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Validation - Conversion Driven Freeze-Out

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Abstract:

1. General Boltzmann Equations

Below we summarize the set of coupled Boltzmann equations and the definitions of the terms entering each of them:

- Entropy:

$$\frac{dN_S}{dx} = \frac{e^{(3x-N_S)}}{HT} \sum_i BR(i,X)\Gamma_i m_i \left(n_i - \mathcal{N}_i^{th}\right) \tag{1.1}$$

- Number densities:

$$\begin{aligned}\frac{dN_i}{dx} &= -3 + \frac{n_i}{H}\left(\frac{\bar{n}_i^2}{n_i^2} - 1\right)\langle\sigma v\rangle_{ii} + \frac{1}{H}\sum_{j\neq i}\left(\frac{\bar{n}_i}{n_i}\bar{n}_j - n_j\right)\langle\sigma v\rangle_{ij} + \frac{n_i}{H}\sum_{j\neq i}\left(\frac{\bar{n}_i^2}{n_i^2}\frac{n_j^2}{\bar{n}_j^2} - 1\right)\langle\sigma v\rangle_{jj} \\ &+ \frac{1}{H}\sum_{j\neq i}\left(\frac{\bar{n}_i}{n_i}\frac{n_j}{\bar{n}_j} - 1\right)\tilde{\Gamma}_{ijSM} - \frac{\Gamma_i}{H}\frac{m_i}{R_i}\left(1 - \frac{\mathcal{N}_i^{th}}{n_i}\right) + \sum_{j\neq i}\mathcal{B}_{ji}^{eff}\frac{\Gamma_j}{H}\frac{m_j}{R_j}\left(\frac{n_j}{n_i} - \frac{\mathcal{N}_{ji}^{th}}{n_i}\right)\end{aligned} \tag{1.2}$$

- Energy density ratios:

$$\frac{dR_i}{dx} = -3\frac{P_i}{n_i} + \sum_{j\neq i}\mathcal{B}_{ji}^{eff}\frac{\Gamma_j}{H}m_j\left(e_{inj}^{ji} - \frac{R_i}{R_j}\right)\left(\frac{n_j}{n_i} - \frac{\mathcal{N}_{ji}^{th}}{n_i}\right) \tag{1.3}$$

where:

$$x = \ln(R/R_0),\ N_S = \ln(S/S_0),\ N_i = \ln(n_i/n_i^0) \text{ and } R_i = \frac{\rho_i}{n_i} \text{ (variables)}$$

$$\langle\sigma v\rangle_{ii} = \frac{1}{\bar{n}_i^2}\int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4\delta^{(4)}(P)|M|^2 e^{-(E_i+E_j)/T} \text{ (self-annihilation to SM)}$$

$$\langle\sigma v\rangle_{ij} = \frac{1}{\bar{n}_i\bar{n}_j}\int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4\delta^{(4)}(P)|M|^2 e^{-(E_i+E_j)/T} \text{ (co-annihilation)}$$

$$\langle\sigma v\rangle_{jj} = \frac{1}{\bar{n}_i^2}\int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4\delta^{(4)}(P)|M|^2 e^{-(E_i+E_j)/T} \text{ (self-annihilation to BSM)}$$

$$\tilde{\Gamma}_{ijSM} = \frac{1}{\bar{n}_i}\int d\Pi_i d\Pi_j d\Pi_a d\Pi_b (2\pi)^4\delta^{(4)}(P)|M|^2 e^{-(E_i+E_j)/T} \text{ (conversion rate)}$$

$$\mathcal{N}_i^{th} = \bar{n}_i\sum_{i\to\ldots} BR(i\to 1+2+\ldots)\prod_{k=1,2,..}\frac{n_k}{\bar{n}_k} \text{ (effective decay density)}$$

$$\mathcal{N}_{ji}^{th} = \frac{\bar{n}_j}{\mathcal{B}_{ji}^{eff}}\sum_{j\to i+\ldots} g_Y BR(j\to g_i i + 1 + \ldots)\left(\frac{n_i}{\bar{n}_i}\right)^{g_i}\prod_{k=1,..}\frac{n_k}{\bar{n}_k} \text{ (effective injection density)}$$

$$\mathcal{B}_{ji}^{eff} = \sum_{j\to i+\ldots} g_i BR(j\to g_i i + 1 + \ldots) \text{ (effective branching ratio)}$$

$$e_{inj}^{ji} = \frac{1}{2}\left(1 + \frac{m_i^2}{m_j^2}\right) \text{ (effective energy injection)}$$

$$BR(i,X) = \frac{E_\gamma}{E_i} \text{ (effective energy injected in the thermal bath)}$$

Finally, we can approximate the P_i/n_i function for all values of R_i by:

$$\frac{P_i}{n_i} = \begin{cases} \frac{2m_i}{3}\left(\frac{R_i}{m_i} - 1\right) + m_i \sum_{n>1} a_n \left(\frac{R_i}{m_i} - 1\right)^n \text{ , for } R_i < \tilde{R} \\ \frac{R_i}{3} \text{ , for } R_i > \tilde{R} \end{cases} \tag{1.4}$$

where the coefficients a_n are given by the numerical fit and $\tilde{R}$ is given by the matching of the two solutions.

2. Simplified Model

The simplified model considered here corresponds to the SM added by a sbottom-like mediator ($\tilde{b}$) and a bino-like dark matter particle (χ). The relevant lagrangian is:

$$\mathcal{L}_{int} = |\mathcal{D}_\mu \tilde{b}|^2 + \lambda_\chi \tilde{b}\bar{b}\frac{1-\gamma_5}{2}\chi + h.c. \tag{2.1}$$

Furthermore the masses are assumed to be: $m_{\tilde{b}} = 510$ GeV, $m_\chi = 500$ GeV. In this case the thermal cross-section $\langle\sigma v\rangle_{\tilde{b}\tilde{b}}$ and the conversion rates $\tilde{\Gamma}_{\tilde{b}\chi SM}$, $\tilde{\Gamma}_{\chi\tilde{b}SM}$are shown in Fig.1.

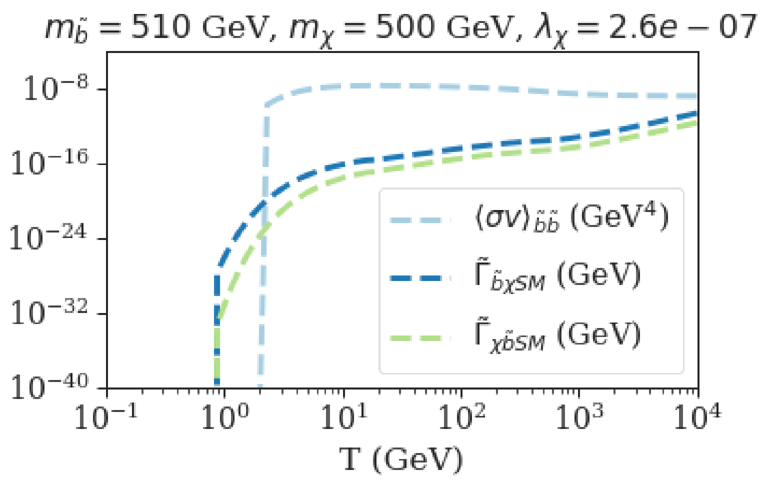

Figure 1: The mediator annihilation rate and the conversion rate for $\tilde{b} + SM \to \chi + SM$ and $\chi + SM \to \tilde{b} + SM$ as function of temperature.

3. Examples

3.1 Simple Freeze-out

In this scenario we assume no conversion rate and $\langle\sigma v\rangle_{\chi\chi} = \langle\sigma v\rangle_{\tilde{b}\tilde{b}} \equiv \langle\sigma v\rangle$, where the latter is the curve shown in Fig.1. We also take the conversion rates ($\tilde{\Gamma}_{ijSM}$) to be zero and the $\tilde{b}$ width to be small, so it decays after freeze-out. Finally, in this simplified scenario, we assume that the entropy injection is negligible ($BR(\tilde{b}, X) = 0$), so we have:

$$
\begin{aligned}
\frac{dN_S}{dx} &= 0 \\
\frac{dN_\chi}{dx} &= -3 + \frac{n_\chi}{H}\left(\frac{\bar{n}_\chi^2}{n_\chi^2} - 1\right)\langle\sigma v\rangle + \frac{\Gamma_{\tilde{b}}}{H}\frac{m_{\tilde{b}}}{R_{\tilde{b}}}\left(\frac{n_{\tilde{b}}}{n_\chi} - \frac{\bar{n}_{\tilde{b}}}{\bar{n}_\chi}\right) \\
\frac{dN_{\tilde{b}}}{dx} &= -3 + \frac{n_{\tilde{b}}}{H}\left(\frac{\bar{n}_{\tilde{b}}^2}{n_{\tilde{b}}^2} - 1\right)\langle\sigma v\rangle - \frac{\Gamma_{\tilde{b}}}{H}\frac{m_{\tilde{b}}}{R_{\tilde{b}}}\left(1 - \frac{\bar{n}_{\tilde{b}} n_\chi}{n_{\tilde{b}}\bar{n}_\chi}\right) \\
\frac{dR_\chi}{dx} &= -3\frac{P_\chi}{n_\chi} + \frac{\Gamma_{\tilde{b}}}{H}m_{\tilde{b}}\left(\frac{1}{2} + \frac{m_\chi^2}{2m_{\tilde{b}}^2} - \frac{R_\chi}{R_{\tilde{b}}}\right)\left(\frac{n_{\tilde{b}}}{n_\chi} - \frac{\bar{n}_{\tilde{b}}}{\bar{n}_\chi}\right) \\
\frac{dR_{\tilde{b}}}{dx} &= -3\frac{P_{\tilde{b}}}{n_{\tilde{b}}}
\end{aligned}
$$