Cosmology Lecture 6

Single-component model universes

Spatially-flat, single component universes

Here, we will only consider spatially-flat model universes since our own Universe is consistent with being flat to within measurable tolerances.

Bear in mind, however, that this may change with future, more accurate, measurements.

Setting $\kappa = 0$ for a spatially-flat universe, the F.E. becomes:

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon(t)a(t)^2$$

From Lecture 5, we know:

$$\varepsilon(t) = \varepsilon_0 a(t)^{-3(1+\omega)}$$

Giving:

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon_0 a(t)^{-(1+3\omega)}$$

Proper distances in single-component universes

Solving the F.E. gives:

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3\omega}}$$

where we again define t_0 such that $a(t_0) = 1$

Recall:

$$d_p = c\int_{t_{\mathrm{em}}}^{t_{\mathrm{ob}}} \frac{dt}{a(t)} = c\int_{t_{\mathrm{em}}}^{t_{\mathrm{ob}}} \left(\frac{t}{t_0}\right)^{\frac{-2}{3+3\omega}} dt$$

$$= ct_{\mathrm{ob}}\frac{3(1+\omega)}{1+3\omega}\left[1 - \left(\frac{t_{\mathrm{em}}}{t_{\mathrm{ob}}}\right)^{\frac{1+3\omega}{3+3\omega}}\right]$$

where we've once again assumed: $t_{\mathrm{ob}} = t_0$ (i.e., we're observing now, at the current age of the universe)

Relating proper distance and redshift

However, we don't immediately know $t_{\rm em}$. Instead, what we measure is redshift.

We use:

$$1 + z = \frac{a(t_{\rm ob})}{a(t_{\rm em})} = \left(\frac{t_{\rm ob}}{t_{\rm em}}\right)^{\frac{2}{3+3\omega}}$$

To get:

$$d_p = ct_{\rm ob}\frac{3(1+\omega)}{1+3\omega}\left[1 - (1+z)^{-(1+3\omega)/2}\right]$$

or:

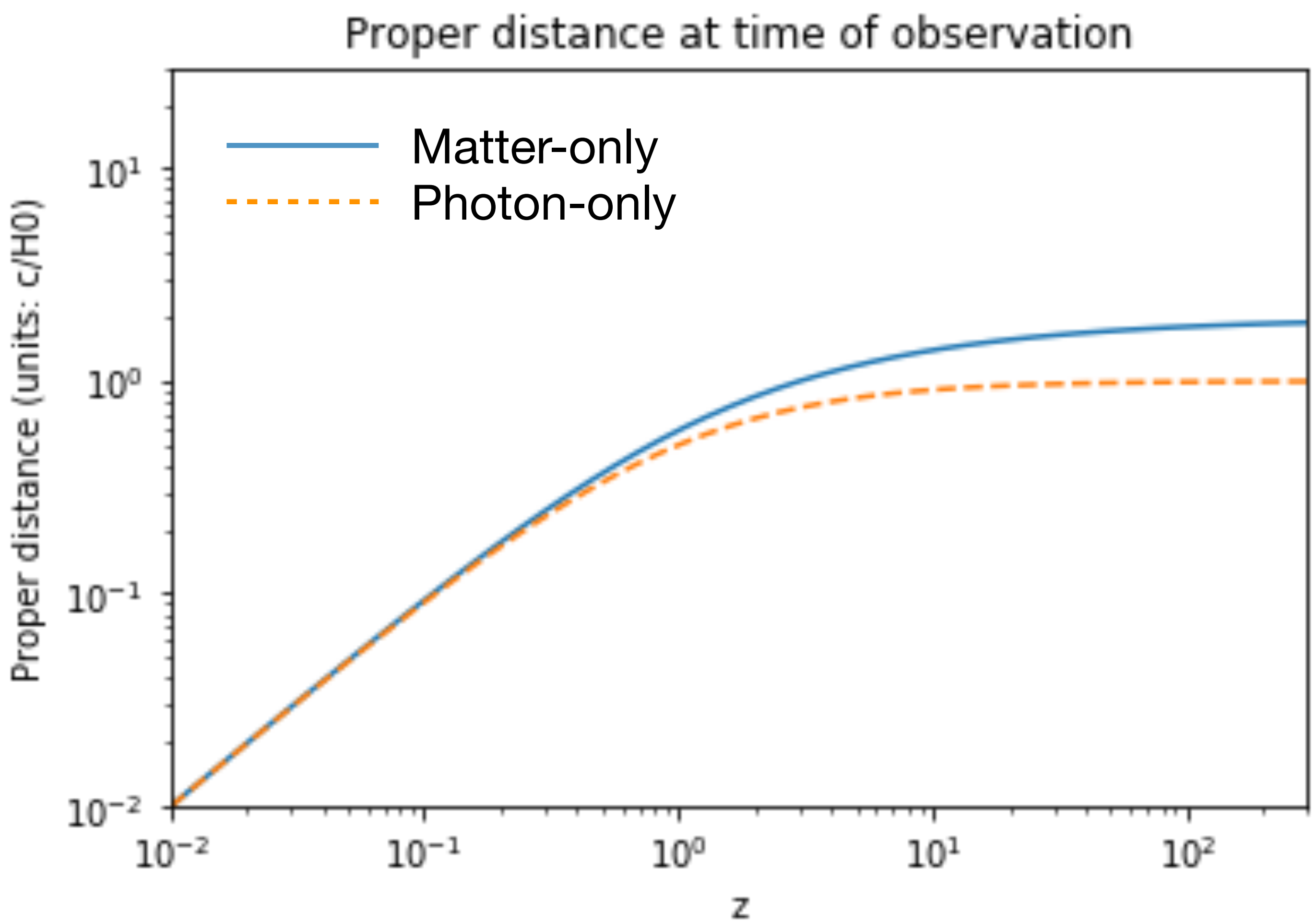
$$d_p = \frac{c}{H_0}\frac{2}{1+3\omega}\left[1 - (1+z)^{-(1+3\omega)/2}\right]$$

Use: $\omega = 0$ for a matter-only universe

$\omega = 1/3$ for a photon-only universe

(see Lecture 4)

How proper distances changes with redshift



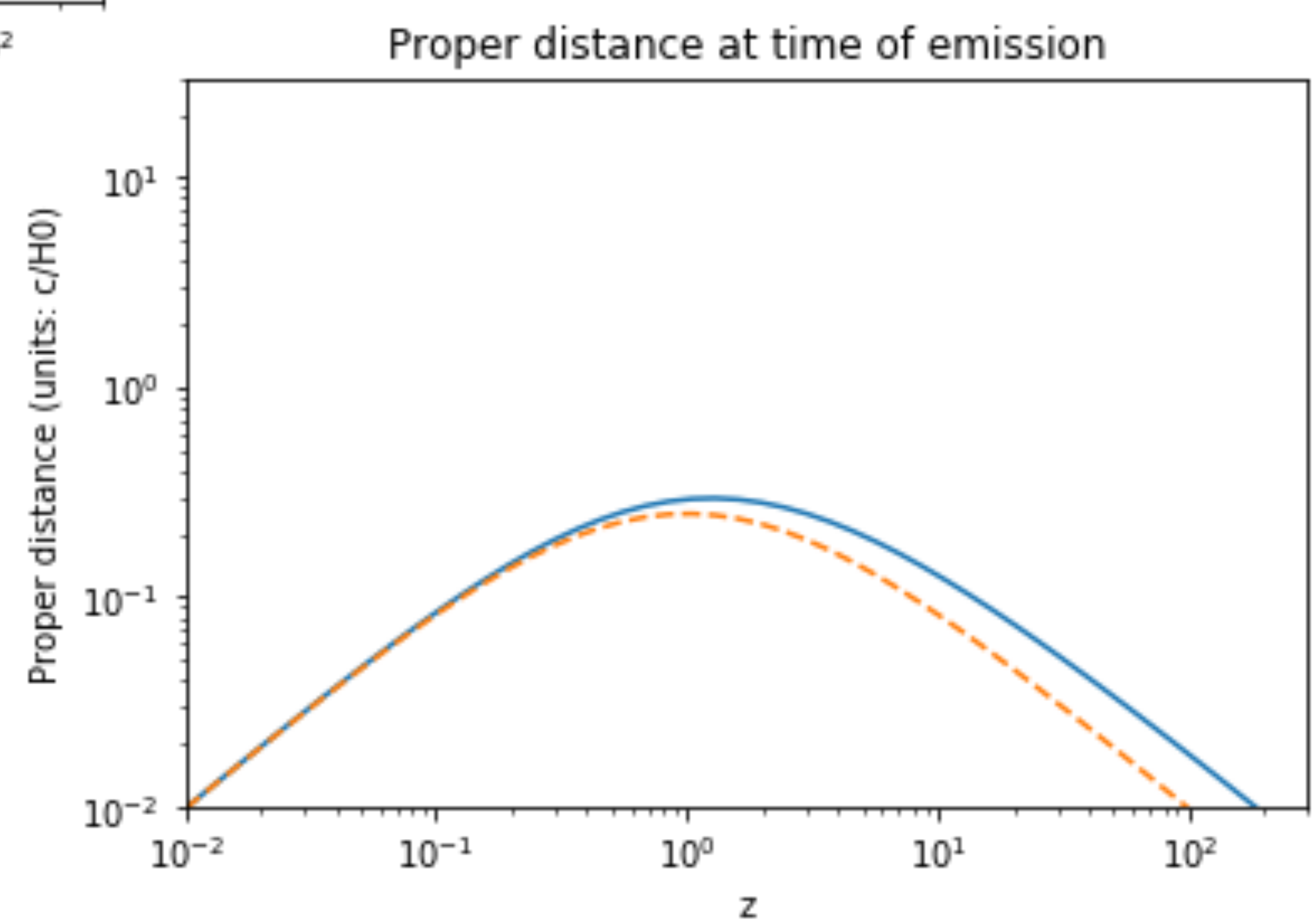
In matter-only and photon-only universes the current proper distance is asymptotic with redshift.

There is a maximum distance we can observe to, defined at $z = \infty$.

This is known as our *Horizon Distance* and defines the extent of the *Observable Universe*.

As with an empty universe, the proper distance at the time of emission increases at low redshifts, before peaking then falling again.

Again, this is a consequence of light travel time - we're seeing the light as it was emitted when the galaxies were closer to us.



What about a Dark Energy-only universe?

We can't use:

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3\omega}}$$

since we saw in Lecture 4 that ω could be -1.

But, we have a saviour in Lecture 5: $\varepsilon_{\mathrm{d}}(a) = \varepsilon_{\mathrm{d},0} a^{-0} = \varepsilon_{\mathrm{d},0}$

i.e., ε_{d} is constant wrt. scale factor and time.

Which gives:

$$\dot{a}^2 = \frac{8\pi G \varepsilon_{\mathrm{d}}}{3c^2} a(t)^2$$

which is straightforward to solve to give:

$$a(t) = e^{H_0(t-t_0)}$$

(remember, $a(t_0) = 1$)

where,

$$H_0 = \left(\frac{8\pi G \varepsilon_{\mathrm{d}}}{3c^2}\right)^{1/2}$$

Proper distances in a Dark Energy-only universe

Recall from Lecture 2 that the current proper distance is given by:

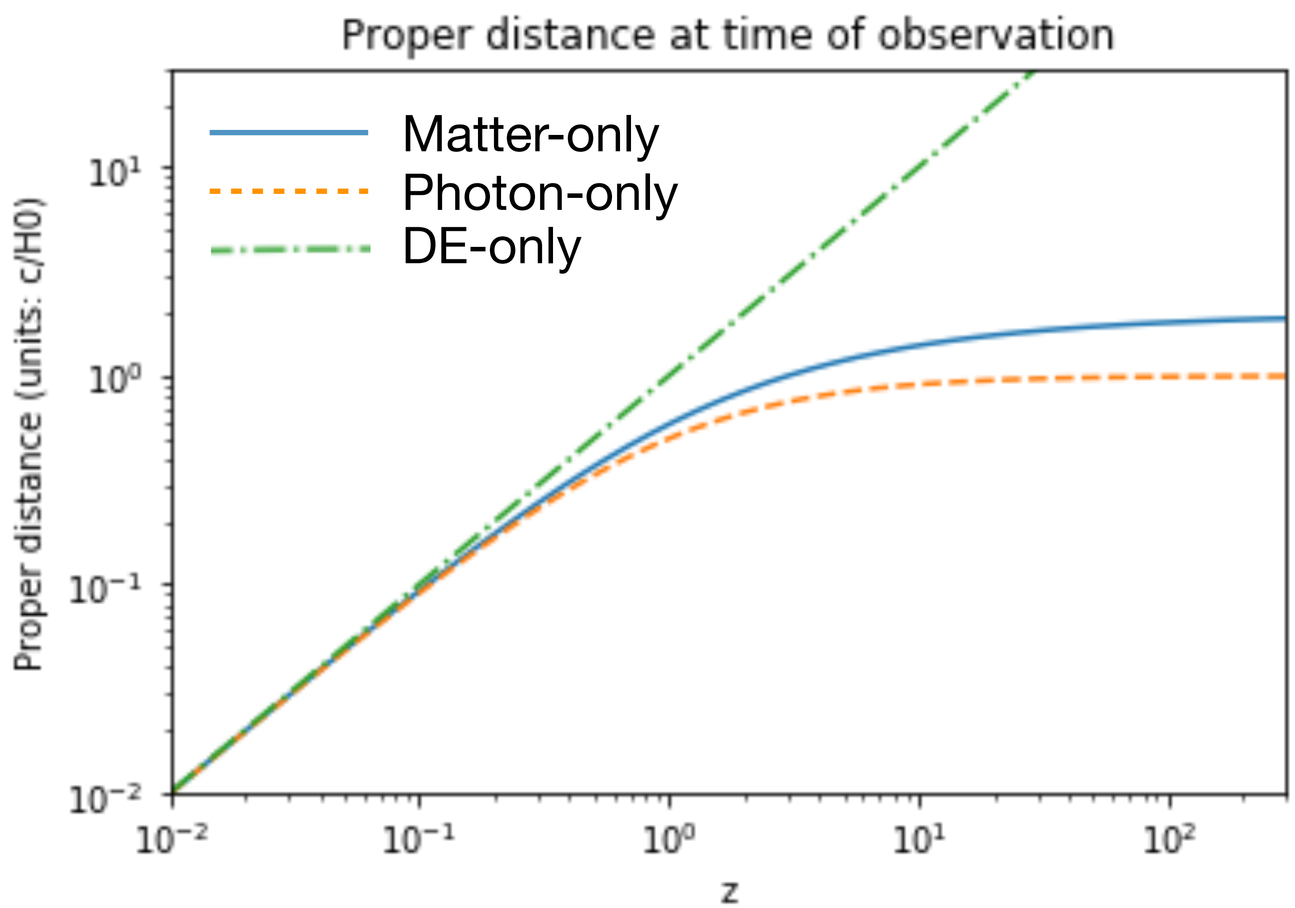
$$
\begin{aligned}
d_p(t_0) &= c\int_{t_{\rm em}}^{t_{\rm ob}} \frac{dt}{a(t)} = c\int_{t_{\rm em}}^{t_{\rm ob}} e^{H_0(t_0-t)}dt \\
&= \frac{c}{H_0}\left(e^{H_0(t_{\rm ob}-t_{\rm em})}-1\right) \\
&= \frac{c}{H_0}\left(\frac{1}{a(t_{\rm em})}-1\right)
\end{aligned}
$$

(since $1+z=\dfrac{1}{a(t_{\rm em})}$)

And that the proper distance at the time of emission is:

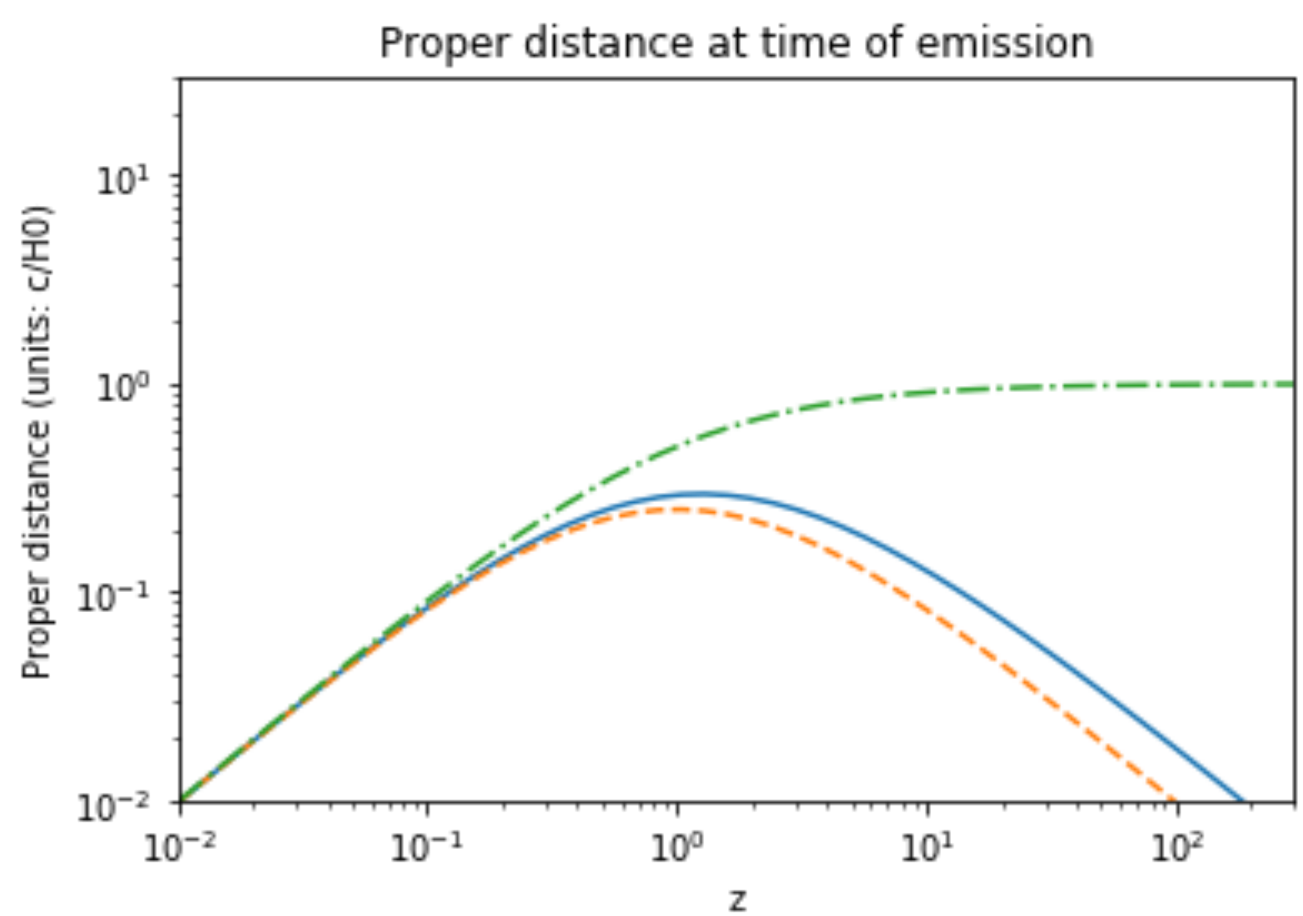
$$
d_p(t_{\rm em}) = d_p(t_{\rm ob})\frac{a(t_{\rm ob})}{a(t_{\rm em})} = \frac{d_p(t_{\rm ob})}{1+z} = \frac{c}{H_0}\frac{z}{1+z}
$$

How proper distance changes with redshift



In a Dark Energy-only universe, current proper distance increases linearly with redshift. We can “see” out to arbitrarily high *current* distances.

However, the proper distance at the time of emission is asymptotic. This means that the most distant galaxy we can observe emitted its light when it was a distance c/H0 away from us. However, in the time between emission and observation, that galaxy has been accelerated away to almost an infinite distance!



Getting the feel of it…

We've now solved the F.E. for four different model universes:
an empty, a matter-only, a radiation-only and a Dark Energy-only universe.

In each case, this has given us an expression for $a(t)$. This expression describes how each universe expands over time.

Knowing $a(t)$ allows us to determine proper distances through *only* knowing redshifts.

There are a few tricks to remember when solving the F.E. or deriving proper distance relationships:

$$H = \frac{\dot{a}}{a} \qquad a(t_0) = 1 \qquad t_{\mathrm{ob}} = t_0$$

$$H_0 = \left(\frac{\dot{a}}{a}\right)_{t=t_0} \qquad \frac{d_p(t_{\mathrm{ob}})}{d_p(t_{\mathrm{em}})} = \frac{a(t_0)}{a(t_{\mathrm{em}})} = \frac{1}{a(t_{\mathrm{em}})} = 1 + z$$