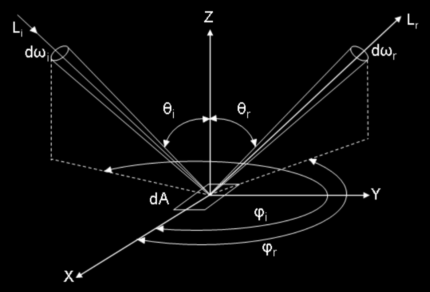
Radiometry and Fresnel Reflectance



70001 – Advanced Computer Graphics: Photographic Image Synthesis

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Lecture 07, Feb. 02nd 2024

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Radiometry & Geometric Optics

- Light transport modeled using geometric or ray optics
  - light as particle, not wave!
  - some exceptions, i.e., polarization
- Basic properties of geometric optics:
  - Linearity
  - Energy conservation

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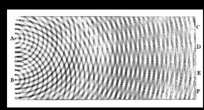
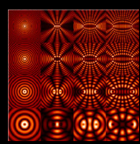
Radiometry & Geometric Optics

- Typical assumptions:
  - No polarization of electromagnetic field
  - No fluorescence
    - wavelength independence
  - Steady state
    - no phosphorescence

- Wave like effects not modeled!
  - diffraction
  - interference





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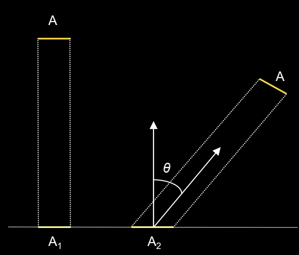
Lambert's Law

- Irradiance E proportional to cosine of the angle between light direction l and surface normal n

$$E = d\Phi/dA,$$

hence $E_1 = \Phi/A,$

and $E_2 = \Phi\cos\theta/A.$



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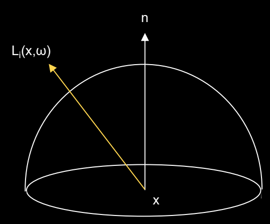
Radiometric Integrals



- $E(x,n) = \int_\Omega L_i(x,\omega) \cos\theta \, d\omega.$
  - Irradiance at surfaces computed over a hemisphere of directions
  - Volumetric integrals computed about a sphere of directions

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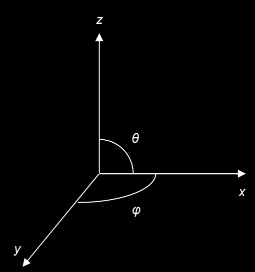
Spherical Coordinates

- Direction vector (x,y,z) related to spherical coordinates (θ, φ):

$$x = \sin\theta \cos\varphi$$

$$y = \sin\theta \sin\varphi$$

$$z = \cos\theta.$$



- Hemispherical integral

$$E(x,n) = \int_0^{2\pi} \int_0^{\pi/2} L_i(x, \theta, \varphi) \cos\theta \sin\theta \, d\theta \, d\varphi.$$

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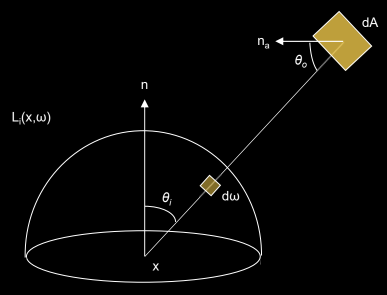
Area Integral



- $E(x,n) = \int_\Omega L \cos\theta_i \, d\omega$
  $= \int_A L \cos\theta_i \cos\theta_o \, dA / r^2$.
  - useful for area light sources!

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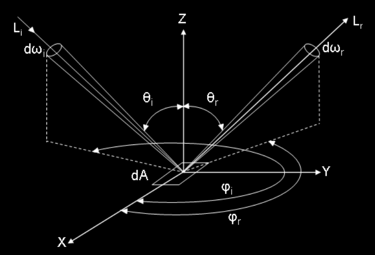
BRDF



- Defined as the ratio of reflected radiance to incident irradiance:
  $f_r(x, \omega_r, \omega_i) = dL_r(x, \omega_r)/dE_i(x, \omega_i)$
  $= dL_r(x, \omega_r)/(L_i(x, \omega_i) \cos\theta \, d\omega_i)$.
  - the units of a BRDF are inverse steradian [1/sr].

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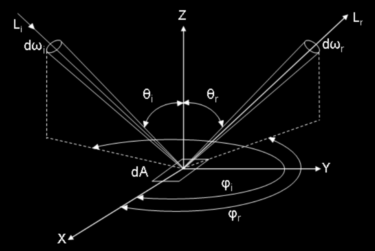
BRDF



- Physically based BRDFs have 2 important properties:

  Helmholtz Reciprocity: $f_r(x, \omega_r, \omega_i) = f_r(x, \omega_i, \omega_r)$.

  and

  Energy Conservation: $\int_\Omega f_r(x, \omega_r, \omega_i) \cos\theta_i \, d\omega_i \leq 1$, for all ω_r in Ω.

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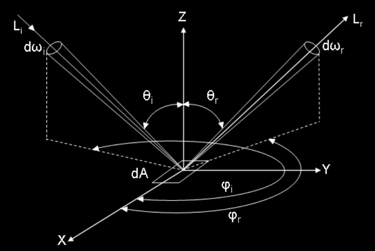
Reflected Radiance



- The Rendering Equation [Kajiya 86]:

  $L_r(x, \omega_r) = \int_\Omega f_r(x, \omega_r, \omega_i) \, L_i(x, \omega_i) \cos\theta_i \, d\omega_i$.

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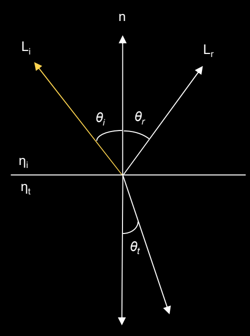
Snell's Law

- Perfect specular reflection: $\theta_i = \theta_r$



- Specular transmission
  - depends on index of refraction

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t.$$

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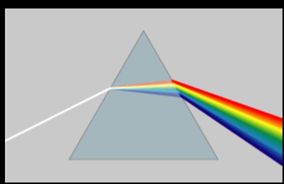
Dispersion

- Index of refraction is wavelength dependent!



  - different wavelengths refract by differing amount causing dispersion of light!

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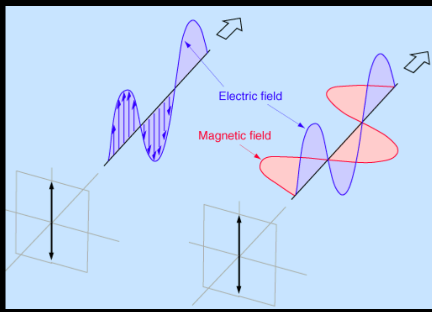
Polarization

- Light a transverse electromagnetic wave
  - natural state un-polarized
- Linear polarization
  - electric field in fixed plane



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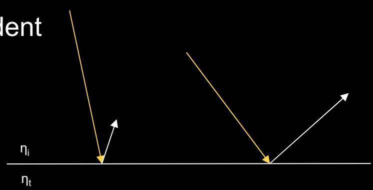
Fresnel Reflectance

- Reflection from a surface is view dependent
- Fresnel equations
  - Maxwell's equations at smooth surfaces
  - index of refraction and polarization!
- Two kinds of Fresnel equations:
  - Dielectric materials (insulators) – reflection & transmission
  - Conductors (metals) – only reflection & some absorption



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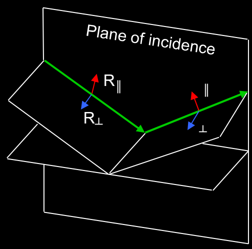
Dielectrics Fresnel



- Fresnel reflectance for parallel polarized light $r_\parallel$:

$$R_\parallel = \left| \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t} \right|^2$$

- Fresnel reflectance for perpendicular polarized light $r_\perp$:

$$R_\perp = \left| \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t} \right|^2$$

- Unpolarized reflectance $F_r = ½(R_\parallel + R_\perp)$.
  - Transmittance $T_r = 1 – F_r$.

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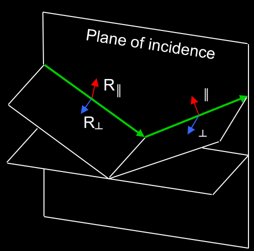
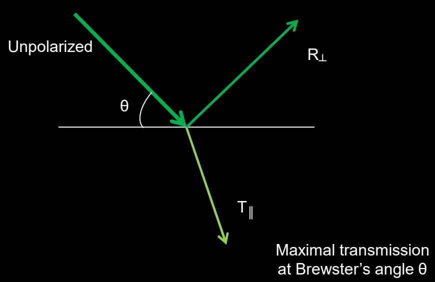
Polarized reflection in dielectrics





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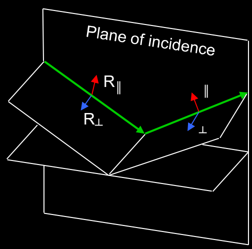
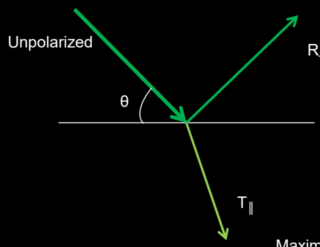
Polarized reflection in dielectrics





Maximal transmission at Brewster's angle θ

0° 90°

Polarized surface reflection on ground surface under natural daylight illumination

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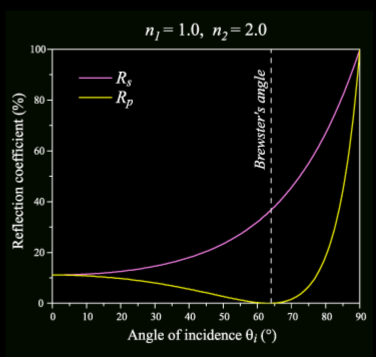
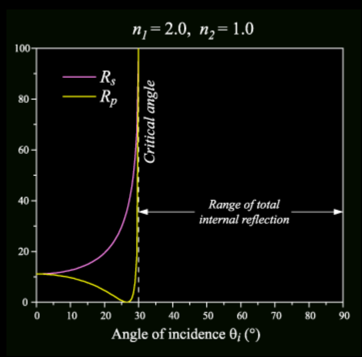
Dielectrics





- R_p – parallel polarized, R_s – perpendicular polarized
- R_p goes to 0 at Brewster's angle
- R_p and R_s go to 1 at Critical angle (total internal reflection)
  - Transmittance violates reciprocity!

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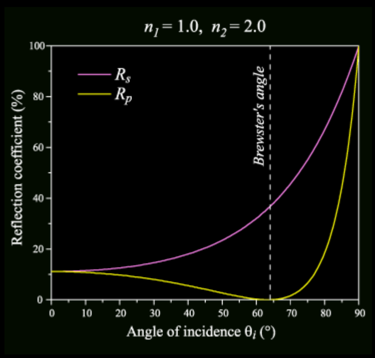
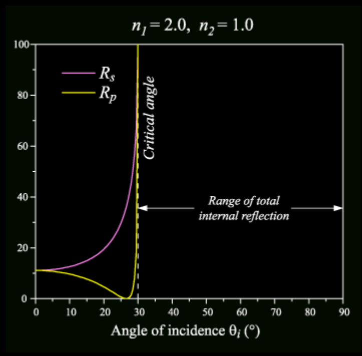
Dielectrics Fresnel angles





- Brewster's angle θ_B : $\tan(\theta_B) = (n_2/n_1)$
- Critical angle θ_C : $\sin(\theta_C) = (n_2/n_1)$

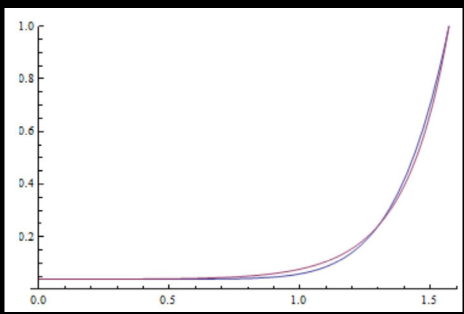
Schlick's Approximation

- Polynomial approximation of unpolarized Fresnel reflectance [Schlick 93] :

  $F_r(\cos\theta) = R_0 + (1 - R_0)(1 - \cos\theta)^5$,

  where R_0 is reflectance at normal incidence

  and $\cos\theta = h \cdot v = h \cdot l$



- Advantage: does not need estimate of index of refraction!

Conductors Fresnel

- No transmission, but some absorption k:

$$R_{\parallel} = \frac{(\eta^2 + k^2)\cos\theta_i^{\,2} - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos\theta_i^{\,2} + 2\eta\cos\theta_i + 1}$$

And

$$R_{\perp} = \frac{(\eta^2 + k^2) - 2\eta\cos\theta_i + \cos\theta_i^{\,2}}{(\eta^2 + k^2) + 2\eta\cos\theta_i + \cos\theta_i^{\,2}}$$

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Conductors

- Much less data on η and k available for conductors
- In computer graphics, approximate one by fixing the other!

if k = 0, and $\cos\theta_i = 1$ (normal incidence),

$$R_{\parallel} = R_{\perp} = \frac{\eta^2 - 2\eta + 1}{\eta^2 + 2\eta + 1} = \frac{(\eta - 1)^2}{(\eta + 1)^2}$$

if reflectance at normal incidence R_0 is known, then

$$\eta = \frac{1 + \sqrt{R_0}}{1 - \sqrt{R_0}}$$

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