Lecture 1:
Fundamental Observations of the Universe

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1 Early history and evidence for a non-static Universe

- Throughout history, various religions have offered their own explanation for what we'd now call the Universe.
- Some of these involve a beginning to the Universe. For example, the "In the beginning, God created heaven and the earth" of the Bible.
- However, another possibility is that the Universe has been around for ever.
- And first impressions suggest that the Universe is static and non-evolving.
- The first documented, logically-sound argument against a non-infinite (in time), evolving Universe is Olber's Paradox.

1.1 Olber's paradox

Olber's paradox concerns the question of why the sky is dark. At first, it may seem obvious why the sky is dark. However, if the Universe is infinite in both size and age (and contains a roughly constant density of stars/galaxies), then there's a scientifically robust proof that the night sky should, in fact, be bright.

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- Imagine looking out into an infinite universe that is filled with stars at some average density (i.e., n_* stars per unit volume).
- Eventually, because this universe is infinite, every line of sight will encounter a star.
- We can calculate the typical distance before our line of sight encounters a star - we'll denote this distance λ.
- To do this, imagine a cyclinder of length λ and radius the same of that of a typical star R_*. The volume of this cylinder is:
$$V = \lambda \pi R_*^2 \tag{1}$$
- The number of stars within this cyclinder is:
$$N_* = n_* \lambda \pi R_*^2 \tag{2}$$

- To calculate the typical distance until our line of sight encouters one star, we set $N* = 1$, giving:

$$\lambda = \frac{1}{n_* \pi R_*^2} \tag{3}$$

- In the current Universe, $n_* \sim 10^9$ Mpc^{-3} and $R*$ is typically about 7×10^8 m, giving $\lambda \sim 10^{18}$ Mpc.

- So our line of sight would typically extend to 10^{18} Mpc before encountering a star.

- This is much larger than the size of the observable Universe, but it is still a finite number.

- In an infinite (in age and size) universe, our line of sight *would* eventually extend to this kind of distance before hitting a star.

- If a star is at distance λ, then its angular area is given by:

$$\Omega_* = \frac{\pi R_*^2}{4\pi\lambda^2} \tag{4}$$

  and the flux we measure is:

$$f_* = \frac{L_*}{4\pi\lambda^2} \tag{5}$$

  where L_* is the star's luminosity.

- Meaning its surface brightness is:

$$\Sigma = \frac{f_*}{\Omega_*} = \frac{L_*}{4\pi\lambda^2}\frac{4\lambda^2}{R_*^2} = \frac{L_*}{\pi R_*^2} \tag{6}$$

- The surface brightness is independent of λ, the distance to the star.

- Since every line of sight eventually hits a star, this means that in an infinite (in age and size) universe, the surface brightness of every part of the sky will be equal to the surface brightness of a typical star (such as our Sun)!

- There is no trickery here, meaning that our assumption of a static, infinite (in age and space) universe is incorrect.

- It turns out that the main reason for Olber's Paradox is that the real Universe is not infinite in age, meaning that the light from the most distant stars has not had time to reach us yet.

2 Isotropy and homogeneity

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- Another key observed property of the Universe is that it is *isotropic* and *homogeneous* on large scales.

- "Large scales" refers to > 100 *Mpc*.

- Isotropic means "no preferred direction"

- Homogeneous means “no preferred location”

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- But this is clearly only the case on vary large scales.

3 Distance is proportional to redshift

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- When we look at galaxies beyond the local group (i.e., beyond the local effects of gravity), we see that they are all moving away from us.
- We know this because the light from these galaxies is *redshifted.*
- Recall, redshift is proportional to velocity - more highly redshifted galaxies are moving away from us at higher velocities.
- We have also measured that more distant galaxies have higher redshifts (i.e., higher relative velocities, v), such that distance, D, is proportional to redshift, z. This is usually expressing in terms of Hubble's law: $v = H_0 D$.

Elastic example

- Imagine a one-dimensional universe - it only has length.
- Galaxies distributed along the universe.
- The location of the galaxies can be described in terms of co-moving coordinates.
- Because the expansion of the universe is isotropic – the same in all directions – the relative positions of the galaxies don't change.
- Their co-moving coordinates don't change.
- The real coordinates of every galaxy in the universe can be expressed as their co-moving coordinates multiplied by a scale factor $a(t)$ *that is the same for every galaxy.*
- Again, because it is isotropic and homogeneous, the expansion of the whole universe is *wholly* encapsulated in $a(t)$.
- As a reference point, we define $a(t)$ at the present moment, t_0, to be 1. (i.e., $a(t_0) = 1$).
- In an expanding universe, $a(t)$ was < 1 in the past, and will be > 1 in the future. It would be the opposite in a contracting universe.

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- Since the real coordinates of every galaxy can be expressed as its co-moving coordinates multiplied by the same scale factor. The real distance between any two galaxies at a given time, $D_{1,2}(t)$, can also be expressed as their distance in co-moving coordinates, $r_{1,2}$, multiplied by the same scale factor, $a(t)$:

$$D_{1,2} = a(t) r_{1,2} \tag{7}$$

- And their relative velocities are given by:

$$v_{1,2} = \frac{dD_{1,2}}{dt} = \frac{da(t)}{dt} r_{1,2} = \dot{a} r_{1,2} \tag{8}$$

- Substituting $r_{1,2} = D_{1,2}/a(t)$ into Eq. 8 gives:

$$v_{1,2} = \frac{\dot{a}}{a} D_{1,2} \tag{9}$$

- **Slide 10** And comparing to $v_{1,2} = H_0 D_{1,2}$ reveals that:

$$H_0 = \frac{\dot{a}}{a} \tag{10}$$

  evaluated at $t = t_0$, i.e., today.

- But, this relationship holds true at any time, meaning that:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{11}$$

- In other words, the Hubble Parameter is the ratio of the relative rate of change of the scale factor.

- This is the first **Important Equation** of the course.

- Note: For historical reasons, H_0 is called the *Hubble Constant*, even though it's not a constant in time (but it is a constant in space)! $H(t)$ (or sometimes just H) is called the *Hubble Parameter*.