

Problems Class I

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Equations and constants

The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \varepsilon - \frac{\kappa c^2}{R_0^2} \frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31, \Omega_{D,0} = 0.69, \Omega_{R,0} = 9 \times 10^{-5}, H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: $1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$

Questions

1. If H were constant with time (i.e., $H = H_0$ always), how long would it take (in years) for the separation between the Milky Way and a very distant galaxy to double in size?
2. If I were to have a 20 cm length of elastic, and I stretched it so that one end moved at 1 cm s^{-1} and the other end remained stationary, what is the value of the Hubble parameter of the elastic (in $\text{km s}^{-1} \text{ Mpc}^{-1}$) at the instant I start stretching it?
3. If I continue to pull the end of the elastic at the same rate (i.e., so that one end moves at a constant 1 cm s^{-1}), will the Hubble parameter of the elastic increase, decrease or stay the same?
4. What will be the value of the Hubble parameter of the elastic after 10 s?
5. Why can a universe only take one of three possible “shapes” (i.e., flat, positively curved, or negatively curved)?
6. If I observe the Ly α emission line (rest wavelength 1216 Å) of a distant galaxy at 1787.52 Å, what was the scale factor of the Universe when those Ly α photons were emitted?