I. EFFECTIVE INTERACTION OF AXION SUPERMULTIPLET

In the following sections, we will calculate the axino and axion partial decay width for various final states. To proceed, we would clarify the convention of Peccei-Quin (PQ) scale, and it will be useful to compare the result with others.

For axino and saxion decay to gauge particles (gauginos and gauge bosons), we start with the interaction lagrangian,

$$\mathcal{L}_{\rm PQ}^{\rm int} = -\frac{\sqrt{2}\alpha_s}{8\pi f_a}\int d^2\theta A W^b W^b + \text{h.c.}, \tag{1}$$

where the A and W are the chiral superfields of axion and gauge field strenth, repectively. Note that in this lagrangian we have an additional factor $\sqrt{2}$ compared to Moroi and Takimoto [**?**]. They wrote the interaction term,

$$\mathcal{L}_{\rm PQ}^{\rm int} = \frac{\alpha_s}{8\pi F_a}\int d^2\theta A W^b W^b + \text{h.c.} \tag{2}$$

Additional minus sign is not important since it does not affect the partial decay width. **These two lagrangians can be matched if we define $\mathbf{f_a/\sqrt{2} = F_a \equiv v_{PQ}}$.** For the component field notation, we can see the details of saxion and axino interactions,

$$\begin{aligned}\mathcal{L}_{\rm PQ}^{\rm int} = \frac{\alpha_s}{8\pi f_a}\Bigg[& s\left(G^{b\mu\nu}G^b_{\mu\nu} - 2D^bD^b - 2i\bar{\tilde{g}}^b_M\gamma^\mu D_\mu \tilde{g}^b_M\right) \\ & + a\left(G^{b\mu\nu}\widetilde{G}^b_{\mu\nu} + 2\bar{\tilde{g}}^b\gamma^\mu\gamma^5 D_\mu\tilde{g}^b_M\right) \\ & - i\bar{\tilde{M}}\frac{[\gamma^\mu,\gamma^{nu}]}{2}\gamma^5\tilde{g}^b_M G^b_{\mu\nu} + 2\bar{\tilde{a}}_M D^b\tilde{g}^b_M\Bigg],\end{aligned} \tag{3}$$

where the subscript M denotes Majorana fermion. This component lagrangian is the same as Graf and Steffen [**?**]. The same is also applied for the $SU(2)_L$ and $U(1)_Y$ gauge interactions.

For the saxion decay to axions and axinos, we use the lagrangian of Graf and Steffen [**?**], which is given by

$$\mathcal{L}_{\rm PQ}^{\rm kin} = \left(1 + \frac{\sqrt{2}\xi}{v_{\rm PQ}}s\right)\left[\frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu s)^2 + i\bar{\tilde{a}}_M\gamma^\mu\partial_\mu\tilde{a}_M\right] + \cdots . \tag{4}$$

Here, the saxion-axino-axino interaction term is taken from Chun and Lukas [**?**]. Remind that $v_{\rm PQ} = f_a/\sqrt{2}$.

II. SAXION DECAYS TO AXIONS AND AXINOS

From the lagrangian (**??**), we obtain

$$\Gamma(s \to a + a) = \frac{\xi^2 m_s^3}{64\pi v_{\rm PQ}^2} = \frac{\xi^2 m_s^3}{32\pi f_{\rm a}^2}. \tag{5}$$

This result agrees with Graf and Steffen [**?**] as well as Moroi and Takimoto [**?**] for $F_a \equiv v_{\rm PQ} = f_a/\sqrt{2}$ as we stress in the previous section. Also, saxion decay to axinos are given by

$$\Gamma(s \to \tilde{a} + \tilde{a}) = \frac{\xi^2}{4\pi v_{\rm PQ}^2} m_{\tilde{a}}^2 m_s \left(1 - \frac{4m_{\tilde{a}}^2}{m_s^2}\right)^{3/2}. \tag{6}$$

III. AXINO DECAY

The axino-gaugino-gauge boson couplings are determined by anomaly contribution of vector-like heavy quarks for KSVZ type axion model and SM quarks for DFSZ type axion models. The axino-gluon-gluino coupling is given by

$$\mathcal{L}_{\tilde{a}\tilde{g}g} = i\frac{\alpha_s}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde{g}_A G_{A\mu\nu}, \tag{7}$$

where N is the number of PQ-charged quark flavor, i.e. $N = N_W N_G$. Here N_W is determined by quark representation of $\mathrm{SU}(2)_W$, e.g. $N_W = 2$ for doublet and $N_W = 1$ for singlet. N_G is the number of copies of PQ-charged quark multiplets. The axino-bino-B_μ coupling is given by

$$\mathcal{L}_{\tilde{a}\widetilde{B}B} = i\frac{\alpha_Y C_{aYY}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{B}B_{\mu\nu}, \tag{8}$$

and the axino-wino-W_μ is given by

$$\mathcal{L}_{\tilde{a}\tilde{\lambda}W} = i\frac{N_c\alpha_2/2}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde{\lambda}_a W_{a\mu\nu}, \tag{9}$$

for $N_W = 2$ and no such interaction exists for $N_W = 1$. Here N_c is the number of color.

In the basis of mass eigenstates, Eqs (**??**) and (**??**) become

$$\mathcal{L}_{\tilde{a}\widetilde{B}B} = i\frac{\alpha_Y C_{aYY} v_4^{(i)}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{Z}_i(\sin\theta_W Z_{\mu\nu} + \cos\theta_W A_{\mu\nu}) \tag{10}$$

$$\begin{aligned}\mathcal{L}_{\tilde{a}\tilde{\lambda}W} = {} & i\frac{N_c(\alpha_2/2)v_3^{(i)}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{Z}_i(-\cos\theta_W Z_{\mu\nu} + \sin\theta_W A_{\mu\nu}) \\ & +i\frac{N_c\alpha_2/2}{16\pi(f_a/N)}\Big[\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\big\{\big(U_{11}P_L\widetilde{W}_2 + U_{12}P_L\widetilde{W}_1\big)W^+_{\mu\nu} \\ & +\big(V_{11}P_R\widetilde{W}_2 + V_{12}P_R\widetilde{W}_1\big)W^-_{\mu\nu}\big\} + \text{h.c.}\Big]\end{aligned} \tag{11}$$

From the above lagrangian, we can obtain $\Gamma(\tilde{a}\to\widetilde{Z}_i+\gamma)$ and $\Gamma(\tilde{a}\to\widetilde{Z}_i+\gamma)$ by replacing

$$\alpha_Y C_{aYY}\cos\theta_W v_4^{(i)} \to \alpha_Y C_{aYY}\cos\theta_W v_4^{(i)} + N_c(\alpha_2/2)v_3^{(i)}\sin\theta_W, \tag{12}$$

$$\alpha_Y C_{aYY}\sin\theta_W v_4^{(i)} \to \alpha_Y C_{aYY}\sin\theta_W v_4^{(i)} - N_c(\alpha_2/2)v_3^{(i)}\cos\theta_W. \tag{13}$$

Therefore, the partial decay widths are

$$\Gamma(\tilde{a}\to\widetilde{Z}_i+\gamma) = \frac{(\alpha_Y C_{aYY}\cos\theta_W v_4^{(i)} + N_c(\alpha_2/2)v_3^{(i)}\sin\theta_W)^2}{128\pi^3(f_a/N)^2}m_{\tilde{a}}^3\left(1-\frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}}\right)^3 \tag{14}$$

$$\begin{aligned}\Gamma(\tilde{a}\to\widetilde{Z}_i+Z) = {} & \frac{(\alpha_Y C_{aYY}\sin\theta_W v_4^{(i)} - N_c(\alpha_2/2)v_3^{(i)}\cos\theta_W)^2}{128\pi^3(f_a/N)^2}m_{\tilde{a}}^3\lambda^{1/2}\left(1,\frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}},\frac{m^2_Z}{m^2_{\tilde{a}}}\right) \\ & \cdot\left\{\left(1-\frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}}\right)^2 + 3\frac{m_{\widetilde{Z}_i}m_Z^2}{m^3_{\tilde{a}}} - \frac{m_Z^2}{2m^2_{\tilde{a}}}\left(1+\frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}}+\frac{m^2_Z}{m^2_{\tilde{a}}}\right)\right\}.\end{aligned} \tag{15}$$

In addition, axino can decay to chargino and W boson if it is kinematically allowed. The partial decay width is given by

$$\begin{aligned}\Gamma(\tilde{a}\to\widetilde{W}_1^\pm+W^\mp) = {} & \frac{N_c^2(\alpha_2/2)^2}{128\pi^3(f_a/N)^2}m_{\tilde{a}}^3\lambda^{1/2}\left(1,\frac{m^2_{\widetilde{W}_1}}{m^2_{\tilde{a}}},\frac{m^2_W}{m^2_{\tilde{a}}}\right) \\ & \cdot\left[\left\{\left(1-\frac{m^2_{\widetilde{W}_1}}{m^2_{\tilde{a}}}\right)^2 - \frac{m_W^2}{2m^2_{\tilde{a}}}\left(1+\frac{m^2_{\widetilde{W}_1}}{m^2_{\tilde{a}}}+\frac{m^2_W}{m^2_{\tilde{a}}}\right)\right\}(|U_{12}|^2+|V_{12}|^2) \right. \\ & \left. +3\frac{m_{\widetilde{W}_1}m_W^2}{m^3_{\tilde{a}}}\big(U^*_{12}V^*_{12}+U_{12}V_{12}\big)\right].\end{aligned} \tag{16}$$

$\Gamma(\tilde{a}\to\widetilde{W}_2^\pm+W^\mp)$ can be obtained by replacing $m_{\widetilde{W}_1}$, U_{12} and V_{12} with $m_{\widetilde{W}_2}$, U_{11} and V_{11}. Note that all notations and conventions follow the reference [**?**].

IV. SAXION DECAY

The saxion-gauge boson-gauge boson couplings are given by

$$\mathcal{L} = \frac{N_c \alpha_2/2}{8\pi(f_a/N)} s W^i_{\mu\nu} W^{i\mu\nu} + \frac{\alpha_Y C_{aYY}}{16\pi(f_a/N)} s B_{\mu\nu} B^{\mu\nu}. \tag{17}$$

From the above lagrangian, we obtain the interaction lagrangian for W boson, Z boson and photon in the mass basis as the following.

$$\mathcal{L}_{sWW} = \frac{N_c \alpha_2/2}{2\pi(f_a/N)} s \left(\partial_\mu W^+_\nu \partial^\mu W^{-\nu} - \partial_\mu W^+_\nu \partial^\nu W^{-\mu}\right), \tag{18}$$

$$\mathcal{L}_{sZZ} = \frac{(N_c \alpha_2/2)\cos^2\theta_W + \alpha_Y C_{aYY} \sin^2\theta_W}{4\pi(f_a/N)} s \left[\partial_\mu Z_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right)\right], \tag{19}$$

$$\mathcal{L}_{s\gamma\gamma} = \frac{(N_c \alpha_2/2)\sin^2\theta_W + \alpha_Y C_{aYY} \cos^2\theta_W}{4\pi(f_a/N)} s \left[\partial_\mu A_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right], \tag{20}$$

$$\mathcal{L}_{sZ\gamma} = \frac{(-N_c \alpha_2/2 + \alpha_Y C_{aYY})\sin\theta_W \cos\theta_W}{2\pi(f_a/N)} s \left[\partial_\mu A_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right)\right]. \tag{21}$$

Note that we neglect the higher order couplings. We obtain the partial decay width of each decay mode.

$$\Gamma(s \to W^\pm + W^\mp) = \frac{(N_c \alpha_2/2)^2}{128\pi^3 (f_a/N)^2} m_s^3 \left(1 - \frac{4m_W^2}{m_s^2}\right)^{1/2} \left(1 - \frac{4m_W^2}{m_s^2} + \frac{6m_W^4}{m_s^4}\right), \tag{22}$$

$$\Gamma(s \to Z + Z) = \frac{\left\{(N_c \alpha_2/2)\cos^2\theta_W + \alpha_Y C_{aYY}\sin^2\theta_W\right\}^2}{256\pi^3 (f_a/N)^2} \cdot m_s^3 \left(1 - \frac{4m_Z^2}{m_s^2}\right)^{1/2} \left(1 - \frac{4m_Z^2}{m_s^2} + \frac{6m_Z^4}{m_s^4}\right), \tag{23}$$

$$\Gamma(s \to \gamma + \gamma) = \frac{\left\{(N_c \alpha_2/2)\sin^2\theta_W + \alpha_Y C_{aYY}\cos^2\theta_W\right\}^2}{256\pi^3 (f_a/N)^2} m_s^3, \tag{24}$$

$$\Gamma(s \to Z + \gamma) = \frac{(-N_c \alpha_2/2 + \alpha_Y C_{aYY})^2 \sin^2\theta_W \cos^2\theta_W}{128\pi^2 (f_a/N)^2} m_s^3 \left(1 - \frac{m_Z^2}{m_s^2}\right)^4 \tag{25}$$

In addition, saxion can decay to neutralinos and charginos. The relevant lagrangian is given by

$$\mathcal{L} = -i\frac{N_c \alpha_2/2}{4\pi(f_a/N)} \left(s\bar{\lambda}_i \gamma^\mu \partial_\mu \lambda_i\right) - i\frac{\alpha_Y C_{aYY}}{4\pi(f_a/N)} \left(s\bar{\lambda}_0 \gamma^\mu \partial_\mu \lambda_0\right) \tag{26}$$

For charginos

$$\mathcal{L}_{s\widetilde{W}\widetilde{W}} = -i\frac{N_c \alpha_2/2}{4\pi(f_a/N)} s \left\{Q^L_{ij} \overline{\widetilde{W}}_i \gamma^\mu P_L \partial_\mu \widetilde{W}_j + Q^R_{ij} \overline{\widetilde{W}}_i \gamma^\mu P_R \partial_\mu \widetilde{W}_j\right\} + \text{h.c.}, \tag{27}$$

where

$$Q^L_{ij} = \begin{pmatrix} |U_{11}|^2 & U^*_{12}U_{11} \\ U^*_{11}U_{12} & |U_12|^2 \end{pmatrix}, \qquad Q^R_{ij} = \begin{pmatrix} |V_{11}|^2 & V^*_{12}V_{11} \\ V^*_{11}V_{12} & |V_12|^2 \end{pmatrix}. \tag{28}$$

For neutralinos,

$$\mathcal{L}_{s\widetilde{Z}\widetilde{Z}} = -i\frac{1}{8\pi(f_a/N)}R_{ij}\overline{\widetilde{Z}}_i\gamma^\mu\partial_\mu\widetilde{Z}_j, \tag{29}$$

where

$$R_{ij} = (N_c\alpha_2/2)v_3^{(i)*}v_3^{(j)} + \alpha_Y C_{aYY}v_4^{(i)*}v_4^{(j)}. \tag{30}$$

The partial decay widths are given by

$$\begin{aligned}\Gamma(s\to\widetilde{W}_i+\widetilde{W}_j) &= \frac{(N_c\alpha_2/2)^2m_s^3}{256\pi^3(f_a/N)^2}\lambda^{1/2}\left(1,\frac{m^2_{\widetilde{W}_i}}{m_s^2},\frac{m^2_{\widetilde{W}_j}}{m_s^2}\right)\\ &\cdot\Bigg[\left(|Q^L_{ij}|^2+|Q^R_{ij}|^2\right)\left\{\left(\frac{m^2_{\widetilde{W}_i}+m^2_{\widetilde{W}_j}}{m_s^2}\right)\left(1-\frac{m^2_{\widetilde{W}_i}+m^2_{\widetilde{W}_j}}{m_s^2}\right)-\frac{4m^2_{\widetilde{W}_i}m^2_{\widetilde{W}_j}}{m_s^4}\right\}\\ &+\left(Q^L_{ij}Q^{R*}_{ij}+Q^{L*}_{ij}Q^R_{ij}\right)\left\{\frac{2m^2_{\widetilde{W}_i}m^2_{\widetilde{W}_j}}{m_s^2}\left(1-2\frac{m^2_{\widetilde{W}_i}+m^2_{\widetilde{W}_j}}{m_s^2}\right)\right\}\Bigg],\end{aligned} \tag{31}$$

$$\begin{aligned}\Gamma(s\to\widetilde{Z}_i+\widetilde{W}_j) &= \frac{|R_{ij}|^2}{128\pi^3(f_a/N)^2}\lambda^{1/2}\left(1,\frac{m^2_{\widetilde{Z}_i}}{m_s^2},\frac{m^2_{\widetilde{Z}_j}}{m_s^2}\right)\left(1-\frac{1}{2}\delta_{ij}\right)\\ &\cdot m_s(m_{\widetilde{Z}_i}+m_{\widetilde{Z}_j})^2\left[1-\left(\frac{m_{\widetilde{Z}_i}+m_{\widetilde{Z}_j}}{m_s}\right)^2\right].\end{aligned} \tag{32}$$

Note that (ij) indices are not summed.

[1] T. Moroi and M. Takimoto, Phys. Lett. B **718**, 105 (2012) [arXiv:1207.4858 [hep-ph]].

[2] P. Graf and F. D. Steffen, arXiv:1208.2951 [hep-ph].

[3] E. J. Chun and A. Lukas, Phys. Lett. B **357**, 43 (1995) [hep-ph/9503233].

[4] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events*, (Cambridge University Press, 2006).