Corporate Credit Rating Analysis Using Markov Chains

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Introduction

This project applies the Markov chain theory to analyse the evolution of corporate credit ratings and to quantify default risk over multiple horizons. Credit ratings provide a discrete representation of a firm's credit quality, and their transitions can be modelled through a probabilistic structure that captures how firms migrate between states such as AAA, BBB, or D. By using a transition matrix as the main input, the model allows for the computation of multi-year transition dynamics, cumulative default probabilities, and the expected performance of a loan with initial capital S_0. The aim is to present a clear and compact methodology for understanding rating-based credit risk, supported by numerical examples and graphical illustrations.

Basic Concepts of Markov Chains

A Markov chain is a stochastic process $\{X_t\}_{t\geq 0}$ taking values in a finite set of states S, where the future evolution of the process depends only on its current state. Formally, the Markov property is:

$$\mathbb{P}(X_{t+1} = j \mid X_t = i, X_{t-1}, \dots, X_0) = \mathbb{P}(X_{t+1} = j \mid X_t = i).$$

This indicates that the conditional distribution of the next state depends solely on the present state. The behaviour of the chain is fully described by the one-step transition probabilities:

$$P_{ij} = \mathbb{P}(X_{t+1} = j \mid X_t = i), \qquad \sum_j P_{ij} = 1.$$

It is also important to define what **homogeneous Markov chains** are: these are chains whose transition probabilities do not depend on time. Formally, a Markov chain is homogeneous if, for any given t,

$$\mathbb{P}(X_{t+m} = j \mid X_t = i) = \mathbb{P}(X_m = j \mid X_0 = i), \qquad \forall\, m \in \mathbb{N},$$

which means that the probability of moving from state i to state j in m steps is the same whether the process starts at time 0, at time t, or at any other point in time.

For credit rating modelling, the state space is:

$$S = \{\text{AAA}, \text{AA}, \text{A}, \text{BBB}, \text{BB}, \text{B}, \text{CCC}, \text{CC}, \text{C}, \text{D}\}.$$

Firms rated AAA exhibit a virtually negligible probability of default, whereas ratings between CCC and C carry substantially higher risk. Finally, the default state D is treated as absorbing:

$$P_{DD} = 1, \qquad P_{Dj} = 0 \text{ for } j \neq D.$$

Multi-step Transition Probabilities

An essential result for Markov chains is the Chapman–Kolmogorov equation:

$$P_{ij}^{(n+m)} = \sum_{k \in S} P_{ik}^{(n)} \, P_{kj}^{(m)}.$$

In matrix form, this implies:

$$P^{(n)} = P^n.$$

These identities hold only for **homogeneous Markov chains**, where transition probabilities do not depend on time as we defined before.

Methodology

1. A transition matrix is used as the core input of the model. In this project an illustrative matrix is provided, but this is not essential: with real data, the only additional step would be estimating the matrix from observed rating changes. Several plots are produced to visualise its implications.

2. To estimate such a matrix from real datasets, the ratings of each firm in two consecutive years are compared, counting transitions from rating i to rating j. Normalising these counts row-wise yields the empirical one-year transition matrix.

3. Using the transition matrix, the model computes multi-year default probabilities and the expected gain of a loan with initial capital S_0 and interest rate r. The code also generates simple plots to illustrate the behaviour of these quantities.

Default Probabilities and Expected Gain

Let P be the one-year transition matrix and P^X the X-year matrix. For a firm initially in rating i:

$$\mathrm{PD}_1(i) = P_{iD}, \qquad \mathrm{PD}_X(i) = P^X_{iD}.$$

Because D is absorbing, $\mathrm{PD}_X(i)$ represents the cumulative probability of default within X years.

Consider now a loan of size S_0 granted at interest rate r for X years. Two outcomes exist:

$$\text{Payoff} = \begin{cases} 0, & \text{if default occurs,} \\ S_0(1+r)^X, & \text{if the firm survives.} \end{cases}$$

Let $q_X(i) = \mathrm{PD}_X(i)$. The expected payoff is:

$$\mathrm{EG}_X(i) = S_0(1 - q_X(i))(1+r)^X.$$

The expected net gain, subtracting the initial outlay, is:

$$\mathrm{ENG}_X(i) = S_0\left[(1 - q_X(i))(1+r)^X - 1\right].$$

These formulas connect default risk with expected credit returns.

Summary

This project shows how a transition matrix and Markov chains theory, whether predefined or estimated from real data, can be used to analyse corporate credit risk. From this structure, multi-year default probabilities and expected gains for a loan of size S_0 are computed, supported by graphical illustrations. The results highlight how rating dynamics evolve over time and how the risk–return profile depends on the firm's initial rating, the interest rate, and the chosen horizon.