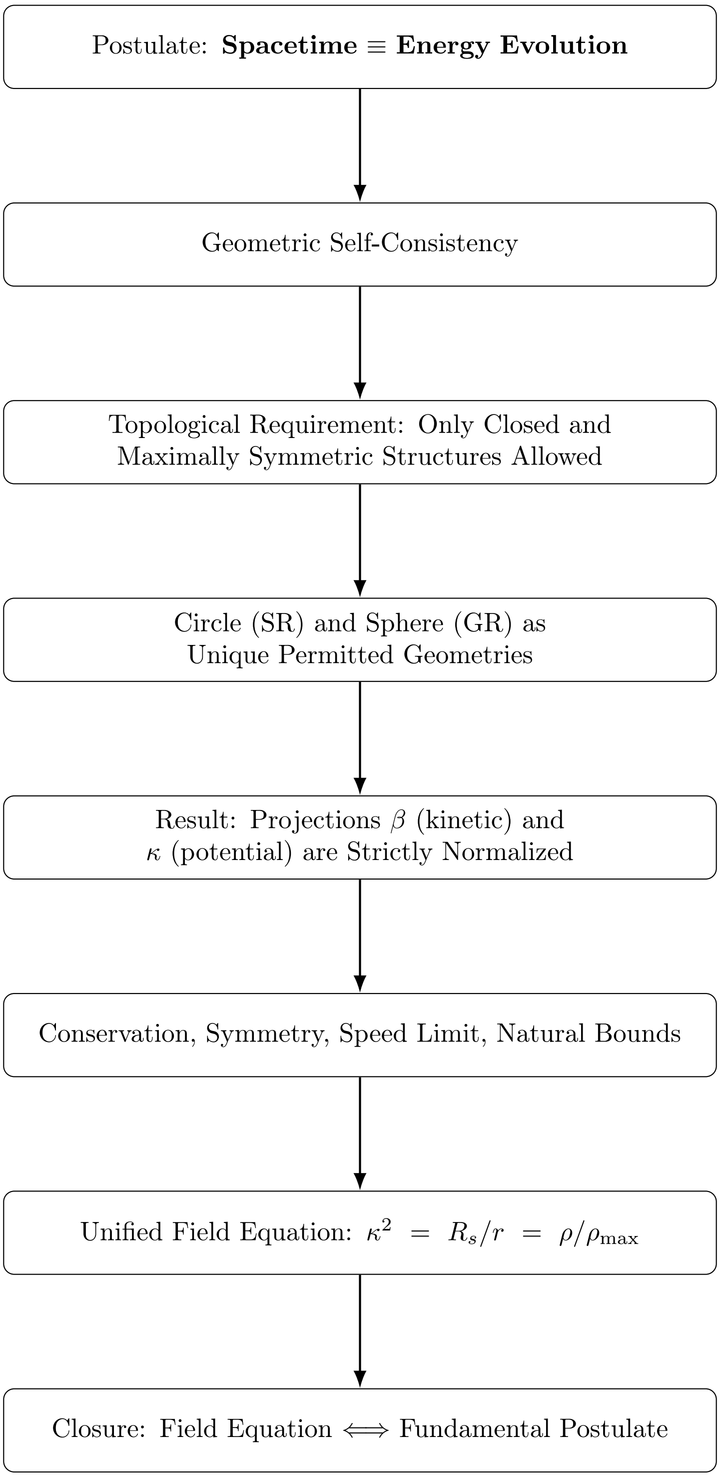
WILL: Unified Framework
PART I - Foundations of Relativity

Anton Rize egeometricity@gmail.com

April 2025



Abstract

This paper, the first in the WILL series, introduces a new theoretical framework that shifts the paradigm from a descriptive to a **generative model** of physics, providing a unified derivation of Special and General Relativity from first principles. The model is founded on the **single postulate** that $\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$, from which all physical laws are deductively derived without free parameters. Key results include: the *ab initio* geometric derivation of the energy-momentum relation ($E^2 = (pc)^2 + (mc^2)^2$); a fundamental unifying equation, $\kappa^2 = 2\beta^2$, which is shown to be a necessary consequence of the topology of the underlying projection manifolds; and an algebraic field equation, $\frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}$, that serves as a local equivalent to Einstein's Field Equations.

By construction, this geometric approach **inherently resolves the problem of singularities** by imposing a natural maximum on curvature, and it successfully **reconstructs the Schwarzschild and Kerr metrics** from its core projections. The framework offers significant **conceptual clarity and computational simplicity** by replacing complex tensor formalism with transparent geometric relations. Furthermore, it demonstrates complete **theoretical closure** (the "Theoretical Uroboros"), wherein the foundational postulate deductively leads to a field equation that fully embodies it. The model's predictions are shown to be in precise agreement with key empirical tests, including the precession of Mercury, GPS time corrections, and the orbital decay of binary pulsars, establishing a robust foundation for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

Contents

1 Foundational Approach

Guiding Principle

Nothing is assumed. Everything is derived.

1.1 Methodological Purity

This framework is built from a single postulate and zero free parameters. This construction is not a simplification — it is a deliberate epistemic constraint. No assumptions are introduced unless they follow strictly from first principles, and no constructs are retained unless they are geometrically or energetically necessary.

No Ontological Commitments

This model makes no ontological claims about the "existence" of space, particles, or fields. Instead, all phenomena are treated as observer-dependent energetic projections.

1.2 Epistemic Hygiene

Modern physics often tolerates hidden assumptions: arbitrary constants, external backgrounds, or abstract entities with ambiguous physical status. Here we enforce **epistemic hygiene**: a refusal to import unjustified assumptions. All physical quantities emerge purely from relational geometry and causal closure.

1.3 Motivation and Core Principle

The standard formulation of General Relativity often relies on the concept of an asymptotically flat spacetime, introducing an implicit external reference frame beyond the physical systems under study. While some modern approaches (e.g., shape dynamics) seek greater relationality, we proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

Principle: *All physical quantities must be defined purely by their relations.* Any introduction of absolute properties or external frames risks reintroducing metaphysical artifacts and contradicts the foundational insight of relativity.

What is Energy?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, quantum — yet all are interchangeable, pointing to a single underlying structure.

Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation.

Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself.

From these empirical and relational facts the definition follows unavoidably:

Energy is the relational measure of difference between possible states,
conserved in any closed whole and guaranteeing the continuity of causal transitions.
It is not an intrinsic property of an object, but **comparative structure**
between states (and observers), always manifesting as the potential for transformation.

Any attempt to treat "spacetime structure" as something separate from "dynamics" already introduces a hidden assumption. It presupposes that reality can be divided into a static container and an evolving content. From the standpoint of epistemic hygiene, such a division is inadmissible: it brings in an ontological artifact without necessity. By removing this assumption, we are compelled to identify structure and dynamics as two aspects of a single entity.

2 Single Unifying Postulate

Once the false division between structure and dynamics is removed, the unique remaining formulation is that the geometry of relations *is* the evolution itself. This leads directly to the unifying postulate:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

Clarification: By "energy evolution" we mean the total structure of possible transitions between observable states—not a process unfolding in spacetime, but the very relational geometry from which both space and time emerge. This is not a derived result, but a foundational postulate, subject to geometric and empirical audit in subsequent sections. "WILL" is used here as a technical term for this unified, emergent structure.

The goal of this framework is not merely to reconstruct the formulas of Special and General Relativity, but to demonstrate that they are the unique and necessary consequences of a single, deeper principle of relational geometry. By showing *why* these specific mathematical structures must arise, the model provides a more fundamental explanation for them than the standard formalisms alone.

2.1 Structure of WILL

SPACETIME ≡ ENERGY EVOLUTION *is not a metaphorical equivalence, but a topologically enforced identification: The closure, isotropy, and conservation of energetic transformations allow only one geometrical substrate: a manifold where structure is transformation.*

1. **Self-contained structure:** Since spacetime and energy evolution are one and the same, no external container can exist. Any attempt to imagine energy as "sitting" inside a background would reintroduce the forbidden separation between structure and dynamics. Therefore the system must be entirely self-contained.

2. **Conservation law:** Because the configuration is closed, nothing enters or leaves. This closure enforces conservation: the total relational energy cannot vary.

3. **Symmetry/isotropy:** Without an external reference, every direction must be equivalent. If one direction were privileged, the hidden background would reappear. Thus perfect symmetry is unavoidable.

4. **Circular/spherical configuration:** Among all possible closed manifolds, only those possessing the **highest degree of symmetry (isotropy)** — where every point and every direction is indistinguishable - can serve as our foundation. If one direction were privileged, a hidden background would reappear. This stringent requirement of perfect isotropy uniquely selects the circle and the surface of a sphere. Bouth exhaust the relational budget without leaving hidden asymmetries. In one dimension the unique possibility is the circle S^1; in two dimensions it is the surface of a sphere S^2. These are not spatial objects but relational manifolds: they encode the closure of energetic relations themselves.

IMPORTANT

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational manifolds encoding conservation. Any reading otherwise is a misinterpretation.

3 Emergence of Spacetime and Geometric Uncertainty

In this construction, "space," "time," and the principle of uncertainty are not treated as separate, fundamental aspects of reality. Instead, they are shown to arise as necessary consequences of a single, underlying principle: the geometry of a closed, relational system.

3.1 The Duality of Evolution

Once the postulate identifies spacetime with energy evolution, the apparent separation of reality into "space" and "time" must be derived. This derivation begins with a critical methodological question: how do we bridge the abstract geometry of the S^1 manifold with physical concepts without resorting to arbitrary naming?

The answer is found within the logical requirements of the term "evolution" itself. Any complete description of change requires two inseparable and complementary measures:

1. A measure for the **extent** of transformation (what has changed).
2. A measure for the **sequence** or internal state of that transformation (how it has changed).

The relational geometry of the circle, S^1, provides the unique, minimal structure that perfectly embodies this required duality. We therefore do not arbitrarily name its orthogonal projections, but *recognize* them as the geometric substrates for these two aspects of evolution. We define them as follows:

- **The Phase Component (β_Y):** This projection represents the *internal structure* of a system. It governs the intrinsic scale of its proper length and proper time units, corresponding to the *sequence* of its evolution. A value of $\beta_Y = 1$ represents a complete and undisturbed manifestation of this internal structure, a state we identify as rest.
- **The Amplitude Component (β_X):** This projection represents the *relational measure* between the system and the observer. It corresponds to the *extent* of transformation, which manifests physically as momentum.

3.2 The Fundamental Law of Conservation

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of reality":

$$\beta_X^2 + \beta_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

3.3 Consequence 1: Relativistic Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of Special Relativity. An increase in the relational measure (β_X, momentum) necessarily requires a decrease in the measure of the internal structure (β_Y). This geometric trade-off is observed physically as length contraction and time dilation. Thus, the geometry of spacetime is nothing but the shadow cast by the geometry of relations.

3.4 Consequence 2: The Geometric Uncertainty Principle

The same conservation law also dictates a geometric analogue to Heisenberg's Uncertainty Principle. The relationship between the Amplitude Component (relational aspect) and the Phase Component (internal aspect) is one of necessary reciprocity.

- To perfectly ascertain a system's **internal structure** ($\beta_Y = 1$), its **relational aspect** must be null ($\beta_X = 0$). This is a state of perfect localization (analogous to position) where the relational momentum is zero.
- To have a maximal **relational aspect** ($\beta_X = 1$), the system's **internal structure** must vanish ($\beta_Y = 0$). This is analogous to a state of perfectly defined momentum where localization is completely lost.

Crucially, unlike the Heisenberg principle which is rooted in the non-commutativity of quantum operators and the constant $\hbar$, this geometric uncertainty arises directly from the single postulate of a closed, relational universe. The model does not *posit* uncertainty as a strange axiom; it *derives* it as a logical inevitability of its geometry. This suggests that the uncertainty seen in quantum mechanics may be a manifestation of this more fundamental, geometric principle of conservation.

3.5 Geometric Form of Uncertainty

Let the relational circle S^1 be parametrized as

$$\beta_X = \cos\theta_1, \qquad \beta_Y = \sin\theta_1, \qquad \beta_X^2 + \beta_Y^2 = 1.$$

Small variations are controlled by a single parameter θ_1:

$$\delta\beta_X = -\sin\theta_1\,\delta\theta_1, \qquad \delta\beta_Y = \cos\theta_1\,\delta\theta_1.$$

Thus the corresponding uncertainties are

$$\Delta\beta_X = |\sin\theta_1|\,\Delta\theta_1, \qquad \Delta\beta_Y = |\cos\theta_1|\,\Delta\theta_1,$$

and their product satisfies the identity

$$\Delta\beta_X\,\Delta\beta_Y = \tfrac{1}{2}\,|\sin 2\theta_1|\,(\Delta\theta_1)^2.$$

In a closed relational system the phase must return to its initial value after a full circuit of 2π. This implies that only an *integer number of phase wraps* can be consistently realized. We denote this integer by n, the **winding number** or **phase index**. It is a purely topological invariant of the closed geometry and not an externally imposed parameter.

Consequently, the minimal angular step that remains physically distinguishable is set by this winding number:

$$\Delta\theta_1 \gtrsim \frac{2\pi}{n}.$$

Substituting yields the **geometric uncertainty relation**:

$$\boxed{\Delta\beta_X\,\Delta\beta_Y \gtrsim \tfrac{1}{2}\,|\sin 2\theta_1|\left(\tfrac{2\pi}{n}\right)^2}$$

Physical meaning: The trade-off between the amplitude component (β_X) and the phase component (β_Y) is not arbitrary but dictated by the closure of the relational circle and the requirement of integer phase winding. This provides a geometric analogue of the Heisenberg uncertainty principle without invoking operator algebra or $\hbar$ as an axiom. In this framework, Planck's constant may be understood as the empirical calibration of the minimal phase grain $(2\pi/n)^2$ to observed dimensional quantities.

4 Relational Energy and Rest State

Since S^1 encodes one-dimensional displacement, the total energy of the system must project consistently onto both axes. Each coordinate (β_X, β_Y) multiplies the same total magnitude E, producing the projection pair

$$E_X = E\beta_X, \qquad E_Y = E\beta_Y.$$

4.1 Rest Energy as Necessary Projection

When $\beta_X = 0$ (the relational displacement vanishes), the circle condition enforces $\beta_Y = 1$. At this point the vertical projection alone remains, and the energy reduces to

$$E_Y = E \cdot 1.$$

This state is uniquely identified as the invariant rest energy:

$$E_0 \equiv E\beta_Y \quad \text{when } \beta_X = 0.$$

4.2 Invariant Relation

Because closure applies for all θ_1, the vertical projection must remain constant across all states. Thus the unique invariant relation is

$$\boxed{E\beta_Y = E_0}.$$

4.3 Total Energy

From the invariant relation $E\beta_Y = E_0$ the total energy follows uniquely as

$$E = \frac{E_0}{\beta_Y}.$$

Since the closure condition enforces $\beta_Y = \sqrt{1-\beta_X^2}$, this can also be written as

$$E = \frac{E_0}{\sqrt{1-\beta_X^2}}.$$

No additional parameter is required. The historical "Lorentz factor" is nothing more than the reciprocal of β_Y and adds no new structure. The full content is already present in the relation $E\beta_Y = E_0$.

4.4 Rest Energy and Mass

In a framework that is genuinely fundamental and free from arbitrary human units, the natural normalization is always given by the unique invariant $c = 1$. With this normalization, the bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ lose all significance. They collapse into the only consistent statement:

$$E_0 = m.$$

Thus mass is nothing beyond the invariant projection of total rest energy. It introduces no new entity and carries no independent meaning apart from E_0. What is conventionally treated as two quantities is in fact one and the same relational invariant.

4.5 Energy–Momentum Relation

From the orthogonal projections it follows that the horizontal component is

$$p \;\equiv\; E\beta_X.$$

Together with the invariant vertical projection $E\beta_Y = E_0$, the circle closure enforces

$$(E\beta_X)^2 + (E\beta_Y)^2 = E^2.$$

Substituting $p = E\beta_X$ and $m = E_0$ yields the unique energy–momentum relation:

$$E^2 = p^2 + m^2.$$

4.6 Geometric Forms

This relation can be written in equivalent ways, making explicit the role of the circle geometry:

$$E^2 = \left(\frac{\beta_X}{\beta_Y} E_0\right)^2 + E_0^2,$$

or, in angular form,

$$E^2 = \big(\cot(\theta_1)\, E_0\big)^2 + E_0^2.$$

These are not alternative parametrizations but the same identity expressed through the coordinates of the relational circle. The algebraic form $E^2 = p^2 + m^2$ is simply the most compact expression of this geometric necessity.

5 Potential Energy Projection on S^2

Analogous to S^1 the relational geometry of the sphere, S^2, provides orthogonal projections, for two aspects of omnidirectional evolution. We define them as follows:

- **The Amplitude Component (κ_Y):** This projection represents the *relational gravitational measure* between the object and the observer. It corresponds to the *extent* of transformation, which manifests physically as gravitation potential. A value of $\kappa_Y = 1$ corresponds precisely to the point where escape velocity equals the speed of light, creating an event horizon. This provides a natural causal limit for our gravitational parameter, analogous to the relative motion it determinant by the same radial normalization r=c=1.

- **The Phase Component (κ_X):** This projection governs the intrinsic scale of its proper length and proper time units, corresponding to the *sequence* of its evolution.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite "budget of reality":

$$\kappa_X^2 + \kappa_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics.

5.1 Consequence: Gravitational Effects

The redistribution of the budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational measure (κ_Y, gravitation potential) necessarily requires a decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length and time corrections. Thus, the geometry of spacetime is nothing but the shadow cast by the geometry of relations.

From here on we will wright $\beta = \beta_X, \quad \kappa = \kappa_Y$ for notation simplicity.

6 Fundamental Relation Between Potential and Kinetic Energy Projections

6.1 The Topological Necessity of S^1 and S^2 Projections

Important clarification:

These are not spatial objects, but abstract topological constructs encoding the conservation law and closure of energy relations.

The postulate SPACETIME ≡ ENERGY EVOLUTION is not a vague metaphor but a precise geometric constraint. It demands that all physical quantities emerge from the relational structure of transitions between observable states. This structure must be:

1. **Self-contained:** There is no external background.
2. **Conservative:** The total "resource" for evolution is fixed.
3. **Maximally Symmetric:** No point or direction is privileged.

These are not additional assumptions; they are the *operational definitions* of what it means for spacetime to be identical to energy evolution. Any geometry violating these principles would reintroduce an absolute background, violating the postulate.

The fundamental question is: *What are the unique geometric arenas that can host such a complete description of physical evolution?*

We must now characterize the types of "energy evolution" that constitute physics. There are two, and only two, fundamental classes:

- **Class I (Directional Evolution):** This describes a transition whose character is defined by a *preferred axis*. The act of measurement is the act of distinguishing this axis (e.g., the line connecting observer and object for relative velocity). The energy measure is inherently tied to a specific direction. This is the domain of kinematics.

- **Class II (Omnidirectional Evolution):** This describes a transition whose character is *inherently without a preferred axis*. The act of measurement is the act of establishing a relation to a center, but the resulting field is defined equally in all directions from that center. This is the domain of a static, central potential.

These are not arbitrary choices. They are an exhaustive catalogue of the fundamental relational categories implied by the postulate. Class I isolates a single degree of freedom for measurement. Class II encompasses the full set of directional possibilities from a point.

The geometry must now accommodate these classes while obeying the three constraints above.

- **For Class I (Directional):** The only closed, 1-dimensional manifold of constant positive curvature is the circle, S^1. It is unique. It is maximally symmetric (all points and rotations are equivalent). It is self-contained and embodies conservation (the fixed radius). **Therefore, the kinematic parameter β must be a projection on S^1. There is no other option.**
- **For Class II (Omnidirectional):** The only closed, simply-connected 2-dimensional manifold of constant positive curvature is the sphere, S^2. It is unique. It is maximally symmetric (all points and rotational axes are equivalent). It is self-contained. **Therefore, the gravitational parameter κ must be a projection on S^2. There is no other option.**

Closed, self-contained universe requires that the "total space of possibilities" for each projection mode be complete. The fundamental, dimensionless measure for the completeness of these geometries is their full angle:

- For the circle (S^1), the total available "configurational space" is its full angle of closure: 2π.
- For the sphere (S^2), the total available "configurational space" is its full angle of closure: 4π.

The ratio of the total angular measures of these two fundamental manifolds is a pure, dimensionless number dictated by their intrinsic topology:

$$\frac{\text{Total Closure of } S^2}{\text{Total Closure of } S^1} = \frac{4\pi}{2\pi} = 2$$

This factor of 2 is a direct and necessary consequence of the dimensional difference between an omnidirectional (2D) and a linear (1D) projection space.

> Here both 2π and 4π are the *complete closure measures* of S^1 and S^2. **Not spatial objects, but abstract topological constructs.** They are universal, dimensionless invariants: the full angle of a closed loop and the full angle of a closed surface. Their ratio is therefore a pure number, reflecting the intrinsic topological difference between one- and two-dimensional projection spaces.

6.2 The Quadratic Nature of Energetic Projections

To translate this purely geometric ratio into a physical law, we must consider how projections relate to energy. In physics, the energetic significance (or "power") of a field, velocity, or wave is proportional not to its amplitude, but to its **amplitude squared**. This quadratic relationship is fundamental, appearing in kinetic energy ($E_k \propto v^2$), potential energy in fields, and the energy density of electromagnetic waves ($E \propto A^2$).

Furthermore, in any projectional geometry, the magnitudes of orthogonal components are related to the whole via a Pythagorean sum of squares. Therefore, the conserved evolution resource is distributed among the squares of its projections. The fundamental relation must connect the ratio of the *energetic significances* of the modes (κ^2 and β^2) to the underlying topological ratio of their geometric arenas.

This leads to the unique and necessary unification equation:

$$\frac{\kappa^2}{\beta^2} = 2 \quad \Longrightarrow \quad \boxed{\kappa^2 = 2\beta^2}$$

Important Clarification. This factor of 2 is not an empirical constant, a Newtonian mechanical result, or a coincidence of orbital dynamics. It is a direct consequence of the ratio of the fundamental angular measures that define the complete projection spaces for omnidirectional and linear phenomena.

Closure Condition. The relation $\kappa^2 = 2\beta^2$ is therefore more than a proportionality: it is the precise criterion for energetic closure. Whenever the equality is satisfied, the system under study is fully self-contained, with no leakage of the conserved resource into unaccounted channels. Whenever the equality *appears* to fail (for instance in highly elliptical binaries or in radiating systems), this does not contradict the principle but indicates that additional energy pathways — such as gravitational-wave emission or cosmological expansion — must be included to restore the balance. In this sense, the relation acts as a diagnostic invariant: it tells us whether our description has captured the full closed geometry or whether unseen flows remain.

6.3 Closure, Falsifiability, and the Conservation of Energy

The relation $\kappa^2 = 2\beta^2$ must be understood not merely as a proportionality, but as the precise geometric criterion for the **energetic closure** of a system. In this framework, it serves as the direct geometric embodiment of the Law of Conservation of Energy.

Any apparent deviation from this equality is therefore not a failure of the principle. Instead, it acts as a powerful diagnostic tool that presents a definitive epistemic choice:

1. **The System is Incompletely Defined:** The observer has failed to account for all energy channels. The deviation is a physical signature of an unaccounted-for process (e.g., gravitational-wave emission, mass accretion, or other energy flows). The relation thus becomes a predictive instrument, compelling the discovery of these flows to restore the geometric balance.

2. **A Violation of Fundamental Conservation:** If, hypothetically, a system were proven to be perfectly closed with all energy flows rigorously accounted for, and the relation $\kappa^2 = 2\beta^2$ was still systematically violated, this would imply a failure of the Law of Conservation of Energy itself.

.

The Principle as a Diagnostic Invariant

The relation $\boxed{\kappa^2 = 2\beta^2}$ holds if and only if the system under study is energetically closed. An apparent violation is not a contradiction of the theory, but a physical prediction that energy is flowing through an unaccounted-for channel. The principle is therefore falsifiable, with its falsification being equivalent to demonstrating a violation of energy conservation.

Illustrative Examples

To clarify the meaning of this closure condition, consider two contrasting cases:

- **Circular Orbit (Closed Subsystem).** For a test body in a perfectly circular orbit around a central mass, the condition $\kappa^2 = 2\beta^2$ is exactly fulfilled. The orbital system can be treated as energetically closed: all of the conserved resource is accounted for between the kinetic projection along the orbit and the gravitational projection toward the center. No external channels are needed, and the equality signals full closure.

- **Radiating Binary (Open Subsystem).** In contrast, for a highly elliptical binary system of compact objects (such as neutron stars. (full calculation in section: Empirical Validation; subsection: Orbital Decay: Binary Pulsar)), the orbital evolution is accompanied by gravitational-wave emission. If one considers only the orbital mechanics, the relation $\kappa^2 = 2\beta^2$ will appear to be violated. Yet this is not a failure of the principle. Instead, it reveals that the subsystem is not closed: part of the resource leaves through radiation. When the gravitational-wave channel is included in the balance, the closure is restored and the equality holds again.

These examples highlight the diagnostic role of the relation: it identifies whether the chosen subsystem is complete, or whether additional flows of the evolution resource must be incorporated to recover closure.

Principle of Closure. The relation $\kappa^2 = 2\beta^2$ holds if and only if the system under consideration is fully closed, with no leakage of energy into channels not included in the description.

The factor of 2 in the fundamental relation $\kappa^2 = 2\beta^2$ is the ratio of the total solid angle of the sphere (4π steradians) to the total planar angle of the circle (2π radians). This ratio is the geometric exchange rate between the two unique manifolds that host the two fundamental categories of energy evolution.

In summary:

1. The postulate defines the universe as a closed, conservative, symmetric relational structure.
2. The physical content of that universe consists of two and only two fundamental types of relations: directional and omnidirectional.
3. The mathematics of geometry dictates that the *only* closed, maximally symmetric arenas for these relations are S^1 and S^2, respectively.
4. Therefore, the projection parameters β and κ are *forced* to live on these **non-spatial** manifolds.

It is the *only geometrical content* that can be housed within the geometric constrains dictated by the fundamental postulate.

Physical Implication. In WILL Geometry, this relation is foundational. It links gravitational and kinetic modes without reference to spacetime as a background fabric, showing that their connection is a matter of geometric and topological necessity, not dynamic coincidence. Classical physics, in its successful predictions, unknowingly traces the consequences of this deeper geometric law.

$$\boxed{\text{Spacetime Geometry } (\kappa^2) \;\equiv\; \text{Kinematic Energy distribution } (\beta^2) \;\times\; 2}$$

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

6.4 Geometric Relationships

The relativistic and gravitational factors can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

$\beta = \beta_X, \quad \kappa = \kappa_Y, \quad \theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa)$	
Algebraic Form	**Trigonometric Form**
$1/\beta_Y = \frac{1}{\sqrt{1-\beta^2}}$	$1/\beta_Y = \frac{1}{\sin(\theta_1)} = \frac{1}{\sin(\arccos(\beta))}$
$1/\kappa_X = \frac{1}{\sqrt{1-\kappa^2}}$	$1/\kappa_X = \frac{1}{\cos(\theta_2)} = \frac{1}{\cos(\arcsin(\kappa))}$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1) = \sin(\arccos(\beta))$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2) = \cos(\arcsin(\kappa))$

Table 1: Unified representation of relativistic and gravitational effects.

6.4.1 The Combined Energy Parameter Q

The total energy projection parameter unifies both aspects:

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{1}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{2}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{3}$$

$$Q_r = \frac{1}{Q_t} \tag{4}$$

These describe the combined effects of relativity and gravity.

Unified Interpretation

The familiar SR and GR factors emerge here as projections of the same conserved geometry. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-evolution constraint revealing their unified origin.

7 Energy–Symmetry Law

7.1 Causal Continuity and Energy Symmetry

Theorem 1 (Energy Symmetry)**.** The energy differences perceived by two observers at different positions balance according to the Energy–Symmetry Law:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{5}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A.
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B.

Each observer perceives energy transfers differently due to gravitational and kinetic differences: From A's perspective (surface $\to$ orbit):

1. Object gains potential energy by moving away from gravitational center.
2. Object gains kinetic energy by accelerating to orbital velocity.

This results in energy difference:

$$\boxed{\Delta E_{A\to B} = \frac{1}{2}\left((\kappa_A^2 - \kappa_B^2) + \beta_B^2\right)} \tag{6}$$

From B's perspective (orbit $\to$ surface):

1. Object loses gravitational potential energy descending into stronger gravitational field.
2. Object loses kinetic energy by reducing velocity from orbital speed to rest.

This results in energy difference:

$$\boxed{\Delta E_{B\to A} = \frac{1}{2}\left((\kappa_B^2 - \kappa_A^2) - \beta_B^2\right)} \tag{7}$$

Summing these transfers:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{8}$$

Lemma 1 (Quadratic budget difference)**.** Define the quadratic budget $Q^2 \equiv \kappa^2 + \beta^2$. For any two relationally defined states A, B, set

$$\Delta E_{A\to B} \equiv \frac{1}{2}\big(Q_A^2 - Q_B^2\big), \qquad \Delta E_{B\to A} \equiv \frac{1}{2}\big(Q_B^2 - Q_A^2\big).$$

Then $\Delta E_{A\to B} + \Delta E_{B\to A} = 0$ identically. This is a purely relational symmetry under the exchange $A \leftrightarrow B$ and does not rely on any specific potential model.

Thus, no net energy is created or destroyed, confirming the Energy–Symmetry Law. □

7.2 Observer-Agnostic Symmetry

Theorem 2 (Observer-Agnostic Energy Symmetry)**.** The Energy–Symmetry Law holds true independent of the specific model of gravitational or kinetic energy:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \quad \forall f(v) \tag{9}$$

Proof. Regardless of the chosen energy function $f(v)$ representing kinetic or gravitational energy differences, consistency demands that:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0. \tag{10}$$

The crucial point is mutual consistency, not specific energy definitions. □

7.3 Physical Meaning of Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ does not originate from classical mechanics, but from the geometric nature of the quadratic form that measures the "evolution budget" $Q^2 = \kappa^2 + \beta^2$.

When comparing two nearby states, the change in this quadratic form is

$$\delta Q^2 = (\kappa_2^2 - \kappa_1^2) + (\beta_2^2 - \beta_1^2).$$

In energetic terms, only *half* of this variation is attributed to the "active" transfer in one direction, because the other half belongs to the reciprocal change seen from the opposite frame. This division is the same geometric reason why kinetic energy in classical mechanics is $K = \frac{1}{2}mv^2$: the quadratic metric is naturally paired with a factor $\frac{1}{2}$ when representing stored or exchanged energy.

Thus:

- Gravitational potential term: $U \propto -\frac{1}{2}\kappa^2$.
- Kinetic term: $K \propto \frac{1}{2}\beta^2$.

The $\frac{1}{2}$ reflects the symmetric partition of a quadratic budget between mutually consistent perspectives, not an externally imposed convention.

7.4 Universal Speed Limit as Consequence of Energy Symmetry

Theorem 3 (Universal Speed Limit)**.** The universal speed limit ($v \leq c$) emerges naturally from Energy–Symmetry conditions.

Proof. Assume an object could exceed the speed of light ($\beta > 1$). In that scenario:

- The kinetic component (β^2) surpasses unity excessively, causing an irreversible imbalance in energy transfer.
- No reciprocal transfer could balance this energy, breaking the fundamental symmetry:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{11}$$

Thus, $\beta \leq 1$ ($v \leq c$) is required intrinsically to preserve causal and energetic consistency. □

In essence, WILL Geometry encodes the principle that:

> The speed of light is the boundary beyond which the energetic symmetry between perspectives breaks down. Causality is not an external rule but a built-in feature of the universe's energetic geometry.

8 Classical Keplerian Energy as a WILL–Minkowski Projection

A striking consequence of the Energy–Symmetry Law is that, when the zero of gravitational potential is chosen on the surface of the central body rather than at infinity, the total specific orbital energy (potential + kinetic, per unit mass) naturally appears in *Minkowski form.*

8.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{12}$$

$$K = \frac{1}{2}m\frac{GM_\oplus}{a}. \tag{13}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{14}$$

8.2 Projection Parameters and Minkowski Form

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{15}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{16}$$

Substituting into (14) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{17}$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{18}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

8.3 Physical Interpretation

In classical derivations, (14) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the WILL framework, (17) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface},\ \text{orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure: the Minkowski–type projection algebra of WILL Geometry.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in a Lorentz–like difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This reframing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy evolution.

9 Derivation of Density, Mass, and Pressure

9.1 Geometric Foundation

From the projective analysis established in the previous sections, the fundamental invariant is

$$\kappa^2 = \frac{R_s}{r_d},$$

where κ emerges from the energy projection on the unit circle, and $R_s = 2Gm_0/c^2$ links to the mass scale factor $m_0 = E_0/c^2$.

9.2 Derivation of Energy Density

From Mass Scale to Volumetric Potential. Starting from the geometric relation,

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G},$$

we associate m_0 with a volumetric proxy r_d^3, obtaining a raw volumetric potential,

$$\frac{m_0}{r_d^3} = \frac{\kappa^2 c^2}{2G r_d^2}.$$

Applying the Geometric Distribution Principle. Because the potential projection κ is distributed over S2 - a 2D spherical manifold, the volumetric expression must be normalized over the unit-sphere area 4π. This yields the physical energy density,

$$\rho = \frac{1}{4\pi}\left(\frac{\kappa^2 c^2}{2Gr_d^2}\right).$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2}.$$

Local Energy Density ≡ Relational Projection

Maximal Density. At $\kappa^2 = R_s/r_d = 1$, the horizon condition is reached, corresponding to the maximal observable energy density at radius r_d:

$$\rho_{\max} = \frac{c^2}{8\pi G r_d^2}.$$

Normalized Relation. Thus the fundamental identification is

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \Rightarrow \quad \kappa^2 \equiv \Omega}.$$

9.3 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G}.$$

From the energy density:

$$m_0 = \alpha\, r_d^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2}$ into $m_0 = \alpha\, r_d^n\, \rho$ gives

$$m_0 = \frac{\alpha \kappa^2 c^2 r_d^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r_d^{n-2}}{8\pi} = \frac{r_d}{2}.$$

Radius independence requires $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$m_0 = 4\pi r_d^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

9.4 Pressure as Surface Curvature Gradient

In the WILL framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\, c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy–geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\, c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energy–geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

10 Rotational Systems (Kerr–Newman Solutions Without Metric)

10.1 Contextual Bounds

- **For a gravitationally closed (static) system,** the physical boundary is defined by the condition $\kappa^2 = 1$. The closure principle ($\kappa^2 = 2\beta^2$) is what dictates that this corresponds to a kinetic state of $\beta^2 = 1/2$.
- **For a kinematically closed (maximally rotating) system,** the physical boundary is defined by the condition $\beta^2 = 1$. The same closure principle ($\kappa^2 = 2\beta^2$) then necessitates that the corresponding gravitational state must be $\kappa^2 = 2$.

For rotating black holes, we establish the connection between relational kinetic projection and the Kerr metric by defining:

$$\beta = \frac{ac}{Gm_0}, \quad \kappa = \sqrt{2}\beta$$

where:

- β is the relational rotation parameter, with $0 \le \beta \le 1$,
- κ is related to the geometry and rotation,
- $R_s = \frac{2Gm_0}{c^2}$ is the Schwarzschild radius,
- $a = \frac{J}{m_0 c}$ is the Kerr rotation parameter,
- J is the angular momentum of the black hole,
- m_0 is the mass of the black hole.

We also derive a key invariant relationship:

$$a_{\max} = \frac{Gm_0}{c^2} = \frac{R_s}{2} = \beta^2_{\max} r_d$$

This relationship holds when $r_d = \frac{R_s}{2\beta^2}$, providing an elegant connection between the parameters.

10.2 Event Horizon

Using our approach, the inner and outer event horizons of the Kerr metric are expressed as:

$$r_{\pm} = \frac{R_s}{2} (1 \pm \beta_Y)$$

For the extreme case where $\beta = 1$ (maximal rotation), the horizons merge at:

$$r_+ = r_- = \frac{R_s}{2}$$

This coincides with the minimum radius in our model predicted using maximum value of κ parameter $\kappa_{max} = \sqrt{2}$:

$$r_{\min} = \frac{1}{\kappa_{max}^2} R_s = \frac{1}{2} R_s$$

10.3 Ergosphere

The radius of the ergosphere in our model is described as:

$$r_{\text{ergo}} = \frac{R_s}{2} \left(1 + \sqrt{1 - \beta^2 \cos^2 \theta}\right)$$

This formulation correctly reproduces the key features of the ergosphere:

- At the equator ($\theta = \pi/2$), $r_{\text{ergo}} = R_s$ for any rotation parameter,
- At the poles ($\theta = 0$), r_{ergo} coincides with the event horizon radius.

10.4 Ring Singularity

Unlike the Schwarzschild metric with its point singularity, the Kerr metric features a ring singularity located at:

$$r = 0, \quad \theta = \frac{\pi}{2}$$

The "size" of this ring is proportional to $a = \frac{Gm_0}{c} \beta$, reaching its maximum for extreme black holes ($\beta = 1$).

10.5 Naked Singularity

For $\beta \leq 1$, a naked singularity does not emerge, aligning with the cosmic censorship hypothesis. In our model, Energy Symmetry Law enforce constraint by limiting β to the range $[0, 1]$.

10.6 The Relationship Between $\kappa > 1$ and Rotation

For extreme rotation ($\beta = 1$), we find $\kappa = \sqrt{2} > 1$, which reflects the displacement of the event horizon and the geometric properties of rotating black holes. This suggests that values of $\kappa > 1$ are inherently connected to the physics of rotation in spacetime.

This connection suggests that the rotation of a black hole can be understood through geometric parameters analogous to orbital mechanics. Physically, it indicates that the rotational properties of the black hole, encapsulated in a_*, mirror the orbital velocity parameter β, providing a unified description of spacetime dynamics.

Philosophically, this reinforces the notion that gravitational phenomena, including rotation, are manifestations of the underlying geometry of the universe. The absence of additional "material" parameters underscores the elegance of general relativity, where the curvature of spacetime alone dictates the behavior of massive rotating objects. This geometric interpretation bridges the gap between the abstract mathematics of the Kerr metric and the intuitive physics of orbital motion, offering a deeper insight into the nature of spacetime.

Physical Interpretation

- **No need for pre-existing spacetime** — geometry emerges from angular energy distributions.
- **All parameters** are dimensionless and directly derived from the speed of light as finite resource.
- **Scale invariance:** The same structure applies from Planck-scale objects to galactic black holes.

11 Unified Geometric Field Equation

11.1 The Theoretical Uroboros

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation:

$$\boxed{\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}} \tag{19}$$

The ratio of geometric scales equals the ratio of energy densities.

This is the unified geometric field equation of WILL Geometry. It expresses the complete equivalence:

GEOMETRY ≡ ENERGY DISTRIBUTION

We have shown that this single postulate, through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with a single fundamental statement about energy and its evolution, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and space, proving the self-consistency of the initial postulate.

img/Theoretical-Ouroboros-min.jpg

The foundational postulate SPACETIME ≡ ENERGY EVOLUTION closes into the unified field equation

$$\frac{R_s}{r_d} = \frac{\rho}{\rho_{\max}},$$

Theoretical Uroboros

The WILL framework exhibits perfect logical closure: the fundamental postulate about the nature of spacetime and energy is proven as the inevitable consequence of geometric consistency.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework — the "Theoretical Uroboros". But regardless of aesthetic beauty of this result let's remain skeptical.

11.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r_d} = \frac{8\pi G}{c^2} r_d^2 \, \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but nonsingular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

12 Beyond Differential Formalism: Structural Dynamics

12.1 Intrinsic Dynamics via Energy Redistribution

The system is not described by differential equations of motion evolving *in* time. Instead, its evolution is dictated by a closed network of algebraic relations that enforce a perpetually balanced configuration. Any change in one parameter necessitates a coordinated shift in all others to maintain geometric self-consistency. What we perceive as "dynamics" is this ordered succession of balanced states. The framework thus inverts the classical paradigm:

The foundation of WILL Geometry lies in the principle that spacetime geometry is fully determined by the distribution of energy, parameterized by the dimensionless quantities β and κ. Any change in the system's energy—due to mass variation, motion, or redistribution—directly reshapes the geometry. This intrinsic linkage ensures that dynamics is embedded within the model itself, without requiring external equations of motion.

This reveals a fundamental inversion of the classical paradigm:

Time does not drive change — instead, change defines time.

Why There Are No Equations of Motion

In classical and relativistic physics, dynamics is formulated through differential equations. These express how physical quantities evolve continuously through time, typically governed by:

- A temporal parameter t,
- A Lagrangian function L,
- A variational principle: $\delta S = 0$, where $S = \int L \, dt$,
- Euler–Lagrange equations that yield the system's path.

This framework assumes:

1. A continuum of possible configurations,
2. That Nature selects one by minimizing action,
3. That time flows independently of the system.

Why This Framework Does Not Apply to WILL Geometry

WILL Geometry begins from a fundamentally different premise.

- There is no "space" of possible paths.
- There is no "freedom to vary."
- The system does not evolve through time — it **defines time** through its structure.

In this model:

- Each observable is locked in a network of algebraic relations.
- Any change in one parameter *necessitates* coherent changes in the others.
- Self-consistency enforces projectional **balance**.

There is only one valid configuration at any moment: the one where all projectional constraints are satisfied. Everything else is not forbidden — it is undefined.

Geometric Principle of Action

In WILL Geometry, there is no equation of motion. There is no Lagrangian. There is no variational calculus.
There is only a closed system of geometric and energetic relationships, and the sequence of valid configurations is what we call *dynamics*.

The only necessary input:

The observable sequence of transformations in the system's energy geometry is the only necessary input for describing its evolution. .

12.2 Time as an Emergent Property

In this framework, time is not a fundamental entity but a derived concept tied to changes in the system's geometry. Similar to time dilation in special relativity, time intervals here are defined by the evolution of geometric parameters like r_d and κ. This eliminates the need for an external clock.

A natural time scale arises from the geometry as $t_d = \frac{r_d}{c}$. For instance, during mass accretion onto a black hole, r_d adjusts as the mass changes, and t_d evolves accordingly. This intrinsic time scale encapsulates the system's dynamics without invoking an independent time variable.

Note for readers accustomed to classical dynamics

Unlike traditional formulations of dynamics based on an external time parameter t, the WILL Geometry framework describes evolution as an intrinsic transformation of the system's geometric structure. Time is not a fundamental variable but an emergent quantity derived from energy redistribution. All physical change is encoded in the interdependence of parameters β, κ, and ρ, etc without requiring differential equations or initial conditions.

This marks a fundamental shift: dynamics here is not a process unfolding "in time," but a change in relational energy geometry itself. The model does not track evolution through an imposed temporal axis — instead, it reveals that what we perceive as temporal progression is a manifestation of continuous geometric reconfiguration. **Thus, prediction becomes a question of geometric continuity, not temporal evolution.**

From Structure to Motion

In the next section, we present the core structural closure of the system — a set of algebraic invariants that together form the backbone of all observed dynamics. These relations are not definitions. They are the **complete geometry of change**, seen from within.

> **Dynamics in WILL Geometry**
>
> Dynamics in WILL Geometry is not described by differential equations but by the ordered succession of globally balanced, algebraically determined configurations.

12.3 Algebraic Closure and Structural Causality

In WILL Geometry, physical dynamics emerges from a set of algebraically closed invariants. Each parameter participates in a self-consistent configuration of relational constraints. There are no functions, no dependent variables, and no variation over time — only balanced configurations.

The following set of relations expresses the minimal algebraic closure of the WILL structure:

$$\begin{cases} \kappa^2 = 2\beta^2 \\ R_s = \dfrac{2Gm_0}{c^2} \\ r_d \cdot \kappa^2 = R_s \\ \rho = \dfrac{\kappa^2 c^2}{8\pi G r_d^2} \\ H = \dfrac{c}{r_d} \\ \Lambda = \dfrac{\kappa^2}{r_d^2} \\ m_0 = 4\pi r_d^3 \cdot \rho \end{cases}$$

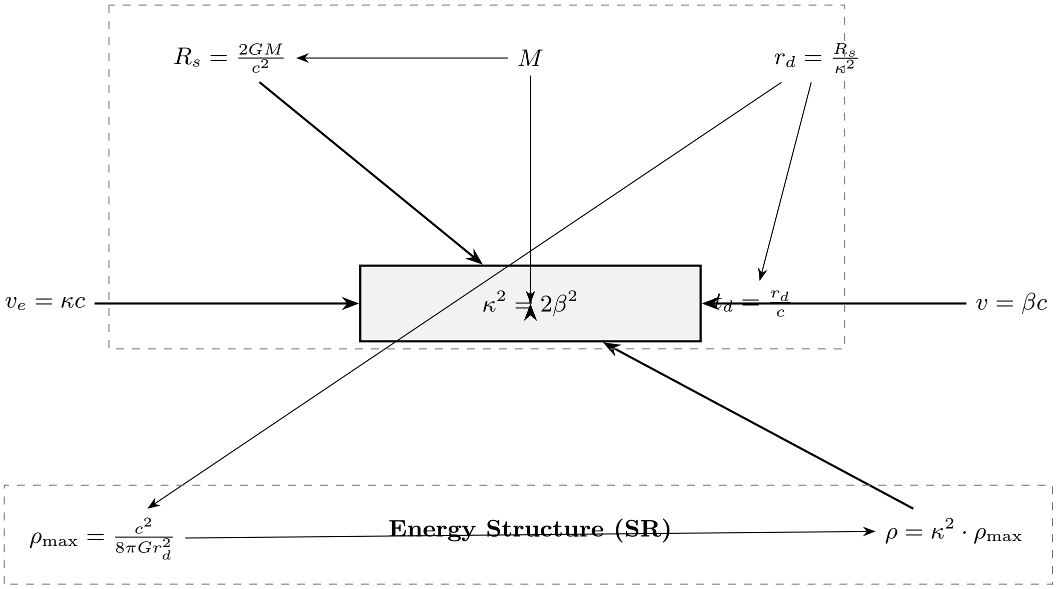
These are not definitions. They are mutual constraints — an algebraic simultaneity. Changing any one parameter necessitates a coordinated shift in all others to maintain validity.

Causal Closure without Circularity

The structure of WILL Geometry is causally closed but not circular. Each parameter is either independently observable or computable from a minimal input pair consisting of one dynamic projection (such as κ or β) and one scale quantity (such as r, M, or ρ).

The system avoids circularity by ensuring that no parameter both defines and is defined by the same input. Instead, values propagate through directed dependencies rooted in physically measurable quantities. Multiple valid entry points exist, but all reduce to consistent, non-redundant relationships governed by the fundamental geometric field equation.

Geometric Structure (GR)



The result is a structure where **causality is internal**, **coherence is enforced**, and **dynamics is simply the shifting of balanced configurations** — not the unfolding of arbitrary functions over time.

12.4 Numerical Example: Accretion onto a Black Hole

Consider a black hole accreting mass from a surrounding disk to illustrate the model's intrinsic dynamics. Let the initial mass be $m_0 = 10, M_\odot$, with a Schwarzschild radius $R_s = \frac{2Gm_0}{c^2} \approx 2.95 \times 10^4, \mathrm{m}$. Suppose $\kappa = 0.1$, so $r_d = \frac{R_s}{\kappa^2} = \frac{2.95 \times 10^4}{0.01} = 2.95 \times 10^6, \mathrm{m}$, and the associated time scale is $t_d = \frac{r_d}{c} \approx 9.83 \times 10^{-3}, \mathrm{s}$.

As the black hole accretes mass, increasing to $m_1 = 10.1, M_\odot$, the Schwarzschild radius becomes $R_s \approx 2.98 \times 10^4, \mathrm{m}$. Assuming κ remains constant for simplicity, $r_d = \frac{2.98 \times 10^4}{0.01} = 2.98 \times 10^6, \mathrm{m}$, and $t_d \approx 9.93 \times 10^{-3}, \mathrm{s}$. This increase in t_d reflects the system's evolution, driven solely by the changing geometry.

No differential equations are required:

Dynamics unfolds as a consequence of relational energy evolution.

12.5 General Principle of Dynamics

The overarching principle in WILL Geometry is that any physical change—be it mass accretion, gravitational collapse, or expansion—manifests as a transformation in the geometric structure. The parameters β, κ, and r_d adjust self-consistently, ensuring that the system's evolution is fully described within the model. This approach eliminates the need for differential equations of motion or external initial conditions, as the geometry at any given "moment" is determined by the current energy configuration.

Thus, the temporal sequence we observe is simply the ordered unfolding of geometric transitions:

$$\boxed{\text{Time} \equiv \text{Change in } (\kappa, \beta, \rho, r_d ...)}$$

This leads to a unified, self-consistent model where **geometry generates both evolution and observability**. There is no external timeline — only evolving curvature.

12.6 Conclusion

In conclusion, dynamics in WILL Geometry emerges naturally from the redistribution of energy within the system's geometric framework. Time arises as a consequence of these geometric changes, providing a unified description of spacetime and energy evolution. This intrinsic approach simplifies the treatment of dynamical processes and offers a novel perspective on the nature of physical systems.

Geometric Principle of Evolution

Physics is not the evolution of a system through time,
but the geometric transformation of the energy landscape,
where *"time"* is simply the name we give to the sequence of such transitions.

13 The Fundamental Invariant $W_{\text{ill}} = 1$

From the geometric closure of WILL framework, we derive a universal dimensionless invariant:

$$W_{ill} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = 1 \tag{20}$$

Proof: Substituting the geometric definitions:

$$W_{\text{ill}} = \frac{\frac{1}{\cos\theta_2} \; m_0 c^2 \cos\theta_2 \frac{r_d^2}{c^2}}{\frac{1}{\sin\theta_1} \; m_0 \sin\theta_1 r_d^2} = \frac{m_0 c^2 r_d^2}{c^2} \cdot \frac{\sin\theta_1}{m_0 r_d^2} \cdot \frac{1}{\sin\theta_1} = 1 \tag{21}$$

This invariant holds universally for all values of m_0, G, c, and κ. Unlike dimensional analysis, this identity emerges from the projectional interdependence of energy-mass (E, M) and spacetime metrics (T, L) within the unified structure.

$$\boxed{\text{The invariant } W_{\text{ill}} = 1 \text{ expresses geometric unity through energetic projection}}$$

13.1 The Name "WILL"

The name WILL reflects both the harmonious unity of the equation and a subtle irony towards the anthropic principle, which often intertwines human existence with the causality of the universe. The equation stands as a testament to the universal laws of physics, transcending any anthropocentric framework.

WILL

$$\boxed{\textbf{It is not the unit of something—it is the unity of everything.}}$$

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY EVOLUTION}}$$

All the derived relations for local energy density, pressure, enclosed mass, and horizon formation are purely logical and mathematical consequences of this unique guiding principle. In this sense, the WILL Geometry framework achieves absolute epistemological cleanliness: it does not introduce any hidden assumptions or coordinate structures. The entire gravitational and relativistic sector (as reconstructed within the WILL Geometry model, from the single postulate without any external assumptions or coordinate backgrounds) — including precise predictions about the onset of horizon formation, radial pressure gradients, and the resolution of classical singularity issues—is reconstructed from this single postulate.

Meaning

Energy is not "inside" space — it defines space through projection.

- **Surface–scaled closure (vs. volume filling).** Mass follows the algebraic closure $m_0 = 4\pi r_d^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r_d^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Algebraic core (vs. differential ansatz).** The core invariants are algebraic and coordinate-free. Differential relations appear only as auxiliary scaffolding (e.g. to express curvature gradients as $P = -\rho c^2$), not as fundamental inputs.
- **Natural bounds.** The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r_d^2)$, avoiding singularities without extra hypotheses.

14 Correspondence with General Relativity

14.1 Equivalence with Schwarzschild Solution

Theorem 4 (Equivalence with Schwarzschild Solution)**.** The WILL Geometry formalism reproduces the Schwarzschild metric in the appropriate limit.

Proof. The Schwarzschild metric in General Relativity is given by:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{22}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the unit sphere.

In WILL Geometry, the key parameters are:

$$\kappa^2 = \frac{R_s}{r} = \frac{2GM}{rc^2} \tag{23}$$

$$\kappa_X = \sqrt{1-\kappa^2} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{24}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{25}$$

The time component of the Schwarzschild metric can be written as:

$$g_{tt} = \left(1 - \frac{2GM}{rc^2}\right) = 1 - \kappa^2 = -\kappa_X^2 \tag{26}$$

And the radial component can be written as:

$$g_{rr} = -\left(1 - \frac{2GM}{rc^2}\right)^{-1} = -\frac{1}{1-\kappa^2} = -\frac{1}{\kappa_X^2} \tag{27}$$

Therefore, in WILL Geometry terms, the Schwarzschild metric takes the form:

$$ds^2 = \kappa_X^2 c^2 dt^2 - \frac{1}{\kappa_X^2} dr^2 - r^2 d\Omega^2 \tag{28}$$

This demonstrates that the WILL Geometry parameters exactly reproduce the Schwarzschild metric. □

14.2 Equivalence with Einstein Field Equations

Theorem 5 (Equivalence with Einstein Field Equations)**.** The geometric field equation of WILL Geometry is equivalent to Einstein's field equations.

Proof. Einstein's field equations in General Relativity are:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{29}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor.

For a spherically symmetric system with perfect fluid, the *tt*-component of Einstein's equations reduces to:

$$\frac{1}{r^2}\frac{d}{dr}\left(r\left(1 - \frac{1}{g_{rr}}\right)\right) = \frac{8\pi G}{c^2}\rho(r) \tag{30}$$

In Energy Geometry terms:

$$g_{rr} = -\frac{1}{\kappa_X^2} \tag{31}$$

$$1 - \frac{1}{g_{rr}} = 1 - (-\kappa_X^2) = 1 + \kappa_X^2 = 1 + (1-\kappa^2) = 2 - \kappa^2 \tag{32}$$

Substituting:

$$\frac{1}{r^2}\frac{d}{dr}(r(2-\kappa^2)) = \frac{8\pi G}{c^2}\rho(r) \tag{33}$$

$$\frac{1}{r^2}\left(\frac{d}{dr}(2r) - \frac{d}{dr}(r\kappa^2)\right) = \frac{8\pi G}{c^2}\rho(r) \tag{34}$$

$$\frac{1}{r^2}\left(2 - \frac{d}{dr}(r\kappa^2)\right) = \frac{8\pi G}{c^2}\rho(r) \tag{35}$$

For a static distribution where κ is only a function of r:

$$-\frac{1}{r^2}\frac{d}{dr}(r\kappa^2) = \frac{8\pi G}{c^2}\rho(r) - \frac{2}{r^2} \tag{36}$$

$$\tag{37}$$

The term $\frac{2}{r^2}$ corresponds to the vacuum solution. For matter content:

$$-\frac{d}{dr}(r\kappa^2) = \frac{8\pi G}{c^2}r^2\rho(r) - 2 \tag{38}$$

$$\tag{39}$$

Taking the derivative of both sides with respect to r and simplifying:

$$-\frac{d^2}{dr^2}(r\kappa^2) = \frac{8\pi G}{c^2}\frac{d}{dr}(r^2\rho(r)) \tag{40}$$

This is directly related to our geometric field equation:

$$\frac{d}{dr}(\kappa^2 r) = \frac{8\pi G}{c^2}r^2\rho(r) \tag{41}$$

Therefore, the WILL Geometry field equation is equivalent to Einstein's field equations. $\square$

Comparison Table: General Relativity (GR) vs WILL Framework

#	Category	**General Relativity (GR)**	**WILL Framework**
1	Nature of Space and Time	Postulated as smooth manifold with metric $g_{\mu\nu}$	Emerges from projection of energy relations (κ, β)
2	Curvature	Defined via $R_{\mu\nu}, R$; second derivatives of the metric	Defined algebraically as $\kappa^2 = \frac{R_s}{r}$
3	Energy and Momentum	Encoded in $T_{\mu\nu}$, requires model of matter	Directly given by $\rho(r)$, $\rho_{\max}(r)$, and $p(r)$
4	Geometry–Matter Relation	$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$; differential equation	$\kappa^2 = \rho/\rho_{\max}$; local proportionality
5	Singularities	Appear when $\rho \to \infty$, $g_{00} \to 0$	Excluded by construction: $\rho \leq \rho_{\max}$, $\kappa^2 \leq 1$
6	Gravitational Limitation	Via metric behavior and horizons	Via geometric constraint $\kappa \in [0, 1]$
7	Density Limit	Not explicitly defined, requires external input (Planck-scale)	Explicitly defined: $\rho_{\max} = \frac{c^2}{8\pi G r^2}$
8	Concept of Time	Coordinate-based, embedded in g_{00}; system-dependent	Physical: β as projection of energy onto temporal axis
9	Dynamics	Via time derivatives and Lagrangians	Via change in energy proportions; no differential equations
10	Formalism	Geometry, tensors, 2nd-order derivatives	Energy projections, circular geometry, algebraic closure
11	Intuitiveness	Low; relies on abstract and heavy formalism	High; built from observable and intrinsic relations
12	Observational Fit	Confirmed (with dark matter/energy assumptions)	Consistent; explains phenomena without "dark entities"

Phenomenon	**Radius** r	κ^2	β^2	**Comment**
Photon sphere	$r = \frac{3}{2}R_s$	$\frac{2}{3}$	$\frac{1}{3}$	Null circular orbits, $Q = 1$, $Q_t = 0$
ISCO (innermost stable orbit)	$r = 3R_s$	$\frac{1}{3}$	$\frac{1}{6}$	Marginal stability of timelike orbits
Static horizon (Schwarzschild)	$r = R_s$	1	$\frac{1}{2}$	Purely gravitational closure, $\kappa^2 = 2\beta^2$
Extremal Kerr horizon	$r = \frac{1}{2}R_s$	2	1	Maximal rotation, $\beta = 1$, merged horizons

Table 2: Critical radii and their projectional parameters in WILL Geometry. All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as special values of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$.

14.3 Asymmetric Generality

The correspondence between these frameworks is fundamentally asymmetric. General Relativity, with its reliance on a pre-supposed metric tensor and the formalism of differential geometry, can be viewed as a specific, parameter-heavy instance of the WILL framework's principles. One can derive GR by adding these additional structures to WILL's minimalist foundation. Therefore, the choice between them is not one of preference, but of logical generality and parsimony, with WILL being presented as the more fundamental underlying structure.

15 Relational Foundation Theorem: WILL vs. GR

Definition 1 (GR Core Axioms)**.** General Relativity (GR) is assumed to rest on the following axioms:

(A1) The spacetime arena is a smooth Lorentzian manifold with metric $g_{\mu\nu}$.

(A2) Diffeomorphism invariance (general covariance): the form of physical laws is independent of coordinates.

(A3) Local Lorentz invariance / Einstein equivalence principle: locally, spacetime is Minkowskian.

(A4) Einstein Field Equations (EFE): $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$.

Definition 2 (WILL Foundational Postulate)**.** The WILL framework is based on a single axiom:

(W1) **Relational Postulate:** All physical magnitudes are defined purely by relations between entities; spacetime is equivalent to energy evolution.

Lemma 2 (Relationality in GR)**.** From A2 and A3 it follows that observable quantities in GR are coordinate-independent and must be expressed relationally. In particular, no absolute magnitudes can serve as observables.

[Bridge: From Relational Postulate to GR Axioms] If the Relational Postulate (W1) were false, then physical magnitudes could in principle be defined in absolute, non-relational terms. Such absolutes would provide a hidden external reference structure. But this contradicts the core of GR:

- It violates diffeomorphism invariance (A2), since coordinate independence presupposes that only relational quantities are observable.
- It undermines the equivalence principle (A3), since local Minkowski structure relies on the impossibility of distinguishing absolute magnitudes from relative ones.

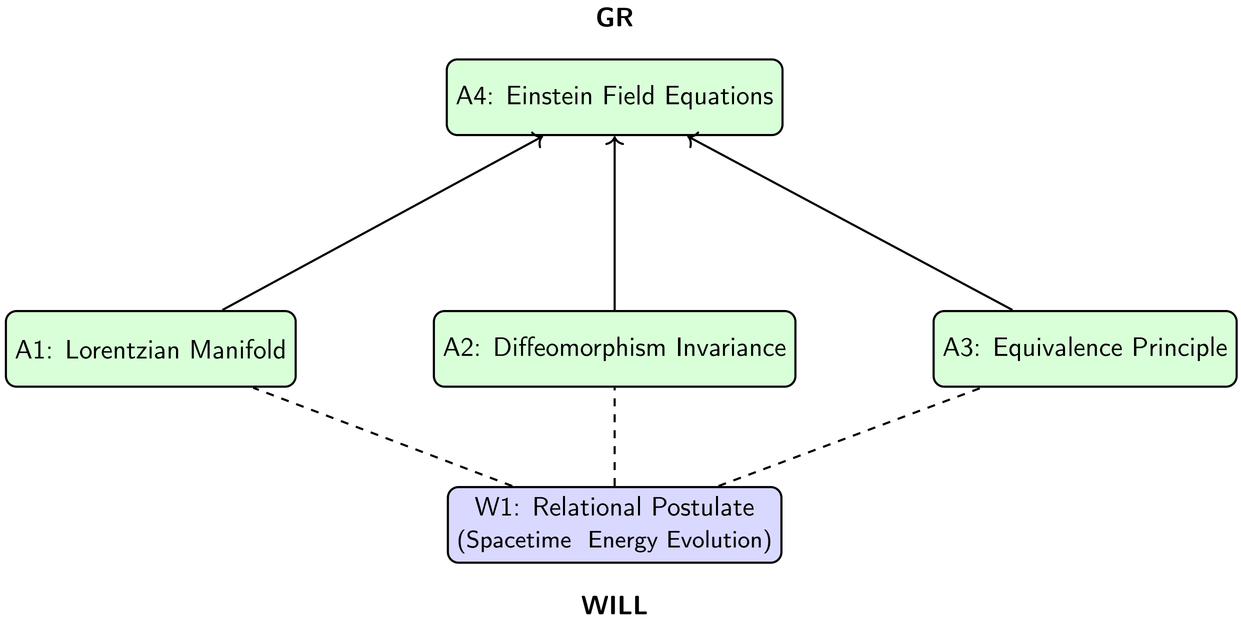
Therefore, the negation of W1 directly negates A2 and A3. This establishes the logical dependency required for the asymmetry theorem below.

Theorem 6 (Asymmetric Falsifiability of GR and WILL)**.** Let GR denote the theory defined by axioms (A1)–(A4), and let WILL denote the theory defined by postulate (W1). Then:

1. If (W1) is empirically falsified, then (A2)–(A3) are also falsified. Hence, GR is necessarily falsified.
2. If any of (A1)–(A3) are empirically falsified, GR collapses, but (W1) may still remain valid as a stand-alone principle.

Therefore, there exist possible empirical scenarios in which GR fails while WILL survives, but there exist no scenarios in which WILL fails while GR survives.

Corollary 1. WILL is axiomatically more fundamental than GR: its sole postulate (W1) is logically included within the core axioms of GR, while GR requires additional ontological structures (metric geometry, equivalence principle, Einstein equations) that are not necessary for the consistency of WILL.



Conclusion (Axiomatic Inclusion and Asymmetric Falsifiability).
WILL rests on the single relational postulate (W1). Core GR assumes additional structures (A1–A4). Hence:

- If **W1** is empirically falsified, GR's core (A2–A3) is undermined; thus GR is falsified.
- If any of **A1–A3** is falsified, GR collapses, while **W1** (and thus WILL) may still hold.

Therefore, there are scenarios where GR fails and WILL survives, but none where WILL fails while GR survives.

Status of General Relativity within WILL

It is important to emphasize that the WILL framework does not *invalidate* the achievements of General Relativity. Rather, it *explains them*. All celebrated predictions of GR — gravitational lensing, perihelion precession, photon spheres, ISCO, horizons — emerge in WILL Geometry as direct consequences of the single closure relation $\kappa^2 = 2\beta^2$.
Thus, GR is not a rival but a **specialized, parameter-heavy realization** of WILL's more general relational principle. In logical terms:

- WILL can stand without GR, but GR cannot stand without the relational postulate (W1).
- The empirical successes of GR are preserved within WILL, but its pathologies (singularities, dependence on dark entities, ambiguous notion of rest) are avoided.

Therefore, GR should be understood as an **effective approximation** embedded in a deeper relational framework. This perspective retains full respect for the historical and observational triumphs of Einstein's theory, while at the same time recognizing its status as a non-fundamental limit of a more parsimonious principle.

16 Empirical Validation

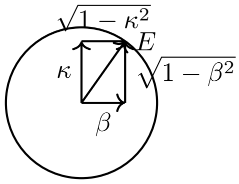
16.1 Geometric Prediction of Photon Sphere and ISCO

Theorem 7 (Critical Radii Emergence)**.** In the WILL Geometry framework, the critical orbital radii of the photon sphere and innermost stable circular orbit (ISCO) emerge naturally from the geometric equilibrium where $\theta_1 = \theta_2$.

Proof. A notable geometric equilibrium occurs at the critical angle

$$\theta_1 = \theta_2 = 54.7356103172^\circ \text{ (balance point for photon sphere and ISCO)} \tag{42}$$

or approximately $\theta_1 = \theta_2 \approx 0.9553$ radians.



This equilibrium yields the fundamental relation:

$$\kappa^2 + \beta^2 = 1, \tag{43}$$

These critical radii emerge spontaneously from the geometry, suggesting inherent spacetime structure without additional assumptions.

16.1.1 Mathematical Derivation of Critical Points

Key critical points include: When:

- $\kappa = \sqrt{\frac{2}{3}} \approx 0.816$ and $\beta = \frac{1}{\sqrt{3}} \approx 0.577$, corresponding to:

$$r = \frac{R_s}{\kappa^2} = \frac{3}{2}R_s = 1.5R_s \quad \text{(radius of the photon sphere).}$$

When:

- $\kappa = \sqrt{\frac{1}{3}} \approx 0.577$, and $\beta = \frac{1}{\sqrt{6}} \approx 0.408$, leading to orbital distance:

$$r = \frac{R_s}{2\beta^2} = \frac{R_s}{2 \cdot \frac{1}{6}} = 3R_s \quad \text{(radius of the innermost stable circular orbit, ISCO).}$$

At the critical point where $\beta = \frac{1}{\sqrt{3}}$ and $\kappa = \sqrt{\frac{2}{3}}$, the following relationships hold:

$$\theta_1 = \theta_2 \tag{44}$$

$$\beta = \kappa_X \tag{45}$$

$$\kappa = \beta_Y \tag{46}$$

$$\cos(\theta_G - \theta_S) = 0 \tag{47}$$

$$Q = \sqrt{\kappa^2 + \beta^2} = 1 \tag{48}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - 3\beta^2} = 0 \quad \text{(Instability threshold)} \tag{49}$$

Critical Radii from the Q–Invariant

The combined projection parameter

$$Q^2 \;=\; \kappa^2 + \beta^2, \qquad Q_t \;=\; \sqrt{1 - Q^2}\,,$$

measures the total causal budget of energy evolution. Critical orbital radii correspond to special conditions of Q and Q_t:

- **Photon Sphere ($r = 1.5R_s$).** At this radius the causal budget is fully exhausted:

$$Q = 1, \qquad Q_t = 0,$$

leaving no timelike separation. Only null (photon) orbits are permitted.

- **Innermost Stable Circular Orbit (ISCO, $r = 3R_s$).** Here the system reaches the stability threshold: Q_t remains nonzero, but its radial slope vanishes,

$$\frac{dQ_t}{dr} = 0, \qquad Q_t = \frac{1}{\sqrt{2}}.$$

This marks the last orbit where timelike geodesics remain stable.

Thus, both the photon sphere and ISCO emerge directly as critical solutions of the same invariant structure (Q, Q_t), without additional assumptions.

Interpretive Note

While the radii $1.5R_s$ (photon sphere) and $3R_s$ (ISCO) are known from classical General Relativity, their spontaneous emergence from angle equality $\theta_1 = \theta_2$ in our geometric framework is not imposed but arises from internal energy projection symmetries. This correspondence reinforces the internal consistency and explanatory power of WILL Geometry.

Projectional Principle

Geometry defines causality before mass, and curvature before gravity.

□

16.2 Energy Symmetry Validation: GPS Satellite and Earth

Theorem 8 (Real-World Energy Symmetry)**.** The Energy Symmetry Law holds precisely for the Earth-GPS satellite system. WILL-invariant ($W_{\text{ill}} = 1$) holds exactly for the Earth–GPS satellite system.

Proof. We verify the Energy Symmetry Law on real orbital data for a GPS satellite and an observer on the Earth's surface, using the following parameters:

Input Data

- Gravitational constant: $G = 6.67430 \times 10^{-11}$ m^3/kg/s^2
- Speed of light: $c = 2.99792458 \times 10^8$ m/s
- Mass of Earth: $M_{Earth} = 5.972 \times 10^{24}$ kg
- Radius of Earth: $R_{Earth} = 6.370 \times 10^6$ m
- Radius of GPS orbit: $r_{GPS} = 2.6571 \times 10^7$ m
- Seconds in 24 hours $D_{ayS} = 86400 \quad$ s
- Microseconds in 1 second $M_{icro} = 10^6 \quad \mu$ s

The orbital velocity of the GPS satellite is:

$$v_{GPS} = \sqrt{\frac{GM_{Earth}}{r_{GPS}}} = \sqrt{\frac{6.67430 \times 10^{-11} \times 5.972 \times 10^{24}}{2.6571 \times 10^7}} = 3873.10090455 \text{ m/s} \tag{50}$$

Converting to dimensionless parameters:

$$\beta_{GPS} = \frac{v_{GPS}}{c} = \frac{3873.10090455}{2.99792458 \times 10^8} = 0.0000129192739884 \tag{51}$$

$$\kappa_{GPS} = \sqrt{\frac{2GM_{Earth}}{c^2 r_{GPS}}} = 0.0000182706124904 \tag{52}$$

$$\frac{\kappa^2_{GPS}}{\beta^2_{GPS}} = \frac{3.3381528077 \times 10^{-10}}{1.6690764039 \times 10^{-10}} = 2 \tag{53}$$

$$Q_{GPS} = \sqrt{\beta^2_{GPS} \ + \kappa^2_{GPS}} = 0.0000223768389448 \tag{54}$$

$$Q_{tGPS} = \sqrt{1 - Q^2_{GPS}} = \sqrt{1 - 3\beta^2_{GPS}} = 0.99999999975 \tag{55}$$

For the Earth's surface:

$$\kappa_{Earth} = \sqrt{\frac{2GM_{Earth}}{c^2 R_{Earth}}} = 0.000037312405944 \tag{56}$$

$$\beta_{Earth} = 0 \text{ (at rest)} \tag{57}$$

$$Q_{Earth} = \sqrt{\beta^2_{Earth} \ + \kappa^2_{Earth}} = 0.000037312405944 \tag{58}$$

$$Q_{tEarth} = \sqrt{1 - Q^2_{Earth}} = 0.999999999304 \tag{59}$$

The daily relativistic time offset between GPS and Earth is:

$$\Delta Q_{t,\text{GPS}\to\text{Earth}} = (1 - \frac{Q_{tEarth}}{Q_{tGPS}}) \cdot D_{ayS} \cdot M_{icro} = 38.52 \ \mu\text{s/day}$$

This **matches the empirical time correction required** for GPS synchronization to high precision.

16.2.1 WILL invariant validation:

W_{ILLGPS} parameters of mass energy time and space:

$\mathrm{M}_{GPS} = T_{dGPS}\beta^2_{GPS}c^2\frac{r_{GPS}}{G}$

$\mathrm{E}_{GPS} = L_{dGPS}\frac{\kappa^2_{GPS}c^4 r_{GPS}}{2G}$

$\mathrm{T}_{GPS} = T_{cGPS}\left(\frac{2GM_{Earth}}{\kappa^2_{GPS}c^3}\right)^2$

$\mathrm{L}_{GPS} = L_{cGPS}\left(\frac{GM_{Earth}}{\beta^2_{GPS}c^2}\right)^2$

Where:

$\mathrm{L}_{cGPS} = \sqrt{1 - \beta^2_{GPS}}$

$\mathrm{T}_{dGPS} = \frac{1}{\sqrt{1-\beta^2_{GPS}}}$

$\mathrm{L}_{dGPS} = \frac{1}{\sqrt{1-\kappa^2_{GPS}}}$

$\mathrm{T}_{cGPS} = \sqrt{1 - \kappa^2_{GPS}}$

The explicit $W_{ILL} = ET^2/ML^2 = 1$ - invariant for the GPS–Earth system, including both GR and SR effects, is:

$$W_{ILLGPS} = \frac{E_{GPS} \cdot T_{GPS}}{M_{GPS} \cdot L_{GPS}} = \frac{L_{dGPS}\frac{\kappa^2_{\text{GPS}}c^4 r_{\text{GPS}}}{2G} \cdot T_{cGPS}\left(\frac{2GM_{\text{Earth}}}{\kappa^2_{\text{GPS}}c^3}\right)^2}{T_{dGPS}\frac{\beta^2_{\text{GPS}}c^2 r_{\text{GPS}}}{G} \cdot L_{cGPS}\left(\frac{GM_{\text{Earth}}}{\beta^2_{\text{GPS}}c^2}\right)^2} = 1$$

All physical quantities cancel identically, leaving $L_{dGPS}T_{cGPS} = T_{dGPS}L_{cGPS}$, which is satisfied by geometric construction.

16.2.2 Energy Symmetry Law validation:

The energy difference from Earth observer to GPS satellite is:

$$\Delta E_{Earth\to GPS} = \frac{1}{2}((\kappa^2_{Earth} - \kappa^2_{GPS}) + \beta^2_{GPS}) = 6.1265399845 \times 10^{-10} \tag{60}$$

The energy difference from GPS satellite to Earth is:

$$\Delta E_{GPS\to Earth} = \frac{1}{2}((\kappa^2_{GPS} - \kappa^2_{Earth}) - \beta^2_{GPS}) = -6.1265399845 \times 10^{-10} \tag{61}$$

Therefore:

$$\Delta E_{GPS\to Earth} + \Delta E_{Earth\to GPS} = -6.1265399845 \times 10^{-10} + 6.1265399845 \times 10^{-10} = 0 \tag{62}$$

This **confirms the Energy Symmetry Law** to machine precision using real-world orbital and physical data.

And lets also compare the results with real total energy. We will take aproximate mass of GPS satellite:

$$m_{sat} = 600 \quad kg$$

Classical potential energy $E_{pGPS} = (-\frac{GM_{Earth}m_{sat}}{r_{GPS}}) - (-\frac{GM_{Earth}m_{sat}}{R_{Earth}})$
Classical kinetic energy $E_{kGPS} = \frac{1}{2}m_{sat}\ v^2_{GPS}$
Classical total energy $E_{tot} = E_{pGPS} + E_{kGPS} = 3.3043450143 \times 10^{10} \quad kg(m^2/s^2)$
Conformation of nontriviality of Energy Symmetry Law.

$$\frac{\frac{E_{tot}}{m_{sat}c^2}}{\Delta E_{Earth\to GPS}} = 1$$

Remarkably, satellite mass m_{sat} does not appear anywhere in the computation of the geometric energy $\Delta E_{Earth\to GPS}$. Yet, when the total physical energy E_{tot} (the sum of kinetic and potential energies, which does depend on m_{sat} is normalized by the satellite's rest energy $E_{0GPS} = m_{sat}c^2$ the ratio exactly matches the unitless geometric value as shown above. This confirms that $\Delta E_{Earth\to GPS}$ precisely encodes the relationship between the system's real energy and the satellite's own rest energy, entirely independent of its absolute mass. In other words, the geometric projection captures the physical "share" of energy—purely as a relational quantity—regardless of the object's individual mass.

Physical Logic:

- All gravitational and velocity (SR+GR) effects are simple projections, not metric-dependent.
- No tensors, no differentials, no explicit metric.
- The universe's "time flow" at each location is just a geometric combination of energy projections.

"Time does not drive change — instead, change defines time."

□

16.3 Relativistic Precession Validation: Mercury and the Sun

Theorem 9 (Relativistic Precession Calculation via WILL Geometry)**.** The relativistic precession of Mercury's orbit matches the classical GR result with high precision, using WILL Geometry projection parameters.

Proof. We verify the precession of Mercury's orbit using WILL Geometry and compare it to the GR prediction.

Input physical parameters:

- Gravitational constant: $G = 6.67430 \times 10^{-11}$ m^3/kg/s^2

- Speed of light: $c = 2.99792458 \times 10^8$ m/s
- Mass of the Sun: $M_{\text{Sun}} = 1.98847 \times 10^{30}$ kg
- Schwarzschild radius of the Sun: $R_{Ssun} = 2.953$ km $= 2953$ m
- Semi-major axis of Mercury: $a_{Merc} = 5.79 \times 10^{10}$ m
- Eccentricity of Mercury's orbit: $e_{Merc} = 0.2056$

Dimensionless projection parameters for Mercury:

$$\kappa_{Merc} = \sqrt{\frac{R_{Ssun}}{a_{Merc}}} = \sqrt{\frac{2953}{5.79 \times 10^{10}}} = 0.000225878693163 \tag{63}$$

$$\beta_{Merc} = \sqrt{\frac{R_{Ssun}}{2a_{Merc}}} = \sqrt{\frac{2953}{2 \times 5.79 \times 10^{10}}} = 0.000159720355661 \tag{64}$$

Combined energy projection parameter:

$$Q_{Merc} = \sqrt{\kappa^2_{Merc} + \beta^2_{Merc}} = 0.000276643771008$$

$$Q^2_{Merc} = 3\beta^2_{Merc} = 3 \times (0.000159720355661)^2 = 7.6531776038 \times 10^{-8} \tag{65}$$

Correction factor for the elliptic orbit divided by 1 orbital period:

$$\frac{1 - e^2_{Merc}}{2\pi} = \frac{1 - (0.2056)^2}{2 \times 3.14159265359} = \frac{0.9577}{6.28318530718} = 0.152427247197 \tag{66}$$

Final WILL Geometry precession result:

$$M_{PWILL} = \frac{3\beta^2_{Merc}}{\frac{1-e^2_{Merc}}{2\pi}} = \frac{2\pi Q^2_{Merc}}{(1 - e^2_{Merc})} = \frac{7.6531776038 \times 10^{-8}}{0.152427247197} = 5.0208724126 \times 10^{-7} \tag{67}$$

Classical GR prediction for precession:

$$M_{PGR} = \frac{3\pi R_{Ssun}}{a_{Merc}(1 - e^2_{Merc})} = \frac{3 \times 3.14159265359 \times 2953}{5.79 \times 10^{10} \times 0.9577} = 5.0208724126 \times 10^{-7} \tag{68}$$

Relative difference:

$$\frac{M_{PGR} - M_{PWILL}}{M_{PGR}} \times 100 = \frac{5.0208724126 \times 10^{-7} - 5.0208724126 \times 10^{-7}}{5.0208724126 \times 10^{-7}} \times 100 \tag{69}$$

$$= 2.1918652104 \times 10^{-10}\% \tag{70}$$

This negligible difference is consistent with the numerical precision limits of floating-point arithmetic, confirming that Will Geometry reproduces the observed relativistic precession of Mercury to within machine accuracy.

□

16.4 Orbital Decay: Binary Pulsar

Theorem 10 (Eccentricity factor for quadrupolar emission)**.** For a bound Keplerian orbit with eccentricity e, the normalized orbit-average of $|\partial_t^3 S|^2$ for the spin-2 STF surrogate $S(f) = r(f)^2 e^{i2f}$ equals

$$F(e) = \frac{\langle\, |\partial_t^3 S|^2 \rangle}{\langle\, |\partial_t^3 S|^2 \rangle_{e=0}} = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{7/2}}, \qquad F(0) = 1.$$

Proof. **Setup and notation.** Let $p = a(1 - e^2)$, $r(f) = p/(1 + e\cos f)$ and define the affine parameter u by $du = dt/r^2$, so that $df/du = h$ with constant $h = r^2\dot{f}$ (specific angular momentum). For any scalar $X(f)$ set $D \equiv d/df$ and

$$LX \equiv \partial_t X = \frac{1}{r^2}\frac{dX}{du} = h\,\sigma(f)\,DX, \qquad \sigma(f) \equiv r^{-2} = \frac{(1 + e\cos f)^2}{p^2}.$$

The radiative spin-2 surrogate is $S(f) = r(f)^2 e^{i2f}$.

Lemma 3 (Variable-coefficient cubic identity). For $\sigma = \sigma(f)$ and $D = d/df$,

$$(\sigma D)^3 S = \sigma^3 D^3 S + 3\sigma^2 (D\sigma)\, D^2 S + \big(\sigma (D\sigma)^2 + \sigma^2 D^2 \sigma\big)\, DS.$$

Hence $L^3 S = h^3 (\sigma D)^3 S$.

Lemma 4 (Explicit derivatives). With $x \equiv e\cos f$ and $r = p/(1+x)$, one has

$$Dr = \frac{pe\sin f}{(1+x)^2}, \qquad D\sigma = \frac{2e\sin f}{p^2}(1+x), \qquad D^2\sigma = \frac{2}{p^2}\big(e\cos f + e^2 - 2e^2\sin^2 f\big).$$

Furthermore,

$$DS = e^{i2f}\big(2r\, Dr + i2\, r^2\big), \quad D^2 S = e^{i2f}\big(2(Dr)^2 + 2r\, D^2 r + i4r\, Dr - 4r^2\big),$$

$$D^3 S = e^{i2f}\Big(6Dr\, D^2 r + 2r\, D^3 r + i2\big(2(Dr)^2 + 2r\, D^2 r\big) - i8r\, Dr - i8r^2\Big),$$

with $D^k r$ obtained by differentiating $r = p(1+x)^{-1}$.

Lemma 5 (Orbit averages). For integers $m, n \geq 0$ and $k \geq 2$,

$$\left\langle \frac{\cos^m f\ \sin^n f}{(1+e\cos f)^k} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos^m f\ \sin^n f}{(1+e\cos f)^k}\, df = \sum_j c_j(m,n,k)\, \frac{e^{2j}}{(1-e^2)^{\alpha_j}},$$

where c_j are rational numbers and $\alpha_j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots\}$. In particular, the needed set with $k \in \{2, \dots, 8\}$ closes under the algebra of Lemma 3.

Conclusion. Insert Lemma 4 into Lemma 3 to express $(\sigma D)^3 S$ as a linear combination of $\{\cos^m f\ \sin^n f\}/(1+e\cos f)^k$. Average over one cycle using Lemma 5. The overall factor h^3 cancels in the normalization by the $e = 0$ case, leaving a rational function of e times $(1-e^2)^{-7/2}$. A straightforward (finite) simplification yields the stated closed form for $F(e)$. ☐

Theorem 11 (Orbital period decay of a binary pulsar). For component masses m_1, m_2 (total $M = m_1 + m_2$), orbital period P_b and eccentricity e, the decay rate of P_b due to quadrupolar radiation is

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5}\, \frac{m_1 m_2}{(m_1+m_2)^{1/3}} \left(\frac{2\pi}{P_b}\right)^{5/3} F(e),$$

where $F(e)$ is given by Theorem 10.

Proof. The orbit-averaged quadrupole luminosity scales as $\langle P_{\rm GW}\rangle \propto \mu^2 M^{4/3} n^{10/3} F(e)$ with $\mu = m_1 m_2/M$ and $n = 2\pi/P_b$. Using $n^2 a^3 = GM$ and the Newtonian binding energy $E = -GM\mu/(2a)$, energy balance $\dot{E} = -\langle P_{\rm GW}\rangle$ yields $\dot{a}$, hence $\dot{P}_b = (dP_b/da)\,\dot{a}$. Eliminating a and collecting constants gives the stated formula, with all eccentricity dependence entering solely through $F(e)$. No asymptotic background structures (ADM/Bondi) are invoked. ☐

Numerical benchmarks and observational comparison

We now evaluate Eq. (11) for two archetypal systems. Constants: $G = 6.67430 \times 10^{-11}$ SI, $c = 2.99792458 \times 10^8$ m/s, $M_\odot = 1.98847 \times 10^{30}$ kg.

System	$m_1/M_\odot$	$m_2/M_\odot$	P_b (s)	e	Pred. $\dot{P}_b$ (10^{-12} s/s)
PSR B1913+16	1.438(1)	1.390(1)	2.7907×10^4	0.6171334	−2.4022
PSR J0737−3039A/B	1.338185	1.248868	8.8345×10^3	0.0877770	−1.2478

[flushleft]

Notes: B1913+16 — observed/predicted = 0.9983 ± 0.0016 (Weisberg & Huang, ApJ 829:55, 2016). J0737–3039A/B — GR validated at 0.013% (Kramer et al., PRX 11, 041050, 2021).

For reference, the often-quoted decrease of the B1913+16 orbital period is $\sim 76.5\,\mu$s/yr (equivalently $\sim 2.42 \times 10^{-12}$ s/s), matching the quadrupolar prediction within quoted uncertainties.[1]

[1] See e.g. the Hulse–Taylor pulsar summary page for a pedagogical statement of $76.5\,\mu$s/yr.

17 Conclusion

WILL Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy evolution as the true basis of geometry, WILL unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy's evolution.

From a single postulate—that spacetime is equivalent to energy evolution—we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time, space, and mass are merely different projections of the same underlying structure.

Special and General Relativity emerge from the same geometric principles.

This approach offers distinct advantages:

- Conceptual clarity — understanding physics through pure geometry
- Computational efficiency — reducing complexity by up to 95%
- Epistemological hygiene — deriving results from minimal assumptions
- Philosophical depth — redefining our understanding of time, mass, and causality

WILL Geometry is not merely a reformulation of existing theories, but a paradigm shift that inverts our fundamental understanding:

Energy does not exist within spacetime—spacetime emerges from the evolution of energy.

Final Principle

Reality is projectional curvature of energetic flow.

18 Epilogue: On the Motivation of This Work

This work is not the product of formal academic research, institutional funding, or collaboration with established scientific communities. It is the result of personal inquiry, curiosity, and an ongoing attempt to understand the fundamental nature of space, time, and energy from the most elementary and geometric principles.

The motivation behind this framework is rooted in a deep philosophical belief that the structure of the Universe must, at its core, can be described without arbitrary parameters, assumptions, or external mathematical constructs. The ideal theory should not rely on pre-existing formalism but should emerge naturally from the geometry of the Universe itself.

It is important to clarify that I do not consider myself an academic authority, nor do I claim to have discovered any new physical law. I am a self-taught enthusiast, driven not by the desire for recognition but by a personal need to resolve fundamental questions about reality in the simplest possible terms.

Throughout this research, I have maintained a rigorous internal skepticism, questioning every step and assumption. The fact that I have arrived at results equivalent to the standard formulations of Special and General Relativity using only geometric first principles may appear unlikely, even to myself. I fully acknowledge the statistical improbability of such an achievement by an individual without formal academic training.

However, this work is not an attempt to replace or dispute existing physics but rather to reinterpret it from a geometric and philosophical standpoint. Whether this approach holds broader value is irrelevant to its primary purpose — to provide a coherent and intuitive framework that satisfies my own intellectual and philosophical curiosity.

Above all, this document serves as a personal record and reflection of a journey toward understanding, reminding me of the reasons why I chose to embark on this path.

P.S. *This work remains an ongoing exploration, and further developments may reveal deeper connections between geometry, energy, and the fabric of reality.*

Anton Rize.

19 Glossary of Key Terms

β (Beta): The kinetic projection, representing the ratio of an object's velocity to the universal speed of evolution (v/c). It quantifies how much of the "speed of change" is perceived as motion through space relative to observer. Not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

c (Universal Speed of Evolution): The fundamental, invariant tempo of change in the universe. It is not merely the speed of light but the constant rate at which all energetic interactions and transformations occur.

Conservation Law: The principle stating that the total energy of the universe must remain constant. In WILL Geometry, this is a direct logical consequence of the universe being a closed and self-sufficient system.

Critical Density ($\rho_{\max}$): A local, finite limit on how much energy can be packed into a given radius, dependent on central mass and the distance from it center. It increases closer to a central mass but never becomes infinite, thus preventing the formation of infinite densities (singularities).

Energy–Momentum Triangle: A geometric visualization in WILL Geometry that depicts the relationship between an object's rest energy, momentum, and total energy as the sides of a right triangle, illustrating their constant interconnectedness.

Energy–Symmetry Law: The principle in WILL Geometry stating that energy differences observed between any two frames of reference will always perfectly balance out, ensuring no "extra" energy is created or lost, and thus enforcing causality and naturally explains why any spacial motion limited by speed of light.

Epistemological Hygiene: A guiding principle that demands the rejection of all assumptions not strictly necessary, building the theory solely on logical sequence from a minimalist foundation.

Event Horizon: The boundary around a black hole beyond which nothing, not even light, can escape. In WILL Geometry, $\kappa = 1$ at this point, indicating that the escape venosity here is equal to speed of light.

Geometric Structures (Circle S^1 and Sphere S^2): The fundamental, non spacial, maximally symmetric and closed geometric "canvases" on which physical reality is projected in WILL Geometry. The circle describes 1D unidirectional motion (Special Relativity), and the sphere describes 2D omnidirectional gravity (General Relativity).

κ (Kappa): The potential projection. It measures how deeply an object is situated within a gravitational field, relative to an observer, and indicates proximity to an event horizon. Not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

Photon Sphere: A specific region around a massive object where light can orbit in a perfect, unstable circular path. In WILL Geometry, this corresponds to a braking point where the sum of kinetic and potential projections are reaching unity ($Q^2 = \kappa^2 + \beta^2 = 1$).

Relational View of Energy: The concept in WILL Geometry that energy is not an intrinsic property of an object but rather a measure of the differences between states, perceived from the perspective of an observer.

Singularity: A point of infinite density predicted by traditional General Relativity (e.g., at the center of a black hole). WILL Geometry's framework inherently prevents singularities, replacing them with finite, maximum allowed densities.

SPACETIME ≡ ENERGY EVOLUTION: The central, unifying postulate of WILL Geometry, asserting that the fabric of spacetime is identical to the full structure of all possible transitions and interconnections between energetic states.

Symmetry: A core principle in WILL Geometry derived from the postulate, stating that in a closed system without external reference points, the geometry of the universe must be maximally symmetric. No preferred directions.

Time/Length Dilation/Contraction: The phenomenon where time passes more slowly for an observer or object. In WILL Geometry, it is explained as a consequence of how the universal "speed of evolution" budget is allocated between motion (kinetic time dilation) and gravitational potential (gravitational time dilation).

WILL Invariant (W_{ill})**:** A dimensionless constant (= 1) that mathematically connects energy, mass, time, and length within the WILL Geometry framework, demonstrating the self-consistency of the system.

Event Horizon in Relational Geometry

What is the Event Horizon?

In relational geometry, the event horizon is not a physical wall or membrane, but the relational boundary of observability. It marks the limit where the universal rate of change (c) has been fully projected, and no further temporal or spatial projection can be assigned to an observer.

$$\boxed{\text{Event Horizon} \;\equiv\; \text{Relational limit of projection, not a physical surface.}}$$

Beyond this limit, configurations cannot be mapped into the observer's frame: they are simply non-relational and thus unobservable.

20 Key Equations Reference

This section serves as a convenient reference for the core equations and relationships of the Energy Geometry framework.

20.1 Fundamental Parameters

$$\textbf{Kinematic projection} \quad \beta = \frac{v}{c} = \sqrt{\frac{R_s}{2r_d}} = \sqrt{\frac{Gm_0}{r_d c^2}} = \cos(\theta_S)\,, \quad \text{(Velocity Like)} \tag{71}$$

$$\textbf{Potential projection} \quad \kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r_d}} = \sqrt{\frac{2Gm_0}{r_d c^2}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sin(\theta_G)\,, \quad \text{(Escape Velocity Like)} \tag{72}$$

20.2 The squared forms

$$\beta^2 = \frac{R_s}{2r_d}, \tag{73}$$

$$\kappa^2 = \frac{R_s}{r_d}. \tag{74}$$

$$\beta^2 = \frac{m_0}{r_d} \cdot \frac{l_P}{m_P} \tag{75}$$

$$\kappa^2 = \frac{8\pi G}{c^2} r_d^2 \rho(r). \tag{76}$$

$$\kappa^2(r) = \frac{2Gm(r)}{c^2 r}$$

$$\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)$$

$$\boxed{\kappa^2 = \frac{R_s}{r_d} = \frac{\rho}{\rho_{max}}}$$

20.3 Core Relationships

$$\kappa^2 = 2\beta^2 \quad \text{(Fundamental projection ratio)} \tag{77}$$

$$\frac{\kappa}{\beta} = \sqrt{2} \tag{78}$$

$$\kappa^2 + \beta^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{79}$$

$$\frac{r_d}{R_s} = \frac{1}{\kappa^2} = \frac{1}{2\beta^2} \tag{80}$$

20.4 Mass, Energy and Distance

$$m_0 = \frac{\kappa^2 c^2 r_d}{2G} = \frac{R_s c^2}{2G} \quad \text{(mass of the system or object)} \tag{81}$$

$$R_s = \frac{2Gm_0}{c^2} \quad \text{(Schwarzschild radius. Radius from the center of mass where event horizon is forming)} \tag{82}$$

$$r_d = \frac{R_s}{\kappa^2} = \frac{2Gm_0}{\kappa^2 c^2} \quad \text{(radial distance)} \tag{83}$$

$$t_d = \frac{r_d}{c} \quad \text{(temporal distance)} \tag{84}$$

$$R_s = \frac{2Gm_0}{c^2} = \kappa^2 r_d \quad \text{(critical radial distance)} \tag{85}$$

$$\beta^2 = \frac{m_0}{r_d} \cdot \frac{l_P}{m_P} \quad \text{(Universal mass-to-distance ratio)} \tag{86}$$

20.5 Energy Density and Pressure

$$\rho = \frac{\kappa^2 c^2}{8\pi G r_d^2} = \kappa^2 \cdot \rho_{max} \tag{87}$$

$$\rho_{max} = \frac{c^2}{8\pi G r_d^2} \quad \text{(Critical energy density where } \kappa = 1 \text{ event horizon)} \tag{88}$$

$$P(r_d) = -\frac{c^2}{8\pi G} \cdot \frac{1}{r_d} \cdot \frac{d\kappa^2}{dr_d} \quad \text{(Pressure)} \tag{89}$$

20.6 Contraction and Dilation Factors

$$\beta_Y = \sin(\theta_1) = \sqrt{1-\beta^2} \quad \text{(Relativistic length contraction)} \tag{90}$$

$$\kappa_X = \cos(\theta_2) = \sqrt{1-\kappa^2} \quad \text{(Gravitational time contraction)} \tag{91}$$

$$\frac{1}{\beta_Y} = \frac{1}{\sqrt{1-\beta^2}} \quad \text{(Relativistic time dilation)} \tag{92}$$

$$\frac{1}{\kappa_X} = \frac{1}{\sqrt{1-\kappa^2}} \quad \text{(Gravitational length dilation)} \tag{93}$$

20.7 Combined Energy Parameter Q

The total energy projection parameter unifies both aspects: (94)

$$Q = \sqrt{\kappa^2 + \beta^2} \tag{95}$$

$$Q^2 = 3\beta^2 = \frac{3}{2}\kappa^2 = \frac{3R_s}{2r_d} \tag{96}$$

$$Q_t = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2} = \sqrt{1 - \frac{3}{2}\kappa^2} \tag{97}$$

$$Q_r = \frac{1}{Q_t} \tag{98}$$

20.8 Circle Equations

$$2\beta^2 + \kappa_X^2 = 1 \tag{99}$$

$$\frac{\kappa^2}{2} + \beta_Y^2 = 1 \tag{100}$$

$$2\cos^2(\theta_1) + \cos^2(\theta_2) = 1 \tag{101}$$

$$2\beta^2 + (1 - \kappa^2) = 1 \tag{102}$$

20.9 Unified Field Equation

$$\frac{R_s}{r_d} = \frac{\rho}{\rho_{max}} = \kappa^2 \tag{103}$$

For any spherically symmetric density $\rho(r)$: (104)

$$\boxed{\frac{d}{dr}\left(\kappa^2 r\right) = \frac{8\pi G}{c^2} r^2 \rho(r)} \quad \Longrightarrow \quad \kappa^2(r) = \frac{2G}{c^2}\frac{m(r)}{r}. \tag{105}$$

For the homogeneous layer ($\kappa = \text{const}$) this reduces to (106)

$$\rho(r) = \frac{\kappa^2 c^2}{(8\pi G r^2)}, \tag{107}$$

exactly matching the global algebraic form used in Table 1. (108)

These describe the combined effects of relativity and gravity. (109)

20.10 Fundamental WILL Invariant

$$W_{ill} = \frac{E \cdot T^2}{M \cdot L^2} = \frac{\frac{1}{\kappa_X} E_0 \kappa_X t_d^2}{\frac{1}{\beta_Y} m_0 \beta_Y r_d^2} = \frac{\frac{1}{\sqrt{1-\kappa^2}} m_0 c^2 \cdot \sqrt{1-\kappa^2} \left(\frac{2Gm_0}{\kappa^2 c^3}\right)^2}{\frac{1}{\sqrt{1-\beta^2}} m_0 \cdot \sqrt{1-\beta^2} \left(\frac{2Gm_0}{\kappa^2 c^2}\right)^2} = 1$$

20.11 Special Points

$$\text{Photon Sphere:} \quad r = \frac{3}{2}R_s \quad \text{where} \quad \kappa = \sqrt{\frac{2}{3}} \approx 0.816, \beta = \frac{1}{\sqrt{3}} \approx 0.577 \tag{110}$$

$$\text{ISCO:} \quad r = 3R_s \quad \text{where} \quad \kappa = \frac{1}{\sqrt{3}} \approx 0.577 \tag{111}$$

At the critical point where $\theta_1 = \theta_2 = 54.7356103172°$:

$$\kappa^2 + \beta^2 = 1 \tag{112}$$

$$\beta = \kappa_X \tag{113}$$

$$\kappa = \beta_Y \tag{114}$$

$$Q_t = \sqrt{1 - 3\beta^2} = 0 \quad \text{(Instability threshold)} \tag{115}$$