Lecture 11:
Nucleosynthesis and thefirst three minutes: Part 1

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1 The temperature of Deuterium synthesis

- The nucleosynthetic form of the Saha equation enables us to determine the ratio of the number of Deuterium nuclei to the number of neutrons as a function of teperature:

$$\frac{n_D}{n_n} = 6n_p \left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right) \tag{1}$$

- We need, however, an expression for n_p - the number densiy of protons.

- As we've seen, after freeze-out, there is 1 neutron for every 5 protons. The total number of protons per Baryon is therefore:

$$\frac{n_p}{n_{\rm Bary}} = \frac{n_p}{n_n + n_p} = \frac{5}{1+5} = \frac{5}{6} = 0.8333 \approx 0.8 \tag{2}$$

- As with recombination, we can use the fact that the number density of Baryons to photons in the Universe is a constant $\eta = n_{\rm Bary}/n_\gamma = 6.1 \times 10^{-10}$. We also know that the number density of blackbody photons is related to Temperature as:

$$n_\gamma = 0.2436 \left(\frac{kT}{\hbar c}\right)^3 \tag{3}$$

- Meaning:

$$n_p = 0.8n_{\rm Bary} = 0.8\eta n_\gamma = 0.8\eta\left[0.2436\left(\frac{kT}{\hbar c}\right)^3\right] \tag{4}$$

- Substituting this expression for n_p into Eq. 1 gives our expression for n_D/n_n that is dependent only on T:

$$\frac{n_D}{n_n} = 6.5\eta \left(\frac{kT}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT}\right) \tag{5}$$

2 From the neutron:proton ratio to the relative mass in He

- Since we know that every He nuclei contains two neutrons and two protons, we can convert the neutron to proton ratio to the relative mass of Baryons in the form of He post-nucleosynthesis:
- Each Helium nuclei contains two neutrons, so we can say:

$$n_{\mathrm{He}} = n_n/2 \tag{6}$$

and each Helium nuclei "soaks up" two protons that can't then be used as Hydrogen nuclei, meaning:

$$n_{\mathrm{H}} = n_p - 2n_{\mathrm{He}} = n_p - n_n \tag{7}$$

- Converting these numbers into mass (and ignoring the slight difference in mass between neutrons and protons), gives:

$$m_{\mathrm{He}} = 4m_p n_{\mathrm{He}} = 2m_p n_n \tag{8}$$

and

$$m_{\mathrm{H}} = m_p(n_p - n_n) \tag{9}$$

and the total Baryon mass as:

$$m_{\mathrm{Bary}} = m_{\mathrm{H}} + m_{\mathrm{He}} = m_p(n_p - n_n + 2n_n) \tag{10}$$

Meaning the total mass of Helium as a proportion of the mass of Baryons is:

$$\frac{m_{\mathrm{He}}}{m_{\mathrm{Bary}}} = \frac{2n_n}{n_p - n_n + 2n_n} = \frac{2n_n}{n_p + n_n} \tag{11}$$

If we now define $r_{np} = n_n/n_p$ (i.e., the ratio of the number of neutrons to protons), we can substitute $n_n = r_{np}n_p$ into Eq. 11 to get:

$$\frac{m_{\mathrm{He}}}{m_{\mathrm{Bary}}} = \frac{2r_{np}n_p}{n_p + r_{np}n_p} = \frac{2r_{np}}{1 + r_{np}} \tag{12}$$