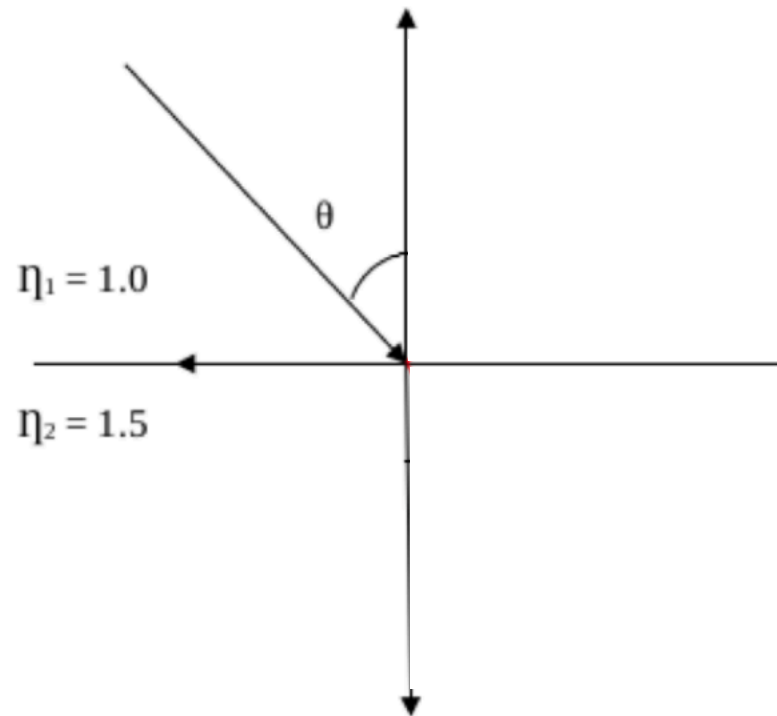
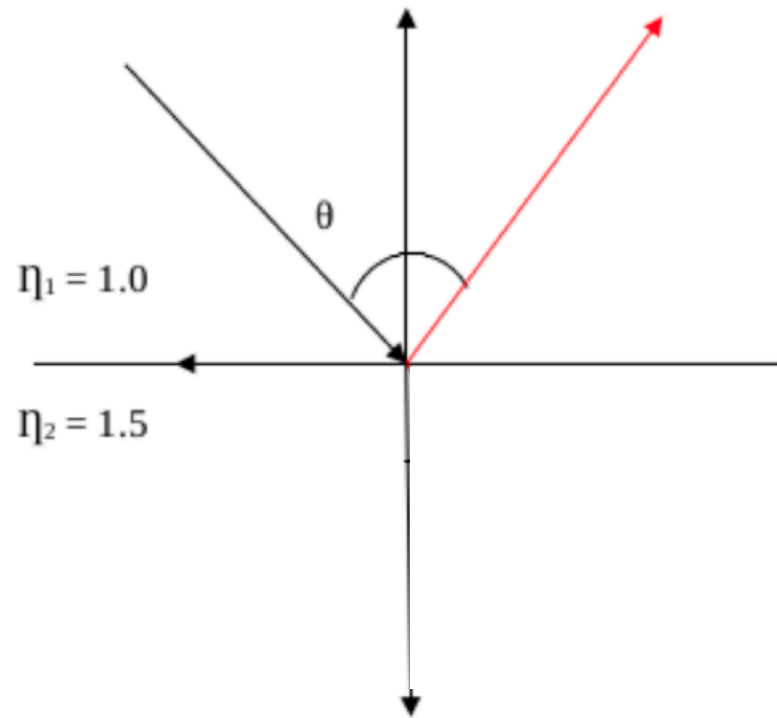


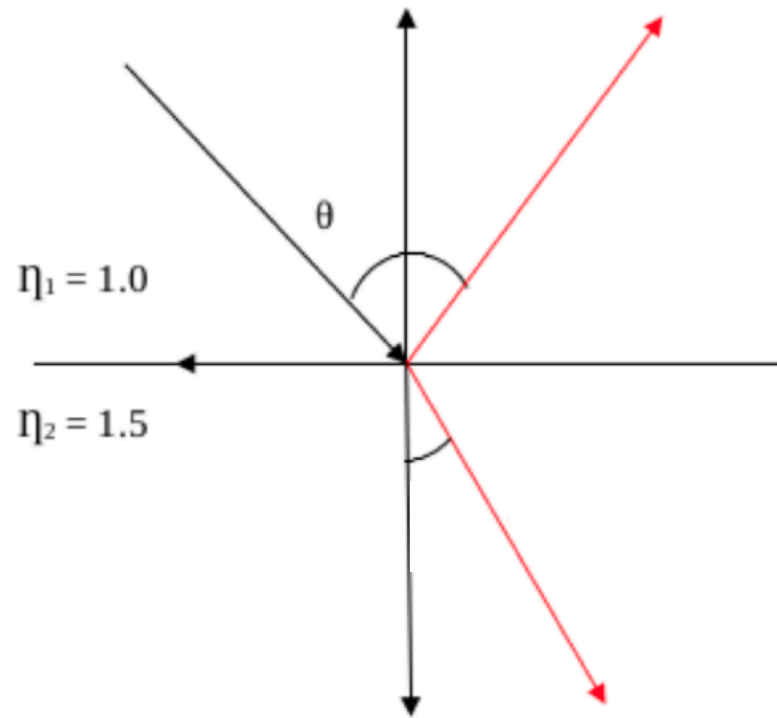
- Draw the light-surface interactions for the following case of a dielectric interface. Also compute the various angles for $\theta = 45^\circ$.



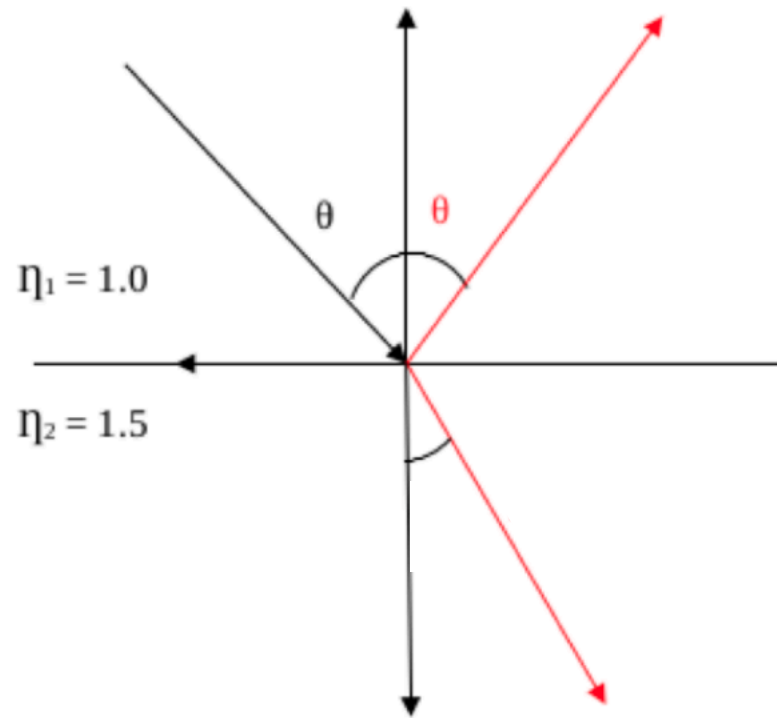
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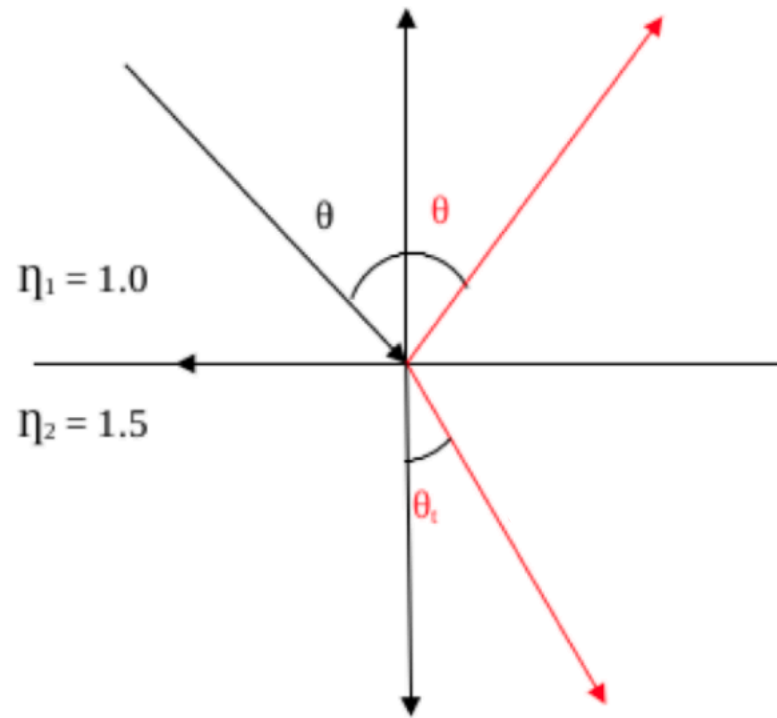
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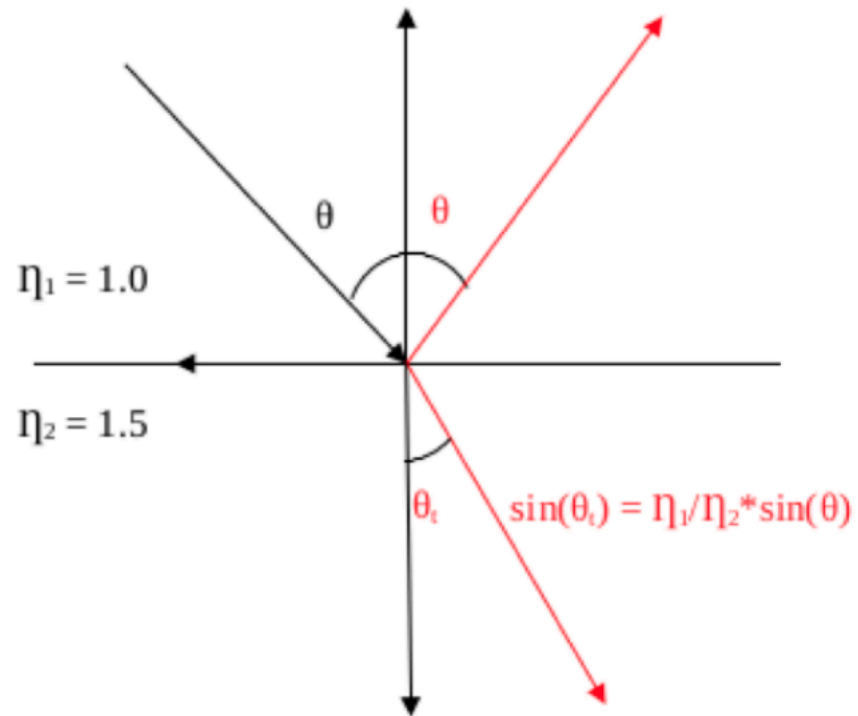
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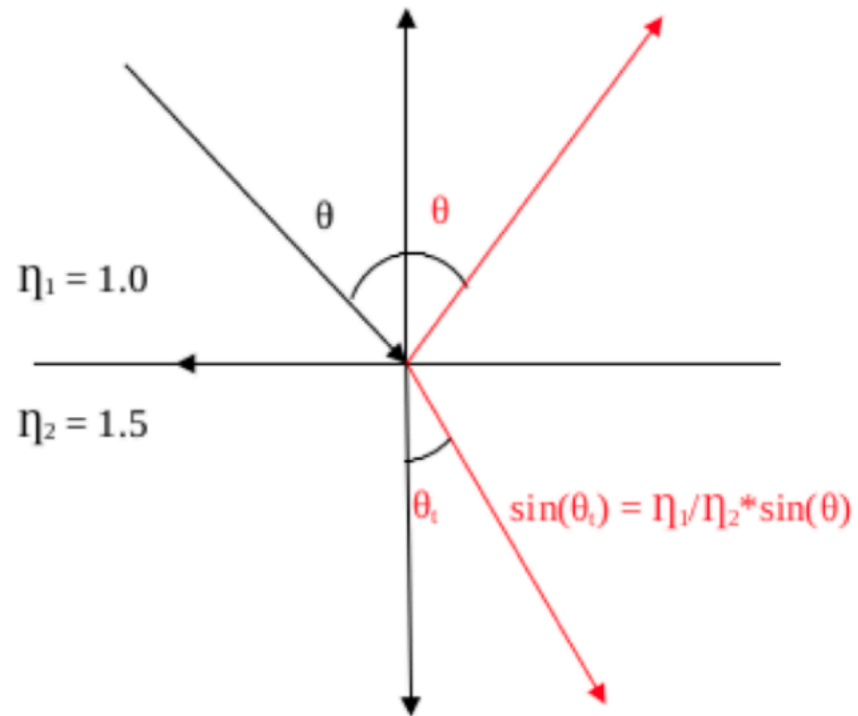
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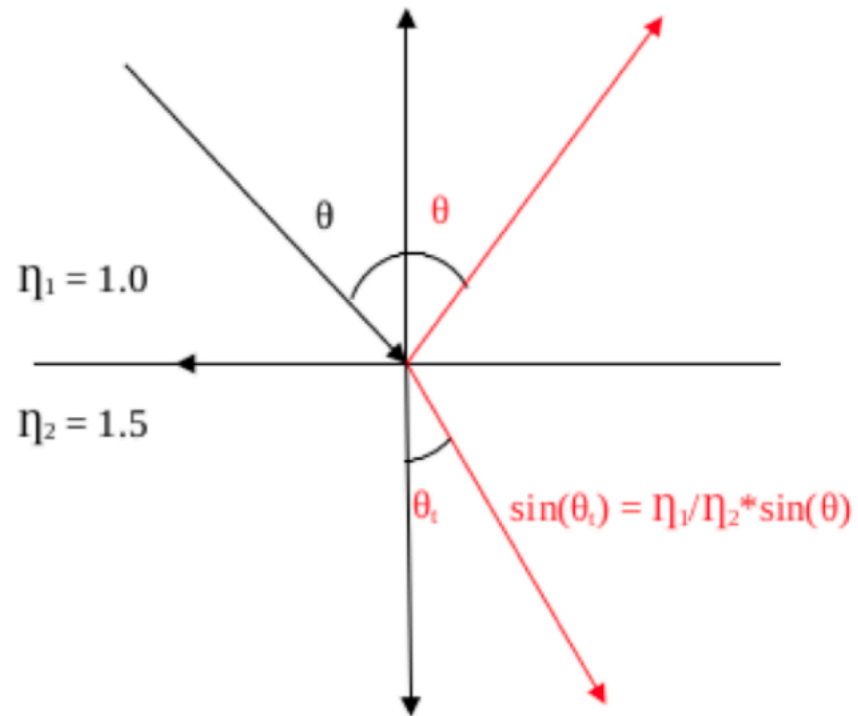


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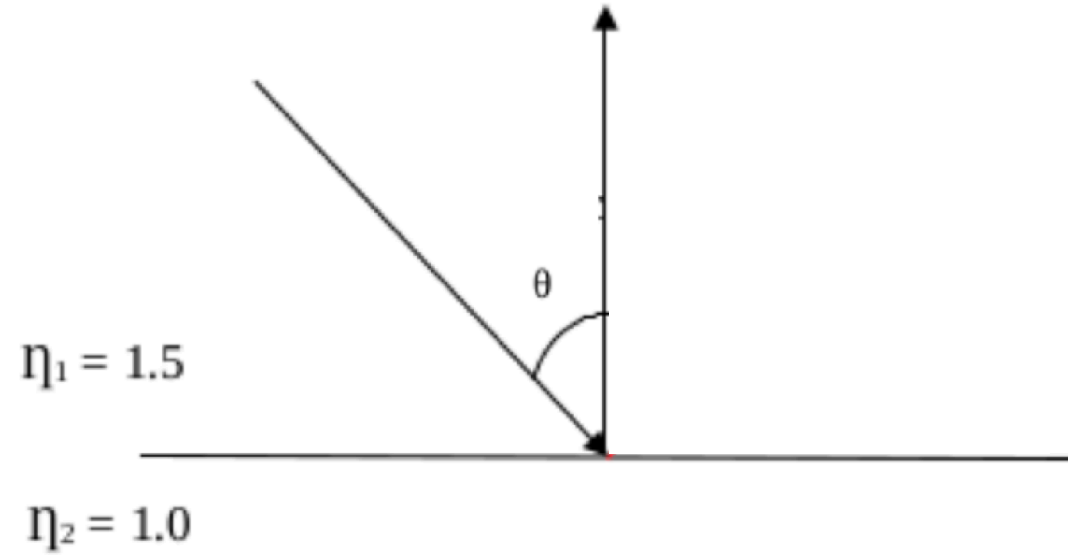
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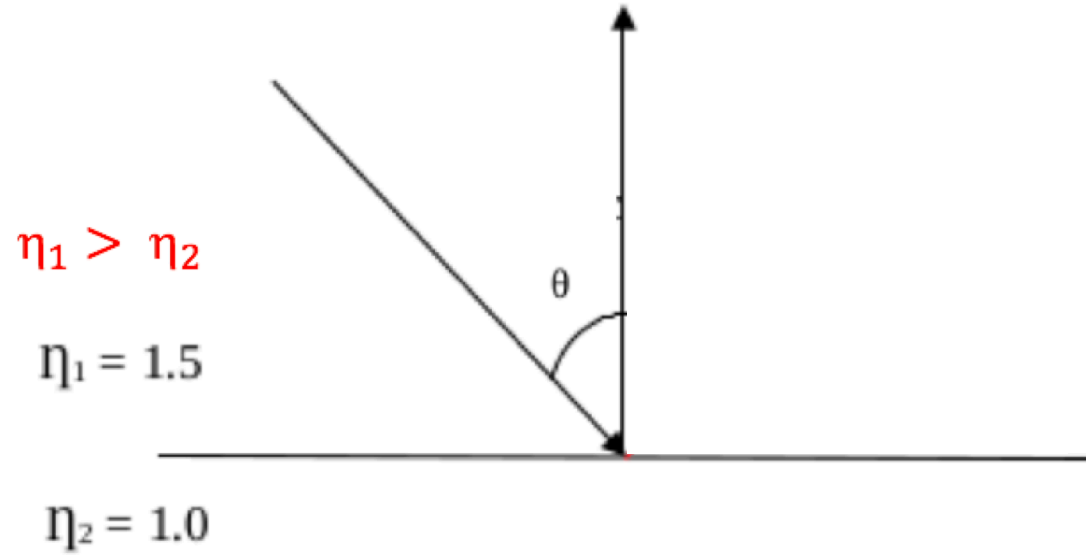


$$\theta_t = \arcsin\left(\frac{n_1}{n_2} * \sin \theta\right) \approx 28 \text{ deg}$$

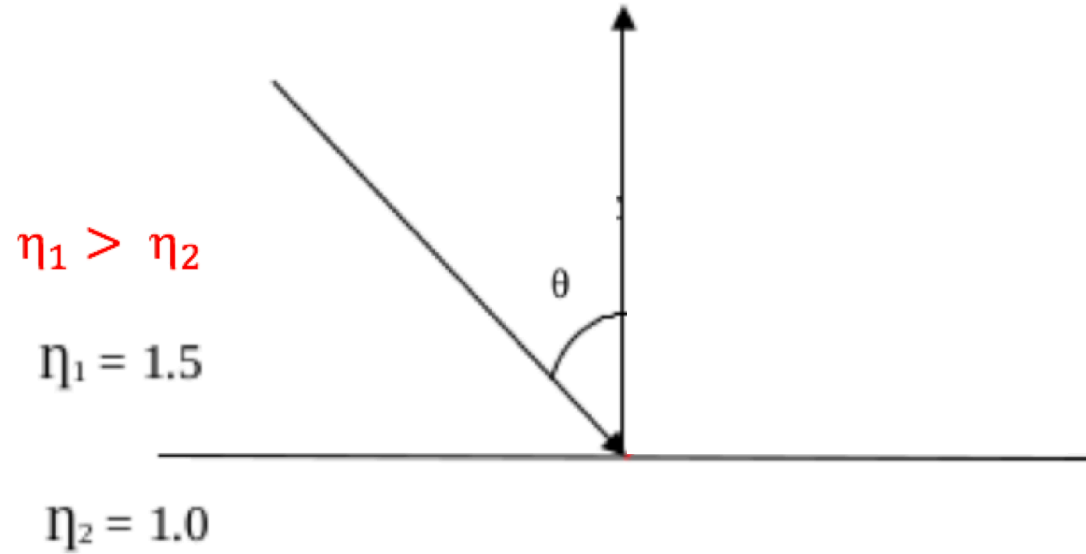
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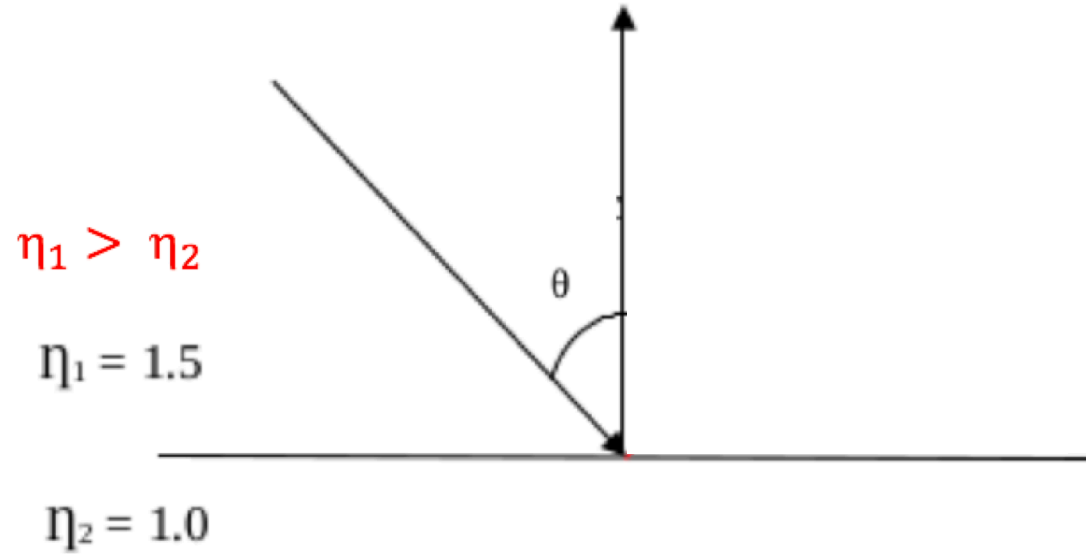
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Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

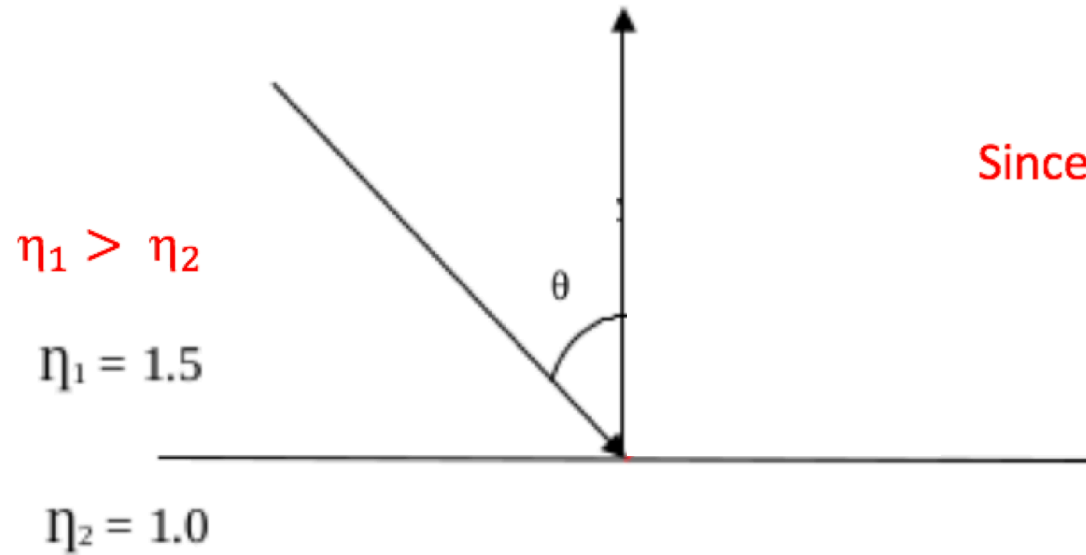
- Draw the light-surface interactions for the following case of a dielectric interface. Also compute the various angles for $\theta = 45^\circ$.



Critical angle:

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) \approx 42 \text{ deg}$$

- Draw the light-surface interactions for the following case of a dielectric interface. Also compute the various angles for $\theta = 45^\circ$.

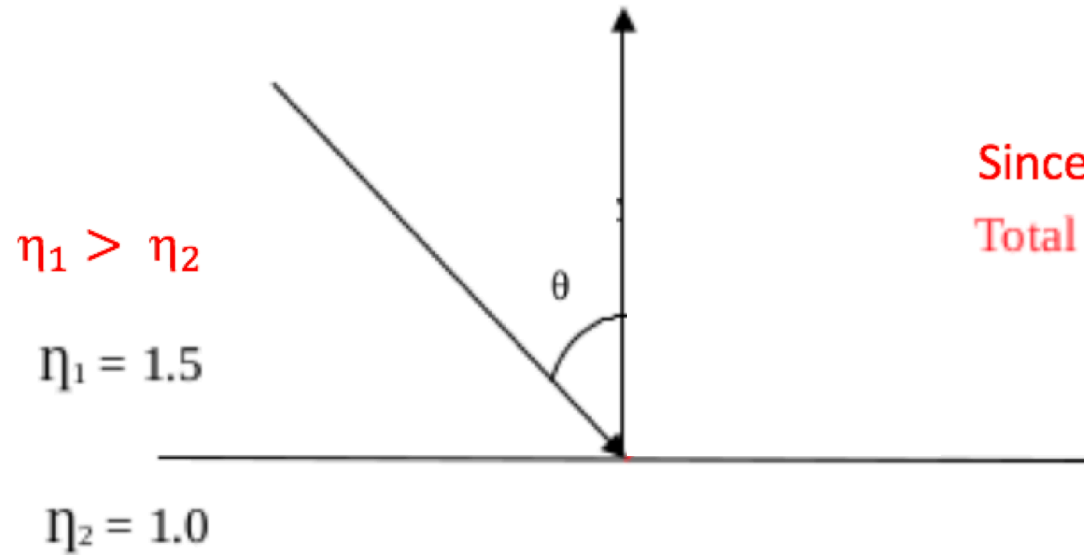


Since $\eta_1 > \eta_2$ and $\theta = 45^\circ > \theta_c = 42^\circ$

Critical angle:

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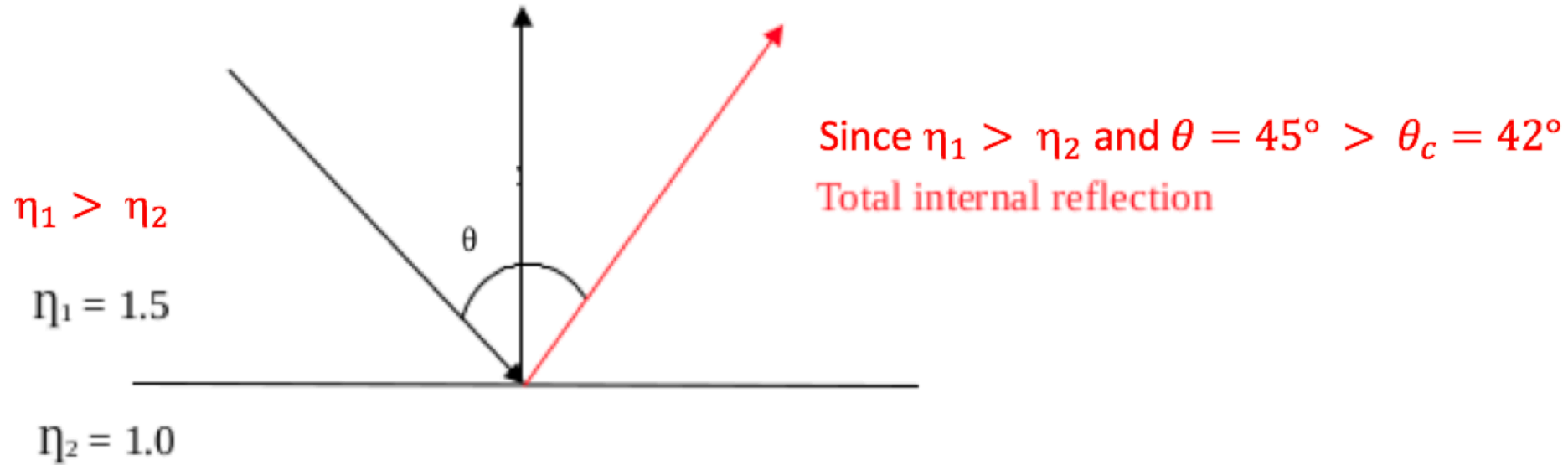


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Total internal reflection

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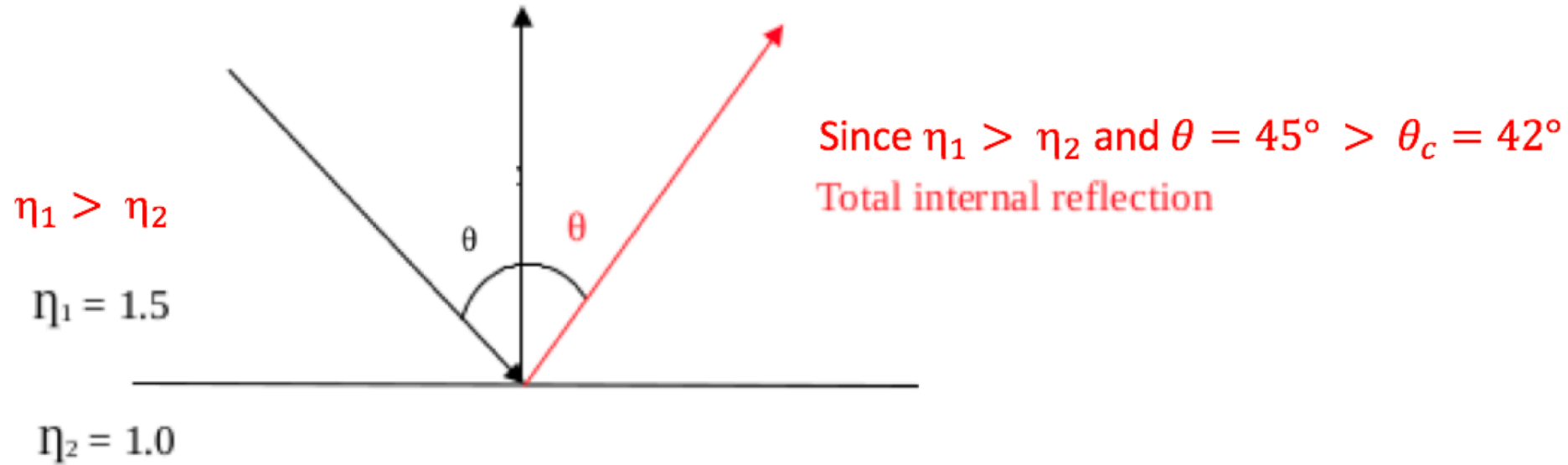
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Given the following environment map L in latlong format with $M \times N$ pixels:



Assume every pixel has unit radiance. Compute the following integral of the environment map: $\mathbf{I} = \int_{\Omega} L_i(\omega_i) d\omega_i$.

Given the following environment map L in latlong format with MxN pixels:



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EM encodes full sphere of lighting directions.

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Assuming a diffuse surface with albedo $\rho_d = 1.0$, compute the irradiance \mathbf{E} on the surface due to illumination from the given environment map \mathbf{L} .

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 &= 2 \int_{\theta_i=0}^{\frac{\pi}{2}} \frac{\sin(2\theta_i)}{2} d\theta_i
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 &= \frac{1}{2} [-\cos(2\theta_i)]_0^{\frac{\pi}{2}} \\
 &= 1
 \end{aligned}$$

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Given a conductor, estimate the index of refraction of the material given measured reflectance at normal incidence $R_r = 0.2$. Assume the coefficient of absorption $k = 0$.

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$$\eta = \frac{1 + \sqrt{F_r}}{1 - \sqrt{F_r}}$$

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$$\eta = \frac{1 + \sqrt{F_r}}{1 - \sqrt{F_r}} \\ \approx 2.62$$