PROBLEMS CLASS 2

Q1

$$H = \frac{\dot{a}}{a} = \frac{da}{dt} \cdot \frac{1}{a}$$

$$H\,dt = \frac{da}{a}$$

$$Ht = \ln a + C$$

$$e^{Ht} = k a$$

$$a(t) = k e^{Ht}$$ — NEED TO GET k

$$a(t_0) = 1$$

$$1 = k e^{Ht_0}$$

$$k = e^{-Ht_0}$$

$$\therefore \quad a(t) = e^{H(t - t_0)}$$

Q2

$$a(t_e) = e^{H(t_e - t_0)}$$

$$\& \quad a(t_e) = \frac{1}{1+z} \qquad \text{SET} \quad 1+z = z_p$$

$$z_p^{-1} = e^{H(t_e - t_0)}$$

$$-\ln z_p = H(t_e - t_0)$$

$$t_e = t_0 - \frac{\ln z_p}{H}$$

Q3

$$d_p = c \int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$d_p = c \int_{t_e}^{t_0} e^{-H(t_w - t_0)} \, dt$$

$$= c\, e^{Ht_0} \int_{t_e}^{t_0} e^{-Ht} \, dt$$

$$= c\, e^{Ht_0} \left[-\frac{1}{H} e^{-Ht} \right]_{t_e = t_0 - \frac{\ln z_p}{H}}^{t_0}$$

$$= -\frac{c\, e^{Ht_0}}{H} \left[e^{-Ht_0} - e^{-H(t_0 - \ln z_p / H)} \right]$$

$$= -\frac{c}{H} \left[\underbrace{e^{\cancel{Ht_0} - \cancel{Ht_0}}}_{1} - \underbrace{e^{\cancel{Ht_0} - \cancel{Ht_0} + \cancel{H} \ln z_p / \cancel{H}}}_{z_p} \right]$$

$$= \frac{-c}{H} \left[\cancel{1} - z \cancel{+1} \right] = \frac{cz}{H}$$

Q4

$$d_p = \frac{cz}{H} \qquad H = 67.7 \text{ Km/s/Mpc}$$

$$= \frac{3\times10^5 \times 1}{6.77\times10^1}$$

Km/s

Km/s/Mpc

$$= \frac{3}{6.77} \times 10^4 \text{ Mpc}$$

$$= 0.44\times10^4 = 4{,}400 \text{ Mpc}$$

$$= 4.4 \text{ Gpc}$$

Q5 – DON'T JUST DO $d_p/c = \frac{z}{H_0}$

INSTEAD, NEED TO DO:

$$t_0 - t_e = \frac{\ln z_p}{H_0} = \frac{\ln z+1}{H_0}$$

$$= \frac{\ln 2}{67.7\times10^3} \times \frac{3.09\times10^{22}}{3\times10^7}$$

$$= \frac{0.69}{\approx 0.69\times10^5} \times \frac{3\times10^{22}}{3\times10^7}$$

$$= 10^{-5} \times 10^{22} \times 10^{-7}$$

$$= 10^{10} \text{ yrs}$$

= 10 BILLION YEARS –

i.e. ~~THE~~ IN A UNIVERSE WHERE H = CONSTANT THE TIME IT TAKES FOR THE UNIVERSE TO DOUBLE IN SIZE FROM z = 1 TO TODAY, IS THE SAME FOR IT TO DOUBLE AGAIN FROM TODAY (I.E. PROBLEMS CLASS 1)