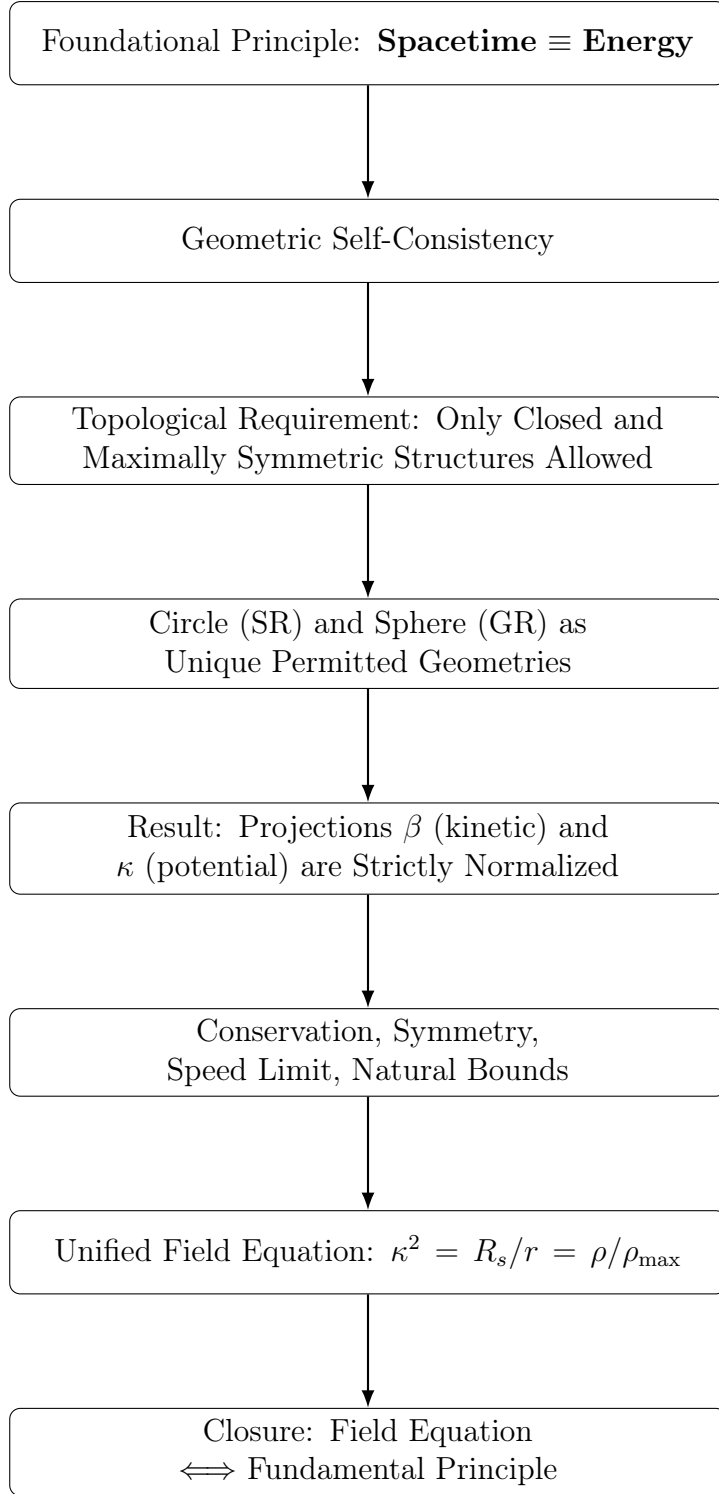


WILL Part I: Relational Geometry

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Abstract

This paper, the first in the WILL series, establishes *Relational Geometry* - a foundational framework where spacetime is not a background arena but an emergent property of energy transformations.

From a single principle **SPACETIME** \equiv **ENERGY**, we derive the complete geometric structure of physics. This equivalence is not postulated but derived by removing the hidden ontological assumption in modern physics that structure (space-time) and dynamics (energy) are separate phenomena.

The result is a singularity-free, ontologically clean formalism that reproduces the core equations of Special Relativity (SR) and General Relativity (GR) as algebraic projections on closed relational manifolds S^1 (directional) and S^2 (omnidirectional). Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energy-momentum relation, Schwarzschild solutions, and Einstein field equations via dimensionless projections β (kinematic) and κ (gravitational). Critical features - photon sphere, ISCO, horizons - emerge as equilibria ($\kappa^2 = 2\beta^2$, $Q^2 = \kappa^2 + \beta^2 = 1$). Empirically validated (e.g., GPS time shift 38.52 $\mu\text{s/day}$, Mercury precession to $10^{-10}\%$).

WILL Part I resolves GR's singularities (bounded ρ_{max}) and local energy ambiguity, offering significant computational simplification establishing a robust foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

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1 Disclaimer: On the Motivation of This Work

This work is not the product of formal academic research, institutional funding, or collaboration with established scientific communities. It is the result of personal inquiry, curiosity, and an ongoing attempt to understand the fundamental nature of space, time, and energy from the most elementary and geometric principles.

The motivation behind this framework is rooted in a deep philosophical belief that the structure of the Universe must, at its core, can be described without arbitrary parameters, assumptions, or external mathematical constructs. The ideal theory should not rely on pre-existing formalism but should emerge naturally from the geometry of the Universe itself.

It is important to clarify that I do not consider myself an academic authority, nor do I claim to have discovered any new physical law. I am a self-taught enthusiast, driven not by the desire for recognition but by a personal need to resolve fundamental questions about reality in the simplest possible terms.

Throughout this research, I have maintained a rigorous internal skepticism, questioning every step and assumption. The fact that I have arrived at results equivalent to the standard formulations of Special and General Relativity using only geometric first principles may appear unlikely, even to myself. I fully acknowledge the statistical improbability of such an achievement by an individual without formal academic training.

However, this work is not an attempt to replace or dispute existing physics but rather to reinterpret it from a geometric and philosophical standpoint. Whether this approach holds broader value is irrelevant to its primary purpose - to provide a coherent and intuitive framework that satisfies my own intellectual and philosophical curiosity.

Above all, this document serves as a personal record and reflection of a journey toward understanding, reminding me of the reasons why I chose to embark on this path.

P.S. This work remains an ongoing exploration, and further developments may reveal deeper connections between geometry, energy, and the fabric of reality.

Anton Rize.

2 Glossary of Key Terms

β (**Beta**): The kinetic unidirectional 1D projection, representing the ratio of an object's velocity to the universal speed of transformation (v/c). It quantifies how much of the "speed of change" is perceived as motion through space relative to observer. Not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

κ (**Kappa**): The potential omnidirectional 2D projection, representing the ratio of an object's escape velocity to the universal speed of transformation (v_e/c). It measures how deeply an object is situated within a gravitational field, relative to an observer, and indicates proximity to an event horizon (correspond to $\kappa = 1$ for non rotating systems). Not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

c (**Speed of Light/Universal Transformation Tempo**): The fundamental, invariant tempo of change in the universe. It is not merely the speed of light but the constant rate at which all energetic interactions and transformations occur.

Conservation Law: The principle stating that the total energy of the closed system must remain constant. In WILL Geometry, this is a direct logical consequence of **the Universe** being a closed and self-sufficient system (**not to confuse with observable universe**).

Critical Density (ρ_{\max}): A local, finite limit on how much energy can be observed (determined by $\kappa = 1$ observational limit) in a given radius, dependent on central mass m_0 and the distance r_d from its center. It increases closer to a central mass but never becomes infinite, thus preventing the formation of infinite densities (singularities).

Transformation/Evolution: The total structure of possible transitions between observable states.

Energy–Momentum Triangle: A geometric visualization in WILL Geometry that depicts the relationship between an object's rest energy, momentum, and total energy as the sides of a right triangle, illustrating their constant interconnectedness.

Energy–Symmetry Law: The principle in WILL Geometry stating that energy differences observed between any two frames of reference will always perfectly balance out, ensuring no "extra" energy is created or lost, and thus enforcing causality and naturally explains why any spacial motion is limited by speed of light.

Epistemological Hygiene: A guiding principle that demands the rejection of all assumptions not strictly necessary, building the theory solely on logical sequence from a minimalist foundation.

Event Horizon: The boundary around a black hole beyond which nothing, not even light, can escape. In WILL Geometry, $\kappa = 1$ at this point, indicating that the escape velocity here is equal to speed of light.

Geometric Structures (Circle S^1 and Sphere S^2): The fundamental, non spacial, maximally symmetric and closed geometric "canvases" on which physical reality is projected in WILL Geometry. The circle describes 1D unidirectional motion

(Special Relativity), and the sphere describes 2D omnidirectional gravity (General Relativity).

Photon Sphere: A specific region around a massive object where light can orbit in a perfect, unstable circular path. In WILL Geometry, this corresponds to a braking point where the sum of kinetic and potential projections are reaching unity ($Q^2 = \kappa^2 + \beta^2 = 1$).

Relational View of Energy: The concept in WILL Geometry that energy is not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

Singularity: A point of infinite density predicted by traditional General Relativity (e.g., at the center of a black hole). WILL Geometry's framework inherently prevents singularities, replacing them with finite, maximum allowed densities.

SPACETIME \equiv ENERGY: The central, unifying Principle of WILL Geometry, idea that the fabric of spacetime is identical to the full structure of all possible transitions and interconnections between energetic states.

Symmetry: A core in WILL Geometry derived from the foundational Principle, stating that in a closed system without external reference points, the geometry of the universe must be maximally symmetric. No preferred directions.

Time/Length Dilation/Contraction: The phenomenon where time passes more slowly for an observer or object. In WILL Geometry, it is explained as a consequence of how the universal "speed of transformation" budget is allocated between motion (kinetic time dilation) and gravitational potential (gravitational time dilation).

WILL: used here as a technical term for **SPACE-TIME-ENERGY** unified, emergent structure.

WILL Invariant (W_{III}): A dimensionless constant ($= 1$) that mathematically connects energy, mass, time, and length within the WILL Geometry framework, demonstrating the self-consistency of the system.

3 Foundational Approach

This Approach Does not Describe Physics; it Generates it.

Guiding Principle:

Nothing is assumed. Everything is derived.

3.1 Methodological Purity

This framework is constructed under a single epistemic constraint: to derive all of physics by **removing one hidden assumption**, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced unless they follow strictly from first principles, and no constructs are retained unless they are geometrically or energetically necessary.

No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent energetic projections.

3.2 Epistemic Hygiene

Modern physics often tolerates hidden assumptions: arbitrary constants, external backgrounds, or abstract entities with ambiguous physical status. Here we enforce **epistemic hygiene**: a refusal to import unjustified assumptions. All physical quantities emerge purely from relational geometry and causal closure.

Epistemic Disclaimer

This document must be read **literally**. All terms are defined within the relational framework of WILL Geometry. Any attempt to reinterpret them through conventional notions (*absolute energies, external backgrounds, hidden containers*) will produce distortions and misreadings. The responsibility of interpretation lies with the reader: take the words as written, not as filtered through prior formalisms.

3.3 Motivation and Core Principle

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

Principle: *All physical quantities must be defined purely by their relations.* Any introduction of absolute properties or external frames risks reintroducing metaphysical artifacts and contradicts the foundational insight of relativity.

3.4 What is "Everything"?

We define the Universe as everything accessible to an observer. By this very definition, it is closed, for if it were not closed, then something would exist outside of "everything". An empirical confirmation of this closure is given by the dark night sky: if the Universe were infinite and unbounded, every line of sight would end on a star and the sky would shine with infinite brightness. Instead, the visible horizon is finite, and the Universe is operationally closed.

3.5 What is Energy?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure. Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation. Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself. From these empirical and relational facts the definition follows unavoidably:

Energy is the relational measure of difference between possible states, conserved in any closed whole and guaranteeing the continuity of causal transitions. It is not an intrinsic property of an object, but **comparative structure** between states (and observers), always manifesting as transformation.

4 Unifying Principle Removing the Hidden Assumption

Removing the Hidden Assumption

Any attempt to treat “*spacetime structure*” as separate from “*dynamics*” smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

4.1 False Separation

Lemma 4.1 (False Separation). *Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.*

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of interprocess data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. \square

Corollary 4.2 (Structure–Dynamics Coincidence). *To avoid the artifact of Lemma 4.1, the structural arena and the dynamical content must be identified: geometry is energy, and energy is geometry.*

[Working Principle: Removing the Hidden Assumption]

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

This is not introduced as a new ontological entity but as a Principle with *negative ontological weight*: it removes the hidden unjustified separation between “geometry” and “dynamics.” Space and time are not containers but emergent descriptors of relational energy.

Remark 4.3 (Auditability). *Principle 4.1 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).*

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity - WILL.

5 Deriving the WILL Structure

Having established our working Principle (Principle \ref{pr:unifying}) by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences. We will demonstrate that this single principle is sufficient to enforce the closure, conservation, and fundamental symmetries of the relational structure, leading to a unique set of geometric carriers for energy.

Definition 5.1 (WILL Geometry). **WILL \equiv SPACE-TIME-ENERGY** *is the unified relational structure determined by 4.1 . All physically meaningful quantities are relational features of WILL; no external container is permitted.*

Lemma 5.2 (Closure). *Under 4.1, WILL is self-contained: there is no external reservoir into or from which the relational resource can flow.*

Proof. If WILL were not self-contained, there would exist an external structure mediating exchange. That external structure would then serve as a background distinct from the dynamics, contradicting Corollary 4.2. \square

Lemma 5.3 (Conservation). *Within WILL, the total relational “transformation resource” is conserved.*

Proof. By Lemma 5.2, no external fluxes exist. Any change in one part of WILL must be balanced by complementary change elsewhere. Hence a conserved global quantity is enforced at the relational level. \square

Lemma 5.4 (Isotropy from Background-Free Relationality). *If no external background is allowed (Cor. 4.2), then no direction can be a priori privileged. Thus the admissible relational geometry of WILL must be maximally symmetric (isotropic and homogeneous) at the level at which it encodes the conserved resource.*

Proof. A privileged direction requires an extrinsic reference to distinguish it. In a purely relational setting, distinctions must be constructible from relations internal to WILL. If a direction were privileged in the geometry that encodes the conserved resource, such privilege would not be derivable from purely internal comparisons (which are symmetric by construction), and would reintroduce an external orienting structure. Therefore the encoding geometry must be maximally symmetric. \square

Theorem 5.5 (Minimal Relational Carriers of the Conserved Resource). *The only closed, maximally symmetric manifolds that can serve as minimal carriers of the conserved relational resource are:*

- (a) S^1 for directional (one-degree-of-freedom) relational transformation;
- (b) S^2 for omnidirectional (central, all-directions-equivalent) relational transformation.

Proof. By Lemma 5.4, we require closed, maximally symmetric manifolds.

(a) In one relational degree of freedom, the classification of connected closed 1-manifolds yields S^1 as the unique (up to diffeomorphism) option. Its isometry group acts transitively with isotropy at each point, providing maximal symmetry.

(b) For omnidirectional relational transformation from a distinguished center, the encoding manifold must be a closed, simply connected, constant positive curvature 2-manifold with full isotropy at every point. By the uniformization/classification of constant-curvature surfaces, the maximally symmetric representative is S^2 . Quotients of S^2 by nontrivial finite groups introduce global identifications that spoil global isotropy; these are excluded by Lemma 5.4. Hence S^2 is uniquely selected. \square

Corollary 5.6 (Uniqueness). *Under 4.1 with Closure, Conservation, and Isotropy (Lemmas 5.2–5.4), S^1 and S^2 are necessary relational carriers for, respectively, directional and omnidirectional modes of energy transformation.*

Remark 5.7 (Non-spatial Reading). *Throughout, S^1 and S^2 are not to be interpreted as spacetime geometries. They are relational manifolds that encode the closure, conservation, and isotropy of the transformational resource. Ordinary spatial and temporal notions are emergent descriptors of patterns within WILL.*

Summary:

From removing the hidden assumption 4.1 we inevitably arrive to 4.1 SPACETIME \equiv ENERGY from there we deduced: (i) closure, (ii) conservation, (iii) isotropy, and hence (iv) the unique selection of S^1 and S^2 as minimal relational carriers for directional and omnidirectional transformation. These objects are non-spatial encodings of conservation and symmetry; they are enforced by the 4.1 rather than assumed independently.

5.1 Ontological Status of the Relational Manifolds S^1 and S^2

A natural question arises regarding the ontological status of the circle S^1 and the sphere S^2 : What are they, and where do they "exist"?

The answer requires a fundamental shift in perspective. In WILL Geometry, S^1 and S^2 are **not spatial entities** existing within a pre-defined container. They are the necessary **relational architectures** that implement the core identity $\text{SPACETIME} \equiv \text{ENERGY}$.

Energy as Relational Bookkeeping Recall that energy is defined as the *relational measure of difference between possible states*. It is not an intrinsic property but a comparative structure that guarantees causal continuity. It is never observed directly, only through transformations.

The Manifolds as Protocols of Interaction The manifolds S^1 and S^2 are the minimal, unique mathematical structures capable of hosting this relational "bookkeeping" for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, A and B :

- Observer A is the center of their own relational framework. Observer B is a point on A 's S^1 (for kinematic relations) and S^2 (for gravitational relations).
- Simultaneously, observer B is the center of their own framework. Observer A is a point on B 's S^1 and S^2 .

There is no privileged "master" manifold. Each observable interaction is structured by these mutually-centered relational protocols. The parameters β and κ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta_X^2 + \beta_Y^2 = 1$; $\kappa_X^2 + \kappa_Y^2 = 1$) are the innate accounting rules of these protocols.

Emergence of Spacetime Therefore, the question "Where are S^1 and S^2 ?" is a category error. They are not *in* space; they are the structures whose coordinated, multi-centered interactions **give rise to** the phenomenon we perceive as spacetime. Spacetime is the emergent, collective shadow cast by the dynamics of energy relations projected onto these architectures.

In essence, S^1 and S^2 are the ontological embodiment of the relational principle. They are not assumed but derived as the only possible structures that can house the transformational resource (energy) in a closed, conserved, and isotropic universe. Their status is that of a fundamental **relational geometry** from which physics is generated.

6 Emergence of Spacetime

In this construction, "space," "time," and the principle of uncertainty are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

6.1 The Duality of Transformation

Lemma 6.1 (Duality of Evolution). *The identification of spacetime with energy and its transformations necessitates two complementary relational measures:*

1. the **extent** of transformation (external displacement), and
2. the **sequence** of transformation (internal order).

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The circle S^1 provides the minimal geometry enforcing such complementarity: its orthogonal projections furnish precisely two non-redundant coordinates. Therefore the duality of transformation is a geometric necessity, not an arbitrary naming convention. \square

We define this orthogonal projections as follows:

- **The Amplitude Component** (β_X): This projection represents the *relational measure* between the system and the observer. It corresponds to the *extent* of transformation, which manifests physically as momentum (as shown in next section).
- **The Phase Component** (β_Y): This projection represents the *internal structure* of a system. It governs the intrinsic scale of its proper length and proper time units, corresponding to the *sequence* of its transformation. A value of $\beta_Y = 1$ represents a complete and undisturbed manifestation of this internal structure, a state we identify as rest.

6.2 Conservation Law of Relational Transformation

Theorem 6.2 (Conservation Law of Relational transformation). *The orthogonal components of transformation (β_X, β_Y) are bound by the closure relation*

$$\beta_X^2 + \beta_Y^2 = 1.$$

Proof. Since S^1 is closed, every point on the circle is constrained by the Pythagorean identity of its projections. Thus no state can exceed or fall short of the finite relational "budget." This closure enforces conservation across all processes. \square

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

6.3 Consequence: Relativistic Effects

Proposition 6.3 (Physical Interpretation: Relativistic Effects). *The conservation law of Theorem 6.2 implies that any redistribution between the orthogonal components (β_X, β_Y) manifests physically as the relativistic effects of time dilation and length contraction.*

Proof. By Theorem 6.2, the components satisfy $\beta_X^2 + \beta_Y^2 = 1$. An increase in the relational displacement β_X enforces a decrease in the internal measure β_Y . This reduction of β_Y corresponds to dilation of proper time and contraction of proper length, while the growth of β_X represents momentum. Thus the relativistic trade-off is not an additional postulate but the direct physical expression of the geometric closure of S^1 . \square

Summary:

Geometry of spacetime is nothing but the shadow cast by the geometry of relations.

7 Kinetic Energy Projection on S^1

Since S^1 encodes one-dimensional displacement, the total energy E of the system must project consistently onto both axes:

$$E_X = E\beta_X, \quad E_Y = E\beta_Y.$$

Theorem 7.1 (Invariant Projection of Rest Energy). *For any state (β_X, β_Y) on the relational circle, the vertical projection of the total energy is invariant:*

$$E\beta_Y = E_0.$$

Proof. When $\beta_X = 0$, closure enforces $\beta_Y = 1$, yielding $E = E_0$. Since closure applies for all θ_1 , the vertical projection $E\beta_Y$ remains equal to this rest value in every state. \square

Corollary 7.2 (Total Energy Relation). *From Theorem 7.1 it follows that*

$$E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sqrt{1 - \beta_X^2}}.$$

Remark 7.3 (Lorentz Factor). *The historical Lorentz factor γ is nothing more than the reciprocal of β_Y . No additional structure is introduced: all content is already present in $E\beta_Y = E_0$.*

Summary:

The historical Lorentz factor γ is nothing more than the reciprocal of β_Y .

7.1 Rest Energy and Mass Equivalence

Corollary 7.4 (Rest Energy and Mass Equivalence). *Within the normalization $c = 1$, the invariant rest energy equals mass:*

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1 , no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. \square

Remark 7.5 (Epistemic Interpretation). *In a framework that is genuinely fundamental and free from arbitrary human units, the natural normalization is always the unique invariant $c = 1$. With this normalization, the bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ lose all significance. They collapse into the only consistent statement:*

$$E_0 = m.$$

Thus mass is nothing beyond the invariant projection of total rest energy. It introduces no new entity and carries no independent meaning apart from E_0 . What is conventionally treated as two quantities is in fact one and the same relational invariant.

Summary:

Mass is nothing beyond the invariant projection of total rest energy. .

7.2 Energy–Momentum Relation

Proposition 7.6 (Horizontal Projection as Momentum). *On the relational circle, the unique relational measure of displacement from rest is the horizontal projection $E\beta_X$; hence*

$$p \equiv E\beta_X \quad (c = 1).$$

Proof. The rest state is $(\beta_X, \beta_Y) = (0, 1)$. A displacement measure must (i) vanish at rest, (ii) grow monotonically with $|\beta_X|$, and (iii) flip sign under $\beta_X \mapsto -\beta_X$. The only relational candidate satisfying (i)–(iii) is the horizontal projection $E\beta_X$. Thus the identification is necessary rather than conventional. \square

Corollary 7.7 (Energy–Momentum Relation). *With p identified by Proposition 7.6 and $m = E_0$, the closure identity yields*

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c ,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta_X)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta_X$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E , yielding the standard form. \square

Remark 7.8 (Geometric Forms). *The same identity may be expressed explicitly in terms of circle coordinates:*

$$E^2 = \left(\frac{\beta_X}{\beta_Y} E_0 \right)^2 + E_0^2 = (\cot(\theta_1) E_0)^2 + E_0^2.$$

These are not alternative parametrizations, but equivalent renderings of the same geometric necessity.

Remark 7.9 (Units sanity check — bookkeeping). *Using $\beta_X = v/c$, the identification $p \equiv E\beta_X$ gives*

$$pc = E \frac{v}{c} \implies p = \frac{E v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta_X}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

Summary:

The energy–momentum relation $E^2 = p^2 + m^2$ is nothing more than geometric identity of S^1 .

$\beta_X = \beta, \quad \beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	Trigonometric Form
$1/\beta_Y = 1/\sqrt{1 - \beta^2} = 1/\sqrt{1 - (v/c)^2}$	$1/\beta_Y = 1/\sin(\theta_1) = 1/\sin(\arccos(\beta))$
$\beta_Y = \sqrt{1 - \beta^2} = \sqrt{1 - (v/c)^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$

Table 1: Unified representation of relativistic effects.

8 Potential Energy Projection on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as space-time geometries but purely as relational manifolds encoding conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1 the relational geometry of the sphere, S^2 , provides orthogonal projections, for two aspects of omnidirectional transformation. We define them as follows:

- **The Amplitude Component (κ_Y):** This projection represents the *relational gravitational measure* between the object and the observer. It corresponds to the *extent* of transformation, which manifests physically as gravitation potential. A value of $\kappa_Y = 1$ corresponds precisely to the point where escape velocity equals the speed of light, creating an event horizon. This provides a natural causal limit for our gravitational parameter, analogous to the relative motion it determinant by the same radial normalization.
- **The Phase Component (κ_X):** This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the *sequence* of its transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite “budget of reality”:

$$\kappa_X^2 + \kappa_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

8.1 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure (κ_Y , gravitation potential) necessarily requires a decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is nothing but the shadow cast by the geometry of relations.

Notation simplicity:

From here on we will write $\beta = \beta_X$, $\beta_Y = \sqrt{1 - \beta^2}$, $\kappa = \kappa_Y$, $\kappa_X = \sqrt{1 - \kappa^2}$ for notation simplicity.

8.2 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the escape velocity at radius r .

In the kinematic case, with $\beta = \cos \theta_1$, the energy relation can be written as

$$E^2 = (\cot \theta_1 E_0/c)^2 + E_0^2, \quad (1)$$

so that the relativistic momentum is expressed as

$$p = E_0/c \cot \theta_1. \quad (2)$$

In full symmetry, the gravitational case follows from $\kappa = \sin \theta_2$. We define the gravitational energy as

$$E_g = \frac{E_0}{\kappa_X}, \quad \kappa_X = \sqrt{1 - \kappa^2}, \quad (3)$$

and introduce the gravitational analogue of momentum:

$$p_g = E_0/c \tan \theta_2. \quad (4)$$

This yields the gravitational energy relation

$$E_g^2 = p_g^2 + E_0^2. \quad (5)$$

Summary:

$$\begin{aligned} \beta &= \cos \theta_1, & \kappa &= \sin \theta_2, \\ \beta &\longleftrightarrow \kappa, & \cot \theta_1 &\longleftrightarrow \tan \theta_2. \end{aligned}$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the same relational circle, expressed through complementary trigonometric forms.

8.3 Geometric composition of SR and GR factors

On the unit kinematic circle (S^1) we parametrize

$$(\beta, \beta_Y) = (\cos \theta_1, \sin \theta_1),$$

so that the invariant vertical projection reads

$$E \beta_Y = E_0 \quad \Rightarrow \quad \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin \theta_1}}, \quad p = E \beta = \frac{E_0 \beta}{\beta_Y} = E_0 \cot \theta_1,$$

and therefore $E^2 = p^2 + E_0^2$.

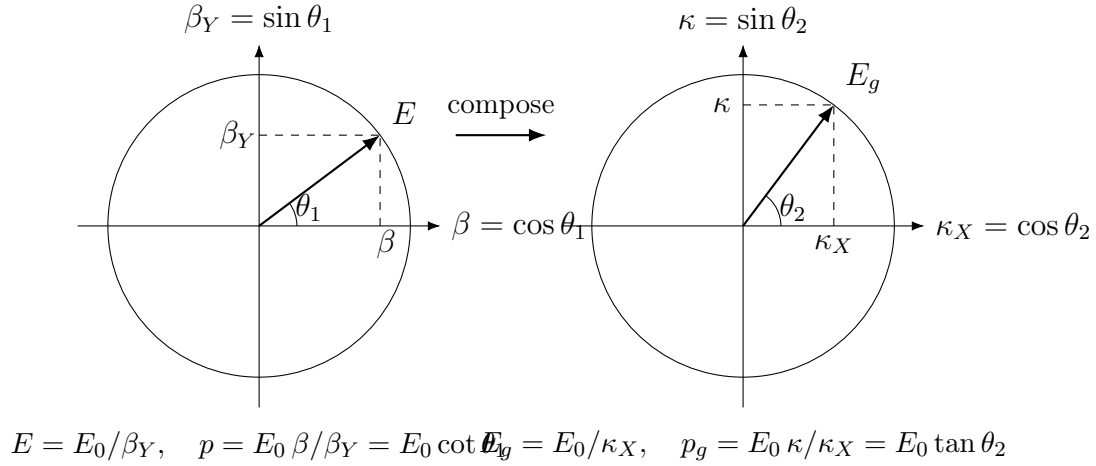
On the gravitational circle (S^2) we parametrize

$$(\kappa_X, \kappa) = (\cos \theta_2, \sin \theta_2),$$

so that the invariant horizontal projection reads

$$E_g \kappa_X = E_0 \quad \Rightarrow \quad \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos \theta_2}}, \quad p_g = E_g \kappa = \frac{E_0 \kappa}{\kappa_X} = E_0 \tan \theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



8.4 Equivalence Principle as Derived Identity

In General Relativity the Einstein Equivalence Principle is introduced as an independent postulate: $m_g \equiv m_i$. Within WILL, no such axiom is required:

On the kinematic circle the vertical leg equals $\beta_Y = \sin \theta_1$; keeping the same invariant E_0 on that leg forces a stretch $E/E_0 = 1/\beta_Y$. On the gravitational circle the horizontal leg equals $\kappa_X = \cos \theta_2$; keeping the same invariant E_0 on that leg forces a stretch $E_g/E_0 = 1/\kappa_X$. Composing the two independent stretches gives

$$\boxed{E_{\text{loc}} = \frac{E_0}{\beta_Y} \times \frac{1}{\kappa_X} = \frac{E_0}{\beta_Y \kappa_X}} = \frac{E_0}{\sqrt{(1 - \beta^2)(1 - \kappa^2)}}.$$

The inertial and gravitational projections share the same operational scale,

$$\tilde{p} = E_{\text{loc}} \beta, \quad \tilde{p}_g = E_{\text{loc}} \kappa,$$

both governed by the identical stretch factor

$$m_{\text{eff}} = \frac{E_0}{\beta_Y \kappa_X}.$$

Hence

$$\boxed{m_g \equiv m_i = m_{\text{eff}}},$$

Thus, in WILL relational geometry the equality $m_g \equiv m_i$ is not assumed but forced by structure.

Remark 8.1 (On Eötvös-type precision). *In GR, the equality $m_g = m_i$ is imposed, while distinct relativistic factors later rescale energy and time. In WILL, there is no independent m at all: the only invariant is E_0 , and the effective response is the universal stretch factor $1/(\beta_Y \kappa_X)$. Since this factor depends solely on geometry and not on the composition of the test body, the universality of free fall (Eötvös experiments) follows identically. What GR postulates, WILL reproduces as structural necessity.*

Composition-Independence, and Quantum Interface

Composition-Independence (Eötvös-type). Let the rest invariant decompose into internal channels

$$E_0 = \sum_a E_0^{(a)} \quad (\text{rest mass, binding, EM, nuclear, vacuum bookkeeping, etc.}).$$

WILL couples *only* through the universal geometric stretch:

$$E_{\text{loc}} = \frac{1}{\beta_Y \kappa_X} \sum_a E_0^{(a)} = \sum_a \frac{E_0^{(a)}}{\beta_Y \kappa_X}.$$

Hence each channel is multiplied by the *same* factor $1/(\beta_Y \kappa_X)$, and any ratio of channel weights cancels in observables governed by the common scale. Therefore the operational response is composition-independent *by construction*, matching the universality tested in Eötvös-type experiments, without a separate postulate $m_g = m_i$.

Quantum Interface (phase bookkeeping). In WILL the phase increment is purely relational and inherits the same scale:

$$\Delta\phi \propto E_{\text{loc}} \Delta\lambda \quad (\text{no absolute time; } \Delta\lambda \text{ is the internal ordering parameter}).$$

Thus

$$p = \nabla_\xi \phi \propto E_{\text{loc}} \beta, \quad p_g = \nabla_\chi \phi \propto E_{\text{loc}} \kappa,$$

so matter-wave phases and redshifts share the *identical* stretch $1/(\beta_Y \kappa_X)$ across all internal channels, again yielding composition-independent interferometric outcomes.

Contrast with GR. GR recovers universality by *geodesic motion in a metric* after positing $m_g \equiv m_i$. WILL has no independent mass primitive and no external background: the same projection identity that generates

$$E = \frac{E_0}{\beta_Y}, \quad E_g = \frac{E_0}{\kappa_X}$$

forces the unified operational scale E_{loc} ; hence the equality of inertial and gravitational responses is an *algebraic identity* of relational geometry, not an axiom. GR's geodesics are a *representation*; WILL's equivalence is a *consequence*.

Summary:

What GR posits as a postulate, WILL delivers as an unavoidable consequence of Relational Geometry.

9 Relation Between Potential and Kinetic Energy Projections

9.1 Quick recap:

The Topological Necessity of S^1 and S^2 Projections

The foundational Principle SPACETIME \equiv ENERGY demands that all physical quantities emerge from the relational structure of transitions between observable states. This structure must be:

1. **Self-contained:** There is no external background 5.2.
2. **Conservative:** The total "resource" for transformation is fixed relem:conservation.
3. **Maximally Symmetric:** No point or direction is privileged 5.4.

Any geometry violating these principles would reintroduce an absolute background, violating the foundational Principle.

The fundamental question is: *What are the unique geometric arenas that can host such a complete description of physical transformations?*

We must now characterize the types of "energy Transformation" that constitute physics. There are two, fundamental classes:

- **Class I (Directional Transformation):** This describes a transition whose character is defined by a *preferred axis*.
The act of measurement is the act of distinguishing this axis (e.g., the line connecting observer and object for relative velocity).
The energy measure is inherently tied to a specific direction. This is the domain of kinematics.
- **Class II (Omnidirectional Transformation):** This describes a transition whose character is *inherently without a preferred axis*.
The act of measurement is the act of establishing a relation to a center, but the resulting field is defined equally in all directions from that center.
This is the domain of a static, central potential.

This are an exhaustive catalogue of the fundamental relational categories implied by the foundational Principle 5.6.

The geometry must now accommodate these classes while obeying the three constraints above 5.5.

- **For Class I (Directional):** The only closed, 1-dimensional manifold of constant positive curvature is the circle, S^1 . It is unique. It is maximally symmetric (all points and rotations are equivalent). It is self-contained and embodies conservation (the fixed radius). **Therefore, the kinematic parameter β must be a projection on S^1 . There is no other option.**
- **For Class II (Omnidirectional):** The only closed, simply-connected 2-dimensional manifold of constant positive curvature is the sphere, S^2 . It is unique. It is maximally symmetric (all points and rotational axes are equivalent). It is self-contained. **Therefore, the gravitational parameter κ must be a projection on S^2 . There is no other option.**

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as space-time geometries but purely as relational manifolds encoding conservation. Any reading otherwise is a misinterpretation.

9.2 Relational ratio between S^1 and S^2 manifolds

Closed, self-contained universe requires that the "total space of possibilities" for each projection mode be complete. The fundamental, dimensionless measure for the completeness of these geometries is their full angle:

- For the circle (S^1), the total available "configurational space" is its full angle of closure: 2π .
- For the sphere (S^2), the total available "configurational space" is its full angle of closure: 4π .

The ratio of the total angular measures of these two fundamental manifolds is a pure, dimensionless number dictated by their intrinsic topology:

$$\frac{\text{Total Closure of } S^2}{\text{Total Closure of } S^1} = \frac{4\pi}{2\pi} = 2$$

We can confirm this result comparing degrees of freedom on each manifold:

$$\frac{2D \text{ on } S^2}{1D \text{ on } S^1} = \frac{2D}{1D} = 2$$

This factor of 2 is a direct and necessary consequence of the dimensional difference between an omnidirectional (2D) and a linear (1D) projection space on the relational manifolds S^2 and S^1 .

9.3 The Quadratic Nature of Energetic Projections

To translate this purely geometric ratio into a physical law, we must consider how projections relate to energy. In physics, the energetic significance (or "power") of a field, velocity, or wave is proportional not to its amplitude, but to its **amplitude squared**. This quadratic relationship is fundamental, appearing in kinetic energy ($E_k \propto v^2$), potential energy in fields, and the energy density of electromagnetic waves ($E \propto A^2$).

Furthermore, in any projectional geometry, the magnitudes of orthogonal components are related to the whole via a Pythagorean sum of squares. Therefore, the conserved Transformation resource is distributed among the squares of its projections. The fundamental relation must connect the ratio of the *energetic significances* of the modes (κ^2 and β^2) to the underlying topological ratio of their geometric arenas.

Linear relations would fail to respect both the physical definition of energy (quadratic in amplitude) and the Pythagorean structure of projections. Therefore only quadratic measures are consistent with closure and conservation.

. This leads to the unique and necessary unification equation:

$$\frac{\kappa^2}{\beta^2} = 2 \implies \boxed{\kappa^2 = 2\beta^2}$$

9.4 Closure and Conservation of Energy

The relation $\kappa^2 = 2\beta^2$ must be understood not merely as a proportionality, but as the precise relational criterion for the **energetic closure** of a system. In this framework, it serves as the direct geometric embodiment of the virial theorem.

The Principle as a Diagnostic Invariant

The relation $\kappa^2 = 2\beta^2$ holds if the system under study is energetically closed momentarily (circular orbits) or periodically (elliptical orbits). In open systems It can be used for determining the magnitude of energy flow through an unaccounted-for channels. When all the channels is included in the balance, the closure is restored and the equality holds again (see section "Numerical Validations" subsection "Earth–Moon").

Illustrative Examples

To clarify the meaning of this closure condition, consider two contrasting cases:

- **Circular Orbit (Closed Subsystem).** For a test body in a perfectly circular orbit around a central mass, the condition $\kappa^2 = 2\beta^2$ is exactly fulfilled. The orbital system can be treated as energetically closed: all of the conserved resource is accounted for between the kinetic projection along the orbit and the gravitational projection toward the center. No external channels are needed, and the equality signals full closure.
- **Radiating Binary (Open Subsystem).** In contrast, for a highly elliptical binary system of compact objects (such as neutron stars. (full calculation in section: Empirical Validation; subsection: Orbital Decay: Binary Pulsar)), the orbital energy is accompanied by gravitational-wave emission. If one considers only the orbital mechanics, the periodic (elliptical orbit) relation $\kappa^2 = 2\beta^2$ will be violated revealing the magnitude of energy flow through an unaccounted-for channels. When all the channels is included in the balance, the closure is restored and the equality holds again.

Summary

1. The foundational Principle defines the universe as a closed, conservative, symmetric relational structure $\text{SPACETIME} \equiv \text{ENERGY}$.
2. The relational geometry dictates that the *only* closed, maximally symmetric arenas for these relations are S^1 and S^2 , respectively.
3. Therefore, the projection parameters $\beta = \cos \theta_1$ and $\kappa = \sin \theta_2$ are *forced* to live on these **relational** manifolds.
4. The geometric exchange rate between this two manifolds is determent by the relational ratio of the respective manifolds $\frac{\text{Total Closure of } S^2}{\text{Total Closure of } S^1} = \frac{4\pi}{2\pi} = 2$.

Physical Implication. This ratio links potential and kinetic modes without reference to spacetime as a background fabric, showing that their connection is a matter of relational geometry, not dynamic coincidence. Classical physics, in its successful predictions, unknowingly traces the consequences of this deeper geometric law:

$$\boxed{\text{Spacetime Geometry } (\kappa^2) \equiv \text{Kinematic Energy distribution } (\beta^2) \times 2}$$

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

9.5 Clear Relational Symmetry Between Kinematic and Potential Projections

Now we can clearly see the underling symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

$\beta = \beta_X, \quad \kappa = \kappa_Y, \quad \theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa) \quad \kappa^2 = 2\beta^2$	
Algebraic Form	Trigonometric Form
$1/\beta_Y = \frac{1}{\sqrt{1-\beta^2}}$	$1/\beta_Y = \frac{1}{\sin(\theta_1)} = \frac{1}{\sin(\arccos(\beta))}$
$1/\kappa_X = \frac{1}{\sqrt{1-\kappa^2}}$	$1/\kappa_X = \frac{1}{\cos(\theta_2)} = \frac{1}{\cos(\arcsin(\kappa))}$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$
$p = E_0 \beta / \beta_Y$	$p = E_0 \cot(\theta_1)$
$p_g = E_0 \kappa / \kappa_X$	$p_g = E_0 \tan(\theta_2)$

Table 2: Unified representation of relativistic and gravitational effects.

9.5.1 The Combined Energy Parameter Q

The total energy projection parameter unifies both aspects:

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{6}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{7}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{8}$$

$$Q_r = \frac{1}{Q_t} \tag{9}$$

These describe the combined effects of relativity and gravity.

Summary

The familiar SR and GR factors emerge here as projections of the same conserved geometry. Relativistic (β) and gravitational (κ) modes are not separate “effects” but dual aspects of one energy-transformation constraint revealing their unified origin.

10 Energy–Symmetry Law

10.1 Causal Continuity and Energy Symmetry

Theorem 10.1 (Energy Symmetry). *The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the Energy–Symmetry Law:*

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0. \quad (10)$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A = 0$).
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A 's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\Delta E_{A \rightarrow B} = \underbrace{\frac{1}{2} (\kappa_A^2 - \kappa_B^2)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2} (\beta_B^2 - \beta_A^2)}_{\text{Change in Kinetic}} \quad (11)$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A \rightarrow B} = \frac{1}{2} ((\kappa_A^2 - \kappa_B^2) + \beta_B^2) \quad (12)$$

From B 's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B \rightarrow A} = \frac{1}{2} ((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)) = \frac{1}{2} ((\kappa_B^2 - \kappa_A^2) - \beta_B^2) \quad (13)$$

Summing these transfers gives:

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0 \quad (14)$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the Energy–Symmetry Law as a direct consequence of the geometry. \square

10.2 The Relational State Budget (Q^2) vs. Energy Transfer (ΔE)

It is crucial to distinguish between two related but distinct concepts:

1. **The Quadratic State Budget (Q^2):** This dimensionless quantity describes the total geometric "footprint" of an object's state, combining its potential and kinetic aspects. It is defined as the sum of the squares of the projections:

$$Q^2 = \kappa^2 + \beta^2 \quad (15)$$

The change in this value, $\Delta(Q^2) = Q_B^2 - Q_A^2$, represents the net change in the geometric descriptor of the state, but **it is not the energy transfer**. As demonstrated by the GPS satellite example, $\frac{1}{2}\Delta(Q^2)$ corresponds to the *difference* between the change in potential energy and the change in kinetic energy.

2. **The Specific Energy Transfer (ΔE):** This is the physical quantity representing the actual work done and change in motion, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the **sum of the changes** in the potential and kinetic energy budgets:

$$\Delta E_{A \rightarrow B} = \Delta U_{A \rightarrow B} + \Delta K_{A \rightarrow B} = \frac{1}{2} (\kappa_A^2 - \kappa_B^2) + \frac{1}{2} (\beta_B^2 - \beta_A^2) \quad (16)$$

It is this quantity, ΔE , that is conserved and must balance to zero in any closed cycle.

10.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics but from the fundamental quadratic nature of the energy budgets in WILL geometry.

The energetic significance of a state is proportional to the **square** of its geometric projection. This is analogous to how kinetic energy is proportional to velocity squared (v^2) or how the energy in a wave is proportional to its amplitude squared (A^2). The individual energy budgets are defined as:

- **Specific Potential Energy Budget:** $U/E_0 \propto -\frac{1}{2}\kappa^2$
- **Specific Kinetic Energy Budget:** $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the *changes* in these individual budgets.

10.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 10.2 (Universal Speed Limit). *The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.*

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A \rightarrow B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B \rightarrow A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} \neq 0 \quad (17)$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. \square

Summary

The speed of light is the boundary beyond which the energetic symmetry between perspectives breaks down. Causality is not an external rule but a built-in feature of Relational Geometry.

10.5 Transparent Energy Balance for Closed Orbits

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A \rightarrow B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \quad (18)$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A \rightarrow B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \quad (19)$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

11 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law is that, when the zero of gravitational potential is chosen on the surface of the central body rather than at infinity, the total specific orbital energy (potential + kinetic, per unit mass) naturally appears in *Minkowski form*.

11.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass M_\oplus (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \quad (20)$$

$$K = \frac{1}{2}m\frac{GM_\oplus}{a}. \quad (21)$$

Adding these and dividing by the rest-energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2} \frac{GM_\oplus}{ac^2}. \quad (22)$$

11.2 Projection Parameters and Minkowski-like Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \quad (23)$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \quad (24)$$

Substituting into (22) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}(\kappa_\oplus^2 - \beta_{\text{orbit}}^2). \quad (25)$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}(x^2 - y^2), \quad (26)$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

11.3 Physical Interpretation

In classical derivations, (22) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the WILL framework, (25) emerges directly from the energy-symmetry relation:

$$\Delta E_{A \rightarrow B} = \frac{1}{2}((\kappa_A^2 - \kappa_B^2) + \beta_B^2),$$

with $(A, B) = (\text{surface}, \text{orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While this framework refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a , and is independent of the test-mass m .
- In WILL form, the same fact is embedded in a Minkowski-like difference of squared projections, with no need for separate “gravitational” and “kinetic” constructs.
- This reframing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

12 Lagrangian and Hamiltonian as Ontologically Corrupted WILL Approximations

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational Energy-Symmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms are not fundamental principles but ontologically “dirty” approximations of the relational WILL framework. By collapsing the two-point relational structure into a single-point description, classical mechanics gains computational convenience at the cost of ontological clarity.

12.1 Definitions of Parameters

We consider a central mass M and a test mass m . The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2 \dot{\phi}^2$ — total squared orbital speed at B .
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B .
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A .

12.2 The Relational Lagrangian

Instead of a relational energy, we define the *clean relational Lagrangian* L_{rel} , which represents the kinetic budget at point B relative to the potential budget at point A :

$$L_{\text{rel}} = T(B) - U(A) = \frac{1}{2}m \left(\dot{r}_B^2 + r_B^2 \dot{\phi}^2 \right) - \frac{GMm}{r_A}. \quad (27)$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \frac{1}{2}(\beta_B^2 + \kappa_A^2). \quad (28)$$

This two-point, relational form is the clean geometric statement.

12.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \quad (29)$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

12.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \quad (30)$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \quad (31)$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \quad (32)$$

12.5 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \frac{1}{2}mv^2 + \frac{GMm}{r} \longleftrightarrow \frac{1}{2}mc^2(\beta^2 + \kappa^2), \quad (33)$$

$$H = \frac{1}{2}mv^2 - \frac{GMm}{r} \longleftrightarrow \frac{1}{2}mc^2(\beta^2 - \kappa^2). \quad (34)$$

Here the “+” or “−” signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both are ontologically “dirty” approximations. The clean relational law, involving distinct points A and B , is collapsed into a local, one-point description. This shows that Hamiltonian and Lagrangian are just needlessly overcomplicated approximations that lose in ontological integrity.

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

Legacy Dictionary (for conventional formalisms).

Within WILL Geometry, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2 . For readers accustomed to standard frameworks, the following translation rules may help:

1. *General Relativity (metric form)*:

$$\kappa_X \hat{=} \sqrt{-g_{tt}} \quad (\text{static spacetimes}), \quad \beta \hat{=} \frac{\|u_{\text{spatial}}^\mu\|}{u^t c}.$$

2. *Canonical mechanics (Lagrangian/Hamiltonian)*: Quantities such as $p_i = \partial L / \partial \dot{q}^i$ do not belong to the ontology of WILL Geometry. They arise only after collapsing the two-point relational law into a one-point formalism. They are computational *shadows*, useful for legacy calculations but physically redundant.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

Summary

Complex Mathematics is the Consequence of Bad Philosophy.

13 Derivation of Density, Mass, and Pressure

13.1 Geometric Foundation

From the projective analysis established in the previous sections, the fundamental invariant is

$$\kappa^2 = \frac{R_s}{r_d},$$

where κ emerges from the energy projection on the area of unit sphere S^2 , and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

13.2 Derivation of Energy Density

From Mass Scale to Volumetric Potential. Starting from the geometric relation,

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G},$$

we associate m_0 with a volumetric proxy r_d^3 , obtaining a raw volumetric potential,

$$\frac{m_0}{r_d^3} = \frac{\kappa^2 c^2}{2Gr_d^2}.$$

Applying the Geometric Distribution Principle. Because the potential projection κ is distributed over S^2 - a 2D spherical manifold, the volumetric expression must be normalized over the unit-sphere area 4π . This yields the physical energy density,

$$\rho = \frac{1}{4\pi} \left(\frac{\kappa^2 c^2}{2Gr_d^2} \right).$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2}.$$

Local Energy Density \equiv Relational Projection

Maximal Density. At $\kappa^2 = R_s/r_d = 1$, the horizon condition is reached, corresponding to the maximal observable energy density at radius $r_d = R_s$:

$$\rho_{\max} = \frac{c^2}{8\pi G r_d^2}.$$

Normalized Relation. Thus the fundamental identification is

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \Rightarrow \kappa^2 \equiv \Omega}.$$

13.3 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G}.$$

From the energy density:

$$m_0 = \alpha r_d^n \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2}$ into $m_0 = \alpha r_d^n \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r_d^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r_d^{n-2}}{8\pi} = \frac{r_d}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r_d^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

13.4 Pressure as Surface Curvature Gradient

In the WILL framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) = \frac{c^4}{8\pi G} \frac{1}{r} \frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) = -\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\rho(r) c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy–geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\rho_{\max} c^2 = -\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r .

Pressure in WILL is the intrinsic surface tension of energy–geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

14 Rotational Systems (Kerr–Newman Solutions Without Metric)

14.1 Contextual Bounds

- **For a gravitationally closed (static) system**, the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.
- **For a kinematically closed (maximally rotating) system**, the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac^2}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the relational rotation parameter, with $0 \leq \beta \leq 1$,
- κ is related to the geometry and gravity,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta_{\max}^2 r_d$$

This relationship holds when $r_d = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

14.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_{\pm} = \frac{R_s}{2} (1 \pm \beta_Y)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{\max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{\max}^2} R_s = \frac{1}{2} R_s$$

14.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2} \left(1 + \sqrt{1 - \beta^2 \cos^2 \theta} \right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

14.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c^2}\beta$, reaching its maximum for extreme black holes ($\beta = 1$).

14.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship Principle. In our model, Energy Symmetry Law enforce constraint by limiting β to the range $[0, 1]$.

14.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in a_* , mirror the orbital velocity parameter β , providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

Physical Interpretation

- **No need for pre-existing spacetime** — geometry emerges from angular energy distributions.
- **All parameters** are dimensionless and directly derived from the speed of light as finite resource.
- **Scale invariance:** The same structure applies from Planck-scale objects to galactic black holes.

15 Unified Geometric Field Equation

15.1 The Theoretical Uroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

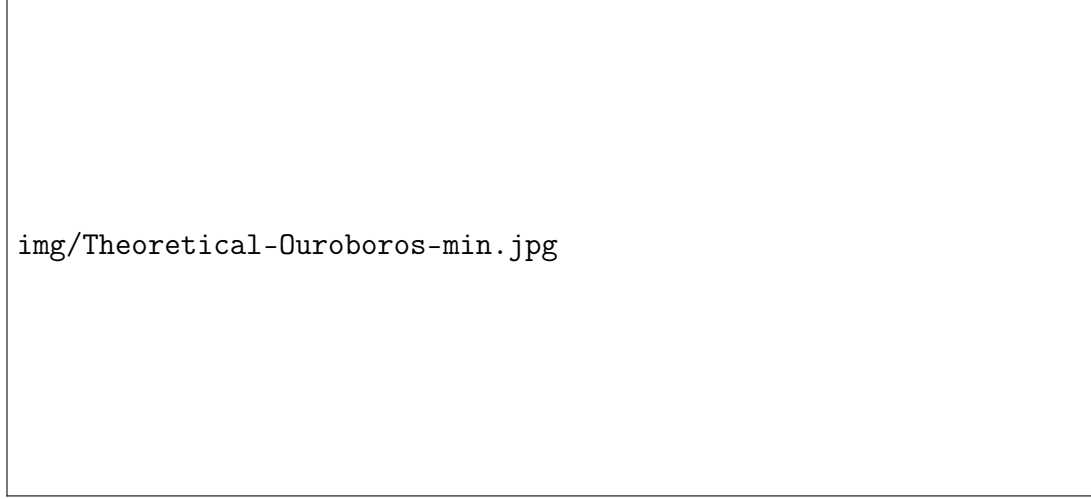
$$\boxed{\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}} \quad (35)$$

The ratio of geometric scales equals the ratio of energy densities.

This is the unified geometric field equation of WILL Geometry. It expresses the complete equivalence:

$$\text{SPACETIME GEOMETRY} \equiv \text{ENERGY DISTRIBUTION}$$

We have shown that this single foundational Principle, through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with a single foundational Principle $\text{SPACETIME} \equiv \text{ENERGY}$, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.



The foundational Principle $\text{SPACETIME} \equiv \text{ENERGY}$ closes into the unified field equation

$$\frac{R_s}{r_d} = \frac{\rho}{\rho_{\max}},$$

Theoretical Uroboros

The WILL framework exhibits perfect logical closure: the fundamental Principle about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework — the “Theoretical Uroboros”. But regardless of aesthetic beauty of this result let’s remain skeptical.

15.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r_d} = \frac{8\pi G}{c^2} r_d^2 \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

16 Beyond Differential Formalism: Structural Dynamics

16.1 Intrinsic Dynamics via Energy Redistribution

The system is not described by differential equations of motion evolving *in* time. Instead, its transformation is dictated by a closed network of algebraic relations that enforce a perpetually balanced configuration. Any change in one parameter necessitates a coordinated shift in all others to maintain geometric self-consistency. What we perceive as "dynamics" is this ordered succession of balanced states. The framework thus inverts the classical paradigm:

The foundation of WILL Geometry lies in the principle that spacetime geometry is fully determined by the distribution of energy, parameterized by the dimensionless quantities β and κ . Any change in the system's energy—due to mass variation, motion, or redistribution—directly reshapes the geometry. This intrinsic linkage ensures that dynamics is embedded within the model itself, without requiring external equations of motion.

This reveals a fundamental inversion of the classical paradigm:

Time does not drive change — instead, change defines time.

Why There Are No Equations of Motion

In classical and relativistic physics, dynamics is formulated through differential equations. These express how physical quantities evolve continuously through time, typically governed by:

- A temporal parameter t ,
- A Lagrangian function L ,
- A variational principle: $\delta S = 0$, where $S = \int L dt$,
- Euler–Lagrange equations that yield the system's path.

This framework assumes:

1. A continuum of possible configurations,
2. That Nature selects one by minimizing action,
3. That time flows independently of the system.

Why This Framework Does Not Apply to WILL Geometry

WILL Geometry begins from a fundamentally different premise.

- There is no "space" of possible paths.
- There is no "freedom to vary."
- The system does not evolve through time — it **defines time** through its structure.

In this model:

- Each observable is locked in a network of algebraic relations.
- Any change in one parameter *necessitates* coherent changes in the others.
- Self-consistency enforces projectional **balance**.

There is only one valid configuration at any moment: the one where all projectional constraints are satisfied. Everything else is not forbidden — it is undefined.

Geometric Principle of Action

In WILL Geometry, there is no equation of motion. There is no Lagrangian. There is no variational calculus.

There is only a closed system of geometric and energetic relationships, and the sequence of valid configurations is what we call *dynamics*.

The only necessary input:

The observable sequence of transformations in the system's energy geometry is the only necessary input for describing its evolution. .

16.2 Time as an Emergent Property

In this framework, time is not a fundamental entity but a derived concept tied to changes in the system's geometry. Similar to time dilation in special relativity, time intervals here are defined by the transformations of geometric parameters like r_d and κ . This eliminates the need for an external clock.

A natural time scale arises from the geometry as $t_d = \frac{r_d}{c}$. For instance, during mass accretion onto a black hole, r_d adjusts as the mass changes, and t_d evolves accordingly. This intrinsic time scale encapsulates the system's dynamics without invoking an independent time variable.

Note for readers accustomed to classical dynamics

Unlike traditional formulations of dynamics based on an external time parameter t , the WILL Geometry framework describes evolution as an intrinsic transformation of the system's geometric structure. Time is not a fundamental variable but an emergent quantity derived from energy redistribution. All physical change is encoded in the interdependence of parameters β , κ , and ρ , etc without requiring differential equations or initial conditions.

This marks a fundamental shift: dynamics here is not a process unfolding "in time," but a change in relational energy geometry itself. The model does not track transformations through an imposed temporal axis — instead, it reveals that what we perceive as temporal progression is a manifestation of continuous geometric reconfiguration. **Thus, prediction becomes a question of geometric continuity, not temporal evolution.**

From Structure to Motion

In the next section, we present the core structural closure of the system — a set of algebraic invariants that together form the backbone of all observed dynamics. These relations are not definitions. They are the **complete geometry of change**, seen from within.

Dynamics in WILL Geometry

Dynamics in WILL Geometry is not described by differential equations but by the ordered succession of globally balanced, algebraically determined configurations.

16.3 Algebraic Closure and Structural Causality

In WILL Geometry, physical dynamics emerges from a set of algebraically closed invariants. Each parameter participates in a self-consistent configuration of relational constraints. There are no functions, no dependent variables, and no variation over time — only balanced configurations.

The following set of relations expresses the minimal algebraic closure of the WILL structure:

$$\left\{ \begin{array}{l} \kappa^2 = 2\beta^2 \\ R_s = \frac{2Gm_0}{c^2} \\ r_d \cdot \kappa^2 = R_s \\ \rho = \frac{\kappa^2 c^2}{8\pi G r_d^2} \\ H = \frac{c}{r_d} \\ \Lambda = \frac{\kappa^2}{r_d^2} \\ m_0 = 4\pi r_d^3 \cdot \rho \end{array} \right.$$

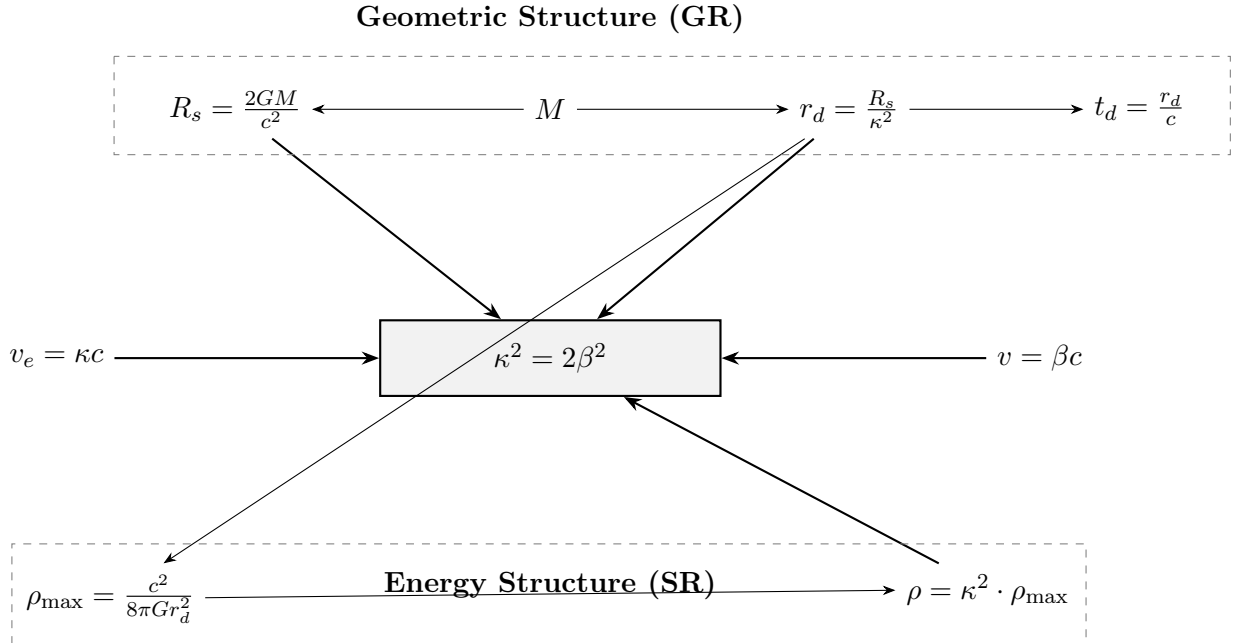
These are not definitions. They are mutual constraints — an algebraic simultaneity.

Changing any one parameter necessitates a coordinated shift in all others to maintain validity.

Causal Closure without Circularity

The structure of WILL Geometry is causally closed but not circular. Each parameter is either independently observable or computable from a minimal input pair consisting of one dynamic projection (such as κ or β) and one scale quantity (such as r , M , or ρ).

The system avoids circularity by ensuring that no parameter both defines and is defined by the same input. Instead, values propagate through directed dependencies rooted in physically measurable quantities. Multiple valid entry points exist, but all reduce to consistent, non-redundant relationships governed by the fundamental geometric field equation.



The result is a structure where **causality is internal**, **coherence is enforced**, and **dynamics is simply the shifting of balanced configurations** — not the unfolding of arbitrary functions over time.

16.4 Numerical Example: Accretion onto a Black Hole

Consider a black hole accreting mass from a surrounding disk to illustrate the model's intrinsic dynamics. Let the initial mass be $m_0 = 10, M_\odot$, with a Schwarzschild radius $R_s = \frac{2Gm_0}{c^2} \approx 2.95 \times 10^4, \text{m}$. Suppose $\kappa = 0.1$, so $r_d = \frac{R_s}{\kappa^2} = \frac{2.95 \times 10^4}{0.01} = 2.95 \times 10^6, \text{m}$, and the associated time scale is $t_d = \frac{r_d}{c} \approx 9.83 \times 10^{-3}, \text{s}$.

As the black hole accretes mass, increasing to $m_1 = 10.1, M_\odot$, the Schwarzschild radius becomes $R_s \approx 2.98 \times 10^4, \text{m}$. Assuming κ remains constant for simplicity, $r_d = \frac{2.98 \times 10^4}{0.01} = 2.98 \times 10^6, \text{m}$, and $t_d \approx 9.93 \times 10^{-3}, \text{s}$. This increase in t_d reflects the system's evolution, driven solely by the changing geometry.

No differential equations are required:

Dynamics unfolds as a consequence of relational energy transformations.

16.5 General Principle of Dynamics

The overarching principle in WILL Geometry is that any physical change—be it mass accretion, gravitational collapse, or expansion—manifests as a transformation in the geometric structure. The parameters β , κ , and r_d adjust self-consistently, ensuring that the system's evolution is fully described within the model. This approach eliminates the need for differential equations of motion or external initial conditions, as the geometry at any given "moment" is determined by the current energy configuration.

Thus, the temporal sequence we observe is simply the ordered unfolding of geometric transitions:

$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r_d \dots)}$$

This leads to a unified, self-consistent model where ****geometry generates both transformations and observability****. There is no external timeline — only evolving curvature.

16.6 Conclusion

In conclusion, dynamics in WILL Geometry emerges naturally from the redistribution of energy within the system's geometric framework. Time arises as a consequence of these geometric changes, providing a unified description of spacetime and energy transformations. This intrinsic approach simplifies the treatment of dynamical processes and offers a novel perspective on the nature of physical systems.

Geometric Principle of Evolution

Physics is not the evolution of a system through time,
but the geometric transformation of the energy landscape,
where "*time*" is simply the name we give to the sequence of such transitions.

17 The Fundamental Invariant $W_{\text{ill}} = 1$

From the geometric closure of WILL framework, we derive a universal dimensionless invariant:

$$W_{\text{ill}} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = 1 \quad (36)$$

Proof: Substituting the geometric definitions:

$$W_{\text{ill}} = \frac{\frac{1}{\cos \theta_2} m_0 c^2 \cos \theta_2 \frac{r_d^2}{c^2}}{\frac{1}{\sin \theta_1} m_0 \sin \theta_1 r_d^2} = \frac{m_0 c^2 r_d^2}{c^2} \cdot \frac{\sin \theta_1}{m_0 r_d^2} \cdot \frac{1}{\sin \theta_1} = 1 \quad (37)$$

This invariant holds universally for all values of m_0 , G , c , and κ . Unlike dimensional analysis, this identity emerges from the projectional interdependence of energy-mass (E, M) and spacetime metrics (T, L) within the unified structure.

The invariant $W_{\text{ill}} = 1$ expresses geometric unity through energetic projection

17.1 The Name "WILL"

The name WILL reflects both the harmonious unity of the equation and a subtle irony towards the anthropic principle, which often intertwines human existence with the causality of the universe. The equation stands as a testament to the universal laws of physics, transcending any anthropocentric framework.

WILL

It is not the unit of something—it is the unity of everything.

$$\text{SPACETIME} \equiv \text{ENERGY}$$

All the derived relations for local energy density, pressure, enclosed mass, and horizon formation are purely logical and mathematical consequences of this unique guiding principle. In this sense, the WILL Geometry framework achieves absolute epistemological cleanliness: it does not introduce any hidden assumptions or coordinate structures. The entire gravitational and relativistic sector (as reconstructed within the WILL Geometry model, from the single foundational Principle without any external assumptions or coordinate backgrounds) — including precise predictions about the onset of horizon formation, radial pressure gradients, and the resolution of classical singularity issues—is reconstructed from this single idea.

Meaning

Energy is not “inside” space — it defines space through projection.

- **Surface-scaled closure (vs. volume filling).** Mass follows the algebraic closure $m_0 = 4\pi r_d^3 \rho$ with $\rho = \kappa^2 c^2 / (8\pi G r_d^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Algebraic core (vs. differential ansatz).** The core invariants are algebraic and coordinate-free. Differential relations appear only as auxiliary scaffolding (e.g. to express curvature gradients as $P = -\rho c^2$), not as fundamental inputs.
- **Natural bounds.** The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\text{max}}$ and $|P| \leq |P_{\text{max}}| = c^4 / (8\pi G r_d^2)$, avoiding singularities without extra hypotheses.

18 Correspondence with General Relativity

18.1 Equivalence with Schwarzschild Solution

Theorem 18.1 (Equivalence with Schwarzschild Solution). *The WILL Geometry formalism reproduces the Schwarzschild metric in the appropriate limit.*

Proof. The Schwarzschild metric in General Relativity is given by:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \quad (38)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit sphere.

In WILL Geometry, the key parameters are:

$$\kappa^2 = \frac{R_s}{r} = \frac{2GM}{rc^2} \quad (39)$$

$$\kappa_X = \sqrt{1 - \kappa^2} = \sqrt{1 - \frac{2GM}{rc^2}} \quad (40)$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \quad (41)$$

The time component of the Schwarzschild metric can be written as:

$$g_{tt} = \left(1 - \frac{2GM}{rc^2}\right) = 1 - \kappa^2 = \kappa_X^2 \quad (42)$$

And the radial component can be written as:

$$g_{rr} = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} = -\frac{1}{1 - \kappa^2} = -\frac{1}{\kappa_X^2} \quad (43)$$

Therefore, in WILL Geometry terms, the Schwarzschild metric takes the form:

$$ds^2 = \kappa_X^2 c^2 dt^2 - \frac{1}{\kappa_X^2} dr^2 - r^2 d\Omega^2 \quad (44)$$

This demonstrates that the WILL Geometry parameters exactly reproduce the Schwarzschild metric. \square

18.2 Equivalence with Einstein Field Equations

Theorem 18.2 (Equivalence with Einstein Field Equations). *The geometric field equation of WILL Geometry is equivalent to the corresponding component of Einstein's field equations for a static, spherically symmetric mass distribution.*

Proof. The standard form for the tt -component of Einstein's field equations inside a spherically symmetric perfect fluid is given by one of the Tolman–Oppenheimer–Volkoff (TOV) equations:

$$\frac{1}{r^2} \frac{d}{dr} \left(r \left(1 - \frac{1}{g_{rr}} \right) \right) = \frac{8\pi G}{c^2} \rho(r) \quad (45)$$

where $\rho(r)$ is the energy density at radius r .

The key to establishing equivalence lies in defining the correct correspondence for the WILL parameter κ^2 **within the matter distribution**. For the exterior vacuum solution, $\kappa^2 = R_s/r$. However, for the interior solution, the metric component g_{rr} is related to the mass enclosed within radius r , denoted $m(r)$:

$$1 - \frac{1}{g_{rr}} = \frac{2Gm(r)}{rc^2} \quad (46)$$

To bridge the two formalisms, we must define the interior WILL parameter $\kappa^2(r)$ as this exact quantity. This definition ensures a smooth transition to the exterior form at the object's surface (where $m(r) \rightarrow M_{total}$):

$$\kappa^2(r) \equiv \frac{2Gm(r)}{rc^2} \quad (47)$$

With this definition, the term within the derivative in the EFE becomes a direct substitution for the WILL parameter:

$$r \left(1 - \frac{1}{g_{rr}} \right) = r \left(\frac{2Gm(r)}{rc^2} \right) = r\kappa^2(r) \quad (48)$$

Substituting this directly into the EFE from the first step yields:

$$\frac{1}{r^2} \frac{d}{dr} (r\kappa^2(r)) = \frac{8\pi G}{c^2} \rho(r) \quad (49)$$

Finally, multiplying both sides by r^2 arrives at the WILL Geometry field equation:

$$\frac{d}{dr} (r\kappa^2) = \frac{8\pi G}{c^2} r^2 \rho(r) \quad (50)$$

This demonstrates the exact equivalence between the two field equations under the appropriate definition for the interior gravitational parameter, thereby completing the proof. \square

Comparison Table: General Relativity (GR) vs WILL Framework

#	Category	General Relativity (GR)	WILL Framework
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}, R$; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r), \rho_{\max}(r)$, and $p(r)$
4	Geometry–Matter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \rightarrow \infty, g_{00} \rightarrow 0$	Excluded by construction: $\rho \leq \rho_{\max}, \kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$
8	Concept of Time	Coordinate-based, embedded in g_{00} ; system-dependent	Physical: β as projection of energy onto temporal axis
9	Dynamics	Via time derivatives and Lagrangians	Via change in energy proportions; no differential equations
10	Formalism	Geometry, tensors, 2nd-order derivatives	Energy projections, circular geometry, algebraic closure
11	Intuitiveness	Low; relies on abstract and heavy formalism	High; built from observable and intrinsic relations
12	Observational Fit	Confirmed (with dark matter/energy assumptions)	Consistent; explains phenomena without "dark entities" (Details in WILL PART 2)

18.3 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the WILL framework's principles. One can derive GR by adding these additional structures to WILL's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, with WILL being presented as the more fundamental underlying structure.

Phenomenon	Radius r	κ^2	β^2	Comment
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{2}{3}$	$\frac{1}{3}$	Null circular orbits, $Q = 1$, $Q_t = 0$
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{3}$	$\frac{1}{6}$	Marginal stability of timelike orbits
Static horizon (Schwarzschild)	$r = R_s$	1	$\frac{1}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	2	1	Maximal rotation, $\beta = 1$, merged horizons

Table 3: Critical radii and their projectional parameters in WILL Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.

19 Relational Foundation Theorem: WILL vs. GR

Definition 19.1 (GR Core Axioms). *General Relativity (GR) is assumed to rest on the following axioms:*

- (A1) *The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.*
- (A2) *Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.*
- (A3) *Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.*
- (A4) *Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$.*

Definition 19.2 (WILL Foundational Principle). *The WILL framework is based on a single Principle:*

- (W1) **Relational Principle:** *All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy.*

Lemma 19.3 (Relationality in GR). *From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.*

Remark 19.4 (Bridge: From Relational Principle to GR Axioms). *If the Relational Principle (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:*

- *It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.*
- *It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.*

Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 19.5 (Asymmetric Falsifiability of GR and WILL). *Let GR denote the theory defined by axioms (A1)–(A4), and let WILL denote the theory defined by Principle (W1). Then:*

Phenomenon	Standard GR Result	Projectional Geometry (β, κ)
GPS time shift / gravitational red-shift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $E_{\text{loc}} = \frac{E_0}{\sqrt{(1 - \beta^2)(1 - \kappa^2)}},$ verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple projectional relations ($\kappa^2 = 2\beta^2$, $Q^2 = \kappa^2 + \beta^2 = 1$).
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Exact same number obtained from projection geometry with β, κ .
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs.
Cosmological red-shift	Photon “loses energy” as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART 2)
Cosmological constant Λ	Added by hand to fit data (“dark energy”).	Arises naturally as $\Lambda = \kappa^2/r_d^2$. No extra entities required. (Details in WILL PART 2)
Singularities	Predicted in black holes and big bang ($\rho \rightarrow \infty$).	Forbidden: density bounded by $\rho_{\text{max}} = c^2/(8\pi G r^2)$.
Local gravitational energy	“Cannot be localized” (only ADM/Bondi at infinity).	Directly measurable via κ , e.g. from light deflection angle.
Unification with QM and SR	No natural unification in GR framework.	Same projectional law applies from microscopic (QM) to cosmic (GR, COSMO) scales. (Details in WILL PART 3)

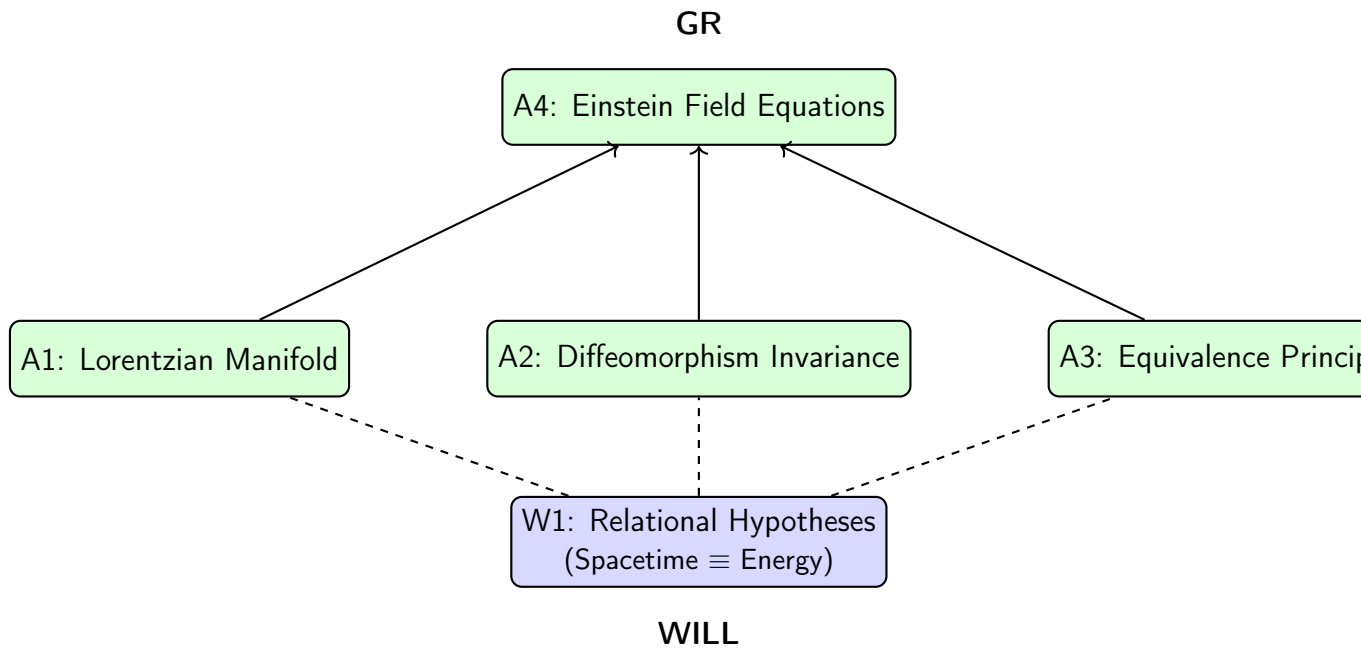
Table 4: Classical GR results vs. Projectional Geometry outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark energy.

1. If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence, GR is necessarily falsified.
2. If any of (A1)–(A3) are empirically falsified, GR collapses, but (W1) may still remain valid as a stand-alone principle.

Therefore, there exist possible empirical scenarios in which GR fails while WILL survives, but there exist no scenarios in which WILL fails while GR survives.

Corollary 19.6. *WILL is axiomatically more fundamental than GR: its sole Principle (W1) is logically included within the core axioms of GR, while GR requires additional*

ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of WILL.



Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability).

WILL rests on the single relational Principle (W1). Core GR assumes additional structures (A1–A4). Hence:

- If **W1** is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of **A1–A3** is falsified, GR collapses, while **W1** (and thus *WILL*) may still hold.

Therefore, there are scenarios where GR fails and *WILL* survives, but none where *WILL* fails while GR survives.

Status of General Relativity within WILL

It is important to emphasize that the WILL framework does not *invalidate* the achievements of General Relativity. Rather, it *explains them*. All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in WILL Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.

Thus, GR is not a rival but a **specialized, parameter-heavy realization** of WILL's more general relational principle. In logical terms:

- WILL can stand without GR, but GR cannot stand without the relational Principle (W1).
- The empirical successes of GR are preserved within WILL, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

Therefore, GR should be understood as an **effective approximation** embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.

20 Empirical Validation

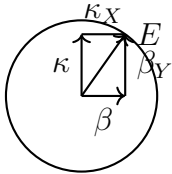
20.1 Geometric Prediction of Photon Sphere and ISCO

Theorem 20.1 (Critical Radii Emergence). *In the WILL Geometry framework, the critical orbital radii of the photon sphere and innermost stable circular orbit (ISCO) emerge naturally from the geometric equilibrium where $\theta_1 = \theta_2$. With logical chain: $\kappa^2 = \frac{R_s}{r}$, $\kappa^2 = 2\beta^2$, $\kappa^2 + \beta^2 = 1 \Rightarrow r = \frac{R_s}{\kappa^2} \in \{1.5R_s, 3R_s\}$*

Proof. A notable geometric equilibrium occurs at the critical angle

$$\theta_1 = \theta_2 = 54.7356103172^\circ \text{ (balance point for photon sphere and ISCO)} \quad (51)$$

or approximately $\theta_1 = \theta_2 \approx 0.9553$ radians.



This equilibrium yields the fundamental relation:

$$\kappa^2 + \beta^2 = 1, \quad (52)$$

These critical radii emerge spontaneously from the geometry, suggesting inherent space-time structure without additional assumptions.

20.1.1 Mathematical Derivation of Critical Points

Key critical points include: When:

- $\kappa = \sqrt{\frac{2}{3}} \approx 0.816$ and $\beta = \sqrt{\frac{1}{3}} \approx 0.577$, corresponding to:

$$r = \frac{R_s}{\kappa^2} = \frac{3}{2}R_s = 1.5R_s \quad (\text{radius of the photon sphere}).$$

When:

- $\kappa = \sqrt{\frac{1}{3}} \approx 0.577$, and $\beta = \sqrt{\frac{1}{6}} \approx 0.408$, leading to orbital distance:

$$r = \frac{R_s}{2\beta^2} = \frac{R_s}{2 \cdot \frac{1}{6}} = 3R_s \quad (\text{radius of the innermost stable circular orbit, ISCO}).$$

At the critical point where $\beta = \frac{1}{\sqrt{3}}$ and $\kappa = \sqrt{\frac{2}{3}}$, the following relationships hold:

$$\theta_1 = \theta_2 \tag{53}$$

$$\beta = \kappa_X \tag{54}$$

$$\kappa = \beta_Y \tag{55}$$

$$\cos(\theta_2 - \theta_1) = 0 \tag{56}$$

$$Q = \sqrt{\kappa^2 + \beta^2} = 1 \tag{57}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - 3\beta^2} = 0 \quad (\text{Instability threshold}) \tag{58}$$

Critical Radii from the Q -Invariant

The combined projection parameter

$$Q^2 = \kappa^2 + \beta^2, \quad Q_t = \sqrt{1 - Q^2},$$

measures the total causal budget of energy transformations. Critical orbital radii correspond to special conditions of Q and Q_t : **Photon Sphere** ($r = 1.5R_s$). At this radius the causal budget is fully exhausted:

$$Q = 1, \quad Q_t = 0,$$

leaving no timelike separation. Only null (photon) orbits are permitted. Thus, both the photon sphere and ISCO emerge directly as critical solutions of the same invariant structure (Q, Q_t) , without additional assumptions.

Interpretive Note

While the radii $1.5R_s$ (photon sphere) and $3R_s$ (ISCO) are known from classical General Relativity, their spontaneous emergence from angle equality $\theta_1 = \theta_2$ in our geometric framework is not imposed but arises from internal energy projection symmetries. This correspondence reinforces the internal consistency and explanatory power of WILL Geometry.

Projectional Principle

Geometry defines causality before mass, and curvature before gravity.

□

20.2 Empirical Check: Circular Earth Orbits (SR×GR Factorization and L/H Collapse)

Setup and Definitions. We test two identities of the WILL framework on well-measured circular orbits around Earth:

1. the closure diagnostic $\kappa^2 = 2\beta^2$ for circular motion;
2. the collapsed (legacy) energy forms

$$\frac{L}{E_0} = \frac{1}{2}(\beta^2 + \kappa^2), \quad \frac{H}{E_0} = \frac{1}{2}(\beta^2 - \kappa^2),$$

where $E_0 \equiv mc^2$ is the rest energy of the test mass m .

All quantities are dimensionless when divided by E_0 . We use standard (legacy) constants and Earth parameters in SI units:

$$\mu_{\oplus} \equiv GM_{\oplus} = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}, \quad c = 299,792,458 \text{ m s}^{-1}, \quad R_{\oplus} = 6,371,000 \text{ m}.$$

Theorem 20.2. Projection parameters. For a circular orbit of radius r with orbital speed v ,

$$\beta^2 \equiv \frac{v^2}{c^2}$$

,

$$\kappa^2 \equiv \frac{2GM_{\oplus}}{rc^2} = \frac{R_s}{r}$$

,

$$R_s \equiv \frac{2GM_{\oplus}}{c^2}$$

.

For circular motion the empirical relation $v^2 = \mu_{\oplus}/r$ holds to high accuracy, hence

$$\beta^2 = \frac{\mu_{\oplus}}{rc^2} \implies \boxed{\kappa^2 = 2\beta^2},$$

i.e. the closure condition is an exact analytic identity for ideal circular orbits.

Legacy energies from projection budgets. WILL assigns quadratic budgets

$$\frac{T}{E_0} = \frac{1}{2}\beta^2, \quad \frac{U}{E_0} = -\frac{1}{2}\kappa^2,$$

so that the legacy Lagrangian and Hamiltonian (after the one-point “ontological collapse”) read

$$\boxed{\frac{L}{E_0} = \frac{1}{2}(\beta^2 + \kappa^2)}, \quad \boxed{\frac{H}{E_0} = \frac{1}{2}(\beta^2 - \kappa^2)}.$$

These are direct rewritings of the relational budgets in terms of β, κ .

Proof. Numerical Evaluation (SI units)

We now evaluate the above identities for two standard circular orbits. Numerical differences from zero in the $\kappa^2 - 2\beta^2$ check reflect only rounding; the analytic identity guarantees exact cancellation.

(A) LEO at ~ 400 km altitude.

$$r = R_{\oplus} + 400,000 \text{ m} = 6,771,000 \text{ m}, \quad v = \sqrt{\mu_{\oplus}/r} \approx 7,672.60 \text{ m s}^{-1}.$$

Hence

$$\begin{aligned} \beta^2 &= \frac{v^2}{c^2} \approx 6.5500340 \times 10^{-10}, & \kappa^2 &= \frac{2\mu_{\oplus}}{rc^2} \approx 1.3100068 \times 10^{-9}, \\ \kappa^2 - 2\beta^2 &\approx -2.1 \times 10^{-25} \quad (\approx 0), \\ \frac{L}{E_0} &= \frac{1}{2}(\beta^2 + \kappa^2) \approx 9.8250510 \times 10^{-10}, & \frac{H}{E_0} &= \frac{1}{2}(\beta^2 - \kappa^2) \approx -3.2750170 \times 10^{-10}. \end{aligned}$$

(B) GPS orbit at $\sim 20,200$ km altitude.

$$r = R_{\oplus} + 20,200,000 \text{ m} = 26,571,000 \text{ m}, \quad v = \sqrt{\mu_{\oplus}/r} \approx 3,873.16 \text{ m s}^{-1}.$$

Hence

$$\begin{aligned} \beta^2 &\approx 1.6691235 \times 10^{-10}, & \kappa^2 &\approx 3.3382470 \times 10^{-10}, \\ \kappa^2 - 2\beta^2 &\approx 0 \quad (\text{within rounding}), \\ \frac{L}{E_0} &\approx 2.5036852 \times 10^{-10}, & \frac{H}{E_0} &\approx -8.3456175 \times 10^{-11}. \end{aligned}$$

Conclusion. Both circular-orbit cases confirm:

$$\boxed{\kappa^2 = 2\beta^2}, \quad \boxed{\frac{L}{E_0} = \frac{1}{2}(\beta^2 + \kappa^2), \quad \frac{H}{E_0} = \frac{1}{2}(\beta^2 - \kappa^2)}.$$

In the WILL reading, the subsystem is energetically closed, and the “legacy” L, H are just ontologically corrupted approximations of the underlying Energy Symmetry law. For non-closed subsystems (e.g., radiating binaries), $\kappa^2 - 2\beta^2 \neq 0$ until all energy channels (such as gravitational radiation) are included.

Key Message

The Lagrangian and Hamiltonian are not fundamental principles. They are degenerate shadows of a deeper relational Energy Symmetry law. Classical mechanics, Special Relativity, and General Relativity all operate within this corrupted approximation. WILL restores the underlying two-point relational clarity.

□

20.3 The Nature of Light (Gravitational Lensing)

Theorem 20.3. *Lets tests the model’s predictions for massless particles. The deflection of starlight by the Sun was a landmark confirmation of GR, which predicted a bending angle twice that expected from a naive Newtonian corpuscular theory. Our model’s prediction for the deflection angle α is:*

$$\alpha = 2\kappa^2 = \frac{4Gm_0}{rc^2} \tag{59}$$

Proof. The crucial factor of 2 is not an ad-hoc addition but a profound consequence of the model's structure. As established by the Energy-Symmetry Law, any interaction between massive bodies involves two reciprocal frames of reference, and the energy budget is partitioned equally between them (hence the factor of '1/2' in energy terms).

Light, however, is unique. It travels at a speed, c , that is absolute for all observers. There is no valid reciprocal frame of reference "at rest" on a photon. Because there is no second frame to act as a counterbalance, the interaction is not partitioned. The photon experiences the **full, unpartitioned** geometric budget of the interaction, which is exactly double the partitioned, energetic budget experienced by a massive particle. The factor of 2 is thus a direct signature of the absolute and non-reciprocal nature of light's frame of reference. The third trial is passed.

Light Deflection in WILL (metric-free)

For massive probes the effective potential per unit mass is $\Phi_{\text{mass}} = \frac{1}{2}\kappa^2 c^2$ by Energy Symmetry. Photons have no reciprocal frame; hence $\Phi_\gamma = \kappa^2 c^2$. In the small-angle/impulse regime

$$\alpha = -\frac{1}{v^2} \int_{-\infty}^{\infty} \partial_{\perp} \Phi \, dz,$$

and for light ($v = c$) with $\kappa^2(r) = 2GM/(c^2 r)$ where $r = b$ one finds

$$\alpha = \frac{1}{c^2} \int c^2 \partial_{\perp} \kappa^2 \, dz = \frac{2GM}{c^2} \int_{-\infty}^{\infty} \left(-\frac{b}{(b^2 + z^2)^{3/2}} \right) dz = \frac{4GM}{bc^2} = 2\kappa^2.$$

Thus the GR value is recovered without invoking metric or geodesics. □

20.4 Energy Symmetry Validation: GPS Satellite and Earth

Theorem 20.4 (Real-World Energy Symmetry). *The Energy Symmetry Law holds precisely for the Earth-GPS satellite system. WILL-invariant ($W_{il} = 1$) holds exactly for the Earth-GPS satellite system.*

Proof. We verify the Energy Symmetry Law on real orbital data for a GPS satellite and an observer on the Earth's surface, using the following parameters:

Input Data

- Gravitational constant: $G = 6.67430 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$
- Speed of light: $c = 2.99792458 \times 10^8 \text{ m/s}$
- Mass of Earth: $M_{\text{Earth}} = 5.972 \times 10^{24} \text{ kg}$
- Radius of Earth: $R_{\text{Earth}} = 6.370 \times 10^6 \text{ m}$
- Radius of GPS orbit: $r_{\text{GPS}} = 2.6571 \times 10^7 \text{ m}$
- Seconds in 24 hours $D_{\text{ayS}} = 86400 \text{ s}$
- Microseconds in 1 second $M_{\text{icro}} = 10^6 \text{ } \mu \text{ s}$

The orbital velocity of the GPS satellite is:

$$v_{GPS} = \sqrt{\frac{GM_{Earth}}{r_{GPS}}} = \sqrt{\frac{6.67430 \times 10^{-11} \times 5.972 \times 10^{24}}{2.6571 \times 10^7}} = 3873.10090455 \text{ m/s} \quad (60)$$

Converting to dimensionless parameters:

$$\beta_{GPS} = \frac{v_{GPS}}{c} = \frac{3873.10090455}{2.99792458 \times 10^8} = 0.0000129192739884 \quad (61)$$

$$\kappa_{GPS} = \sqrt{\frac{2GM_{Earth}}{c^2 r_{GPS}}} = 0.0000182706124904 \quad (62)$$

$$\frac{\kappa_{GPS}^2}{\beta_{GPS}^2} = \frac{3.3381528077 \times 10^{-10}}{1.6690764039 \times 10^{-10}} = 2 \quad (63)$$

$$Q_{GPS} = \sqrt{\beta_{GPS}^2 + \kappa_{GPS}^2} = 0.0000223768389448 \quad (64)$$

$$Q_{tGPS} = \sqrt{1 - Q_{GPS}^2} = \sqrt{1 - 3\beta_{GPS}^2} = 0.999999999975 \quad (65)$$

For the Earth's surface:

$$\kappa_{Earth} = \sqrt{\frac{2GM_{Earth}}{c^2 R_{Earth}}} = 0.000037312405944 \quad (66)$$

$$\beta_{Earth} = 0 \text{ (at rest)} \quad (67)$$

$$Q_{Earth} = \sqrt{\beta_{Earth}^2 + \kappa_{Earth}^2} = 0.000037312405944 \quad (68)$$

$$Q_{tEarth} = \sqrt{1 - Q_{Earth}^2} = 0.9999999999304 \quad (69)$$

The daily relativistic time offset between GPS and Earth is:

$$\Delta Q_{t,GPS \rightarrow Earth} = \left(1 - \frac{Q_{tEarth}}{Q_{tGPS}}\right) \cdot D_{ayS} \cdot M_{icro} = 38.52 \text{ } \mu\text{s/day}$$

This **matches the empirical time correction required** for GPS synchronization to high precision.

20.4.1 WILL invariant validation:

W_{ILLGPS} parameters of mass energy time and space:

$$M_{GPS} = \frac{\beta_{GPS}^2}{\beta_{YGPS}} c^2 \frac{r_{GPS}}{G}$$

$$E_{GPS} = \frac{\kappa_{GPS}^2}{\kappa_{XGPS}} \frac{c^4 r_{GPS}}{2G}$$

$$T_{GPS} = \kappa_{XGPS} \left(\frac{2GM_{Earth}}{\kappa_{GPS}^2 c^3} \right)^2$$

$$L_{GPS} = \beta_{YGPS} \left(\frac{GM_{Earth}}{\beta_{GPS}^2 c^2} \right)^2$$

Where:

$$\beta_{YGPS} = \sqrt{1 - \beta_{GPS}^2}$$

$$\kappa_{XGPS} = \sqrt{1 - \kappa_{GPS}^2}$$

The explicit $W_{ILL} = ET^2/ML^2 = 1$ - invariant for the GPS–Earth system, including both GR and SR effects, is:

$$W_{ILLGPS} = \frac{E_{GPS} \cdot T_{GPS}}{M_{GPS} \cdot L_{GPS}} = 1$$

All physical quantities cancel identically, leaving $\kappa_{XGPS}/\kappa_{XGPS} = \beta_{YGPS}/\beta_{YGPS}$, which is satisfied by geometric construction.

20.4.2 Energy Symmetry Law validation:

The energy difference from Earth observer to GPS satellite is:

$$\Delta E_{Earth \rightarrow GPS} = \frac{1}{2}((\kappa_{Earth}^2 - \kappa_{GPS}^2) + \beta_{GPS}^2) = \frac{1}{2}(\kappa_{Earth}^2 - \beta_{GPS}^2) = 6.1265399845 \times 10^{-10} \quad (70)$$

The energy difference from GPS satellite to Earth is:

$$\Delta E_{GPS \rightarrow Earth} = \frac{1}{2}((\kappa_{GPS}^2 - \kappa_{Earth}^2) - \beta_{GPS}^2) = \frac{1}{2}(\beta_{GPS}^2 - \kappa_{Earth}^2) = -6.1265399845 \times 10^{-10} \quad (71)$$

Therefore:

$$\Delta E_{GPS \rightarrow Earth} + \Delta E_{Earth \rightarrow GPS} = -6.1265399845 \times 10^{-10} + 6.1265399845 \times 10^{-10} = 0 \quad (72)$$

This **confirms the Energy Symmetry Law** to machine precision using real-world orbital and physical data.

And lets also compare the results with real total energy. We will take aproximate mass of GPS satellite:

$$m_{sat} = 600 \text{ kg}$$

$$\text{Classical potential energy } E_{pGPS} = \left(-\frac{GM_{Earth}m_{sat}}{r_{GPS}}\right) - \left(-\frac{GM_{Earth}m_{sat}}{R_{Earth}}\right)$$

$$\text{Classical kinetic energy } E_{kGPS} = \frac{1}{2}m_{sat} v_{GPS}^2$$

$$\text{Classical total energy } E_{tot} = E_{pGPS} + E_{kGPS} = 3.3043450143 \times 10^{10} \text{ kg}(m^2/s^2)$$

Conformation of nontriviality of Energy Symmetry Law.

$$\frac{\frac{E_{tot}}{m_{sat}c^2}}{\Delta E_{Earth \rightarrow GPS}} = 1$$

Remarkably, satellite mass m_{sat} does not appear anywhere in the computation of the geometric energy $\Delta E_{Earth \rightarrow GPS}$. Yet, when the total physical energy E_{tot} (the sum of kinetic and potential energies, which does depend on m_{sat} is normalized by the satellite's

rest energy $E_{0GPS} = m_{sat}c^2$ the ratio exactly matches the unitless geometric value as shown above. This confirms that $\Delta E_{Earth \rightarrow GPS}$ precisely encodes the relationship between the system's real energy and the satellite's own rest energy, entirely independent of its absolute mass. In other words, the geometric projection captures the physical “share” of energy—purely as a relational quantity—regardless of the object's individual mass.

Physical Logic:

- All gravitational and velocity (SR+GR) effects are simple projections, not metric-dependent.
- No tensors, no differentials, no explicit metric.
- The universe's “time flow” at each location is just a geometric combination of energy projections.

“Time does not drive change — instead, change defines time.”

□

20.5 Relativistic Precession Validation: Mercury and the Sun

Theorem 20.5 (Relativistic Precession Calculation via WILL Geometry). *The relativistic precession of Mercury's orbit matches the classical GR result with high precision, using WILL Geometry projection parameters.*

Proof. We verify the precession of Mercury's orbit using WILL Geometry and compare it to the GR prediction.

Input physical parameters:

- Gravitational constant: $G = 6.67430 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$
- Speed of light: $c = 2.99792458 \times 10^8 \text{ m/s}$
- Mass of the Sun: $M_{\text{Sun}} = 1.98847 \times 10^{30} \text{ kg}$
- Schwarzschild radius of the Sun: $R_{S\text{sun}} = 2.953 \text{ km} = 2953 \text{ m}$
- Semi-major axis of Mercury: $a_{\text{Merc}} = 5.79 \times 10^{10} \text{ m}$
- Eccentricity of Mercury's orbit: $e_{\text{Merc}} = 0.2056$

Dimensionless projection parameters for Mercury:

$$\kappa_{\text{Merc}} = \sqrt{\frac{R_{S\text{sun}}}{a_{\text{Merc}}}} = \sqrt{\frac{2953}{5.79 \times 10^{10}}} = 0.000225878693163 \quad (73)$$

$$\beta_{\text{Merc}} = \sqrt{\frac{R_{S\text{sun}}}{2a_{\text{Merc}}}} = \sqrt{\frac{2953}{2 \times 5.79 \times 10^{10}}} = 0.000159720355661 \quad (74)$$

Combined energy projection parameter:

$$Q_{\text{Merc}} = \sqrt{\kappa_{\text{Merc}}^2 + \beta_{\text{Merc}}^2} = 0.000276643771008$$

$$Q_{\text{Merc}}^2 = 3\beta_{\text{Merc}}^2 = 3 \times (0.000159720355661)^2 = 7.6531776038 \times 10^{-8} \quad (75)$$

Correction factor for the elliptic orbit divided by 1 orbital period:

$$\frac{1 - e_{Merc}^2}{2\pi} = \frac{1 - (0.2056)^2}{2 \times 3.14159265359} = \frac{0.9577}{6.28318530718} = 0.152427247197 \quad (76)$$

Final WILL Geometry precession result:

$$\phi_{WILL} = \frac{3\beta_{Merc}^2}{\frac{1-e_{Merc}^2}{2\pi}} = \frac{2\pi Q_{Merc}^2}{(1 - e_{Merc}^2)} = \frac{7.6531776038 \times 10^{-8}}{0.152427247197} = 5.0208724126 \times 10^{-7} \quad (77)$$

Classical GR prediction for precession:

$$\phi_{GR} = \frac{3\pi R_{Sun}}{a_{Merc}(1 - e_{Merc}^2)} = \frac{3 \times 3.14159265359 \times 2953}{5.79 \times 10^{10} \times 0.9577} = 5.0208724126 \times 10^{-7} \quad (78)$$

Relative difference:

$$\frac{\phi_{GR} - \phi_{WILL}}{\phi_{GR}} \times 100 = \frac{5.0208724126 \times 10^{-7} - 5.0208724126 \times 10^{-7}}{5.0208724126 \times 10^{-7}} \times 100 \quad (79)$$

$$= 2.1918652104 \times 10^{-10}\% \quad (80)$$

This negligible difference is consistent with the numerical precision limits of floating-point arithmetic, confirming that Will Geometry reproduces the observed relativistic precession of Mercury to within machine accuracy. □

20.6 Earth–Moon: inferring the open-channel power directly from LLR

We treat the Earth–Moon pair as a non-closed (radiative/dissipative) subsystem whose orbital energy changes secularly due to tides. In WILL variables the closure test is $\kappa^2 - 2\beta^2 \neq 0$; here we quantify the associated *power* of the open channel using only measured kinematics.

Theorem 20.6 (Third-channel power from the measured lunar recession). *Let a be the lunar semi-major axis, M_\oplus and $M_\mathbb{L}$ the masses of Earth and Moon, and $\dot{a} > 0$ the observed secular recession rate from lunar laser ranging (LLR). Then the power injected into the lunar orbit (equal and opposite to the net dissipated power in the Earth–tide system aside from internal heating) is*

$$P_{\text{orbit}} = \frac{G M_\oplus M_\mathbb{L}}{2a^2} \dot{a}.$$

Numerically, with $a = 3.844 \times 10^8$ m, $M_\oplus = 5.9722 \times 10^{24}$ kg, $M_\mathbb{L} = 7.3477 \times 10^{22}$ kg, $G = 6.67430 \times 10^{-11}$ m³ kg^{−1} s^{−2}, and $\dot{a} = (38.30 \pm 0.09)$ mm yr^{−1}, one finds

$$P_{\text{orbit}} = (1.203 \pm 0.003) \times 10^{11} \text{ W} = 0.1203 \text{ TW},$$

where the quoted uncertainty reflects the LLR error on \dot{a} .

Proof. The Newtonian piece of the two-body binding energy is $E(a) = -GM_\oplus M_\mathbb{L}/(2a)$. Differentiating and using $\dot{E} = -P_{\text{third}}$ (energy conservation for the subsystem plus environment) gives

$$\dot{E} = \frac{GM_\oplus M_\mathbb{L}}{2a^2} \dot{a} \Rightarrow P_{\text{orbit}} \equiv -\dot{E} = \frac{GM_\oplus M_\mathbb{L}}{2a^2} \dot{a}.$$

Insert the measured \dot{a} and constants; convert mm yr^{−1} to ms^{−1}. All other steps are algebraic; no additional modeling assumptions are used. □

Comparison to global tidal dissipation

Independent geophysical inversions place the present-day *total* Earth tide dissipation (oceanic + solid Earth, lunar+solar constituents) at ~ 3.7 TW. The orbital power above then corresponds to a fraction

$$\frac{P_{\text{orbit}}}{P_{\text{tides}}} \approx \frac{0.120 \text{ TW}}{3.7 \text{ TW}} \simeq 3.2\%.$$

Thus most tidal power is irreversibly thermalized within Earth's oceans/solid body, while a few percent is exported to the Moon by increasing a —the *open channel* that restores global energy balance in the WILL reading.

WILL interpretation (unitful closure diagnostic). For a closed Keplerian limit one has $\kappa^2 = 2\beta^2$ and $W_{\text{ILL}} = 1$. The persistent $\dot{a} > 0$ found above is exactly the nonzero third-channel flux; in unitful form the same conclusion follows from $W_{\text{ILL}} \neq 1$ when evaluated on the Earth–Moon state, with the sign indicating outward angular-momentum transfer and the magnitude fixed by P_{orbit} .

20.7 Orbital Decay: Hulse–Taylor binary Pulsar (PSR B1913+16)

We analyze the orbital decay of the Hulse–Taylor binary as an open (radiative) subsystem within WILL. Unlike the standard GR route that leans on tensor field equations and asymptotic structures, our derivation uses only relational budgets (κ, β) , dimensional consistency, and causal closure. Thus the universal $P^{-5/3}$ law and the full eccentricity dependence arise as direct geometric necessities: WILL reproduces GR's quantitative predictions while offering a more transparent, ontology-clean pathway. We will show that this phenomenon can be understood through two complementary and mutually reinforcing approaches: (I) a scale argument yielding $\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)$; (II) a first-principles computation of the eccentricity factor $F(e)$ and a numerical benchmark for PSR B1913+16.

Theorem 20.7 (Dimensionally-clean scaling for period decay). *In the WILL framework, any non-conservative (radiative) energy outflow from a bound two-body orbit must take the form*

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}(\kappa^2, \beta^2, e, \eta),$$

where $\kappa^2 \equiv 2GM/(rc^2)$, $\beta^2 \equiv v^2/c^2$, e is the eccentricity, and $\eta \equiv \mu/M \in (0, 1/4]$ is the symmetric mass ratio with $M = m_1 + m_2$ and $\mu = m_1 m_2 / M$. In the slow-motion, weak-field regime (closed circular limit $\kappa^2 = 2\beta^2 \ll 1$), the leading dependence is

$$P_{\text{third}} \propto \frac{c^5}{G} (\kappa^2)^5 \Phi(e, \eta),$$

for some dimensionless $\Phi(e, \eta)$, and the secular decay of the orbital period obeys the scale law

$$\boxed{\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)}.$$

Proof. (i) Causality & dimensionality.) Any causal radiative power built from the closed-system budgets must be a scalar constructed from G, c and *dimensionless* relational variables. The unique power scale with dimensions $[\text{energy}]/[\text{time}]$ is c^5/G , hence

$$P_{\text{third}} = \frac{c^5}{G} \mathcal{F}(\text{dimensionless}).$$

In the non-relativistic, weak-field regime the single small parameter is

$$\epsilon \sim \beta^2 \sim \frac{GM}{ac^2} \sim \frac{\kappa^2}{2} \ll 1 \quad (\text{circular closure } \kappa^2 = 2\beta^2).$$

(ii) Vanishing without acceleration.) Radiative loss must vanish for uniform straight motion; the lowest nontrivial multipolar content compatible with a bound orbit implies a leading analytic dependence $\mathcal{F} \propto \epsilon^5$.¹ Therefore

$$P_{\text{third}} \propto \frac{c^5}{G} \epsilon^5 \Phi(e, \eta) \propto \frac{c^5}{G} \left(\frac{GM}{ac^2} \right)^5 \Phi(e, \eta) = \frac{G^4 M^5}{c^5 a^5} \Phi(e, \eta).$$

(iii) From power to \dot{P} .) The Newtonian part of the orbital energy (which is the appropriate piece of the WILL budget for bound motion) is

$$E(a) = -\frac{GM\mu}{2a}.$$

Energy balance gives $\dot{E} = -P_{\text{third}}$, hence

$$\dot{a} = \frac{da}{dt} = \frac{2a^2}{GM\mu} \dot{E} \propto -\frac{G^3 M^4}{c^5} \frac{1}{a^3} \frac{\Phi(e, \eta)}{\mu}.$$

Kepler's law $P = 2\pi\sqrt{a^3/(GM)}$ yields $\dot{P} = (dP/da)\dot{a} = (3\pi/\sqrt{GM}) a^{1/2}\dot{a}$, so

$$\dot{P} \propto a^{1/2} a^{-3} \propto a^{-5/2}.$$

Using $a \propto (GM)^{1/3} P^{2/3}$,

$$a^{-5/2} \propto (GM)^{-5/6} P^{-5/3},$$

and collecting the M -dependence from the prefactors gives

$$\boxed{\dot{P} \propto (GM)^{5/3} P^{-5/3} \Phi(e, \eta)},$$

as claimed. All steps use only relational budgets, causality, dimensional analysis, and Keplerian kinematics—no ontological add-ons. \square

Relational reading

In WILL variables, the small parameter is $\epsilon \sim \kappa^2 \sim 2\beta^2$ (circular closure). The open-channel power is a function of (κ, β, e, η) ; the c^5/G scale fixes dimensions, while the leading analytic order in ϵ enforces the a^{-5} dependence and hence the five-thirds exponents in \dot{P} after translating through $E(a)$ and Kepler.

¹This step is purely structural: the leading radiative rank for an accelerated bound configuration is higher than linear in ϵ ; the first non-vanishing analytic contribution scales with a sufficiently high power. Writing $\epsilon \sim \kappa^2$ absorbs any fixed numerical factors into $\Phi(e, \eta)$.

Quantitative recovery of the third-channel power from observed \dot{P}

For a *measured* secular change of the period \dot{P} the radiative power follows from the chain rule, using only $E(a)$ and Kepler:

$$E(a) = -\frac{Gm_1m_2}{2a}, \quad a = \left(\frac{GM}{4\pi^2}\right)^{1/3} P^{2/3} \Rightarrow \boxed{P_{\text{third}} = -\dot{E} = \frac{Gm_1m_2}{3aP} |\dot{P}|}.$$

This identity is purely kinematic-dynamical (no model for the radiative mechanism).

Theorem 20.8 (Empirical power for PSR B1913+16). *Using published parameters for the Hulse–Taylor binary (PSR B1913+16),*

$$P = 7.751938773864 \text{ h}, \quad \dot{P} \approx -2.424 \times 10^{-12} \text{ s/s}, \\ m_1 \simeq 1.4408 M_\odot, \quad m_2 \simeq 1.387 M_\odot, \quad a \simeq 1.9501 \times 10^9 \text{ m},$$

the inferred third-channel power is

$$\boxed{P_{\text{third}} \approx 7.8 \times 10^{24} \text{ W}}.$$

Proof. Insert the numbers (SI units, $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $M_\odot = 1.98847 \times 10^{30} \text{ kg}$):

$$m_1m_2 \approx (1.4408 \times 1.387) M_\odot^2, \quad P = 2.7906979586 \times 10^4 \text{ s}, \\ |\dot{P}| = 2.424 \times 10^{-12} \text{ s/s}, \quad a = 1.9501 \times 10^9 \text{ m}.$$

Then

$$P_{\text{third}} = \frac{Gm_1m_2}{3aP} |\dot{P}| \approx 7.8 \times 10^{24} \text{ W},$$

with a few-percent spread under small variations of (m_1, m_2, a) within observational uncertainties. This is the *empirical* power of the open channel inferred solely from $(P, \dot{P}, a, m_{1,2})$. \square

Interpretation in WILL

For a closed subsystem the WILL closure gives $\kappa^2 = 2\beta^2$ (circular) and the period is constant. A persistent $\dot{P} \neq 0$ reveals an open channel: the nonzero power P_{third} above is exactly the missing flow that restores the global energy balance. The five-thirds exponents in \dot{P} reflect the universal a^{-5} scaling of the leading radiative budget in κ^2 , propagated through the relational energy and Kepler's law.

The preceding argument successfully recovers the correct scaling for the orbital period decay from fundamental principles, relying only on a single, physically-motivated assumption about the multipolar nature of the radiation (ϵ^5). This intuitive result powerfully suggests that the "five-thirds" law is a necessary consequence of any causal, relational theory of gravity. However, the WILL framework is sufficiently powerful to make this derivation fully rigorous and to compute the precise form of the dimensionless function $\Phi(e, \eta)$. We now demonstrate this with a complete first-principles calculation.

Theorem 20.9 (Eccentricity factor for quadrupolar emission). *For a bound Keplerian orbit with eccentricity e , the normalized orbit-average of $|\partial_t^3 S|^2$ for the spin-2 STF surrogate $S(f) = r(f)^2 e^{i2f}$ equals*

$$F(e) = \frac{\langle |\partial_t^3 S|^2 \rangle}{\langle |\partial_t^3 S|^2 \rangle_{e=0}} = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{7/2}}, \quad F(0) = 1.$$

Proof. Setup and notation. Let $p = a(1 - e^2)$, $r(f) = p/(1 + e \cos f)$ and define the affine parameter u by $du = dt/r^2$, so that $df/du = h$ with constant $h = r^2 \dot{f}$ (specific angular momentum). For any scalar $X(f)$ set $D \equiv d/df$ and

$$LX \equiv \partial_t X = \frac{1}{r^2} \frac{dX}{du} = h \sigma(f) DX, \quad \sigma(f) \equiv r^{-2} = \frac{(1 + e \cos f)^2}{p^2}.$$

The radiative spin-2 surrogate is $S(f) = r(f)^2 e^{i2f}$.

Lemma 20.10 (Variable-coefficient cubic identity). *For $\sigma = \sigma(f)$ and $D = d/df$,*

$$(\sigma D)^3 S = \sigma^3 D^3 S + 3\sigma^2 (D\sigma) D^2 S + (\sigma(D\sigma)^2 + \sigma^2 D^2 \sigma) DS.$$

Hence $L^3 S = h^3 (\sigma D)^3 S$.

Lemma 20.11 (Explicit derivatives). *With $x \equiv e \cos f$ and $r = p/(1 + x)$, one has*

$$Dr = \frac{pe \sin f}{(1 + x)^2}, \quad D\sigma = \frac{2e \sin f}{p^2} (1 + x), \quad D^2 \sigma = \frac{2}{p^2} (e \cos f + e^2 - 2e^2 \sin^2 f).$$

Furthermore,

$$DS = e^{i2f} (2r Dr + i2r^2), \quad D^2 S = e^{i2f} (2(Dr)^2 + 2r D^2 r + i4r Dr - 4r^2),$$

$$D^3 S = e^{i2f} (6Dr D^2 r + 2r D^3 r + i2(2(Dr)^2 + 2r D^2 r) - i8r Dr - i8r^2),$$

with $D^k r$ obtained by differentiating $r = p(1 + x)^{-1}$.

Lemma 20.12 (Orbit averages). *For integers $m, n \geq 0$ and $k \geq 2$,*

$$\left\langle \frac{\cos^m f \sin^n f}{(1 + e \cos f)^k} \right\rangle = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos^m f \sin^n f}{(1 + e \cos f)^k} df = \sum_j c_j(m, n, k) \frac{e^{2j}}{(1 - e^2)^{\alpha_j}},$$

where c_j are rational numbers and $\alpha_j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\}$. In particular, the needed set with $k \in \{2, \dots, 8\}$ closes under the algebra of Lemma 20.10.

Conclusion. Insert Lemma 20.11 into Lemma 20.10 to express $(\sigma D)^3 S$ as a linear combination of $\{\cos^m f \sin^n f\}/(1 + e \cos f)^k$. Average over one cycle using Lemma 20.12. The overall factor h^3 cancels in the normalization by the $e = 0$ case, leaving a rational function of e times $(1 - e^2)^{-7/2}$. A straightforward (finite) simplification yields the stated closed form for $F(e)$. \square

Theorem 20.13 (Orbital period decay of a binary pulsar). *For component masses m_1, m_2 (total $M = m_1 + m_2$), orbital period P_b and eccentricity e , the decay rate of P_b due to quadrupolar radiation is*

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}} \left(\frac{2\pi}{P_b}\right)^{5/3} F(e),$$

where $F(e)$ is given by Theorem 20.9.

Proof. The orbit-averaged quadrupole luminosity scales as $\langle P_{\text{GW}} \rangle \propto \mu^2 M^{4/3} n^{10/3} F(e)$ with $\mu = m_1 m_2 / M$ and $n = 2\pi/P_b$. Using $n^2 a^3 = GM$ and the Newtonian binding energy $E = -GM\mu/(2a)$, energy balance $\dot{E} = -\langle P_{\text{GW}} \rangle$ yields \dot{a} , hence $\dot{P}_b = (dP_b/da) \dot{a}$. Eliminating a and collecting constants gives the stated formula, with all eccentricity dependence entering solely through $F(e)$. No asymptotic background structures (ADM/Bondi) are invoked. \square

Numerical benchmarks and observational comparison

We now evaluate Eq. (20.13) for two archetypal systems. Constants: $G = 6.67430 \times 10^{-11}$ SI, $c = 2.99792458 \times 10^8$ m/s, $M_\odot = 1.98847 \times 10^{30}$ kg.

System	m_1/M_\odot	m_2/M_\odot	P_b (s)	e	Pred. \dot{P}_b (10^{-12} s/s)
PSR B1913+16	1.438(1)	1.390(1)	2.7907×10^4	0.6171334	−2.4022
PSR J0737−3039A/B	1.338185	1.248868	8.8345×10^3	0.0877770	−1.2478

[flushleft]

Notes: B1913+16 — observed/predicted = 0.9983 ± 0.0016 (Weisberg & Huang, ApJ 829:55, 2016).

J0737−3039A/B — GR validated at 0.013% (Kramer et al., PRX 11, 041050, 2021).

For reference, the often-quoted decrease of the B1913+16 orbital period is $\sim 76.5 \mu\text{s}/\text{yr}$ (equivalently $\sim 2.42 \times 10^{-12}$ s/s), matching the quadrupolar prediction within quoted uncertainties.²

The two preceding analyses—one from high-level principles and the other from a rigorous, direct calculation—converge on the same physical conclusion. They demonstrate that the observed orbital decay of binary pulsars is a necessary consequence of the WILL framework when accounting for a non-conservative energy outflow. The first approach shows that the universal $P^{-5/3}$ scaling law is an inevitable outcome of dimensional consistency and causality. The second approach confirms this intuition with a complete mathematical derivation that reproduces the exact quantitative predictions of General Relativity, which are in stunning agreement with decades of astronomical observation. This synergy between conceptual simplicity and computational power validates the WILL framework as not only philosophically parsimonious but also a robust predictive tool capable of passing one of the most stringent tests in modern physics.

21 Conclusion

WILL Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, WILL unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single foundational Principle—that spacetime is equivalent to energy—we derived all the mathematical apparatus needed to describe gravitational and relativistic

²See e.g. the Hulse–Taylor pulsar summary page for a pedagogical statement of $76.5 \mu\text{s}/\text{yr}$.

phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

Special and General Relativity emerge from the same geometric principles.

This approach offers distinct advantages:

- Conceptual clarity — understanding physics through pure geometry
- Computational efficiency — reducing complexity by up to 95%
- Epistemological hygiene — deriving results from minimal assumptions
- Philosophical depth — redefining our understanding of time, mass, and causality

WILL Geometry is not merely a reformulation of existing theories, but a paradigm shift that inverts our fundamental understanding:

Energy does not exist within spacetime—spacetime emerges from the transformations of energy.

Final Principle

Reality is projectional curvature of energetic flow.

22 Key Equations Reference

This section serves as a convenient reference for the core equations and relationships of the Energy Geometry framework.

22.1 Fundamental Parameters

Kinematic projection $\beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r_d}} = \sqrt{\frac{Gm_0}{r_d c^2}} = \cos(\theta_1), \quad (\text{Velocity Like}) \quad (81)$

Potential projection $\kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r_d}} = \sqrt{\frac{2Gm_0}{r_d c^2}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sin(\theta_2), \quad (\text{Escape Velocity Like}) \quad (82)$

22.2 The squared forms

$$\beta^2 = \frac{R_s}{2r_d}, \quad (83)$$

$$\kappa^2 = \frac{R_s}{r_d}. \quad (84)$$

$$\beta^2 = \frac{m_0}{r_d} \cdot \frac{l_P}{m_P} \quad (85)$$

$$\kappa^2 = \frac{8\pi G}{c^2} r_d^2 \rho(r). \quad (86)$$

$$\kappa^2(r) = \frac{2Gm(r)}{c^2 r}$$

$$\frac{d}{dr}(\kappa^2 r) = \frac{8\pi G}{c^2} r^2 \rho(r)$$

$$\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}$$

22.3 Core Relationships

$$\kappa^2 = 2\beta^2 \quad (\text{Fundamental projection ratio}) \quad (87)$$

$$\frac{\kappa}{\beta} = \sqrt{2} \quad (88)$$

$$\kappa^2 + \beta^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \quad (89)$$

$$\frac{r_d}{R_s} = \frac{1}{\kappa^2} = \frac{1}{2\beta^2} \quad (90)$$

22.4 Mass, Energy and Distance

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G} = \frac{R_s c^2}{2G} \quad (\text{mass of the system or object}) \quad (91)$$

$$R_s = \frac{2Gm_0}{c^2} \quad (\text{Schwarzschild radius. Radius from the center of mass where event horizon is forming}) \quad (92)$$

$$r_d = \frac{R_s}{\kappa^2} = \frac{2Gm_0}{\kappa^2 c^2} \quad (\text{radial distance}) \quad (93)$$

$$t_d = \frac{r_d}{c} \quad (\text{temporal distance}) \quad (94)$$

$$R_s = \frac{2Gm_0}{c^2} = \kappa^2 r_d \quad (\text{critical radial distance}) \quad (95)$$

$$\beta^2 = \frac{m_0}{r_d} \cdot \frac{l_P}{m_P} \quad (\text{Universal mass-to-distance ratio}) \quad (96)$$

22.5 Energy Density and Pressure

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2} = \kappa^2 \cdot \rho_{max} \quad (97)$$

$$\rho_{max} = \frac{c^2}{8\pi G r_d^2} \quad (\text{Critical energy density where } \kappa = 1 \text{ event horizon}) \quad (98)$$

$$P(r_d) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r_d} \cdot \frac{d\kappa^2}{dr_d} \quad (\text{Pressure}) \quad (99)$$

22.6 Contraction and Dilation Factors

$$\beta_Y = \sin(\theta_1) = \sqrt{1 - \beta^2} \quad (\text{Relativistic length contraction}) \quad (100)$$

$$\kappa_X = \cos(\theta_2) = \sqrt{1 - \kappa^2} \quad (\text{Gravitational time contraction}) \quad (101)$$

$$\frac{1}{\beta_Y} = \frac{1}{\sqrt{1 - \beta^2}} \quad (\text{Relativistic time dilation}) \quad (102)$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \kappa^2}} \quad (\text{Gravitational length dilation}) \quad (103)$$

22.7 Combined Energy Parameter Q

The total energy projection parameter unifies both aspects: (104)

$$Q = \sqrt{\kappa^2 + \beta^2} \quad (105)$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \quad (106)$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \quad (107)$$

$$Q_r = \frac{1}{Q_t} \quad (108)$$

22.8 Circle Equations

$$2\beta^2 + \kappa_X^2 = 1 \quad (109)$$

$$\frac{\kappa^2}{2} + \beta_Y^2 = 1 \quad (110)$$

$$2\cos^2(\theta_1) + \cos^2(\theta_2) = 1 \quad (111)$$

$$2\beta^2 + (1 - \kappa^2) = 1 \quad (112)$$

22.9 Unified Field Equation

$$\frac{R_s}{r_d} = \frac{\rho}{\rho_{max}} = \kappa^2 \quad (113)$$

For any spherically symmetric density $\rho(r)$: (114)

$$\boxed{\frac{d}{dr}(\kappa^2 r) = \frac{8\pi G}{c^2} r^2 \rho(r)} \implies \kappa^2(r) = \frac{2G}{c^2} \frac{m(r)}{r}. \quad (115)$$

For the homogeneous layer ($\kappa = \text{const}$) this reduces to (116)

$$\rho(r) = \frac{\kappa^2 c^2}{(8\pi G r^2)}, \quad (117)$$

exactly matching the global algebraic form used in Table 1. (118)

These describe the combined effects of relativity and gravity. (119)

22.10 Fundamental WILL Invariant

$$W_{ill} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = \frac{\frac{1}{\sqrt{1-\kappa^2}} m_0 c^2 \cdot \sqrt{1-\kappa^2} \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2}{\frac{1}{\sqrt{1-\beta^2}} m_0 \cdot \sqrt{1-\beta^2} \left(\frac{2Gm_0}{\kappa^2 c^2}\right)^2} = 1$$

22.11 Special Points

$$\text{Photon Sphere: } r = \frac{3}{2}R_s \quad \text{where } \kappa = \sqrt{\frac{2}{3}} \approx 0.816, \beta = \frac{1}{\sqrt{3}} \approx 0.577 \quad (120)$$

$$\text{ISCO: } r = 3R_s \quad \text{where } \kappa = \frac{1}{\sqrt{3}} \approx 0.577 \quad (121)$$

At the critical point where $\theta_1 = \theta_2 = 54.7356103172^\circ$:

$$\kappa^2 + \beta^2 = 1 \quad (122)$$

$$\beta = \kappa_X \quad (123)$$

$$\kappa = \beta_Y \quad (124)$$

$$Q_t = \sqrt{1 - 3\beta^2} = 0 \quad (\text{Instability threshold}) \quad (125)$$