Cosmology
Lecture 11

Nucleosynthesis and the
first three minutes: Part 1

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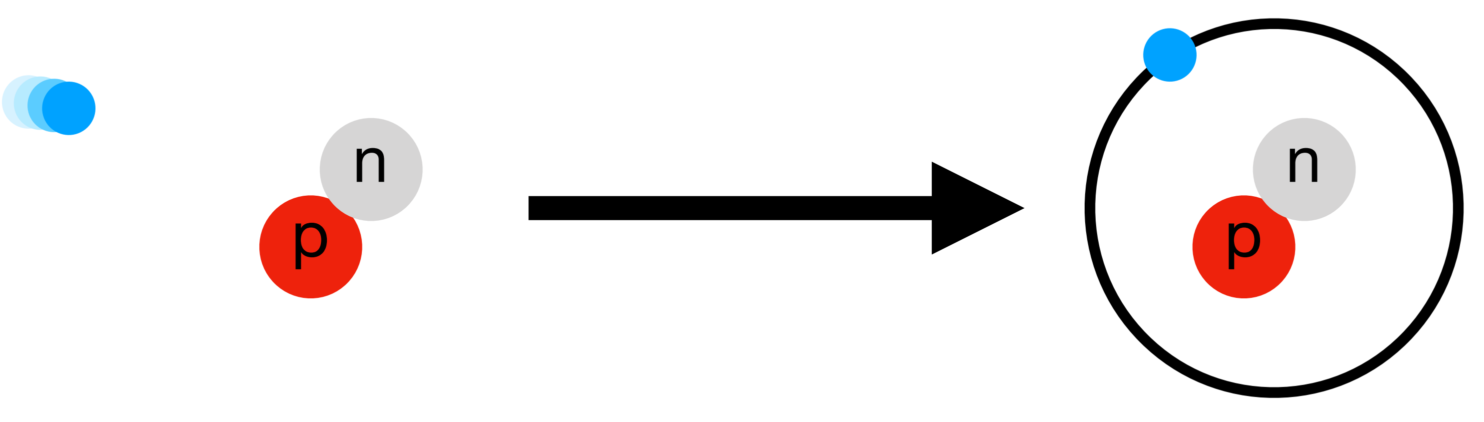
What we'll cover in this lecture

- The physics that is important in producing the first atomic nuclei;
- The production of deuterium;
- What governed the ratio of Hydrogen to Helium in the Universe that still persists today.

Nucleosynthesis vs. recombination

Nucleosynthesis (NS) is the joining of subatomic particles to form atomic nuclei.

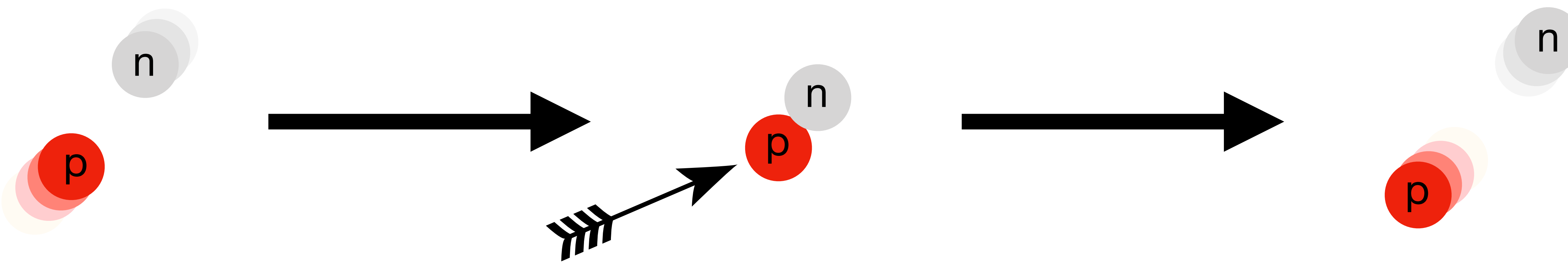
Recombination is the joining of these atomic nuclei and electrons to form neutral atoms.

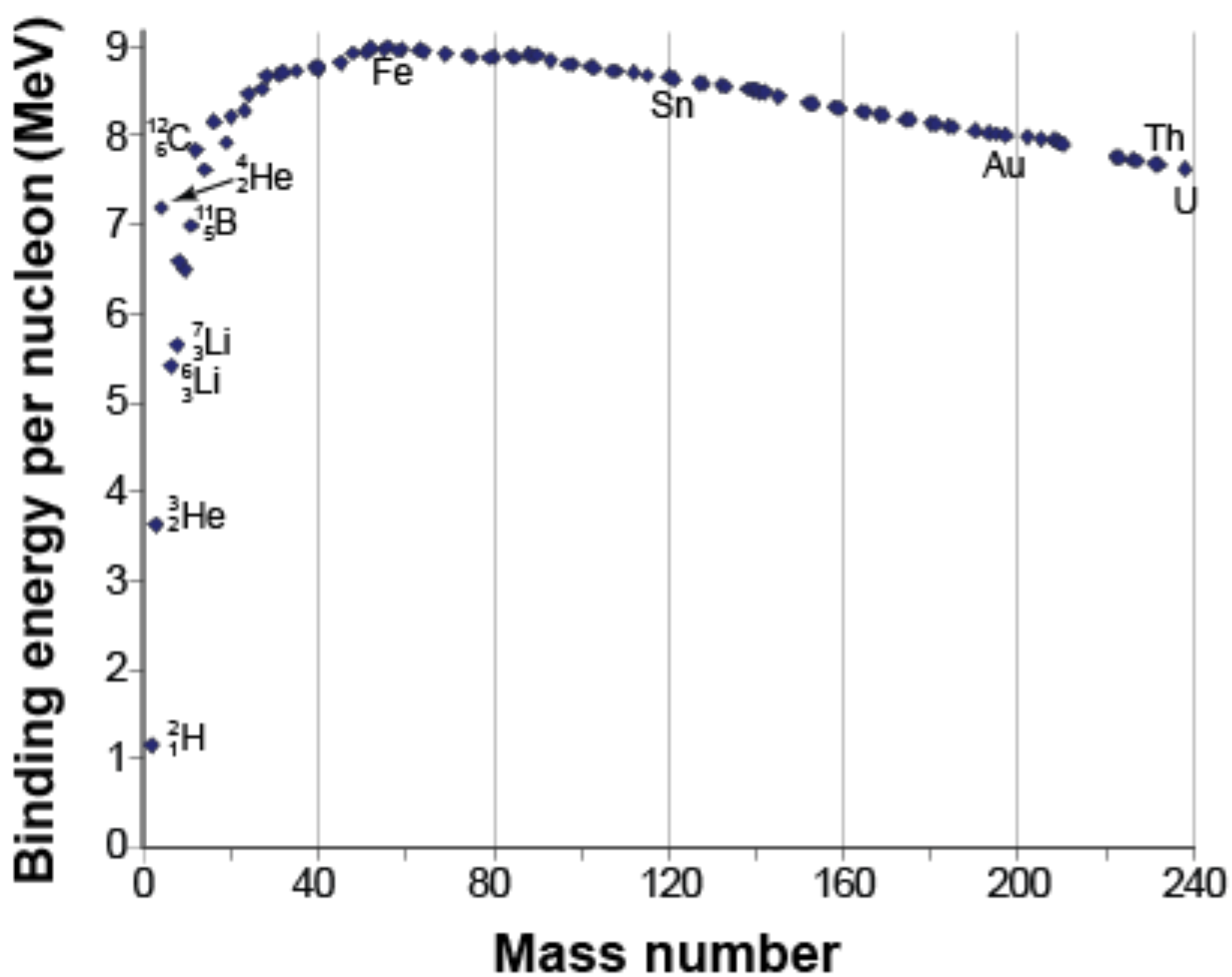


As we saw in previous lectures, recombination takes place when typical photon energies drop to eV levels (particularly 13.6eV), since these are the energies at which atoms are ionised.

Nucleosynthesis vs. recombination

Similarly, for NS to occur, typical photon energies need to drop to below the binding energies of atomic nuclei, otherwise:

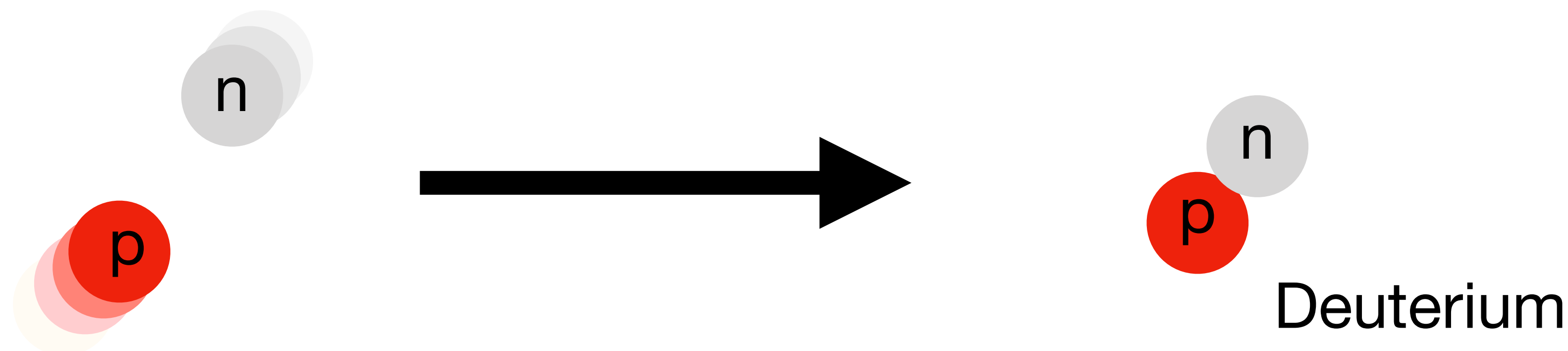




This occurs around the MeV scale.

Around when did nucleosynthesis occur?

The first NS was between a proton and neutron to produce deuterium.



Deuterium has a binding energy of 2.22 MeV - a factor of 1.6×10^5 higher than the 13.6eV binding energy of atomic Hydrogen.

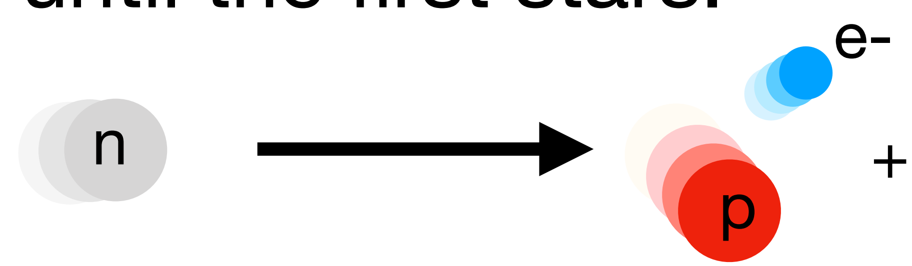
As a rough guess, then, the temperature of the Universe at the time of NS is 1.6×10^5 higher than the temperature of recombination: $1.6 \times 10^5 \times 3760$ K $= 6 \times 10^8$ K

Since temperature is proportional to a^{-1}, we can use the Benchmark model to work out that the Universe was ~300 s (5 minutes) old when this happened *(but this is only approximate)*.

Protons and neutrons prior to nucleosynthesis

NS involves the joining of protons and neutrons.

The half-life of a free neutron is about 15 minutes so, if NS hadn't taken place within the first few hours after the Big Bang, we'd have no atomic nuclei until the first stars.



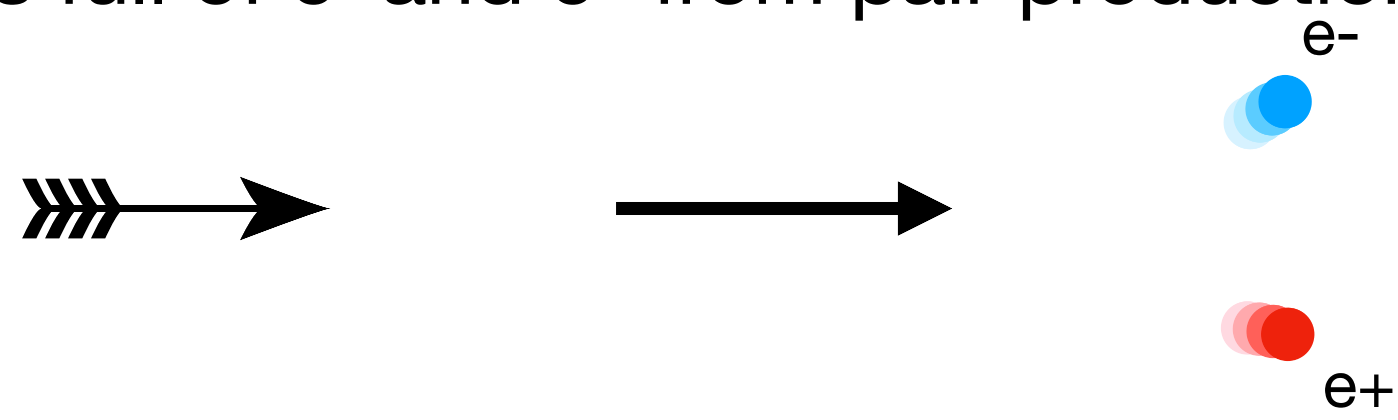
$+ \quad \bar{\nu}_e$

This wasn't the case, however; there *were* neutrons around when the Universe was cool enough to form stable atomic nuclei.

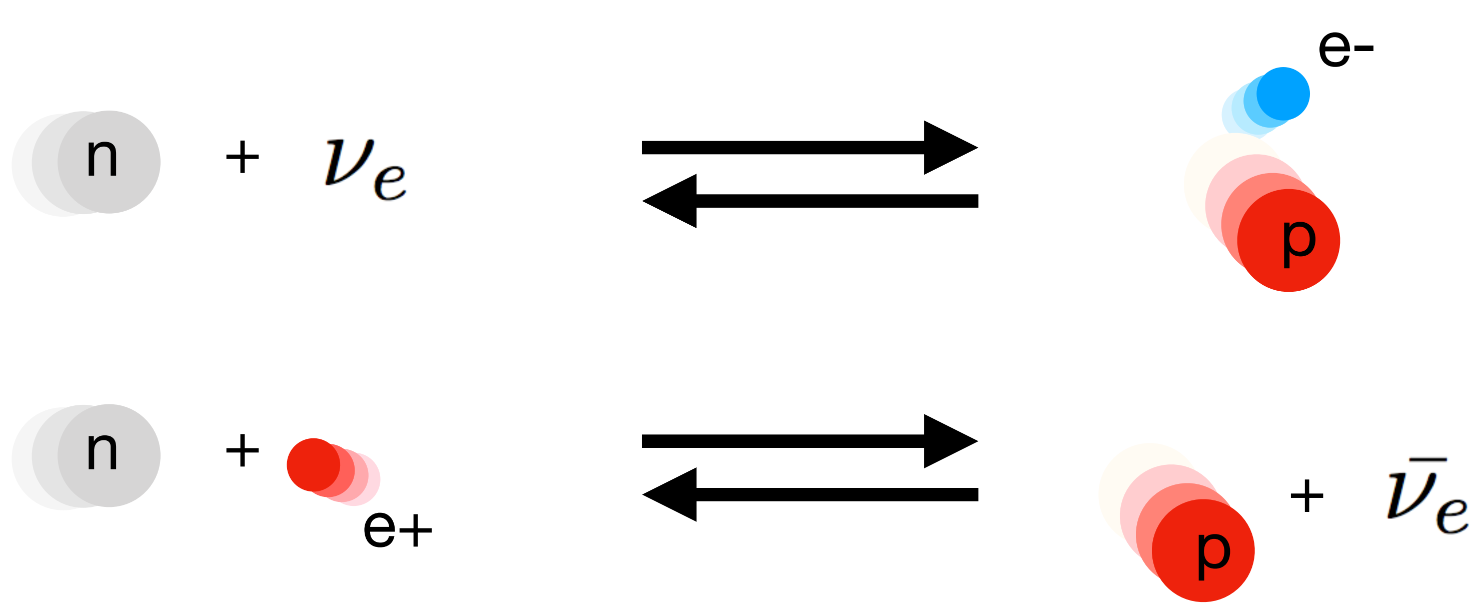
What determines the relative numbers of atomic nuclei after NS is the relative numbers of protons and neutrons just prior to NS.

Calculating the relative numbers of p and n

At around t~0.1s, typical photon energies were higher than the rest-mass energy of electrons and positrons, so the Universe was full of e^- and e^+ from pair production.



With all these e^-, e^+ and p and n around, the numbers of p and n would be in equilibrium:



Calculating the relative numbers of p and n

From statistical mechanics, the number densities of n and p are given by:

$$n_n = g_n \left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_n c^2}{kT}\right)$$

$$n_p = g_p \left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_p c^2}{kT}\right)$$

where g_n and g_p are the statistical weights of n and p, which are both equal to 2 (corresponding to the two spin states of these particles).

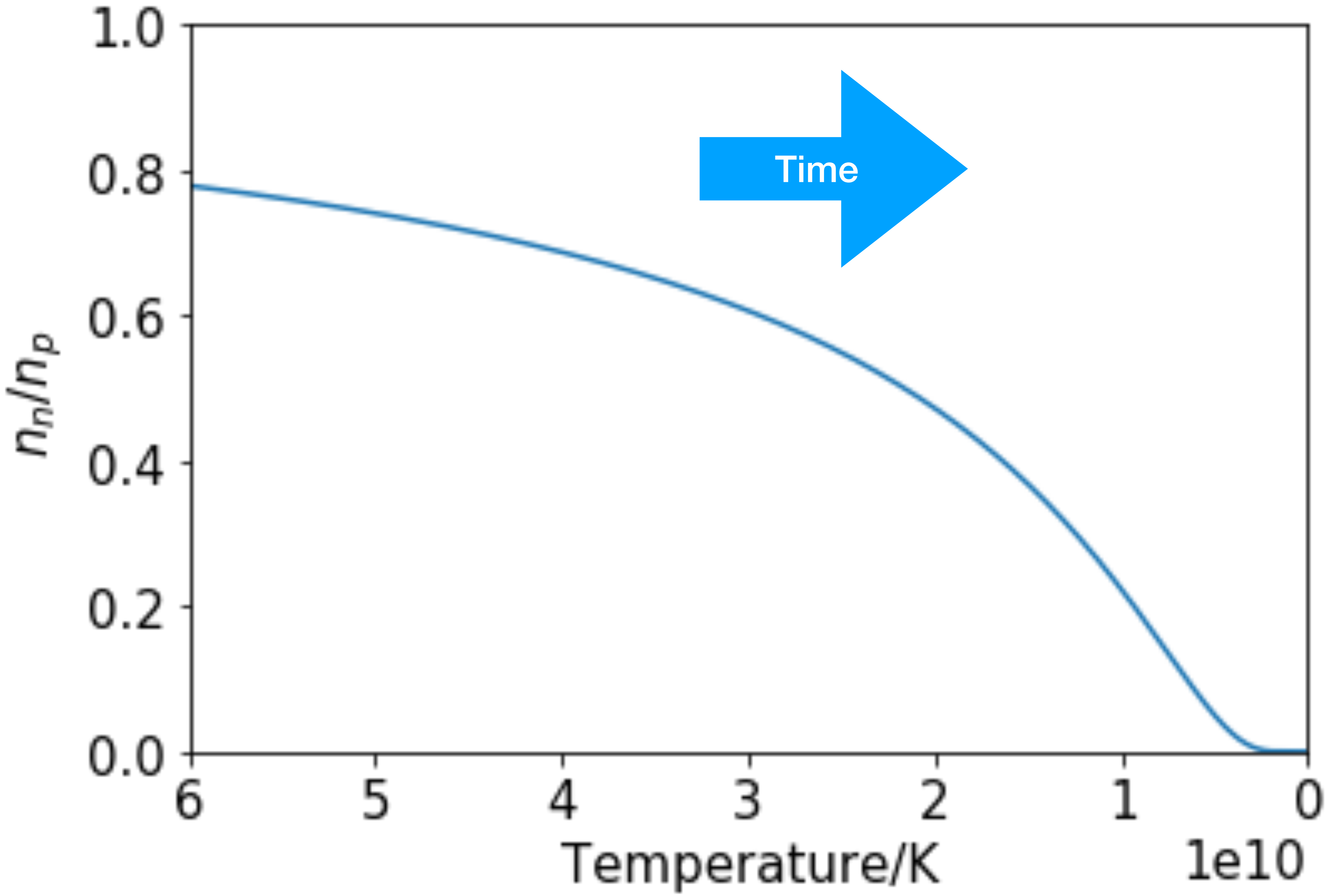
Thus, the relative number of n and p is given by:

1.29 MeV

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{(m_n - m_p)c^2}{kT}\right)$$

and we can say $m_n/m_p \approx 1$ with no significant lack of accuracy.

The neutron to proton ratio in the early Universe

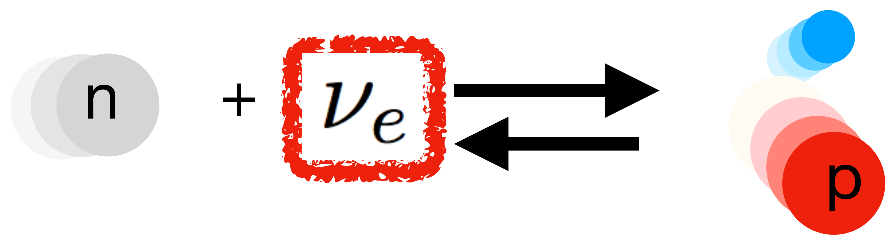


If protons and neutrons stayed in equilibrium, then within six minutes of the Big Bang, there'd only be one neutron for every million protons.

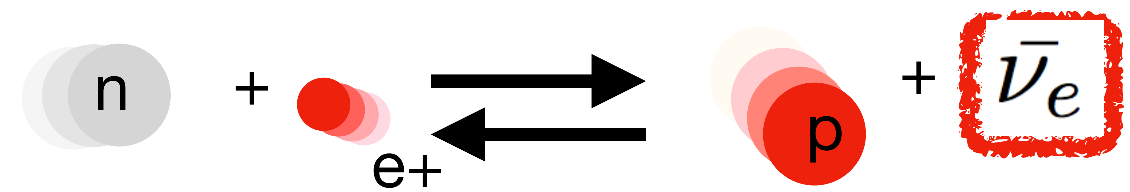
Proton-neutron freeze-out

But, the ratio of n to p doesn't fall forever…

Notice:



and



Thus, we require neutrinos to interact with nucleons for the equilibrium to continue.

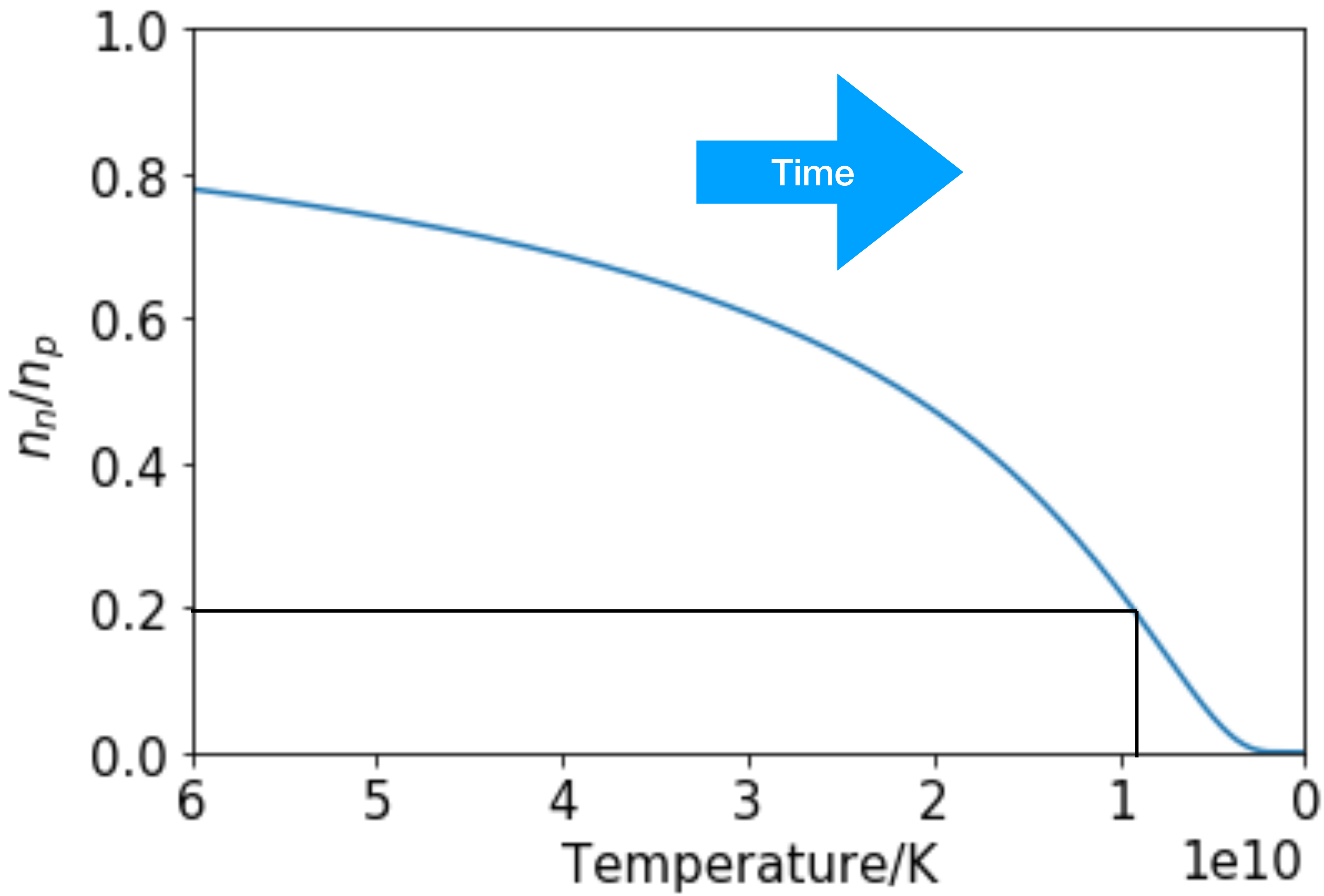
The cross section of this interaction is given by: $\sigma_w \sim 10^{-47}\ \mathrm{m}^2 \left(\frac{kT}{1\ \mathrm{MeV}}\right)^2$

and since $T \propto a^{-1}$ and $a \propto t^{1/2}$ at this time when the Universe was radiation-dominated, and the number density of neutrinos fall as a^{-3}, then the rate of neutrino interaction fell quickly with time:

$$\Gamma \propto t^{-5/2}$$

Eventually, at around 1s after the Big Bang (when kT~0.8 MeV), the temperature and density of neutrinos falls to the point where equilibrium cannot be sustained, and the relative numbers of p and n are "frozen".

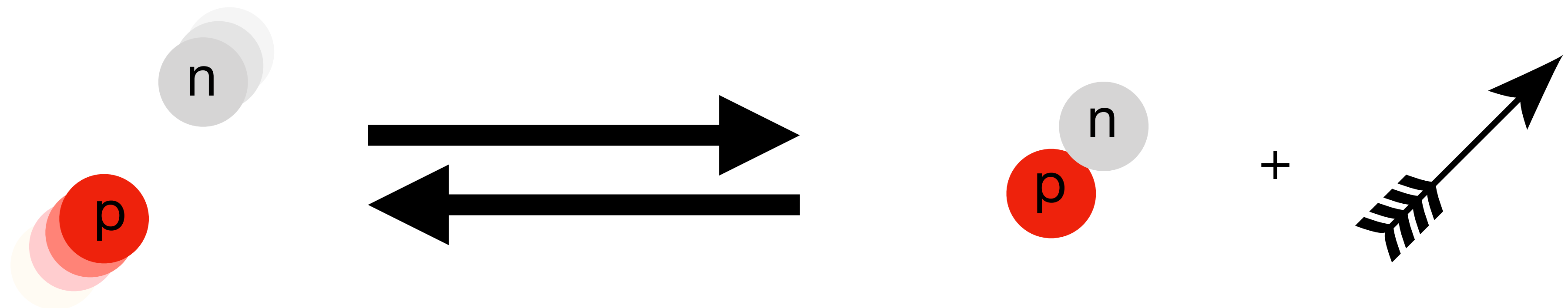
The neutron to proton ratio after “freeze-out”.



0.8 MeV corresponds to 9.3x10^9 K via E=kT. This defines the “freeze-out” temperature, which corresponds to at neutron-to-proton number ratio of 1/5.

Deuterium formation

After p-n freeze-out the first stable atomic nuclei to form is deuterium:



A deuterium nuclei will be destroyed by a photon with energy > 2.2 MeV.

As such, the numbers of deuterium nuclei will increase as the typical photon energy decreases with decreasing temperature of the Universe.

The epoch of Deuterium synthesis is not instantaneous - it happens over an extended period of time.

The epoch of deuterium formation

Just as we did with recombination, we can determine the temperature during the epoch of deuterium synthesis.

$$\text{Temperature} \xrightarrow{T \propto a^{-1}} \text{Scale factor} \xrightarrow[\text{model}]{\text{Benchmark}} \text{Age}$$

Since it's not instantaneous, we need to set an acceptable criterion that the "epoch of deuterium synthesis" must satisfy.

We'll use: $n_D/n_n = 1$ (i.e., half of all neutrons are in D)

And just as with recombination, we can use this criterion in the Saha equation to determine the temperature at this time:

$$\frac{n_D}{n_p n_n} = 6 \left(\frac{m_n kT}{\pi \hbar^2} \right)^{-3/2} \exp \left(\frac{B_D}{kT} \right)^{-3/2} e$$

$$\boxed{\begin{array}{l} g_D = 3 \qquad g_p = g_n = 2 \\ B_D = (m_p + m_n - m_D)c^2 \\ m_p = m_n = m_D/2 \end{array}}$$

The temperature at Deuterium synthesis

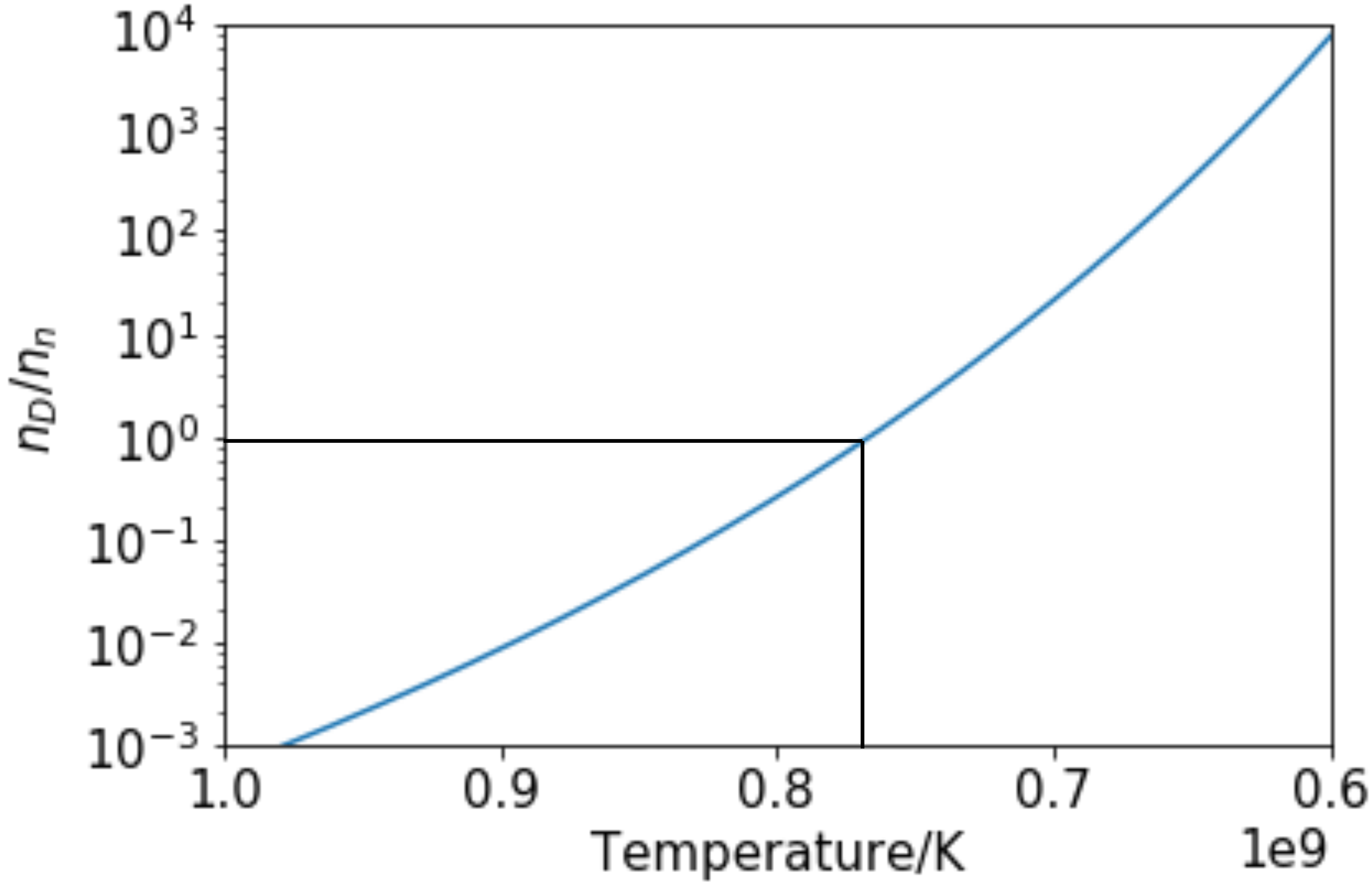
We can therefore obtain an expression for n_D/n_n as a function of temperature:

$$\frac{n_D}{n_n} = 6n_p \left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right)$$

Which, when we assume $n_p = 0.8n_{\rm bary}$ becomes:

$$\frac{n_D}{n_n} = 6.5\eta \left(\frac{kT}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT}\right)$$

The temperature at Deuterium synthesis



The number of Deuterium nuclei increases with decreasing temperature. If we define the epoch of Deuterium nucleosynthesis as when half of all neutrons in the Universe are in Deuterium, this occurs when the temperature of the Universe is 760 million K at a time of **200s** since the Big Bang**.**

An updated neutron to proton ratio

With the time to Deuterium synthesis being ~200 s, and the decay time of neutrons being 800 s, some neutrons will decay before forming Deuterium.

During this time, the ratio of neutrons to protons will fall from 1/5 at the time of freeze-out (i.e., 1 s) to:

$$r_{np} = \frac{n_n}{n_p} = \frac{\exp(-200/800)}{5 + (1 - \exp(-200/800))}$$

As a consequence, there are fewer neutrons available to form He, ultimately leading to a He mass ratio of:

$$Y_{\mathrm{He}} = \frac{m_{\mathrm{He}}}{m_{\mathrm{Bary}}} = \frac{2r_{np}}{r_{np} + 1} \approx 0.26$$

Comparison with observations

As partial proof of this, when we observe the Universe, we see no non-fractionated regions with a He mass fraction lower than 0.26.

Getting the feel for it…

- Even if the numbers of protons and neutrons started off the same, the slightly lower mass of protons causes that to be preferred as temperature drops.

- The proton/neutron ratio “freezes out” when the neutrinos stop being in equilibrium.

- Next, it’s a race for Deuterium to form *before* the unstable neutrons decay to protons.

- Deuterium forms after around 200 s, paving the way for the next atomic nuclei to form…