1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

a) Assume that the mutation is proposed with uniform random perturbation.

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \rightarrow x)}{f(x)T(x \rightarrow x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \rightarrow x') = 0.3$ and $T(x' \rightarrow x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

$$a(x \to x') = min(1, \frac{f(x')}{f(x)})$$

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$
$$a(x \to x') = min(1, \frac{f(x')}{f(x)}) = min(1, 1.4)$$

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

$$a(x \to x') = min(1, \frac{f(x')}{f(x)}) = min(1, 1.4) = 1$$

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation $\mathbf{f(x) = 0.5}$ at position x and another neighboring function evaluation $\mathbf{f(x') = 0.7}$ at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

$$a(x \to x') = min(1, \frac{f(x')}{f(x)}) = min(1, 1.4) = 1$$

b) Assume that the mutation is proposed with a transition probability $\mathbf{T}$ such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

$$a(x \to x') = min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation **f(x) = 0.5** at position x and another neighboring function evaluation **f(x') = 0.7** at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

$$a(x \to x') = min(1, \frac{f(x')}{f(x)}) = min(1, 1.4) = 1$$

b) Assume that the mutation is proposed with a transition probability **T** such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

$$a(x \to x') = min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')}) = min(1, 0.5)$$

1 Metropolis-Hastings Mutation Strategy

Given a function evaluation **f(x) = 0.5** at position x and another neighboring function evaluation **f(x') = 0.7** at position x', what is the probability of accepting a mutation from x to x' according to Metropolis-Hastings algorithm in the following two cases:

The general formula is :

$$min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')})$$

a) Assume that the mutation is proposed with uniform random perturbation.

$$T(x' \to x) = T(x \to x')$$

$$a(x \to x') = min(1, \frac{f(x')}{f(x)}) = min(1, 1.4) = 1$$

b) Assume that the mutation is proposed with a transition probability **T** such that $T(x \to x') = 0.3$ and $T(x' \to x) = 0.1$

$$a(x \to x') = min(1, \frac{f(x')T(x' \to x)}{f(x)T(x \to x')}) = min(1, 0.5) = 0.5$$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

a) 2 bounces. $p(2) =$

b) 3 bounces. $p(3) =$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

a) 2 bounces. $p(2) =$

b) 3 bounces. $p(3) =$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: $q = 0.75$

Further tracing probability: $(1-q) = 0.25$

a) 2 bounces. $p(2) =$

b) 3 bounces. $p(3) =$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75$

b) 3 bounces. $p(3) =$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) =$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) = .25 * .25 * .75$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) = .25 * .25 * .75 \approx .047$

c) 4 bounces. $p(4) =$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) = .25 * .25 * .75 \approx .047$

c) 4 bounces. $p(4) = .25 * .25 * .25 * .75$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) = .25 * .25 * .75 \approx .047$

c) 4 bounces. $p(4) = .25 * .25 * .25 * .75 \approx .012$

2 Russian Roulette

If the path termination probability q=0.75, then what is the probability of the path tracing to stop after the following number of bounces:

Termination probability: q = 0.75

Further tracing probability: (1-q) = 0.25

a) 2 bounces. $p(2) = .25 * .75 \approx 0.1875$

b) 3 bounces. $p(3) = .25 * .25 * .75 \approx .047$

c) 4 bounces. $p(4) = .25 * .25 * .25 * .75 \approx .012$

The general formula is:

$$p(n) = (1-q)^{n-1}q$$

3 Photon Mapping

Assume that 50 photons are deposited on a surface within a radius r = 0.5 m. Each photon is carrying a flux $\Delta\Phi = 2Watt$. Compute the reflected radiance estimate at the surface assuming a diffuse BRDF of albedo $\rho_d = 0.5$.

We have the equation

$$L(x, \vec{\omega}) = \sum_{p=1}^{n} f_r(x, \vec{\omega_p}', \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega_p}')}{\pi r^2}$$

3 Photon Mapping

Assume that 50 photons are deposited on a surface within a radius r = 0.5 m. Each photon is carrying a flux $\Delta\Phi = 2Watt$. Compute the reflected radiance estimate at the surface assuming a diffuse BRDF of albedo $\rho_d = 0.5$.

We have the equation

$$L(x, \vec{\omega}) = \sum_{p=1}^{n} f_r(x, \vec{\omega_p}', \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega_p}')}{\pi r^2}$$

$$f_r(x, \vec{\omega_p}', \vec{\omega}) = \frac{\rho_d}{\pi}$$

3 Photon Mapping

Assume that 50 photons are deposited on a surface within a radius r = 0.5 m. Each photon is carrying a flux $\Delta\Phi = 2Watt$. Compute the reflected radiance estimate at the surface assuming a diffuse BRDF of albedo $\rho_d = 0.5$.

We have the equation

$$L(x, \vec{\omega}) = \sum_{p=1}^{n} f_r(x, \vec{\omega_p}', \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega_p}')}{\pi r^2}$$

$$f_r(x, \vec{\omega_p}', \vec{\omega}) = \frac{\rho_d}{\pi}$$

Substitute $\rho_d = 0.5$, $\Delta\Phi_p(x, \vec{\omega_p}') = 2$, $r = 0.5$ and $n = 50$

3 Photon Mapping

Assume that 50 photons are deposited on a surface within a radius r = 0.5 m. Each photon is carrying a flux $\Delta\Phi = 2Watt$. Compute the reflected radiance estimate at the surface assuming a diffuse BRDF of albedo $\rho_d = 0.5$.

We have the equation

$$L(x, \vec{\omega}) = \sum_{p=1}^{n} f_r(x, \vec{\omega_p}', \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega_p}')}{\pi r^2}$$

$$f_r(x, \vec{\omega_p}', \vec{\omega}) = \frac{\rho_d}{\pi}$$

Substitute $\rho_d = 0.5$, $\Delta\Phi_p(x, \vec{\omega_p}') = 2$, $r = 0.5$ and $n = 50$

$$L(x, \vec{\omega}) = 50 * \frac{0.5}{\pi} * \frac{2}{\pi * 0.5^2}$$

3 Photon Mapping

Assume that 50 photons are deposited on a surface within a radius r = 0.5 m. Each photon is carrying a flux $\Delta\Phi = 2Watt$. Compute the reflected radiance estimate at the surface assuming a diffuse BRDF of albedo $\rho_d = 0.5$.

We have the equation

$$L(x, \vec{\omega}) = \sum_{p=1}^{n} f_r(x, \vec{\omega_p}', \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega_p}')}{\pi r^2}$$

$$f_r(x, \vec{\omega_p}', \vec{\omega}) = \frac{\rho_d}{\pi}$$

Substitute $\rho_d = 0.5$, $\Delta\Phi_p(x, \vec{\omega_p}') = 2$, $r = 0.5$ and $n = 50$

$$L(x, \vec{\omega}) = 50 * \frac{0.5}{\pi} * \frac{2}{\pi * 0.5^2} = 20.264\, W/m^2 sr$$