Devising an explicit algorithm based on simple rules is difficult! L1 reg: $\ell = err(y, \hat{y}) + \lambda \sum_{i=1}^{N} |w_i|$ favours few non-0 coefs, L2 favours small coefs
under-fitting → high bias (high training, high test error) → add features, decrease regularization term λ, increase degree of polynomial)
over-fitting → high variance (low training, high test error) → get more data, remove features, increase regularization term λ, decrease degree of polynom)

Challenges in Semantic Segmentation (every pixel in an image belongs to a class)

- *noise* – high-frequency pixel variability (not relevant/may obscure target)
- *partial volume* – quantized version of object (pixels may contain mix of two objects and both contribute to pixel value) and object may be elevated (unclear where to begin/end object)
- *intensity inhomogeneities* – varying contrast and intensity differences across the image plain
- *anisotropic resolution* – (not isotropic, where voxels are cubes) causes ↓ clarity in coarse dims
- *imaging artifacts* – implants may interfere with imaging modality
- *limited contrasts* – different tissues may have similar physical properties and leak boundaries
- *morphological variability* – variability in physiological conditions or imaging modalities

Pitfalls in Segmentation Evaluation

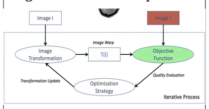
- *Structure Size* – equal differences between small and big structures change spatial overlap lots
- *Structure Shape* – spatial overlap metrics are unaware of complex shapes
- *Spatial alignment* – HD & DSC & IoU don't capture object centre point alignment
- *Holes* – Boundary-based metrics ignore overlap between structures
- *Noise* – Affects HD as it is spiked by a far away FP
- *Empty Label-Maps* – scores of 0 or NaN for each method with combo of empty ref. or predict.
- *Resolution* – same prediction shapes at different resolutions give different results
- *Over vs Under-segmentation* – for equal HD, DSC may be better for over than undersegment.

Segmentation Methods

- *Intensity Segmentation:* (hist) × regions must be homogenous, leakages, threshold loc. Hard
- *Region Based*: (start from seed) × requires user points, leakages, assumed homogeneity
- *Atlas Based:* (averaged templates) *Registration:* mutate multiple atlases into target and fuse labels (majority voting) (this saves pdf indicating contention between sources). √ robust, accurate, automatic × comput, expensive, poor for abnormalities, not for tumour segmentation.
- *Random Forests:* different modalities of 1 image, construct a tree to classify a pixel based on rules. Ensemble it by averaging answer of many trees. × no hierarchal features √ || & accurate

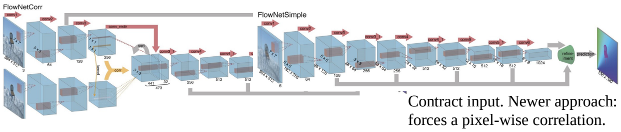
- (up)pooling and max (un)pooling with stored spatial location
- Transposed Convolution:
$$(M-1) \times S - 2P + D \times (K-1) + P_{output} + 1$$
- Convolution: $\left\lfloor \frac{M+2P-D*(K-1)-1}{S} \right\rfloor + 1$
- Atrous spatial pyramid pooling: repeated max-pooling and striding reduces spatial resolution of the resulting feature map
- Padding during upsampling may introduce artifacts

Registration iterative process



Gradient descent or SGD.

FlowNet: tries to predict dense displacement field between two video frames.



Contract input. Newer approach: forces a pixel-wise correlation.

Then upsample for p.w. displacement. Train w/ flying rendered chairs (you know Ground Truth). Next evolution of **FlowNet2.0** passes image through once, applies the translation, and passes it into the next layer for further fine-tuning. In parallel, there is detailed matching, then concat all.
Optical Flow with Semantic Segmentation and Localized Layers: segment 'Things', 'Planes' & 'Stuff'. Then perform flow estimation on segmented objects for a sharper answer.
Non-rigid Image Registration Using Multi-scale 3D CNNs: randomly deform an image, then train a model to predict your known deformation. To use this network, you need to slide this network accross the image and generate for each pixel a displacement vector.
Spatial Transformer Networks: takes feature map (original or pre-processed) and predicts transformation and transform the image according to this transform map. There is a localisation net which trains θ to then deform the grid.
Unsupervised Deformable Image Registration: Two images are fed into an NN to predict deformation. Then feed into spatial transformer, this transforms input and calculates sim. metric.
Voxel Morph: u-net architecture which produces a dense displacement field. Then it uses the spacial transformer to warp the image to the fixed image then minimise the loss to the network.

Output size = , # of param $C \times K \times K$

Expert Gold Standard: × training, tedious, intra (same dude) + inter (diff dude) observability variability, √ multiple segmentations, agreement can be quantified

- specificity = $\frac{TN}{N} = \frac{TN}{TN+FP}$
- Fβ = $(1+\beta^2)\frac{Precision \cdot Recall}{(\beta^2 \cdot Precision)+Recall}$
- Jaccard Index/ IoU = $\frac{|S_g \cap S_p|}{|S_g \cup S_p|} = \frac{DSC}{2-DSC}$
- Dice Sim. Coeff. = $2\frac{|S_g \cap S_p|}{|S_g|+|S_p|} = F_1$
- Volume Sim = $1 - \frac{||S_g|-|S_p||}{|S_g|+|S_p|} = 1 - \frac{|FN-FP|}{2TP+FP+FFN}$

surface distance measure

- Hausdorff Distance = $\max(h(A,B), h(B,A)), h(A,B) = \max_{a \in A}\min_{b \in B}||a-b||$
- Average Surface Distance (create map and swap) $\frac{d(A,B)+d(B,A)}{2}, d(A,B) = \frac{1}{N}\sum_{a \in A}\min_{b \in B}||a-b||$

Multi-scale processing: 4 layers of 5^3 kernels followed by 1^3 kernel for classification. Multiple pathways for different sized snippets of the image. Then we concat. Feature maps from both pathways
Vision transformers: split image into patches, encode location, get hidden feature after convolutions, linear layer and pass through attention network similar to nlp. Upsample in U-net fashion with connections.

Linear Transformations

$$\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = \underbrace{\overbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}^{\text{translation vectors}} \overbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{direction vectors}} \overbrace{\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{\text{scaling matrix}}}_{T_{ItW}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

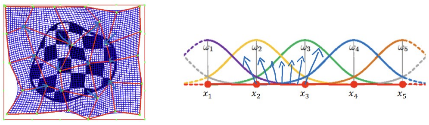
If there is more than one world, then:

$$\begin{pmatrix} x_A \\ y_A \end{pmatrix} = \mathrm{T}^A_{WtI} T_{BtA} T^B_{ItW} \begin{pmatrix} x_B \\ y_B \end{pmatrix}$$

In 3D, there are identity (0 dof) rigid (translation and rotation, 3+3 dof), similar (scaling, 3+3+1 dof), affine (shear 12). In affine, if two lines are || , after affine T, its still true.

$$\text{shearing} = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Non-linear Transformations



Embed image onto grid, prescribe motion at each grid point (red) and underlying blue grid is as a result of linear interpolation.
Medical Applications: Cardiac Motion and Respiration Motion tracking, Multi-modal Image Fusion, Pre- and Post-op comparison
Intra-Subject Registration: create an atlas and transform.

Objective: $C(T) = D(I \circ T, J)$ (**T**ransformation, **D**issimilarity measure, (**J**) Fixed image, $(I \circ T)$ Moving Image
Optimization: $\hat{T} = \arg\min_T C(T)$
Mono-modal Registration: Image intensities are related by a (simple) function. Assumption: the identity relationship between intensity distributions. Not good when brightness changes and subtraction is no longer the best metric.

- Sum of squared differences: $D_{SSD}(I \circ T, J) = \frac{1}{N}\sum_{i=1}^{N}(I(T(x_i)) - J(x_i))^2$
- Sum of absolute differences: $D_{SAD}(I \circ T, J) = \frac{1}{N}\sum_{i=1}^{N}|I(T(x_i)) - J(x_i)|$
- Correlation Coefficient: $D_{CC}(I \circ T, J) = -\frac{\overbrace{\frac{1}{N}\sum_{i=1}^{N}(I(T(x_i)) - \mu_I)(J(x_i) - \mu_I)}^{cov(I,J)}}{\underbrace{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(I(T(x_i)) - \mu_I)^2}}_{}\underbrace{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(J(x_i) - \mu_J)^2}}_{}}$

Multi-modal Registration: Image intensities are related by a complex function or statistical relationship. Measures "When are these two images the most statistically aligned". To avoid local minimas: increase dof, or gaussian smoothing. May require linear interpolation

- Intensity Histograms: plot intensities of both images on x and y, discretize histogram with bins. A Registered image will have more clustered regions (identity will be line $x = y$)
  - $p(i,j) = \frac{h(i,j)}{N}$ at a point, in one image i and other image j counts in the histogram. $p(i) = \sum_j p(i,j)$ sim for $p(j)$
  - Shannon Entropy: $H(I) = -\sum_i p(i)\log p(i)$ low value if every pixel has the same value, or high if randomness
  - Joint Entropy: $H(I,J) = -\sum_i\sum_j p(i,j)\log p(i,j)$ measures how clustered a space is, and minimising that entropy is a good criterium
  - Mutual Information: $MI(I,J) = H(I) + H(J) - H(I,J)$ describes how well one image can be explained by another image. $MI(I,J) = \sum_i\sum_j p(i,j)\log\frac{p(i,j)}{p(i)p(j)}$ with dissimilarity: $D_{MI}(I \circ T, J) = -MI(I \circ T, J)$
  - Normalized Mutual Information: $NMI(I,J) = \frac{H(I)+H(J)}{H(I,J)}$ with dissimilarity similar to above