

Validation - Conversion Driven Freeze-Out

Andre Lessa

ABSTRACT:

1. General Boltzmann Equations

Below we summarize the set of coupled Boltzmann equations and the definitions of the terms entering each of them:

- Entropy:

$$\frac{dN_S}{dx} = \frac{e^{(3x-N_S)}}{HT} \sum_i BR(i, X) \Gamma_i m_i (n_i - \mathcal{N}_i^{th}) \quad (1.1)$$

- Number densities:

$$\begin{aligned} \frac{dN_i}{dx} = & -3 + \frac{n_i}{H} \left(\frac{\bar{n}_i^2}{n_i^2} - 1 \right) \langle \sigma v \rangle_{ii} + \frac{1}{H} \sum_{j \neq i} \left(\frac{\bar{n}_i}{n_i} \bar{n}_j - n_j \right) \langle \sigma v \rangle_{ij} + \frac{n_i}{H} \sum_{j \neq i} \left(\frac{\bar{n}_i^2}{n_i^2} \frac{n_j^2}{\bar{n}_j^2} - 1 \right) \langle \sigma v \rangle_{jj} \\ & + \frac{1}{H} \sum_{j \neq i} \left(\frac{\bar{n}_i}{n_i} \frac{n_j}{\bar{n}_j} - 1 \right) \tilde{\Gamma}_{ijSM} - \frac{\Gamma_i}{H} \frac{m_i}{R_i} \left(1 - \frac{\mathcal{N}_i^{th}}{n_i} \right) + \sum_{j \neq i} \mathcal{B}_{ji}^{eff} \frac{\Gamma_j}{H} \frac{m_j}{R_j} \left(\frac{n_j}{n_i} - \frac{\mathcal{N}_{ji}^{th}}{n_i} \right) \end{aligned} \quad (1.2)$$

- Energy density ratios:

$$\frac{dR_i}{dx} = -3 \frac{P_i}{n_i} + \sum_{j \neq i} \mathcal{B}_{ji}^{eff} \frac{\Gamma_j}{H} m_j \left(e_{inj}^{ji} - \frac{R_i}{R_j} \right) \left(\frac{n_j}{n_i} - \frac{\mathcal{N}_{ji}^{th}}{n_i} \right) \quad (1.3)$$

where:

$$\begin{aligned} x &= \ln(R/R_0), \quad N_S = \ln(S/S_0), \quad N_i = \ln(n_i/n_i^0) \text{ and } R_i = \frac{\rho_i}{n_i} \text{ (variables)} \\ \langle \sigma v \rangle_{ii} &= \frac{1}{\bar{n}_i^2} \int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4 \delta^{(4)}(P) |M|^2 e^{-(E_i+E_j)/T} \text{ (self-annihilation to SM)} \\ \langle \sigma v \rangle_{ij} &= \frac{1}{\bar{n}_i \bar{n}_j} \int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4 \delta^{(4)}(P) |M|^2 e^{-(E_i+E_j)/T} \text{ (co-annihilation)} \\ \langle \sigma v \rangle_{jj} &= \frac{1}{\bar{n}_j^2} \int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4 \delta^{(4)}(P) |M|^2 e^{-(E_i+E_j)/T} \text{ (self-annihilation to BSM)} \\ \tilde{\Gamma}_{ijSM} &= \frac{1}{\bar{n}_i} \int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4 \delta^{(4)}(P) |M|^2 e^{-(E_i+E_j)/T} \text{ (conversion rate)} \\ \mathcal{N}_i^{th} &= \bar{n}_i \sum_{i \rightarrow \dots} BR(i \rightarrow 1 + 2 + \dots) \prod_{k=1,2,\dots} \frac{n_k}{\bar{n}_k} \text{ (effective decay density)} \\ \mathcal{N}_{ji}^{th} &= \frac{\bar{n}_j}{\mathcal{B}_{ji}^{eff}} \sum_{j \rightarrow i + \dots} g_Y BR(j \rightarrow g_i i + 1 + \dots) \left(\frac{n_i}{\bar{n}_i} \right)^{g_i} \prod_{k=1,\dots} \frac{n_k}{\bar{n}_k} \text{ (effective injection density)} \\ \mathcal{B}_{ji}^{eff} &= \sum_{j \rightarrow i + \dots} g_i BR(j \rightarrow g_i i + 1 + \dots) \text{ (effective branching ratio)} \\ e_{inj}^{ji} &= \frac{1}{2} \left(1 + \frac{m_i^2}{m_j^2} \right) \text{ (effective energy injection)} \\ BR(i, X) &= \frac{E_\gamma}{E_i} \text{ (effective energy injected in the thermal bath)} \end{aligned}$$

Finally, we can approximate the P_i/n_i function for all values of R_i by:

$$\frac{P_i}{n_i} = \begin{cases} \frac{2m_i}{3} \left(\frac{R_i}{m_i} - 1 \right) + m_i \sum_{n>1} a_n \left(\frac{R_i}{m_i} - 1 \right)^n, & \text{for } R_i < \tilde{R} \\ \frac{R_i}{3}, & \text{for } R_i > \tilde{R} \end{cases} \quad (1.4)$$

where the coefficients a_n are given by the numerical fit and \tilde{R} is given by the matching of the two solutions.

2. Simplified Model

The simplified model considered here corresponds to the SM added by a sbottom-like mediator (\tilde{b}) and a bino-like dark matter particle (χ). The relevant lagrangian is:

$$\mathcal{L}_{int} = |\mathcal{D}_\mu \tilde{b}|^2 + \lambda_\chi \tilde{b} \tilde{b}^\dagger \frac{1 - \gamma_5}{2} \chi + h.c. \quad (2.1)$$

Furthermore the masses are assumed to be: $m_{\tilde{b}} = 510$ GeV, $m_\chi = 500$ GeV. In this case the thermal cross-section $\langle \sigma v \rangle_{\tilde{b}\tilde{b}}$ and the conversion rates $\tilde{\Gamma}_{\tilde{b}\chi SM}$, $\tilde{\Gamma}_{\chi\tilde{b} SM}$ are shown in Fig.1.

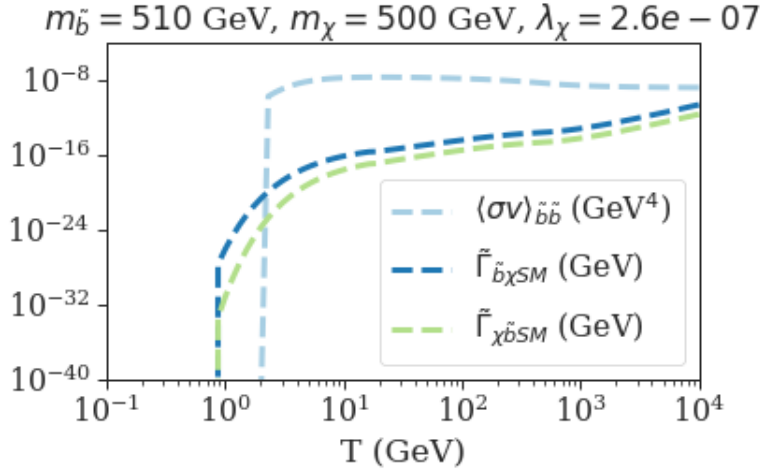


Figure 1: The mediator annihilation rate and the conversion rate for $\tilde{b} + SM \rightarrow \chi + SM$ and $\chi + SM \rightarrow \tilde{b} + SM$ as function of temperature.

3. Examples

3.1 Simple Freeze-out

In this scenario we assume no conversion rate and $\langle \sigma v \rangle_{\chi\chi} = \langle \sigma v \rangle_{\tilde{b}\tilde{b}} \equiv \langle \sigma v \rangle$, where the latter is the curve shown in Fig.1. We also take the conversion rates ($\tilde{\Gamma}_{ij SM}$) to be zero and the \tilde{b} width to be small, so it decays after freeze-out. Finally, in this simplified scenario, we assume that the entropy injection is negligible ($BR(\tilde{b}, X) = 0$), so we have:

$$\begin{aligned}
\frac{dN_S}{dx} &= 0 \\
\frac{dN_\chi}{dx} &= -3 + \frac{n_\chi}{H} \left(\frac{\bar{n}_\chi^2}{n_\chi^2} - 1 \right) \langle \sigma v \rangle + \frac{\Gamma_{\tilde{b}}}{H} \frac{m_{\tilde{b}}}{R_{\tilde{b}}} \left(\frac{n_{\tilde{b}}}{n_\chi} - \frac{\bar{n}_{\tilde{b}}}{\bar{n}_\chi} \right) \\
\frac{dN_{\tilde{b}}}{dx} &= -3 + \frac{n_{\tilde{b}}}{H} \left(\frac{\bar{n}_{\tilde{b}}^2}{n_{\tilde{b}}^2} - 1 \right) \langle \sigma v \rangle - \frac{\Gamma_{\tilde{b}}}{H} \frac{m_{\tilde{b}}}{R_{\tilde{b}}} \left(1 - \frac{\bar{n}_{\tilde{b}} n_\chi}{n_{\tilde{b}} \bar{n}_\chi} \right) \\
\frac{dR_\chi}{dx} &= -3 \frac{P_\chi}{n_\chi} + \frac{\Gamma_{\tilde{b}}}{H} m_{\tilde{b}} \left(\frac{1}{2} + \frac{m_\chi^2}{2m_{\tilde{b}}^2} - \frac{R_\chi}{R_{\tilde{b}}} \right) \left(\frac{n_{\tilde{b}}}{n_\chi} - \frac{\bar{n}_{\tilde{b}}}{\bar{n}_\chi} \right) \\
\frac{dR_{\tilde{b}}}{dx} &= -3 \frac{P_{\tilde{b}}}{n_{\tilde{b}}}
\end{aligned}$$