Kernel Methods and Automata

Anton Xue

1 Introduction

In this sketch we study the relation between graph kernels and finite state machines. We study previous work in algebraic formulation for automata theory and kernel methods, with an interest in how these techniques may be extended to formal methods.

2 Preliminaries

2.1 Algebraic Automata Theory

Previous work in formulation in algebraic foundations for automata theory exist in literature [2], and much of our notation is taken from [1]. We overload notation for addition ($+$) and multiplication ($\cdot$) in algebraic structures when possible to avoid clutter. Similarly, the additive identity (0) and multiplicative identity (1) are also overloaded when possible.

Definition 1 (Weighted Finite State Transducer)**.** *A weighted finite state transducer over a semiring $\mathbb{K}$ is an 8-tuple $(\Sigma_I, \Sigma_O, Q, I, F, \Delta, \lambda, \rho)$ where*

- Σ_I *is a finite input alphabet.*
- Σ_O *is a finite output alphabet.*
- Q *is a finite set of states.*
- $I \subseteq Q$ *is the set of starting states.*
- $F \subseteq Q$ *is the set of final states.*
- $\Delta \subseteq Q \times (\Sigma_I \cup \{\varepsilon\}) \times \mathbb{K} \times (\Sigma_O \cup \{\varepsilon\}) \times Q$ *is the transition function weighted by* $\mathbb{K}$.
- $\lambda : I \to \mathbb{K}$ *is the initial state weight function.*
- $\rho : F \to \mathbb{K}$ *is the final state weight function.*

2.2 Reproducing Kernel Hilbert Spaces

3 Automata Embeddings

4 Bounding Inner Products

References

[1] Corinna Cortes, Patrick Haffner, and Mehryar Mohri. "Rational kernels: Theory and algorithms". In: *Journal of Machine Learning Research* 5.Aug (2004), pp. 1035–1062.

[2] Werner Kuich and Arto Salomaa. *Semirings, automata, languages*. Vol. 5. Springer Science & Business Media, 2012.