Abstract:

Keywords: .

1. Production of DFSZ axino and saxion

According to Ref. [1], we can obtain the simple formula for the axino/saxion yield from Boltzmann equation. It is given by

$$Y_{\tilde{a}(s)} = \int_{T_0}^{T_R} dT \frac{1}{s(T)H(T)T} \left[\sum_I \langle \Gamma_{(I\to\tilde{a}(s)+\cdots)} \rangle n_I + \sum_{I,J} \langle \sigma_{(I+J\to\tilde{a}(s)+\cdots)} \rangle n_I n_J \right]. \quad (1.1)$$

The thermal averaged decay width and scattering cross section can be written as

$$\langle \Gamma_{(I\to\tilde{a}(s)+\cdots)} \rangle n_I = \frac{mT^2}{2\pi^2} \Gamma \int_{m/T}^{\infty} dx \frac{\left(x^2 - m^2/T^2\right)^{1/2}}{e^x \mp 1}, \quad (1.2)$$

$$\begin{aligned} \langle \sigma_{(I+J\to\tilde{a}(s)+\cdots)} \rangle n_I n_J &= \frac{T^2}{16\pi^4} \int_{(m_1+m_2)/T}^{\infty} dx K_1(x) \sigma\left(x^2T^2\right) \\ &\quad \times \left[\left(x^2T^2 - m_1^2 - m_2^2\right)^2 - 4m_1^2 m_2^2 \right], \end{aligned} \quad (1.3)$$

where Γ is the decay width in the rest frame and K_1 is the Bessel function.

For the DFSZ axino production, we calculate both the scattering and decay processes. We first consider the scattering case. Scattering processes in the realistic model parameters are very messy so that it is very hard to make these handy formulae. Instead of that, in the limiting case of SUSY preserving, we can simplify the scattering cross section. From Ref. [2], we can write a simple approximate formula as

$$\sigma_{(I+J\to\tilde{a}+\cdots)}(s) \sim \frac{1}{16\pi s} |\mathcal{M}|^2 \sim \frac{\xi_{IJ} g^2 c_H^2 \left|T_{ij}(\Phi)^a\right|^2}{2\pi s} \frac{M_\Phi^2}{v_{PQ}^2}, \quad (1.4)$$

where g is the gauge coupling constant, $T_{ij}(\Phi)^a$ is the gauge-charge matrix of Φ, M_Φ is the mass of Φ, and ξ_{IJ} is $\mathcal{O}(1)$ constant (see the Table 1 of [2]). We obtain that

$$\langle \sigma_{(I+J\to\tilde{a}+\cdots)} \rangle n_I n_J \sim \frac{\xi_{IJ} g^2 c_H^2 \left|T_{ij}(\Phi)^a\right|^2}{32\pi^5} \frac{M_\Phi^2}{v_{PQ}^2} T^4 \int_{M/T}^{\infty} dx K_1(x) x^2 \quad (1.5)$$

where M is the threshold scale of the process. For the DFSZ SUSY axion model, the heaviest PQ charged superfield is Higgs doublets, so g is the $SU(2)$ gauge coupling, $M_\Phi = \mu$, and $|T_{ij}(\Phi)^a|^2 = (N^2-1)/2 = 3/2$.

For the heavy particle decay, we can refer the Ref. [3]. If heavy Higgs can decay into axino and light higgsino (possible in SOA), it is also important production channel of axinos. The partial decay width is given by

$$\Gamma \sim \frac{c_H^2}{2\pi} \frac{\mu^2}{f_a^2} m_H. \quad (1.6)$$

Then we obtain

$$\langle \Gamma_{(H\to\tilde{a}+\widetilde{H})} \rangle n_H \sim \frac{c_H^2}{4\pi^3} \frac{\mu^2}{f_a^2} m_H^2 T^2 \int_{m_H/T}^{\infty} dx \frac{\left(x^2 - m^2/T^2\right)^{1/2}}{e^x \mp 1}. \quad (1.7)$$

From these thermal averaged rates, we can calculate the axino yield,

$$\begin{aligned}
Y_{\tilde{a}}^{\rm scat} &= \sum_{I,J} \int_{T_0}^{T_R} dT \frac{\langle \sigma_{IJ} \rangle n_I n_J}{s(T) H(T) T} \\
&\simeq \frac{\bar{g} M_P g^2 c_H^2}{32\pi^5} \frac{3}{2} \frac{\mu^2}{f_a^2} \sum_{I,J} \xi_{IJ} \int_{T_0}^{T_R} dT \frac{1}{T^2} \int_{M/T}^{\infty} dx\, x^2 K_1(x) \\
&\simeq 3.7 \times 10^9 \sum_{I,J} \xi_{IJ} \left(\frac{\mu}{f_a} \right)^2 \left(\frac{\rm TeV}{M} \right), \qquad (1.8)
\end{aligned}$$

where $\bar{g} = 135\sqrt{10}/(2\pi^3 g_*^{3/2})$ and we use the fact that for $T_R \gg M$

$$\int_{T_0}^{T_R} dT \frac{1}{T^2} \int_{M/T}^{\infty} dx\, x^2 K_1(x) \simeq \frac{4.7}{M}. \qquad (1.9)$$

If the threshold scale M is of TeV order, we obtain that

$$Y_{\tilde{a}}^{\rm scat} \simeq 10^{-7} \xi \left(\frac{\mu}{\rm TeV} \right)^2 \left(\frac{10^{12}\ {\rm TeV}}{f_a} \right)^2, \qquad (1.10)$$

where ξ is $\mathcal{O}(1)$ constant. Here we use the fact that $\sum \xi_{IJ} \simeq \mathcal{O}(10)$. On the other hand, the decay contribution becomes

$$\begin{aligned}
Y_{\tilde{a}}^{\rm dec} &= \sum_{I} \int_{T_0}^{T_R} dT \frac{\langle \Gamma_I \rangle n_I}{s(T) H(T) T} \\
&\simeq \frac{\bar{g} M_P c_H^2}{4\pi^3} \frac{\mu^2}{f_a^2} m_H^2 \int_{T_0}^{T_R} dT \frac{1}{T^4} \int_{m_H/T}^{\infty} dx \frac{\left(x^2 - m_H^2/T^2\right)^{1/2}}{e^x \mp 1} \\
&\simeq 2.2 \times 10^{11} \left(\frac{\mu}{f_a} \right)^2 \left(\frac{\rm TeV}{m_H} \right). \qquad (1.11)
\end{aligned}$$

Thus if m_H is of TeV order, we obtain

$$Y_{\tilde{a}}^{\rm dec} \sim 10^{-7} \left(\frac{\mu}{\rm TeV} \right)^2 \left(\frac{10^{12}\ {\rm TeV}}{f_a} \right)^2. \qquad (1.12)$$

Also, there are additional contributions from the top Yukawa coupling, which are of the same order of the above two contributions. Therefore, we can easily conclude

$$Y_{\tilde{a}} \simeq 10^{-7} \zeta_{\tilde{a}} \left(\frac{\mu}{\rm TeV} \right)^2 \left(\frac{10^{12}\ {\rm TeV}}{f_a} \right)^2, \qquad (1.13)$$

For the saxion production, the same arguments can be applied. Let us consider an example of the scattering process, $\widetilde{W} + h \to s + \widetilde{H}$. This $2 \to 2$ process involves $SU(2)$ gauge coupling and μ/f_a. Hence we obtain the similar result with $Y_{\tilde{a}}^{\rm scat}$ and we can write

$$Y_s \simeq 10^{-7} \zeta_s \left(\frac{\mu}{\rm TeV} \right)^2 \left(\frac{10^{12}\ {\rm TeV}}{f_a} \right)^2. \qquad (1.14)$$

References

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