WILL Part I: Relational Geometry

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Abstract

This paper, the first in the WILL series, applies extreme methodological constraints in constructing a theory and establishes Relational Geometry (RG): a foundational framework where spacetime is not a background arena but an emergent property of energy transformations.

From a single ontological principle, SPACETIME $\equiv$ ENERGY, it aims to derive the complete geometric structure of physics across physical domains. This equivalence is not postulated but derived by removing the hidden ontological assumption, implicit in modern physics, that structure (spacetime) and dynamics (energy) are separate phenomena.

This shift establishes an ontological transition from descriptive to generative physics: instead of introducing laws to model observations, it derives them as necessary consequences of RG itself - turning physics from a catalogue of phenomena into the logical unfolding of a single principle.

The result is a singularity-free, ontologically clean formalism that reproduces the core equations of Special Relativity (SR) and General Relativity (GR) as geometric projections on closed relational manifolds S^1 (directional) and S^2 (omnidirectional). Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energy-momentum relation, Schwarzschild and Kerr solutions, and Einstein field equations via the dimensionless projections β (kinematic) and κ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$ (geometrically derived, virial-like). All results are empirically validated (e.g., GPS time shift 38.52 μs/day, Mercury precession to 10^{-10}%, and others listed in Appendix I).

WILL Part I offers solutions to several long-standing problems, including:

- Resolution of GR singularities (via naturally bounded $\rho_{\max}$),
- Derivation of the equality of gravitational and inertial masses (from the common channel of rest-invariant scaling),
- Removal of local energy ambiguity $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$,
- Revelation of a clear relational symmetry between kinematic and potential projections,
- Establishment of a computationally simpler and ontologically consistent foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

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Guiding Principles

$\boxed{\text{Relational, Epistemic and Ontological Minimalism}}$

False Ontological Split:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

Ontological Principle: Spacetime $\equiv$ Energy

Topological Requirement: Only Closed and Maximally Symmetric Relational Carriers Allowed:

Relational Carriers: Circle and Sphere with Projections Closure ($S^1 \Rightarrow \beta^2 + \beta_Y^2 = 1$) and ($S^2 \Rightarrow \kappa_X^2 + \kappa^2 = 1$) as Minimal Permitted Topologies

"Exchange Rate" as Ratio of degrees of freedoms $\mathcal{R} = \frac{\text{d.o.f.}(S^2)}{\text{d.o.f.}(S^1)} = \frac{2}{1} = 2 \Rightarrow$ $\boxed{\kappa^2 = 2\beta^2}$ Energetic Closure Condition as Relational Analogue of the Virial Theorem

Conservation, Energy Symmetry, Speed Limit, Causality, Natural Bounds

Unified Field Equation: $\kappa^2 = R_s/r = \rho/\rho_{\max}$

Closure:
Field Equation $\Longleftrightarrow$ Ontological Principle

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> "There is no such thing as an empty space, i.e., a space without field. . . . Space-time does not claim existence on its own, but only as a structural quality of the field."

— Albert Einstein, Relativity: The Special and the General Theory (Appendix V: "Relativity and the Problem of Space"), 1952 edition, Methuen (London), p. 155; based on earlier 1920 additions.

IMPORTANT:

This document must be read literally. All terms are defined within the relational framework of WILL Relational Geometry. Any attempt to reinterpret them through conventional notions (absolute energies, external backgrounds, hidden containers) will produce distortions and misreadings. Just like responsibility of formulating lies with the author, the responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

1 Foundational Approach

This Approach Does not Describe Physics; it Generates it. 24.1

Guiding Principle:

Nothing is assumed. Everything is derived.

1.1 Epistemic Hygiene as Refusal to Import Unjustified Assumptions.

This framework is constructed under a single epistemic constraint: to derive all of physics by removing one hidden assumption, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced and no constructs are retained unless they are geometrically or energetically necessary.

Principle 1.1 (Ontological Minimalism). Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities. This principle is not a statement about the nature of reality, but a rule of logical hygiene for constructing a theory.

No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent relational projections.

Principle 1.2 (Relational Origin). All physical quantities must be defined by their relations. Any introduction of absolute properties risks reintroducing metaphysical artefacts and contradicts the foundational insight of relativity.

1.2 Mathematical Transparency

Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.

1. Every mathematical phrase, operational choice, or identity caries its on ontological statement.
2. Each mathematical object must correspond to explicitly identifiable relation between observers with transparent ontological origin.
3. Every symbol must be anchored to unique physical idea.
4. Introducing symbols without explicit necessity constitutes semantic inflation: the proliferation of symbols without corresponding physical meaning.
5. Number of symbols = Number of independent physical ideas.

IMathematical hygiene

$\boxed{\text{Mathematical hygiene is the geometry of reason}}$

2 Ontological Blind Spot In Modern Physics

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

2.1 Historical Pattern: breakthroughs delete, not add

- Copernicus eliminated the Earth/cosmos separation.
- Newton eliminated the terrestrial/celestial law separation.
- Einstein eliminated the space/time separation.
- Maxwell eliminated the electricity/magnetism separation.

Each step widened the relational circle and reduced the number of unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

2.2 The contemporary split: an unpaid ontological bill

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a bivariable syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate inside the split. The split is therefore not an empirical discovery but an unpaid ontological debt.

2.3 Empirical bankruptcy of the separation

- Local energy conservation verified only after the metric is declared fixed; no experiment varies the volume of flat space and checks calorimetry.
- Universality of free fall tests $m_i = m_g$ numerically, not the claim that inertia resides in the object rather than in a geometric scaling relation.
- Gravitational-wave polarisations test spin content, not ontology; extra modes can still be called "matter on spacetime".
- Casimir/Lamb shift measure differences of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

In short, every "test" is an internal consistency check of a formalism that already presupposes two substances. None constitute positive evidence for the split.

2.4 Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an un-evidenced metaphysical postulate—the last geocentric epicycle in physics.

Summary

Any attempt to treat "spacetime structure" as separate from "dynamics" smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

Ontological Minimalism:

If no empirical or logical ground justifies the distinction between (structure) and (dynamics), the distinction must be dissolved.

$$\text{SPACETIME} \equiv \text{ENERGY}$$

This equivalence is not algebraic but ontological;
spacetime and energy are two descriptive projections of a single invariant entity we call:

WILL

3 Unifying Principle Removing the Hidden Assumption

3.1 False Separation

Lemma 3.1 (False Separation). Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of interprocess data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. □

Corollary 3.2 (Structure–Dynamics Coincidence). To avoid the artifact of Lemma 3.1, the structural arena and the dynamical content must be identified: geometry is energy, and energy is geometry.

Principle 3.3 (Ontological Principle: Removing the Hidden Assumption).

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight: it removes the hidden unjustified separation between "geometry" and "dynamics." Space and time are not containers but emergent descriptors of relational energy.

Remark 3.4 (Auditability). Principle 3.3 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call - WILL.

3.2 What is Energy in Relational Framework?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure. Crucially, this quantity is never observed directly, but only through differences between states: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation. Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself. From these empirical and relational facts the definition follows unavoidably:

Energy :

Energy is the relational measure of difference between possible states, conserved in any closed whole.
It is not an intrinsic property of an object, but comparative structure between states (and observers), always manifesting as transformation.

4 Deriving the WILL Structure

Having established our Principle 3.3 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences. We will demonstrate that this single principle is sufficient to enforce the closure, conservation, and isotropy of the relational structure, leading to a unique set of geometric carriers for energy.

Definition 4.1 (WILL). WILL $\equiv$ SPACE-TIME-ENERGY is the technical term we use for unified relational structure determined by 3.3 . All physically meaningful quantities are relational features of WILL; no external container is permitted.

Lemma 4.2 (Closure). Under 3.3, WILL is self-contained: there is no external reservoir into or from which the relational resource can flow.

Proof. If WILL were not self-contained, there would exist an external structure mediating exchange. That external structure would then serve as a background distinct from the dynamics, contradicting Corollary 3.2. □

Lemma 4.3 (Conservation). Within WILL, the total relational "transformation resource" (energy) is conserved.

Proof. By Lemma 4.2, no external fluxes exist. Any change in one part of WILL must be balanced by complementary change elsewhere. Hence a conserved global quantity is enforced at the relational level. □

Lemma 4.4 (Isotropy from Background–Free Relationality). If no external background is allowed (Cor. 3.2), then no direction can be a priori privileged. Thus the admissible relational geometry of WILL must be maximally symmetric (isotropic and homogeneous) at the level at which it encodes the conserved resource.

Proof. A privileged direction requires an extrinsic reference to distinguish it. In a purely relational setting, distinctions must be constructible from relations internal to WILL. If a direction were privileged in the geometry that encodes the conserved resource, such privilege would not be derivable from purely internal comparisons (which are symmetric by construction), and would reintroduce an external orienting structure. Therefore the encoding geometry must be maximally symmetric. □

4.1 Derivation of the Relational Carriers

The lemmas of Closure, Conservation, and Isotropy (Lemmas 4.2–4.4) establish the *necessary properties* of any geometric carrier of the relational resource. We must now identify these carriers.

To do this, we must first reject the substantivalist or "God's-eye view" common in physics, which illegitimately postulates an external 3D coordinate system ("State C") from which to describe the interaction between two states ("A" and "B"). Per Principle 1.2 (Relational Origin), such an external frame is an ontological speculation.

In a purely relational framework, only the participants (A and B) exist. All physics must be described *only* from their mutual perspectives. This methodological constraint is not a simplification; it is an ontological necessity.

Theorem 4.5 (Minimal Relational Carriers of the Conserved Resource). The only relational manifolds satisfying the derived constraints of Closure, Conservation, and maximal Symmetry (Lemmas 4.2–4.4) are:

(a) S^1 for directional (Kinematic) relational transformation;

(b) S^2 for omnidirectional (Gravitational) relational transformation.

Proof. The proof proceeds by classifying the minimal types of relations and applying the derived Lemmas:

(a) Directional (Kinematic) Relation: This is the simplest non-trivial relation: a transformation from State A to State B. Per the Principle of Relational Origin, this interaction can only be described from the frame of A or B. From the perspective of B, any complex 3D motion of A (including transverse motion) is operationally perceived only as a change in the rate of approach or recession. Thus, the fundamental, operational description of a two-state transformation is necessarily one-dimensional (1D).

Applying the Lemmas: By Lemma 4.2 this 1D geometry must be closed. By Lemma 4.4 it must be maximally symmetric. The classification of connected, closed, 1-manifolds yields S^1 as the unique (up to diffeomorphism) carrier satisfying these constraints.

(b) Omnidirectional (Gravitational) Relation: This is the other minimal relation type: a central state (A) relating to the locus of all equidistant states (e.g., an orbit). This describes a "center-to-orbit" relationship. By Lemma _reflem:isotropy (Isotropy), the conserved WILL resource must be distributed uniformly across all possible orientations from this center. The minimal manifold required to describe all possible orientations from a center is a two-dimensional (2D) surface.

Applying the Lemmas: By Lemma 4.2, this 2D surface must be closed. By Lemma 4.4, it must be maximally symmetric (isotropic from the center). By the classification of constant-curvature surfaces, the unique closed, simply connected, maximally symmetric 2-manifold is the 2-sphere (S^2).

The Principle of Ontological Minimalism (1.1), combined with the derived constraints, thus uniquely and necessarily selects S^1 and S^2 as the minimal relational carriers. □

Corollary 4.6 (Uniqueness). Under 3.3 with Closure, Conservation, and Isotropy (Lemmas 4.2–4.4), S^1 and S^2 are necessary relational carriers for, respectively, directional and omnidirectional modes of energy transformation.

Remark 4.7 (Non-spatial Reading). Throughout, S^1 and S^2 are not to be interpreted as spacetime geometries. They are relational manifolds that encode the closure, conservation, and isotropy of the transformational resource. Ordinary spatial and temporal notions are emergent descriptors of patterns within WILL.

Summary:

From removing the hidden assumption 3.1 we inevitably arrive to 3.3 SPACETIME $\equiv$ ENERGY from there we deduced: (i) closure, (ii) conservation, (iii) isotropy, and hence (iv) the unique selection of S^1 and S^2 as minimal relational carriers for directional and omnidirectional transformation. These objects are non-spatial encodings of conservation and symmetry; they are enforced by the3.3 rather than assumed independently.

4.2 Ontological Status of the Relational Manifolds S^1 and S^2

A natural question arises regarding the ontological status of the circle S^1 and the sphere S^2: What are they, and where do they "exist"?

The answer requires a fundamental shift in perspective. In WILL Relational Geometry, S^1 and S^2 are not spatial entities existing within a pre-defined container. They are the necessary relational architectures that implement the core identity SPACETIME $\equiv$ ENERGY.

Energy as Relational Bookkeeping Recall that energy is defined as the relational measure of difference between possible states. It is not an intrinsic property but a comparative structure that guarantees causal continuity. It is never observed directly, only through transformations.

The Manifolds as Protocols of Interaction The manifolds S^1 and S^2 are the minimal, unique mathematical structures capable of hosting this relational "bookkeeping" for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, A and B:

- Observer A is the center of their own relational framework. Observer B is a point on A's S^1 (for kinematic relations) and S^2 (for gravitational relations).
- Simultaneously, observer B is the center of their own framework. Observer A is a point on B's S^1 and S^2.

There is no privileged "master" manifold. Each observable interaction is structured by these mutually-centered relational protocols. The parameters β and κ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta_X^2 + \beta_Y^2 = 1; \quad \kappa_X^2 + \kappa_Y^2 = 1$) are the innate accounting rules of these protocols.

Emergence of Spacetime Therefore, the question "Where are S^1 and S^2?" is a category error. They are not in space; they are the structures whose coordinated, multi-centered interactions give rise to the phenomenon we perceive as spacetime. Spacetime is the emergent, collective shadow cast by the dynamics of energy relations projected onto these architectures.

In essence, S^1 and S^2 are the ontological embodiment of the relational principle. They are derived as the only possible structures that can house the transformational resource (energy) in a closed, conserved, and isotropic system. Their status is that of a fundamental relational geometry from which physics is generated.

5 Emergence of Spacetime

In this construction, "space," "time," are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

5.1 The Duality of Transformation

Lemma 5.1 (Duality of Evolution). The identification of spacetime with energy and its transformations necessitates two complementary relational measures:

1. the extent of transformation (external displacement), and
2. the sequence of transformation (internal order).

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The circle S^1 provides the minimal geometry enforcing such complementarity: its orthogonal projections furnish precisely two non-redundant coordinates. □

We define this orthogonal projections as follows:

- The Amplitude Component (β_X): This projection represents the relational measure between the system and the observer. It corresponds to the extent of transformation, which manifests physically as momentum (as shown in next section).
- The Phase Component (β_Y): This projection represents the internal structure of a system. It governs the intrinsic scale of its proper space and proper time units, corresponding to the sequence of its transformation. A value of $\beta_Y = 1$ represents a complete and undisturbed manifestation of this internal structure, a state we identify as rest.

5.2 Conservation Law of Relational Transformation

Theorem 5.2 (Conservation Law of Relational transformation). The orthogonal components of transformation (β_X, β_Y) are bound by the closure relation

$$\beta_X^2 + \beta_Y^2 = 1.$$

Proof. Since S^1 is closed, every point on the circle is constrained by the Pythagorean identity of its projections. Thus no state can exceed or fall short of the finite relational "budget." This closure enforces conservation across all processes. □

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

5.3 Consequence: Relativistic Effects

Proposition 5.3 (Physical Interpretation: Relativistic Effects). The conservation law of Theorem 5.2 implies that any redistribution between the orthogonal components (β_X, β_Y) manifests physically as the relativistic effects of time dilation and length contraction.

Proof. By Theorem 5.2, the components satisfy $\beta_X^2 + \beta_Y^2 = 1$. An increase in the relational displacement β_X enforces a decrease in the internal measure β_Y. This reduction of β_Y corresponds to dilation of proper time and contraction of proper length, while the growth of β_X represents momentum. Thus the relativistic trade-off is the direct physical expression of the geometric closure of S^1. □

Summary:

Geometry of spacetime is the shadow cast by the geometry of relations.

6 Kinetic Energy Projection on S^1

Since S^1 encodes one-dimensional displacement, the total energy E of the system must project consistently onto both axes:

$$E_X = E\beta_X, \qquad E_Y = E\beta_Y.$$

Theorem 6.1 (Invariant Projection of Rest Energy). For any state (β_X, β_Y) on the relational circle, the vertical projection of the total energy is invariant:

$$E\beta_Y = E_0.$$

Proof. When $\beta_X = 0$, closure enforces $\beta_Y = 1$, yielding $E = E_0$. Since closure applies for all θ_1, the vertical projection $E\beta_Y$ remains equal to this rest value in every state. □

Corollary 6.2 (Total Energy Relation). From Theorem 6.1 it follows that

$$E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sqrt{1-\beta_X^2}}.$$

Remark 6.3 (Lorentz Factor). The historical Lorentz factor γ is nothing more than the reciprocal of β_Y. No additional structure is introduced: all content is already present in $E\beta_Y = E_0$.

Summary:

The historical Lorentz factor γ is the reciprocal of β_Y. $\gamma = 1/\beta_Y$

6.1 Rest Energy and Mass Equivalence

Corollary 6.4 (Rest Energy and Mass Equivalence). Within the normalization $c = 1$, the invariant rest energy equals mass:

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1, no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. □

Summary:
Mass is the invariant projection of total rest energy.

6.2 Energy–Momentum Relation

Proposition 6.5 (Horizontal Projection as Momentum). On the relational circle, the unique relational measure of displacement from rest is the horizontal projection $E\beta_X$; hence

$$p \equiv E\beta_X \quad (c = 1).$$

Proof. The rest state is $(\beta_X, \beta_Y) = (0, 1)$. A displacement measure must (i) vanish at rest, (ii) grow monotonically with $|\beta_X|$, and (iii) flip sign under $\beta_X \mapsto -\beta_X$. The only relational candidate satisfying (i)-(iii) is the horizontal projection $E\beta_X$. Thus the identification is necessary rather than conventional. □

Corollary 6.6 (Energy–Momentum Relation). With p identified by Proposition 6.5 and $m = E_0$, the closure identity yields

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta_X)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta_X$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E, yielding the standard form. □

Remark 6.7 (Geometric Forms). The same identity may be expressed explicitly in terms of circle coordinates:

$$E^2 = \left(\tfrac{\beta_X}{\beta_Y} E_0\right)^2 + E_0^2 = \big(\cot(\theta_1)\, E_0\big)^2 + E_0^2.$$

These are equivalent renderings of the same geometric necessity.

Remark 6.8 (Units sanity check - bookkeeping). Using $\beta_X = v/c$, the identification $p \equiv E\beta_X$ gives

$$pc = E\,\frac{v}{c} \quad \Longrightarrow \quad p = \frac{E\,v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta_X}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

Summary
The energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$ is geometric identity of S^1.

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$\beta_X = \beta, \quad \beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	Trigonometric Form
$1/\beta_Y = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(v/c)^2}$	$1/\beta_Y = 1/\sin(\theta_1) = 1/\sin(\arccos(\beta))$
$\beta_Y = \sqrt{1-\beta^2} = \sqrt{1-(v/c)^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$

Table 1: Geometric representation of relativistic effects.

7 Potential Energy Projection on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational manifolds encoding energy conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1 the relational geometry of the sphere, S^2, provides orthogonal projections, for two aspects of omnidirectional transformation. We define them as follows:

- The Amplitude Component (κ_Y): This projection represents the relational gravitational measure between the object and the observer. It corresponds to the extent of transformation, which manifests physically as gravitation potential. A value of $\kappa = 1$ denotes saturation: the entire relational resource of the system has been allocated into the gravitational channel. No residual capacity remains for kinematic projection. This condition defines the relational horizon.
- The Phase Component (κ_X): This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the sequence of its transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of transformation":

$$\kappa_X^2 + \kappa_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

7.1 Gravitational Meridional Section of S^2

By isotropy the omnidirectional carrier is S^2, but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of S^2 with the parametrization $(\kappa_X, \kappa) = (\cos\theta_2, \sin\theta_2)$.

7.2 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure (κ_Y, gravitation potential) necessarily requires a decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is the shadow cast by the geometry of relations.

Notation simplicity:

From here on we will write $\beta = \beta_X, \quad \beta_Y = \sqrt{1-\beta^2}, \quad \kappa = \kappa_Y, \qquad \kappa_X = \sqrt{1-\kappa^2}$ for notation simplicity.

7.3 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the escape velocity at radius r.

In the kinematic case, with $\beta = \cos\theta_1$, the energy relation can be written as

$$E^2 = \left(\cot\theta_1\, E_0\right)^2 + E_0^2, \tag{1}$$

so that the relativistic momentum is expressed as

$$p = E_0/c\,\cot\theta_1. \tag{2}$$

In full symmetry, the gravitational case follows from $\kappa = \sin\theta_2$. We define the gravitational energy as

$$E_g = \frac{E_0}{\kappa_X}, \qquad \kappa_X = \sqrt{1-\kappa^2}, \tag{3}$$

and introduce the gravitational analogue of momentum:

$$p_g \;=\; E_0/c\,\tan\theta_2. \tag{4}$$

This yields the gravitational energy relation

$$E_g^2 = (p_g c)^2 + (mc^2)^2. \tag{5}$$

Summary:

$$\beta = \cos\theta_1, \qquad \kappa = \sin\theta_2,$$
$$\beta \;\longleftrightarrow\; \kappa, \qquad \cot\theta_1 \;\longleftrightarrow\; \tan\theta_2.$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the same relational circle, expressed through complementary trigonometric forms.

7.4 Geometric composition of SR and GR factors

On the unit kinematic circle (S^1) we parametrize

$$(\beta, \beta_Y) = (\cos\theta_1,\ \sin\theta_1),$$

so that the invariant vertical projection reads

$$E\,\beta_Y = E_0 \quad\Rightarrow\quad \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin\theta_1}}, \qquad p = \frac{E}{c}\,\beta = \frac{E_0\,\beta}{\beta_Y} = E_0\cot\theta_1,$$

and therefore $E^2 = (pc)^2 + E_0^2$.

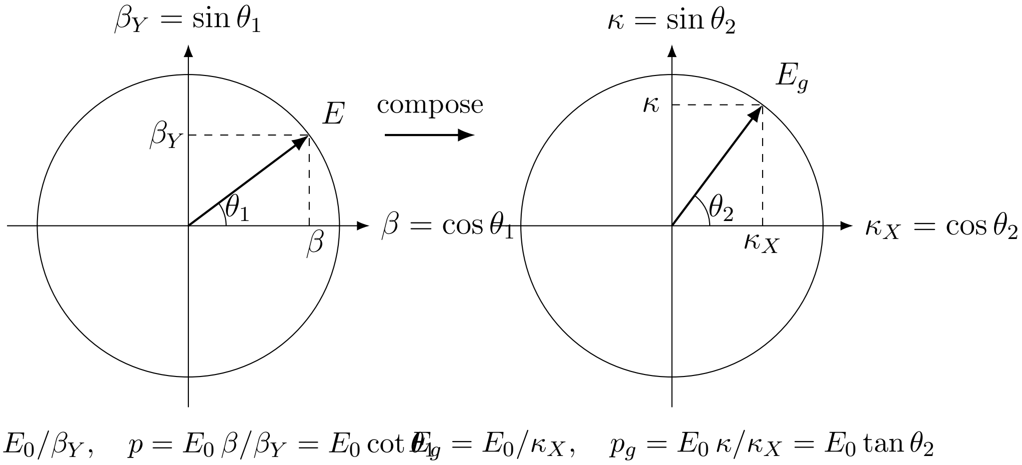
On the gravitational circle (S^2) we parametrize

$$(\kappa_X, \kappa) = (\cos\theta_2,\ \sin\theta_2),$$

so that the invariant horizontal projection reads

$$E_g\,\kappa_X = E_0 \quad \Rightarrow \quad \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos\theta_2}}, \qquad p_g = E_g\,\kappa = \frac{E_0\,\kappa}{\kappa_X} = E_0\tan\theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



7.5 Clear Relational Symmetry Between Kinematic and Potential Projections

Now we can clearly see the underling symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

$\theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa),\ \kappa^2 = 2\beta^2$	
Algebraic Form	Trigonometric Form
$\beta = v/c$	$\beta = cos(\theta_1)$
$\kappa = \sqrt{R_s/r}$	$\kappa = sin(\theta_2)$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$
$p = E_0/c \cdot \beta/\beta_Y$	$p = E_0/c \cdot \cot(\theta_1)$
$p_g = E_0/c \cdot \kappa/\kappa_X$	$p_g = E_0/c \cdot \tan(\theta_2)$
$\tau = \beta_Y \kappa_X$	$\tau = \sin(\theta_1)\cos(\theta_2)$
$Q = \sqrt{\kappa^2 + \beta^2} = \sqrt{3}\beta$	$Q = \sqrt{3}\cos(\theta_1)$
$Q_t = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2}$	$Q_t = \sqrt{1-3\cos^2(\theta_1)}$

Table 2: Unified representation of relativistic and gravitational effects for closed systems.

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Summary

The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-transformation constraint revealing their unified origin.

8 Operational Independence and the Role of Constants

A common objection from conventional frameworks (such as GR) is that any model using the Schwarzschild radius (R_s) must be fundamentally dependent on the Newtonian constants G and m_0 (mass) This implies that G and m_0 are hidden inputs or postulates.

We will now prove that this is incorrect. In RG, G and m_0 are not inputs to the theory; they are outputs derived from calibration. The parameter κ is operationally measurable without prior knowledge of G or m_0.

Theorem 8.1 (Operational Measurability of κ). The relational potential κ is a direct, empirical measurable (a pure dimensionless number) that is operationally independent of G and m_0.

Proof.

1. The Observable: The most direct observable of a gravitational field is gravitational time dilation, κ_X (or its equivalent, gravitational redshift, z). This κ_X is an empirically measurable, pure dimensionless number (e.g., by comparing clock frequencies at two different potentials).

2. The RG Definition: From Theorem 9.2, the total local time dilation for a stationary observer ($\beta = 0 \Rightarrow \beta_Y = 1$) is defined by relational geometry as:

$$\tau = \beta_Y \kappa_X = \kappa_X$$

3. The Geometric Constraint: The S^2 carrier (Theorem 4.5) enforces a non-negotiable quadratic closure (a Pythagorean identity) on its projections:

$$\kappa_X^2 + \kappa^2 = 1$$

4. The Algebraic Consequence: By substituting (2) into (3), we find the direct relationship between the observable (τ) and the parameter (κ):

$$\tau^2 + \kappa^2 = 1 \quad \Longrightarrow \quad \boxed{\kappa = \sqrt{1 - \tau^2}}$$

5. Conclusion: An observer can empirically measure the pure number τ and algebraically find the pure number κ. This entire operation requires zero knowledge of G, c, or m_0. This proves that κ is a operationally accessible quantity.

□

Remark 8.2 (The Role of G as a "Translation Constant"). The objection from conventional physics arises from a categorical error: it mistakes a calibration tool for an axiom.

The formulas $R_s = 2Gm_0/c^2$ and $\kappa^2 = R_s/r$ are not postulates of RG. They form a "translation bridge" constructed after the theory is established.

The correct, relational procedure is:

1. We measure the dimensionless κ (via redshift, as shown above).
2. We measure the relational scale r (e.g., via parallax, radar, etc.).
3. We calculate the system's geometric scale: $R_s = \kappa^2 r$.
4. Finally, if we wish to translate this scale R_s into the historical, "cultural" unit of the kilogram (m_0), we use the *converter* G:

$$m_0 \equiv \frac{R_s c^2}{2G}$$

Thus, G is not a "hidden input" to the theory; it is a "legacy converter" used to map our geometry back to Newtonian "cultural heritage" units.

9 Equivalence Principle as Derived Identity

Lemma 9.1 (Unified Relational Scaling). Within the relational framework of WILL, both kinematic (S^1) and gravitational (S^2) transformations act as independent projections of the same invariant energy E_0. Each projection rescales the observable quantities by its respective geometric factor:

$$E = \frac{E_0}{\beta_Y}, \qquad E_g = \frac{E_0}{\kappa_X}.$$

Proof. On the kinematic circle S^1, the invariant vertical projection corresponds to $\beta_Y = \sin\theta_1$. Preserving the same invariant leg E_0 forces the stretch $E/E_0 = 1/\beta_Y$. On the gravitational sphere S^2, the invariant horizontal projection is $\kappa_X = \cos\theta_2$, forcing $E_g/E_0 = 1/\kappa_X$. These transformations are independent and commute, each preserving the closure identity of its respective manifold. □

Theorem 9.2 (Equivalence of Inertial and Gravitational Response). Composing the independent relational stretches of Lemma 9.1 yields the total local energy scale

$$E_{\text{loc}} = \frac{E_0}{\tau} = \frac{E_0}{\beta_Y\,\kappa_X} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The corresponding inertial and gravitational projections share a single operational factor,

$$\tilde{p} = \frac{E_{\text{loc}}}{c}\beta, \qquad \tilde{p}_g = \frac{E_{\text{loc}}}{c}\kappa,$$

both governed by the same effective mass

$$m_{\text{eff}} = \frac{E_0}{\beta_Y\,\kappa_X\,c^2} = \frac{E_0}{\tau c^2}.$$

Therefore,

$$\boxed{m_g \equiv m_i \equiv m_{\text{eff}}},$$

and the Einstein equivalence principle follows as a necessary structural identity of WILL.

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Corollary 9.3 (Mass Invariance under Relational Scaling). The invariant core E_0 denotes the complete internal equilibrium state ($\beta_Y = \kappa_X = 1$). Relational factors β_Y and κ_X rescale only external manifestations (energy, momentum, and rates), while E_0 remains unchanged. Hence,

$$\boxed{m_g \equiv m_i \equiv m = E_0/c^2},$$

is not a dynamical statement but the definition of rest invariance itself.

Remark 9.4 (Composition-Independence). Decomposing the invariant rest energy into internal channels,

$$E_0 = \sum_a E_0^{(a)},$$

each term couples identically through the same geometric stretch:

$$E_{\mathrm{loc}} = \sum_a \frac{E_0^{(a)}}{\tau}.$$

Since all channels scale by the same factor $1/\tau = 1/(\beta_Y \kappa_X)$, ratios between channels cancel in all observables. Therefore, composition-independence of motion (Etvs universality) follows identically, without requiring a postulate $m_g = m_i$.

Remark 9.5 (Quantum Interface). The relational phase increment inherits the same scaling:

$$\Delta\phi \propto E_{\mathrm{loc}}\, \Delta\lambda,$$

where $\Delta\lambda$ is the internal ordering parameter. Thus both kinematic and gravitational phase shifts share the same stretch $1/\tau = 1/(\beta_Y \kappa_X)$, yielding composition-independent matter-wave interference patterns.

Summary:

In WILL, the equivalence of inertial and gravitational mass is not assumed but follows necessarily from the compositional closure of relational geometry. What General Relativity posits as a postulate, WILL reveals as a corollary.

10 Unification of Projections: The Geometric Exchange Rate

Having established that directional (kinematic) and omnidirectional (gravitational) relations are carried by the unique manifolds S^1 and S^2 respectively, we now derive the relationship that unifies them.

10.1 Derivation of the Energetic Closure Condition

The Principle of a unified relational resource (Energy) requires a self-consistent "exchange rate" between its different modes of expression. The closed system must partition its total transformation resource into all available modes. If we have a 2D carrier and a 1D carrier, the resource budget must be shared in proportion to their capacity to host transformations. The simplest symmetric partition is linear in the number of independent directions. but is dictated by the intrinsic geometry of the relations themselves:

Remark 10.1 (From slice to whole S^2). Although we parametrise a single meridional great circle (κ_X, κ) for algebraic convenience, the energetic quantity κ^2 in Theorem 10.2 refers to the total S^2 budget, not to the single slice. The factor of 2 precisely accounts for reconstructing the full sphere from one great circle.

Theorem 10.2 (Energetic Closure Condition). Since energy is quadratic in its relational amplitudes, the unique quadratic balance between directional (S^1) and omnidirectional (S^2) resource distributions compatible with closure is

$$\boxed{\kappa^2 = 2\beta^2.}$$

Proof. The relational resource (Energy) must be conserved and interchangeable between its two fundamental carriers, S^1 and S^2. We must therefore establish the "exchange rate" between them.

At this foundational (pre-geometric) level, we have established (Theorem 4.5) that these carriers are topologically distinct, possessing 1 and 2 degrees of freedom, respectively.

Crucially, as per the Principle of Ontological Minimalism (1.1), this ratio of dimensionalities (2:1) is the only distinguishable property between the carriers from which a non-arbitrary exchange rate ($\mathcal{R}$) can be derived. To postulate any other rate would be an ad-hoc, unjustifiable assumption. Therefore, the only logical and non-arbitrary exchange rate is:

$$\mathcal{R} = \frac{\text{d.o.f.}(S^2)}{\text{d.o.f.}(S^1)} = \frac{2}{1} = 2.$$

Furthermore, we have established that the only conserved, invariant "currency" for these carriers is their quadratic form (β^2, κ^2), as this is the only form that satisfies their intrinsic closure identity (e.g., $\beta^2 + \beta_Y^2 = 1$).

To find the Energetic Closure Condition, we simply equate the two "currencies" using this necessary "exchange rate." This dictates that the omnidirectional (2D) budget must be balanced by twice the directional (1D) budget:

$$\boxed{\kappa^2 = \mathcal{R} \cdot \beta^2 \quad \Longrightarrow \quad \kappa^2 = 2\beta^2.}$$

This shows that the exchange rate factor 2 is deductive necessity from the topological ratio of the minimal carriers. □

Definition 10.3 (Closure Defect). $\delta \equiv \frac{\kappa^2}{2\beta^2}$ A subsystem is energetically closed if $\langle \delta \rangle_{\text{cycle}} = 1$. For circular orbits, $\delta \equiv 1$.

Corollary 10.4 (Energetic Closure Criterion). Closed systems (momentary or periodic) satisfy $\kappa^2 = 2\beta^2$ identically. Open systems display $\delta \neq 1$, the magnitude of which quantifies the energy flow through unaccounted channels. When all channels are included, closure is restored.

Diagnostic Invariant:

The condition $\kappa^2 = 2\beta^2$ defines energetic closure. In closed systems (circular or periodic), it holds exactly; in open systems, its deviation δ measures interaction with external channels.

Remark 10.5 (Physical Interpretation). The exchange rate between the kinematic and gravitational projections corresponds to the ratio of their relational dimensions. This purely geometric constant (2) replaces the empirical proportionalities of classical dynamics. It is the relational analogue of the virial theorem: the kinetic and potential aspects of WILL maintain closure through the invariant ratio

$$\boxed{\kappa^2 = 2\beta^2.}$$

Illustrative Examples.

- Circular Orbit (Closed). A body in circular orbit exactly satisfies $\kappa^2 = 2\beta^2$. The entire conserved resource is partitioned between kinetic and gravitational projections; no external channel exists.
- Radiating Binary (Open). An elliptical compact binary violates $\kappa^2 = 2\beta^2$ when only orbital degrees of freedom are counted, the closure defect δ quantifying energy lost to gravitational radiation. Including all channels restores closure.

Summary:

1. WILL defines the universe as a closed relational structure, SPACETIME $\equiv$ ENERGY.
2. The simplest maximally symmetric carriers of these relations are S^1 and S^2.
3. The parameters $\beta = \cos\theta_1$ and $\kappa = \sin\theta_2$ are thus constrained to these manifolds.
4. The geometric exchange rate between these modes equals the ratio of their relational dimensionalities: 2.

Remark 10.6 (Geometric Origin of Physical Law). The relation between kinetic and potential energy is not an empirical coincidence but a geometric necessity of relational closure. Classical mechanics merely approximates this deeper invariant. Explicitly,

$$\boxed{\text{Geometric Distribution } (\kappa^2) \;\equiv\; 2 \times \text{Kinetic Distribution } (\beta^2).}$$

11 Energy-Symmetry Law

In RG, every transformation is bidirectional: each change observed by A corresponds to an equal and opposite change observed by B. This reciprocity is the algebraic form of causal continuity, and its geometric expression is the Energy-Symmetry Law.

11.1 Causal Continuity and Energy Symmetry

Theorem 11.1 (Energy Symmetry). The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the Energy-Symmetry Law:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{6}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A = 0$).
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A\to B} = \underbrace{\frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}\left(\beta_B^2 - \beta_A^2\right)}_{\text{Change in Kinetic}}} \tag{7}$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right) \tag{8}$$

From B's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)\right) = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right) \tag{9}$$

Summing these transfers gives:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{10}$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the Energy-Symmetry Law as a direct consequence of the closed geometry. □

11.2 The Specific Energy Transfer (ΔE):

This is the physical quantity representing the actual work done and change in motion, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the sum of the changes in the potential and kinetic energy budgets:

$$\Delta E_{A\to B} = \Delta U_{A\to B} + \Delta K_{A\to B} = \frac{1}{2}\left(\kappa_A^2 - \kappa_B^2\right) + \frac{1}{2}\left(\beta_B^2 - \beta_A^2\right) \tag{11}$$

It is this quantity, ΔE, that is conserved and must balance to zero in any closed cycle.

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

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Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \tag{12}$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \tag{13}$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

11.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics but from the fundamental quadratic nature of the energy budgets in RG.

The energetic significance of a state is proportional to the square of its geometric projection. This is analogous to how kinetic energy is proportional to velocity squared (v^2) or how the energy in a wave is proportional to its amplitude squared (A^2). The individual energy budgets are defined as:

- Specific Potential Energy Budget: $U/E_0 \propto -\frac{1}{2}\kappa^2$
- Specific Kinetic Energy Budget: $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the changes in these individual budgets.

11.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 11.2 (Universal Speed Limit). The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A\to B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B\to A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{14}$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. □

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11.5 Single-Axis Energy Transfer and the Nature of Light

Theorem 11.3 (Single-Axis Transformation Principle). For light, the kinematic projection reaches its full extent:

$$\boxed{\beta = 1 \;\Rightarrow\; \beta_Y = 0.}$$

This means that all transformation of the relational energy occurs along a single orthogonal axis. The complementary branch of the bidirectional energy exchange is absent, and the total resource of transformation is entirely expressed on one geometric component.

Proof. For massive systems, the Energy-Symmetry Law distributes the total energy exchange evenly between two orthogonal projections:

$$U/E_0 = -\tfrac{1}{2}\kappa^2, \qquad K/E_0 = +\tfrac{1}{2}\beta^2.$$

The symmetry of exchange arises because both branches — (κ, κ_X) and (β, β_Y) — coexist and compensate each other. Each side carries one half of the total transformation resource, ensuring

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

For light, however, $\beta = 1$ implies $\beta_Y = 0$. The complementary projection disappears; there is no dual observer-frame available for symmetric partition. As a result, the transformation cannot be divided between two orthogonal branches. The full relational resource of the interaction is realised on a single axis.

Therefore, the specific energy potential for light is not halved but complete:

$$\boxed{\Phi_\gamma = \kappa^2 c^2,}$$

while for a massive body the potential remains partitioned,

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This explains why light experiences a total geometric effect exactly twice that of a massive particle in the same field, without introducing any auxiliary approximations. □

> Interpretive Note Light occupies the boundary state where relational reciprocity collapses into self-reference. It is not a "massless limit" but a distinct single-axis state of the energy geometry. A photon is simultaneously its own counter-frame and its own anti-state. The factor of two that appears in gravitational deflection and frequency shift is a direct signature of this one-axis transformation.

Summary

Light has no rest frame. The Speed of Light is the boundary beyond which the energy symmetry law breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

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12 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law (Section 11) emerges when analysing the total specific orbital energy. Since energy in RG is defined relationally, as the measure of difference between two states, we naturally select these two states (e.g., the surface of the central body 'A' and the orbit 'B') as the reference points for the potential and kinetic energy budgets. Under this relational approach, the total specific orbital energy (potential + kinetic, per unit rest mass) naturally appears in a form structurally identical to the Minkowski interval.

12.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{15}$$

$$K = \frac{1}{2} m \frac{GM_\oplus}{a}. \tag{16}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{17}$$

12.2 Projection Parameters and Minkowski-like Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{18}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{19}$$

Substituting into (17) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{20}$$

This is already in the form of a hyperbolic difference of squares: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{21}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\kappa^2$ and $K/E_0 = \frac{1}{2}\beta^2$ as budgets. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer ΔE is the signed sum of budget changes.

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12.3 Physical Interpretation

In classical derivations, (17) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG, (20) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface, orbit})$, and is invariantly expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While this framework refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in a Minkowski–like difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This re framing answers why the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

13 Lagrangian and Hamiltonian as Ontologically Corrupted RG Approximations

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational Energy-Symmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms are not fundamental principles but ontologically "dirty" approximations of the relational WILL framework. By collapsing the two-point relational structure into a single-point description, classical mechanics gains computational convenience at the cost of ontological clarity.

13.1 Definitions of Parameters

We consider a central mass M and a test mass m. The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2\dot{\phi}^2$ — total squared orbital speed at B.
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B.
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A.

13.2 The Relational Lagrangian

Instead of a relational energy, we define the clean relational Lagrangian L_{rel}, which represents the kinetic budget at point B relative to the potential budget at point A:

$$L_{\text{rel}} = T(B) - U(A) = \tfrac{1}{2}m\left(\dot{r}_B^2 + r_B^2\dot{\phi}^2\right) - \frac{GMm}{r_A}. \tag{22}$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \tfrac{1}{2}\big(\beta_B^2 + \kappa_A^2\big). \tag{23}$$

This two-point, relational form is the clean geometric statement.

13.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{24}$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

13.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \tag{25}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \tag{26}$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \tfrac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \frac{GMm}{r}. \tag{27}$$

13.4.1 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \tfrac{1}{2}mv^2 + \frac{GMm}{r} \quad\longleftrightarrow\quad \tfrac{1}{2}mc^2\big(\beta^2 + \kappa^2\big), \tag{28}$$

$$H = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad\longleftrightarrow\quad \tfrac{1}{2}mc^2\big(\beta^2 - \kappa^2\big). \tag{29}$$

Here the "+" or "−" signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point

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formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both are ontologically "dirty" approximations. The clean relational law, involving distinct points A and B, is collapsed into a local, one-point description. This shows that Hamiltonian and Lagrangian are just needlessly overcomplicated approximations that lose in ontological integrity.

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2. For readers accustomed to standard frameworks, the following translation rules may help:

1. General Relativity (metric form):

$$\kappa_X \hat{=} \sqrt{-g_{tt}} \quad \text{(static spacetimes)}, \qquad \beta \hat{=} \frac{\|u^\mu_{\text{spatial}}\|}{u^t c}.$$

2. Canonical mechanics (Lagrangian/Hamiltonian): Quantities such as $p_i = \partial L/\partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are computational shadows, useful for legacy calculations but physically redundant.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

13.5 Third Ontological Collapse: Derivation of Newton's Third Law

We now demonstrate that Newton's Third Law, like the Lagrangian and Hamiltonian, is not a fundamental principle but another "degenerate shadow" of the WILL framework. It arises as a necessary mathematical consequence of the same ontological collapse — forcing a two-point relational law into a single-point, instantaneous formalism.

Theorem 13.1 (Newton's Third Law as a Degenerate Consequence). The Energy–Symmetry Law ($\Delta E_{A\to B} + \Delta E_{B\to A} = 0$) mathematically necessitates Newton's Third Law ($\vec{F}_{AB} = -\vec{F}_{BA}$) in the classical, non-relativistic limit where the two-point relational energy budget is collapsed into a single-point potential function $U(\vec{r})$.

Proof. We begin with the foundational Energy–Symmetry Law (Section 11), the principle of causal balance for state transitions:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

In the classical, non-relativistic limit, this two-point relational law is "ontologically corrupted" into a single-point potential energy function, U. This function is assumed to depend only on the relative positions of the two interacting entities, A and B:

$$U = U(\vec{r}) \quad \text{where} \quad \vec{r} = \vec{r}_B - \vec{r}_A.$$

This $U(\vec{r})$ is the classical approximation of the system's relational energy budget. In this collapsed formalism, the force $\vec{F}$ is defined as the negative gradient of this potential.

(1) Force on B by A ($\vec{F}_{AB}$): This force is found by taking the gradient with respect to B's coordinates:

$$\begin{aligned}
\vec{F}_{AB} &= -\nabla_B U(\vec{r}_B - \vec{r}_A) && (30)\\
&= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_B}\right) && (31)\\
&= -\nabla U(\vec{r}) \cdot (\mathbf{I}) && (32)\\
&= -\nabla U(\vec{r}) && (33)
\end{aligned}$$

(2) Force on A by B ($\vec{F}_{BA}$): This force is found by taking the gradient with respect to A's coordinates:

$$\begin{aligned}
\vec{F}_{BA} &= -\nabla_A U(\vec{r}_B - \vec{r}_A) && (34)\\
&= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_A}\right) && (35)\\
&= -\nabla U(\vec{r}) \cdot (-\mathbf{I}) && (36)\\
&= +\nabla U(\vec{r}) && (37)
\end{aligned}$$

(3) Conclusion: By direct comparison of the results, we find:

$$\vec{F}_{AB} = -\nabla U(\vec{r}) \quad \text{and} \quad \vec{F}_{BA} = +\nabla U(\vec{r}).$$

Therefore, it is a mathematical tautology of the collapsed formalism that:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

This completes the proof. Newton's Third Law is not an independent physical axiom, but the built-in mathematical consequence of approximating the Energy Symmetry Law with a single potential function. The law of "equal and opposite forces" is revealed to be a degenerate approximation of the more fundamental, generative law of Relational Geometry. □

13.6 General Consequence

Bad philosophy, in RG sense, has three measurable effects:

1. Inflated Formalism: Equations multiply to compensate for ontological error.
2. Loss of Transparency: Physical meaning becomes hidden behind coordinate dependencies.
3. Empirical Fragmentation: Each domain (cosmology, quantum, gravitation) requires separate constants.

By contrast, good philosophy-epistemic hygiene-enforces relational closure and yields simplicity through necessity, not through approximation.

In short:

$$\boxed{\text{Bad philosophy creates complexity} \quad \text{Good philosophy reveals geometry.}}$$

Daring Remark

The historical escalation of mathematical complexity in physics did not reveal deeper reality - it compensated for a philosophical mistake. Once the ontological symmetry is restored, Nature's laws reduce to algebraic self-consistency.

$$\boxed{\text{Bad Philosophy}} \Rightarrow \boxed{\text{Ontological Duplication}} \Rightarrow \boxed{\text{Mathematical Inflation}}$$

$$\boxed{\text{Mathematical complexity is the symptom of philosophical negligence.}}$$

14 Relational Orbital Mechanics Without Mass or G

Thesis

Orbital dynamics requires no mass, no G, no metric, and no spacetime geometry. All observable orbital structure follows from two directly measurable frequency projections:

κ (gravitational projection from redshift), β (kinematic projection from Doppler).

Everything else is algebra.

14.1 Exact Operational Inputs

A physical observer measures exactly three quantities at the orbital periapsis (subscript p). We discard weak-field approximations in favor of the exact algebraic relations derived from the S^2 closure identity.

(1) Exact Gravitational Projection (κ_p) Directly derived from the measured gravitational redshift z:

$$1 + z = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} = \frac{1}{\kappa_X}.$$

Substituting into the closure identity $\kappa^2 = 1 - \kappa_X^2$:

$$\boxed{\kappa_p^2 = \frac{z(2+z)}{(1+z)^2}} \tag{38}$$

This formula is exact for any field strength, from Earth orbit to the Event Horizon.

(2) Kinematic Doppler Projection (β_p)

$$\boxed{\beta_p = \frac{v_{max}}{c}}$$

obtained from frequency drift or phase tracking at periapsis.

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(3) Periapsis Radius (r_p) determined astrometrically.

No mass or gravitational constant is ever used. The physics relies solely on the exact geometry of light frequency ratios.

14.2 Constructing the Schwarzschild Scale

The effective Schwarzschild scale is obtained directly from the measurement:

$$\boxed{R_s = \kappa_p^2 r_p} \tag{39}$$

R_s is an operational constant defined entirely from observed frequency ratios, independent of mass or G.

14.3 Closure Factor: The Origin of Orbital Asymmetry

The relational parameter governing the orbital shape is defined at the point of maximum interaction:

$$\boxed{\delta_p := \frac{\kappa_p}{\sqrt{2}\,\beta_p}} \qquad (0 < \delta_p \leq 1). \tag{40}$$

Interpretation.

- $\delta_p = 1$ — perfect closure of energy projections: circular orbit.
- $\delta_p < 1$ — closure deficit: two turning points appear (elliptic orbit).

14.4 Turning-Point Structure and Angular Invariant

Turning points arise because for $\delta_p < 1$ the radial and tangential modes cannot simultaneously satisfy closure. At the apsides, the radial mode collapses, giving the relational angular projection invariant:

$$\boxed{h = r\beta = \text{const at turning points.}} \tag{41}$$

This is the relational equivalent of angular momentum conservation, derived purely from S^2 symmetry.

14.5 Relational Eccentricity and Shape Reconstruction

Thesis

Geometric eccentricity emerges algebraically from the ratio of projections at periapsis.

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Derivation. For any bound system governed by the omnidirectional projection ($\kappa^2 \propto 1/r$), the kinematic projection at periapsis is related to the potential projection by the geometric identity $2\beta_p^2 = \kappa_p^2(1+e)$. This is the relational form of the vis-viva equation 10.2.

Substituting this into the definition of the closure factor ($\delta_p^2 = \kappa_p^2/2\beta_p^2$):

$$\delta_p^2 = \frac{\kappa_p^2}{\kappa_p^2(1+e)} = \frac{1}{1+e}.$$

Solving this identity for e, we obtain the exact relation:

$$\boxed{e = \frac{1}{\delta_p^2} - 1 = \frac{1-\delta_p^2}{\delta_p^2}} \qquad (0 \le e < 1). \tag{42}$$

This demonstrates that eccentricity is strictly a measure of the energetic deviation from the closure equilibrium ($\delta = 1$).

Once e is derived, the aphelion radius r_a follows algebraically:

$$\boxed{r_a = r_p\left(\frac{1+e}{1-e}\right) = r_p\left(\frac{1}{2\delta_p^2 - 1}\right)} \tag{43}$$

14.6 Example 1: Reconstruction of Mercury's Orbit

Inputs at perihelion:

$$r_p = 4.600 \times 10^{10}\,\mathrm{m}, \qquad \beta_p = 1.967 \times 10^{-4}, \qquad \kappa_p^2 = \frac{R_s}{r_p} \approx 6.41 \times 10^{-8}.$$

Closure and Shape: Calculating the closure factor:

$$\delta_p = \frac{\kappa_p}{\sqrt{2}\beta_p} \approx 0.9108.$$

Reconstructing eccentricity via Eq. (42):

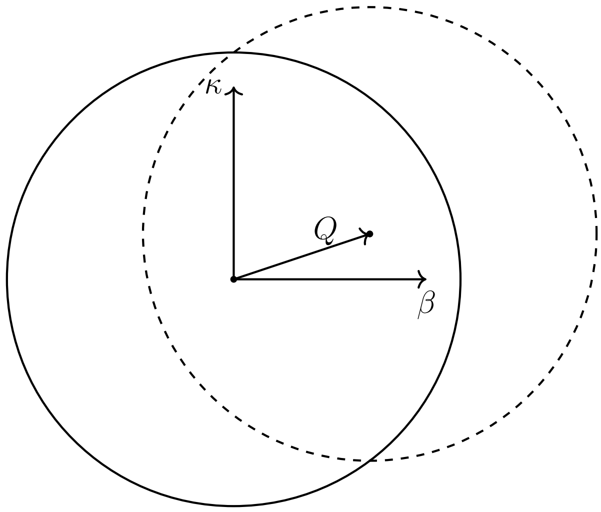
$$e = \frac{1}{(0.9108)^2} - 1 \simeq 0.2056.$$

Reconstructing aphelion via Eq. (43):

$$r_a = 4.600 \times 10^{10}\left(\frac{1}{2(0.9108)^2 - 1}\right) \simeq 6.976 \times 10^{10}\,\mathrm{m}.$$

This matches astronomical ephemerides precisely.

15 Relational Displacement and Orbital Precession



When an observer observes another system, they assign to it a relational displacement norm Q:

$$\boxed{Q^2 = \beta^2 + \kappa^2} \tag{44}$$

Relational reciprocity is the invariance of this norm under the self-centering operation of each observer.

Each observer places itself at the relational origin

$$(\beta, \kappa) = (0, 0).$$

If the other system now looks back, it again self–centres at $(0, 0)$ and applies the same rule. It measures the observer's (β, κ) and again obtains

$$Q^2 = \beta^2 + \kappa^2.$$

Thus Q is the norm of relational displacement, not a spatial distance. Geometrically, the observer is always at the centre of its own S^1 (or S^2) carrier, and any external system is a point (β, κ) on that plane. The scalar Q measures the total deviation from the observer's relational origin.

Remark 15.1 (Closure-specific simplification). Under energetic closure $\kappa^2 = 2\beta^2$ (circular/periodic systems), the norm reduces to $Q^2 = 3\beta^2$.

In general (open or elliptic) configurations, the full definition $Q^2 = \beta^2 + \kappa^2$ must be used.

15.1 Principle of Relational Reciprocity

Self–centering reciprocity. Every observer performs self–centering:

$$(\beta, \kappa) = (0, 0).$$

When I observe another system, I assign to it (β, κ) and therefore a displacement

$$Q^2 = \beta^2 + \kappa^2.$$

When that system observes me, it again self–centres and obtains the same form. It assigns to me some (β, κ) and again computes $Q^2 = \beta^2 + \kappa^2$.

Reciprocity is therefore not a vector symmetry in a shared space. It is a symmetry of the self–centering operation: each observer applies the same rule and only the norm of displacement is invariant.

Summary

$$\boxed{\text{Relational reciprocity} \;=\; \text{invariance of the norm } Q \text{ under self–centering.}}$$

There is no common background arena. There are only mutual displacement magnitudes Q computed in each observer's own relational origin.

15.2 The Universal Precession Law: Derivation via Q_a

For an elliptic orbit, the net angular mismatch (precession) is derived from the displacement norm Q measured at the system's characteristic scale. We select the semi-major axis a as this reference scale, defining the norm Q_a.

At the scale $r = a$, the closure condition ($\kappa^2 = 2\beta^2$) implies the specific distribution of the invariant Schwarzschild scale R_s:

$$\kappa^2(a) = \frac{R_s}{a}, \qquad \beta^2(a) = \frac{R_s}{2a}.$$

Substituting these into the definition of the displacement norm $Q^2 = \beta^2 + \kappa^2$:

$$Q_a^2 = \frac{R_s}{2a} + \frac{R_s}{a} = \frac{3R_s}{2a}. \tag{45}$$

The general geometric precession formula is:

$$\Delta\varphi = \frac{2\pi\, Q_a^2}{1 - e^2}. \tag{46}$$

Substituting Q_a^2, we recover the standard form purely algebraically:

$$\Delta\varphi = \frac{2\pi}{1 - e^2}\left(\frac{3R_s}{2a}\right) = \frac{3\pi R_s}{a(1 - e^2)}.$$

15.2.1 Transformation to Periapsis Observables

To eliminate the abstract parameters R_s, a, e in favor of direct observables, we map this expression to the periapsis (p), where interaction is maximal. Using the identities $R_s = \kappa_p^2 r_p$ and $a(1-e^2) = r_p(1+e)$, and the closure relation $(1+e) = 1/\delta_p^2 = 2\beta_p^2/\kappa_p^2$, we arrive at the ultimate operational reduction.

The secular evolution of an orbit is determined solely by the ratio of the gravitational redshift to the Doppler shift at the point of closest approach:

$$\boxed{\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2}} \tag{47}$$

This equation replaces the complex dynamical derivation with a direct comparison of light interactions.

$$\boxed{\text{No differential equations. No metric. Pure algebra of light red vs. blue ratio at one point.}}$$

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15.3 Verification A: Mercury (Direct Substitution)

We test the law using the precise operational inputs for Mercury at perihelion.

$$\kappa_p^4 \approx 4.11 \times 10^{-15}, \qquad \beta_p^2 \approx 3.87 \times 10^{-8}.$$

Plugging these values directly into Eq. (47):

$$\Delta\varphi = \frac{3\pi}{2}\left(\frac{4.11 \times 10^{-15}}{3.87 \times 10^{-8}}\right).$$

$$\Delta\varphi \approx 4.712 \times (1.062 \times 10^{-7}) \approx 5.00 \times 10^{-7}\ \text{rad/orbit}.$$

This corresponds exactly to the observed 43 arcseconds per century. The physics is exact.

15.4 Verification B: Strong Field Test (Star S2)

For distant stars like S2 (orbiting Sgr A*), we reconstruct the potential depth κ_p purely from the visible orbital shape (e) and velocity (β_p), using the relation derived from Eq. (42):

$$\kappa_p = \beta_p\sqrt{\frac{2}{1+e}}.$$

Data (GRAVITY Collaboration):

$$e \simeq 0.8846, \qquad \beta_p \simeq 0.0255\,(7650\ \text{km/s}).$$

Prediction: 1. Reconstruct Potential: $\kappa_p \approx 0.02627$. 2. Calculate Precession Ratio using Eq. (47):

$$\frac{\kappa_p^4}{\beta_p^2} = \frac{(0.02627)^4}{(0.0255)^2} \approx 7.32 \times 10^{-4}.$$

3. Result:

$$\Delta\varphi = \frac{3\pi}{2}(7.32 \times 10^{-4}) \approx 3.45 \times 10^{-3}\ \text{rad} \approx \mathbf{11.85'}.$$

Comparison:

- WILL prediction: $\approx 11.89'$.
- Observed shift: $12' \pm 1.5'$.

The result lies well within observational uncertainty.

15.4.1 Unique Predictive Power

Unlike General Relativity, which requires mass M extracted from decades of monitoring. WILL reconstructs κ_p instantly from (e, β_p) providing a Snapshot Prediction. It asserts that precession is determined entirely by the instantaneous kinematic state (β_p) and the orbital shape (e) at periapsis.

Interpretation

This section demonstrates that the full structure of orbital dynamics — including turning points, eccentricity, radial asymmetry, and periapsis precession — can be reconstructed exactly from the two directly observable projection parameters κ and β, without introducing mass, G, metrics, manifolds, or any additional geometric assumptions.
Orbital phenomena therefore require no spacetime curvature and no dynamical field equations; they arise entirely from algebraic relations among observable frequency projections.

16 Rotational Systems (Kerr Without Metric)

16.1 Contextual Bounds

- For a gravitationally closed (static) system, the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.
- For a kinematically closed (maximally rotating) system, the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac^2}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the relational rotation parameter, with $0 \leq \beta \leq 1$,
- κ is related to the geometry and gravity,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta^2_{\max} r$$

This relationship holds when $r = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

16.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_\pm = \frac{R_s}{2}\left(1 \pm \beta_Y\right)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

16.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2}\left(1 + \sqrt{1 - \beta^2 \cos^2\theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

16.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c^2}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

16.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship Principle. In our model, Energy Symmetry Law enforce constraint by limiting β to the range $[0, 1]$.

16.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the

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rotational properties of the black hole, encapsulated in a_*, mirror the orbital velocity parameter β, providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

Physical Interpretation

- No need for pre-existing spacetime - geometry emerges from angular energy distributions.
- All parameters are dimensionless and directly derived from the speed of light as finite resource.
- Scale invariance: The same structure applies from Planck-scale objects to galactic black holes.

17 W_{ILL} - Unity of Relational Structure

From the geometric closure of WILL we derive the universal invariant connecting mass, energy, time, and length. Each is a projection of the same relational whole: $M = \frac{\beta^2}{\beta_Y}\,\frac{c^2 r}{G}, \quad E = \frac{\kappa^2}{\kappa_X}\,\frac{c^4 r}{2G}, \quad T = \kappa_X \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2, \quad L = \beta_Y \left(\frac{Gm_0}{\beta^2 c^2}\right)^2.$

Combining these projections yields the universal dimensionless invariant:

$$\boxed{W_{\mathrm{ILL}} = \frac{E\,T}{M\,L} = \frac{\frac{E_0}{\kappa_X}\,\kappa_X t^2}{\frac{m_0}{\beta_Y}\,\beta_Y r^2} = 1.}$$

All dimensional constants and parameters cancel identically. This equality is not a result of unit choice, but the manifestation of structural closure:

$$\boxed{\mathrm{WILL} = 1.}$$

17.1 Interpretive Note: The Name "WILL"

The term WILL stands for SPACE-TIME-ENERGY. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the cosmos does not possess "will" - yet through WILL, it manifests all that is.

Summary

$$\boxed{\text{WILL} \equiv \frac{ET}{ML} = 1 \quad \Longleftrightarrow \quad \text{Geometry} = \text{Energy} = \text{Causality.}}$$

It is not the unit of something - it is the unity of everything.

18 Derivation of Density, Mass, and Pressure

18.1 Derivation of Density

Translating RG (2D) to Conventional Density (3D). In RG κ^2 is the 2D parameter defined in the relational manifold S^2 . In conventional physics, the source term is volumetric density ρ, a 3D concept defined by the "cultural artifact" (a Newtonian "cannonball" model) of mass-per-volume .

To bridge our 2D theory with 3D empirical data, we must create a "translation interface". We do this by explicitly adopting the conventional (Newtonian) definition of density, $\rho \propto m_0/r^3$, as our "translation target".

From the projective analysis established in the previous sections:

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

This leads to mass definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}$$

To translate this into a volumetric density, we first adopt the conventional 3D (volumetric) proxy, r^3. This is not a postulate of RG, but the first step in applying the legacy (3D) definition of density:

$$\frac{m_0}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}$$

This expression, however, is incomplete. Our κ^2 "lives" on the 2D surface S^2 (which corresponds to 4π), while the r^3 proxy implicitly assumes a 3D volume. To correctly normalize the 2D parameter κ^2 against the 3D volume, we must apply the geometric normalization factor of the S^2 carrier by deviding on to area of the sphere, which is $1/4\pi$.

This normalization is the necessary geometric step to interface the 2D relational carrier (S^2) with the 3D legacy definition of density:

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr^2}\right)$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$$

Local Density $\equiv$ Relational Projection

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Maximal Density. At $\kappa^2 = 1$ (the horizon condition (for non rotating systems), $r = R_s$), this density reaches its natural bound, $\rho_{\max}$, which is derived purely from geometry:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}$$

Normalized Relation. Thus, our "translation" reveals an identity: the geometric projection κ^2 is simply the ratio of density to the maximal density:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}$$

18.2 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $m_0 = \alpha\, r^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

18.3 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\,\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\, c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy-geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\,c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energy-geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

19 Unified Geometric Field Equation

19.1 The Theoretical Ouroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation linking the geometric scale to the energy density ratio:

$$\boxed{\kappa^2 = \frac{\rho_{\text{field}}}{\rho_{\max}}}$$

This identity defines the local energy state of the geometry itself. Here $\rho_{\max} = c^2/(8\pi G r^2)$ is the saturation density limit, and ρ_{field} is the effective energy density of the relational curvature.

19.2 Field Equation and Matter Sources

For a static, spherically symmetric configuration containing matter with density $\rho_{\text{matter}}(r)$, the relationship is governed by the differential accumulation of the potential:

$$\boxed{\frac{d}{dr}\big(r\,\kappa^2\big) = \frac{8\pi G}{c^2}\,r^2\rho_{\text{matter}}(r)} \tag{48}$$

This expression reproduces the tt-component of the Einstein field equations.

The Vacuum Solution ($\rho_{\text{matter}} = 0$). In the vacuum region outside a central mass, the source density vanishes ($\rho_{\text{matter}} = 0$). The field equation implies conservation of the projection budget:

$$\frac{d}{dr}(r\kappa^2) = 0 \quad \Longrightarrow \quad r\kappa^2 = \text{const} = R_s.$$

Thus, we recover the fundamental potential law of WILL:

$$\boxed{\kappa^2 = \frac{R_s}{r}.}$$

Resolution of Roles

1. The Identity $\kappa^2 = \rho/\rho_{\max}$ describes the state of the field geometry.
2. The Equation $(r\kappa^2)' \sim \rho_{\text{matter}}$ describes how matter generates that geometry.

In vacuum, the generator is zero, but the field persists as the algebraic structure $\kappa^2 = R_s/r$.

20 Unified Geometric Field Equation

20.1 The Theoretical Ouroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

The ratio of geometric scales equals the ratio of energy densities.

This is the unified geometric field equation of WILL Relational Geometry. It expresses the complete equivalence:

$$\text{SPACETIME GEOMETRY} \equiv \text{ENERGY DISTRIBUTION}$$

We have shown that this single Ontological Principle, through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with a single foundational Principle SPACETIME $\equiv$ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

> Theoretical Ouroboros
>
> The RG framework exhibits perfect logical closure: the fundamental Principle about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework - the "Theoretical Ouroboros". But regardless of aesthetic beauty of this result let's remain sceptical.

20.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2 \, \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- Surface-scaled closure (vs. volume filling). Mass follows the algebraic closure $m_0 = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- Natural bounds. The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

Summary

The WILL framework postulates no external laws or assumptions. All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

21 Local Cosmological Term Λ in RG

Lemma 21.1 (Normalization Identity). In WILL Relational Geometry, local energy density and its maximal counterpart are related by

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2}, \tag{49}$$

$$\rho_{\max}(r) = \frac{c^2}{8\pi G r^2}. \tag{50}$$

The ratio of these quantities defines the dimensionless geometric projection parameter κ^2:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}}}$$

Theorem 21.2 (Unified Geometric Field Equation). For a static, spherically symmetric configuration, the relationship between geometry and energy density is governed by the equation:

$$\boxed{\frac{d}{dr}\big(r\,\kappa^2\big) = \frac{8\pi G}{c^2}\,r^2\rho(r)}. \tag{51}$$

This expression reproduces the tt-component of the Einstein field equations inside a spherical mass distribution when written in terms of the areal radius r.

Proof. Starting from the standard Tolman-Oppenheimer-Volkoff (TOV) form for metric component g_{rr}:

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1 - \frac{1}{g_{rr}}\right)\right] = \frac{8\pi G}{c^2}\rho(r). \tag{52}$$

Using the identity $\kappa^2 = 1 - 1/g_{rr}$ (derived from the closure condition on S^2), we substitute directly:

$$\frac{1}{r^2}\frac{d}{dr}\big[r\,\kappa^2\big] = \frac{8\pi G}{c^2}\rho(r).$$

Multiplying by r^2 yields Eq. (51). □

21.1 Derivation of Vacuum Density

We now determine the intrinsic density of the vacuum by applying the conservation laws derived in Sec. 4 to the Universe as a whole.

Theorem 21.3 (Vacuum Energy Partition). In a globally closed relational system in equilibrium, the effective vacuum energy density ρ_Λ is geometrically constrained to exactly two-thirds of the saturation limit $\rho_{\max}$.

$$\rho_\Lambda(r) = \frac{2}{3}\rho_{\max}(r).$$

Proof. We treat the vacuum as a self-contained relational system subject to the Lemmas of Closure (4.2) and Conservation (4.3).

1. Total Projection Budget (Q^2). The total transformation resource available to the system is the sum of its relational carriers projections:

$$Q^2 = \kappa^2 + \beta^2.$$

2. Equilibrium Condition. According to the Energetic Closure Theorem (10.2), a stable, closed system must satisfy the invariant exchange rate:

$$\kappa^2 = 2\beta^2.$$

3. The Structural Share. To find the fraction of the total resource allocated strictly to the structural (potential) sector, we substitute the closure condition into the total budget equation:

$$\frac{\kappa^2}{Q^2} = \frac{\kappa^2}{\kappa^2 + \beta^2} = \frac{2\beta^2}{2\beta^2 + \beta^2} = \frac{2}{3}.$$

4. Density Mapping. Since the local energy density ρ is linearly proportional to the squared projection κ^2 (via the Unified Field Equation), the density of the vacuum ρ_Λ must represent the same 2/3 proportion of the maximal density $\rho_{\max}$.

Thus,

$$\rho_\Lambda(r) = \frac{2}{3}\rho_{\max}(r) = \frac{2}{3}\frac{c^2}{8\pi G r^2}.$$

□

21.2 Derivation of Vacuum Pressure (Equation of State)

Unlike in standard cosmology, where the equation of state $w = -1$ is assumed, in RG it is derived from the tension of the geometric field.

Theorem 21.4 (Vacuum Pressure). The intrinsic pressure of the vacuum geometry is negative and proportional to its density:

$$P_\Lambda(r) = -\,\rho_\Lambda(r)\,c^2. \tag{53}$$

Proof. Pressure in a static field arises from the gradient of the potential. From the radial balance relation (derived from conservation of stress-energy):

$$P(r) = \frac{c^4}{8\pi G}\frac{1}{r}\frac{d\kappa^2}{dr}.$$

Substituting the vacuum potential $\kappa^2 = R_s/r$:

$$\frac{d\kappa^2}{dr} = -\frac{R_s}{r^2} = -\frac{\kappa^2}{r}.$$

Therefore:

$$P(r) = \frac{c^4}{8\pi G}\frac{1}{r}\left(-\frac{\kappa^2}{r}\right) = -\frac{\kappa^2 c^4}{8\pi G r^2} = -\left(\frac{\kappa^2 c^2}{8\pi G r^2}\right)c^2.$$

Recognizing the term in parentheses as density $\rho(r)$, we obtain:

$$P(r) = -\rho(r)c^2.$$

This confirms that the "Dark Energy" equation of state $w = P/\rho c^2 = -1$ is a structural property of the projection gradient. □

21.3 Legacy Correspondence: Mapping to General Relativity

To demonstrate consistency with the standard cosmological model (ΛCDM), we translate our scalar results into the tensor formalism of General Relativity.

Remark 21.5 (Translation to Metric Formalism). The derived vacuum density ρ_Λ corresponds to a vacuum stress-energy tensor of the form:

$$T^{(\mathrm{vac})}_{\mu\nu} \mathrel{\hat{=}} -\,\rho_\Lambda(r)\,c^2\,g_{\mu\nu}. \tag{54}$$

Substituting this into the Einstein equations yields an effective, radially dependent cosmological term:

$$\Lambda(r) = \frac{8\pi G}{c^4}(\rho_\Lambda c^2) = \frac{8\pi G}{c^2}\left(\frac{2}{3}\frac{c^2}{8\pi G r^2}\right) = \frac{2}{3r^2}. \tag{55}$$

Summary

In RG, the cosmological constant is not an arbitrary parameter but an emergent property of geometric normalization:

$$\boxed{\Lambda(r) = \frac{2}{3r^2}.}$$

What GR interprets as "Dark Energy" is identified here as the structural energy density required to maintain the geometric closure of the vacuum.

21.4 Geometric Signature of Spatial Dimension

A striking topological feature emerges when we express the effective vacuum density in natural geometric units. Substituting $\rho_\Lambda = \frac{2}{3}\rho_{\max}$ into the explicit definition of $\rho_{\max}$:

$$\rho_\Lambda(r) = \frac{2}{3} \cdot \frac{c^2}{8\pi G r^2} = \frac{c^2}{12\pi G r^2}. \tag{56}$$

Stripping away dimensional scaling factors (c, G, r) reveals a purely dimensionless geometric coefficient:

$$\boxed{\hat{\rho}_\Lambda = \frac{1}{12\pi} = \frac{1}{3 \times 4\pi}} \tag{57}$$

This factorization suggests a profound geometric origin for 3D space:

- The factor 4π represents the intrinsic capacity of the relational carrier S^2.
- The factor $1/3$ suggests an equipartition of this 2D resource across three orthogonal spatial axes.

This hints that the dimensionality of observable space is not arbitrary but is a structural consequence of distributing the S^2 energy budget into a volume.

22 Beyond Differential Formalism: The Structure of Reality

22.1 Intrinsic Dynamics via Energy Redistribution

In Relational Geometry (RG), dynamics is not the evolution of quantities in time, but the continuous re-balancing of a closed network of algebraic relations. What appears as "motion" is the ordered succession of states satisfying all projectional constraints.

Classical physics describes systems through differential evolution ($\delta S = 0, S = \int L\,dt$), assuming an independent temporal parameter and variational freedom over possible trajectories. In RG, none of these assumptions hold:

- There is no continuum of possible paths.
- There is no freedom to vary arbitrarily.
- The system itself defines temporal order.

Each observable is locked into a closed web of relational equations. A valid state is oneand only onewhere all constraints are simultaneously satisfied.

23 Allowed Free WILL and Structural Dynamics

In RG, there is no external spacetime arena. A physical situation is a self-consistent assignment of relational projections (β, β_Y, κ) and scales (r, ρ, t) such that the following closure relations hold simultaneously:

$$\text{(i)} \quad \beta^2 + \beta_Y^2 = 1 \qquad \text{(Kinematic Closure on } S^1\text{)} \tag{58}$$

$$\text{(ii)} \quad \kappa^2 = R_s/r \qquad \text{(Geometric Field Identity)} \tag{59}$$

$$\text{(iii)} \quad W_{ILL} \equiv \frac{ET}{ML} = 1 \qquad \text{(Global Unity Invariant)} \tag{60}$$

Definition 23.1 (Allowed Free WILL State). A configuration is an Allowed Free WILL state if and only if all relational quantities satisfy relations (i)–(iii) simultaneously. Any configuration violating these is physically non-existent.

Theorem 23.2 (Structural Dynamics Theorem). For any closed WILL system with fixed rest energy E_0 and fixed geometric scale R_s:

1. No Extra Freedom. Once a single projection (e.g., β) is changed, all other quantities $(\beta_Y, \kappa, r, \rho, t)$ are forced to readjust to maintain closure. There is no independent freedom to vary them.

2. Dynamics as Redistribution. Every physically admissible "evolution" is a continuous redistribution of the relational budgets (β, β_Y, κ). No differential equation of motion in an external time parameter is required.

3. Orbit Example. For a bound orbit, specifying a single turning point (r_p, β_p) fixes all other quantities algebraically:

$$r_p, \beta_p, R_s \implies r_a, a, e, Q_{\text{orbit}}, \Delta\varphi.$$

   No trajectory differential equation is solved; the orbit is completely encoded in the closure of the relational budgets.

Proof. (1) Kinematic closure (i) fixes $\beta_Y = \sqrt{1-\beta^2}$. Geometric closure (ii) ties κ^2 to r. The global invariant $W_{ILL} = 1$ fixes the joint scaling of (E, T, M, L). Thus, changing one variable forces a correlated shift in all others. There is no "slot" for an independent time function.

(2) A physical process is a succession of Allowed Free WILL states. If β increases, β_Y decreases; if κ changes, r follows. The universe moves between self-consistent configurations.

(3) In the orbital case, starting from (r_p, β_p), the invariants directly yield the quadratic relation for r_a and the precession $\Delta\varphi$ without integration. This confirms that dynamical quantities are fixed algebraically by closure. □

24 "Stretching" of Simultaneity as the Origin of Time and the Null-Interval

The kinematic circle enforces $\beta^2 + \beta_Y^2 = 1$, where β measures relational displacement and β_Y measures the internal phase component carrying proper time.

Theorem 24.1 (Origin of Time in Relational Geometry). Temporal order exists if and only if the Phase Component satisfies $\beta_Y > 0$.

- $\beta_Y = 0 \implies$ No internal ticking. Complete simultaneity. No time.
- $\beta_Y > 0 \implies$ Internal sequence of states exists. Time emerges as order.

Proof. From the invariant $E\beta_Y = E_0$, a massive system (defined by $\beta < 1, \beta_Y > 0$) possesses an internal structure capable of distinguishing states ("ticks"). Structural Dynamics orders these states, and time emerges as their index.

In the limiting state $\beta = 1 \implies \beta_Y = 0$ (light), the internal phase vanishes. The invariant $E\beta_Y$ holds only in a degenerate sense. Such a system has no internal "before" or "after"; it connects events without temporal extension. □

24.1 The Ontological Status of the Universe

We distinguish between the fundamental structure and the observed projection.

Definition 24.2 (The Fundamental Null-Interval). The state $\beta_Y = 0$ describes the fundamental connectivity of the universe. For this state, the separation between causally connected events is identically zero.

Definition 24.3 (The Temporal Projection). A massive observer ($\beta_Y > 0$), by definition, cannot inhabit the null-interval. Its internal structure forces a separation, projecting the point-like unity into an extended sequence ($\Delta t > 0, \Delta x > 0$).

Ontological Consequence

$$\boxed{\text{Reality} \equiv \text{The Null-Interval (The Point)}}$$

$$\boxed{\text{Spacetime} \equiv \text{The Projection (The Stretching)}}$$

We do not experience the fundamental "point-like" reality. We experience our own β_Y-driven projection of that reality as a temporal sequence.

Time does not drive change - instead, change defines time.

Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as “laws of nature.”

3. Mathematical formalisms are constructed to describe these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are generated as inevitable consequences of relational geometry:

- There are no independent axioms such as "inertial mass equals gravitational mass."
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls "laws of nature" are secondary shadows of the single relational principle:

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

Summary

Standard Physics: Laws describe what we observe.
Relational Geometry: Laws are generated as necessary products of closure and self-consistency.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

Descriptive Physics (Standard)	Generative Physics (WILL)
Phenomena are observed first, then summarized into empirical laws.	Laws emerge as inevitable consequences of relational geometry.
Physical laws are assumptions introduced to model reality.	Physical laws are identities, enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are projections of energy relations.
Dynamics = evolution of states in time.	Dynamics = ordered succession of balanced configurations; time is emergent.
Goal: describe what is observed.	Goal: show why nothing else is possible.

Table 3: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

25 Axiomatic Foundations Theorem:WILL Relational Geometry (RG) and General Relativity (GR)

"This logical asymmetry does not imply physical superiority a priori; it only states that any empirical support for GR already presupposes relational invariance."

Definition 25.1 (GR Core Axioms). General Relativity (GR) is assumed to rest on the following axioms:

(A1) The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.

(A2) Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.

(A3) Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.

(A4) Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Definition 25.2 (RG One Principle). RG is based on a single Principle:

(W1) Relational Principle: All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy.

Lemma 25.3 (Relationality in GR). From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.

Remark 25.4 (Bridge: From Relational Principle to GR Axioms). If the Relational Principle (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:

- It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.
- It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.

Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 25.5 (Asymmetric Falsifiability of GR and RG). Let GR denote the theory defined by axioms (A1)–(A4), and let RG denote the theory defined by Principle (W1). Then:

1. If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence, GR is necessarily falsified.
2. If any of (A1)–(A3) are empirically falsified, GR collapses, but (W1) may still remain valid as a stand-alone principle.

Therefore, there exist possible empirical scenarios in which GR fails while RG survives, but there exist no scenarios in which RG fails while GR survives.

Corollary 25.6. RG is axiomatically more fundamental than GR: its sole Principle (W1) is logically included within the core axioms of GR, while GR requires additional ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of RG.

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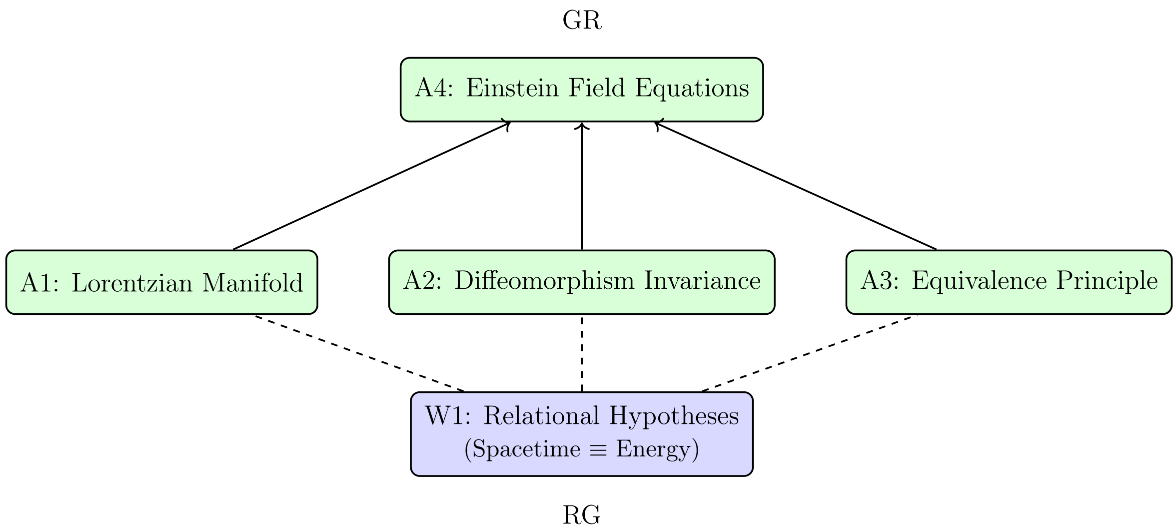

Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability).
RG rests on the single relational Principle (W1). Core GR assumes additional structures (A1–A4). Hence:

- If W1 is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of A1–A3 is falsified, GR collapses, while W1 (and thus RG) may still hold.

Therefore, there are scenarios where GR fails and RG survives, but none where RG fails while GR survives.

Figure 1: Axiomatic Structure: RG vs GR

Status of General Relativity within RG

It is important to emphasize that the RG framework does not invalidate the achievements of General Relativity. Rather, it explains them. All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in Relation Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.
Thus, GR is not a rival but a specialized, parameter-heavy realization of RG's more general principle. In logical terms:

- Relational Geometry can stand without GR, but GR cannot stand without the relational Principle (W1).
- The empirical successes of GR are preserved within RG, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

Therefore, GR should be understood as an effective approximation embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.

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Comparison Table: General Relativity (GR) vs WILL Relational Geometry (RG)

#	Category	General Relativity (GR)	Relational Geometry (RG)
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}, R$; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$
4	Geometry-Matter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \to \infty$, $g_{00} \to 0$	Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$
8	Concept of Time	Coordinate-based, embedded in g_{00}; system-dependent	Physical: β as projection of energy onto temporal axis
9	Dynamics	Via time derivatives and Lagrangians	Via change in energy proportions; no differential equations
10	Formalism	Geometry, tensors, 2nd-order derivatives	Energy projections, circular geometry, algebraic closure
11	Intuitiveness	Low; relies on abstract and heavy formalism	High; built from observable and intrinsic relations
12	Observational Fit	Confirmed (with dark matter/energy assumptions)	Consistent; explains phenomena without "dark entities" (Details in WILL PART II)

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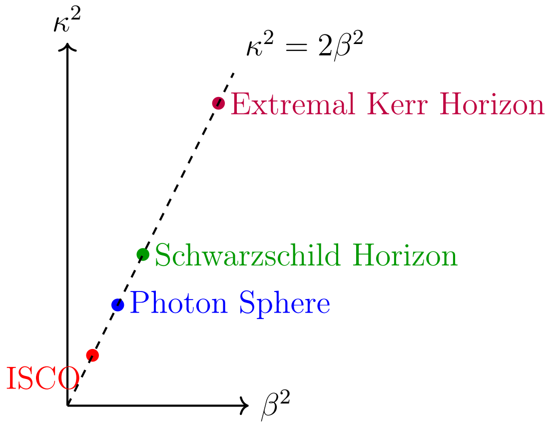
Phenomenon	Radius r	β^2	κ^2	Q^2	Comment
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal stability of timelike orbits $Q = Q_t$
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Null circular orbits, $\theta_1 = \theta_2$ $Q = 1$, $Q_t = 0$
Static horizon (Schwarzschild)	$r = R_s$	$\frac{1}{2}$	1	$\frac{3}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 4: Critical radii and their projectional parameters in WILL Relational Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.



25.1 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the RG's principles. One can derive GR by adding these additional structures to RG's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, where RG provides the logical foundation upon which GR can be consistently constructed.

25.2 Epistemological Role of General Relativity

General Relativity occupies a unique historical position. It is the first theory to recognize geometry as the carrier of energy, yet it stops one step short of full equivalence. By treating the metric $g_{\mu\nu}$ as an independent entity that responds to energy–momentum, GR still separates cause and effect. In WILL Relational Geometry this distinction dissolves: geometry is energy, not its consequence.

Thus, GR should be seen as the transitional language between the descriptive physics of the nineteenth century and the generative physics of the twenty-first. It already encodes the relational structure implicitly, but expresses it through redundant coordinates and differential machinery. RG reveals the algebraic heart of GR stripped of these redun-

Phenomenon	Standard (GR) Result	Relational Geometry (RG)
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2$, ISCO: $Q = Q_t$).
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Exact same number obtained from R G with $\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2}$.
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs.
Cosmological redshift	Photon "loses energy" as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological constant Λ	Added by hand to fit data ("dark energy").	Arises naturally as $\Lambda = 2/3r^2$. No extra entities required. (More details in WILL PART II)
Singularities	Predicted in black holes and big bang ($\rho \to \infty$).	Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$.
Local gravitational energy	"Cannot be localized" (only ADM/Bondi at infinity).	Directly measurable via κ, e.g. from light deflection angle.
Unification with QM and SR	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 5: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

dancies:

$$\boxed{G_{\mu\nu} \ \Leftrightarrow \ \kappa^2}, \qquad \boxed{T_{\mu\nu} \ \Leftrightarrow \ \rho}.$$

The celebrated field equations of Einstein then reduce to the unified geometric identity

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} = \frac{R_s}{r}},$$

which is the simplest, most symmetric realization of the same principle that GR only encodes indirectly.

Phenomenon	Empirical Benchmark	WILL Prediction
GPS satellite time dilation (SR + GR)	38.52 μs/day (observed)	38.52 μs/day
Mercury perihelion precession	43”/century (observed)	43”/century
Solar light deflection	1.75 arcsec (observed)	1.75 arcsec
Schwarzschild photon sphere	$r = 1.5R_s$ (GR prediction)	$r = 1.5R_s$
Schwarzschild ISCO	$r = 3R_s$ (GR prediction)	$r = 3R_s$
Hulse–Taylor pulsar period decay	$\Delta P \approx -2.42 \times 10^{-12}$ s/s (observed)	$\Delta P \approx -2.40 \times 10^{-12}$ s/s
Earth–Moon tidal power (LLR recession)	0.120 TW orbital power (measured)	0.120 TW orbital power (predicted)
Galactic rotation curves	"Dark Matter" speculations.	RMSE=20.23 km/s from projection law for 175 SPARC Galaxies with 0 free parameters (Details in WILL PART II).
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, ΛCDM fits	Emergent from any one scale input only (Details in WILL PART II).

Table 6: Empirical validation of WILL Relational Geometry across classical relativistic tests, orbital dynamics, astrophysical observations, and cosmology. See details in "Appendix I"

Historical Function of GR

General Relativity is not wrong - it is prematurely general. It describes through differentials what WILL generates through relations. It built the bridge; WILL walks across it.

26 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single Ontological Principle-that spacetime is equivalent to energy-we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

$$\boxed{\text{Special and General Relativity emerge from the same geometric principles.}}$$

This approach offers distinct advantages:

- Conceptual clarity - understanding physics through pure geometry
- Computational efficiency - significantly reducing complexity
- Epistemological hygiene - deriving results from minimal assumptions
- Philosophical depth - redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

$$\boxed{\text{Spacetime and energy are mutually defining aspects of a single relational structure.}}$$

Final Summary

$$\text{SPACETIME} \equiv \text{ENERGY}.$$

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