Lecture 15: Structure Formation II Statistical Properties, Hot and Cold Dark Matter, and BAOs

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1 Matter fluctuations

- If you consider lots of large spheres within the Universe, because of the density fluctations, they'd all contain very slightly different masses.
- The average mass, however, would be:
$$\langle M \rangle = \frac{4\pi}{3} r^3 \rho_0 = 1.67 \times 10^{11}\ \mathrm{M}_\odot \left(\frac{r}{1\ \mathrm{Mpc}}\right)^3 \tag{1}$$
- but their actual masses would vary slightly from this mean by:
$$\left\langle \left(\frac{M - \langle M \rangle}{\langle M \rangle}\right)^2 \right\rangle = \frac{V}{2\pi^2} \int P(k) \left[\frac{3 j_1(kr)}{kr}\right]^2 k^2 dk \tag{2}$$
where $j_1(x) = (\sin x - x\cos x)/x^2$ is the spherical Bessel function.
- If we then assume $P(k) \propto k^n$, and $u = kr$, this reduces to:
$$\left\langle \left(\frac{M - \langle M \rangle}{\langle M \rangle}\right)^2 \right\rangle = \frac{9V}{2\pi^2} r^{-3-n} \int_0^\infty u^n j_1(u)^2 du \tag{3}$$
- And since any definite integral just becomes a constant, we can say that:
$$\frac{\delta M}{M} = \left\langle \left(\frac{M - \langle M \rangle}{\langle M \rangle}\right)^2 \right\rangle^{1/2} \propto r^{-(3+n)/2} \tag{4}$$
- And since $M \propto r^{1/3}$, we can also say:
$$\frac{\delta M}{M} \propto M^{-(3+n)/6} \tag{5}$$
- If you scattered galaxies throughout the universe entirely at random, then you'd get a Poissonian mass distribution, whereby $\delta M/M \propto N^{-1/2}$, where N is the *expected* number of galaxies per sphere.
- And since $M \propto N$, for a random distribution of galaxies, $\delta M/M \propto M^{-1/2}$, and $n = 0$.
- Therefore, an $n = 1$ will produce $\delta M/M \propto M^{-4/6}$, and thus will lead to more power in small mass scales than a random distribution.

2 Hot vs. Cold Dark Matter

- Depending on the mass of Dark Matter particles, then they will become non-relativistic at different temperatures:
$$T_h \approx \frac{m_h c^2}{3k} \approx 12000 \text{ K} \left(\frac{m_h c^2}{3 \text{ eV}}\right) \tag{6}$$
corresponding to different times:
$$t_h \approx 42000 \text{ yr} \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-2} \tag{7}$$

- At times earlier than this, the Dark Matter particles will travel at in random directions at relativistic speeds - a process known as *free streaming.* In the time t_h, the hot Dark Matter particles will travel:
$$d_{\text{min}} \approx ct_h \approx 13 \text{ kpc} \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-2} \tag{8}$$

- Which, today, corresponds to:
$$r_{\text{min}} = \frac{d_{\text{min}}}{a(t_h)} \approx \left(\frac{T_h}{2.7 \text{ K}}\right) \approx 55 \text{ Mpc} \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-1} \tag{9}$$

- This "free streaming" acts to smooth out any density fluctuations on scales smaller than this, which delays the formation of smaller structures, such as galaxies or groups.

- Today, a sphere of radius r_{min} contains a mass of:
$$M_{\text{min}} = \frac{4\pi}{3} r_{\text{min}}^3 \rho_{m,0} \approx 2.7 \times 10^{16} \; M_\odot \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-3} \tag{10}$$

- This means that Dark Matter particles with masses of around 50 eV will supress the formation of structures smaller than the Local Group of galaxies, those with masses of around 8 eV will supress the formation of structures smaller than clusters of galaxies, and those with masses of around 3 eV will suppress the formation of structures smaller than superclusters.

- So, if most of the Dark Matter in the Universe were ~ 3 eV, and thus hot, then the earliest structures to form would be superclusters (because things smaller than superclisters are supressed). Galaxies would eventually form as the Dark Matter cooled, but they'd be recent creations.

- By contrast, in the real Universe, we see galaxies forming at very early times, yet superclusters are have only recently started to form.

- As such, cosmologists believe that most of the Dark Matter in the Universe is dominated by massive, and hence "cold" dark matter.

3 Baryone Acoustic Oscillations

- The size of the "speckles" on the CMB is set by the sound horizon distance:

$$d_s = 0.145 \text{ Mpc} \tag{11}$$

- Since that time, the Universe has expanded by a factor of $1/a(t_{\text{CMB}}) \approx 1090$, meaning those density fluctations have grown in size to:

$$r_s = d_s \times 1090 \approx 158 \text{ Mpc} \tag{12}$$

  which corresponds to a mass of:

$$M_s = \frac{4\pi}{3} r_s^3 \rho_{m_0} \approx 7 \times 10^{17}\ M_\odot \tag{13}$$