Chapter 1

The road to Geneva

1.1 The scientific roots

1.1.1 Fusion energy in stars

With thermodynamics, nineteenth-century physics and chemistry seemed to have achieved their ultimate unifying theory and there was little doubt that its validity would extend over the neighbouring fields of astronomy and geology as well. Its basic principle, conservation of energy, therefore required that the Sun's radiation be accounted for in terms of an equivalent source. Chemical fuels were inadequate and Helmholtz found that gravitational contraction could account for only some 20 million years of the Sun's radiation output. Kelvin estimated the time it would have taken the Earth to cool down to its current state at one hundred million years, and although geologists could accept this figure—their own estimates also tended to converge towards it—even half a billion years was not enough for Charles Darwin to comply with his evolution theory. Kelvin had convinced the geologists that the Earth could not have existed for an indefinite length of time, as had been their accepted doctrine. But he kept revising his figures downwards, and when he eventually arrived at an estimate of 20–40 million years they could no longer stay with him. The physicists for whom it had been heresy to question the axiom of the immutability of the atom or to contemplate the existence of as yet unknown forms of energy, had overplayed their hand. Surely, thermodynamics would stand the test of time, but there would turn out to be 'more in Heaven and Earth'. Around the turn of the century, Becquerel demonstrated that the physics of Helmholtz and Kelvin was incomplete. Radioactivity at once turned out to be the key, both to determining the ages of rocks and to explaining the Earth's hidden source of heat. Now the ball was in the astronomers' court. With physics, geology and biology agreeing that the age of the Earth was in fact several billion years [1], the question was from where the Sun, which clearly could not be younger than the Earth, obtained its heat.

Einstein's equivalence of mass and energy shed a new light on the problem and the thought arose that 'atomic transmutations' might be the source of solar and stellar energy. Immediately after Aston had measured the mass defect of helium relative to four hydrogen atoms with his mass spectroscope, Arthur Eddington, one of the leading astronomers of his time, put forward the conversion of hydrogen into helium [2]:

> *A star is drawing on some vast reservoir of energy by means unknown to us. This reservoir can scarcely be other than the sub-atomic energy which, it is known, exists abundantly in all matter; we sometimes dream that man will one day learn how to release it and use it for his service.... I think that the suspicion has been generally entertained that the stars are the crucibles in which the lighter atoms, which abound in the nebulae, are compounded into more complex elements.*

This was in 1920. Russell had by then established a connection between the liberation of energy and the temperature inside a star and when Eddington refined the existing model for the stellar interior by including radiation pressure and radiative energy transport, he found that the observational data—the Herzsprung–Russell diagram in which each star is represented by a point on a luminosity versus surface temperature diagram—suggested a central energy source, rather than a more uniform source like gravitation [3]. Eddington concluded that the energy release must set in at a critical temperature, for which his model yielded 40 million Kelvin (MK) (it was later adjusted to 15–20 MK). He considered fusion of hydrogen into helium the most plausible reaction, although there was a problem in that the Sun was supposed to contain only a few per cent hydrogen. A few years later, Atkinson and Houtermans invoked Gamov's theory of barrier penetration, suggesting the fusion of hydrogen into helium through successive proton and electron captures in as yet unidentified heavier nuclei, followed by α-decay [4].

At that time, the nucleus was thought of as an assembly of protons and electrons, or of alpha particles built up from these constituents. So, precisely which reactions might be involved remained unclear. In an astounding development, nuclear physics in hardly a decade matured to the point where it was possible to identify the appropriate thermonuclear reactions and to calculate their reaction rates. Important stepping stones were the introduction of particle accelerators in 1930, the discovery of deuterium and of the neutron, both in 1932, and the theory of beta decay in 1934 [5]. In 1936, Atkinson suggested the beta decay of colliding protons as the first step towards helium production via the proton–proton chain:*

* Here, the three hydrogen isotopes are represented by the nuclear physicist's symbols p, d and t. Later on, when our emphasis shifts from the nuclear processes to the fuels burnt and ashes produced, we shall use the chemical symbols H, D and T, in accordance with most recent fusion literature.

$$p + p \longrightarrow d + e^+$$

$$p + d \longrightarrow {}^3\mathrm{He}$$

$${}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2p$$

The reaction rate was calculated by Bethe and Critchfield [6]. Von Weizsäcker and Bethe independently recognized the CNO cycle [7]:

$${}^{12}\mathrm{C} + p \longrightarrow {}^{13}\mathrm{N} \longrightarrow {}^{13}\mathrm{C} + e^+$$

$${}^{13}\mathrm{C} + p \longrightarrow {}^{14}\mathrm{N}$$

$${}^{14}\mathrm{N} + p \longrightarrow {}^{15}\mathrm{O} \longrightarrow {}^{15}\mathrm{N} + e^+$$

$${}^{15}\mathrm{N} + p \longrightarrow {}^{12}\mathrm{C} + {}^4\mathrm{He}$$

as the one for which Atkinson and Houtermans had been looking and, again, Bethe calculated the reaction rate. In 1939 he wrote a classic paper [8], in which he concluded that the p–p chain and the CNO cycle are equally probable at a solar temperature of 16 MK, and that the latter would be dominant at the assumed 19 MK. A review of the subject in which several side branches of these reaction sequences are taken into consideration shows that, on present knowledge, conditions in the Sun favour the p–p chain [9].

The subject of energy release in stars is, of course, intimately related to that of nucleosynthesis. Aside from the early reviews by von Weizsäcker [10], who clearly posed the outstanding problems, and Bethe, who quantitatively evaluated the relevant reaction rates, articles by Burbidge *et al.* and Cameron stand as landmarks in the development of this field [11].

Although burning ordinary hydrogen can comfortably sustain a main-sequence star's radiation flux for billions of years, the process is utterly inadequate for earthly purposes: the reaction rates and, hence, the power production per unit volume are too low for economic purposes. Even at a pressure as high as that in the Sun, the average power density is $0.27\,\mathrm{W/m^3}$ against some $\mathrm{MW/m^3}$ required for a commercial reactor.

Stars differ from prospective fusion reactors in two other aspects: confinement and radiative equilibrium. In stars, the nuclear fuel is held together by gravity. As to radiation, bremsstrahlung in a fusion reactor counts as an immediate plasma heat loss because it has a mean free path for reabsorption far in excess of the dimensions of any conceivable earthly magnetic-confinement reactor.* In stars, on the other hand, this reabsorption length is small compared with the star's size, so that radiation from the centre diffuses slowly towards the photosphere and the reaction temperature can be sustained with the power density referred to above.

* The situation is different in inertial confinement of pellets compressed to extremely high densities, a subject not dealt with in this review.

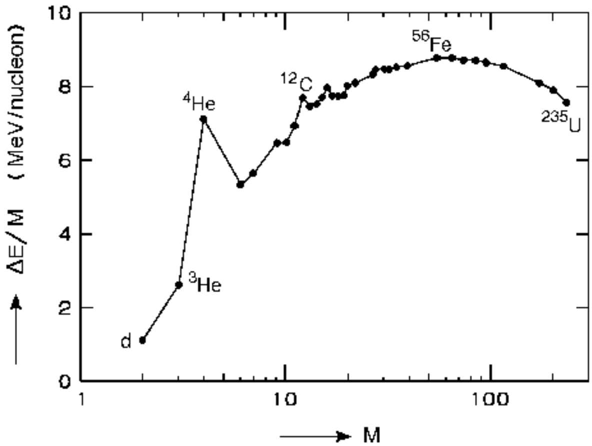

Figure 1.1. Binding energy or mass defect per nucleon, $\Delta E/M$, of stable nuclei as a function of nuclear mass number M. Nuclear energy is released when reactions lead to an increase in the binding energy, which is negative potential energy.

1.1.2 Fusion reactions on Earth

When astrophysics and nuclear physics pointed towards the conversion of nuclear binding energy into heat,* the thought immediately arose [12] that this '*latent power*' could be controlled '*for the well-being of the human race—or for its suicide*'. Hydrogen presented itself as the first candidate for nuclear fuel, because the repulsive Coulomb-barrier varies as the product of the nuclear charge numbers of the reactants. And since, among the lightest nuclei, ^{4}He stands out with as much as 7.1 MeV binding energy per nucleon (figure 1.1), thoughts turned towards the conversion of hydrogen isotopes into helium.

When systematic investigations of nuclear reactions became possible by the introduction of particle accelerators, it appeared that in particular the loosely bound deuteron gave prolific reactions on various target nuclei [13]. The reaction may involve the formation of a compound nucleus or proceed through a mechanism called 'deuteron stripping'. In either case, one constituent of the deuteron is trapped in the target nucleus and the other flies off, carrying most of the reaction energy (the difference between the reaction products' and the reactants' binding energies) as kinetic energy. But when later considered as a source of nuclear energy, these reactions were named 'fusion', because the alliteration with 'fission' accentuates the difference and because the more descriptive term 'thermonuclear reaction' in people's minds was associated with the hydrogen bomb.

* Commonly referred to as 'energy production'.

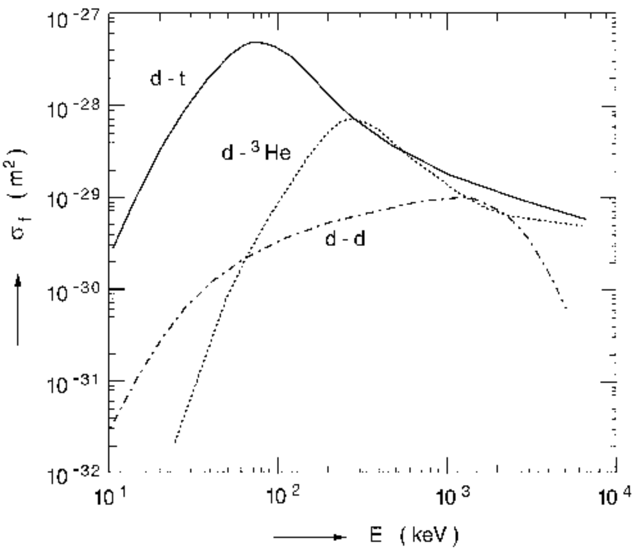

Figure 1.2. Cross-section, σ_f, for the d–d, d–t and d–^{3}He fusion reactions as a function of kinetic energy, E, of the relative motion of the colliding nuclei. The curve marked d–d indicates the total cross-section for the two reactions d + d mentioned in the text.

Most of what is now called fusion research aims at exploiting deuteron break-up by letting deuterium react with itself, with tritium, or with ^{3}He. The latter are produced in d–d reactions, but may also be supplied as fuel along with the deuterium. The reactions in question are:

$$\mathrm{d} + \mathrm{d} \longrightarrow \mathrm{t} + \mathrm{p} + 4.0\,\mathrm{MeV}$$

$$\mathrm{d} + \mathrm{d} \longrightarrow {}^3\mathrm{He} + \mathrm{n} + 3.3\,\mathrm{MeV}$$

$$\mathrm{d} + \mathrm{t} \longrightarrow {}^4\mathrm{He} + \mathrm{n} + 17.6\,\mathrm{MeV}$$

$$\mathrm{d} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + \mathrm{p} + 18.3\,\mathrm{MeV}$$

of which the first and second have approximately equal probability (figure 1.2). The high cross-section and the high energy yield of the d–t reaction make it the favourite candidate for terrestrial fusion.

To confuse the terminology still further, even a fission, or spallation reaction like

$${}^{11}\mathrm{B} + \mathrm{p} \longrightarrow 3 \times {}^4\mathrm{He} + 8.7\ \mathrm{MeV}$$

has been labelled ‘fusion’, which must now be understood to include all energy-producing reactions between light nuclei.

Reactions between lithium isotopes and normal hydrogen have been considered but have far lower reaction rates than those with d, t, and ^{3}He. Lithium, however, is important for the neutron-induced reactions

$$^6\text{Li} + \text{n} \longrightarrow {}^4\text{He} + \text{t} + 4.8\,\text{MeV}$$

$$^7\text{Li} + \text{n} \longrightarrow {}^4\text{He} + \text{t} + \text{n} - 2.5\,\text{MeV}$$

which can serve to breed tritium. In contrast to charged-particle reactions, these neutron reactions do not require high temperatures. In a deuterium–tritium fusion reactor, the tritium will therefore be bred in a lithium-containing blanket surrounding the thermonuclear burning chamber. In this blanket, both the d–t neutron energy and the net energy produced in the lithium reactions will be converted into heat at a temperature suitable for driving a gas or steam turbine. The ^{7}Li reaction is endothermic and requires a neutron energy of at least 2.5 MeV, whereas the ^{6}Li reaction is an additional source of heat. Deuterium and lithium are extremely rare in the Sun's interior because, when formed, they react very rapidly. But on Earth, there are ample supplies of these fuels, left as debris from cosmological processes and supernova explosions of early massive stars. Some 150 ppm* of natural hydrogen is deuterium, and even with this small amount one litre of ocean water contains as much energy as three hundred litres of gasoline. Such comparisons triggered wild fantasies of virtually inexhaustible energy supplies. Also lithium, which has an abundance of 0.17 ppm in ocean water and is available in widely scattered salt deposits and brines, can support any foreseeable demand from fusion energy for centuries.

The initial supply of tritium can be extracted from fission reactors, whereupon a fusion reactor fitted with a lithium-containing blanket can breed its own fuel, each triton burned producing one neutron and this, in turn, producing one triton from ^{6}Li. Inevitable neutron or tritium loss can be compensated by first letting fast neutrons from the d–t reaction interact with ^{7}Li, which gives a triton plus a slow neutron, and then capturing the latter in ^{6}Li. Alternatively, one may use (n, 2n) reactions with beryllium or a heavy element, such as lead, as a blanket material to generate more neutrons. Finally there is the possibility to use fissionable materials as neutron multipliers. But although the tritium–lithium cycle was already considered in the 1950s [14], thoughts on fusion energy were initially focused on deuterium, with tritium and helium-3 only as intermediate products [15], even though the temperature† required for d–d reactions is of the order of 100 keV, against 20 keV for d–t.

* Parts per million.

† Generally, we use MKS units, except that, henceforth, temperatures are mostly expressed in eV or keV. The unit eV corresponds with 11.6×10^3 K. With this convention, Boltzmann's constant becomes $k = 1.6 \times 10^{-19}$ J/eV $= 1.6 \times 10^{-16}$ J/keV.

The world became acutely aware of the devastating power of nuclear energy when it learned of the two fission weapons exploded over Japanese cities towards the end of the Second World War and of the first thermonuclear explosions in 1952. But there was also the fission reactor, which was seen as a great hope for cheap and abundant energy. Physicists who had worked on the bomb were among the first, along with the astrophysicists, to tackle the problem of completing the quartet by developing a fusion reactor [16].

Although it is relatively easy to accelerate nuclei to the energies required for fusion and to demonstrate the reactions by shooting particle beams onto solid targets, it was quickly realized that this would not be a feasible way to generate energy. Most of the accelerated nuclei give up their energy to electrons in the target and relatively few approach the target nuclei sufficiently close to undergo fusion. The energy gain from fusion does not match that invested in accelerating the much larger number of nuclei that are stopped in the target. Shooting a beam into a plasma is, under special conditions (box 8.1), a viable option, but thoughts on fusion turned primarily towards reactions in a thermal plasma.

1.1.3 The origins of plasma physics*

The word 'plasma' entered the vocabulary of physics through Irving Langmuir's description of the positive column of a gas discharge [17]. One interpretation is that he saw an analogy with blood plasma in that both were carrying particles, but it seems more plausible that he thought of the Greek word for 'to mould', because the luminous substance tended to take the shape of its glass container [18]. The plasma, an example of which is the light-emitting gas in a fluorescent lamp, is electrically quasi-neutral and highly conductive, hence nearly electric-field-free (in contrast to the region in front of the cathode—the cathode fall—in which there is a steep potential gradient). The study of atomic, molecular, and surface phenomena determining the state of ionization in these discharges had started with the discovery of the electron and would develop into a mature science in the years before the Second World War [19]. But the collective motions of ions and electrons, which are what plasma physics in the narrow sense is about (a wider sense includes the ionization phenomena) were only beginning to be studied towards the end of the pre-war period.

Langmuir introduced two important concepts in plasma physics—the Debye screening distance and the plasma frequency. He applied the Debye–Hückel theory of electrolytes to describe the space-charge sheath in front of an electrode and called the thickness of this sheath the Debye

* The reader who does not wish to go too deeply into plasma physics may choose to skip the boxes.

Box 1.1 Debye length and plasma frequency

The basic parameters characterizing collective phenomena in a non-magnetized plasma are the Debye length

$$\lambda_d = (\varepsilon_0 k T_e / n_e e^2)^{1/2}$$

which is normally very small, and the plasma, or Langmuir frequency

$$\omega_p = (n_e e^2 / \varepsilon_0 m_e)^{1/2}.$$

Here, n_e is the density, T_e is the temperature, e is the charge and m_e is the mass, all of the electrons, while k is Boltzmann's constant and ε_0 is the permittivity of free space. The product of λ_d and ω_p equals the electron thermal velocity.

If, in the positive column of a gas discharge, n_i is the density of an ion species with charge number Z_i, the deviation from neutrality, $(\sum Z_i n_i - n_e)/n_e$, is of the order of $(\lambda_d/a)^2$, where a is the radius of the column. The Debye length is also called the Debye screening distance because it is a measure for the thickness of the layer outside which a positive charge immersed in a plasma is screened by a locally increased electron density—or a negative charge by a decreased electron density. In this layer, the electrostatic field, determined by Poisson's equation, makes equilibrium with the pressure gradient in the electron gas. The plasma frequency appears when the electrostatic field and the electron mass together govern electron-density oscillations. If there is a magnetic field, the gyroradius and the gyrofrequency (see box 1.2) also come into play.

Since $m_i \gg m_e$, the ions may often be thought of as only a fixed neutralizing background for high-frequency fluctuations in the electron gas, particularly in classical gas discharges where $T_i \ll T_e$. In this case, one may speak of an electron plasma. In the same vein, electrons in solid conductors or semi-conductors can behave as a plasma. For a complete description of low-frequency phenomena in ionized gases one needs to introduce also the ion Debye length and the ion plasma frequency, λ_{di} and ω_{pi}, in which case the corresponding electron parameters defined above are written as λ_{de} and ω_{pe}.

length. This appears also as a characteristic length in the description of the positive column (box 1.1). Langmuir also explained [20] the high-frequency noise observed by Penning [21] as emitted by oscillations resulting from collective interactions of ions and electrons through self-generated electric fields.

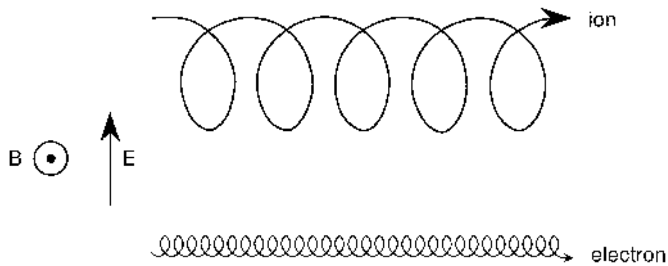

Figure 1.3. The **E** × **B** drift of ions and electrons (not to scale). An ion gains speed and the radius of curvature of its orbit increases when it moves in the direction of **E**; the inverse holds for an electron. The magnetic field points out of the page.

Electrostatic waves* were found to occur in electronic tubes and were exploited in devices like klystrons and magnetrons. This subject was elaborated particularly in the wartime development of radar systems [22]. In the same context, gas discharges found applications in switching devices like spark gaps and thyratrons. And before that time, Heaviside and others had already studied the transmission of radio waves through and their reflection from ionized layers in the upper atmosphere [23].

Collective interactions involving magnetic fields were first recognized in 1934 by Bennett [24]. In its original version, the Bennett-pinch was a pair of counter-streaming relativistic electron and ion beams whose relatively small random transverse velocities were contained by the self-magnetic field of the beams. Tonks [25], who apparently was unaware of Bennett's work, considered the opposite case of a magnetically constricted plasma current carried by ions or electrons with an axial drift velocity small compared with the thermal speed, and came to essentially the same result (box 1.4). Before we continue with the action of a magnetic field on a plasma, we must say something about the motion of individual particles.

An electron or an ion in a uniform and constant magnetic field will describe a spiral orbit, the radius and the frequency of which are designated with the prefix gyro, cyclotron or Larmor—although Larmor did not study the orbits of free electrons [26]. Electrons spiral clockwise and ions anticlockwise when viewed in the conventional direction of the field. If in addition there is an electric or a gravitational field perpendicular to the magnetic field, the particles respond not by falling in the direction of this force but by moving sideways, that is in the third perpendicular direction (figure 1.3). In crossed electric and magnetic fields, **E** and **B**, positive and negative charges drift in the same direction with a velocity equal to E/B (box 1.2). So if these fields extended to infinity, a plasma would move

* Conventionally, an electric field is called electrostatic if it is associated with an electric charge (not necessarily constant), and electrodynamic if it is induced by a time-varying magnetic field.

Box 1.2 Plasma in uniform crossed fields

To understand the collective behaviour of charged particles, one may start from the motions of individual particles in electric and magnetic fields, **E** and **B**, governed by the Lorentz force. In the non-relativistic approximation

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F}_{\mathrm{L}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

where m, q and $\mathbf{v}$ are the mass, charge and velocity of the particle. In uniform and constant crossed fields ($\mathbf{E} \perp \mathbf{B}$), this equation is satisfied by $\mathbf{v} = \mathbf{v}_{\mathrm{d}}$, where $\mathbf{v}_{\mathrm{d}}$ is the drift velocity

$$\mathbf{v}_{\mathrm{d}} = \mathbf{E} \times \mathbf{B}/B^2$$

which is independent of mass and charge. J J Thomson's velocity selector was based on this relation, which reappears as the drift of a plasma in crossed electric and magnetic fields. Superimposed on this drift may be a random velocity in the form of a spiralling motion with arbitrary radius and pitch, that is with arbitrary parallel and perpendicular velocities, and with angular frequency

$$\omega_{\mathrm{c}} = qB/m.$$

The reader who is familiar with linear differential equations will recognize a special solution of the inhomogeneous, and the general solution of the homogeneous equation.

sideways without a current being induced. But in a gravitational field the ions and electrons move in opposite directions, producing precisely the current density, **j**, which by the $\mathbf{j} \times \mathbf{B}$ force supports the plasma against the force of gravity.

A charged particle in a circular orbit has a magnetic moment—the ring current times the surface area. This moment is directed against the field, so it is diamagnetic. And as a paramagnetic material is attracted by a magnet, so is the diamagnetic plasma repelled by a magnetic field. More precisely, a charged particle experiences a force directed against the gradient of the field strength. The component of this force perpendicular to the magnetic field causes a sideways drift, but the parallel component repels the particle from the stronger field. So wherever field lines converge or diverge, the particle will be pushed back from the stronger field.

The complicated motions that arise in non-uniform and time-dependent fields were studied first in astrophysical contexts. Not surprisingly, Scandinavian physicists, who had personal experiences of the beauty and marvel of aurora borealis, played a leading role in studying the phenomenon. Birkeland fired electron beams at magnetized spheres to simulate the behaviour of solar-wind particles in the Earth's field. Inspired by these 'terrella' experiments, Störmer made extensive calculations of such orbits [27]. He distinguished allowed and forbidden regions and found impact zones explaining the geographical location of auroras. Alfvén developed the concept of a guiding-centre orbit [28], about which the particle gyrates with approximately constant magnetic moment (box 1.3, figure 1.4).

Another interesting aspect of the motion of a single particle is the ponderomotive force exerted on electrons by radiofrequency waves (box 3.3). This is proportional to the square of the amplitude, so that it appears as a non-linear effect in plasma–wave interactions, notably in laser-produced plasmas; it has also played a role in fusion research as a possible means for helping to confine a plasma. It was first addressed, in different parts of the world, in the context of work on high frequency generating tubes but was soon considered for thermonuclear applications as well [29].

To study particle orbits in given fields is only one step towards understanding plasma behaviour. We have already seen some collective effects in which particles alter, if not shape, the fields through space charges and currents. A more general treatment had to be based on statistical mechanics. The first point that needed clarification was how the concept of a collision as an instantaneous exchange of momentum translates to what happens in a plasma, in which a particle at any time interacts with many others through long-range forces. Building upon the Liouville and Boltzmann equations which describe the evolution of the electron velocity-distribution function, Landau [30] derived a formula for the collision term, while Vlasov [31] proposed to include the smoothed self-fields of the particles. Landau [32] predicted collisionless damping of waves in plasma by resonant particles with thermal velocity near the phase velocity. The kinetic theory of gases and its application to plasmas was reviewed in Chapman and Cowling's book *The Mathematical Theory of Non-Uniform Gases* [33].

Viewed from a different angle, the plasma may be seen as a conducting fluid. In 1950, the Swedish astrophysicist Hannes Alfvén, who received the Nobel prize for physics in 1970, published his book *Cosmical Electrodynamics*, in which he argued that, on astronomical scales, electrical forces are usually small, but that magnetic forces can have a dominant effect on fluid motion [34]. Hence, the name hydromagnetic or magneto-hydrodynamic (MHD) theory. In fact, Alfvén showed that cosmical plasmas can often be considered as ideal conductors—with negligible resistivity. This led him to the concept of frozen-in magnetic fields [35] and to his prediction of the existence of MHD waves (box 7.2). These are now known as Alfvén waves and their phase velocity is called the Alfvén velocity. MHD shock waves were studied by de Hoffmann and Teller [36]. The idea of a magnetic field moving around with a conducting fluid—from which magnetic field lines derive an identity—was

Box 1.3 Guiding-centre motion

The magnetic moment of a particle with perpendicular velocity $v_\perp$ relative to the direction of the field

$$\mu = mv_\perp^2/2B$$

is constant to a high approximation while the motion is affected by the space and time derivatives of the field strength. This allows the motion to be treated as the superposition of a gyration and the motion of the guiding centre—the centre of curvature of the gyration.

In a constant field, the kinetic energy $mv^2/2$ is also constant, so that the particle exhausts its parallel energy when travelling towards a stronger field—a magnetic mirror. Reflection occurs where the guiding-centre velocity parallel to the magnetic field, $v_{||}$, equals zero, that is where

$$B_{\text{refl}} = mv^2/2\mu.$$

In analogy with the cross-field drift caused by a perpendicular electric field (box 1.2), a perpendicular gradient of the magnetic field strength causes a guiding centre drift in a direction perpendicular to both **B** and the gradient, again with approximately constant magnetic moment. And a similar drift is caused by curvature of field lines

$$v_{\text{d}} = \frac{mv_\perp^2}{2qB^3}\,\mathbf{B} \times \nabla B + \frac{mv_{||}^2}{qB^2R^2}\,\mathbf{R} \times \mathbf{B}$$

where **R** is the radius of curvature of the field line at the guiding centre. The first term may be understood as a response to the time-averaged force on a particle gyrating in a non-uniform field, whereas the second arises from the centrifugal force resulting from its motion along curved field lines. In a toroidal vacuum field, the gradient is connected with the curvature and the drift becomes

$$\mathbf{v}_{\text{d}} = \frac{w_\perp + 2w_{||}}{qB^2R^2}\,\mathbf{R} \times \mathbf{B}$$

where $w_\perp$ and $w_{||}$ stand for the kinetic energy in the perpendicular and the parallel velocity.

In a time-dependent field, the constancy of μ implies that the perpendicular energy varies with B while the parallel energy stays constant. This can be exploited as a heating mechanism—magnetic pumping.

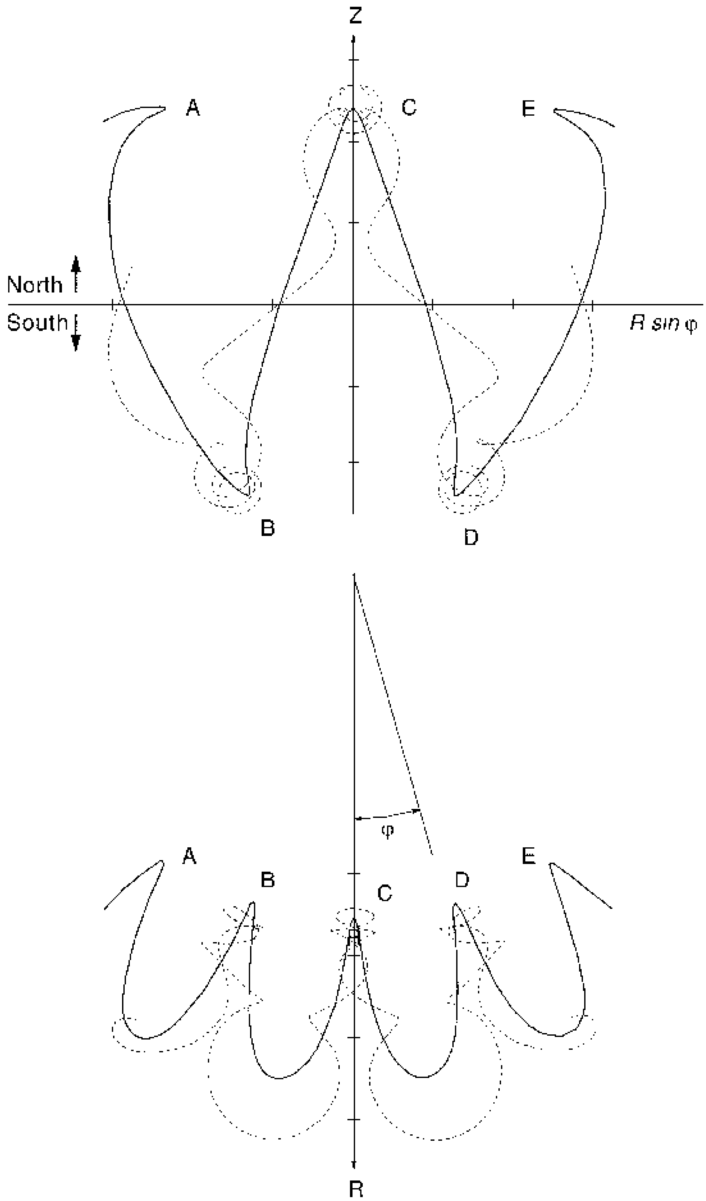

Figure 1.4. Motion of a charged particle in the Earth's dipole field, calculated by numerical integration (broken lines) and by guiding centre theory (solid lines). R, φ and Z are cylindrical coordinates. Above: projection on the $\varphi = \pm\pi/2$ plane through the polar axis, below: projection upon the equatorial plane ($Z = 0$). The orbit displays both repeated reflections from the magnetic mirrors in the northern and southern hemispheres (points A–E), and azimuthal drift due to the gradient of the field strength and the curvature of the field lines.

foreign to Maxwell's electromagnetic theory. It is valid only in an approximation that neglects high-frequency phenomena in which the displacement current plays a role, and Alfvén has stressed that the coupling can indeed be broken, also in cosmical phenomena. Yet this way of looking at the dynamics of magnetic fields and plasma has turned out to be extremely fruitful.

A related concept emerging from MHD theory is the magnetic pressure. Plasma can disturb a magnetic field—push it aside, bend or twist it or constrict the field lines. Conversely, the field can accelerate the plasma, shape or confine it. For magnetic confinement it is necessary that there is a pressure equilibrium (box 1.4) between the plasma and the field; and for the equilibrium to be a stable one, it must be such that no small disturbance can grow and destroy the equilibrium. The first example of such a pressure equilibrium was the pinch, to which we shall return in many places hereafter.

After the Second World War, Germany's leading physicist, Werner Heisenberg, encouraged the astrophysicist Ludwig Biermann to assemble a group of young theoreticians in Göttingen to look into fusion from the vantage point of astronomy. Germany was not permitted by the occupying powers to have a nuclear energy programme, so the work stayed at the level of basic science and the results obtained by Arnulf Schlüter, Reimar Lüst and others were freely published. There had been much discussion, both among gas discharge physicists and astrophysicists, about how to reconcile the individual particle description and the fluid description. One question was, for example, whether a magnetic field, which inhibits electron flow perpendicular to it, gives rise to a highly anisotropic conductivity. Schlüter developed a two-fluid theory [37] which brought light in such controversies.

We have already noted Landau's treatment of long-distance interactions in a plasma, in which the strength of the interaction varies as $1/r^2$ while the number of particles in a shell with radius r varies as r^2. Astronomers had encountered the same problem when they studied gravitational interactions between stars. Lyman Spitzer [38] computed the resistivity of a fully ionized gas caused by electrons losing their directed velocities in multiple small-angle scattering ('diffusion in velocity space'), and summarized the work on the effects of collisions in his seminal monograph *Physics of Fully Ionized Gasses,* which appeared in 1956. Because the cross-section for collisions between charged particles—the Rutherford cross-section—scales as T_e^{-2}, the resistivity of a fully ionized plasma scales as $T_e^{-3/2}$. (A hydrogenic plasma of $T_e = 1.5\,\text{keV}$ already conducts as well as copper at room temperature, so that Ohmic dissipation becomes ineffective for heating as thermonuclear temperatures are approached.)

Also in 1956, Richard Post [39] published a survey of the fusion-relevant unclassified physics known at that time. Like Spitzer, he had to stop short of revealing secrets of the thermonuclear programme in which, as it turned out later, both were deeply involved. The constancy of the magnetic moment and the action of a magnetic mirror were well known at that time and Fermi [40]

Box 1.4 Magnetic pressure

By summing up the contributions of individual particles to the local current density in a steady-state plasma, one finds the diamagnetic current density associated with a temperature or density gradient,

$$\mathbf{j} = (\mathbf{B} \times \nabla p)/B^2.$$

In a macroscopic picture, this is the current through which, in static equilibrium, the magnetic field exerts a balancing pressure perpendicular to $\mathbf{B}$:

$$\mathbf{j} \times \mathbf{B} = -\nabla p.$$

In a rectilinear magnetic field the sum of the plasma pressure, $p = n_e kT_e + \sum n_i kT_i$, and the pressure or energy density, $B^2/2\mu_0$, of the magnetic field is uniform; in a curvilinear field there is a magnetic tension along $\mathbf{B}$ of the same magnitude which provides the restoring force in Alfvén waves. The parameter β is defined as the ratio

$$\beta = 2\mu_0 p/B^2$$

and when this refers to a magnetically confined plasma, p is meant to be the maximum pressure while B is usually measured outside the plasma. In a curvilinear system, there is some arbitrariness in where precisely B is measured, so that one may even find instances where β is said to exceed one.

The pressure equilibrium between the self-field of a current in a plasma and the plasma pressure is known as the Bennett relation; in Tonks' version it reads

$$I^2 = 2 \times 10^7 \left(N_e kT_e + \sum N_i kT_i \right)$$

where I is the discharge current in amps, N_e is the number of electrons per metre length of the plasma column and T_e is the electron temperature (in eV), while N_i and T_i refer to the various ion species that may constitute the plasma. In a sharp-boundary model, in which the plasma is separated from the field by a thin current sheath, one finds this result by posing an arbitrary plasma radius, calculating the magnetic field and the plasma pressure, and noting that the radius drops out of the equation for the pressure equilibrium. Its validity can also be demonstrated, however, for arbitrary pressure profiles and the corresponding profiles of current density.

had even proposed repeated reflections between opposite mirrors moving towards each other as a mechanism for accelerating particles in cosmic magnetic fields. Yet secrecy prevented mirrors as means for confining plasma from being mentioned in Post's review article and Spitzer's book. Despite such limitations, these works defined the scientific background against which the groups that began to look into the subject around that time started their work. For those who entered the new field with a background of knowledge about individual particles and their motions, and naïvely thought that it would be just a small step to come to grips with their collective behaviour, the plasma had surprises in store. Alfvén, Spitzer and Post were the guides who led the emerging fusion community to the new way of thinking. (Post made up for his earlier omission by publishing in 1987 a monumental review of mirror confinement, and in 1995 he looked back at twentieth century plasma physics [41]).

1.2 In and out of secrecy

1.2.1 Programmes taking shape

Although the physics community was at first divided about the possibility of exploiting nuclear energy on earth—Rutherford [42] had called it '*moonshine*'—speculation on the subject abounded from the days when it was suspected that nuclear processes might be important for the stars. Remarkably, cold fusion was among the first schemes that were considered. Already in 1926, German chemists had reported the production of helium in hydrogen-loaded palladium, but before long they had to retract their claim [43]. Tandberg, a Swedish engineer, nevertheless continued along this line until the 1930s [44]. This topic returned briefly to prominence in 1989 with the much-publicized claims that cold fusion had been observed in electrolytic cells, but the results did not stand up to close scrutiny [45].

Magnetic confinement of hot plasma was actively pursued in 1938 by Kantrowitz and Jacobs [46] from the US National Advisory Commission for Aeronautics, who attempted heating a plasma by radio-frequency power in a toroidal magnetic field, but had to give up for lack of support. And in 1937, the German physicist Fritz Houtermans had made plans for work in the Ukrainian Physico-Technical Institute in Kharkov before he became one of Stalin's political prisoners.

In Britain, thoughts on thermonuclear reactors began to take shape shortly after the war. In 1946, George P Thomson and Moses Blackman at Imperial College in London had registered a patent for a thermonuclear reactor. This was based on magnetic forces—the pinch effect—for the confinement of electrons and on electrostatic forces, resulting from the space charge of these electrons, for the confinement of ions. In a torus of

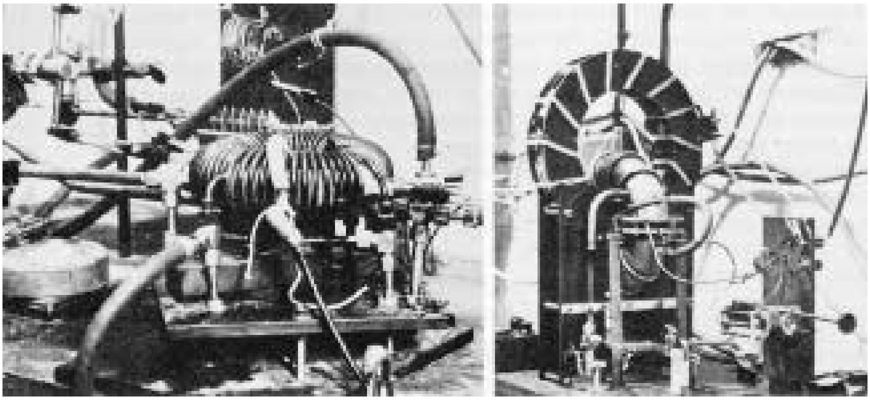

Figure 1.5. Two of Thonemann's early tori at the Clarendon Laboratory, Oxford (*circa* 1949–50). The torus on the left is of Pyrex with a water-cooled coil wound around it to provide a toroidal magnetic field. The plasma current (about 10 A) was induced via RF coils and a transformer core (most of which has been removed for clarity in the picture). The torus on the right is of copper and was used to study the pinch effect at currents up to 2 kA.

major radius $R_0 = 1.3$ m and minor radius $a = 0.3$ m, a ring current of 0.5 MA was to have been established by betatron acceleration and to be maintained by a travelling wave fed by 3 GHz magnetrons. Ions would be heated by collisions with electrons and, with a postulated space-charge voltage of 0.5 MV, their temperature could run up to several hundred keV. The fuel would be deuterium and the fusion energy yield 9 MW(th), to be exploited as heat or as neutrons for breeding fissile materials. This proposal provoked much discussion within British atomic energy circles, but little action so that it would not be until 1948 [47] that Alan Ware could start experimental fusion research at Imperial College. The patent was classified, so the details were not made public at that time, and three years later this work was moved for security reasons to Associated Electrical Industries (AEI) in Aldermaston.

A parallel initiative had been started in 1946 in the Clarendon Laboratory at Oxford University by Peter Thonemann, who built a series of glass tori in which alternating current discharges could be induced electromagnetically with initially a 5 MHz and later a 100 kHz power source (figure 1.5). But only when the radio frequency source was used to drive a direct current in the discharge (an arrangement similar to the current drive that will be discussed in section 7.3) was the pinch effect clearly seen. From 1953 on, Thonemann used an iron-core pulse transformer, in which a unidirectional primary current pulse was driven by a switched capacitor bank and the plasma loop acted as the secondary winding (figure 1.6), the technique adopted earlier by Ware as well as by Russians and Americans. After

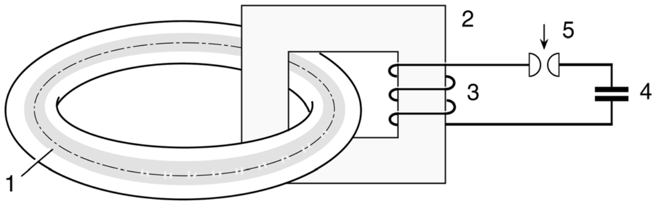

Figure 1.6. Transformer-driven toroidal discharge. The plasma loop,1, acts as the secondary winding of a transformer, 2, whose primary winding, 3, is fed by a capacitor bank, 4, discharged through a spark gap, 5.

Thonemann's work had been transferred to Harwell, it rapidly expanded to culminate in the famous ZETA experiment. A third British programme developed in the mid-1950s in the Atomic Weapons Research Establishment, also at Aldermaston but quite separate from the AEI programme referred to earlier. The AWRE fusion work was originally concerned with imploding shock waves and linear pinches, but later expanded in other directions as well. These separate elements of the UK fusion research were brought together at the new Culham Laboratory established in the early 1960s.

Meanwhile, both Ware and Thonemann [48] had published papers on the subject which until 1956 would remain the most relevant places in the literature. Since all this work became classified in 1951, little was heard about it until 1956, except that in 1951 an Austrian physicist working in Argentina moved his benefactor, President Perón, to announce a breakthrough in fusion research. The scientific merits of this work were subsequently disproved, but Perón did manage to draw the attention of both scientists and government officials in the USA to the subject and so became the catalyst that activated their fusion research [49].

Fusion energy had been discussed in Los Alamos already before the end of the war, and some of the basic physics was recognized at that time, but it had remained dormant until an ambitious classified programme was launched by the Atomic Energy Commission (AEC) in 1952 and 1953. This started with the stellarator, conceived by Lyman Spitzer at Princeton as a steady-state magnetic field with closed magnetic surfaces on which the field lines had 'rotational transform' (section 2.4). Spitzer, himself a theoretician, attracted further mathematical support from Martin Kruskal and Martin Schwartzschild. Next, J L (Jim) Tuck, an Englishman who had already worked in Los Alamos during the war and had returned there from Oxford, initiated work on pinches. He started with an inductively driven discharge, named Perhapsatron, but before long Los Alamos had linear pinches as well. Richard Post, who first worked at Berkeley but then moved to Livermore, proposed mirrors. Initially, the Berkeley Laboratory's director, Herbert York, had considered a cylindrical system with an axial magnetic field plugged at the ends with radio-frequency fields, but this soon gave way to the mirrors. Later, Livermore broadened its programme with pinch work by Sterling Colgate and a relativistic-electron ring, Astron, invented by Nicholas Christofilos, while Oak Ridge entered the field with a study by Albert Simon of magnetized arc discharges, later to be followed by work on injection of ions into mirror machines [50]. Edward Teller, then already a senior physicist, and Admiral Lewis Strauss, chairman of the US AEC, were influential supporters of the American programme.

In the Soviet Union, Oleg Lavrentiev—a soldier in the far-east army without even a high-school diploma—appears to have been the first to draw the attention of his government to controlled thermonuclear reactions [51]. The letters he wrote in 1949 and 1950 were passed on to Igor Tamm, who in turn asked Andrej Sakharov (both of them worked in the nuclear weapons programme) to look into the problem.* They saw that Lavrentiev's suggestion to confine ions in electrostatic fields (section 3.5) was rather naïve, and developed their own ideas about toroidal magnetic thermonuclear reactors (MTR) [52]. In May 1951 the Soviet Government formally launched the controlled fusion programme and established a Council on MTR headed by Kurchatov, director of the Institute of Atomic Energy (IAE) in Moscow—which was later named after him. Lev Artsimovich became responsible for the experimental programme in the IAE, while Mikhail Leontovich directed theoretical studies. Leontovich was a gifted teacher, whose school brought forth eminent theoreticians who rose to prominent positions in their country. Kurchatov also drew institutes in Moscow (besides the IAE), Kharkov, Leningrad, Sukhumi, and later Novosibirsk into the work on plasma physics and controlled fusion which, like that in the US and Britain, was initially kept secret.

The early experimental work in the IAE was organized in three groups under Andrianov, Golovin, and Yavlinsky. Artsimovich first stimulated work on fast linear pinches, done by Filippov in Andrianov's group. Inductive discharges in strong toroidal fields as proposed by Sakharov and Tamm—known as tokamaks from 1959 on—initially were a common programme of Golovin's and Yavlinsky's groups. Osovets studied confinement by the time-averaged magnetic pressure exerted by a travelling wave. Later, when Vedenov and his theoretical colleagues had understood ponderomotive forces (box 3.3), RF confinement took the form of repelling the axial flow of a plasma in a straight field. In 1956, after Budker [53] had pointed out that drift orbits would lead to enhanced diffusion in toroidal systems and

* In 1950, the Harwell theoretician Klaus Fuchs was arrested for espionage on behalf of the Soviet Union and another theoretician, Bruno Pontecorvo, defected to Moscow. They knew about the work by Thonemann and Ware, and likely passed on information about this before it was published. There is, however, no evidence that this had any influence on Tamm and Sakharov, whose first ideas show little resemblance to the British work.

had proposed confining high-energy ions between magnetic mirrors, M S Ioffe formed a group to study such ‘adiabatic traps’, and started the PR series of experiments that would eventually run to PR-7. Artsimovich had become concerned about current-driven instability and tended to favour Ioffe’s mirror experiments, but when he grew convinced of the potential of toroidal discharges, he encouraged Yavlinsky to attract a strong group of young physicists to this work.

The hierarchy in the IAE was complicated in that Igor Golovin was until 1958 a deputy to Kurchatov, which gave him an opportunity to promote a large mirror facility, OGRA, against the will of Artsimovich who wanted to conduct initial experiments on a smaller scale first. Kurchatov then formed an independent group under Golovin to build OGRA, but after Kurchatov’s death in 1960, Artsimovich became the undisputed leader of all fusion research in the Soviet Union. He held important positions in the IAE, the Academy of Sciences and Moscow University which, however, did not keep him from taking direct charge of the tokamak group when in 1962 Yavlinsky died in a plane crash. By his deep insight in both the theoretical and the experimental aspects of the problem, his sober-minded and critical assessment of results and his sharp judgement of what were the most promising developments, Lev Artsimovich grew to become the world’s leading authority on fusion research. He was a gifted lecturer and author of one of the most comprehensive books [54] on fusion research that were to appear in the early 1960s.

1.2.2 Looking behind the curtain

The first UN Conference on the Peaceful Uses of Atomic Energy, *Atoms for Peace*, convened in Geneva in 1955 upon the initiative of US President Eisenhower. Here, fusion was mentioned only in the opening address of the president of the conference, Homi Bhabha, who ventured his much-quoted prediction:

> *It is well known that atomic energy can be obtained by fusion processes as in the H-bomb and there is no basic scientific knowledge in our possession today to show that it is impossible for us to obtain this energy from the fusion process in a controlled manner. The technical problems are formidable, but one should remember that it is not yet fifteen years since atomic energy was released in an atomic pile for the first time by Fermi. I venture to predict that a method will be found for liberating fusion energy in a controlled manner within the next two decades. When that happens the energy problem of the world will truly have been solved forever for the fuel will be as plentiful as the heavy hydrogen in the oceans.*

These words stimulated laboratories around the world to take up the subject. Several contributions to the 1957 Venice, 1958 Geneva and 1959 Uppsala

Conferences report work started around 1956. In France, for example, where the fusion research that Meunier had started in 1952 had been abandoned when he left Fontenay-aux-Roses, a new programme was initiated in 1956 by Hubert, with support from Vendryès, Yvon and Francis Perrin. In Japan, the Universities of Tokyo, Nagoya, Osaka and Kyoto started pioneering research [55]. Special committees of the Japanese Atomic Energy Commission and the Science Council of Japan charted out a course in which first the scientific ground would be explored before the Japan Atomic Energy Research Institute (JAERI) would become involved. This led to the foundation of the Institute of Plasma Physics, affiliated to Nagoya University, in 1961. For its first twelve years this institute was directed by Kodi Husimi, a man of dignity and refinement who after his retirement continued to serve as the president of the Science Council.

Well before the advent of fusion research, gas discharge physics had been an active field of study, both in universities and in industrial laboratories. The highest energy densities were found in arcs, struck between metal or carbon electrodes. But in a classical arc, the degree of ionization is low and the ion temperature cannot rise much above the temperature of the neutral gas, which in turn is cooled by the surroundings. To overcome this barrier one had to resort to entirely different techniques. As a first step, the pinch effect would isolate a fully ionized plasma from the wall; the next step would be to bend the discharge tube into a torus so as to avoid heat loss to the electrodes. Naturally, linear and toroidal pinches were among the first schemes to be considered for the production of thermonuclear plasmas.

The simplest way to produce a high-current arc is to discharge a high-voltage capacitor bank through a low-inductance circuit into a pre-ionized gas. Under these conditions the formation of the plasma involves shock-wave heating and adiabatic compression, both of which are more efficient than Ohmic (or Joule) heating at elevated temperatures. Moreover, shock waves tend to heat the ions preferentially. So, although most groups saw this scheme as only a stepping stone towards toroidal discharges, one could envisage also very fast, very high-density linear pinches being established and conditions for fusion energy gain being reached on a time scale shorter than those of instabilities and axial heat loss.

When the first experiments on magnetic confinement were initiated, it had already been recognized that the pressure equilibrium could be unstable [56]. As a first step towards a more comprehensive MHD-stability theory, Kruskal and Schwartzschild [57] described the flute instability of a plasma supported against gravity by magnetic pressure, as well as the kink instability*

* Unstable deformations with amplitude varying as $\exp[i(m\theta + kz)]$ in cylindrical coordinates or $\exp[i(m\theta + n\phi)]$ in toroidal coordinates are called sausage instabilities if $m = 0$ and kink instabilities if $m = 1$.

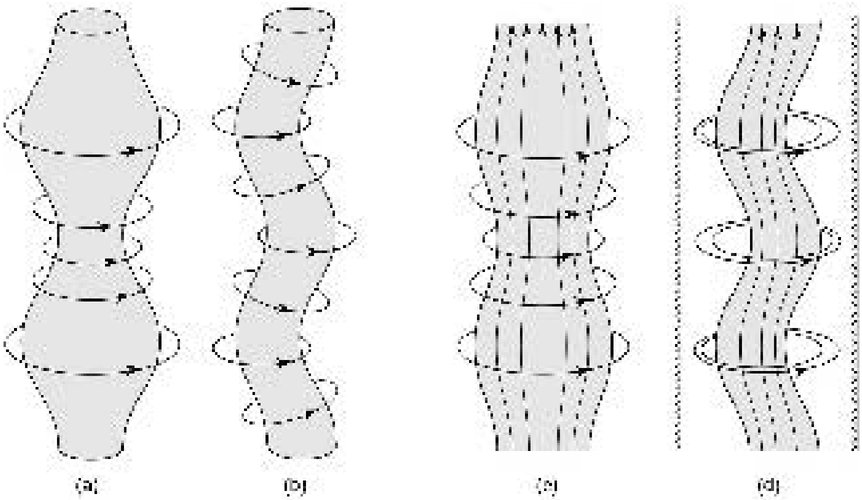

Figure 1.7. Sausage and kink instabilities. (a) and (b) Unstabilized, the field is stronger at the neck of the sausage than at the bulge and it is stronger at the inside of the kink than at the outside. (c) The sausage is stabilized by a trapped axial field; both the azimuthal field of the plasma current and the trapped field are strongest in the neck, but the former goes as r^{-1}, while the latter goes as r^{-2}. The kink is stabilized by the stress of the trapped field, combined with the compression of the pinch field between the plasma and the metal wall.

(figure 1.7) of a pinched discharge (box 1.5). Their paper had escaped censorship, but further studies of pinch stability in the US, Britain and the Soviet Union, which included the effects of trapped magnetic fields, metal shields, finite boundary layers and arbitrary distortions, were classified.

The first glance behind the curtains of secrecy was given on the occasion of a Soviet state visit to Britain in April 1956. As a member of the delegation, Kurchatov [58] was allowed to present a lecture at Harwell, where he surprised the world by bringing the Russian experiments on discharges in straight tubes, filled with deuterium or some other gas, into the open. Kurchatov reported neutrons [59], but made it clear that they were produced by deuterons colliding with other deuterons—possibly adsorbed at the electrodes or at the wall—after having been accelerated over several tens of kilovolts, rather than by true thermonuclear reactions. Neutron pulses were always coincident with bursts of hard (up to 300–400 keV) X-rays. Kurchatov expressed doubt about the possibility of keeping the pinched column from coming in contact with the walls and attached particular interest '*to methods in which stationary processes may be employed*', without elaborating on this subject. In the discussion [60] that followed the lecture it became clear that they were also working on toroidal pinches, but he may as well have been referring to the mirror devices on which his laboratory had just started experimental work.

Box 1.5 Stability of the pinched plasma column

The simplest form of a pinch, a straight plasma cylinder carrying an axial current whose azimuthal field balances the plasma pressure, is first of all unstable against sausage deformations. The field is strongest in the constrictions, squeezing the plasma farther into the bulges. A trapped axial field resists both the contraction and the expansion because its strength varies as the square of the radius, whereas the confining field varies linearly. In a kink deformation the driving force comes from the crowding of the azimuthal field lines on the inside and the thinning out on the outside. The stabilizing force is the tension of the trapped axial field, or the increase of its energy associated with the stretching.

The earliest theoretical work on these 'stabilized pinches' was based on a normal-mode analysis and dealt with a sharp-boundary model in which the plasma current was restricted to an infinitely thin layer. It showed that a trapped field would be effective for stabilizing short wavelengths while a conducting wall outside the plasma would suppress long-wavelength deformations by image currents or, what amounts to the same, by the field of the plasma current being compressed between the plasma and the wall.

So far, no toroidal effects had been considered, other than the requirement that the pitch of a disturbance must be an integer fraction of the length of the cylinder for the endplanes to match when it is bent into a torus or toroid. But then the equilibrium is lost, so one first had to see how this could be restored before one could turn to the stability of toroidal systems. Shafranov described the equilibrium of the toroidal pinch, after which he and Grad independently derived the general equation for the equilibrium in a closed system from which subsequent stability analysis had to start.

A series of papers published in *Atomnaya Energya* [61] described the Russian pinch work in considerable detail. They were summarized by Artsimovich and Golovin in an extraordinary session of the Stockholm Symposium on Electromagnetic Phenomena in Cosmical Physics of the International Astronomical Union in August of 1956 [62]. Artsimovich reviewed the evidence for the occurrence of both sausage and kink instabilities. He estimated the ion temperature at 100 eV, thus excluding a thermonuclear origin of the observed neutron flux. In an impressive performance, he then went on to present Shafranov's paper, which extended earlier theoretical work on the kink instability by including an axial (B_z) magnetic field.

Golovin described more details of the experimental work. Artsimovich's paper opened with an introduction in which he argued that the energy balance of a reacting plasma is determined by the lifetime of an ion in it. From this he derived a criterion for power generation in a D–D reactor: $10^{-15} n\tau \approx 1$ at $T \approx 10^8$ K, where n is the density of the plasma in cm^{-3}, τ is the lifetime of an ion in the system in seconds and T is the temperature. This is, in essence, the criterion derived a few years earlier by Lawson from Harwell, but not published because of secrecy until 1957. There is no trace of this result in the original Russian papers, so one must assume that it reflected how Artsimovich himself had thought about the thermonuclear problem. Yet he never claimed credit for having derived the Lawson criterion independently.

At this Stockholm conference, the only *acte de présence* of fusion research in the Western countries was by R S (Bas) Pease from Harwell, who derived the pinch current (1–2 MA) at which bremsstrahlung makes equilibrium with Ohmic dissipation [63]. Meanwhile, pressure towards declassification built up, both in the scientific community and in public opinion, and some fusion work began to trickle into scientific journals also west of the Iron Curtain.

Harwell responded to Kurchatov's challenge by producing a series of papers for the January 1957 issues of the *Philosophical Magazine* and the *Proceedings of the Physical Society*. Here, they established the Lawson criterion and the Pease current, described theoretical work on stability and on MHD shocks, and published the first photographs (figure 1.8) of an unstable toroidal pinch. Elsewhere, W B Thompson published a brief theoretical introduction to the thermonuclear problem [64].

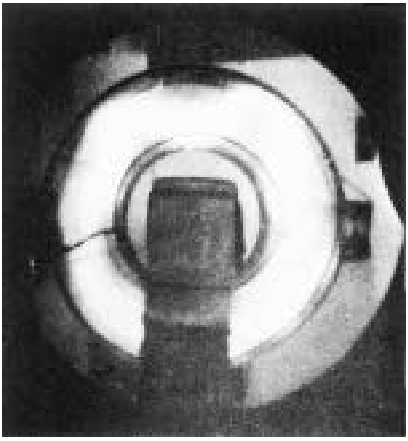

Figure 1.8. Photograph of an unstable toroidal pinch discharge with current 1.3 kA in a xenon plasma. The glass tube has a major radius of about 0.3 m.

When a plasma is confined in a magnetic field, MHD instability may be driven by two sources of free energy, the pressure gradient and the magnetic energy associated with the deviation from the vacuum field. The second cause had been studied—in an astronomical context—by Lundquist [65], who had derived an energy principle for a non-vacuum field in a pressureless conducting fluid (one in which the current is everywhere parallel to the magnetic field). Several groups had undertaken to extend this treatment, to take account of the free energy residing in the plasma pressure. While most of the earliest work was concentrated on the kink instability of the pinch [66], Rosenbluth and Longmire [67] had studied flute instabilities in geometries with curved field lines, recognizing the unfavourable curvature in mirror devices and throwing a new light on the sausage instability of the pinch. Further work along these lines eventually led to the general energy principle laid down in the definitive 1957 BFKK (Bernstein, Frieman, Kruskal and Kulsrud) [68] paper, but independently derived in essence by Kadomtsev, by Hain, Lüst and Schlüter and by Berkowitz *et al.*

Not yet having their own channels of communication, fusion physicists from East and West set out to invade the Third International Conference on Ionization Phenomena in Gases, held in Venice in June 1957 [69]. Groups from Harwell and from the Centre d'Etudes Nucléaire in Saclay, France, reported on highly unstable toroidal pinches, but Harwell saw the first signs of stabilization by an 'axial' or toroidal magnetic field. Aldermaston, Berkeley and Saclay also had work to report on linear pinches, not unlike that disclosed earlier by the Russians, except that Aldermaston ventured to claim a possible thermonuclear origin for their neutrons. Both Shafranov and the Göttingen group discussed toroidal equilibria; the latter had derived a general stability theory and proposed magnetic pumping as a heating mechanism. Papers from Los Alamos, Harwell and CERN (Geneva) discussed runaway electrons, a phenomenon caused by fast electrons having a longer mean-free path for collisions with ions than slow ones, so that above a critical field strength—the Dreicer limit—there develops a beam of fast electrons. Los Alamos, Harwell and Aachen expanded the theory of the B_z-stabilized pinch; Kruskal from Princeton presented his theory of adiabatic invariance of the magnetic moment.

The next step in the declassification process was preceded by a series of papers on equilibrium and stability from the Göttingen group, who were not restricted by rules of secrecy, in *Zeitschrift für Naturforschung* [70]. Then on 25 January 1958 the British journal *Nature* [71] published a series of papers that represented the outcome of British–American exchanges of information and negotiations about declassification [72]. Optimism ran high in Britain and the United Kingdom Atomic Energy Authority (UKAEA) had been anxious to open up their thermonuclear file. Harwell had recently installed the large toroidal pinch apparatus ZETA (Zero-Energy Thermonuclear Assembly, a name alluding to fission terminology in which a low-power critical assembly was called a zero-energy pile), and had neutron fluxes suggestive of a temperature of 0.4 to 0.5 keV. In the US, Los Alamos had comparable results from their Perhapsatron S-3 toroidal pinch, which gave neutron yields consistent with 0.5 keV ion temperatures. There were also reports about toroidal pinch work in the AEI laboratory at Aldermaston, as well as stabilized linear pinch work from Los Alamos. In the same *Nature* issue, Spitzer sounded a warning by pointing out that the ion temperatures claimed for toroidal pinches could not be explained on the basis of Ohmic heating, and elsewhere [73] Livermore disclosed neutron-spectroscopic measurements on a linear pinch which showed that deuterons had been accelerated axially to energies over 50 keV, presumably by sausage instabilities.

Despite these critical notes, the ZETA results gave rise to great expectations. A journalist enticed Harwell's director Sir John Cockroft to say that he was ninety per cent certain that the neutrons had a thermonuclear origin. But a few months later, in May 1958, nuclear physicists from Harwell [74] reported anisotropic neutron spectra showing the presence of deuteron streams and invalidating the thermonuclear claim. About the same time, the Americans disclosed much of their non-pinch work in the Spring Conference of the American Physical Society (APS) [75]. Spitzer presented the stellarator and discussed the main methods—figure-eight tubes and helical windings—to produce rotational transform. Albert Simon introduced the Oak Ridge approach towards trapping injected ions in a mirror machine and Post described the Livermore pulsed-mirror or pyrotron line of experiments. Details were withheld for the forthcoming second Atoms for Peace Conference, however. Thus, the ball was again in the Soviet court, but their side deferred further declassification until the conference.

1.2.3 The road to travel

Let us recall at this point what could be said around that time, based upon sources in the open literature such as the books by Alfvén and Spitzer and the 1956 review article by Post, about the problem of building a thermonuclear reactor.

By considering the yield-to-loss ratio,* R, in a pulsed reactor system—which did not rely on ignition—Lawson had derived general criteria for the temperature, density and confinement time required for a reactor to become energetically self-sustaining (box 1.6). The reaction-rate parameters as known at this time are shown in figure 1.9 [76].

In a steady state, τ assumes the character of an energy-confinement time, τ_E. (In early work particle loss went much faster than heat conduction, so that the energy loss was primarily the energy carried off by the particles and τ_E was essentially the particle confinement time, τ_p.) The energy confinement time is

* Elsewhere, we use the symbol R to denote the major radius of a torus.

Box 1.6 Lawson's criteria

The fusion power density generated in a D–D plasma is

$$P_\mathrm{f} = \tfrac{1}{2} n_\mathrm{i}^2 \langle \sigma v_\mathrm{i} \rangle E_\mathrm{DD}$$

where $\langle \sigma v_\mathrm{i} \rangle$ is the reaction cross-section averaged over the relative velocity of colliding ions and E_DD is the mean reaction energy of the two primary D–D reactions ($E_\mathrm{DD} = 3.65\,\mathrm{MeV}$). In a D–T plasma with equal amounts of D and T,

$$P_\mathrm{f} = \tfrac{1}{4} n_\mathrm{i}^2 \langle \sigma v_\mathrm{i} \rangle E_\mathrm{DT}$$

where $E_\mathrm{DT} = 17.6\,\mathrm{MeV}$.

The bremsstrahlung power loss scales as

$$P_\mathrm{B} \propto n_\mathrm{e}^2 T_\mathrm{e}^{1/2}$$

so that the ratio of the reaction yield to the bremsstrahlung loss is a function of the temperature only. For net energy production, this ratio has to be at least in the order of one, the precise requirement depending on the fuel cycle and the energy conversion efficiencies. This defines a lower limit for the operating temperature.

In a pulsed system, the energy balance must also account for the thermal energy invested in the plasma,

$$W = \tfrac{3}{2}(n_\mathrm{i} k T_\mathrm{i} + n_\mathrm{e} k T_\mathrm{e}) \approx 3nkT.$$

If τ is the duration of the pulse, the yield-to-investment ratio, R, is a function of $n\tau$:

$$R = \frac{\tau P_\mathrm{f}}{W} = \frac{n\tau f(T)}{3kT}.$$

Lawson concentrated on pulsed systems and drew curves of constant $n\tau$ in an R versus T diagram, but plots of $n\tau$ versus T or of $nT\tau$ versus T for constant R are also called Lawson diagrams. For a pulsed reactor with an efficiency of 33% for conversion of heat to electricity, Lawson gave criteria both for the minimum temperature and for the minimum $n\tau$ value at the optimum temperature:

$$T \geq 20\,\mathrm{keV} \text{ and } n\tau \geq 10^{22}\,\mathrm{m}^{-3}\,\mathrm{s} \text{ at } T \approx 100\,\mathrm{keV} \text{ for DD}$$

or

$$T \geq 3\,\mathrm{keV} \text{ and } n\tau \geq 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s} \text{ at } T \approx 30\,\mathrm{keV} \text{ for DT.}$$

For an energy-producing reactor it is of course not sufficient that the system can sustain its own energy demand; to arrive at an economically acceptable level of circulating power, it must exceed Lawson's $n\tau$ limit by nearly one order of magnitude.

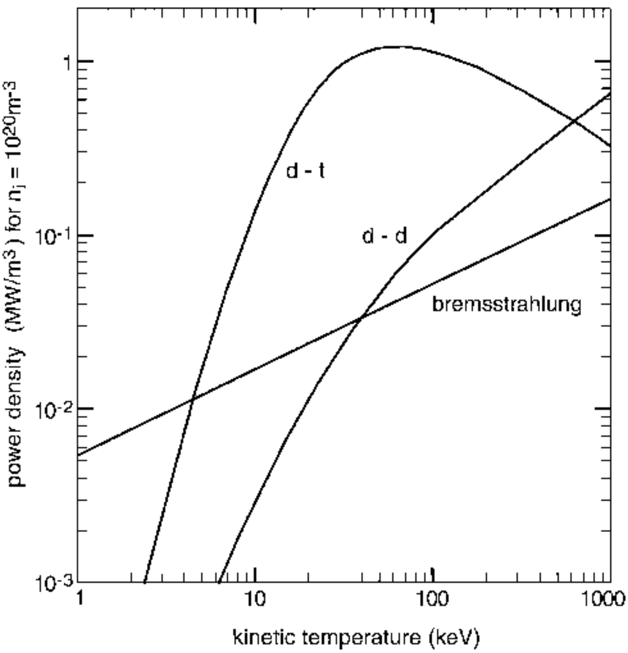

Figure 1.9. Power density of the fusion reaction in pure deuterium and in a 50% D plus 50% T mixture as a function of plasma temperature compared with bremsstrahlung.

related to the heating power P_H needed to maintain the plasma at the temperature T through $P_H = 3VnkT/\tau_E$, where V is the volume of the plasma and the factor 3 expresses that the ions and the electrons both have $\frac{3}{2}kT$ as mean energy. Then, the power loss scales as nT/τ_E and the power yield as $n^2\langle\sigma v\rangle E_r$, so that R is proportional to $n\tau_E f(T)$. Neglecting all losses other than bremsstrahlung and assuming equal ion and electron temperatures, Post estimated the ignition point to lie at 35 keV in deuterium and 4 keV in a 50 : 50 deuterium–tritium mixture. To find the optimum values for $n\tau$ (or $n\tau_E$) and T, one must of course specify the mode of operation, pulsed or steady state, and the efficiencies of various energy conversion steps in more detail. Moreover, the way in which τ_E depends on n and T affects the optimization. But whether τ is taken to be a pulse duration, a particle-confinement time or an energy-confinement time,* the outcome is not greatly different, so that Lawson's figure $n\tau = 10^{14}\,\mathrm{cm^{-3}\,s}$ ($10^{20}\,\mathrm{m^{-3}\,s}$ as we quote it), stuck in people's minds and became the milestone against which progress in fusion research was measured, regardless of the precise meaning of τ.

* For the present discussion, energy confinement is of course what counts. But the charm of the $n\tau$ criterion is its global nature, which obviates the need to define precisely whether τ represents energy transport, particle loss or pulse duration.

In Alfvén's terminology, magnetic confinement is an example of a field 'frozen' in a plasma, although in this case one should perhaps say frozen out. Such a state will decay towards thermodynamic equilibrium, in which the temperature and the density of the plasma are uniform and the field has the vacuum configuration consistent with the currents in external coils; the decay time is given in box 1.7. The time constant characterizing this decay would then figure as the energy-confinement time in a Lawson criterion. Assuming this classical loss rate, one can calculate that a D–D reactor operating at $T \approx 50\,\text{keV}$ would need $Ba \approx 1\,\text{T m}$ (Tesla-meter), with the additional requirement that B has to satisfy the pressure balance (the optimum temperatures under these conditions are about a factor 2 lower than in Lawson's pulsed model). For a D–T reactor at 15 keV this figure would be only $10^{-1}\,\text{T m}$, but to confine 3.5 MeV α-particles with $B\rho = 0.27\,\text{T m}$, one would still need Ba in the order of 1 T m.

All this seemed well within the range of technical feasibility. It was clear, however, that classical confinement could not be taken for granted.

Box 1.7 Classical loss rate

To estimate the decay time of a plasma column in a magnetic field, one first calculates the E/B drift velocity (box 1.2) associated with the diamagnetic current (box 1.4)

$$\mathbf{v}_\mathrm{d} = -\frac{\eta \nabla p}{B^2}$$

where η is the resistivity of the plasma, and obtains as an estimate of the plasma or particle confinement time

$$\tau_\mathrm{p} \approx \frac{B^2 \Lambda^2}{\eta p}$$

Λ being the scale length of the pressure profile. As a first estimate, one may put $\Lambda \approx a/2.4$, where a is the plasma radius. (The factor 2.4 is appropriate for a Bessel function profile in a plasma cylinder; it would be $\pi/2$ for a cosine profile in a plane slab.)

The Spitzer resistivity of a hydrogenic plasma is given by

$$\eta \approx 3 \times 10^{-8} T_\mathrm{e}^{-3/2}\ \text{ohm-m} \quad [T_\mathrm{e}\ \text{in keV}]$$

so that, if this 'classical' resistive particle diffusion were the dominant loss mechanism, a thermonuclear plasma with $T_\mathrm{e} \approx T_\mathrm{i} \approx T$ and $p = 2n_\mathrm{i}kT$ would be confined with

$$n\tau_\mathrm{p} \approx 1.8 \times 10^{22} (Ba)^2 T^{1/2}.$$

Switching from an MHD to a particle point of view, one can show that, in binary collisions, the guiding centres of the two particles make equal or opposite jumps, depending on whether the charges are opposite or equal. Thus, ion–electron collisions (responsible for Spitzer resistivity in the MHD picture) give rise to equal diffusion rates for ions and electrons (ambipolar diffusion). Classical diffusion may be shown to correspond with

$$D_{\mathrm{ci}} = \frac{\eta(p_{\mathrm{e}} + p_{\mathrm{i}})}{B^2},$$

which has an $nB^{-2}T^{-1/2}$ dependence.

This classical picture of cross-field diffusion, however, fails to account for the actually observed losses of magnetically confined plasmas. Anomalous loss caused by collectively generated field disturbances was first encountered in the context of the wartime development of ion sources for isotope separators [77]. Bohm proposed a threatening semi-empirical formula for the diffusion coefficient

$$D_{\perp \mathrm{B}} = \frac{kT}{16eB}.$$

The numerical factor is 1/10 in Bohm's paper, but is usually quoted as 1/16, in agreement with the companion papers co-authored by Burhop and Massey. This 'drain diffusion' was suspected of being connected with microscopic plasma turbulence (box 3.3) and was long feared to be an intrinsic property of magnetized plasma, even though Simon [78] had shown that, at least in arc discharges like those in the isotope separators, which were what Bohm diffusion was really about, the loss rate could be explained by assuming that the process was not really an instance of cross-field diffusion of the plasma, but came about by ions diffusing radially across the magnetic field and electrons being driven out axially by the resulting electric field. But when in later years stellarators once more displayed anomalous loss, such an explanation was not immediately at hand and in this area Bohm's (or by then rather Spitzer's) conjecture was already living its own life. In fact, we shall see in later chapters that something akin to it, albeit at a far lower level, continues to play a role in discussions about anomalous loss in tokamaks and stellarators.

Bohm diffusion as first hypothesized would lead to a staggering requirement of $B^{3/2}a > 100\,\mathrm{T}^{3/2}\,\mathrm{m}$ for a D–T reactor* and to even an order of magnitude more for D–D. So, where Lawson had set the goal and Spitzer's resistivity had defined the shortest possible path along which to reach it, Bohm had placed a seemingly insurmountable roadblock. Until the issue was finally settled at the 1968 IAEA Conference in Novosibirsk, Bohm diffusion haunted fusion research like an evil spirit.

* Strictly, the requirement would be: $B^{3/2}a \approx 3.4 \times 10^{-10}(n\tau)^{1/2}T/\beta^{1/2}$. With reasonable estimates for a D–T reactor, like $n\tau = 10^{21}$, $\beta = 0.1$ and $T = 15\,\mathrm{keV}$, this would translate into $B^{3/2}a \approx 500\,\mathrm{T}^{3/2}\,\mathrm{m}$. (Note: T represents *temperature*, T the unit of Tesla.)