Problems Class I

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Equations and constants

The Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$

The Fluid Equation:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Cosmological parameter values in The Benchmark Model:

$$\Omega_{M,0} = 0.31,\ \Omega_{D,0} = 0.69,\ \Omega_{R,0} = 9 \times 10^{-5},\ H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Parsec in SI units: $1 \text{ pc} = 3.09 \times 10^{16}$ m

Questions

1. In the matter-only and radiation-only models of the universe, the scale factor, a, at a given time is given by:

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3\omega}}. \tag{1}$$

   Starting with the above equation, obtain expressions for how the angular distance changes as a function of redshift for (a) matter-only and (b) radiation-only universes.

2. Assuming that the distribution of Dark Matter around a galaxy is spherically symmetric, obtain an expression for how the density of dark matter must depend on radius to ensure a flat galaxy rotation curve.

3. So far in the course, we've only considered flat universes. However, it's not much of a challenge to extend our models to include curvature. To demonstrate this, show that the Hubble parameter of a curved, radiation-dominated universe can be described as:

$$\frac{H^2}{H_0^2} = \frac{\Omega_0}{a^4} + \frac{1-\Omega_0}{a^2} \tag{2}$$

   What will be the maximum scale factor of this universe?

4. (a) Explain to a colleague (sitting next to you if you're in the problems class) why assuming that recombination occurs when the *average* photon energy is 13.6 eV gives an innacurate estimate of when this occurs. Include in your explanation whether this assumption leads to too early or too late an estimate compared to the real time of recombination.

   (b) In return, your colleague should explain to you how cosmologists obtain a more precise estimate of the time of recombination. They should include in their answer how cosmologists obtain the scale factor of the Universe at the time of recombination.

5. The Friedmann Equation tells us the *rate of change* of the scale factor, a. By differentiating it with respect to time, we obtain an expression for the *acceleration* of the Universe. Show that this expression can be written as:

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}\left(\frac{\dot{\varepsilon}a}{\dot{a}} + 2\varepsilon\right) \tag{3}$$