Problems Class VI

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April 4, 2019

Equations and constants
The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: 1 pc $= 3.09 \times 10^{16}$ m

1. What was the age of a $H_0 = 67.74$ km s^{-1} Mpc^{-1}, matter-dominated universe at redshift $z = 1$?

2. What was the horizon distance, in Mpc, of that same universe at redshift $z = 1$?

3. So far in the course, we've only considered flat universes. However, it's not much of a challenge to extend our models to include curvature. To demonstrate this, show that the Hubble parameter of a curved, radiation-dominated universe can be described as:

$$\frac{H^2}{H_0^2} = \frac{\Omega_0}{a^x} + \frac{1 - \Omega_0}{a^y} \tag{1}$$

giving the values for x and y.

4. What will be the maximum scale factor of such a (radiation-dominated, curved) universe if the radiation energy density is half that of the critical density?