# Problems Class II

Dr. James Mullaney

February 13, 2019

**Equations and constants**
The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: 1 pc $= 3.09 \times 10^{16}$ m

**Prologue**
In this Problems Class, we will go through the process of calculating the proper distance, $d_p$, to a galaxy given its redshift, $z$. For now, we will assume a time-invariant Hubble parameter. **While this assumption is incorrect for our Universe**, the take-away message is the same: all you need to calculate proper distance is an expression for the (time-dependent) scale factor (i.e., $a(t)$), and a redshift.

**Questions**

1. Assuming a time-invariant Hubble parameter (i.e., $H(t) =$ constant), derive the expression for the time-varying scale-parameter (i.e., $a(t)$) in terms of $H$, $t$, and $t_0$ (where $t_0$ represents the present).

2. Using your answer to Question 1, and the formula that relates redshift, $z$, to the scale factor at the time of emission, $a(t_{\text{em}})$ (see L2), obtain an expression that relates $t_{\text{em}}$ to redshift.

3. Using your answers to questions 1 and 2, obtain the expression for the proper distance, $d_p$, to a galaxy, given its redshift, $z$.

4. Assuming $H(t) = H_0$, calculate the proper distance, in Mpc, to a galaxy observed at redshift $z = 1$.

5. [And if we have time]: When we observe light from a $z = 1$ galaxy today, how many years ago was that light emitted (again, assuming $H(t) = H_0$)?