Problems Class III

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Equations and constants
The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: $1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$

Prologue
This was Problems Class 3 in 2019. I've since changed the Problems Class 3 to cover more topics, but the exercises in this homework are still useful to get you more familiar with manipulating the Friedmann Equation and deriving its alternative form. I therefore recommend you attempt these questions as an unassessed homework. Answers can be found in the same folder.

Questions

1. Derive the expression for ϵ_{m}, the evolving matter energy density, in terms of scale factor, $a(t)$, the current critical energy density, $\epsilon_{\text{c},0}$, and the current matter parameter, $\Omega_{\text{m},0}$.
2. Write the expressions for the evolving radiation energy and dark energy densities, ϵ_{p} and ϵ_{D}, in the same respective terms.
3. Derive the expression for $\frac{-\kappa c^2}{R_0^2}$ in terms of the Hubble constant, H_0, and the current energy parameter, Ω_0.
4. Putting the answers from all the above questions together, obtain the expression for $\dot{a}$ in terms of the Hubble constant, H_0, the various energy parameters, $\Omega_{\text{i},0}$, and the scale factor, $a(t)$.
5. And if we have time: by differentiating the answer you obtained in question 4 with respect to time, obtain the expresson for the current accelation of the Universe. Is the Universe currently accelerating, decelerating, or neither?