70001 Tutorial 3

1 Sampling a 1D function

You are given a distribution of 10 values as follows: D = [15, 5, 5, 25, 5, 10, 15, 5, 5, 10].
Given four randomly drawn variables: $\mu_1 = 0.5, \mu_2 = 0.25, \mu_3 = 0.9, \mu_4 = 0.65$, what will be the correspondingly selected values when sampled with the given random variables?

The first step is to get a pdf from the given distrbution, by dividing each value by the sum of all values in the series:

$$p(x) = \frac{D}{\sum_i D_i} = [.15, .05, .05, .25, .05, .1, .15, .05, .05, .1]$$

From there, we can construct the corresponding cdf and apply the inversion method:

$$P_i(x) = \sum_{j=0}^{i} p_j(x)$$

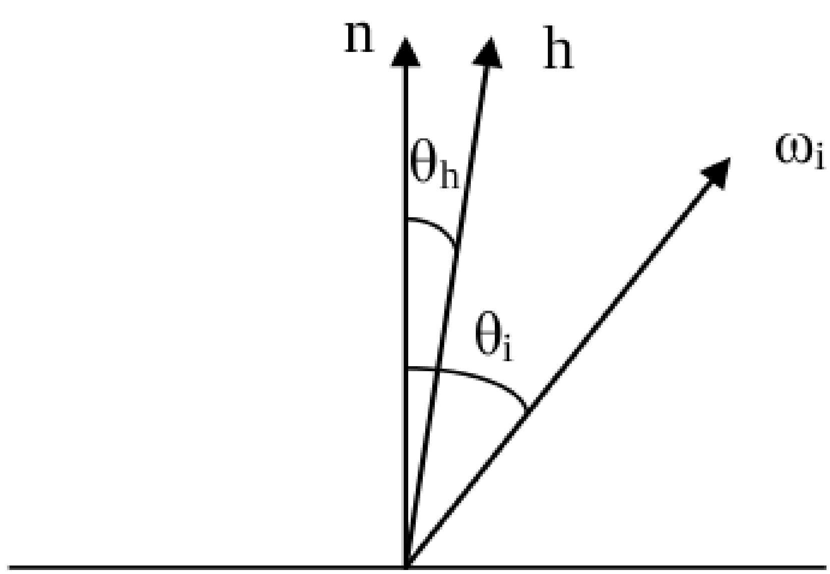
$$P(x) = [.15, .2, .25, .5, .55, .65, .8, .85, .9, 1]$$

The inversion method, given a random number $u \sim U[0, 1]$, consists of finding the entry in the cdf (P(x)) that is closest to u. The index of that entry in the cdf can then be used to look up a value in the distribution D, that is distributed according to p(x):

- $\mu_1 \to D_4 = 25$
- $\mu_2 \to D_3 = 5$
- $\mu_3 \to D_9 = 5$
- $\mu_4 \to D_6 = 10$

2 Microfacet BRDF

Given a microfacet BRDF, assume a sample h being drawn from the proposal distribution has probability $p(\omega_h) = 0.5$. Compute the final weight of the sample $\boldsymbol{p(\omega_r)}$ for Monte Carlo rendering of direct illumination. Assume the following configuration:



Here, **n**, **h** and $\boldsymbol{\omega_i}$ are in the same plane and $\boldsymbol{\arccos(n.h) = \theta_h = 15\tilde{A}\hat{A}}$, and $\boldsymbol{\arccos(n.\omega_i) = \theta_i = 45\tilde{A}\hat{A}}$.

$$\begin{aligned} \boldsymbol{p(\omega_r)} &= \boldsymbol{\frac{p(\omega_h)}{4\cos(\theta_i - \theta_h)}} \\ &\approx \mathbf{.144} \end{aligned}$$

Also derive the relation between p($\boldsymbol{\omega_h}$) and p($\boldsymbol{\omega_r}$) using change of variables. (**Hint:** compute $\boldsymbol{\frac{d\omega_h}{d\omega_i}}$).

By definition, ω_h is the vector halfway between ω_i and ω_r, hence if we define θ_h to be the angle between ω_i and ω_h and θ_i to be the angle between ω_i and ω_r, we have:

$$\begin{aligned} \theta_i &= 2\theta_h \\ \phi_i &= \phi_h \quad \text{(All three vectors lie in the same plane)} \end{aligned}$$

We have:

$$p(\omega_r) = p(\omega_h)\frac{d\omega_h}{d\omega_i}$$

$$\begin{aligned}
\frac{d\omega_h}{d\omega_i} &= \frac{\sin\theta_h \; d\theta_h \; d\phi_h}{\sin\theta_i \; d\theta_i \; d\phi_i} \\
&= \frac{\sin\theta_h \; d\theta_h \; d\phi_h}{\sin 2\theta_h \; 2d\theta_h \; d\phi_h} \qquad (\theta_i = 2\theta_h, \phi_i = \phi_h) \\
&= \frac{\sin\theta_h}{4\cos\theta_h \sin\theta_h} \qquad (2\sin 2\theta_h = 4\cos\theta_h \sin\theta_h) \\
&= \frac{1}{4(\omega_i.\omega_h)} = \frac{1}{4(\omega_r.\omega_h)} \quad ((\omega_i.\omega_h) = (\omega_r.\omega_h))
\end{aligned}$$

It then follows that $p(\omega_r)$ and $p(\omega_h)$ are related by:

$$p(\omega_r) = \frac{p(\omega_h)}{4(\omega_r.\omega_h)}$$

3 BRDF Importance Sampling - Ward isotropic lobe

Given an isotropic Ward BRDF with distribution defined as: $p(\omega_h) \sim \exp\frac{-\tan^2\theta_h}{\alpha^2}$, where ω_h is the half vector and α is the specular roughness. Given two random variables $u_1 = 0.4$ and $u_2 = 0.2$, compute the sampled half vector ω_h in terms of its spherical coordinates (θ_h, ϕ_h) for a Ward lobe with roughness $\alpha = 0.2$.

$$\theta_h = \arctan\alpha\sqrt{-log u_1} \approx 10.84^\circ$$
$$\phi_h = 2\pi u_2 \approx 72^\circ$$