The University Of Sheffield.

Data Provided:
A formula sheet and table of physical constants is attached to this paper.

Ancillary Material:
Cosmological formulae and constants are provided on the first page of the question paper.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring 2020

Introduction to Cosmology
Class Test 1 — 50 minutes

Instructions:
Answer ALL questions

All questions are marked out of five.

Please clearly indicate the question numbers on which you would like to be examined on the front cover of your answer book. Cross through any work that you do not wish to be examined.

Cosmological formulae and constants

The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}.$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0.$$

Cosmological parameter values in The Benchmark Model of our own Universe:

$\Omega_{M,0} = 0.31;\ \Omega_{D,0} = 0.69;\ \Omega_{R,0} = 9 \times 10^{-5};\ H_0 = 67.74 \pm 0.46$ km s^{-1} Mpc$^{-1}, \kappa = 0.$

Parsec in SI units: 1 pc $= 3.09 \times 10^{16}$ m

Answer ALL questions

1. For the case of The Benchmark Model, calculate the:

(a) current critical density in units of MeV m^{-3}; **[1]**

(b) redshift at which the energy densities of dark energy and matter were equal; **[1]**

(c) energy density of matter at the time of dark energy-matter equality, in MeV m^{-3}; **[1]**

(d) redshift at which the energy densities of matter and radiation were equal; **[1]**

(e) energy density of matter at the time of matter-radiation equality, in MeV m^{-3}. **[1]**

2. Around ten billion years ago the scale factor, a, of our Universe was a third of today's value. Calculate the value of the Hubble parameter at that time, giving your answer in km s^{-1} Mpc^{-1}. **[5]**

3. To a good approximation, the scale factor of a strongly matter-dominated universe is given by

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}. \quad (1)$$

If an observer in such a universe measures a Hubble constant of $H_0 = 67.74$ km s^{-1} Mpc^{-1}, what is the age of that universe when they make that measurement? Please give your answer in years. **[5]**

4. In a flat, dark energy-dominated universe, show that the measured flux of a constant-luminosity source (i.e., a standard candle) changes with redshift as

$$F \propto (z + z^2)^{\beta}, \quad (2)$$

and provide the value of β. **[5]**

END OF EXAMINATION PAPER