

Automata and Some Generalizations

0.1 Deterministic Finite Automata

A deterministic finite automata is often presented as a quintuple:

$$\mathcal{A} = (\Sigma, Q, \delta, s, F)$$

Where Σ is a finite alphabet, Q is a finite set of states, $\delta: \Sigma \times Q \rightarrow Q$ is the transition function, $s \in Q$ is a special initial state, and $F \subseteq Q$ is the set of final states.

One particular view may see δ as a matrix M , where the entry $M_{i,j} \subseteq \Sigma$ is the letters of Σ that takes state $q_i \in Q$ to $q_j \in Q$.

0.2 Non-deterministic Finite Automata

A non-deterministic finite automata may also be presented as a quintuple:

$$\mathcal{A} = (\Sigma, Q, \Delta, s, F)$$

As with before, Σ is a finite alphabet, Q is a finite set of states, $s \in Q$ is a special start state, and $F \subseteq Q$ is the set of final states. What is different here from deterministic finite automata, is that the transition function $\Delta: \Sigma \times Q \rightarrow \mathcal{P}(Q)$ maps into a set of states rather than a single state. This represents the “non-deterministic” nature of a transition, where a single letter read may induce multiple possible states.

As with before, Δ may be seen as a matrix M . The presentation is the same as that of a deterministic finite automata's, where $M_{i,j} \subseteq \Sigma$ is the letters of Σ that take $q_i \in Q$ to $q_j \in Q$.

0.3 Semiring Automata

The semiring automata is a formalization of a non-deterministic finite automata using algebraic structures. Take the semiring automata to be a quintuple:

$$\mathcal{A} = (\Sigma, Q, \Delta, s, F)$$

Like before, Σ is a finite alphabet that generates the free semiring R , Q is a countable set of states, $\Delta: R \times Q \rightarrow \mathcal{P}(Q)$ is the transition function, $s \in Q$ is the special start state, and $F \subseteq Q$ is the set of final states.

Similar to before, if Q is finite, the matrix representation M of the transition function Δ has entry $M_{i,j} \in R$ as the elements that map state $q_i \in Q$ to $q_j \in Q$. Note that we now permit each entry of the transition matrix to be an arbitrary element of R rather than a set of single letters of Σ .

0.4 Division Semiring Automata

The division semiring automata introduces the notion of “negative” words through multiplicative inverses. Similar to the semiring automata, we take the division semiring automata as the quintuple:

$$\mathcal{A} = (\Sigma, Q, \Delta, s, F)$$

Here, Σ is a finite alphabet that generates the free division semiring R , Q is a countable set of states, $\Delta: R \times Q \rightarrow \mathcal{P}(Q)$ is the transition function, $s \in Q$ is the special start state, and $F \subseteq Q$ is the set of final states.

If Q is finite, let M be the matrix representation of the transition function Δ , where $M_{i,j} \in R$ maps state $q_i \in Q$ to $q_j \in Q$.