Problems Class III

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Equations and constants
The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: $1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$

Prologue
The idea behind this problems class is to get you more familiar with working with the Friedmann Equation, and deriving its different (but equivalent) forms.

Questions

1. Derive the expression for ϵ_{m}, the evolving matter energy density, in terms of scale factor, $a(t)$, the current critical energy density, $\epsilon_{\text{c},0}$, and the current matter parameter, $\Omega_{\text{m},0}$.

2. Write the expressions for the evolving radiation energy and dark energy densities, ϵ_{p} and ϵ_{D}, in the same respective terms.

3. Derive the expression for $\frac{-\kappa c^2}{R_0^2}$ in terms of the Hubble constant, H_0, and the current energy parameter, Ω_0.

4. Putting the answers from all the above questions together, obtain the expression for $\dot{a}$ in terms of the Hubble constant, H_0, the various energy parameters, $\Omega_{\text{i},0}$, and the scale factor, $a(t)$.

5. And if we have time: by differentiating the answer you obtained in question 4 with respect to time, obtain the expresson for the current accelation of the Universe. Is the Universe currently accelerating, decelerating, or neither?