogen atom. We have demonstrated that quantization, the Bohr radius, the fine structure constant, and the discrete energy spectrum are not independent physical postulates but are all necessary and inevitable consequences of a

single, unified principle: that reality is described by a closed, self-consistent relational geometry.

The framework requires no classical force analogues, no probabilistic axioms, and no external postulates. Atomic structure is shown to be a direct manifestation of the geometry of energy itself.