Lecture 13:
Inflation

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1 The flatness problem

- As the section title above suggests, this is all to do with the flatness of the Universe.
- Recall from Lecture 4 that:

$$1 - \Omega(t) = -\kappa \left(\frac{c}{a(t)H(t)R_0} \right)^2 \tag{1}$$

and, when the same equation is applied to today:

$$1 - \Omega_0 = -\kappa \left(\frac{c}{H_0 R_0} \right)^2 \tag{2}$$

- That our measurements of the curvature of today's Universe indicate that it is flat to within an accuracy of 0.5% implies:

$$1 - \Omega_0 < 0.005 \tag{3}$$

- But what about at earlier times, has this level of flatness stayed the same?
- Combining Eqs. 1 and 2 gives:

$$1 - \Omega(t) = \left(\frac{H_0}{H(t)} \right)^2 \frac{1 - \Omega_0}{a(t)^2} \tag{4}$$

- When the Universe was radiation-dominated:

$$\left(\frac{H(t)}{H_0} \right)^2 = \frac{\Omega_0}{a(t)^4} \tag{5}$$

Meaning:

$$1 - \Omega(t) = a(t)^2 \frac{(1 - \Omega_0)}{\Omega_0} \tag{6}$$

- Remember, $1 - \Omega(t)$ *is* the curvature at time t.
- So, as we go back in time and a(t) gets smaller and smaller, the Universe was flatter and flatter.
- Thus, in order for the curvature of the Universe to be < 0.005 today,at the time of Big Bang nucleosynthesis when $a(t_{\rm nuc}) = 3.6 \times 10^{-9}$, it must have been:

$$1 - \Omega(t_{\rm nuc}) < 7 \times 10^{-16} \tag{7}$$

- Or flat to within one part in one quintillion!

2 How does inflation solve the flatness problem?

- One theory of inflation states that the Universe was dominated by a "cosmological constant" akin to, but much larger than, today's Dark Energy.

- Just like Dark Energy, this would have had an energy density that was constant with respect to scale factor:

$$\varepsilon_\Lambda = \Lambda \tag{8}$$

meaning:

$$H^2 = \frac{8\pi G}{3c^2}\Lambda = \text{constant} \tag{9}$$

- If this dominates over all other energies in the Universe at that time, then, as we saw in Lecture 6, the scale factor goes as:

$$a(t) \propto e^{Ht} \tag{10}$$

- How does this solve the flatness problem? Well, recall that Eq. 4 states:

$$1 - \Omega(t) = \left(\frac{H_0}{H(t)}\right)^2 \frac{1-\Omega_0}{a(t)^2} \tag{11}$$

where the RHS *is* the measure of curvature of the Universe and $H(t)$ is constant.

- So, during inflation Eq. 4 becomes:

$$1 - \Omega(t) \propto e^{-2Ht} \tag{12}$$

or

$$1 - \Omega(t) = Ke^{-2Ht} \tag{13}$$

- And we'll define K – the constant of proportionality – by considering the Universe just before inflation, at t_i. This means that, just after inflation, at t_f, the curvature of the Universe is given by:

$$1 - \Omega(t_f) = e^{-2H(t_f - t_i)}(1 - \Omega(t_i)) \tag{14}$$

- If we assume that the Universe is highly curved just before inflation (i.e., $1 - \Omega(t_i) \sim 1$), then all we need is for $H(t_f - t_i)$ to be large and the Universe will flatten itself.

- How large?

- Well, this theory states that inflation stopped when the Universe was 10^{-34} s old. Using Eq. 6, today's measured curvature of $< 0.5\%$ and $a(t_f) \approx 1.6 \times 10^{-27}$, we can calculate that the curvature of the Universe just after inflation was:

$$1 - \Omega(t_f) = (1.6 \times 10^{-27})^2 \frac{0.005}{0.995} = 1.2 \times 10^{-56} \tag{15}$$

- If $(1 - \Omega(t_i)) = 1$, then:

$$e^{-2H(t_f - t_i)} = \frac{1 - \Omega(t_f)}{1 - \Omega(t_i)} = 1.2 \times 10^{-56} \tag{16}$$

- Meaning:
$$H(t_f - t_i) = 64 \tag{17}$$

- Which doesn't look too bad, until you consider $t_f - t_i = 10^{-34}$ s, meaning:
$$H = \frac{64}{10^{-34}} \sim 10^{36}\ \mathrm{s}^{-1} \tag{18}$$

- Meaning that:
$$\varepsilon_\Lambda = \frac{3c^2}{8\pi G} H^2 \sim 10^{105}\ \mathrm{TeV\ m}^{-3} \tag{19}$$

- Compared to today's density of Dark Energy of 0.0034 TeV m^{-3}.

- So, inflation is able to very quickly flatten the Universe, but only if it is driven by a *very* strong cosmological constant.

3 The effects of inflation

- During the period of inflation, the scale factor of the Universe increased by a factor of $e^{H(t_f - t_i)} = e^{64} = 6.2 \times 10^{27}$.

- Just after inflation, the scale factor of the Universe was $a(t_f) \approx 1.6 \times 10^{-27}$, meaning that today's entire observable Universe - a sphere of diameter 28.8 Gpc (with the Milky Way at the centre) - was crammed into a sphere with a diameter of only 1.4 m.

- However, inflation had already expanded the observable Universe to this size - prior to inflation, the observable Universe fit within a sphere of diameter:
$$D_i = \frac{1.4}{6.2 \times 10^{27}} = 2.3 \times 10^{-28}\ \mathrm{m} \tag{20}$$

- Which is a factor of 10^{13} smaller than the diameter of a proton.