I. SUPERSYMMETRIC DFSZ AXION MODEL

Supersymmetric DFSZ axion model has PQ-charged Higgs supermultiplets instead of exotic heavy quarks. The relevant Lagrangian is given by [1]

$$\mathcal{L} = \int d^2\theta(1+B\theta^2)\mu e^{c_H A/v_{PQ}} H_u H_d, \tag{1}$$

where B denotes SUSY breaking parameter and c_H is determined by PQ charges of Higgs multiplets. Here A denotes axion superfield,

$$A = \frac{s+ia}{\sqrt{2}} + \sqrt{2}\theta\tilde{a} + \theta^2\mathcal{F}_A. \tag{2}$$

From the above Lagrangian and soft terms for Higgses and soft mass of saxion, we obtain the scalar potential,

$$\begin{aligned} V = &\left(\mu^2 e^{\sqrt{2}c_H s/v_{PQ}} + m_{H_u}^2\right)\left|h_u^0\right|^2 + \left(\mu^2 e^{\sqrt{2}c_H s/v_{PQ}} + m_{H_d}^2\right)\left|h_d^0\right|^2 + \frac{1}{2}m_s^2 s^2 \\ &- \left\{B\mu e^{c_H(s+ia)/\sqrt{2}v_{PQ}} h_u^0 h_d^0 + \text{h.c.}\right\} \\ &+ \frac{1}{8}\left(g'^2+g^2\right)\left(\left|h_u^0\right|^2 - \left|h_d^0\right|^2\right)^2 + \left(\frac{c_H\mu}{v_{PQ}}\right)^2 e^{\sqrt{2}c_H s/v_{PQ}}\left|h_u^0 h_d^0\right|^2. \end{aligned} \tag{3}$$

We ignore here the charged components of Higgses since they do not develop the vacuum expectation values (VEVs). We will deal with the charged components to calculate saxion decay width. Note that we consider CP-conserving Lagrangian so that only CP-violation comes only from quark Yukawa sector. As we can see, we can obtain the above scalar potential by replacing

$$\mu \longrightarrow \mu\exp\left\{\frac{c_H(s+ia)}{\sqrt{2}v_{PQ}}\right\}, \qquad B\mu \longrightarrow B\mu\exp\left\{\frac{c_H(s+ia)}{\sqrt{2}v_{PQ}}\right\}. \tag{4}$$

To derive the saxion interaction with MSSM particles, we can approximate the exponential to the linear order of $1/v_{PQ}$. Similarly, we also obtain the axino interaction by approximating the Lagrangian (1),

$$\mu\exp\left(\frac{c_H A}{v_{PQ}}\right) \longrightarrow \mu\left(1+\frac{c_H A}{v_{PQ}}\right). \tag{5}$$

Before discussing the interaction Lagrangian for saxion and axino, we check if the axion remains

[1] Following the convention of Ref. [1], we use $H_{da} = \epsilon_{ab}H_d^a$ instead of H_d^a.

massless when the electroweak symmetry is broken. Taking the Higgs VEVs, we can write

$$h_u^0 = v_u + \frac{1}{\sqrt{2}}\left(h_{uR} + i h_{uI}\right), \qquad h_d^0 = v_d + \frac{1}{\sqrt{2}}\left(h_{dR} + i h_{dI}\right). \tag{6}$$

From the potential we obtain the electroweak symmetry breaking (EWSB) conditions,

$$\frac{\partial V}{\partial h_{uR}} = \sqrt{2}\left(\mu^2 + m_{H_u}^2\right) v_u - \sqrt{2} B\mu v_d + \frac{\sqrt{2}}{4}\left(g'^2 + g^2\right)\left(v_u^2 - v_d^2\right) v_u + \sqrt{2}\lambda^2 v_u v_d^2 = 0, \tag{7}$$

$$\frac{\partial V}{\partial h_{dR}} = \sqrt{2}\left(\mu^2 + m_{H_d}^2\right) v_d - \sqrt{2} B\mu v_u + \frac{\sqrt{2}}{4}\left(g'^2 + g^2\right)\left(v_d^2 - v_u^2\right) v_d + \sqrt{2}\lambda^2 v_u^2 v_d = 0, \tag{8}$$

where $\lambda = c_H \mu / v_{PQ}$. Equivalently,

$$\mu^2 + m_{H_u}^2 = B\mu \cot\beta + \frac{1}{2} M_Z^2 \cos 2\beta - \lambda^2 v^2 \cos^2\beta, \tag{9}$$

$$\mu^2 + m_{H_d}^2 = B\mu \tan\beta - \frac{1}{2} M_Z^2 \cos 2\beta - \lambda^2 v^2 \sin^2\beta, \tag{10}$$

where we take the relations, $v^2 = v_u^2 + v_d^2 = (174 \text{ GeV})^2$, $\tan\beta = v_u/v_d$ and $M_Z^2 = (g'^2 + g^2)v^2/2$. It is worth noting that saxion s also develops a VEV by the EWSB, i.e.

$$\frac{\partial V}{\partial s} = m_s^2 v_s + \sqrt{2}\lambda\mu v^2 e^{\sqrt{2} c_H v_s / v_{PQ}} - \sqrt{2}\lambda \left(\frac{B\mu}{\mu}\right) v_u v_d e^{c_H v_s / \sqrt{2} v_{PQ}} + \sqrt{2}\lambda^3 \left(\frac{v_u^2 v_d^2}{\mu}\right) e^{\sqrt{2} c_H v_s / v_{PQ}} = 0. \tag{11}$$

From the above equation, we obtain the VEV of saxion,

$$v_s \approx \sqrt{2}\lambda \left(\frac{1}{m_s^2}\right)\left[\left(\frac{B\mu}{\mu}\right) v_u v_d - \mu v^2\right] \sim \text{keV} \tag{12}$$

if $m_S \sim \mu \sim B\mu/\mu \sim$ TeV and $v_{PQ} \sim 10^{10}$ GeV. Hence we neglect the VEV of saxion from now on.

From these conditions we obtain the mass matrix of CP-odd scalars,

$$\mathcal{M}_p^2 = B\mu \begin{pmatrix} \cot\beta & 1 & \lambda v \cos\beta/\mu \\ & \tan\beta & \lambda v \sin\beta/\mu \\ & & \lambda^2 v^2 \sin 2\beta / 2\mu^2 \end{pmatrix}, \tag{13}$$

in the basis of (h_{uI},h_{dI},a). Firstly, we consider the basis change

$$\begin{pmatrix} G^0 \\ A \\ a \end{pmatrix} = \mathcal{P}_1 \begin{pmatrix} h_{uI} \\ h_{dI} \\ a \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_{uI} \\ h_{dI} \\ a \end{pmatrix}. \tag{14}$$

Then the mass matrix becomes

$$\mathcal{P}_1 \mathcal{M}_p^2 \mathcal{P}_1^T = B\mu \begin{pmatrix} 0 & 0 & 0 \\ & 2/\sin 2\beta & \lambda v/\mu \\ & & \lambda^2 v^2 \sin 2\beta/2\mu^2 \end{pmatrix}. \tag{15}$$

Again we consider the mixing

$$\begin{pmatrix} G^0 \\ A' \\ a' \end{pmatrix} = \mathcal{P}_2 \begin{pmatrix} G^0 \\ A \\ a \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\rho & \sin\rho \\ 0 & -\sin\rho & \cos\rho \end{pmatrix} \begin{pmatrix} G^0 \\ A \\ a \end{pmatrix}, \tag{16}$$

where $\tan\rho = \lambda v \sin 2\beta/2\mu$ and $0 < \rho < \pi/2$. The mass matrix becomes

$$\mathcal{P}_2 \mathcal{P}_1 \mathcal{M}_p^2 \mathcal{P}_1^T \mathcal{P}_2^T = \begin{pmatrix} 0 & & \\ & m_A^2 & \\ & & 0 \end{pmatrix}, \qquad \text{where} \qquad m_A^2 = 2B\mu(1+\tan^2\rho)/\sin 2\beta, \tag{17}$$

and the overall mixing matrix of pseudoscalar is given by

$$\mathcal{P} = \mathcal{P}_2 \mathcal{P}_1 = \begin{pmatrix} \sin\beta & -\cos\beta & 0 \\ \cos\beta\cos\rho & \sin\beta\cos\rho & \sin\rho \\ -\cos\beta\sin\rho & \sin\beta\sin\rho & \cos\rho \end{pmatrix}. \tag{18}$$

We have *an additional massless pseudoscalar* field which corresponds to goldstone boson for broken $U(1)_{PQ}$ symmetry. Note that the mixing between A and a is very small, so $A' \simeq A$ and $a' \simeq a$. Therefore, we just use the notation A and a for mass eigenstates of CP-odd Higgs and axion.

We can also obtain the scalar mass matrix,

$$\mathcal{M}^2_{s,11} = M_Z^2 \sin^2\beta + m_A^2 \cos^2\rho \cos^2\beta + \Delta^2_{11}, \tag{19}$$

$$\mathcal{M}^2_{s,22} = M_Z^2 \cos^2\beta + m_A^2 \cos^2\rho \sin^2\beta + \Delta^2_{22}, \tag{20}$$

$$\mathcal{M}^2_{s,12} = -\frac{1}{2}\left(M_Z^2 + m_A^2\cos^2\rho\right)\sin 2\beta + 4\mu^2\left(\frac{\tan^2\rho}{\sin 2\beta}\right) + \Delta^2_{12}, \tag{21}$$

$$\mathcal{M}^2_{s,33} = m_s^2 + m_A^2 \sin^2\rho + 8\mu^2\left(\frac{\tan^2\rho}{\sin^2 2\beta}\right)\left(1+\tan^2\rho\right), \tag{22}$$

$$\mathcal{M}^2_{s,13} = -m_A^2\cos^2\rho\tan\rho\cos\beta + 2\mu^2\left(\frac{\tan\rho}{\cos\beta}\right)\left(1+\frac{\tan^2\rho}{\sin^2\beta}\right), \tag{23}$$

$$\mathcal{M}^2_{s,23} = -m_A^2\cos^2\rho\tan\rho\sin\beta + 2\mu^2\left(\frac{\tan\rho}{\sin\beta}\right)\left(1+\frac{\tan^2\rho}{\cos^2\beta}\right), \tag{24}$$

in the basis of (h_{uR}, h_{dR}, s). Here we add loop corrections, Δ^2_{ij} for doublet Higgs components. In principle, saxion sector can also have loop corrections but they are at most $\mathcal{O}(v^2/8\pi v^2_{PQ})$. Therefore, we neglect them in this note.

In order to diagonalize this matrix, we first consider upper (2×2) part as for pseudoscalar case. In the same way as the MSSM, we invoke the mixing,

$$\begin{pmatrix} h \\ H \\ s \end{pmatrix} = \mathcal{S}_1 \begin{pmatrix} h_{uR} \\ h_{dR} \\ s \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_{uR} \\ h_{dR} \\ s \end{pmatrix}, \tag{25}$$

where $0 < \alpha < \pi/2$. Then mass matrix becomes

$$\mathcal{M}'^2_s \equiv \mathcal{S}_1 \mathcal{M}^2_s \mathcal{S}_1^T, \tag{26}$$

with

$$\begin{aligned} \mathcal{M}'^2_{s,11} \equiv m_h^2 = \; & M_Z^2\sin^2(\beta-\alpha) + m_A^2\cos^2\rho\cos^2(\alpha+\beta) \\ & +\Delta^2_{11}\cos^2\alpha + \Delta^2_{22}\sin^2\alpha + \Delta^2_{12}\sin 2\alpha + 4\mu^2\left(\frac{\tan^2\rho}{\sin 2\beta}\right)\sin 2\alpha, \end{aligned} \tag{27}$$

$$\begin{aligned} \mathcal{M}'^2_{s,22} \equiv m_H^2 = \; & M_Z^2\cos^2(\beta-\alpha) + m_A^2\cos^2\rho\sin^2(\alpha+\beta) \\ & +\Delta^2_{11}\sin^2\alpha + \Delta^2_{22}\cos^2\alpha - \Delta^2_{12}\sin 2\alpha - 4\mu^2\left(\frac{\tan^2\rho}{\sin 2\beta}\right)\sin 2\alpha, \end{aligned} \tag{28}$$

$$\begin{aligned} \mathcal{M}'^2_{s,12} \;\; = 0 = \; & \frac{1}{2}\sin 2\alpha\left\{-\left(m_A^2\cos^2\rho - M_Z^2\right)\cos 2\beta + \left(\Delta^2_{22}-\Delta^2_{11}\right)\right\} \\ & -\cos 2\alpha\left\{-\frac{1}{2}\left(m_A^2\cos^2\rho + M_Z^2\right)\sin 2\beta + \Delta^2_{12} + 4\mu^2\left(\frac{\tan^2\rho}{\sin 2\beta}\right)\right\}, \end{aligned} \tag{29}$$

and

$$\mathcal{M}'^2_{s,33} = \mathcal{M}^2_{s,33}, \tag{30}$$

$$\mathcal{M}'^2_{s,13} = -m_A^2 \tan\rho \cos(\beta-\alpha) + 4\mu^2 \left(\frac{\tan\rho}{\sin 2\beta}\right) \left\{\sin(\alpha+\beta) + \frac{2\tan^2\rho}{\sin 2\beta}\cos(\beta-\alpha)\right\}, \tag{31}$$

$$\mathcal{M}'^2_{s,23} = -m_A^2 \tan\rho \sin(\beta-\alpha) + 4\mu^2 \left(\frac{\tan\rho}{\sin 2\beta}\right) \left\{\cos(\alpha+\beta) + \frac{2\tan^2\rho}{\sin 2\beta}\sin(\beta-\alpha)\right\}. \tag{32}$$

Secondly, we consider the mixing between doublet Higgses and saxion as follows.

$$\begin{pmatrix} h' \\ H' \\ s' \end{pmatrix} = \mathcal{S}_2 \begin{pmatrix} h \\ H \\ s \end{pmatrix}. \tag{33}$$

Mixing is suppressed by v_{PQ} so that we can approximately obtain $\mathcal{S}_2$ by perturbation theory, which is given by

$$\mathcal{S}_{2,nm} = \frac{\mathcal{M}'^2_{s,mn}}{\mathcal{M}'^2_{s,nn} - \mathcal{M}'^2_{s,mm}}. \tag{34}$$

Then we obtain the matrix elements at the linear order of $\tan\rho$,

$$\mathcal{S}_{2,13} = -\mathcal{S}_{2,31} \equiv -\epsilon_h = \left(\frac{\tan\rho}{m_h^2 - m_s^2}\right) \left\{-m_A^2 \cos(\beta-\alpha) + \frac{4\mu^2 \sin(\alpha+\beta)}{\sin 2\beta}\right\}, \tag{35}$$

$$\mathcal{S}_{2,23} = -\mathcal{S}_{2,32} \equiv -\epsilon_H = \left(\frac{\tan\rho}{m_H^2 - m_s^2}\right) \left\{-m_A^2 \sin(\beta-\alpha) + \frac{4\mu^2 \cos(\alpha+\beta)}{\sin 2\beta}\right\}, \tag{36}$$

where we use the relations, $\tan\rho = \lambda v \sin 2\beta / 2\mu$ and $\lambda = c_H \mu / v_{PQ}$. We obtain the final mixing matrix,

$$\begin{aligned} \mathcal{S} = \mathcal{S}_2 \mathcal{S}_1 &\approx \begin{pmatrix} 1 & 0 & -\epsilon_h \\ 0 & 1 & -\epsilon_H \\ \epsilon_h & \epsilon_H & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos\alpha & \sin\alpha & -\epsilon_h \\ -\sin\alpha & \cos\alpha & -\epsilon_H \\ \epsilon_h \cos\alpha - \epsilon_H \sin\alpha & \epsilon_h \sin\alpha + \epsilon_H \cos\alpha & 1 \end{pmatrix}. \end{aligned} \tag{37}$$

II. INTERACTIONS AND PARTIAL DECAY WIDTHS OF SAXION

A. Higgs final states

Since PQ scale v_{PQ} is much larger than weak scale parameters and soft SUSY breaking terms, we keep only the linear order of $1/v_{PQ}$ in this section. We rewrite the scalar potential (3),

$$\begin{aligned} V_{shh} \simeq & \frac{\sqrt{2}c_H\mu^2}{v_{PQ}} s\left(|h_u^0|^2+|h_d^0|^2+|h_u^+|^2+|h_d^-|^2\right) \\ & -\frac{c_H B\mu}{\sqrt{2}v_{PQ}}(s+ia)\left(h_u^+h_d^-+h_u^0h_d^0\right)+\text{h.c.}, \end{aligned} \tag{38}$$

where the charged Higgs sector is added to consider saxion decay to charged Higgs. Setting aside the mass mixing terms and some unnecessary terms, we can write

$$\begin{aligned} V_{shh} \simeq & \frac{\sqrt{2}c_H\mu^2}{v_{PQ}} s\left\{\frac{1}{2}\left(h_{uR}^2+h_{dR}^2+h_{uI}^2+h_{dI}^2\right)+|h_u^+|^2+|h_d^-|^2\right\} \\ & -\frac{c_H B\mu}{\sqrt{2}v_{PQ}} s\left(h_{uR}h_{dR}-h_{uI}h_{dI}\right)+\frac{c_H B\mu}{\sqrt{2}v_{PQ}} a\left(h_{uR}h_{dI}+h_{uI}h_{dR}\right), \\ & -\left[\frac{c_H B\mu}{\sqrt{2}v_{PQ}}(s+ia)h_u^+h_d^-+\text{h.c.}\right]. \end{aligned} \tag{39}$$

From now on, we ignore the axion interaction,

$$V_{ahh} = \frac{c_H B\mu}{\sqrt{2}v_{PQ}} a\left(h_{uR}h_{dI}+h_{uI}h_{dR}\right)-\frac{ic_H B\mu}{\sqrt{2}v_{PQ}} a\left(h_u^+h_d^- - h_u^{+*}h_d^{-*}\right). \tag{40}$$

Imposing the ordinary MSSM mixing matrices, (14), (25) and mixing matrix for charged Higgs,

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_d^{-*} \\ h_u^+ \end{pmatrix}, \tag{41}$$

we obtain

$$\begin{aligned} V_{shh} \simeq & \frac{\sqrt{2}c_H\mu^2}{v_{PQ}} s\left\{\frac{1}{2}\left(h^2+H^2+A^2+G^{0^2}\right)+G^+G^-+H^+H^-\right\} \\ & -\frac{c_H B\mu}{\sqrt{2}v_{PQ}} s\left\{\frac{1}{2}\sin 2\alpha\left(h^2-H^2\right)+\cos 2\alpha hH+\frac{1}{2}\sin 2\beta\left(G^{0^2}-A^2\right)+\cos 2\beta G^0 A\right\} \\ & -\frac{c_H B\mu}{\sqrt{2}v_{PQ}} s\left\{\sin 2\beta\left(G^+G^- - H^+H^-\right)+\cos 2\beta\left(H^+G^- + G^+H^-\right)\right\}. \end{aligned} \tag{42}$$

Moreover, saxion has mixing with CP-even Higgs states, h and H by mixig matrix $\mathcal{S}_2$, so couples with every SM particles that couple with CP-even Higgses. According to Ref. [2], Feynman rules for Higgs self couplings in the unit of $-iM_Z^2/\sqrt{2}v$ are given by

$$hhh \;:\; 3\cos 2\alpha \sin(\beta-\alpha) \quad , \quad Hhh \;:\; -2\sin 2\alpha \sin(\beta-\alpha) - \cos 2\alpha \cos(\beta-\alpha), \tag{43}$$

$$HHH \;:\; 3\cos 2\alpha \cos(\beta-\alpha) \quad , \quad hHH \;:\; 2\sin 2\alpha \cos(\beta-\alpha) - \cos 2\alpha \sin(\beta-\alpha), \tag{44}$$

$$hAA \;:\; \cos 2\beta \sin(\beta-\alpha) \quad , \quad HAA \;:\; -\cos 2\beta \cos(\beta-\alpha). \tag{45}$$

Feynman rules for charged Higgs are given by

$$hH^+H^- \;:\; \cos 2\beta \sin(\beta-\alpha) + 2\cos^2\theta_W \sin(\beta+\alpha), \tag{46}$$

$$HH^+H^- \;:\; -\cos 2\beta \cos(\beta-\alpha) + 2\cos^2\theta_W \cos(\beta+\alpha). \tag{47}$$

Together with the saxion potential (42), we obtain the following Feynman rules for saxion (in the unit of $-i$).

$$\begin{aligned}\lambda_{shh} =& \left(\frac{\sqrt{2}c_H\mu^2}{v_{PQ}}\right)\left\{1-\frac{1}{4}\left(\frac{m_A^2}{\mu^2}\right)\sin 2\beta \sin 2\alpha\right\} \\ &+\left(\frac{M_Z^2}{\sqrt{2}v}\right)\cos 2\alpha \sin(\beta-\alpha)\left[3\epsilon_h - \epsilon_H\left\{2\tan 2\alpha + \cot(\beta-\alpha)\right\}\right],\end{aligned} \tag{48}$$

$$\begin{aligned}\lambda_{sHH} =& \left(\frac{\sqrt{2}c_H\mu^2}{v_{PQ}}\right)\left\{1+\frac{1}{4}\left(\frac{m_A^2}{\mu^2}\right)\sin 2\beta \sin 2\alpha\right\} \\ &+\left(\frac{M_Z^2}{\sqrt{2}v}\right)\cos 2\alpha \sin(\beta-\alpha)\left[3\epsilon_H \cot(\beta-\alpha) + \epsilon_h\left\{2\tan 2\alpha \cot(\beta-\alpha) - 1\right\}\right],\end{aligned} \tag{49}$$

$$\begin{aligned}\lambda_{shH} =& -\left(\frac{c_H m_A^2}{2\sqrt{2}v_{PQ}}\right)\sin 2\beta \cos 2\alpha \\ &+\left(\frac{M_Z^2}{\sqrt{2}v}\right)\cos 2\alpha \sin(\beta-\alpha)\left[-\epsilon_h\left\{2\tan 2\alpha + \cot(\beta-\alpha)\right\}\right. \\ &\left.+\epsilon_H\left\{2\tan 2\alpha \cot(\beta-\alpha) - 1\right\}\right],\end{aligned} \tag{50}$$

$$\begin{aligned}\lambda_{sAA} =& \left(\frac{\sqrt{2}c_H\mu^2}{v_{PQ}}\right)\left\{1+\frac{1}{4}\left(\frac{m_A^2}{\mu^2}\right)\sin^2 2\beta\right\} \\ &+\left(\frac{M_Z^2}{\sqrt{2}v}\right)\cos 2\beta \sin(\beta-\alpha)\left\{\epsilon_h - \epsilon_H \cot(\beta-\alpha)\right\},\end{aligned} \tag{51}$$

$$\begin{aligned}\lambda_{sH^+H^-} =& \left(\frac{\sqrt{2}c_H\mu^2}{v_{PQ}}\right)\left\{1+\frac{1}{4}\left(\frac{m_A^2}{\mu^2}\right)\sin^2 2\beta\right\} \\ &+\left(\frac{M_Z^2}{\sqrt{2}v}\right)\left[\cos 2\beta \sin(\beta-\alpha)\left\{\epsilon_h - \epsilon_H \cot(\beta-\alpha)\right\}\right. \\ &\left.+2\cos^2\theta_W \sin(\beta+\alpha)\left\{\epsilon_h + \epsilon_H \cot(\beta+\alpha)\right\}\right].\end{aligned} \tag{52}$$

From the above Feynman rules, we obtain the partial decay width into Higgs states,

$$\Gamma(s \to \phi_i\phi_j) = \frac{\lambda^2_{s\phi_i\phi_j}}{16\pi m_s}\lambda^{1/2}\left(1, \frac{m^2_{\phi_i}}{m^2_s}, \frac{m^2_{\phi_j}}{m^2_s}\right)\left(1 - \frac{1}{2}\delta_{ij}\right), \tag{53}$$

where $\phi_i = h, H, A, H^+, H^-$.

B. gauge/Higgs boson final states

Feynman rules for relevant Higgs couplings to gauge bosons are given by

$$Z_\mu Z_\nu h \ : \ ig_Z M_Z \sin(\beta+\alpha) g_{\mu\nu} \quad , \quad Z_\mu Z_\nu H \ : \ ig_Z M_Z \cos(\beta+\alpha) g_{\mu\nu}, \tag{54}$$

$$W^+_\mu W^-_\nu h \ : \ ig_W M_W \sin(\beta+\alpha) g_{\mu\nu} \quad , \quad W^+_\mu W^-_\nu H \ : \ ig_W M_W \cos(\beta+\alpha) g_{\mu\nu}, \tag{55}$$

$$Z_\mu h A \ : \ +\frac{g_Z}{2}\cos(\beta+\alpha)(p+p')_\mu \quad , \quad Z_\mu H A \ : \ -\frac{g_Z}{2}\sin(\beta+\alpha)(p+p')_\mu, \tag{56}$$

$$W^\pm_\mu H^\mp h \ : \ \mp i\frac{g_W}{2}\cos(\beta+\alpha)(p+p')_\mu \quad , \quad W^\pm_\mu H^\mp H \ : \ \pm i\frac{g_W}{2}\sin(\beta+\alpha)(p+p')_\mu, \tag{57}$$

where $g_W = g$ and $g_Z = g/\cos\theta_W$. To obtain saxion couplings, we can replace h and H with $\epsilon_h s$ and $\epsilon_H s$, respectively.

From these Feynman rules, we can obtain the saxion partial decay width for gauge boson final states,

$$\Gamma(s \to VV) = \frac{g^2_V g^2_{sVV}}{16\pi} m_s \left\{3\frac{M^2_V}{m^2_s} + \frac{m^2_s}{4M^2_V}\left(1 - \frac{4M^2_V}{m^2_s}\right)\right\}\left(1 - \frac{4M^2_V}{m^2_s}\right)^{1/2}\left(1 - \frac{1}{2}\delta_{VZ}\right) \tag{58}$$

with

$$g_{sVV} = \epsilon_h g_{hVV} + \epsilon_H g_{HVV} = \epsilon_h \sin(\beta+\alpha) + \epsilon_H \cos(\beta+\alpha), \tag{59}$$

where $\delta_{VZ} = 1, 0$ for $V = Z, W$. Also, we have the case of one gauge boson and one Higgs boson final states, its decay width is given by

$$\begin{aligned}\Gamma(s \to V\phi) =& \frac{g^2_V g^2_{sV\phi}}{32\pi}\frac{m^3_s}{M^2_V}\left\{\left(1 - \frac{m^2_\phi}{m^2_s}\right)^2 - 2\frac{M^2_V}{m^2_s}\left(1 + \frac{m^2_\phi}{m^2_s}\right) + \frac{M^2_V}{m^2_s}\right\} \\ &\times \lambda^{1/2}\left(1, \frac{M^2_V}{m^2_s}, \frac{m^2_\phi}{m^2_s}\right)\left(1 - \frac{1}{2}\delta_{VZ}\right),\end{aligned} \tag{60}$$

with

$$g_{sV\phi} = \epsilon_h g_{hV\phi} + \epsilon_H g_{HV\phi} = \epsilon_h \cos(\beta+\alpha) - \epsilon_H \sin(\beta+\alpha), \tag{61}$$

for $V\phi = ZA, W^{\pm}H^{\mp}$.

C. fermion final states

Feynman rules are given by

$$huu \ : \ -i\frac{m_u}{\sqrt{2}v}\frac{\cos\alpha}{\sin\beta}, \qquad Huu \ : \ i\frac{m_u}{\sqrt{2}v}\frac{\sin\alpha}{\sin\beta}, \qquad Auu \ : \ -\frac{m_u}{\sqrt{2}v}\cot\beta\gamma_5, \tag{62}$$

$$hdd \ : \ -i\frac{m_d}{\sqrt{2}v}\frac{\sin\alpha}{\cos\beta}, \qquad Hdd \ : \ -i\frac{m_d}{\sqrt{2}v}\frac{\cos\alpha}{\cos\beta}, \qquad Auu \ : \ -\frac{m_d}{\sqrt{2}v}\tan\beta\gamma_5. \tag{63}$$

Partial decay width of saxion into fermions is given by

$$\Gamma(s\to f\bar{f}) = \frac{N_c}{16\pi}\frac{m_f^2}{v^2}g_{sff}^2 m_s\left(1-\frac{4m_f^2}{m_s^2}\right)^{3/2} \tag{64}$$

with

$$g_{sff} = \epsilon_h g_{hff} + \epsilon_H g_{Hff} = \begin{cases} \frac{1}{\sin\beta}\left(-\epsilon_h\cos\alpha + \epsilon_H\sin\alpha\right), & \text{for up-type fermions,} \\ \frac{1}{\cos\beta}\left(-\epsilon_h\sin\alpha - \epsilon_H\cos\alpha\right), & \text{for down-type fermions.} \end{cases} \tag{65}$$

D. neutralino and chargino final states

According to Ref. [1], Higgs interactions with neutralinos and charginos are given by

$$\begin{aligned}\mathcal{L} =& g\sqrt{2}S_1^h\overline{\widetilde{W}}_1\widetilde{W}_1 h + g\sqrt{2}S_1^h\overline{\widetilde{W}}_2\widetilde{W}_2 h + \left[\frac{g}{\sqrt{2}}\overline{\widetilde{W}}_1\left(S^h + P^h\gamma_5\right)\widetilde{W}_2 h + \text{h.c.}\right] \\ &+ \left[\sum_{i,j} X_{i,j}^h \overline{\widetilde{Z}}_i(-i\gamma_5)^{\theta_i+\theta_j}\widetilde{Z}_j h + \text{h.c.}\right],\end{aligned} \tag{66}$$

where

$$S_1^h = \frac{1}{2}(-1)^{\theta_{\widetilde{W}_1}} \left[\sin\alpha \sin\gamma_R \cos\gamma_L + \cos\alpha \sin\gamma_L \cos\gamma_R\right], \tag{67}$$

$$S_2^h = \frac{1}{2}(-1)^{\theta_{\widetilde{W}_2}+1}\theta_x\theta_y \left[\sin\alpha \cos\gamma_R \sin\gamma_L + \cos\alpha \cos\gamma_L \sin\gamma_R\right], \tag{68}$$

$$\begin{aligned} S^h = \frac{1}{2}\Big[&-(-1)^{\theta_{\widetilde{W}_1}}\theta_x \sin\gamma_R \sin\gamma_L \sin\alpha + (-1)^{\theta_{\widetilde{W}_1}}\theta_x \cos\gamma_R \cos\gamma_L \cos\alpha \\ &-(-1)^{\theta_{\widetilde{W}_2}}\theta_y \sin\gamma_R \sin\gamma_L \cos\alpha + (-1)^{\theta_{\widetilde{W}_2}}\theta_y \cos\gamma_R \cos\gamma_L \sin\alpha\Big], \end{aligned} \tag{69}$$

$$\begin{aligned} P^h = \frac{1}{2}\Big[&+(-1)^{\theta_{\widetilde{W}_1}}\theta_x \sin\gamma_R \sin\gamma_L \sin\alpha - (-1)^{\theta_{\widetilde{W}_1}}\theta_x \cos\gamma_R \cos\gamma_L \cos\alpha \\ &-(-1)^{\theta_{\widetilde{W}_2}}\theta_y \sin\gamma_R \sin\gamma_L \cos\alpha + (-1)^{\theta_{\widetilde{W}_2}}\theta_y \cos\gamma_R \cos\gamma_L \sin\alpha\Big], \end{aligned} \tag{70}$$

$$X_{ij}^h = -\frac{1}{2}(-1)^{\theta_i+\theta_j}\left(v_2^{(i)}\sin\alpha - v_1^{(i)}\cos\alpha\right)\left(g v_3^{(j)} - g' v_4^{(j)}\right). \tag{71}$$

The couplings of the heavy scalar H can be obtained from those of h by replacing $\cos\alpha \to -\sin\alpha$ and $\sin\alpha \to \cos\alpha$. In addition, saxion-higgsino-higgsino coupling from PQ sector is derived by replacing $\mu \to \mu\exp(c_H A/v_{PQ})$,

$$\mathcal{L} = -\frac{1}{2}\frac{c_H\mu}{v_{PQ}}\frac{s}{\sqrt{2}}\left\{\left(\bar{\psi}_{h_u^0}\psi_{h_d^0} + \bar{\psi}_{h_d^0}\psi_{h_u^0}\right) + \left(\bar{\psi}_{h_u^+}\psi_{h_d^-} + \bar{\psi}_{h_d^-}\psi_{h_u^+}\right)\right\}. \tag{72}$$

From the neutralino mixing,

$$\psi_{h_u^0} = \sum_{i=1}^{4} v_1^{(i)}\,(i\gamma_5)^{\theta_i}\,\widetilde{Z}_i, \qquad \psi_{h_d^0} = \sum_{j=1}^{4} v_2^{(j)}\,(i\gamma_5)^{\theta_j}\,\widetilde{Z}_j, \tag{73}$$

and chargino mixing

$$\psi_{h_d^-} = -\theta_x(-1)^{\theta_{\widetilde{W}_2}}\sin\gamma_L P_L\widetilde{W}_2 + (-1)^{\theta_{\widetilde{W}_1}}\cos\gamma_L P_L\widetilde{W}_1, \tag{74}$$

$$\psi_{h_u^+} = \theta_y \sin\gamma_R P_R\widetilde{W}_2 - \cos\gamma_R P_R\widetilde{W}_1, \tag{75}$$

we obtain

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2\sqrt{2}}\frac{c_H\mu}{v_{PQ}}s\left[\sum_{i,j}\overline{\widetilde{Z}}_i\,(-i\gamma_5)^{\theta_i+\theta_j}\,\widetilde{Z}_j X_{ij}^s + \text{h.c.}\right] \\ &-\frac{1}{2\sqrt{2}}\frac{c_H\mu}{v_{PQ}}s\left[S_1^s\overline{\widetilde{W}}_1\widetilde{W}_1 + S_2^s\overline{\widetilde{W}}_2\widetilde{W}_2 + \left\{\overline{\widetilde{W}}_1\,(S^s + P^s\gamma_5)\,\widetilde{W}_2 + \text{h.c.}\right\}\right], \end{aligned} \tag{76}$$

where we define

$$X^s_{ij} = (-1)^{\theta_i+\theta_j} v_1^{(i)} v_2^{(j)}, \tag{77}$$

$$S^s_1 = (-1)^{\theta_{\widetilde{W}_1}} \cos\gamma_L \cos\gamma_R, \tag{78}$$

$$S^s_2 = -(-1)^{\theta_{\widetilde{W}_2}} \theta_x\theta_y \sin\gamma_L \sin\gamma_R, \tag{79}$$

$$S^s = \frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_1}} \theta_y \cos\gamma_L \sin\gamma_R - (-1)^{\theta_{\widetilde{W}_2}} \theta_x \sin\gamma_L \cos\gamma_R\right\} \tag{80}$$

$$P^s = \frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_1}} \theta_y \cos\gamma_L \sin\gamma_R + (-1)^{\theta_{\widetilde{W}_2}} \theta_x \sin\gamma_L \cos\gamma_R\right\}. \tag{81}$$

Then we can finally obtain physical saxion interaction up to $\mathcal{O}(1/v_{PQ})$,

$$\begin{aligned}\mathcal{L} =& g\sqrt{2}\Sigma^s_1 \overline{\widetilde{W}}_1 \widetilde{W}_1 s + g\sqrt{2}\Sigma^s_1 \overline{\widetilde{W}}_2 \widetilde{W}_2 s + \left[\frac{g}{\sqrt{2}}\overline{\widetilde{W}}_1 \left(\Sigma^s + \Pi^s\gamma_5\right)\widetilde{W}_2 s + \text{h.c.}\right] \\ &+ \left[\sum_{i,j} \Xi^h_{i,j} \overline{\widetilde{Z}}_i (-i\gamma_5)^{\theta_i+\theta_j} \widetilde{Z}_j s + \text{h.c.}\right],\end{aligned} \tag{82}$$

where

$$\Sigma^s_1 = \epsilon_h S^h_1 + \epsilon_H S^H_1 - \frac{c_H\mu}{4gv_{PQ}} S^s_1, \tag{83}$$

$$\Sigma^s_2 = \epsilon_h S^h_2 + \epsilon_H S^H_2 - \frac{c_H\mu}{4gv_{PQ}} S^s_2, \tag{84}$$

$$\Sigma^s = \epsilon_h S^h + \epsilon_H S^H - \frac{c_H\mu}{2gv_{PQ}} S^s, \tag{85}$$

$$\Pi^s = \epsilon_h P^h + \epsilon_H P^H - \frac{c_H\mu}{2gv_{PQ}} P^s, \tag{86}$$

$$\Xi^s_{ij} = \epsilon_h X^h_{ij} + \epsilon_H X^H_{ij} - \frac{c_H\mu}{2\sqrt{2}v_{PQ}} X^s_{ij}. \tag{87}$$

Partial decay width into charginos is given by

$$\Gamma(s \to \widetilde{W}_i^+ \widetilde{W}_i^-) = \frac{g^2}{4\pi}\left|\Sigma^s_i\right|^2 m_s \left(1 - \frac{4m^2_{\widetilde{W}_i}}{m_s^2}\right)^{3/2} \tag{88}$$

and

$$\begin{aligned}\Gamma(s \to \widetilde{W}_1^+ \widetilde{W}_2^-) =& \frac{g^2}{16\pi} m_s \lambda^{1/2}\left(1, \frac{m^2_{\widetilde{W}_1}}{m_s^2}, \frac{m^2_{\widetilde{W}_2}}{m_s^2}\right) \\ &\times \left[\left|\Sigma^s\right|^2 \left\{1 - \left(\frac{m_{\widetilde{W}_2} + m_{\widetilde{W}_1}}{m_s}\right)^2\right\} + \left|\Pi^s\right|^2 \left\{1 - \left(\frac{m_{\widetilde{W}_2} - m_{\widetilde{W}_1}}{m_s}\right)^2\right\}\right].\end{aligned} \tag{89}$$

Partial width into neutralinos is given by

$$\begin{aligned}\Gamma(s \to \widetilde{Z}_i \widetilde{Z}_j) =& \frac{1}{8\pi} m_s \left(\Xi^s_{ij} + \Xi^s_{ji}\right)^2 \left[1 - \left\{\frac{m_{\widetilde{Z}_i} + (-1)^{\theta_i+\theta_j} m_{\widetilde{Z}_j}}{m_s}\right\}^2\right] \\ & \times \lambda^{1/2}\left(1, \frac{m^2_{\widetilde{Z}_i}}{m_s^2}, \frac{m^2_{\widetilde{Z}_j}}{m_s^2}\right)\left(1 - \frac{1}{2}\delta_{ij}\right).\end{aligned} \tag{90}$$

E. sfermion final states

From the Higgs decays to sfermions in Ref. [1], partial decay withs of saxion to sfermions are give by

$$\Gamma(s \to \tilde{f}_i \tilde{f}_j) = \frac{\left|\epsilon_h \mathcal{A}^h_{\tilde{f}_i\tilde{f}_j} + \epsilon_H \mathcal{A}^H_{\tilde{f}_i\tilde{f}_j}\right|^2}{16\pi m_s} N_c(f) \lambda^{1/2}\left(1, \frac{m^2_{\tilde{f}_i}}{m_s^2}, \frac{m^2_{\tilde{f}_j}}{m_s^2}\right), \tag{91}$$

where

$$\mathcal{A}^{h,H}_{\tilde{f}_1\tilde{f}_1} = \mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_L}\cos^2\theta_f + \mathcal{A}^{h,H}_{\tilde{f}_R\tilde{f}_R}\sin^2\theta_f - 2\mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_R}\cos\theta_f\sin\theta_f, \tag{92}$$

$$\mathcal{A}^{h,H}_{\tilde{f}_2\tilde{f}_2} = \mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_L}\sin^2\theta_f + \mathcal{A}^{h,H}_{\tilde{f}_R\tilde{f}_R}\cos^2\theta_f + 2\mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_R}\cos\theta_f\sin\theta_f, \tag{93}$$

$$\mathcal{A}^{h,H}_{\tilde{f}_1\tilde{f}_2} = \mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_L}\cos\theta_f\sin\theta_f - \mathcal{A}^{h,H}_{\tilde{f}_R\tilde{f}_R}\cos\theta_f\sin\theta_f + 2\mathcal{A}^{h,H}_{\tilde{f}_L\tilde{f}_R}\cos 2\theta_f, \tag{94}$$

$$\mathcal{A}^{h,H}_{\tilde{f}_2\tilde{f}_1} = \mathcal{A}^{h,H}_{\tilde{f}_1\tilde{f}_2} \tag{95}$$

with

$$\mathcal{A}^h_{\tilde{u}_L\tilde{u}_L} = g\left[M_W\left(\frac{1}{2} - \frac{1}{6}\tan^2\theta_W\right)\sin(\beta-\alpha) - \frac{m_u^2\cos\alpha}{M_W\sin\beta}\right], \tag{96}$$

$$\mathcal{A}^h_{\tilde{u}_R\tilde{u}_R} = g\left[\frac{2}{3}M_W\tan^2\theta_W\sin(\beta-\alpha) - \frac{m_u^2\cos\alpha}{M_W\sin\beta}\right], \tag{97}$$

$$\mathcal{A}^h_{\tilde{u}_L\tilde{u}_R} = \frac{g m_u}{2M_W\sin\beta}\left(-\mu\sin\alpha + A_u\cos\alpha\right), \tag{98}$$

$$\mathcal{A}^H_{\tilde{u}_L\tilde{u}_L} = g\left[-M_W\left(\frac{1}{2} - \frac{1}{6}\tan^2\theta_W\right)\cos(\beta-\alpha) + \frac{m_u^2\sin\alpha}{M_W\sin\beta}\right], \tag{99}$$

$$\mathcal{A}^H_{\tilde{u}_R\tilde{u}_R} = g\left[-\frac{2}{3}M_W\tan^2\theta_W\cos(\beta-\alpha) + \frac{m_u^2\sin\alpha}{M_W\sin\beta}\right], \tag{100}$$

$$\mathcal{A}^H_{\tilde{u}_L\tilde{u}_R} = \frac{g m_u}{2M_W\sin\beta}\left(-\mu\cos\alpha - A_u\sin\alpha\right), \tag{101}$$

$$\mathcal{A}^h_{\tilde{d}_L\tilde{d}_L} = g\left[M_W\left(-\frac{1}{2}-\frac{1}{6}\tan^2\theta_W\right)\sin(\beta-\alpha)-\frac{m_d^2\sin\alpha}{M_W\cos\beta}\right], \tag{102}$$

$$\mathcal{A}^h_{\tilde{d}_R\tilde{d}_R} = g\left[-\frac{1}{3}M_W\tan^2\theta_W\sin(\beta-\alpha)-\frac{m_d^2\sin\alpha}{M_W\cos\beta}\right], \tag{103}$$

$$\mathcal{A}^h_{\tilde{d}_L\tilde{d}_R} = \frac{gm_d}{2M_W\cos\beta}\left(-\mu\cos\alpha+A_d\sin\alpha\right), \tag{104}$$

$$\mathcal{A}^H_{\tilde{d}_L\tilde{d}_L} = g\left[M_W\left(\frac{1}{2}+\frac{1}{6}\tan^2\theta_W\right)\cos(\beta-\alpha)-\frac{m_d^2\cos\alpha}{M_W\cos\beta}\right], \tag{105}$$

$$\mathcal{A}^H_{\tilde{d}_R\tilde{d}_R} = g\left[\frac{1}{3}M_W\tan^2\theta_W\cos(\beta-\alpha)-\frac{m_d^2\cos\alpha}{M_W\cos\beta}\right], \tag{106}$$

$$\mathcal{A}^H_{\tilde{d}_L\tilde{d}_R} = \frac{gm_d}{2M_W\cos\beta}\left(\mu\sin\alpha+A_d\cos\alpha\right). \tag{107}$$

III. INTERACTIONS AND PARTIAL DECAY WIDTH OF AXINO

From the potential (1), we can obtain the Lagrangian for axino interactions with charginos and neutralinos,

$$\begin{aligned}\mathcal{L} = &-\frac{c_H\mu}{v_{PQ}}\left(\bar{\tilde{a}}P_L\psi_{h_u^0}h_d^0+\tilde{a}P_R\psi_{h_u^0}h_d^{0*}+\bar{\tilde{a}}P_L\psi_{h_d^0}h_u^0+\tilde{a}P_R\psi_{h_d^0}h_u^{0*}\right)\\ &-\frac{c_H\mu}{v_{PQ}}\left(\bar{\tilde{a}}P_L\tilde{\chi}h_u^+-\bar{\tilde{a}}P_R\tilde{\chi}h_d^{-*}+\text{h.c.}\right),\end{aligned} \tag{108}$$

where

$$\tilde{\chi} = P_L\psi_{h_d^-}-P_R\psi_{h_u^+} \tag{109}$$

and $\tilde{a}$ and ψ_i for $i = h_u^0, h_d^0, h_u^+, h_d^-$ are all Majorana spinors.

Taking the vevs of h_u^0 and h_d^0, the neutralino sector become

$$\mathcal{L} = -\frac{c_H\mu}{v_{PQ}}\left(v_d\bar{\tilde{a}}\psi_{h_u^0}+v_u\bar{\tilde{a}}\psi_{h_d^0}\right). \tag{110}$$

Together with the MSSM neutralino mass matrix, we can write (5×5) mass matrix in the basis of $(\tilde{a}, \psi_{h_u^0}, \psi_{h_d^0}, \lambda_3, \lambda_0)$,

$$\mathcal{M}_{\text{neutral}} = \begin{pmatrix} m_{\tilde{a}} & c_H \mu v_d / v_{PQ} & c_H \mu v_u / v_{PQ} & 0 & 0 \\ & 0 & \mu & -g v_u/\sqrt{2} & g' v_u/\sqrt{2} \\ & & 0 & g v_d/\sqrt{2} & -g' v_d/\sqrt{2} \\ & & & M_2 & 0 \\ & & & & M_1 \end{pmatrix} \tag{111}$$

Similar to the MSSM neutralino mixing, we can write

$$\begin{pmatrix} \tilde{a} \\ \psi_{h_u^0} \\ \psi_{h_d^0} \\ \lambda_3 \\ \lambda_0 \end{pmatrix} = \begin{pmatrix} v_0^{(0)} & v_0^{(1)} & v_0^{(2)} & v_0^{(3)} & v_0^{(4)} \\ v_1^{(0)} & v_1^{(1)} & v_1^{(2)} & v_1^{(3)} & v_1^{(4)} \\ v_2^{(0)} & v_2^{(1)} & v_2^{(2)} & v_2^{(3)} & v_2^{(4)} \\ v_3^{(0)} & v_3^{(1)} & v_3^{(2)} & v_3^{(3)} & v_3^{(4)} \\ v_4^{(0)} & v_4^{(1)} & v_4^{(2)} & v_4^{(3)} & v_4^{(4)} \end{pmatrix} \begin{pmatrix} \widetilde{Z}'_0 \\ \widetilde{Z}'_1 \\ \widetilde{Z}'_2 \\ \widetilde{Z}'_3 \\ \widetilde{Z}'_4 \end{pmatrix} . \tag{112}$$

We define the mass eigenstates such that

$$\widetilde{Z}_i = (-i\gamma_5)^{\theta_i} \widetilde{Z}'_i \tag{113}$$

with θ_i equals 0(1) if the eigenvalue corresponding to $\widetilde{Z}'_i$ is positive(negative). In PQMSSM case, since v_{PQ} is much larger than EW scale, $\widetilde{Z}_0 = \widetilde{Z}'_0 \simeq \tilde{a}$.

A. axino-neutralino-neutral Higgs

Lagrangian (108) becomes

$$\begin{aligned} \mathcal{L} \simeq & -\frac{c_H \mu}{v_{PQ}} \sum_{i=1}^{4} \bar{\tilde{a}} (-i\gamma_5)^{\theta_{\tilde{a}}} \left\{ v_1^{(i)} \left(P_L h_d^0 + P_R h_d^{0*} \right) + v_2^{(i)} \left(P_L h_u^0 + P_R h_u^{0*} \right) \right\} (i\gamma_5)^{\theta_i} \widetilde{Z}_i \\ = & -\frac{c_H \mu}{\sqrt{2} v_{PQ}} \sum_{i=1}^{4} \overline{\widetilde{Z}}_i (-i\gamma_5)^{\theta_i + \theta_{\tilde{a}}} \tilde{a} \left(T^i_{\tilde{a}\tilde{Z}h} h + T^i_{\tilde{a}\tilde{Z}H} H \right) \\ & -\frac{c_H \mu}{\sqrt{2} v_{PQ}} \sum_{i=1}^{4} \overline{\widetilde{Z}}_i (-i\gamma_5)^{\theta_i + \theta_{\tilde{a}} + 1} \tilde{a} \left(T^i_{\tilde{a}\tilde{Z}G^0} G^0 + T^i_{\tilde{a}\tilde{Z}A} A \right) , \end{aligned} \tag{114}$$

where

$$T^i_{\tilde{a}\tilde{Z}h} = (-1)^{\theta_i+\theta_{\tilde{a}}} \left(v_1^{(i)} \sin\alpha + v_2^{(i)} \cos\alpha\right), \tag{115}$$

$$T^i_{\tilde{a}\tilde{Z}H} = (-1)^{\theta_i+\theta_{\tilde{a}}} \left(v_1^{(i)} \cos\alpha - v_2^{(i)} \sin\alpha\right), \tag{116}$$

$$T^i_{\tilde{a}\tilde{Z}G^0} = (-1)^{\theta_i+\theta_{\tilde{a}}+1} \left(v_1^{(i)} \cos\beta - v_2^{(i)} \sin\beta\right), \tag{117}$$

$$T^i_{\tilde{a}\tilde{Z}A} = (-1)^{\theta_i+\theta_{\tilde{a}}+1} \left(-v_1^{(i)} \sin\beta + v_2^{(i)} \cos\beta\right). \tag{118}$$

Note that we keep only $\mathcal{O}(1/v_{PQ})$ so that we impose $\widetilde{Z}_0 \simeq \tilde{a}$. Recalling the neutralino interaction of the MSSM, we can write

$$\begin{aligned} \mathcal{L} =& \sum_{i=1}^{4} \overline{\widetilde{Z}}_i(-i\gamma_5)^{\theta_i+\theta_{\tilde{a}}} \tilde{a} \left\{ \left(X^h_{i0} + X^h_{0i}\right) h + \left(X^H_{i0} + X^H_{0i}\right) H \right\} \\ &+ \sum_{i=1}^{4} \overline{\widetilde{Z}}_i(-i\gamma_5)^{\theta_i+\theta_{\tilde{a}}+1} \tilde{a} \left(X^A_{i0} + X^A_{0i}\right) A, \end{aligned} \tag{119}$$

where

$$X^A_{ij} = \frac{1}{2}(-1)^{\theta_i+\theta_j} \left(v_2^{(i)} \sin\beta - v_1^{(i)} \cos\beta\right) \left(g v_3^{(j)} - g' v_4^{(j)}\right). \tag{120}$$

with $\theta_0 = \theta_{\tilde{a}}$. Combining these, we obtain

$$\mathcal{L} = \sum_{i=1}^{4} \overline{\widetilde{Z}}_i(-i\gamma_5)^{\theta_i+\theta_{\tilde{a}}} \tilde{a} \left(\Lambda^i_{\tilde{a}\tilde{Z}h} h + \Lambda^i_{\tilde{a}\tilde{Z}H} H\right) + \sum_{i=1}^{4} \overline{\widetilde{Z}}_i(-i\gamma_5)^{\theta_i+\theta_{\tilde{a}}+1} \tilde{a} \Lambda^i_{\tilde{a}\tilde{Z}A} A, \tag{121}$$

where

$$\Lambda^i_{\tilde{a}\tilde{Z}h} = X^h_{i0} + X^h_{0i} - \frac{c_H \mu}{\sqrt{2} v_{PQ}} T^i_{\tilde{a}\tilde{Z}h}, \tag{122}$$

$$\Lambda^i_{\tilde{a}\tilde{Z}H} = X^H_{i0} + X^H_{0i} - \frac{c_H \mu}{\sqrt{2} v_{PQ}} T^i_{\tilde{a}\tilde{Z}H}, \tag{123}$$

$$\Lambda^i_{\tilde{a}\tilde{Z}A} = X^A_{i0} + X^A_{0i} - \frac{c_H \mu}{\sqrt{2} v_{PQ}} T^i_{\tilde{a}\tilde{Z}A}. \tag{124}$$

Partial decay width is given by

$$\begin{aligned} \Gamma(\tilde{a} \to \widetilde{Z}_i \phi) =& \frac{1}{16\pi} \left(\Lambda^i_{\tilde{a}\widetilde{Z}\phi}\right)^2 m_{\tilde{a}} \lambda^{1/2} \left(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}}, \frac{m^2_\phi}{m^2_{\tilde{a}}}\right) \\ &\times \left[\left(1 + \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{a}}} - \frac{m^2_\phi}{m^2_{\tilde{a}}}\right) + 2(-1)^{\theta_i} \left(1 - 2\delta_{A\phi}\right) \frac{m_{\widetilde{Z}_i}}{m_{\tilde{a}}} \right], \end{aligned} \tag{125}$$

for $\phi = h, H, A$.

B. axino-chargino-charged Higgs

For chargino sector, we can rewrite the Lagrangian (108),

$$\begin{aligned}\mathcal{L} = \frac{c_H\mu}{v_{PQ}}(i)^{\theta_{\tilde{a}}}\bar{\tilde{a}}\Big\{-P_L\cos\beta\left(-\theta_x\sin\gamma_L\widetilde{W}_2+\cos\gamma_L\widetilde{W}_1\right)\\ -P_R(-1)^{\theta_{\tilde{a}}}\sin\beta\left(-\theta_y\sin\gamma_R(-1)^{\theta_{\widetilde{W}_2}}\widetilde{W}_2+\cos\gamma_R(-1)^{\theta_{\widetilde{W}_1}}\widetilde{W}_1\right)\Big\}H^+ + \text{h.c.}\end{aligned} \tag{126}$$

Note that we neglect the goldstone sector. In addition, from the MSSM interaction, we obtain

$$\begin{aligned}\mathcal{L} =&\bar{\tilde{a}}(i)^{\theta_{\tilde{a}}}\Big\{-P_L(-1)^{\theta_{\tilde{a}}}\sin\beta\left(A_3^{(0)}\theta_x\widetilde{W}_2+A_4^{(0)}\widetilde{W}_1\right)\\ &+P_R\cos\beta\left(A_1^{(0)}\theta_y(-1)^{\theta_{\widetilde{W}_2}}\widetilde{W}_2+A_2^{(0)}(-1)^{\theta_{\widetilde{W}_1}}\widetilde{W}_1\right)\Big\}H^+ + \text{h.c.}\end{aligned} \tag{127}$$

where

$$A_1^{(0)} = -\frac{1}{\sqrt{2}}\left(gv_3^{(0)}+g'v_4^{(0)}\right)\sin\gamma_R - gv_1^{(0)}\cos\gamma_R, \tag{128}$$

$$A_2^{(0)} = \frac{1}{\sqrt{2}}\left(gv_3^{(0)}+g'v_4^{(0)}\right)\cos\gamma_R - gv_1^{(0)}\sin\gamma_R, \tag{129}$$

$$A_3^{(0)} = -\frac{1}{\sqrt{2}}\left(gv_3^{(0)}+g'v_4^{(0)}\right)\sin\gamma_L + gv_2^{(0)}\cos\gamma_L, \tag{130}$$

$$A_4^{(0)} = \frac{1}{\sqrt{2}}\left(gv_3^{(0)}+g'v_4^{(0)}\right)\cos\gamma_L + gv_2^{(0)}\sin\gamma_L. \tag{131}$$

Then we obtain

$$\begin{aligned}\mathcal{L} =&\bar{\tilde{a}}\Big\{-P_L(-1)^{\theta_{\tilde{a}}}\sin\beta\left(\Lambda_3^{(0)}\theta_x\widetilde{W}_2+\Lambda_4^{(0)}\widetilde{W}_1\right)\\ &+P_R\cos\beta\left(\Lambda_1^{(0)}\theta_y(-1)^{\theta_{\widetilde{W}_2}}\widetilde{W}_2+\Lambda_2^{(0)}(-1)^{\theta_{\widetilde{W}_1}}\widetilde{W}_1\right)\Big\}H^+ + \text{h.c.}\end{aligned} \tag{132}$$

where

$$\Lambda_1^{(0)} = A_1^{(0)} + \frac{c_H\mu}{v_{PQ}}(-1)^{\theta_{\tilde{a}}}\tan\beta\sin\gamma_R, \tag{133}$$

$$\Lambda_2^{(0)} = A_2^{(0)} - \frac{c_H\mu}{v_{PQ}}(-1)^{\theta_{\tilde{a}}}\tan\beta\cos\gamma_R, \tag{134}$$

$$\Lambda_3^{(0)} = A_3^{(0)} - \frac{c_H\mu}{v_{PQ}}(-1)^{\theta_{\tilde{a}}}\cot\beta\sin\gamma_L, \tag{135}$$

$$\Lambda_4^{(0)} = A_4^{(0)} + \frac{c_H\mu}{v_{PQ}}(-1)^{\theta_{\tilde{a}}}\cot\beta\cos\gamma_L. \tag{136}$$

Partial decay width is given by

$$\begin{aligned}\Gamma(\tilde{a}\to\widetilde{W}_i^-H^+)=&\frac{1}{16\pi}m_{\tilde{a}}\lambda^{1/2}\left(1,\frac{m_{\widetilde{W}_i}^2}{m_{\tilde{a}}^2},\frac{m_{H^+}^2}{m_{\tilde{a}}^2}\right)\\&\times\left[\left(a_i^2+b_i^2\right)\left(1+\frac{m_{\widetilde{W}_i}^2}{m_{\tilde{a}}^2}-\frac{m_{H^+}^2}{m_{\tilde{a}}^2}\right)+2\left(a_i^2-b_i^2\right)\frac{m_{\widetilde{W}_i}}{m_{\tilde{a}}}\right],\end{aligned}\tag{137}$$

where

$$a_1=\frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_1}}\cos\beta\Lambda_2^{(0)}-(-1)^{\theta_{\tilde{a}}}\sin\beta\Lambda_4^{(0)}\right\},\tag{138}$$

$$b_1=\frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_1}}\cos\beta\Lambda_2^{(0)}+(-1)^{\theta_{\tilde{a}}}\sin\beta\Lambda_4^{(0)}\right\},\tag{139}$$

$$a_2=\frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_2}}\theta_y\cos\beta\Lambda_1^{(0)}-(-1)^{\theta_{\tilde{a}}}\theta_x\sin\beta\Lambda_3^{(0)}\right\},\tag{140}$$

$$b_2=\frac{1}{2}\left\{(-1)^{\theta_{\widetilde{W}_2}}\theta_y\cos\beta\Lambda_1^{(0)}+(-1)^{\theta_{\tilde{a}}}\theta_x\sin\beta\Lambda_3^{(0)}\right\}.\tag{141}$$

Note that $\Gamma(\tilde{a}\to\widetilde{W}_i^-H^+)=\Gamma(\tilde{a}\to\widetilde{W}_i^+H^-)$.

C. axino-neutralino-Z boson

From the gauge interaction of Higgsinos, we obtain

$$\mathcal{L}=\sum_{i=1}^{4}W_{i0}\overline{\widetilde{Z}}_i\gamma_\mu(\gamma_5)^{\theta_i+\theta_{\tilde{a}}+1}\tilde{a}Z^\mu+\text{h.c.},\tag{142}$$

where

$$W_{i0}=\frac{1}{4}\sqrt{g^2+g'^2}(-i)^{\theta_i}(i)^{\theta_{\tilde{a}}}\left(v_1^{(i)}v_1^{(0)}-v_2^{(i)}v_2^{(0)}\right).\tag{143}$$

Partial decay width is given by

$$\begin{aligned}\Gamma(\tilde{a}\to\widetilde{Z}_iZ)=&\frac{1}{4\pi}|W_{i0}|^2m_{\tilde{a}}\lambda^{1/2}\left(1,\frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2},\frac{M_Z^2}{m_{\tilde{a}}^2}\right)\\&\times\left[\left(1+\frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2}-2\frac{M_Z^2}{m_{\tilde{a}}^2}\right)+\left(\frac{m_{\tilde{a}}^2}{M_Z^2}\right)\left(1-\frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{a}}^2}\right)^2+6(-1)^{\theta_i}\frac{m_{\widetilde{Z}_i}}{m_{\tilde{a}}}\right].\end{aligned}\tag{144}$$

D. axino-chargino-W boson

From the gauge interaction of Higgsinos, we obtain

$$\mathcal{L} = -g(-i)^{\theta_{\tilde{a}}} \sum_{i=1}^{2} \overline{\widetilde{W}}_i \left(X_i^0 + Y_i^0 \gamma_5\right) \gamma_\mu \tilde{a} W^\mu + \text{h.c.}, \tag{145}$$

where

$$X_1^0 = \frac{1}{2}\left[(-1)^{\theta_{\widetilde{W}_1}\theta_{\tilde{a}}} \left(\frac{\cos\gamma_R}{\sqrt{2}} v_1^{(0)} + \sin\gamma_R v_3^{(0)}\right) - \frac{\cos\gamma_L}{\sqrt{2}} v_2^{(0)} + \sin\gamma_L v_3^{(0)}\right], \tag{146}$$

$$X_2^0 = \frac{1}{2}\left[(-1)^{\theta_{\widetilde{W}_2}+\theta_{\tilde{a}}} \theta_y \left(\frac{-\sin\gamma_R}{\sqrt{2}} v_1^{(0)} + \cos\gamma_R v_3^{(0)}\right) + \theta_x \left(\frac{\sin\gamma_L}{\sqrt{2}} v_2^{(0)} + \cos\gamma_L v_3^{(0)}\right)\right], \tag{147}$$

$$Y_1^0 = \frac{1}{2}\left[-(-1)^{\theta_{\widetilde{W}_1}+\theta_{\tilde{a}}} \left(\frac{\cos\gamma_R}{\sqrt{2}} v_1^{(0)} + \sin\gamma_R v_3^{(0)}\right) - \frac{\cos\gamma_L}{\sqrt{2}} v_2^{(0)} + \sin\gamma_L v_3^{(0)}\right], \tag{148}$$

$$Y_2^0 = \frac{1}{2}\left[-(-1)^{\theta_{\widetilde{W}_2}+\theta_{\tilde{a}}} \theta_y \left(\frac{-\sin\gamma_R}{\sqrt{2}} v_1^{(0)} + \cos\gamma_R v_3^{(0)}\right) + \theta_x \left(\frac{\sin\gamma_L}{\sqrt{2}} v_2^{(0)} + \cos\gamma_L v_3^{(0)}\right)\right]. \tag{149}$$

Partial decay width is given by

$$\begin{aligned}\Gamma(\tilde{a} \to \widetilde{W}_i^- W^+) =& \frac{1}{16\pi} m_{\tilde{a}} \lambda^{1/2}\left(1, \frac{m_{\widetilde{W}_i}^2}{m_{\tilde{a}}^2}, \frac{M_W^2}{m_{\tilde{a}}^2}\right) \\ & \times \left[\left(X_i^{0\,2} + Y_i^{0\,2}\right)\left\{\left(1 + \frac{m_{\widetilde{W}_i}^2}{m_{\tilde{a}}^2} - 2\frac{M_W^2}{m_{\tilde{a}}^2}\right) + \left(\frac{m_{\tilde{a}}^2}{M_W^2}\right)\left(1 - \frac{m_{\widetilde{W}_i}^2}{m_{\tilde{a}}^2}\right)^2\right\}\right. \\ & \left. - 6\left(X_i^{0\,2} - Y_i^{0\,2}\right)\frac{m_{\widetilde{W}_i}}{m_{\tilde{a}}}\right].\end{aligned} \tag{150}$$

Note that $\Gamma(\tilde{a} \to \widetilde{W}_i^- W^+) = \Gamma(\tilde{a} \to \widetilde{W}_i^+ W^-)$.

E. axino-sfermion-fermion

Following the neutralino decays into sfermion and fermion in Ref. [1], axino decays into sfermions and fermions are given by

$$\begin{aligned}\Gamma(\tilde{a} \to f\tilde{f}_k) =& \frac{N_c m_{\tilde{a}}}{16\pi} \lambda^{1/2}\left(1, \frac{m_{\tilde{f}_k}^2}{m_{\tilde{a}}^2}, \frac{m_f^2}{m_{\tilde{a}}^2}\right) \\ & \times \left[|a_f^k|^2 \left\{\left(1 + \frac{m_f}{m_{\tilde{a}}}\right)^2 - \frac{m_{\tilde{f}_k}^2}{m_{\tilde{a}}^2}\right\} + |b_f^k|^2 \left\{\left(1 - \frac{m_f}{m_{\tilde{a}}}\right)^2 - \frac{m_{\tilde{f}_k}^2}{m_{\tilde{a}}^2}\right\}\right],\end{aligned} \tag{151}$$

where

$$a_u^1 = \frac{1}{2}\left[\left\{iA_{\tilde a}^u - (i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\cos\theta_u - \left\{iB_{\tilde a}^u - (-i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\sin\theta_u\right], \tag{152}$$

$$b_u^1 = \frac{1}{2}\left[\left\{-iA_{\tilde a}^u - (i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\cos\theta_u - \left\{iB_{\tilde a}^u + (-i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\sin\theta_u\right], \tag{153}$$

$$a_d^1 = \frac{1}{2}\left[\left\{iA_{\tilde a}^d - (i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\cos\theta_d - \left\{iB_{\tilde a}^d - (-i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\sin\theta_d\right], \tag{154}$$

$$b_d^1 = \frac{1}{2}\left[\left\{-iA_{\tilde a}^d - (i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\cos\theta_d - \left\{iB_{\tilde a}^d + (-i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\sin\theta_d\right], \tag{155}$$

$$a_u^2 = \frac{1}{2}\left[\left\{iA_{\tilde a}^u - (i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\sin\theta_u + \left\{iB_{\tilde a}^u - (-i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\cos\theta_u\right], \tag{156}$$

$$b_u^2 = \frac{1}{2}\left[\left\{-iA_{\tilde a}^u - (i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\sin\theta_u + \left\{iB_{\tilde a}^u + (-i)^{\theta_{\tilde a}} f_u v_1^{(0)}\right\}\cos\theta_u\right], \tag{157}$$

$$a_d^2 = \frac{1}{2}\left[\left\{iA_{\tilde a}^d - (i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\sin\theta_d + \left\{iB_{\tilde a}^d - (-i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\cos\theta_d\right], \tag{158}$$

$$b_d^2 = \frac{1}{2}\left[\left\{-iA_{\tilde a}^d - (i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\sin\theta_d + \left\{iB_{\tilde a}^d + (-i)^{\theta_{\tilde a}} f_d v_2^{(0)}\right\}\cos\theta_d\right], \tag{159}$$

with

$$A_{\tilde a}^u = \frac{(-i)^{\theta_{\tilde a}-1}}{\sqrt{2}}\left[g v_3^{(0)} + \frac{g'}{3} v_4^{(0)}\right], \tag{160}$$

$$A_{\tilde a}^d = \frac{(-i)^{\theta_{\tilde a}-1}}{\sqrt{2}}\left[-g v_3^{(0)} + \frac{g'}{3} v_4^{(0)}\right], \tag{161}$$

$$B_{\tilde a}^u = \frac{4}{3\sqrt{2}} g'(i)^{\theta_{\tilde a}-1} v_4^{(0)}, \tag{162}$$

$$B_{\tilde a}^d = -\frac{2}{3\sqrt{2}} g'(i)^{\theta_{\tilde a}-1} v_4^{(0)}. \tag{163}$$

IV. APPROXIMATE FORMULAE FOR AXINO AND SAXION DECAY

A. axino-neutralino mixing matrix

To make senses for the decay temperatures of axino and saxion, we need to consider crude expressions for axino and saxion decays. First of all, let us consider the mixing matrix of axino-neutralino. Full mass matrix of axino-neutralino is given in (111),

$$\mathcal{M}_{\text{neutral}} = \begin{pmatrix} m_{\tilde{a}} & c_H \mu v_d / v_{PQ} & c_H \mu v_u / v_{PQ} & 0 & 0 \\ & 0 & \mu & -g v_u/\sqrt{2} & g' v_u/\sqrt{2} \\ & & 0 & g v_d/\sqrt{2} & -g' v_d/\sqrt{2} \\ & & & M_2 & 0 \\ & & & & M_1 \end{pmatrix}. \tag{164}$$

To diagonalize the mass matrix, we first consider the rotation of higgsino sector by orthogonal matrix,

$$N_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{165}$$

The mass matrix becomes

$$\begin{aligned} \mathcal{M}'_{\text{neutral}} =& N_1 \mathcal{M}_{\text{neutral}} N_1^T \\ =& \begin{pmatrix} m_{\tilde{a}} & \lambda(v_d - v_u)/\sqrt{2} & \lambda(v_d + v_u)/\sqrt{2} & 0 & 0 \\ & -\mu & 0 & -g(v_d+v_u)/2 & g'(v_d+v_u)/2 \\ & & \mu & g(v_d - v_u)/2 & -g'(v_d - v_u)/2 \\ & & & M_2 & 0 \\ & & & & M_1 \end{pmatrix}, \end{aligned} \tag{166}$$

where $\lambda = c_H \mu / v_{PQ}$ as before. This rotated matrix can be perturbatively diagonalized and the mixin matrix is given by

$$(N_2)_{nm} \simeq \frac{(\mathcal{M}'_{\text{neutral}})_{mn}}{(\mathcal{M}'_{\text{neutral}})_{nn} - (\mathcal{M}'_{\text{neutral}})_{mm}}. \tag{167}$$

At the first order approximation, we obtain

$$(N_2)_{01} = \frac{c_H \mu v(c_\beta - s_\beta)}{\sqrt{2} v_{PQ}(m_{\tilde{a}} + \mu)}, \tag{168}$$

$$(N_2)_{02} = \frac{c_H \mu v(c_\beta + s_\beta)}{\sqrt{2} v_{PQ}(m_{\tilde{a}} - \mu)}, \tag{169}$$

$$(N_2)_{13} = \frac{g v(c_\beta + s_\beta)}{2(M_2 + \mu)}, \tag{170}$$

$$(N_2)_{14} = -\frac{g' v(c_\beta + s_\beta)}{2(M_1 + \mu)}, \tag{171}$$

$$(N_2)_{23} = -\frac{g v(c_\beta - s_\beta)}{2(M_2 - \mu)}, \tag{172}$$

$$(N_2)_{24} = \frac{g v(c_\beta - s_\beta)}{2(M_1 - \mu)}, \tag{173}$$

$$(N_2)_{00} = (N_2)_{11} = (N_2)_{22} = (N_2)_{33} = (N_2)_{44} = 1, \tag{174}$$

$$(N_2)_{03} = (N_2)_{04} = (N_2)_{12} = (N_2)_{34} = 0, \tag{175}$$

$$(N_2)_{ij} = -(N_2)_{ji} \qquad \text{for} \qquad i \neq j. \tag{176}$$

Overall mixing matrix N is $N_2 N_1$. The matrix components are

$$N_{00} = 1, \tag{177}$$

$$N_{01} = \frac{c_H \mu v(m_{\tilde{a}} c_\beta + \mu s_\beta)}{v_{PQ}(m_{\tilde{a}}^2 - \mu^2)}, \tag{178}$$

$$N_{02} = \frac{c_H \mu v(m_{\tilde{a}} s_\beta + \mu c_\beta)}{v_{PQ}(m_{\tilde{a}}^2 - \mu^2)}, \tag{179}$$

$$N_{03} = 0, \tag{180}$$

$$N_{04} = 0, \tag{181}$$

$$N_{10} = -\frac{c_H \mu v(c_\beta - s_\beta)}{\sqrt{2} v_{PQ}(m_{\tilde{a}} + \mu)}, \tag{182}$$

$$N_{11} = \frac{1}{\sqrt{2}}, \tag{183}$$

$$N_{12} = -\frac{1}{\sqrt{2}}, \tag{184}$$

$$N_{13} = \frac{g v(c_\beta + s_\beta)}{2(M_2 + \mu)}, \tag{185}$$

$$N_{14} = -\frac{g' v(c_\beta + s_\beta)}{2(M_1 + \mu)}, \tag{186}$$

$$N_{20} = -\frac{c_H \mu v (c_\beta + s_\beta)}{\sqrt{2} v_{PQ} (m_{\tilde{a}} - \mu)}, \tag{187}$$

$$N_{21} = \frac{1}{\sqrt{2}}, \tag{188}$$

$$N_{22} = \frac{1}{\sqrt{2}}, \tag{189}$$

$$N_{23} = -\frac{g v (c_\beta - s_\beta)}{2(M_2 - \mu)}, \tag{190}$$

$$N_{24} = \frac{g v (c_\beta - s_\beta)}{2(M_1 - \mu)}, \tag{191}$$

$$N_{30} = 0, \tag{192}$$

$$N_{31} = \frac{g v (\mu c_\beta - M_2 s_\beta)}{\sqrt{2}(M_2^2 - \mu^2)}, \tag{193}$$

$$N_{32} = -\frac{g v (\mu s_\beta - M_2 c_\beta)}{\sqrt{2}(M_2^2 - \mu^2)}, \tag{194}$$

$$N_{33} = 1, \tag{195}$$

$$N_{34} = 0, \tag{196}$$

$$N_{40} = 0, \tag{197}$$

$$N_{41} = -\frac{g' v (\mu c_\beta - M_2 s_\beta)}{\sqrt{2}(M_1^2 - \mu^2)}, \tag{198}$$

$$N_{42} = \frac{g' v (\mu s_\beta - M_2 c_\beta)}{\sqrt{2}(M_1^2 - \mu^2)}, \tag{199}$$

$$N_{43} = 0, \tag{200}$$

$$N_{44} = 1. \tag{201}$$

Note that this mixing matrix do not arrange eigen states according to the size of eigenvalues.

If the lightest neutralino is Higgsino-like, we can determine which eigenvalue is the smallest. At the leading order,

$$\left(\mathcal{M}_{\text{neutral}}^{diag}\right)_{11} = -\mu - \frac{(v_d + v_u)^2}{4}\left(\frac{g^2}{M_2 + \mu} + \frac{g'^2}{M_1 + \mu}\right), \tag{202}$$

$$\left(\mathcal{M}_{\text{neutral}}^{diag}\right)_{22} = \mu - \frac{(v_d - v_u)^2}{4}\left(\frac{g^2}{M_2 - \mu} + \frac{g'^2}{M_1 - \mu}\right). \tag{203}$$

The absolute value of (22) component is smaller than (11) component so the lightest neutralino is (22)-like and the second lightest is (11)-like. In addition, if we assume the gaugino mass unification at the GUT scale, $M_2 \simeq 2M_1$, so the 3rd lightest is Bino-like and the 4th lightest is Wino-like.

Note that we consider 0th mode is axino-like independently of its mass. From these argument, we obtain the real mixing matrix that makes neutralino states in increasing order of mass, by replacing

$$N_{1i} \to N_{2i}, \qquad N_{2i} \to N_{1i}, \qquad N_{3i} \to N_{4i}, \qquad N_{4i} \to N_{3i} \qquad \text{for} \qquad i = 0, \cdots, 4. \tag{204}$$

From this, obtain $v_i^{(j)}$'s,

$$v_0^{(0)} = 1, \tag{205}$$
$$v_1^{(0)} = \frac{c_H \mu v (m_{\tilde{a}} c_\beta + \mu s_\beta)}{v_{PQ}(m_{\tilde{a}}^2 - \mu^2)}, \tag{206}$$
$$v_2^{(0)} = \frac{c_H \mu v (m_{\tilde{a}} s_\beta + \mu c_\beta)}{v_{PQ}(m_{\tilde{a}}^2 - \mu^2)}, \tag{207}$$
$$v_3^{(0)} = 0, \tag{208}$$
$$v_4^{(0)} = 0, \tag{209}$$

$$v_0^{(1)} = -\frac{c_H \mu v (c_\beta + s_\beta)}{\sqrt{2} v_{PQ}(m_{\tilde{a}} - \mu)}, \tag{210}$$
$$v_1^{(1)} = \frac{1}{\sqrt{2}}, \tag{211}$$
$$v_2^{(1)} = \frac{1}{\sqrt{2}}, \tag{212}$$
$$v_3^{(1)} = -\frac{g v (c_\beta - s_\beta)}{2(M_2 - \mu)}, \tag{213}$$
$$v_4^{(1)} = \frac{g v (c_\beta - s_\beta)}{2(M_1 - \mu)}, \tag{214}$$

$$v_0^{(2)} = -\frac{c_H \mu v (c_\beta - s_\beta)}{\sqrt{2} v_{PQ}(m_{\tilde{a}} + \mu)}, \tag{215}$$
$$v_1^{(2)} = \frac{1}{\sqrt{2}}, \tag{216}$$
$$v_2^{(2)} = -\frac{1}{\sqrt{2}}, \tag{217}$$
$$v_3^{(2)} = \frac{g v (c_\beta + s_\beta)}{2(M_2 + \mu)}, \tag{218}$$
$$v_4^{(2)} = -\frac{g' v (c_\beta + s_\beta)}{2(M_1 + \mu)}, \tag{219}$$

$$v_0^{(3)} = 0, \tag{220}$$
$$v_1^{(3)} = -\frac{g' v(\mu c_\beta - M_2 s_\beta)}{\sqrt{2}(M_1^2 - \mu^2)}, \tag{221}$$
$$v_2^{(3)} = \frac{g' v(\mu s_\beta - M_2 c_\beta)}{\sqrt{2}(M_1^2 - \mu^2)}, \tag{222}$$
$$v_3^{(3)} = 0, \tag{223}$$
$$v_4^{(3)} = 1. \tag{224}$$

$$v_0^{(4)} = 0, \tag{225}$$
$$v_1^{(4)} = \frac{g v(\mu c_\beta - M_2 s_\beta)}{\sqrt{2}(M_2^2 - \mu^2)}, \tag{226}$$
$$v_2^{(4)} = -\frac{g v(\mu s_\beta - M_2 c_\beta)}{\sqrt{2}(M_2^2 - \mu^2)}, \tag{227}$$
$$v_3^{(4)} = 1, \tag{228}$$
$$v_4^{(4)} = 0. \tag{229}$$

B. dominant saxion decays to standard model particles

We will obtain simple approximation of saxion decay in this subsection. For the practical point of view, we consider Higgsino-like neutralino dark matter case in RNS2 benchmark point (BM2). If saxion is much heavier than doublet Higgs masses, i.e. $m_s^2 \gg m_h^2, m_H^2$, we can guess saxion-higgs mixing is very small, as shown in Eqs. (35) and (36). Therefore, the dominant decays come from the DFSZ interaction (1). Those are decays into decays into gauge/higgs bosons and neutralino/chargino.

In our parameter space of BM2 and very heavy saxion, λ_{shH} is the largest coupling of cubic scalar interactions, Eqs. (48)-(52). The reason is that it comes form DFSZ coupling is sH_uH_d and $H_u \approx h$ and $H_d \approx H$. The partial decay width for $s \to hH$ is approximately obtained from Eq. (53), and is given by

$$\Gamma^s_{mboxhiggs} \approx \Gamma(s \to hH) \approx \frac{1}{16\pi m_s} \frac{c_H^2 m_A^4}{8 v_{PQ}^2} \sin^2 2\beta = \frac{c_H^2}{128\pi} \left(\frac{m_A^2 \sin 2\beta}{v_{PQ}} \right)^2 \frac{1}{m_s}. \tag{230}$$

For the gauge boson final states (WW and ZZ) and the gauge/Higgs final states (ZA and $W^\pm H^\mp$), we can apply similar arguments. The longitudinal mode of gauge bosons are mostly charged component of H_u for large $\tan\beta$, so gauge/Higgs decay modes are much larger than gauge boson decay modes. It is very easily seen in interaction Lagrangian (42). Comparing the terms of GG, AA or H^+H^- and GA or G^+H^-, we know that saxion-Goldstone-Higgs interactions are relatively larger by factor $\cot 2\beta$. In order to obtain partial decay width, we know that saxion-Higgs mixing, Eqs. (35) and (36) since saxion in SM singlet. In the decoupling limit of Higgs sector, we know that the tree-level relation,

$$\frac{\sin 2\alpha}{\sin 2\beta} = \frac{m_H^2 + m_h^2}{m_H^2 - m_h^2} \approx 1 + \frac{2m_h^2}{m_H^2}. \tag{231}$$

For large $\tan\beta$, $\alpha + \beta \simeq \pi/2$, so we obtain

$$\sin\alpha \simeq \left(1 + \frac{2m_h^2}{m_H^2}\right)\cos\beta. \tag{232}$$

From the above relation and the fact that $m_A^2 \simeq m_H^2$,

$$\epsilon_h \approx -\left(\frac{\tan\rho}{m_s^2}\right)\left(-2m_A^2\cos\beta + \frac{2\mu^2}{\cos\beta}\right), \tag{233}$$

$$\epsilon_H \approx \left(\frac{\tan\rho}{m_s^2}\right) m_A^2, \tag{234}$$

where $\tan\rho = \lambda v \sin 2\beta/2\mu$ and $\lambda = c_H\mu/v_{PQ}$. For our purpose, we assume that $m_A^2\cos^2\beta < \mu^2$. Actually, it is not plausible assumption for BM2 benchmark ($m_A^2\cos^2\beta \sim 0.6\mu^2$). However, we might consider larger mu to avoid direct/indirect detection bounds of neutralino. We approximate once more,

$$\epsilon_h \approx -\left(\frac{\tan\rho}{m_s^2}\right)\left(\frac{2\mu^2}{\cos\beta}\right). \tag{235}$$

Then we obtain the partial decay width from Eq. (60),

$$\begin{aligned}\Gamma(s\to V\phi) \approx& \frac{g_V^2 g_{sV\phi}^2}{32\pi}\frac{m_s^2}{M_V^2}\left(1-\frac{1}{2}\delta_{VZ}\right)\\ =&\frac{g_V^2}{32\pi}\frac{m_s^2}{M_V^2}\left(\frac{c_H v}{v_{PQ}}\right)^2\left(\frac{m_A^2}{m_s^2}\right)^2\cos^2\beta\left(1-\frac{1}{2}\delta_{VZ}\right)\\ =&\frac{c_H^2}{16\pi}\frac{m_A^4\cos^2\beta}{v_{PQ}^2}\frac{1}{m_s}\left(1-\frac{1}{2}\delta_{VZ}\right).\end{aligned} \tag{236}$$

Including $s \to ZA$ and $s \to W^\pm H^\mp$,

$$\Gamma^s_{\text{gauge/higgs}} \approx \frac{3c_H^2}{32\pi}\frac{m_A^4\cos^2\beta}{v_{PQ}^2}\frac{1}{m_s}. \tag{237}$$

Observing Eqs. (230) and (237), we can see that the sum of them are exactly decay width of singlet scalar into two doublet scalar, whose coupling constant is $c_H B\mu/v_{PQ}$.

C. dominant saxion decays to superparticles

For the same reason discussed in the previous subsection, dominant final states are higgsino-like neutralinos and charginos. They are simply obtained from Eqs. (88) and (90)

$$\Gamma_{\text{neutralino}} \approx \frac{c_H^2}{64\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_s, \tag{238}$$

$$\Gamma_{\text{chargino}} \approx \frac{c_H^2}{64\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_s. \tag{239}$$

The above two are the same since these are decay widths of singlet scalar into doublet Dirac fermions (the one is isospin-up part and the other is isospin-down part).

D. dominant axino decays

As the saxion decay, if we consider very heavy axino, we can neglect axino-neutralino mixing. In such case, dominant axino decays come from DFSZ interaction (1).

Firstly we consider $\tilde{a} \to \widetilde{Z}_i\phi$ decay. It is obtained from the Eq. (125). Since axino-neutralino mixing is negligible, $X_{i0}^{\phi} + X_{0i}$ is very small. Then we simply obtain the decay widths only using $T^i_{\tilde{a}\widetilde{Z}\phi}$. In addition, $\widetilde{Z}_{1,2}$ are higgsino-like, so it is enough to include decays into these states.

$$\sum_{i=1,2}\sum_{\phi=h,H,A}\Gamma(\tilde{a} \to \widetilde{Z}_i\phi) \approx 2\times 3\times\frac{c_H^2}{64\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}}. \tag{240}$$

For the similar reasons, $\tilde{a} \to \widetilde{Z}_i Z$ is also simplified. It is obtained from Eq. (144). Using the neutralino mixing obtained in the previous subsection, we can simply obtain

$$\sum_{i=1,2}\Gamma(\tilde{a} \to \tilde{Z}_i Z) \approx 2\times\frac{c_H^2}{64\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}}. \tag{241}$$

Including these components, partial decay width of axino into neutralinos is

$$\Gamma^{\tilde{a}}_{\text{neutralino}} = \frac{c_H^2}{8\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}}. \tag{242}$$

In the same way, we can also obtain axino decays into charginos from Eqs. (137) and (150),

$$\begin{aligned}\Gamma^{\tilde{a}}_{\text{chargino}} \approx&\Gamma(\tilde{a} \to \widetilde{W}_1^{\pm} H^{\mp}) + \Gamma(\tilde{a} \to \widetilde{W}_1^{\pm} W^{\mp}) \\ \approx&\frac{c_H^2}{16\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}} + \frac{c_H^2}{16\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}} \\ =&\frac{c_H^2}{8\pi}\left(\frac{\mu}{v_{PQ}}\right)^2 m_{\tilde{a}}.\end{aligned} \tag{243}$$

These results are reasonable since we assume very large axino and higgsino-like neutralino and chargino. Therefore the results should be approximately same as those only from interaction (1) without any EWSB effects.

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