Cosmology Lecture 13

Inflation

In this lecture

- Problems with The Benchmark Model;
- What is inflation?;
- How a period of rapid inflation solves these problems;
- Problems with the inflationary model.

The success of The Benchmark Model

To date, we've only considered the successes of the Benchmark/Big Bang model of the Universe:

- its successful prediction of the CMB;
- its agreement with Type 1a SN data;
- its ability to reproduce acoustic "wiggles" in the CMB;
- nucleosynthesis and the lack of sub-26% He regions.

Despite this…all is not well with the Big Bang model as we've currently described it.

Three problems with the Big Bang model

Soon after its strengths were appreciated, cosmologists realised there were three key problems with the Big Bang model:

- The flatness problem;
  "The Universe is nearly flat today, and was even flatter in the past."

- The horizon problem;
  "The Universe is nearly homogeneous and isotropic today, despite being causally unconnected."

- The monopole problem.
  "Where are all the magnetic monopoles!"

The flatness problem

Recall from Lecture 4 that:

$$1 - \Omega(t) = -\kappa\left(\frac{c}{a(t)H(t)R_0}\right)^2 \quad \text{and} \quad 1 - \Omega_0 = -\kappa\left(\frac{c}{H_0 R_0}\right)^2$$

According to our measurements, the current Universe is flat to within 0.5%, meaning: $1 - \Omega_0 < 0.005$

Equivalent to having an A4 sheet of paper, and curving it up so its edges are about 1mm higher than the centre.

But, what does this <0.5% mean for earlier times? Has the curvature remained constant?

Placing limits on curvature at earlier times

Recall from Lecture 4 that:

$$1 - \Omega(t) = -\kappa\left(\frac{c}{a(t)H(t)R_0}\right)^2 \quad \text{and} \quad 1 - \Omega_0 = -\kappa\left(\frac{c}{H_0 R_0}\right)^2$$

Subbing one into the other gives: $1 - \Omega(t) = \left(\frac{H_0}{H(t)}\right)^2 \frac{1-\Omega_0}{a(t)^2}$

At early times, the Universe was radiation-dominated, meaning:

$$\left(\frac{H(t)}{H_0}\right)^2 = \frac{\Omega_0}{a(t)^4} \quad \text{so:} \quad 1 - \Omega(t) = a(t)^2\frac{(1-\Omega_0)}{\Omega_0}$$

As we go further and further back in time, a(t) gets smaller, meaning the Universe gets flatter and flatter.

By nucleosynthesis, when $a(t_{nuc}) = 3.6\times10^{-9}$:

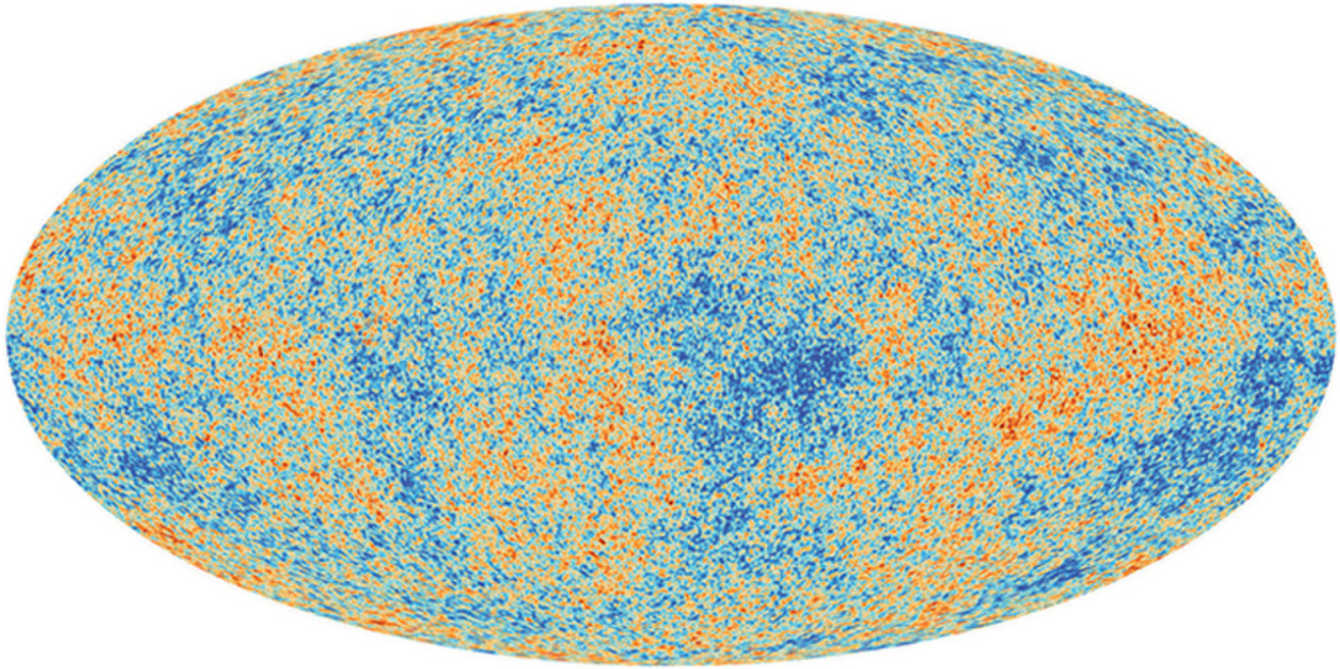
$$1 - \Omega(t_{\mathrm{nuc}}) < 7 \times 10^{-16}$$

The flatness of the Universe at early times…

Equivalent to having an A4 sheet of paper, and curving it up so its edges are about 1 *proton's diameter* higher than the centre.

The Horizon Problem

Consider the Cosmic Microwave Background…

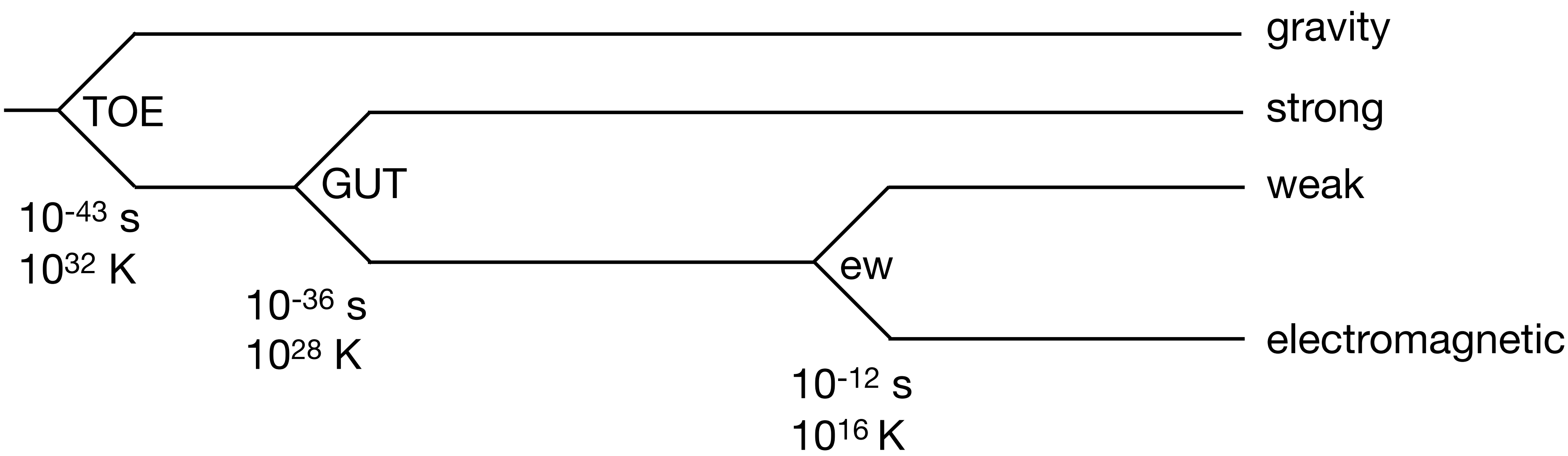


- Recall from Lecture 11 that only points within 1 degree of each other were causally connected at the time of recombination.
- Yet, the whole Universe was the same temperature to within one part in 10^5, despite only those points within 1 degree of each other being able to reach thermal equilibrium.
- There are 40,000 1degree patches of the CMB, and they're all the same temperature to within 1 part in 10^5.
- That's like asking every person in a large sports stadium to pick a card from a deck containing 10^5 *different* cards, and they all picking the same card!

The Monopole Problem

The Monopole Problem comes from a prediction of the Grand Unified Theory (GUT).

GUT predicts that - at high enough energies - the strong, weak and electromagnetic forces unify as a single force.



Each time a force splits from the others, the Universe goes through a phase transition as symmetry is lost due to lowering temperatures.

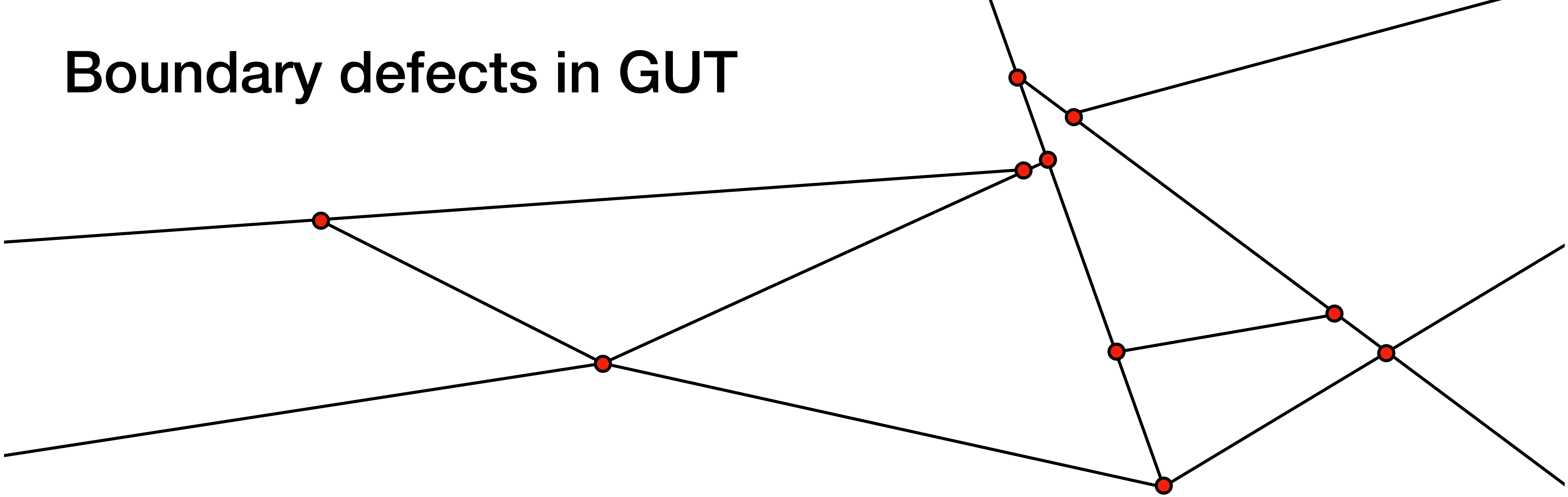
WTF????

"…the Universe goes through a phase transition as symmetry is lost due to lowering temperatures."

A liquid has a high degree of symmetry, no matter how much you rotate it, it looks the same.

But as it cools and forms crystals, it loses that symmetry, and forms boundaries between crystals.

Boundary defects in GUT



GUT predicts that the strong-weak phase transition creates point-like defects that act as *magnetic monopoles*.

It also predicts that these magnetic monopoles are very massive; around 10^{12} TeV (about the mass of a bacterium).

And at the time of the GUT phase transition, there should be one per horizon volume - about 10^{82} m^{-3}.

What density does that correspond to today? Clue: $T_{GUT}=10^{28}$ K

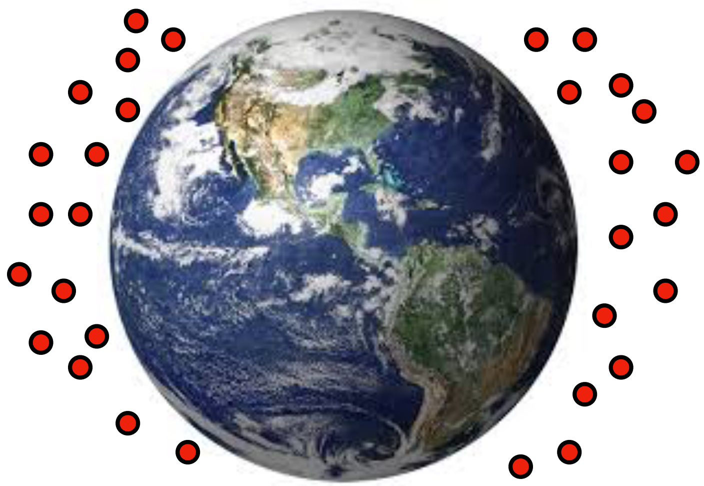
Today's magnetic monopoles

GUT predicts that there should be, on average, 0.2 magnetic monopoles per m^3 today.

Compared to an average Baryonic density of 0.25 protons or neutrons per cubic m.

But, magnetic monopoles are very massive, so they'll cluster in response to gravity like Baryons do.

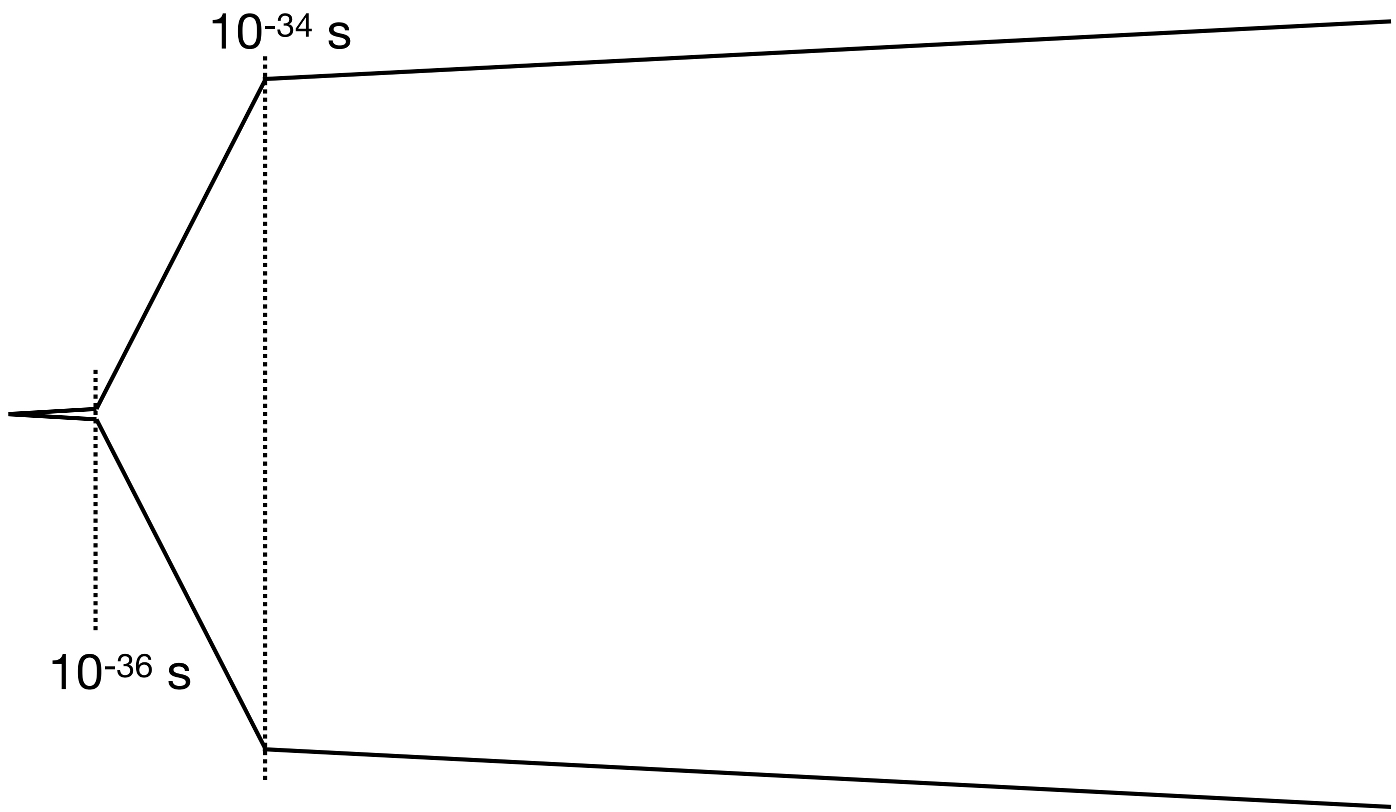
So the density of magnetic monopoles should roughly be the same as Baryons in the vicinity of the Earth.



Inflation: A solution to the problems

The Theory of Inflation was devised to solve the flatness, horizon and monopole problems all in one go.

It states that very early-on in the history of the Universe, just after the GUT time, it was dominated by a positive "cosmological constant" akin to, but much larger than, today's Dark Energy.



How does inflation solve the flatness problem?

Inflation's cosmological constant would have an energy density:

$$\varepsilon_\Lambda = \Lambda \quad \text{meaning} \quad H^2 = \frac{8\pi G}{3c^2}\Lambda = \text{constant}$$

And from Lecture 6, we know that a constant energy density gives:

$$a(t) \propto e^{Ht}$$

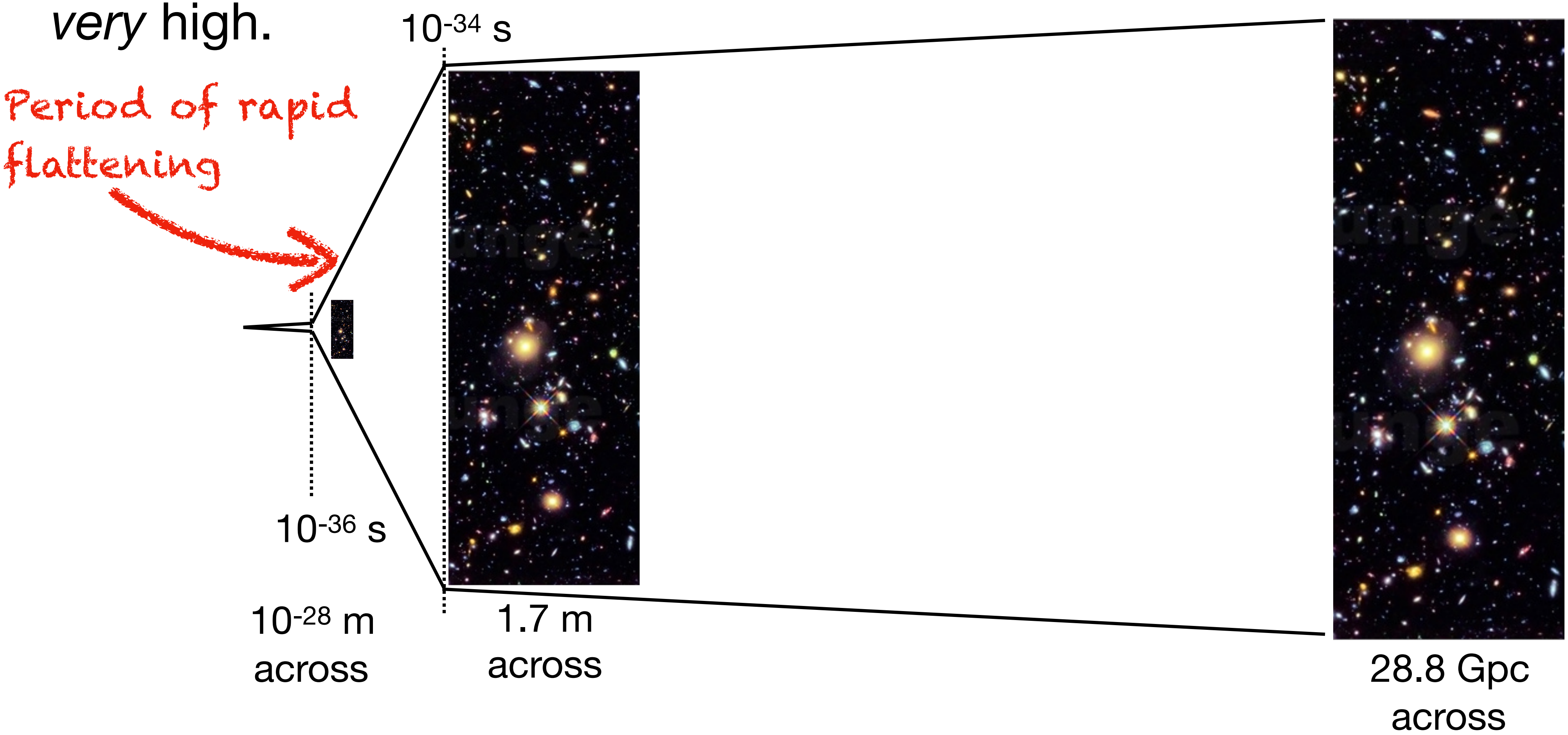
Recall that curvature is given by:

$$1 - \Omega(t) = \left(\frac{H_0}{H(t)}\right)^2 \frac{1 - \Omega_0}{a(t)^2}$$

With H(t) = constant.

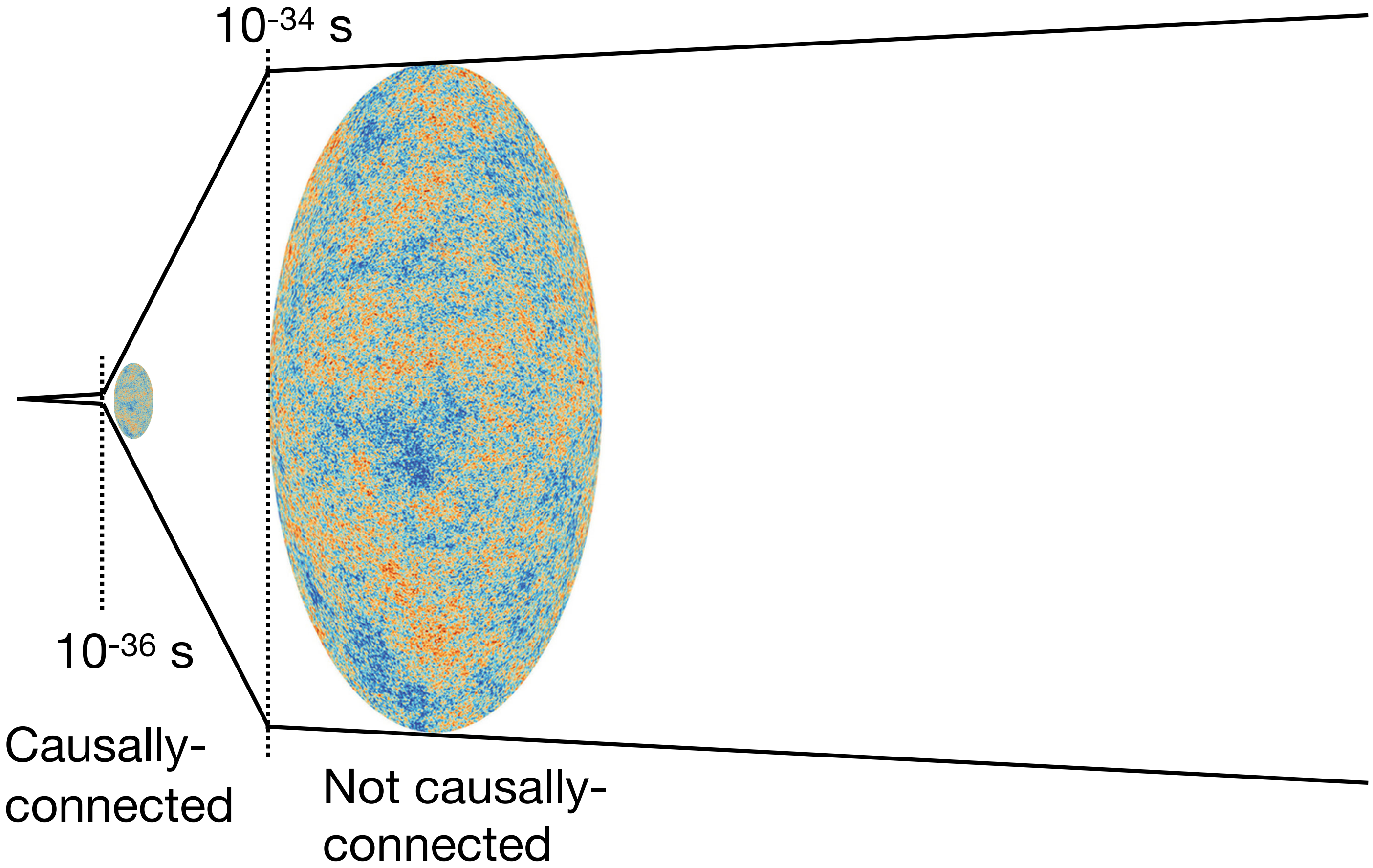
How does inflation solve the flatness problem?

Inflation is able to flatten the Universe very effectively in a short period of time, provided that the energy density during inflation is *very* high.



During the 10^{-34} s epoch of inflation, the scale factor of the Universe increased by a factor of e64=6.2x10^{27}. Coincidentally, this is roughly the same factor increase as over the past 13.6 Gyr!

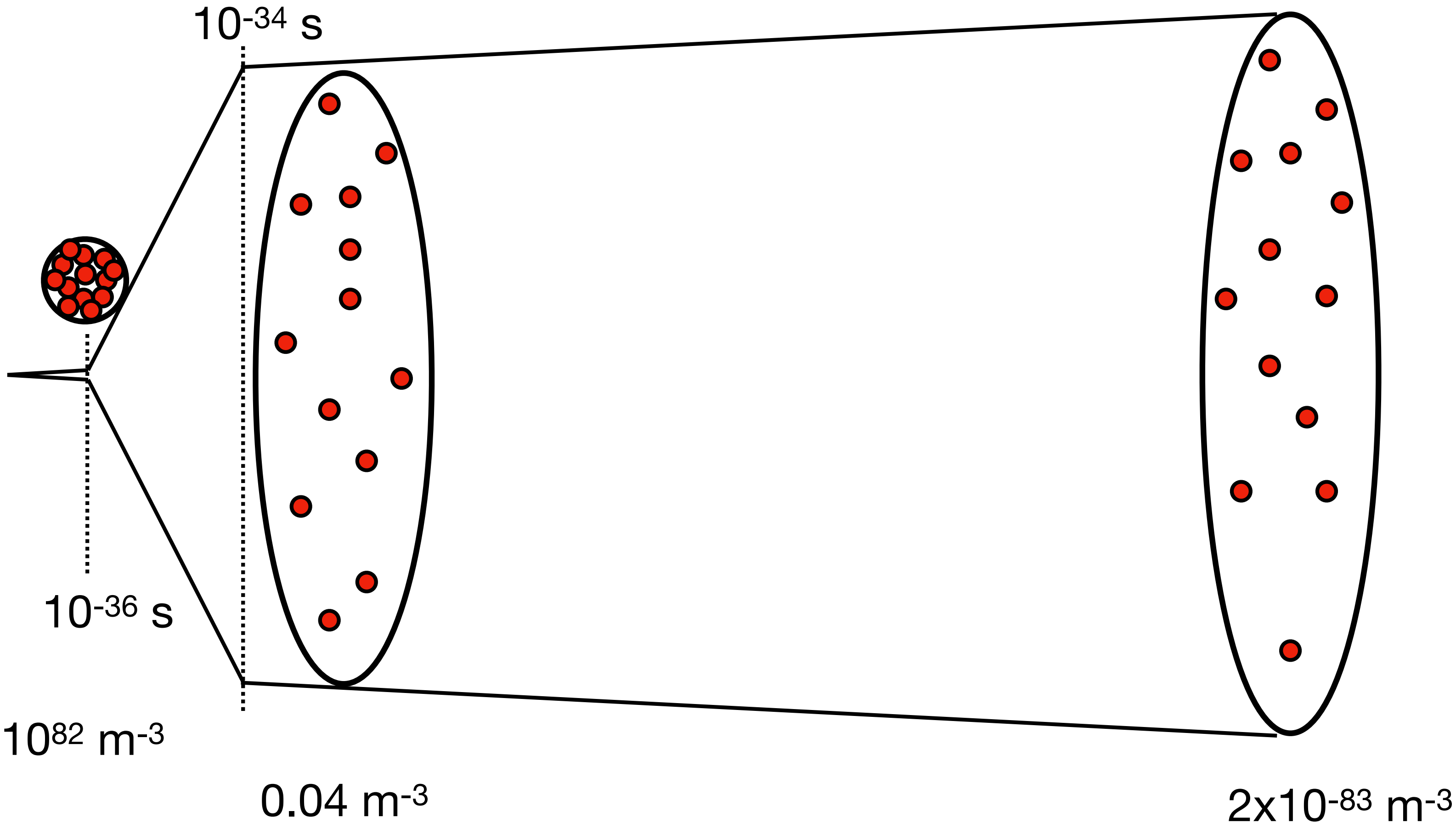
How does inflation solve the Horizon problem?



In the 10^{-36} s prior to inflation, the growth of the Universe was slow enough that the whole observable Universe was able to exchange information with itself.

Inflation then blasted these regions apart, out of causal connection with each other, but maintained the imprint of isotropy.

How does inflation solve the Monopole Problem?



Since magnetic monopoles are a form of matter, their density falls as a^{-3}.

The factor of 6.2×10^{27} increase in scale factor during the epoch of inflation reduces their density from 10^{82} m^{-3} to 0.04 m^{-3}.

The post-inflation expansion of the Universe reduces this further to 1 monopole per 10^{82} m^3. The volume of the observable Universe is 3×10^{80} m^3!

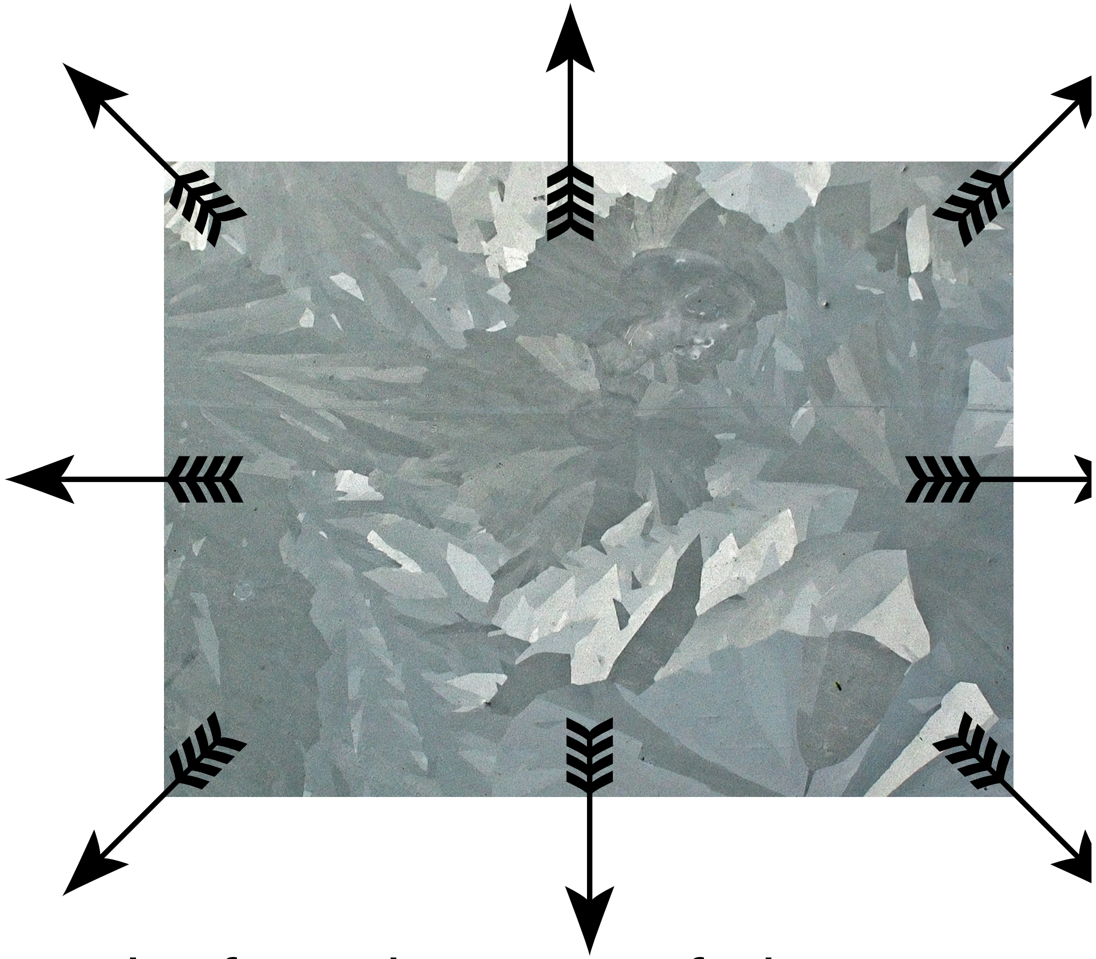
Why isn't the Universe cold and empty?

Inflation is very good at addressing the flatness, horizon and monopole problems.

Immediately, however, it faces two problems:

- Why doesn't it cause the density of all other particles, including photons, to drop catastrophically?
- If the temperature of the Universe prior to inflation was 10^{28} K, how come it wasn't $10^{28}/6.2\text{x}10^{27}=1.6$ K after inflation?

Phase transitions again…



It is thought that inflation is the result of another type of phase transition.

When phase transitions occur, energy is released. This is the same as when energy is released when water freezes to ice.

This phase transition pumped energy back into the Universe as photons, which then pair-produced baryons, electrons etc.

Getting the feel of it…

- The flatness of today's Universe corresponds to 1 part in about 10^{15} at the time of nucleosynthesis.
- The Universe is isotropic to 1 part in 10^{5}, even though it's not causally-connected.
- There are no magnetic monopoles.
- These are all problems for the standard Big Bang Model.
- Inflation solves this by increasing the scale factor of the Universe by a factor of 6.2×10^{27} within 10^{-34} s.
- This flattens the Universe, causes once causally-connected regions of the Universe to become non-causally connected, and drops the density of magnetic monopoles to undetectable levels.