# WILL Part I: Relational Geometry

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Foundational Principle: Spacetime $\equiv$ Energy

$\downarrow$

Geometric Self-Consistency

$\downarrow$

Topological Requirement: Only Closed and Maximally Symmetric Structures Allowed

$\downarrow$

Relational Circle (SR) and Relational Sphere (GR) as Unique Permitted Geometries

$\downarrow$

Result: Projections $\beta$ (kinetic) and $\kappa$ (potential) are Strictly Normalized

$\downarrow$

Conservation, Symmetry, Speed Limit, Natural Bounds

$\downarrow$

Unified Field Equation: $\kappa^2 = R_s/r = \rho/\rho_{\max}$

$\downarrow$

Closure: Field Equation $\Longleftrightarrow$ Fundamental Principle

Abstract

This paper, the first in the WILL series, applies extreme methodological constraints in constructing a theory and establishes Relational Geometry (RG): a foundational framework where spacetime is not a background arena but an emergent property of energy transformations.

From a single principle, SPACETIME $\equiv$ ENERGY, it aims to derive the complete geometric structure of physics across physical domains. This equivalence is not postulated but derived by removing the hidden ontological assumption, implicit in modern physics, that structure (spacetime) and dynamics (energy) are separate phenomena.

This shift establishes an ontological transition from descriptive to generative physics: instead of introducing laws to model observations, it derives them as necessary consequences of RG itself  turning physics from a catalogue of phenomena into the logical unfolding of a single principle.

The result is a singularity-free, ontologically clean formalism that reproduces the core equations of Special Relativity (SR) and General Relativity (GR) as geometric projections on closed relational manifolds $S^1$ (directional) and $S^2$ (omnidirectional). Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energymomentum relation, Schwarzschild and Kerr solutions, and Einstein field equations via the dimensionless projections $\beta$ (kinematic) and $\kappa$ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of $(\kappa, \beta)$ from the single closure law $\kappa^2 = 2\beta^2$ (geometrically derived, virial-like). All results are empirically validated (e.g., GPS time shift 38.52 $\mu$s/day, Mercury precession to $10^{-10}$%, and others listed in Appendix I).

WILL Part I offers solutions to several long-standing problems, including:

- Resolution of GR singularities (via naturally bounded $\rho_{\max}$),
- Derivation of the equality of gravitational and inertial masses (from the common channel of rest-invariant scaling),
- Removal of local energy ambiguity $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$,
- Revelation of a clear relational symmetry between kinematic and potential projections,
- Establishment of a computationally simpler and ontologically consistent foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

# Contents

**Epistemic Disclaimer**

This document must be read literally. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (absolute energies, external backgrounds, hidden containers) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

# 1 Foundational Approach

This Approach Does not Describe Physics; it Generates it. 18.1

**Guiding Principle:**

Nothing is assumed. Everything is derived.

## 1.1 Methodological Purity

This framework is constructed under a single epistemic constraint: to derive all of physics by removing one hidden assumption, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced unless they follow strictly from first principles, and no constructs are retained unless they are geometrically or energetically necessary.

**Core Methodological Principle**

Principle 1.1 (Methodological Minimalism). Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities. This principle is not a statement about the nature of reality, but a rule of logical hygiene for constructing a theory.

## No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent relational projections.

## 1.2 Epistemic Hygiene

Modern physics often tolerates hidden assumptions: arbitrary constants, external backgrounds, or abstract entities with ambiguous physical status. Here we enforce epistemic hygiene: a refusal to import unjustified assumptions.

Principle: All physical quantities must be defined purely by their relations. Any introduction of absolute properties or external frames risks reintroducing metaphysical artifacts and contradicts the foundational insight of relativity.

# 2 False Separation: Ontological Blind Spot In Modern Physics

**Removing the Hidden Assumption**

Any attempt to treat "spacetime structure" as separate from "dynamics" smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

# 3 False Separation: the Ontological Blind-Spot of Modern Physics

## Historical Pattern: breakthroughs delete, not add

- Copernicus eliminated the Earth/cosmos separation.
- Newton eliminated the terrestrial/celestial law separation.
- Einstein eliminated the space/time separation.
- Maxwell eliminated the electricity/magnetism separation.

Each step widened the relational circle and reduced the number of unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

## The contemporary split: an unpaid ontological bill

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a bi-variable syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate inside the split. The split is therefore not an empirical discovery but an unpaid ontological debt.

## Empirical bankruptcy of the separation

- Local energy conservation verified only after the metric is declared fixed; no experiment varies the volume of flat space and checks calorimetry.
- Universality of free fall tests $m_i = m_g$ numerically, not the claim that inertia resides in the object rather than in a geometric scaling relation.

- Gravitational-wave polarisations test spin content, not ontology; extra modes can still be called “matter on spacetime”.
- Casimir/Lamb shift measure differences of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

In short, every “test” is an internal consistency check of a formalism that already presupposes two substances. None constitute positive evidence for the split.

### Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an un-evidenced metaphysical postulate—the last geocentric epicycle in physics (hopefully).

**Summary**

Every major advance in physics has come not from adding new ontological furniture but from removing a false separation.
Todays silent assumption - that “spacetime structure” and “energy/dynamics” are two distinct substances - is the last such artefact (hopefully). It survives only because no experiment has ever varied the amount of space while holding everything else constant.

# 4 Unifying Principle Removing the Hidden Assumption

## 4.1 False Separation

Lemma 4.1 (False Separation). Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of interprocess data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. □

Corollary 4.2 (Structure–Dynamics Coincidence). To avoid the artifact of Lemma 4.1, the structural arena and the dynamical content must be identified: geometry is energy, and energy is geometry.

Principle 4.3 (Working Principle: Removing the Hidden Assumption).

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight: it removes the hidden unjustified separation between geometry and dynamics. Space and time are not containers but emergent descriptors of relational energy.

Remark 4.4 (Auditability). Principle 4.3 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).

| Summary: |
| --- |
| This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call - WILL. |

## 4.2 What is Energy in Relational Framework?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure. Crucially, this quantity is never observed directly, but only through differences between states: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation. Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself. From these empirical and relational facts the definition follows unavoidably:

| |
| --- |
| Energy is the relational measure of difference between possible states, conserved in any closed whole.<br>It is not an intrinsic property of an object, but comparative structure between states (and observers), always manifesting as transformation. |

# 5 Deriving the WILL Structure

Having established our Principle 4.3 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences. We will demonstrate that this single principle is sufficient to enforce the closure, conservation, and isotropy of the relational structure, leading to a unique set of geometric carriers for energy.

Definition 5.1 (WILL). WILL $\equiv$ SPACE-TIME-ENERGY is the technical term we use for unified relational structure determined by 4.3 . All physically meaningful quantities are relational features of WILL; no external container is permitted.

Lemma 5.2 (Closure). Under 4.3, WILL is self-contained: there is no external reservoir into or from which the relational resource can flow.

Proof. If WILL were not self-contained, there would exist an external structure mediating exchange. That external structure would then serve as a background distinct from the dynamics, contradicting Corollary 4.2. □

Lemma 5.3 (Conservation). Within WILL, the total relational "transformation resource" (energy) is conserved.

Proof. By Lemma 5.2, no external fluxes exist. Any change in one part of WILL must be balanced by complementary change elsewhere. Hence a conserved global quantity is enforced at the relational level. □

Lemma 5.4 (Isotropy from Background–Free Relationality). If no external background is allowed (Cor. 4.2), then no direction can be a priori privileged. Thus the admissible relational geometry of WILL must be maximally symmetric (isotropic and homogeneous) at the level at which it encodes the conserved resource.

Proof. A privileged direction requires an extrinsic reference to distinguish it. In a purely relational setting, distinctions must be constructible from relations internal to WILL. If a direction were privileged in the geometry that encodes the conserved resource, such privilege would not be derivable from purely internal comparisons (which are symmetric by construction), and would reintroduce an external orienting structure. Therefore the encoding geometry must be maximally symmetric. □

## 5.1 Classification of Minimal Relational Transformations

The lemmas of Closure, Conservation, and Isotropy establish the necessary properties of any geometric carrier of the relational resource. To identify these carriers, we must first classify the simplest possible types of physical relations that can exist in a background-free framework, according to the Principle of Methodological Minimalism (1.1).

(a) Directional (Kinematic) Relation: The simplest possible non-trivial relation is between two distinct states (A and B). The minimal description of this directed relation requires only a single degree of freedom (the axis connecting A and B). Per Principle 1.1, introducing additional dimensions would be an unjustified complication. For this description to be self-contained (Lemma 5.2), the 1D geometry must be closed. This uniquely specifies the circle ($S^1$) as the necessary and sufficient carrier for minimal directional relations.

(b) Omnidirectional (Gravitational) Relation: The simplest possible isotropic relation is between a central state (A) and the locus of all states equidistant to it. The minimal description for all directions of this relation requires a surface representing all possible orientations from the center. Such a surface has two degrees of freedom (corresponding to the two angles needed to specify a direction). For this description to be closed and maximally symmetric (Lemmas 5.2, 5.4), the geometry must be that of a 2-sphere ($S^2$). It is the necessary and sufficient carrier for minimal omnidirectional relations.

The Principle of Methodological Minimalism thus restricts the admissible relational geometries to these two minimal classes. The following theorem provides their formal identification.

Theorem 5.5 (Minimal Relational Carriers of the Conserved Resource). The only closed, maximally symmetric manifolds that can serve as minimal carriers of the conserved relational resource are:

(a) $S^1$ for directional (one-degree-of-freedom) relational transformation;

(b) $S^2$ for omnidirectional (central, all-directions-equivalent) relational transformation.

Proof. By Lemma 5.4, we require closed, maximally symmetric manifolds.

(a) In one relational degree of freedom, the classification of connected closed 1-manifolds yields $S^1$ as the unique (up to diffeomorphism) option. Its isometry group acts transitively with isotropy at each point, providing maximal symmetry.

(b) For omnidirectional relational transformation from a distinguished center, the encoding manifold must be a closed, simply connected, constant positive curvature 2-manifold with full isotropy at every point. By the uniformization/classification of constant-curvature surfaces, the maximally symmetric representative is $S^2$. Quotients of $S^2$ by nontrivial finite groups introduce global identifications that spoil global isotropy; these are excluded by Lemma 5.4. Hence $S^2$ is uniquely selected. □

Corollary 5.6 (Uniqueness). Under 4.3 with Closure, Conservation, and Isotropy (Lemmas 5.2–5.4), $S^1$ and $S^2$ are necessary relational carriers for, respectively, directional and omnidirectional modes of energy transformation.

Remark 5.7 (Non-spatial Reading). Throughout, $S^1$ and $S^2$ are not to be interpreted as spacetime geometries. They are relational manifolds that encode the closure, conservation, and isotropy of the transformational resource. Ordinary spatial and temporal notions are emergent descriptors of patterns within WILL.

Summary:

From removing the hidden assumption 4.1 we inevitably arrive to 4.3 SPACETIME $\equiv$ ENERGY from there we deduced: (i) closure, (ii) conservation, (iii) isotropy, and hence (iv) the unique selection of $S^1$ and $S^2$ as minimal relational carriers for directional and omnidirectional transformation. These objects are non-spatial encodings of conservation and symmetry; they are enforced by the4.3 rather than assumed independently.

## 5.2 Ontological Status of the Relational Manifolds $S^1$ and $S^2$

A natural question arises regarding the ontological status of the circle $S^1$ and the sphere $S^2$: What are they, and where do they "exist"?

The answer requires a fundamental shift in perspective. In WILL Relational Geometry, $S^1$ and $S^2$ are not spatial entities existing within a pre-defined container. They are the necessary relational architectures that implement the core identity SPACETIME $\equiv$ ENERGY.

Energy as Relational Bookkeeping Recall that energy is defined as the relational measure of difference between possible states. It is not an intrinsic property but a comparative structure that guarantees causal continuity. It is never observed directly, only through transformations.

The Manifolds as Protocols of Interaction The manifolds $S^1$ and $S^2$ are the minimal, unique mathematical structures capable of hosting this relational "bookkeeping" for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, $A$ and $B$:

- Observer $A$ is the center of their own relational framework. Observer $B$ is a point on $A$'s $S^1$ (for kinematic relations) and $S^2$ (for gravitational relations).
- Simultaneously, observer $B$ is the center of their own framework. Observer $A$ is a point on $B$'s $S^1$ and $S^2$.

There is no privileged "master" manifold. Each observable interaction is structured by these mutually-centered relational protocols. The parameters $\beta$ and $\kappa$ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta_X^2 + \beta_Y^2 = 1; \quad \kappa_X^2 + \kappa_Y^2 = 1$) are the innate accounting rules of these protocols.

Emergence of Spacetime Therefore, the question "Where are $S^1$ and $S^2$?" is a category error. They are not in space; they are the structures whose coordinated, multi-centered interactions give rise to the phenomenon we perceive as spacetime. Spacetime is the emergent, collective shadow cast by the dynamics of energy relations projected onto these architectures.

In essence, $S^1$ and $S^2$ are the ontological embodiment of the relational principle. They are derived as the only possible structures that can house the transformational resource (energy) in a closed, conserved, and isotropic system. Their status is that of a fundamental relational geometry from which physics is generated.

# 6 Emergence of Spacetime

In this construction, “space,” “time,” are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

## 6.1 The Duality of Transformation

Lemma 6.1 (Duality of Evolution). The identification of spacetime with energy and its transformations necessitates two complementary relational measures:

1. the extent of transformation (external displacement), and
2. the sequence of transformation (internal order).

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The circle $S^1$ provides the minimal geometry enforcing such complementarity: its orthogonal projections furnish precisely two non-redundant coordinates. □

We define this orthogonal projections as follows:

- The Amplitude Component ($\beta_X$): This projection represents the relational measure between the system and the observer. It corresponds to the extent of transformation, which manifests physically as momentum (as shown in next section).
- The Phase Component ($\beta_Y$): This projection represents the internal structure of a system. It governs the intrinsic scale of its proper space and proper time units, corresponding to the sequence of its transformation. A value of $\beta_Y = 1$ represents a complete and undisturbed manifestation of this internal structure, a state we identify as rest.

### 6.2 Conservation Law of Relational Transformation

Theorem 6.2 (Conservation Law of Relational transformation). The orthogonal components of transformation $(\beta_X, \beta_Y)$ are bound by the closure relation

$$\beta_X^2 + \beta_Y^2 = 1.$$

Proof. Since $S^1$ is closed, every point on the circle is constrained by the Pythagorean identity of its projections. Thus no state can exceed or fall short of the finite relational "budget." This closure enforces conservation across all processes. □

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

### 6.3 Consequence: Relativistic Effects

Proposition 6.3 (Physical Interpretation: Relativistic Effects). The conservation law of Theorem 6.2 implies that any redistribution between the orthogonal components $(\beta_X, \beta_Y)$ manifests physically as the relativistic effects of time dilation and length contraction.

Proof. By Theorem 6.2, the components satisfy $\beta_X^2 + \beta_Y^2 = 1$. An increase in the relational displacement $\beta_X$ enforces a decrease in the internal measure $\beta_Y$. This reduction of $\beta_Y$ corresponds to dilation of proper time and contraction of proper length, while the growth of $\beta_X$ represents momentum. Thus the relativistic trade-off is not an additional postulate but the direct physical expression of the geometric closure of $S^1$. □

| Summary: |
| --- |
| Geometry of spacetime is nothing but the shadow cast by the geometry of relations. |

## 7 Kinetic Energy Projection on $S^1$

Since $S^1$ encodes one-dimensional displacement, the total energy $E$ of the system must project consistently onto both axes:

$$E_X = E\beta_X, \qquad E_Y = E\beta_Y.$$

Theorem 7.1 (Invariant Projection of Rest Energy). For any state $(\beta_X, \beta_Y)$ on the relational circle, the vertical projection of the total energy is invariant:

$$E\beta_Y = E_0.$$

Proof. When $\beta_X = 0$, closure enforces $\beta_Y = 1$, yielding $E = E_0$. Since closure applies for all $\theta_1$, the vertical projection $E\beta_Y$ remains equal to this rest value in every state. □

Corollary 7.2 (Total Energy Relation). From Theorem 7.1 it follows that

$$E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sqrt{1-\beta_X^2}}.$$

Remark 7.3 (Lorentz Factor). The historical Lorentz factor $\gamma$ is nothing more than the reciprocal of $\beta_Y$. No additional structure is introduced: all content is already present in $E\beta_Y = E_0$.

**Summary:**

The historical Lorentz factor $\gamma$ is nothing more than the reciprocal of $\beta_Y$.

## 7.1 Rest Energy and Mass Equivalence

Corollary 7.4 (Rest Energy and Mass Equivalence). Within the normalization $c = 1$, the invariant rest energy equals mass:

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of $S^1$, no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. □

Remark 7.5 (Epistemic Interpretation). In a framework that is genuinely fundamental and free from arbitrary human units, the natural normalization is always the unique invariant $c = 1$. With this normalization, the bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ lose all significance. They collapse into the only consistent statement:

$$E_0 = m.$$

Thus mass is nothing beyond the invariant projection of total rest energy. It introduces no new entity and carries no independent meaning apart from $E_0$. What is conventionally treated as two quantities is in fact one and the same relational invariant.

**Summary:**

Mass is nothing beyond the invariant projection of total rest energy.

## 7.2 Energy–Momentum Relation

Proposition 7.6 (Horizontal Projection as Momentum). On the relational circle, the unique relational measure of displacement from rest is the horizontal projection $E\beta_X$; hence

$$p \equiv E\beta_X \quad (c = 1).$$

Proof. The rest state is $(\beta_X, \beta_Y) = (0, 1)$. A displacement measure must (i) vanish at rest, (ii) grow monotonically with $|\beta_X|$, and (iii) flip sign under $\beta_X \mapsto -\beta_X$. The only relational candidate satisfying (i)(iii) is the horizontal projection $E\beta_X$. Thus the identification is necessary rather than conventional. □

Corollary 7.7 (Energy–Momentum Relation). With $p$ identified by Proposition 7.6 and $m = E_0$, the closure identity yields

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring $c$,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta_X)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta_X$ and $m = E_0$ proves the claim. Restoring $c$ is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while $E$ remains $E$, yielding the standard form. □

Remark 7.8 (Geometric Forms). The same identity may be expressed explicitly in terms of circle coordinates:

$$E^2 = \left(\tfrac{\beta_X}{\beta_Y} E_0\right)^2 + E_0^2 = \big(\cot(\theta_1)\, E_0\big)^2 + E_0^2.$$

These are not alternative parametrizations, but equivalent renderings of the same geometric necessity.

Remark 7.9 (Units sanity check  bookkeeping). Using $\beta_X = v/c$, the identification $p \equiv E\beta_X$ gives

$$pc = E\,\frac{v}{c} \quad \Longrightarrow \quad p = \frac{E\,v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta_X}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

| $\beta_X = \beta, \quad \beta = v/c \quad \theta_1 = \arccos(\beta)$ | |
|---|---|
| Algebraic Form | Trigonometric Form |
| $1/\beta_Y = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(v/c)^2}$ | $1/\beta_Y = 1/\sin(\theta_1) = 1/\sin(\arccos(\beta))$ |
| $\beta_Y = \sqrt{1-\beta^2} = \sqrt{1-(v/c)^2}$ | $\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$ |

Table 1: Unified representation of relativistic effects.

Summary:

The energy–momentum relation $E^2 = p^2 + m^2$ is nothing more then geometric identity of $S^1$.

# 8 Potential Energy Projection on $S^2$

IMPORTANT:

Throughout this work, $S^1$ and $S^2$ are not to be interpreted as spacetime geometries but purely as relational manifolds encoding conservation. Any reading otherwise is a misinterpretation.

Analogous to $S^1$ the relational geometry of the sphere, $S^2$, provides orthogonal projections, for two aspects of omnidirectional transformation. We define them as follows:

- The Amplitude Component ($\kappa_Y$): This projection represents the relational gravitational measure between the object and the observer. It corresponds to the extent of transformation, which manifests physically as gravitation potential. A value of $\kappa_Y = 1$ corresponds precisely to the point where escape velocity equals the speed of light, creating an event horizon. This provides a natural causal limit for our gravitational parameter, analogous to the relative motion it determinant by the same radial normalization.
- The Phase Component ($\kappa_X$): This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the sequence of its transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of reality":

$$\kappa_X^2 + \kappa_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

### 8.1 Gravitational Meridional Section of $S^2$

By isotropy the omnidirectional carrier is $S^2$, but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of $S^2$ with the parametrization $(\kappa_X, \kappa) = (\cos\theta_2, \sin\theta_2)$.

## 8.2 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure ($\kappa_Y$, gravitation potential) necessarily requires a decrease in the measure of the internal structure ($\kappa_X$). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is nothing but the shadow cast by the geometry of relations.

Notation simplicity:

From here on we will write $\beta = \beta_X, \quad \beta_Y = \sqrt{1-\beta^2}, \quad \kappa = \kappa_Y, \qquad \kappa_X = \sqrt{1-\kappa^2}$ for notation simplicity.

## 8.3 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where $v_e$ is the escape velocity at radius $r$.

In the kinematic case, with $\beta = \cos\theta_1$, the energy relation can be written as

$$E^2 = \left(\cot\theta_1\, E_0/c\right)^2 + E_0^2, \tag{1}$$

so that the relativistic momentum is expressed as

$$p = E_0/c\, \cot\theta_1. \tag{2}$$

In full symmetry, the gravitational case follows from $\kappa = \sin\theta_2$. We define the gravitational energy as

$$E_g = \frac{E_0}{\kappa_X}, \qquad \kappa_X = \sqrt{1-\kappa^2}, \tag{3}$$

and introduce the gravitational analogue of momentum:

$$p_g \;=\; E_0/c\, \tan\theta_2. \tag{4}$$

This yields the gravitational energy relation

$$E_g^2 = p_g^2 + E_0^2. \tag{5}$$

**Summary:**

$$\beta = \cos\theta_1, \qquad \kappa = \sin\theta_2,$$

$$\beta \longleftrightarrow \kappa, \qquad \cot\theta_1 \longleftrightarrow \tan\theta_2.$$

Kinematic momentum $p$ and gravitational momentum $p_g$ are thus dual projections of the same relational circle, expressed through complementary trigonometric forms.

## 8.4 Geometric composition of SR and GR factors

On the unit kinematic circle ($S^1$) we parametrize

$$(\beta, \beta_Y) = (\cos\theta_1,\ \sin\theta_1),$$

so that the invariant vertical projection reads

$$E\,\beta_Y = E_0 \quad \Rightarrow \quad \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin\theta_1}}, \qquad p = E\,\beta = \frac{E_0\,\beta}{\beta_Y} = E_0 \cot\theta_1,$$

and therefore $E^2 = p^2 + E_0^2$.

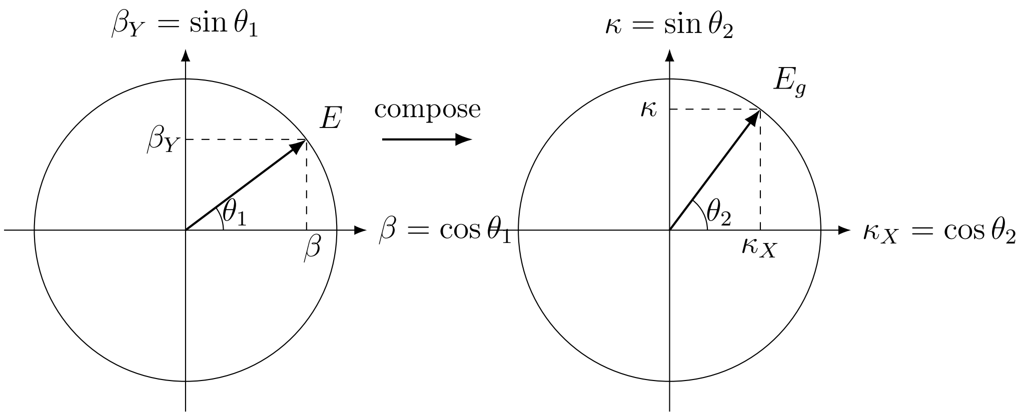
On the gravitational circle ($S^2$) we parametrize

$$(\kappa_X, \kappa) = (\cos\theta_2,\ \sin\theta_2),$$

so that the invariant horizontal projection reads

$$E_g\,\kappa_X = E_0 \quad \Rightarrow \quad \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos\theta_2}}, \qquad p_g = E_g\,\kappa = \frac{E_0\,\kappa}{\kappa_X} = E_0 \tan\theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.




$E = E_0/\beta_Y, \quad p = E_0\,\beta/\beta_Y = E_0 \cot\theta_1 \qquad E_g = E_0/\kappa_X, \quad p_g = E_0\,\kappa/\kappa_X = E_0 \tan\theta_2$

## 8.5 Clear Relational Symmetry Between Kinematic and Potential Projections

Now we can clearly see the underling symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

| $\beta = \beta_X, \quad \kappa = \kappa_Y, \quad \theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa)\ \kappa^2 = 2\beta^2$ | |
|---|---|
| Algebraic Form | Trigonometric Form |
| $1/\beta_Y = \frac{1}{\sqrt{1-\beta^2}}$ | $1/\beta_Y = \frac{1}{\sin(\theta_1)} = \frac{1}{\sin(\arccos(\beta))}$ |
| $1/\kappa_X = \frac{1}{\sqrt{1-\kappa^2}}$ | $1/\kappa_X = \frac{1}{\cos(\theta_2)} = \frac{1}{\cos(\arcsin(\kappa))}$ |
| $\beta_Y = \sqrt{1-\beta^2}$ | $\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$ |
| $\kappa_X = \sqrt{1-\kappa^2}$ | $\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$ |
| $p = E_0\,\beta/\beta_Y$ | $p = E_0\ \cot(\theta_1)$ |
| $p_g = E_0\,\kappa/\kappa_X$ | $p_g = E_0 \tan(\theta_2)$ |

Table 2: Unified representation of relativistic and gravitational effects.

### 8.5.1 The Combined Energy Parameter $Q$

The total energy projection parameter unifies both aspects:

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{6}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r} \tag{7}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{8}$$

$$Q_r = \frac{1}{Q_t} \tag{9}$$

These describe the combined effects of relativity and gravity.

**Summary**

The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic ($\beta$) and gravitational ($\kappa$) modes are not separate effects but dual aspects of one energy-transformation constraint revealing their unified origin.

## 8.6 Equivalence Principle as Derived Identity

In General Relativity the Einstein Equivalence Principle is introduced as an independent postulate: $m_g \equiv m_i$. Within WILL, no such axiom is required:

On the kinematic circle the vertical leg equals $\beta_Y = \sin\theta_1$; keeping the same invariant $E_0$ on that leg forces a stretch $E/E_0 = 1/\beta_Y$. On the gravitational circle the horizontal leg equals $\kappa_X = \cos\theta_2$; keeping the same invariant $E_0$ on that leg forces a stretch $E_g/E_0 = 1/\kappa_X$. Composing the two independent stretches gives

$$\boxed{E_{\text{loc}} = \frac{E_0}{\beta_Y} \times \frac{1}{\kappa_X} = \frac{E_0}{\beta_Y\,\kappa_X}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The inertial and gravitational projections share the same operational scale,

$$\tilde{p} = E_{\text{loc}}\,\beta, \qquad \tilde{p}_g = E_{\text{loc}}\,\kappa, \quad (c=1).$$

both governed by the identical stretch factor

$$m_{\text{eff}} = \frac{E_0}{\beta_Y\,\kappa_X} \quad (c=1).$$

Hence

$$\boxed{m_g \;\equiv\; m_i \;=\; m_{\text{eff}}},$$

Corollary 8.1 (Mass Invariance under Relational Scaling). Since mass denotes the invariant rest measure corresponding to the complete internal state of equilibrium ($\beta_Y = \kappa_X = 1$), it does not participate in relational scaling. The parameters $\beta_Y$ and $\kappa_X$ rescale only the external manifestations (energy, momentum, and temporal or spatial rates), while the invariant core $E_0$ remains unchanged. Therefore the equality of inertial and gravitational mass,

$$\boxed{m_g \equiv m_i \equiv m = E_0/c^2,}$$

is not a dynamical statement but the definition of the invariant rest condition itself.

Thus, in RG the equality $m_g \equiv m_i$ is not assumed but forced by structure.

Remark 8.2 (On Eцtvцs-type precision). In GR, the equality $m_g = m_i$ is imposed, while distinct relativistic factors later rescale energy and time. In WILL, there is no independent $m$ at all: the only invariant is $E_0$, and the effective response is the universal stretch factor $1/(\beta_Y\kappa_X)$. Since this factor depends solely on geometry and not on the composition of the test body, the universality of free fall (Eцtvцs experiments) follows identically. What GR postulates, WILL reproduces as structural necessity.

## Composition-Independence, and Quantum Interface

Composition-Independence (Eцtvцs-type). Let the rest invariant decompose into internal channels

$$E_0 = \sum_a E_0^{(a)} \quad \text{(rest mass, binding, EM, nuclear, vacuum bookkeeping, etc.).}$$

WILL couples only through the universal geometric stretch:

$$E_{\text{loc}} = \frac{1}{\beta_Y\,\kappa_X} \sum_a E_0^{(a)} \;=\; \sum_a \frac{E_0^{(a)}}{\beta_Y\,\kappa_X}.$$

Hence each channel is multiplied by the same factor $1/(\beta_Y \kappa_X)$, and any ratio of channel weights cancels in observables governed by the common scale. Therefore the operational response is compositionindependent by construction, matching the universality tested in Eцtvцs-type experiments, without a separate postulate $m_g = m_i$.

Quantum Interface (phase bookkeeping). In WILL the phase increment is purely relational and inherits the same scale:

$$\Delta\phi \;\propto\; E_{\text{loc}}\,\Delta\lambda \qquad \text{(no absolute time; } \Delta\lambda \text{ is the internal ordering parameter).}$$

Thus

$$p = \nabla_\xi \phi \;\propto\; E_{\text{loc}}\,\beta, \qquad p_g = \nabla_\chi \phi \;\propto\; E_{\text{loc}}\,\kappa,$$

so matter-wave phases and redshifts share the identical stretch $1/(\beta_Y \kappa_X)$ across all internal channels, again yielding compositionindependent interferometric outcomes.

Contrast with GR. GR recovers universality by geodesic motion in a metric after positing $m_g \equiv m_i$. WILL has no independent mass primitive and no external background: the same projection identity that generates

$$E = \frac{E_0}{\beta_Y}, \quad E_g = \frac{E_0}{\kappa_X}$$

forces the unified operational scale $E_{\text{loc}}$; hence the equality of inertial and gravitational responses is an algebraic identity of relational geometry, not an axiom. GRs geodesics are a representation; WILLs equivalence is a consequence.

Summary:

What GR posits as a postulate, WILL delivers as an unavoidable consequence of Relational Geometry.

# 9 Unification of Projections: The Geometric Exchange Rate

Having established that directional (kinematic) and omnidirectional (gravitational) relations are carried by the unique manifolds $S^1$ and $S^2$ respectively, we now derive the relationship that unifies them.

## 9.1 Derivation of the Energetic Closure Condition

The Principle of a unified relational resource (Energy) requires a self-consistent "exchange rate" between its different modes of expression. This rate is not an arbitrary parameter but is dictated by the intrinsic geometry of the relations themselves:

1. The Ratio of Relational Degrees of Freedom. Distinction between the two relational modes is their dimensionality. An omnidirectional spherical relation ($\kappa^2$ on $S^2$) requires two degrees of freedom (2D) to specify a direction from its centre, while a directional circular relation ($\beta^2$ and $S^1$) requires only one (1D). The intrinsic ratio between these modes of resource distribution is therefore:

$$\frac{\text{Omnidirectional Degrees of Freedom}}{\text{Directional Degrees of Freedom}} = \frac{2\text{D}}{1\text{D}} = 2$$

This intuitive ratio finds its formal mathematical realization in the topology of the unique geometric carriers, expressed as the ratio of their total solid angles:

$$\frac{\text{Total Closure of } S^2}{\text{Total Closure of } S^1} = \frac{4\pi}{2\pi} = 2$$

2. The Quadratic Nature of Energetic Realization. To translate this dimensionless geometric ratio into a physical law, we must recognise that energy is fundamentally quadratic in nature. The energetic significance of a state is proportional not to the amplitude of a projection ($\beta$, $\kappa$), but to its square ($\beta^2$, $\kappa^2$). This principle is ubiquitous, reflected in kinetic energy ($E_k \propto v^2$) and in the Pythagorean (sum-of-squares) structure of the relational manifolds themselves.

Definition 9.1 (Closure defect). $\delta \equiv \kappa^2 - 2\beta^2$. A subsystem is energetically closed iff $\langle \delta \rangle_{cycle} = 0$. For circular orbits, $\delta \equiv 0$.

Proposition 9.2 (Energetic closure criterion). In closed (momentary or periodic) regimes, the unique quadratic balance compatible with directional ($S^1$) vs omnidirectional ($S^2$) resource splitting is $\boxed{\kappa^2 = 2\beta^2}$.

**The Principle as a Diagnostic Invariant**

The relation $\boxed{\kappa^2 = 2\beta^2}$ holds if the system is energetically closed momentarily (circular orbits) or periodically (elliptical orbits). In open systems It can be used for determining the magnitude of energy flow through an unaccounted-for channels. When all the channels is included in the balance, the closure is restored and the equality holds again (see section "Numerical Validations" subsection "Earth–Moon").

Conclusion: The Unification Equation. Therefore, the exchange rate between the energetic significance of the gravitational and kinematic modes must equal the fundamental ratio of their relational dimensions. This leads to equation:

$$\frac{\kappa^2}{\beta^2} = 2 \quad \Longrightarrow \quad \boxed{\kappa^2 = 2\beta^2}$$

This relation is not merely a proportionality but the precise criterion for the energetic closure of a system. It serves as the direct geometric embodiment of the virial theorem within the RG framework, linking the potential and kinetic aspects of a closed system through a fundamental constant derived from pure geometry.

## Illustrative Examples

To clarify the meaning of this closure condition, consider two contrasting cases:

- Circular Orbit (Closed Subsystem). For a test body in a perfectly circular orbit around a central mass, the condition $\kappa^2 = 2\beta^2$ is exactly fulfilled. The orbital system can be treated as energetically closed: all of the conserved resource is accounted for between the kinetic projection along the orbit and the gravitational projection toward the center. No external channels are needed, and the equality signals full closure.

- Radiating Binary (Open Subsystem). In contrast, for a highly elliptical binary system of compact objects (such as neutron stars. (full calculation in section: Empirical Validation; subsection: Orbital Decay: Binary Pulsar)), the orbital energy is accompanied by gravitational-wave emission. If one considers only the orbital mechanics, the periodic (elliptical orbit) relation $\kappa^2 = 2\beta^2$ will be violated revealing the magnitude of energy flow through an unaccounted-for channels. When all the channels is included in the balance, the closure is restored and the equality holds again.

**Summary**

1. The foundational Principle defines the universe as a closed, conservative, symmetric relational structure SPACETIME $\equiv$ ENERGY.

2. The relational geometry dictates that the only closed, maximally symmetric arenas for these relations are $S^1$ and $S^2$, respectively.

3. Therefore, the projection parameters $\beta = \cos\theta_1$ and $\kappa = \sin\theta_2$ are forced to live on these relational manifolds.

4. The exchange rate between these modes is dictated by the ratio of their relational degrees of freedom (2D/1D), a principle formally realized in the topology of the manifolds as the ratio of their total solid angles ($4\pi/2\pi = 2$).

Physical Implication: The Geometric Origin of Physical Law. This unification of projections reveals that the relationship between gravitational potential and kinetic energy is not an incidental feature of dynamics, but a direct consequence of the underlying relational geometry. Classical physics, in its successful predictions, unknowingly traces this deeper law. The identity SPACETIME $\equiv$ ENERGY is thus explicitly resolved as a precise geometric equivalence:

$$\boxed{\text{Geometric Distribution } (\kappa^2) \;\equiv\; \text{Kinetic Distribution } (\beta^2) \;\times\; 2}$$

# 10 EnergySymmetry Law

## 10.1 Causal Continuity and Energy Symmetry

Theorem 10.1 (Energy Symmetry). The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the EnergySymmetry Law:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{10}$$

Proof. Consider two observers:

- Observer $A$ at rest on the surface at radius $r_A$ (state defined by $\kappa_A, \beta_A = 0$).
- Observer $B$ orbiting at radius $r_B > r_A$ with orbital velocity $v_B$ (state defined by $\kappa_B, \beta_B$).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From $A$'s perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A\to B} = \underbrace{\frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}\left(\beta_B^2 - \beta_A^2\right)}_{\text{Change in Kinetic}}} \tag{11}$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right) \tag{12}$$

From $B$'s perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)\right) = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right) \tag{13}$$

Summing these transfers gives:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{14}$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the EnergySymmetry Law as a direct consequence of the geometry. □

## 10.2 The Relational State Budget ($Q^2$) vs. Energy Transfer ($\Delta E$)

It is crucial to distinguish between two related but distinct concepts:

1. The Quadratic State Budget ($Q^2$): This dimensionless quantity describes the total geometric "footprint" of an object's state, combining its potential and kinetic aspects. It is defined as the sum of the squares of the projections:

$$Q^2 = \kappa^2 + \beta^2 \tag{15}$$

   The change in this value, $\Delta(Q^2) = Q_B^2 - Q_A^2$, represents the net change in the geometric descriptor of the state, but it is not the energy transfer. As demonstrated by the GPS satellite example, $\frac{1}{2}\Delta(Q^2)$ corresponds to the difference between the change in potential energy and the change in kinetic energy.

2. The Specific Energy Transfer ($\Delta E$): This is the physical quantity representing the actual work done and change in motion, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the sum of the changes in the potential and kinetic energy budgets:

$$\Delta E_{A\to B} = \Delta U_{A\to B} + \Delta K_{A\to B} = \frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right) + \frac{1}{2}\left(\beta_B^2 - \beta_A^2\right) \tag{16}$$

   It is this quantity, $\Delta E$, that is conserved and must balance to zero in any closed cycle.

## 10.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics but from the fundamental quadratic nature of the energy budgets in RG.

The energetic significance of a state is proportional to the square of its geometric projection. This is analogous to how kinetic energy is proportional to velocity squared ($v^2$) or how the energy in a wave is proportional to its amplitude squared ($A^2$). The individual energy budgets are defined as:

- Specific Potential Energy Budget: $U/E_0 \propto -\frac{1}{2}\kappa^2$
- Specific Kinetic Energy Budget: $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the changes in these individual budgets.

## 10.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 10.2 (Universal Speed Limit). The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A\to B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B\to A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{17}$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. □

## 10.5 Transparent Energy Balance for Closed Orbits

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where $E_{0B}$ is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \tag{18}$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \tag{19}$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms ($\kappa^2$) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

## 10.6 SingleAxis Energy Transfer and the Nature of Light

Theorem 10.3 (SingleAxis Transformation Principle). For light, the kinematic projection reaches its full extent:

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

This means that all transformation of the relational energy occurs along a single orthogonal axis. The complementary branch of the bidirectional energy exchange is absent, and the total resource of transformation is entirely expressed on one geometric component.

Proof. For massive systems, the EnergySymmetry Law distributes the total energy exchange evenly between two orthogonal projections:

$$U/E_0 = -\tfrac{1}{2}\kappa^2, \qquad K/E_0 = +\tfrac{1}{2}\beta^2.$$

The symmetry of exchange arises because both branches — $(\kappa, \kappa_X)$ and $(\beta, \beta_Y)$ — coexist and compensate each other. Each side carries one half of the total transformation resource, ensuring

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

For light, however, $\beta = 1$ implies $\beta_Y = 0$. The complementary projection disappears; there is no dual observer-frame available for symmetric partition. As a result, the transformation cannot be divided between two orthogonal branches. The full relational resource of the interaction is realized on a single projection.

Therefore, the specific energy potential for light is not halved but complete:

$$\boxed{\Phi_\gamma = \kappa^2 c^2,}$$

while for a massive body the potential remains partitioned,

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This explains why light experiences a total geometric effect exactly twice that of a massive particle in the same field, without introducing any auxiliary approximations. □

> Interpretive Note Light occupies the boundary state where relational reciprocity collapses into self-reference. It is not a massless limit but a distinct single-axis state of the energy geometry. A photon is simultaneously its own counter-frame and its own anti-state. The factor of two that appears in gravitational deflection and frequency shift is a direct signature of this one-axis transformation.

**Summary**

Light has no rest frame. The Speed of Light is the boundary beyond which the energy symmetry law breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

# 11 Beyond Differential Formalism: Structural Dynamics

## 11.1 Intrinsic Dynamics via Energy Redistribution

The system is not described by differential equations of motion evolving in time. Instead, its transformation is dictated by a closed structure of algebraic relations that enforce a perpetually balanced configuration. What we perceive as "dynamics" is this ordered succession of balanced states.

## 11.2 Time as an Emergent Property

In this framework, time is not a fundamental entity but a derived concept tied to changes in the system's geometry. Similar to time dilation, time intervals here are defined by the transformations of geometric parameters like and . A natural time scale arises from the geometry as . This intrinsic time scale encapsulates the system's dynamics without invoking an independent time variable.

**Summary**

Time does not drive change  instead, change defines time.

## 11.3 Why There Are No Equations of Motion

In classical and relativistic physics, dynamics is formulated through differential equations. These express how physical quantities evolve continuously through time, typically governed by:

- A temporal parameter $t$,
- A Lagrangian function $L$,
- A variational principle: $\delta S = 0$, where $S = \int L\, dt$,
- EulerLagrange equations that yield the systems path.

This framework assumes:

1. A continuum of possible configurations,
2. That Nature selects one by minimizing action,
3. That time flows independently of the system.

### Why This Framework Does Not Apply to Relational Geometry

RG begins from a fundamentally different premise.

- There is no "space" of possible paths.
- There is no "freedom to vary."
- The system does not evolve through time — it defines time through its structure.

In RG:

- Each observable is locked in a network of algebraic relations.
- Any change in one parameter necessitates coherent changes in the others.
- Self-consistency enforces projectional balance.

**One valid configuration**

There is only one valid configuration at any moment: the one where all projectional constraints are satisfied. Everything else is not forbidden  it is physically meaningless.

### 11.4 Metric is physically meaningless

In classical frameworks, geometry is parameterized by a metric $g_{\mu\nu}$, assumed to exist independently of the systems energetic configuration. In Relational Geometry, such an entity has no physical meaning: there is no background to be measured, and nothing external to define distance. The relations between $\beta$ and $\kappa$ already determine all observable structure. Introducing a metric tensor would only restate these relations in redundant coordinates. Therefore, RG contains no metric  because it never existed.

**Summary**

In RG, there is no equation of motion. There is no Lagrangian. There is no variational calculus. There is no metric There is only a closed structure of geometric and energetic relationships, and the sequence of valid configurations is what we call dynamics.

## 12 $W_{\text{ill}}$ = Unity

From the geometric closure of RG framework, we derive a universal dimensionless invariant:

$W_{ILL}$ parameters of mass energy time and length:

M $= \frac{\beta^2}{\beta_Y} c^2 \frac{r}{G}$

E $= \frac{\kappa^2}{\kappa_X} \frac{c^4 r}{2G}$

T $= \kappa_X \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2$

L $= \beta_Y \left(\frac{Gm_0}{\beta^2 c^2}\right)^2$

The explicit - invariant for any energetically closed system, including both GR and SR effects, is:

$$W_{ILL} = \frac{E \cdot T}{M \cdot L} = \frac{\frac{1}{\kappa_X} E_0 \cdot \kappa_X t^2}{\frac{1}{\beta_Y} m_0 \cdot \beta_Y r^2} = 1$$

All physical quantities cancel identically, leaving $WILL = 1$, which is satisfied by geometric closure.

This invariant holds universally for all values of $m_0$, $G$, $c$, and $\kappa$. Unlike dimensional analysis, this identity emerges from the projectional interdependence of energy-mass $(E, M)$ and spacetime metrics $(T, L)$ within the unified structure.

The invariant $W_{\text{ILL}} = 1$ expresses geometric unity through energetic projection

## 12.1 The Name "WILL"

The name WILL reflects both the harmonious unity of the equation and a subtle irony towards the anthropic principle, which often intertwines human existence with the causality of the universe. The equation stands as a testament to the universal laws of physics, transcending any anthropocentric framework.

**WILL**

It is not the unit of somethingit is the unity of everything.

# 13 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law is that, when the zero of gravitational potential is chosen on the surface of the central body rather than at infinity, the total specific orbital energy (potential + kinetic, per unit mass) naturally appears in Minkowski form.

## 13.1 Classical Result with Surface Reference

For a test body of mass $m$ on a circular orbit of radius $a$ about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{20}$$

$$K = \frac{1}{2} m \frac{GM_\oplus}{a}. \tag{21}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{22}$$

## 13.2 Projection Parameters and Minkowski-like Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{23}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{24}$$

Substituting into (22) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{25}$$

This is already in the form of a hyperbolic difference of squares: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\left(x^2 - y^2\right), \tag{26}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\kappa^2$ and $K/E_0 = \frac{1}{2}\beta^2$ as budgets. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer $\Delta E$ is the signed sum of budget changes.

## 13.3 Physical Interpretation

In classical derivations, (22) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG framework, (25) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A,B) = (\text{surface, orbit})$, and is invariantly expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While this framework refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

**Why This Matters**

- In classical form, the total orbital energy per unit mass depends only on $GM$ and $a$, and is independent of the test–mass $m$.
- In WILL form, the same fact is embedded in a Minkowski–like difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This reframing answers why the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

# 14 Lagrangian and Hamiltonian as Ontologically Corrupted WILL Approximations

The following section present philosophical and algebraic demonstration: the standard $L$ and $H$ arise as degenerate limits of the relational EnergySymmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms are not fundamental principles but ontologically "dirty" approximations of the relational WILL framework. By collapsing the two-point relational structure into a single-point description, classical mechanics gains computational convenience at the cost of ontological clarity.

## 14.1 Definitions of Parameters

We consider a central mass $M$ and a test mass $m$. The state of the test mass is described in polar coordinates $(r, \phi)$ relative to the central mass.

- $r_A$ — reference radius associated with observer $A$ (e.g., planetary surface).
- $r_B$ — orbital radius of the test mass $m$ (position of observer $B$).
- $v_B^2 = \dot{r}_B^2 + r_B^2\dot{\phi}^2$ — total squared orbital speed at $B$.
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at $B$.
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at $A$.

## 14.2 The Relational Lagrangian

Instead of a relational energy, we define the clean relational Lagrangian $L_{\text{rel}}$, which represents the kinetic budget at point $B$ relative to the potential budget at point $A$:

$$L_{\text{rel}} = T(B) - U(A) = \tfrac{1}{2}m\left(\dot{r}_B^2 + r_B^2\dot{\phi}^2\right) - \frac{GMm}{r_A}. \tag{27}$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \tfrac{1}{2}\left(\beta_B^2 + \kappa_A^2\right). \tag{28}$$

This two-point, relational form is the clean geometric statement.

## 14.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{29}$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

## 14.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \tag{30}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \tag{31}$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \tfrac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \frac{GMm}{r}. \tag{32}$$

## 14.5 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \tfrac{1}{2}mv^2 + \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 + \kappa^2\big), \tag{33}$$

$$H = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 - \kappa^2\big). \tag{34}$$

Here the "+" or "−" signs do not come from $\kappa^2$ itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both are ontologically "dirty" approximations. The clean relational law, involving distinct points $A$ and $B$, is collapsed into a local, one-point description. This shows that Hamiltonian and Lagrangian are just needlessly overcomplicated approximations that lose in ontological integrity.

**Key Message**

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections $\beta$ and $\kappa$ on $S^1$ and $S^2$. For readers accustomed to standard frameworks, the following translation rules may help:

1. General Relativity (metric form):

$$\kappa_X \mathrel{\hat{=}} \sqrt{-g_{tt}} \quad \text{(static spacetimes)}, \qquad \beta \mathrel{\hat{=}} \frac{\|u^\mu_{\text{spatial}}\|}{u^t c}.$$

2. Canonical mechanics (Lagrangian/Hamiltonian): Quantities such as $p_i = \partial L/\partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are computational shadows, useful for legacy calculations but physically redundant.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

**Summary**

Complex Mathematics is the Consequence of Bad Philosophy.

# 15 Derivation of Density, Mass, and Pressure

## 15.1 Geometric Foundation

From the projective analysis established in the previous sections, the fundamental invariant is

$$\kappa^2 = \frac{R_s}{r},$$

where $\kappa$ emerges from the energy projection on the area of unit sphere $S^2$, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

## 15.2 Derivation of Energy Density

From Mass Scale to Volumetric Potential. Starting from the geometric relation,

$$m_0 = \frac{\kappa^2 c^2 r}{2G},$$

we associate $m_0$ with a volumetric proxy $r^3$, obtaining a raw volumetric potential,

$$\frac{m_0}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}.$$

Applying the Geometric Distribution Principle. Because the potential projection $\kappa$ is distributed over S2 - a 2D spherical manifold, the volumetric expression must be normalized over the unit-sphere area $4\pi$. This yields the physical energy density,

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr^2}\right).$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}.$$

Local Energy Density $\equiv$ Relational Projection

Maximal Density. At $\kappa^2 = R_s/r = 1$, the horizon condition is reached, corresponding to the maximal observable energy density at radius $r = R_s$:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}.$$

Normalized Relation. Thus the fundamental identification is

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}.$$

## 15.3 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $m_0 = \alpha\, r^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

## 15.4 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\,\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\, c^2}.$$

Interpretation. $P$ is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energygeometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of $\kappa$ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\, c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale $r$.

Pressure in WILL is the intrinsic surface tension of energygeometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

# 16 Rotational Systems (Kerr Without Metric)

## 16.1 Contextual Bounds

- For a gravitationally closed (static) system, the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.
- For a kinematically closed (maximally rotating) system, the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac^2}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- $\beta$ is the relational rotation parameter, with $0 \leq \beta \leq 1$,
- $\kappa$ is related to the geometry and gravity,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- $J$ is the angular momentum of the black hole,
- $m_0$ is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta^2_{\max} r$$

This relationship holds when $r = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

## 16.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_{\pm} = \frac{R_s}{2}(1 \pm \beta_Y)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of $\kappa$ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

## 16.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2}\left(1 + \sqrt{1 - \beta^2 \cos^2\theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), $r_{\text{ergo}}$ coincides with the event horizon radius.

## 16.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c^2}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

## 16.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship Principle. In our model, Energy Symmetry Law enforce constraint by limiting $\beta$ to the range $[0, 1]$.

## 16.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in $a_*$, mirror the orbital velocity parameter $\beta$, providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

### Physical Interpretation

- No need for pre-existing spacetime geometry emerges from angular energy distributions.
- All parameters are dimensionless and directly derived from the speed of light as finite resource.
- Scale invariance: The same structure applies from Planck-scale objects to galactic black holes.

# 17 Unified Geometric Field Equation

## 17.1 The Theoretical Ouroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}} \tag{35}$$

The ratio of geometric scales equals the ratio of energy densities.

This is the unified geometric field equation of WILL Relational Geometry. It expresses the complete equivalence:

SPACETIME GEOMETRY $\equiv$ ENERGY DISTRIBUTION

We have shown that this single foundational Principle, through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with a single foundational Principle SPACETIME $\equiv$ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

> Theoretical Ouroboros
>
> The RG framework exhibits perfect logical closure: the fundamental Principle about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

IMAGES/1_WILL-Relational-Geometry.png

The foundational Principle SPACETIME $\equiv$ ENERGY closes into the unified field equation

$$\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}},$$

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework  the Theoretical Ouroboros. But regardless of aesthetic beauty of this result lets remain skeptical.

## 17.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2\ \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- Surfacescaled closure (vs. volume filling). Mass follows the algebraic closure $m_0 = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the $4\pi$ is the spherical projection measure, not a Newtonian volume average.

- Natural bounds. The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

Summary

The WILL framework postulates no external laws or assumptions. All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

# 18 Algebraic Closure and Structural Dynamics

Physical dynamics in WILL Relational Geometry (RG) emerges not from temporal evolution equations but from a set of algebraically closed invariants. Each parameter participates in a selfconsistent configuration of relational constraints. Changing any one parameter necessitates a coordinated shift in all others to maintain validity.

Definition 18.1 (Algebraic Closure of the WILL Structure). The minimal algebraic closure of the RG system is expressed by the set:

$$\begin{cases} \kappa^2 = 2\beta^2, \\ R_s = \dfrac{2Gm_0}{c^2}, \\ r = \dfrac{R_s}{\kappa^2}, \\ \rho = \dfrac{\kappa^2 c^2}{8\pi G r^2}, \\ \rho_{\max} = \dfrac{c^2}{8\pi G r^2}, \\ t = \dfrac{r}{c}, \\ m_0 = 4\pi r^3 \rho. \end{cases}$$

These relations are not independent definitions but mutual constraints. Each variable acquires meaning only within the self-consistent closure of the entire system.

No differential equations are required:

Dynamics unfolds as a consequence of relational energy transformations.

Lemma 18.2 (Causal Closure without Circularity). Every observable quantity in RG can be determined from a minimal input pair composed of one dynamic projection (e.g. $\kappa$ or $\beta$) and one scale parameter (e.g. $r$, $M$, or $\rho$). No parameter both defines and is defined by the same input. Thus, the system avoids circularity while remaining causally closed.

Theorem 18.3 (Structural Dynamics). Within the closed algebraic configuration $(\kappa, \beta, \rho, r, t)$, any local variation of one parameter entails a coherent transformation of all others, such that the algebraic closure is preserved:

$$\delta\Phi(\kappa, \beta, \rho, r, t) = 0, \qquad \Phi \text{ representing the constraint manifold.}$$

Therefore, dynamics is the propagation of constraint consistency rather than temporal evolution.

Corollary 18.4 (Emergent Time). Since every admissible configuration is defined by its energygeometry relations, the parameter conventionally called time is nothing more than an ordered index of transitions between self-consistent configurations:

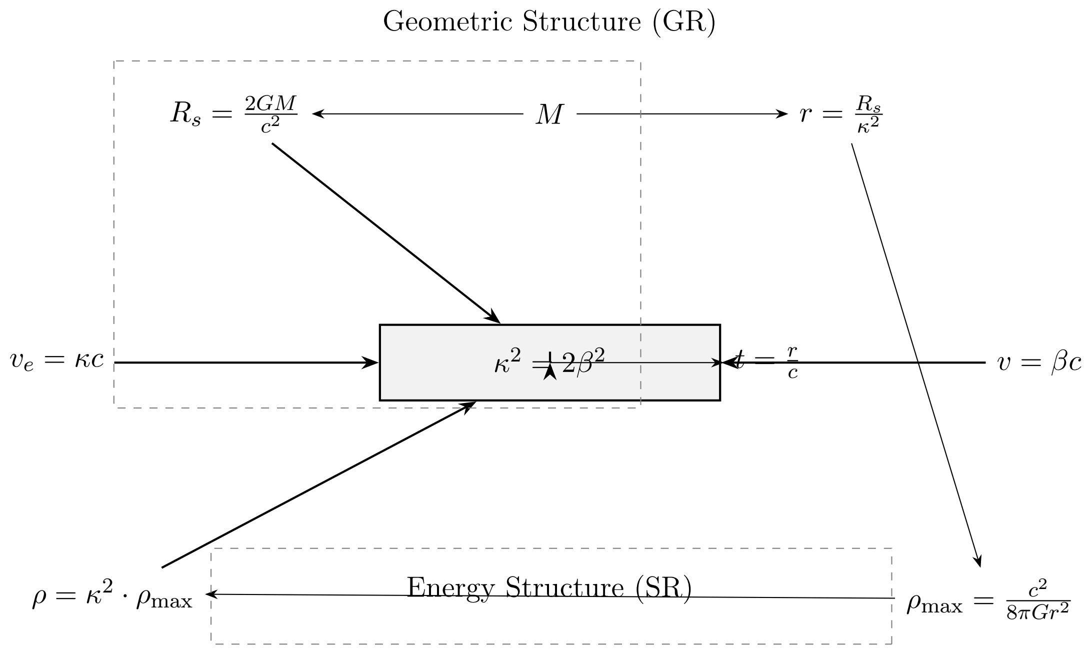
$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r, \ldots)}.$$

Thus, causality in RG is structural, not temporal; it refers to coherence of relations rather than to sequences of events.

**Interpretation**

Dynamics without Differentials: In RG, no differential equations of motion are required. Physical change is expressed through algebraic redistribution of geometric and energetic quantities. Time is emergent, coherence is enforced, and the universe evolves as a continuous sequence of balanced configurations.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$




The result is a structure where causality is internal, coherence is enforced, and dynamics is simply the shifting of balanced configurations — not the unfolding of arbitrary functions over time.

## 18.1 Numerical Example: Accretion onto a Black Hole

Consider a black hole accreting mass from a surrounding disk to illustrate the model's intrinsic dynamics. Let the initial mass be $m_0 = 10, M_\odot$, with a Schwarzschild radius $R_s = \frac{2Gm_0}{c^2} \approx 2.95 \times 10^4, \mathrm{m}$. Suppose $\kappa = 0.1$, so $r = \frac{R_s}{\kappa^2} = \frac{2.95\times10^4}{0.01} = 2.95 \times 10^6, \mathrm{m}$, and the associated time scale is $t = \frac{r}{c} \approx 9.83 \times 10^{-3}, \mathrm{s}$.

As the black hole accretes mass, increasing to $m_1 = 10.1, M_\odot$, the Schwarzschild radius becomes $R_s \approx 2.98 \times 10^4, \mathrm{m}$. Assuming $\kappa$ remains constant for simplicity, $r = \frac{2.98\times10^4}{0.01} = 2.98 \times 10^6, \mathrm{m}$, and $t \approx 9.93 \times 10^{-3}, \mathrm{s}$. This increase in $t$ reflects the system's evolution, driven solely by the changing geometry.

**Summary**

In Relational Geometry, causality is not temporal but structural. Each variable $(\kappa, \beta, \rho, r, t)$ participates in an algebraically closed configuration that defines a self-consistent physical state. Any local variation of one variable entails a coherent transformation of all others, preserving the closure of the system. Hence, dynamics is the propagation of constraint consistency, and time is the ordered index of these transformations.

$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r, ...)}$$

# Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as "laws of nature."
3. Mathematical formalisms are constructed to describe these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are generated as inevitable consequences of relational geometry:

- There are no independent axioms such as "inertial mass equals gravitational mass."
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls "laws of nature" are secondary shadows of the single relational principle:

$$\text{SPACETIME} \ \equiv \ \text{ENERGY}.$$

**Key Distinction**

Standard Physics: Laws describe what we observe.
RG Framework: Laws are generated as necessary products of relational geometry.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

| Descriptive Physics (Standard) | Generative Physics (WILL) |
|---|---|
| Phenomena are observed first, then summarized into empirical laws. | Laws emerge as inevitable consequences of relational geometry. |
| Physical laws are assumptions introduced to model reality. | Physical laws are identities, enforced by geometric self-consistency. |
| Time and space are treated as external backgrounds. | Time and space are projections of energy relations. |
| Dynamics = evolution of states in time. | Dynamics = ordered succession of balanced configurations; time is emergent. |
| Goal: describe what is observed. | Goal: show why nothing else is possible. |

Table 3: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

# 19 Axiomatic Foundations Theorem:WILL Relational Geometry (RG) and General Relativity (GR)

> This logical asymmetry does not imply physical superiority a priori; it only states that any empirical support for GR already presupposes relational invariance.

Definition 19.1 (GR Core Axioms). General Relativity (GR) is assumed to rest on the following axioms:

(A1) The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.

(A2) Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.

(A3) Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.

(A4) Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Definition 19.2 (RG One Principle). RG is based on a single Principle:

(W1) Relational Principle: All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy.

Lemma 19.3 (Relationality in GR). From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.

Remark 19.4 (Bridge: From Relational Principle to GR Axioms). If the Relational Principle (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:

- It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.
- It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.

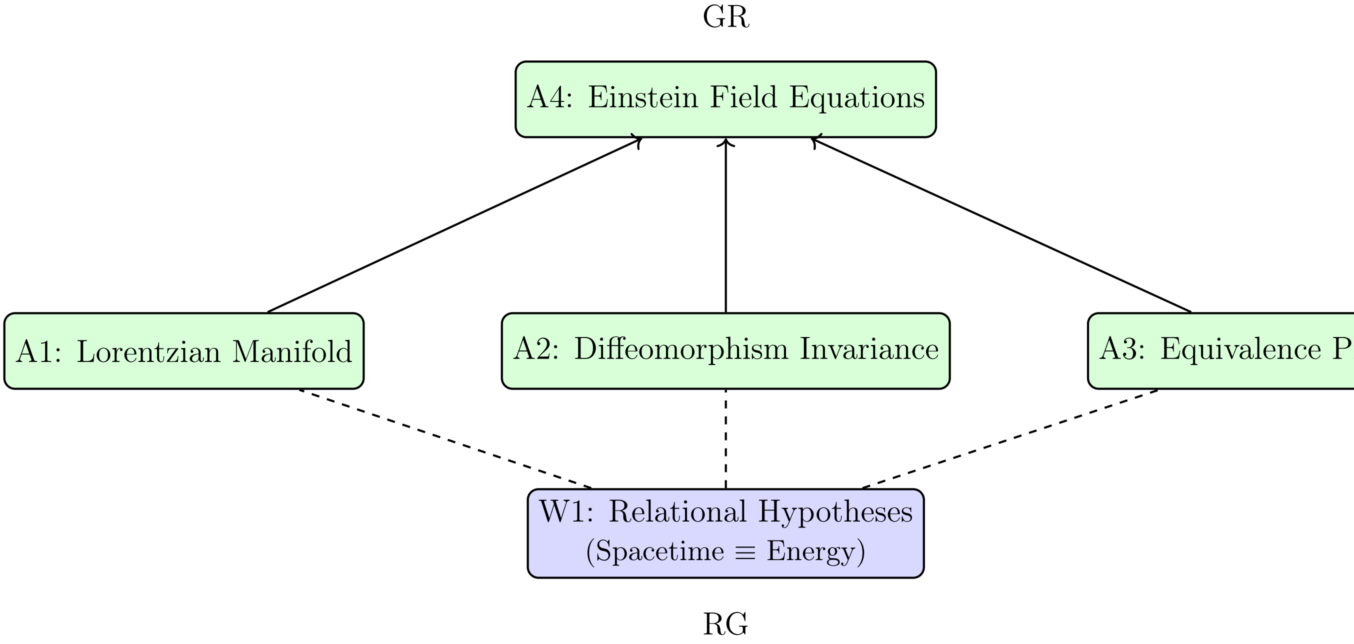
Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 19.5 (Asymmetric Falsifiability of GR and RG). Let $\mathsf{GR}$ denote the theory defined by axioms (A1)–(A4), and let $\mathsf{RG}$ denote the theory defined by Principle (W1). Then:

1. If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence, $\mathsf{GR}$ is necessarily falsified.
2. If any of (A1)–(A3) are empirically falsified, $\mathsf{GR}$ collapses, but (W1) may still remain valid as a stand-alone principle.

Therefore, there exist possible empirical scenarios in which GR fails while RG survives, but there exist no scenarios in which RG fails while GR survives.

Corollary 19.6. RG is axiomatically more fundamental than GR: its sole Principle (W1) is logically included within the core axioms of GR, while GR requires additional ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of RG.




Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability). RG rests on the single relational Principle (W1). Core GR assumes additional structures (A1–A4). Hence:

- If W1 is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of A1–A3 is falsified, GR collapses, while W1 (and thus RG) may still hold.

Therefore, there are scenarios where GR fails and RG survives, but none where RG fails while GR survives.

### Status of General Relativity within RG

It is important to emphasize that the RG framework does not invalidate the achievements of General Relativity. Rather, it explains them. All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in Relation Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.
Thus, GR is not a rival but a specialized, parameter-heavy realization of RG's more general principle. In logical terms:

- Relational Geometry can stand without GR, but GR cannot stand without the relational Principle (W1).
- The empirical successes of GR are preserved within RG, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

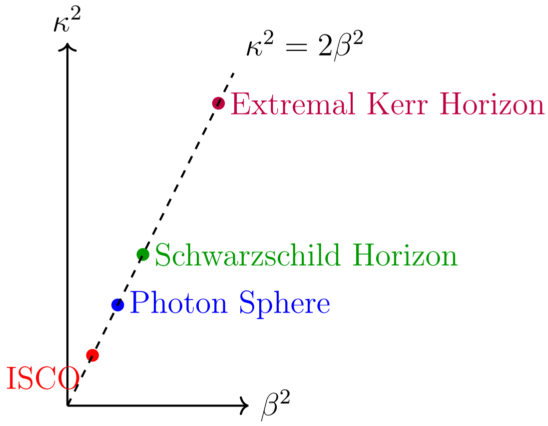
Therefore, GR should be understood as an effective approximation embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.

Comparison Table: General Relativity (GR) vs WILL Relational Geometry (RG)

| # | Category | General Relativity (GR) | Relational Geometry (RG) |
|---|---|---|---|
| 1 | Nature of Space and Time | Postulated as smooth manifold with metric $g_{\mu\nu}$ | Emerges from projection of energy relations $(\kappa, \beta)$ |
| 2 | Curvature | Defined via $R_{\mu\nu}, R$; second derivatives of the metric | Defined algebraically as $\kappa^2 = \frac{R_s}{r}$ |
| 3 | Energy and Momentum | Encoded in $T_{\mu\nu}$, requires model of matter | Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$ |
| 4 | GeometryMatter Relation | $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation | $\kappa^2 = \rho/\rho_{\max}$; local proportionality |
| 5 | Singularities | Appear when $\rho \to \infty$, $g_{00} \to 0$ | Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$ |
| 6 | Gravitational Limitation | Via metric behavior and horizons | Via geometric constraint $\kappa \in [0, 1]$ |
| 7 | Density Limit | Not explicitly defined, requires external input (Planck-scale) | Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$ |
| 8 | Concept of Time | Coordinate-based, embedded in $g_{00}$; system-dependent | Physical: $\beta$ as projection of energy onto temporal axis |
| 9 | Dynamics | Via time derivatives and Lagrangians | Via change in energy proportions; no differential equations |
| 10 | Formalism | Geometry, tensors, 2nd-order derivatives | Energy projections, circular geometry, algebraic closure |
| 11 | Intuitiveness | Low; relies on abstract and heavy formalism | High; built from observable and intrinsic relations |
| 12 | Observational Fit | Confirmed (with dark matter/energy assumptions) | Consistent; explains phenomena without "dark entities" (Details in WILL PART 2) |

| Phenomenon | Radius $r$ | $\beta^2$ | $\kappa^2$ | $Q^2$ | Comment |
|---|---|---|---|---|---|
| ISCO (innermost stable orbit) | $r = 3R_s$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | Marginal stability of timelike orbits |
| Photon sphere | $r = \frac{3}{2}R_s$ | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 | Null circular orbits, $Q = 1$, $Q_t = 0$ |
| Static horizon (Schwarzschild) | $r = R_s$ | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | Purely gravitational closure, $\kappa^2 = 2\beta^2$ |
| Extremal Kerr horizon | $r = \frac{1}{2}R_s$ | 1 | 2 | 3 | Maximal rotation, $\beta = 1$, merged horizons |

Table 4: Critical radii and their projectional parameters in WILL Relational Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of $(\kappa, \beta)$ from the single closure law $\kappa^2 = 2\beta^2$.




## 19.1 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the RG's principles. One can derive GR by adding these additional structures to RG's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, where RG provides the logical foundation upon which GR can be consistently constructed.

# 20 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry,

| Phenomenon | Standard GR Result | Projectional Geometry $(\beta, \kappa)$ |
|---|---|---|
| GPS time shift / gravitational redshift | Frequency shift = combination of kinetic (SR) and gravitational (GR) effects. | Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ verified directly with GPS satellites. Photon sphere, ISCO, horizons & Derived by solving geodesic equations in Schwarzschild metric. & Critical radii emerge from simple projectional relations ($\kappa^2 = 2\beta^2$, $Q^2 = \kappa^2 + \beta^2 = 1$, $Q^2 = \kappa^2 + \beta^2 = \frac{1}{2}$). |
| Mercurys perihelion precession | Complex expansion of Einstein field equations. | Exact same number obtained from projection geometry with $\beta, \kappa$. |
| Binary pulsar orbital decay | Explained via quadrupole radiation formula; requires asymptotic Bondi mass. | Emerges from balance of projection invariants without asymptotic constructs. |
| Cosmological redshift | Photon loses energy as universe expands. | Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART 2) |
| Cosmological constant $\Lambda$ | Added by hand to fit data (dark energy). | Arises naturally as $\Lambda = \kappa^2/r^2$. No extra entities required. (Details in WILL PART 2) |
| Singularities | Predicted in black holes and big bang ($\rho \to \infty$). | Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$. |
| Local gravitational energy | Cannot be localized (only ADM/Bondi at infinity). | Directly measurable via $\kappa$, e.g. from light deflection angle. |
| Unification with QM and SR | No natural unification in GR framework. | Same projectional law applies from microscopic (QM) to cosmic (GR, COSMO) scales. (Details in WILL PART 3) |

Table 5: Classical GR results vs. Projectional Geometry outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark energy.

RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single foundational Principlethat spacetime is equivalent to energywe derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different

| Phenomenon | Empirical Benchmark | WILL Predicti |
| --- | --- | --- |
| GPS satellite time dilation (SR + GR) | 38.52 $\mu$s/day (observed) | 38.52 $\mu$s/day |
| Mercury perihelion precession | 43”/century (observed) | 43”/century |
| Solar light deflection | 1.75 arcsec (observed) | 1.75 arcsec |
| Schwarzschild photon sphere | $r = 1.5R_s$ (GR prediction) | $r = 1.5R_s$ |
| Schwarzschild ISCO | $r = 3R_s$ (GR prediction) | $r = 3R_s$ |
| Hulse–Taylor pulsar period decay | $\Delta P \approx -2.42 \times 10^{-12}$ s/s (observed) | $\Delta P \approx -2.40 \times$ |
| Earth–Moon tidal power (LLR recession) | 0.120 TW orbital power (measured) | 0.120 TW |
| Galaxy rotation curves (Milky Way example) | Flat curves beyond $\sim$10 kpc | Flat curves fro |
| Cosmological absolute scale (Supernovae fit) | Hubble-like expansion, $\Lambda$CDM fits | Emergent abso |

Table 6: Empirical validation of WILL Relational Geometry across classical relativistic tests, orbital dynamics, astrophysical observations, and cosmology. See details in "Appendix 1"

projections of the same underlying structure.

$$\boxed{\text{Special and General Relativity emerge from the same geometric principles.}}$$

This approach offers distinct advantages:

- Conceptual clarity  understanding physics through pure geometry
- Computational efficiency  significantly reducing complexity
- Epistemological hygiene  deriving results from minimal assumptions
- Philosophical depth  redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

$$\boxed{\text{Spacetime and energy are mutually defining aspects of a single relational structure.}}$$

**Final Summary**

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

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