Problems Class III

Dr. James Mullaney

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Equations and constants
The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{\mathrm{M},0} = 0.31,\ \Omega_{\mathrm{D},0} = 0.69,\ \Omega_{\mathrm{R},0} = 9 \times 10^{-5},\ H_0 = 67.7\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

Parsec in SI units: $1\ \mathrm{pc} = 3.09 \times 10^{16}\ \mathrm{m}$

Questions

1. Re-write the expressions for $\epsilon_{\mathrm{m}}(t)$, $\epsilon_{\mathrm{r}}(t)$ and $\epsilon_{\mathrm{d}}(t)$ in terms of the current critical energy density, $\epsilon_{\mathrm{c},0}$, the scale factor, $a(t)$, and their respective current density parameters (i.e., $\Omega_{\mathrm{M},0}$, $\Omega_{\mathrm{R},0}$ and $\Omega_{\mathrm{D},0}$).

2. Using the answers to Q1, calculate the redshifts at which the:

   (a) dark energy and matter energy densities;

   (b) radiation and matter energy densities

   were equal in a universe in which $\Omega_{\mathrm{M},0} = 0.25$, $\Omega_{\mathrm{R},0} = 0.05$, and $\Omega_{\mathrm{D},0} = 0.7$.

3. In Lecture 6, we saw that both matter and radiation-dominated universes have a *horizon distance* (i.e., a maximum proper distance corresponding to $z = \infty$).[1] Calculate the current horizon distance in a matter-only universe in which $H_0 = 67.7\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$. Give your answer in Gpc.

4. We saw in the last problems class that $a(t) = e^{H_0(t-t_0)}$ in a Dark Energy-only universe. Derive the expression for how angular distance changes with redshift for such a universe. Your answer should include the following terms: H_0, c, and z.

[1] The horizon distance corresponds to the furthest observable distance in a universe and thus defines the extent of the observable universe.