PROBLEMS CLASS 3

1. COULD START BY SOLVING THE FLUID EQUATION, BUT UNLESS SPECIFICALLY ASKED OTHERWISE, IT'S OK TO USE:

$$\varepsilon_m = \varepsilon_{m,0}\, a^{-3} \qquad \Omega_{m,0} = \frac{\varepsilon_{m,0}}{\varepsilon_{c,0}}$$

$$\therefore \quad \varepsilon_{m,0} = \Omega_{m,0}\, \varepsilon_{c,0}$$

GIVING

$$\varepsilon_m = \Omega_{m,0}\, \varepsilon_{c,0}\, a^{-3}$$

2

$$\varepsilon_p = \Omega_{p,0}\, \varepsilon_{c,0}\, a^{-4}$$

$$\varepsilon_D = \Omega_{D,0}\, \varepsilon_{c,0}$$

3 START WITH F.E.

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{8\pi G\varepsilon}{3c^2} - \frac{\kappa c^2}{R_0^2}\cdot\frac{1}{a^2}$$

$$\varepsilon = \varepsilon_c\, \Omega$$

$$H^2 = \frac{8\pi G \varepsilon_c\, \Omega}{3c^2} - \frac{\kappa c^2}{R_0^2}\cdot\frac{1}{a^2}$$

BY DEFINITION: $H^2 = \dfrac{8\pi G\varepsilon_c}{3c^2} \Rightarrow \varepsilon_c = \dfrac{3c^2H^2}{8\pi G}$

$$H^2 = \frac{\cancel{8\pi G}}{\cancel{3c^2}}\,\frac{\cancel{3c^2}H^2}{\cancel{8\pi G}}\,\Omega - \frac{\kappa c^2}{R_0^2}\cdot\frac{1}{a^2}$$

$$H^2(1-\Omega) = -\frac{\kappa c^2}{R_0^2}\cdot\frac{1}{a^2}$$

at $a = 1$ $H = H_0$, $\Omega = \Omega_0$ $\quad H_0^2(1-\Omega_0) = -\dfrac{\kappa c^2}{R_0^2}$

4 START WITH F.E.

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(\varepsilon_m + \varepsilon_r + \varepsilon_p\right) - \frac{Kc^2}{R_0^2}\frac{1}{a^2}$$

FROM Q1, 2 & 3

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3c^2}\,\varepsilon_{c,0}\left(\Omega_{m,0}\,a^{-3} + \Omega_{r,0}\,a^{-4} + \Omega_{d,0}\right)$$

$$+ H_0^2\,(1-\Omega_0)\cdot\frac{1}{a^2}$$

RECALL $\quad \varepsilon_{c,0} = \dfrac{3c^2 H_0^2}{8\pi G}$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left(\Omega_{m,0}\,a^{-3} + \Omega_{r,0}\,a^{-4} + \Omega_{d,0} + \frac{1-\Omega_0}{a^2}\right)$$

$$\dot a^2 = H_0^2\left(\frac{\Omega_{m,0}}{a} + \frac{\Omega_{r,0}}{a^2} + a^2\,\Omega_{d,0} + 1 - \Omega_0\right)$$

5 TO GET $\ddot a$, DIFFERENTIATE F.E. WRT t:

LHS: $$\frac{d\dot a^2}{dt} = \frac{d\dot a^2}{d\dot a}\cdot\frac{d\dot a}{dt} = 2\dot a\,\ddot a$$

RHS: $$\frac{d\,\frac{1}{a}}{dt} = \frac{d\,\frac{1}{a}}{da}\cdot\dot a = -a^{-2}\,\dot a$$

$$\frac{d a^{-2}}{dt} = \frac{d a^{-2}}{da}\cdot\dot a = -2a^{-3}\,\dot a$$

$$\frac{d a^{2}}{dt} = \frac{d a^{2}}{da}\cdot\dot a = 2a\,\dot a$$

5 CONTD:

$$2\dot{a}\ddot{a} = H_0^2\left(2\Omega_{d,0}\, a\dot{a} \cancel{+} - \Omega_{m,0}\, a^{-2}\dot{a} - 2\Omega_{r,0}\, a^{-3}\dot{a}\right)$$

$$\ddot{a} = H_0^2\left(\Omega_{d,0}\, a - \frac{\Omega_{m,0}}{2a^2} - \frac{\Omega_{r,0}}{a^3}\right)$$

AT $t = t_0$, $a = 1$

$$\ddot{a} = H_0^2\left(\Omega_{d,0} - \frac{\Omega_{m,0}}{2} - \Omega_{r,0}\right)$$

$\Rightarrow$ UNIVERSE IS ACCELERATING.