Decay of an ion density discontinuity in a collisionless plasma

Article *in* Plasma Physics and Controlled Fusion · February 1997

DOI: 10.1088/0741-3335/39/2/006

1 author:

Yury Vasilevich Medvedev
Joint Institute for High Temperatures

Decay of an ion density discontinuity in a collisionless plasma

1997 Plasma Phys. Control. Fusion 39 291

(http://iopscience.iop.org/0741-3335/39/2/006)

Plasma Phys. Control. Fusion **39** (1997) 291–300. Printed in the UK PII: S0741-3335(97)77518-3

Decay of an ion density discontinuity in a collisionless plasma

Yu V Medvedev
Institute for High Temperatures of Russian Academy of Sciences, Moscow, 127412, Russia

Received 2 September 1996, in final form 26 November 1996

Abstract. The decay of an ion density discontinuity in a collisionless plasma is considered when allowance is made for the effect of ion temperature on the process. A proper rarefaction wave is given. It is shown that the process can be described without recourse to complicated computations. The theoretical, numerical and experimental results are found to be in reasonable agreement.

1. Introduction

An extensive literature has evolved which traces the decay of an arbitrary initial discontinuity of density in a collisionless plasma. Particular attention has been given to the case of expansion of a plasma into a vacuum. The pioneering investigation of this problem was carried out by Gurevich *et al* (1966), who found a self-similar solution of the collisionless kinetic equation. A number of theoretical studies of plasma expansion into either a vacuum (Gurevich *et al* 1968, 1973, Allen and Andrews 1970, Widner *et al* 1971, Crow *et al* 1975, Anisimov and Medvedev 1979, Denavit 1979, Gurevich and Mescherkin 1981, 1984, Sack and Schamel 1985) or a low-density plasma (Mason 1971, Anisimov *et al* 1978, 1979, Singh *et al* 1986, Anisimov and Medvedev 1988) were later reported. Also certain experimental results were gathered (Plyutto 1961, Hendel and Reboul 1962, Tyulina 1965, Andersen *et al* 1967, 1968, Alikhanov *et al* 1970, Korn *et al* 1970, Michelsen and Pécseli 1973, Eselevich and Fainshtein 1979, 1980, 1981, Chan *et al* 1984) and a few reviews were published (Gurevich and Pitaevskii 1975, 1986, Sack and Schamel 1987, Gurevich *et al* 1989).

The expansion of a plasma into a vacuum is the best studied process, on condition that the initial ion temperature T_{i0} is negligible compared to the initial electron temperature T_{e0} (the cold ion limit). In this case the fluid description may be used and, in particular, a self-similar solution can be obtained in the quasineutral approximation. However, analytical consideration of the fluid equations fails if charge separation is taken into account and one must resort to numerical investigation in such a case.

It turns out that use of the rather complicated numerical methods is practically inevitable for the case of $T_{i0} \neq 0$. Examples are found in papers (Gurevich *et al* 1966, 1968) where self- similar motions of the isothermal plasma ($T_{i0} = T_{e0}$) were studied. Several versions of the plasma expansion into a vacuum (Anisimov and Medvedev 1979, Denavit 1979) or into a low-density plasma (Mason 1971, Anisimov *et al* 1978, 1979, Singh *et al* 1986, Gurevich *et al* 1989) were considered by kinetic simulations for only some particular values of T_{i0}. Therefore, any new approach which makes it possible to describe the process without recourse to elaborate computations is of interest.

The purpose of this paper is to examine the effect of T_{i0} on the plasma expansion by using the rarefaction wave which has been deduced formerly (Medvedev 1993). In this case the solution can be found from some simple relations. Comparison of such solutions with data of computer simulations by particles shows good agreement up to $T_{i0} = T_{e0}$. In section 2 the basic equations and a theoretical analysis are given. The theoretical values are compared with those obtained by our numerical simulations in section 3. In section 4 some experimental results are discussed from our standpoint. A summary is given in section 5.

2. Basic equations

We consider a one-dimensional expansion of a collisionless plasma. At time $t = 0$ an initial density discontinuity is given at a point $x = 0$ and the ion density takes the form

$$n_i(x, 0) = \begin{cases} n_{i0} & x < 0 \\ n'_{i0} & x > 0 \end{cases} \tag{1}$$

where $n'_{i0} < n_{i0}$. The electrons are assumed to obey the Boltzmann distribution and the electron density n_e can be written as

$$n_e = n_{e0} \exp(e\phi/T_{e0}) \tag{2}$$

where ϕ is the electrostatic potential, n_{e0} is the electron density in the unperturbed plasma ($\phi = 0$) and $-e$ is the electron charge. Here the electron temperature T_{e0} and the ion temperature T_{i0} both are given in energy units. Using the distribution (2) is justified because we consider hydrodynamic plasma flows in which the electrons have time to come into local equilibrium.

The kinetic approach is common for a description of a collisionless plasma. However, in this case one can only obtain numerical solutions. Below we present results of our kinetic simulations of the processes under consideration mainly in order to test the validity of the more simple theory.

The theory is based on the set of equations for the first three moments of the ion distribution function. The set can be obtained on the assumption that thermal flux is equal to zero (Gurevich and Pitaevskii 1975) and may be written as

$$\frac{\partial n_i}{\partial t} + V_i \frac{\partial n_i}{\partial x} + n_i \frac{\partial V_i}{\partial x} = 0 \tag{3}$$

$$\frac{\partial V_i}{\partial t} + V_i \frac{\partial V_i}{\partial x} + \frac{1}{n_i m_i} \frac{\partial}{\partial x}(n_i T_i) + \frac{Ze}{m_i} \frac{\partial \phi}{\partial x} = 0 \tag{4}$$

$$T_i/n_i^2 = T_{i0}/n_{i0}^2 \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e[Z n_i - n_{e0} \exp(e\phi/T_{e0})] \tag{6}$$

where Ze and m_i are the ion charge and the ion mass, respectively. The moments n_i, V_i and T_i are defined as

$$n_i = \int_{-\infty}^{\infty} f_i(x, v, t)\, dv$$

$$V_i = \frac{1}{n_i} \int_{-\infty}^{\infty} v f_i(x, v, t)\, dv$$

$$T_i = \frac{m_i}{n_i} \int_{-\infty}^{\infty} (v - V_i)^2 f_i(x, v, t)\, dv$$

where v is velocity and $f_i(x, v, t)$ is the ion distribution function. Quasineutrality is assumed in the unperturbed plasma ($n_{e0} = Zn_{i0}$).

In the cold ion limit $T_{i0} = 0$ the set of equations (3)–(6) transforms into that of dispersive hydrodynamic which has been studied in detail in the context of the plasma expansion problem. Specifically, when characteristic lengths of motion are much more than the Debye length, the quasineutral approximation can be used. In this case the motion turns out to be self-similar and the plasma expansion into a vacuum ($n'_{i0} = 0$) can be described by the rarefaction wave (Gurevich *et al* 1966):

$$n_i = \exp(-1-\tau) \qquad V_i = 1+\tau \qquad \phi = -(1+\tau) \tag{7}$$

for $\tau = x/t \geqslant -1$.

All quantities in (7) and further are given in normalized form. We take the Debye length D, the values of $c_e = (ZT_{e0}/m_i)^{1/2}$, D/c_e, n_{i0}, ZT_{e0}, T_{e0}/e, $Zen_{i0}c_e$, and $(1/2)m_i n_{i0} c_e^3$ as the scales of length, velocity, time, density, temperature, potential, ion current and energy flux, respectively. The notation remains as before.

In a previous paper (Medvedev 1993) we have established that in the quasineutral approximation a solution of the set of equations (3)–(6) can also be found in the form of a self- similar rarefaction wave. For our problem the wave becomes:

$$\begin{aligned} &T_i = T_{i0} n_i^2 \qquad c_i = (1+3T_i)^{1/2} \\ &V_i = V_0 + c_{i0} - c_i + \frac{1}{2}\left(\ln\frac{c_{i0}-1}{c_{i0}+1} - \ln\frac{c_i - 1}{c_i+1}\right) \\ &\phi = \ln n_i \qquad \tau = V_i - c_i \end{aligned} \tag{8}$$

for $\tau = x/t \geqslant -c_{i0}$. Here, we assume $Z = 1$, V_0 is the drift ion velocity in the unperturbed plasma, c_i and $c_{i0} = (1 + 3T_{i0})^{1/2}$ designate the sound speed in the wave and in the unperturbed plasma, respectively. Implicit relations between the plasma parameters and the self-similar variable τ can be seen from equations (8). Calculation of a particular solution presents no special problems.

By assuming that the ion velocity distribution in the wave is close to the Maxwellian one, we can estimate the ion current j_i and the kinetic energy flux q_i:

$$j_i = n_i V_i \qquad q_i = n_i V_i (V_i^2 + 3T_i). \tag{9}$$

From relations (8) it follows that the ion velocity increases and the ion density decreases with rising τ. The curve $j_i(\tau)$ has a peak at some τ. The same is true for the curve $q_i(\tau)$. Examination of these functions gives $\tau = 0$ and $\tau \approx 2$ for the locations of the maximum of j_i and the maximum of q_i, respectively.

The above relations are directly applicable to the problem of the expansion of a plasma into a vacuum. The use of the rarefaction wave (8) for the description of the plasma expansion into a low-density plasma is less appropriate. As a first approximation, we construct the solution as a sum of two rarefaction waves. The first wave arises from the expansion of the plasma of high density n_{i0} into the vacuum region $x > 0 (n'_{i0} = 0)$ and the second wave arises from the expansion of the plasma of low density n'_{i0} into the vacuum region $x < 0 (n_{i0} = 0)$. This procedure takes no account of the interaction of the flows. Clearly the approximation is the more accurate the lower density n'_{i0}.

3. Numerical results

In this section we present a comparison between results following from relations (8) and data obtained by the particle-in-cell computer simulations in which the ions are simulated

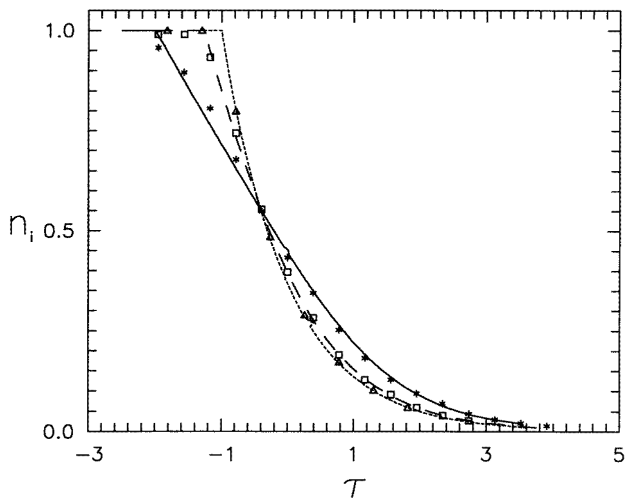

Figure 1. Functions $n_i(\tau)$ obtained from relations (8) (curves) and from simulations (symbols) at several T_{i0}: 0.001 (short-dashed curve and triangles), 0.2 (long-dashed curve and squares), and 1.0 (full curve and asterisks).

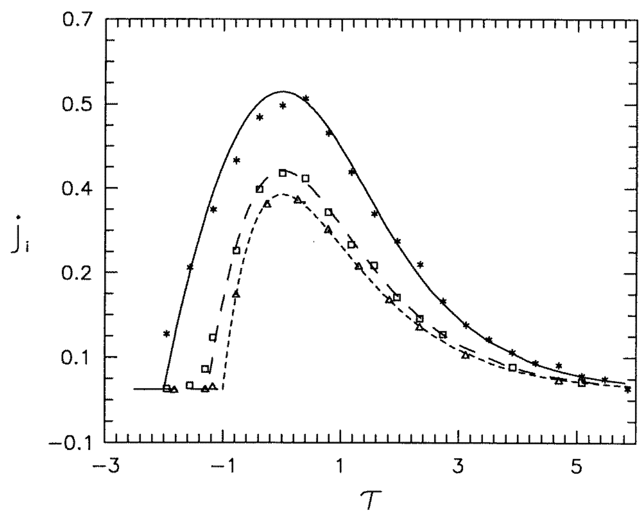

Figure 2. Functions $j_i(\tau)$ obtained from relations (8) and (9) (curves) and from simulations (symbols) at several T_{i0}: 0.001 (short-dashed curve and triangles), 0.2 (long-dashed curve and squares), and 1.0 (full curve and asterisks).

by particles and the electrons obey the Boltzmann distribution (2). Charge separation is taken into account and the nonlinear Poisson's equation (6) is used to find the potential.

Figures 1–3 show the ion density, the current and the energy flux, respectively, as functions of the self-similar variable τ for three values of T_{i0}. These curves may be thought of as profiles with the exception that positions are scaled by factor $1/t$. The rarefaction wave profiles are found to be in good agreement with the data of our computer simulations. A

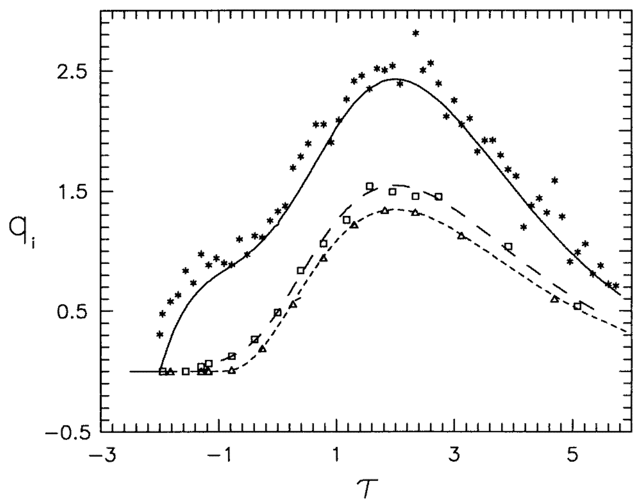

Figure 3. Functions $q_i(\tau)$ obtained from relations (8) and (9) (curves) and from simulations (symbols) at several T_{i0}: 0.001 (short-dashed curve and triangles), 0.2 (long-dashed curve and squares), and 1.0 (full curve and asterisks).

slight difference between two solutions can be seen near the leading front of the rarefaction wave at $\tau = -c_{i0}$, a fact which is due to the approach used. Indeed, the leading front is the point which joins the rarefaction wave and the undisturbed plasma, i.e. the position of the weak discontinuity. Naturally, this discontinuity appears to be somewhat expanded by thermal motion and the accurate solution lacks it.

The temperature dependence of the maxima of the ion current and the ion energy flux is plotted in figure 4. It is seen that the use of equations (8) and (9) provides a good fit to the simulation data. The values obtained largely differ from the free-streaming values given by

$$j_f \approx 0.399 T_{i0}^{1/2} \qquad q_f \approx 0.798 T_{i0}^{3/2}. \tag{10}$$

Note that formulae (10) are derived without regard for a hydrodynamic motion.

Consider the applicability of the rarefaction wave (8) for the study of the expansion of a plasma into a low-density plasma. Here some restrictions are imposed. The flow structure has been found to depend on the ratio of n_{i0}/n'_{i0} and on T_{i0} (Mason 1971, Gurevich *et al* 1989). In particular, a complicated structure that involves such elements as a self-similar expansion region, a plateau and a collisionless shock develops at small T_{i0} and moderate n_{i0}/n'_{i0}. Obviously any effort to construct the flow profile by using the rarefaction wave alone fails in this case.

The solution transforms to the smooth self-similar one with increasing the ion temperature to $T_{i0} \sim 1$. The ion density and the ion velocity are monotonic functions of position, and the profiles do not contain the plateau and shock regions. It is in this case that the rarefaction wave (8) can be used to find an approximate solution.

As an example suitable for the following comparison with experiments, let us consider the expansion of the high-density plasma $n_{i0} = 1$ into the low-density plasma $n'_{i0} = 0.24$ in the isothermal case ($T_{i0} = 1$). The initial drift velocities of the plasmas are both equal to 2.3. We can fulfil the plan given at the end of the previous section and construct the solution by summing the rarefaction waves originated from the expansion of the high-density plasma

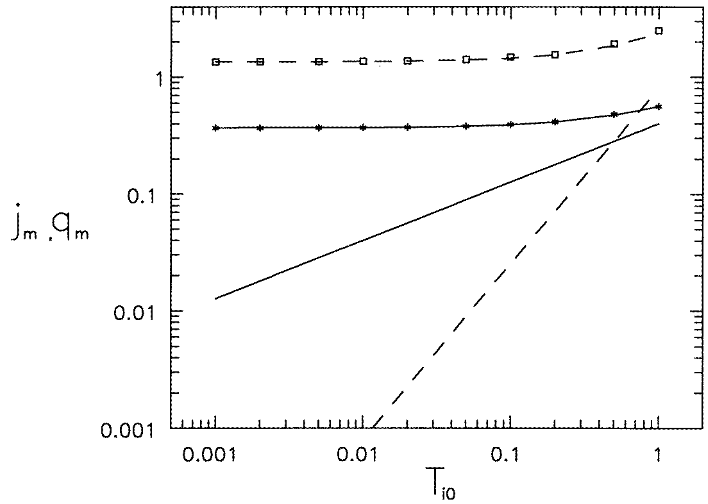

Figure 4. Temperature dependence of the maximum ion current (top full curve) and the maximum ion energy flux (top broken curve). The free-streaming ion current j_f is given by the bottom full curve and the free-streaming energy flux q_f is shown by the bottom broken curve. Simulation data are averaged over time and presented by asterisks (current) and squares (energy flux).

and from the expansion of the low-density one. The solution obtained needs to be compared with the results of the numerical simulation. Figure 5 gives the ion density profiles of both the high-density plasma n_i and the low-density plasma n_i' together with the profile of the total ion density. As is seen from the graph the true distributions n_i and n_i' slightly differ from the corresponding rarefaction waves due to the interaction between the flows. However, the total ion density distribution agrees closely with that predicted from the theory except for the neighbourhood of the weak discontinuities at $\tau = V_0 \pm c_{i0}$.

4. Comparison with experiments

Some experimental investigations of a plasma expansion into a vacuum or into a low-density plasma have been carried out on condition that T_{i0} is not small or even comparable to T_{e0}. Let us compare the experimental data with the results of the foregoing consideration.

The plasma expansion into a vacuum has been studied by Eselevich and Fainshtein (1981) in the range $T_{i0} \simeq 0.25$–0.5. The data obtained were compared with the theory developed for the ion cold limit $T_{i0} = 0$. In this case some disagreement was found. Whereas the measured value of T_{e0} was equal to 6.5 eV, the value of T_{e0} estimated by using the slope for the semi-logarithmic scale of the ion density dependence on the square root of the ion energy (an equivalent to the self-similar variable τ) was equal to 5 eV. It should be noted that the disagreement cannot be ascribed to the effect of non-zero ion temperature. As illustrated in figure 1, in the experimental studied region of $\tau \geqslant 3$ the main effect of T_{i0} lies in increase of the ion density (the density at $T_{i0} = 1$ is approximately 1.8 times greater than one at $T_{i0} = 0$). However, the slope of the curve $n_i(\tau)$ for the semi-logarithmic scale remains practically constant regardless of T_{i0}. Hence the effect of T_{i0} on the process cannot be extracted from these experimental data. Comparison measurements of the plasma parameters at different T_{i0} are required.

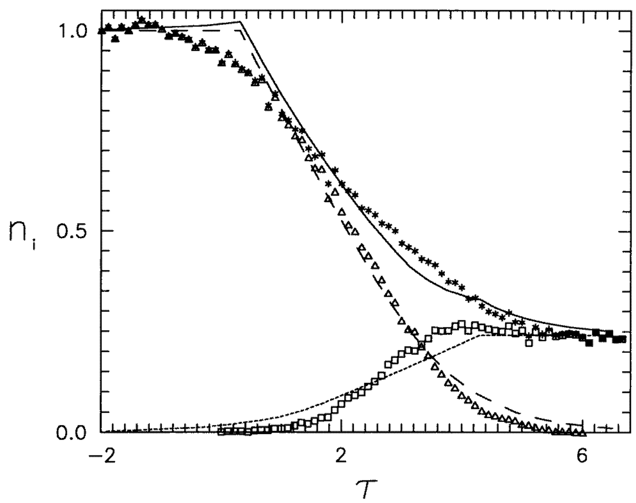

Figure 5. Functions $n_i(\tau)$ obtained from relations (8) (curves) and from simulations (symbols) at the version $n_{i0} = 1, n'_{i0} = 0.24, T_{i0} = 1, V_0 = 2.3$: n_i (long-dashed curve and triangles), n'_i (short-dashed curve and squares) and $n_i + n'_i$ (full curve and asterisks).

An experimental verification of the self-similarity of the plasma expansion into a vacuum has been obtained by Chan *et al* (1984). The potential profile was measured and compared with the expression for the potential in the cold ion limit (7). Note that the measured value of T_{i0} was equal to 0.15. Here, too, however, effort to extract the ion temperature effect on the process fails because T_{i0} is too small. This is apparent from figure 1. The ion density profiles at various T_{i0} differ slightly up to $T_{i0} \simeq 0.2$. As for the corresponding potential profiles, they coincide very closely.

The evolution of an initial ion density discontinuity in a Q-device has been studied by Andersen *et al* (1967, 1968). At $T_{i0} \simeq 1$ they observed the smooth profiles without any shock. For comparison we rearrange the experimental ion density profiles to present data as functions of the self-similar variable at several times (figure 6). The experimental distributions differ by a shift along the density direction. When it is considered that the shifts are small and the values of n_i are given in arbitrary units, one can suggest that these shifts are due to minor changes of the density scale in passing from one time to another and the experimental data fall on the common self-similar dependence, i.e. at $T_{i0} = 1$ the process is the self-similar one (Gurevich *et al* 1989).

Since the accurate values of n_{i0}, n'_{i0}, and V_0 were not specified in the papers, we plot on the graph the function $n_i(\tau)$ for the version discussed at the end of the previous section (figure 5). This version seems to agree closely with the experimental parameters.

As seen in figure 6, the simulation data as well as the theoretical curve provide a reasonable fit to the results of the experiment. From this one can come to recognize that the plasma moves with the drift velocity $V_0 \simeq 2.3$. Note that the authors of the papers assumed that the plasma drift was possible. This fact was verified in subsequent experiments (Korn *et al* 1970, Andersen *et al* 1971, Michelsen and Pécseli 1973).

In particular, the drift velocity V_0 has been even measured in much the same conditions by Korn *et al* (1970), who studied a decay of an ion density discontinuity in a single-ended Q-machine at $T_{i0} = 1$. In the process the ion flux profiles were determined at several times. We retain their normalization of the ion flux such that the flux $n_{i0}V_0$ in the unperturbed

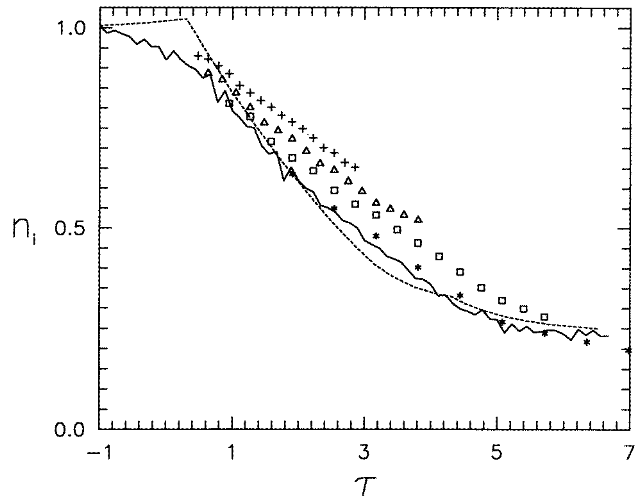

Figure 6. Comparison of functions $n_i(\tau)$ obtained from relations (8) (dashed curve) and from simulations (full curve) at the version $n_{i0} = 1, n'_{i0} = 0.24, T_{i0} = 1, V_0 = 2.3$ with the experimental data of Andersen *et al* (1968) at several time: asterisks, $t = 0.2$ ms; squares: $t = 0.4$ ms; triangles, $t = 0.6$ ms; and plus; $t = 0.8$ ms.

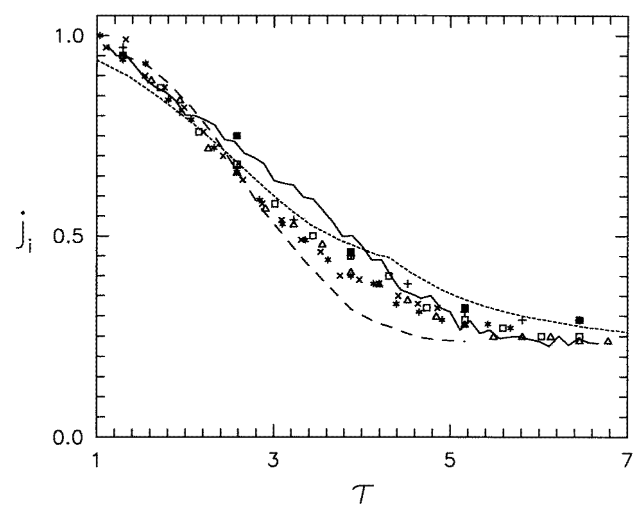

Figure 7. Comparison of functions $j_i(\tau)$ obtained from relations (8) and (9) (short-dashed curve) and from simulations (full curve) at the version $n_{i0} = 1, n'_{i0} = 0.24, T_{i0} = 1, V_0 = 2.3$ with the experimental data of Korn *et al* (1970) at several time: closed squares, $t = 10$ μs; plus, $t = 20$ μs; open squares, $t = 30$ μs; triangles, $t = 40$ μs; asterisks, $t = 50$ μs; multiplication sign, $t = 60$ μs. The free-streaming curve of Korn *et al* (1970) is shown by the long-dashed curve.

high-density plasma is used as the unit flux and plot the experimental fluxes as functions of the self-similar variable τ (figure 7). Also the corresponding simulation and theoretical curves are shown in figure 7. (Obviously the corresponding normalized values of the ion flux and the ion current coincide and we denote them by j_i.)

It is of interest that the scatter of the experimental points which are associated with different times is small enough, i.e. the motion shows the self-similar behaviour. Some discrepancy between the experiment and our results is observed in the limited range of τ. Except for the sign, nearly the same difference is found between the experimental values and the free-streaming solution of Korn *et al* (1970) almost in the same range of τ.

As indicated by figure 7, the weak discontinuities at $\tau = V_0 \pm c_i$ manifest themselves not only in the ion density profiles but in the ion flux ones as well.

5. Summary

We have studied the evolution of an initial ion density discontinuity in a collisionless plasma when the ion temperature effect on the process is taken into account.

The solution of the problem has been proposed by using the rarefaction wave. The wave is written as some implicit relations between the plasma parameters and the self-similar variable. A particular solution can readily be obtained without recourse to complicated computations.

The theoretical findings are presented in comparison with the results of our numerical simulations by particles. Except for slight variations in the vicinity of the weak discontinuities, good agreement is found. The available experimental data give no way of determining the ion temperature effect on the plasma expansion into a vacuum. However, the theory is verified by comparison with experiments on the expansion of one plasma into another. Reasonable agreement is observed.

Acknowledgment

The author is grateful to Professor A V Gurevich for valuable discussions and support.

References

Alikhanov S G, Belan V G, Kichigin G N and Chebotaev P Z 1970 *Zh. Eksp. Teor. Fiz.* **59** 1961
Allen J E and Andrews J G 1970 *J. Plasma Phys.* **4** 187
Andersen H K, D'Angelo N, Michelsen P and Nielsen P 1967 *Phys. Rev. Lett.* **19** 149
——1968 *Phys. Fluids* **11** 606
Andersen S A, Jensen V O, Michelsen P and Nielsen P 1971 *Phys. Fluids* **14** 728
Anisimov S I and Medvedev Yu V 1979 *Sov. Phys.–JETP* **49** 62
——1988 *Zh. Tekh. Fiz.* **58** 1846
Anisimov S I, Medvedev Yu V and Pitaevskii L P 1978 *Dokl. Akad. Nauk SSSR* **241** 563
——1979 *Zh. Prik. Mekh. Tekh. Fiz.* **3** 87
Chan C, Hershkowitz N, Ferreira A, Intrator T, Nelson B and Lonngren K 1984 *Phys. Fluids* **27** 266
Crow J E, Auer P L and Allen J E 1975 *J. Plasma Phys.* **14** 65
Denavit J 1979 *Phys. Fluids* **22** 1384
Eselevich V G and Fainshtain V G 1979 *Sov. Phys.–Dokl.* **24** 114
——1980 *Sov. Phys.–JETP* **52** 441
——1981 *Sov. J. Plasma Phys.* **7** 271
Gurevich A V and Meshcherkin A P 1981 *Sov. Phys.–JETP* **53** 937
——1984 *Sov. Phys.–JETP* **60** 732
Gurevich A V and Pitaevskii L P 1975 *Prog. Aerospace Sci.* **16** 227
——1986 *Reviews of Plasma Physics* vol 10 (New York: Consultants Bureau)
Gurevich A V, Pariiskaya L V and Pitaevskii L P 1966 *Sov. Phys.–JETP* **22** 449
——1968 *Sov. Phys.–JETP* **27** 476
——1973 *Sov. Phys.–JETP* **36** 274

Gurevich A V, Sagdeev R Z, Anisimov S I and Medvedev Yu V 1989 *Sov. Sci. Rev. A. Phys.* **13** (2) 1
Hendel H W and Reboul T T 1962 *Phys. Fluids* **5** 360
Korn P, Marshall T C and Schlesinger S P 1970 *Phys. Fluids* **13** 517
Mason R J 1971 *Phys. Fluids* **14** 1943
Medvedev Yu V 1993 *Plasma Phys. Control. Fusion* **35** 1103
Michelsen P and Pécseli H L 1973 *Phys. Fluids* **16** 221
Plyutto A A 1961 *Sov. Phys.–JETP* **12** 1106
Sack Ch and Schamel H 1985 *Plasma Phys. Control. Fusion* **27** 717
——1987 *Phys. Rep.* **156** 311
Singh N, Thiemann H and Schunk R W 1986 *Phys. Scr.* **33** 355
Tyulina M A 1965 *Sov. Phys.– Tech. Phys.* **10** 396
Widner M, Alexeff I and Jones W D 1971 *Phys. Fluids* **14** 795