I. AXINO DECAY

The axino-gaugino-gauge boson couplings are determined by anomaly contribution of vector-like heavy quarks for KSVZ type axion model and SM quarks for DFSZ type axion models. The axino-gluon-gluino coupling is given by

$$\mathcal{L}_{\tilde{a}\tilde{g}g} = i\frac{\alpha_s}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde{g}_A G_{A\mu\nu}, \tag{1}$$

where N is the number of PQ-charged quark flavor, i.e. $N = N_W N_G$. Here N_W is determined by quark representation of $\mathrm{SU}(2)_W$, e.g. $N_W = 2$ for doublet and $N_W = 1$ for singlet. N_G is the number of copies of PQ-charged quark multiplets. The axino-bino-B_μ coupling is given by

$$\mathcal{L}_{\tilde{a}\widetilde{B}B} = i\frac{\alpha_Y C_{aYY}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{B}B_{\mu\nu}, \tag{2}$$

and the axino-wino-W_μ is given by

$$\mathcal{L}_{\tilde{a}\tilde{\lambda}W} = i\frac{N_c\alpha_2/2}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde{\lambda}_a W_{a\mu\nu}, \tag{3}$$

for $N_W = 2$ and no such interaction exists for $N_W = 1$. Here N_c is the number of color.

In the basis of mass eigenstates, Eqs (2) and (3) become

$$\mathcal{L}_{\tilde{a}\widetilde{B}B} = i\frac{\alpha_Y C_{aYY} v_4^{(i)}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{Z}_i(\sin\theta_W Z_{\mu\nu} + \cos\theta_W A_{\mu\nu}) \tag{4}$$

$$\begin{aligned}\mathcal{L}_{\tilde{a}\tilde{\lambda}W} &= i\frac{N_c(\alpha_2/2)v_3^{(i)}}{16\pi(f_a/N)}\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\widetilde{Z}_i(-\cos\theta_W Z_{\mu\nu} + \sin\theta_W A_{\mu\nu}) \\ &\quad + i\frac{N_c\alpha_2/2}{16\pi(f_a/N)}\Big[\bar{\tilde{a}}\gamma_5[\gamma^\mu,\gamma^\nu]\big\{\big(U_{11}P_L\widetilde{W}_2 + U_{12}P_L\widetilde{W}_1\big)W^+_{\mu\nu} \\ &\quad + \big(V_{11}P_R\widetilde{W}_2 + V_{12}P_R\widetilde{W}_1\big)W^-_{\mu\nu}\big\} + \mathrm{h.c.}\Big]\end{aligned} \tag{5}$$

From the above lagrangian, we can obtain $\Gamma(\tilde{a}\to\widetilde{Z}_i+\gamma)$ and $\Gamma(\tilde{a}\to\widetilde{Z}_i+\gamma)$ by replacing

$$\alpha_Y C_{aYY}\cos\theta_W v_4^{(i)} \to \alpha_Y C_{aYY}\cos\theta_W v_4^{(i)} + N_c(\alpha_2/2)v_3^{(i)}\sin\theta_W, \tag{6}$$

$$\alpha_Y C_{aYY}\sin\theta_W v_4^{(i)} \to \alpha_Y C_{aYY}\sin\theta_W v_4^{(i)} - N_c(\alpha_2/2)v_3^{(i)}\cos\theta_W. \tag{7}$$

Therefore, the partial decay widths are

$$\Gamma(\tilde{a} \to \widetilde{Z}_i + \gamma) = \frac{(\alpha_Y C_{aYY} \cos\theta_W v_4^{(i)} + N_c(\alpha_2/2) v_3^{(i)} \sin\theta_W)^2}{128\pi^3 (f_a/N)^2} m_{\tilde{a}}^3 \left(1 - \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2}\right)^3 \tag{8}$$

$$\Gamma(\tilde{a} \to \widetilde{Z}_i + Z) = \frac{(\alpha_Y C_{aYY} \sin\theta_W v_4^{(i)} - N_c(\alpha_2/2) v_3^{(i)} \cos\theta_W)^2}{128\pi^3 (f_a/N)^2} m_{\tilde{a}}^3 \lambda^{1/2}\left(1, \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2}, \frac{m_Z^2}{m_{\tilde{a}}^2}\right)$$
$$\cdot \left\{ \left(1 - \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2}\right)^2 + 3\frac{m_{\widetilde{Z}_i} m_Z^2}{m_{\tilde{a}}^3} - \frac{m_Z^2}{2m_{\tilde{a}}^2}\left(1 + \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2} + \frac{m_Z^2}{m_{\tilde{a}}^2}\right)\right\}. \tag{9}$$

In addition, axino can decay to chargino and W boson if it is kinematically allowed. The partial decay width is given by

$$\Gamma(\tilde{a} \to \widetilde{W}_1^{\pm} + W^{\mp}) = \frac{N_c^2(\alpha_2/2)^2}{128\pi^3 (f_a/N)^2} m_{\tilde{a}}^3 \lambda^{1/2}\left(1, \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{a}}^2}, \frac{m_W^2}{m_{\tilde{a}}^2}\right)$$
$$\cdot \left[\left\{ \left(1 - \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{a}}^2}\right)^2 - \frac{m_W^2}{2m_{\tilde{a}}^2}\left(1 + \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{a}}^2} + \frac{m_W^2}{m_{\tilde{a}}^2}\right)\right\} (|U_{12}|^2 + |V_{12}|^2)\right.$$
$$\left. + 3\frac{m_{\widetilde{W}_1} m_W^2}{m_{\tilde{a}}^3} \left(U_{12}^* V_{12}^* + U_{12} V_{12}\right)\right]. \tag{10}$$

$\Gamma(\tilde{a} \to \widetilde{W}_2^{\pm} + W^{\mp})$ can be obtained by replacing $m_{\widetilde{W}_1}$, U_{12} and V_{12} with $m_{\widetilde{W}_2}$, U_{11} and V_{11}. Note that all notations and conventions follow the reference [1].

II. SAXION DECAY

The saxion-gauge boson-gauge boson couplings are given by

$$\mathcal{L} = \frac{N_c \alpha_2/2}{8\pi (f_a/N)} \sigma W_{\mu\nu}^i W^{i\mu\nu} + \frac{\alpha_Y C_{aYY}}{16\pi (f_a/N)} \sigma B_{\mu\nu} B^{\mu\nu}. \tag{11}$$

From the above lagrangian, we obtain the interaction lagrangian for W boson, Z boson and photon in the mass basis as the following.

$$\mathcal{L}_{sWW} = \frac{N_c \alpha_2/2}{2\pi (f_a/N)} \left(\partial_\mu W_\nu^+ \partial^\mu W^{-\nu} - \partial_\mu W_\nu^+ \partial^\nu W^{-\mu}\right), \tag{12}$$

$$\mathcal{L}_{sZZ} = \frac{(N_c \alpha_2/2)\cos^2\theta_W + \alpha_Y C_{aYY} \sin^2\theta_W}{4\pi (f_a/N)} \left[\partial_\mu Z_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right)\right], \tag{13}$$

$$\mathcal{L}_{s\gamma\gamma} = \frac{(N_c \alpha_2/2)\sin^2\theta_W + \alpha_Y C_{aYY} \cos^2\theta_W}{4\pi (f_a/N)} \left[\partial_\mu A_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right], \tag{14}$$

$$\mathcal{L}_{sZ\gamma} = \frac{(-N_c \alpha_2/2 + \alpha_Y C_{aYY})\sin\theta_W \cos\theta_W}{2\pi (f_a/N)} \left[\partial_\mu A_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right)\right]. \tag{15}$$

Note that we neglect the higher order couplings. We obtain the partial decay width of each decay mode.

$$\Gamma(s \to W^{\pm} + W^{\mp}) = \frac{(N_c\alpha_2/2)^2}{128\pi^3 (f_a/N)^2} m_s^3 \left(1 - \frac{4m_W^2}{m_s^2}\right)^{1/2} \left(1 - \frac{4m_W^2}{m_s^2} + \frac{6m_W^4}{m_s^4}\right), \tag{16}$$

$$\Gamma(s \to Z + Z) = \frac{\left\{(N_c\alpha_2/2)\cos^2\theta_W + \alpha_Y C_{aYY}\sin^2\theta_W\right\}^2}{256\pi^3 (f_a/N)^2} \cdot m_s^3 \left(1 - \frac{4m_Z^2}{m_s^2}\right)^{1/2} \left(1 - \frac{4m_Z^2}{m_s^2} + \frac{6m_Z^4}{m_s^4}\right), \tag{17}$$

$$\Gamma(s \to \gamma + \gamma) = \frac{\left\{(N_c\alpha_2/2)\sin^2\theta_W + \alpha_Y C_{aYY}\cos^2\theta_W\right\}^2}{256\pi^3 (f_a/N)^2} m_s^3, \tag{18}$$

$$\Gamma(s \to Z + \gamma) = \frac{(-N_c\alpha_2/2 + \alpha_Y C_{aYY})^2 \sin^2\theta_W \cos^2\theta_W}{128\pi^2 (f_a/N)^2} m_s^3 \left(1 - \frac{m_Z^2}{m_s^2}\right)^4 \tag{19}$$

In addition, saxion can decay to neutralinos and charginos. The relevant lagrangian is given by

$$\mathcal{L} = -i\frac{N_c\alpha_2/2}{4\pi(f_a/N)}\left(s\bar{\lambda}_i\gamma^\mu\partial_\mu\lambda_i\right) - i\frac{\alpha_Y C_{aYY}}{4\pi(f_a/N)}\left(s\bar{\lambda}_0\gamma^\mu\partial_\mu\lambda_0\right) \tag{20}$$

For charginos

$$\mathcal{L}_{s\widetilde{W}\widetilde{W}} = -i\frac{N_c\alpha_2/2}{4\pi(f_a/N)} s\left\{Q^L_{ij}\overline{\widetilde{W}}_i\gamma^\mu P_L\partial_\mu\widetilde{W}_j + Q^R_{ij}\overline{\widetilde{W}}_i\gamma^\mu P_R\partial_\mu\widetilde{W}_j\right\} + \text{h.c.}, \tag{21}$$

where

$$Q^L_{ij} = \begin{pmatrix} |U_{12}|^2 & U^*_{12}U_{11} \\ U^*_{11}U_{12} & |U_{11}|^2 \end{pmatrix}, \qquad Q^R_{ij} = \begin{pmatrix} |V_{12}|^2 & V^*_{12}V_{11} \\ V^*_{11}V_{12} & |V_{11}|^2 \end{pmatrix}. \tag{22}$$

For neutralinos,

$$\mathcal{L}_{s\widetilde{Z}\widetilde{Z}} = -i\frac{1}{8\pi(f_a/N)} R_{ij}\overline{\widetilde{Z}}_i\gamma^\mu\partial_\mu\widetilde{Z}_j, \tag{23}$$

where

$$R_{ij} = (N_c\alpha_2/2)v_3^{(i)*}v_3^{(j)} + \alpha_Y C_{aYY} v_4^{(i)*}v_4^{(j)}. \tag{24}$$

The partial decay widths are given by

$$\Gamma(s \to \widetilde{W}_i + \widetilde{W}_j) = \frac{(N_c \alpha_2/2)^2 m_s^3}{256\pi^3 (f_a/N)^2} \lambda^{1/2}\left(1, \frac{m^2_{\widetilde{W}_i}}{m_s^2}, \frac{m^2_{\widetilde{W}_j}}{m_s^2}\right)$$
$$\cdot\left[\left(|Q^L_{ij}|^2 + |Q^R_{ij}|^2\right)\left\{\left(\frac{m^2_{\widetilde{W}_i} + m^2_{\widetilde{W}_j}}{m_s^2}\right)\left(1 - \frac{m^2_{\widetilde{W}_i} + m^2_{\widetilde{W}_j}}{m_s^2}\right) - \frac{4m^2_{\widetilde{W}_i} m^2_{\widetilde{W}_j}}{m_s^4}\right\}\right.$$
$$\left.+\left(Q^L_{ij} Q^{R*}_{ij} + Q^{L*}_{ij} Q^R_{ij}\right)\left\{\frac{2m_{\widetilde{W}_i} m_{\widetilde{W}_j}}{m_s^2}\left(1 - 2\frac{m^2_{\widetilde{W}_i} + m^2_{\widetilde{W}_j}}{m_s^2}\right)\right\}\right], \tag{25}$$
$$\Gamma(s \to \widetilde{Z}_i + \widetilde{W}_j) = \frac{|R_{ij}|^2}{128\pi^3 (f_a/N)^2} \lambda^{1/2}\left(1, \frac{m^2_{\widetilde{Z}_i}}{m_s^2}, \frac{m^2_{\widetilde{Z}_j}}{m_s^2}\right)\left(1 - \frac{1}{2}\delta_{ij}\right)$$
$$\cdot m_s (m_{\widetilde{Z}_i} + m_{\widetilde{Z}_j})^2 \left[1 - \left(\frac{m_{\widetilde{Z}_i} + m_{\widetilde{Z}_j}}{m_s}\right)^2\right]. \tag{26}$$

Note that (ij) indices are not summed.

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[1] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events*, (Cambridge University Press, 2006).