Problems Class IV

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Equations and constants

The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: $1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$

1. Assuming that the dark matter halo of the Milky Way is spherically symmetric, derive an expression that describes how the density distribution of dark matter, $\rho(r)$, changes with radius, r, to ensure a flat rotation curve, i.e., $v = const.$

2. Given that the Sun lies 8.5 kpc from the centre of the Milky Way and orbits the centre at a speed of 235 km s^{-1}, calculate the density of dark matter within the Solar System in units of TeV m^{-3}. Do you expect this to be higher, lower, or the same as the critical density of the Universe?

3. The Friedmann Equation can be written as

$$\dot{a}^2 = H_0^2 \left(\frac{\Omega_{\mathrm{r},0}}{a^2} + \frac{\Omega_{\mathrm{m},0}}{a} + \Omega_{\kappa,0} + \Omega_{\mathrm{D},0} a^2 \right). \tag{1}$$

   By differentiating the above equation with respect to time, determine the redshift at which a universe with $H_0 = 100$ km s^{-1} Mpc^{-1} consisting of 50% matter and 50% dark energy switches from decelerating to accelerating.

4. (a) Explain to a colleague (sitting next to you if you're in the problems class) why assuming that recombination occurs when the *average* photon energy is 13.6 eV gives an innacurate estimate of when this occurs. Include in your explanation whether this assumption leads to too early or too late an estimate compared to the real time of recombination.

   (b) In return, your colleague should explain to you how cosmologists obtain a more precise estimate of the time of recombination. They should include in their answer how cosmologists obtain the scale factor of the Universe at the time of recombination.