PC 3

1 $\varepsilon_m = \dfrac{\varepsilon_{m,0}}{a^3}$ & $\Omega_{m,0} = \dfrac{\varepsilon_{m,0}}{\varepsilon_{c,0}}$

$\varepsilon_{m,0} = \Omega_{m,0}\, \varepsilon_{c,0}$

$\therefore \quad \varepsilon_m = \dfrac{\Omega_{m,0}}{a^3} \varepsilon_{c,0}$

Using the same approach:

$\varepsilon_r = \dfrac{\Omega_{r,0}\, \varepsilon_{c,0}}{a^4}$ & $\varepsilon_\Lambda = \Omega_{\Lambda,0}\, \varepsilon_{c,0}$

2 a) $\varepsilon_m = \varepsilon_\Lambda$

$\dfrac{\Omega_{m,0}}{a^3} = \Omega_{\Lambda,0}$ $\qquad a = \left(\dfrac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3}$

$= \left(\dfrac{0.25}{0.7}\right) = 0.71$

$1 + z = \dfrac{1}{a} = 1.41$

$z = 0.41$

b) $\dfrac{\Omega_{r,0}}{a^{\cancel{4}}} = \dfrac{\Omega_{m,0}}{\cancel{a^3}}$ $\qquad a = \dfrac{\Omega_{r,0}}{\Omega_{m,0}} = 5$

$1 + z = 5$

$z = 4$

3 $\cancel{d}$ $a = \left(\frac{t}{t_0}\right)^{2/3}$

$$d_p = c \int_{t_e}^{t_0} \frac{1}{a}\, dt$$

$$= c\, t_0^{2/3} \int_{t_e}^{t_0} t^{-2/3}\, dt$$

$$= c\, t_0^{2/3} \left[3 t^{1/3} \right]_{t_e}^{t_0}$$

$$= 3c\, t_0^{2/3} \left[t_0^{1/3} - t_e^{1/3} \right]$$

AT $z = \infty$ $t_e = \phi$

$$d_p = 3c\, t_0^{2/3}\, t_0^{1/3} = 3c\, t_0$$

Need to determine t_0

$$H_0 = \dot{a}\Big|_{t=t_0} = \frac{2}{3} \cdot \frac{1}{t_0^{2/3}}\, t_0^{-1/3} = \frac{2}{3}\, t_0^{-2/3-1/3} = \frac{2}{3} t_0^{-1}$$

$$= \frac{2}{3t_0}$$

$$t_0 = \frac{2}{3H_0}$$

$$d_p = 3c \cdot \frac{2}{3H_0}$$

$$= \frac{2c}{H_0}$$

$$d_p = \frac{2 \times 3 \times 10^5}{67.7}\,\mathrm{Mpc} = 8.9\ \mathrm{Gpc}$$

4 $$a(t) = e^{H_0(t-t_0)}$$

$$d_A = \frac{d_p}{1+z}$$

$$d_p = c\int_{t_e}^{t_0} \frac{1}{a}\, dt$$

$$= c\int_{t_e}^{t_0} e^{-H_0(t-t_0)}\, dt$$

$$= -\frac{c}{H_0}\left[e^{-H_0(t-t_0)}\right]_{t_e}^{t_0}$$ SINCE $\frac{1}{a} = e^{-H(t-t_0)}$

$$= \frac{-c}{H_0}\left[\frac{1}{\underbrace{a(t_0)}_{1}} - \frac{1}{a(t_e)}\right]$$

$$= \frac{-c}{H_0}\left[1 - (1+z)\right]$$

$$= \frac{-c}{H_0}\cdot -z = \frac{cz}{H_0}$$

$$d_A = \frac{cz}{H_0(1+z)}$$