70001 Tutorial 5

1 Volume scattering

Compute the differential radiance for light transport through a volume of length 1m with the following properties: $\sigma_a = 0.2m^{-1}, \sigma_s = 0.3m^{-1}$ and uniform isotropic phase function.

We assume unit radiance and a non-emissive participating medium. We also assume that the volume considered is a sphere.

$$\begin{aligned}
dL_o(x,\omega) &= \int_0^1 \left(-\sigma_t(x,\omega)Li(x,-\omega) \;+\; \sigma_s(x,\omega)\int_\Omega p(x,-\omega' \to \omega)L_i(x,\omega')d\omega' \right) dt \\
&= \int_0^1 \left(-(\sigma_a(x,\omega)+\sigma_s(x,\omega)) + \frac{\sigma_s(x,\omega)}{4\pi} \underbrace{\int_0^{2\pi}\int_0^{\pi} \sin\theta' d\theta' d\phi'}_{4\pi} \right) dt \\
&= \int_0^1 (-0.5+0.3)\, dt \\
&= \int_0^1 -0.2dt \\
&= -0.2
\end{aligned}$$

2 Subsurface scattering

Given a layered scattering medium with two layers and individual layer reflectance and transmittance parameters as follows: $R_1 = 0.1, T_1 = 0.7, T_2 = 0.6, R_2 = 0.4$. Compute the total transmittance T_{12} due to the two layers according to the Kubelka-Munk model.

$$T_{12} = \frac{T_1 T_2}{1 - R_1 R_2} = \frac{0.7 * 0.6}{1 - 0.1 * 0.4} \approx 0.44$$

3 Extinction

Given a homogeneous isotropic participating media and following parameters of absorption cross-section $\sigma_a = 0.1m^{-1}$, and out-scattering coefficient $\sigma_s = 0.2m^{-1}$, compute the transport coefficient T_r for energy transfer along a ray of distance $0.5m$ through the volume.

Extinction cross-section:
$\sigma_t = \sigma_a + \sigma_s = 0.1 + 0.2 = 0.3m^{-1}$.

Transport coefficient in homogeneous media:
$T_r = exp(-\sigma_t d) = exp(-0.3 * 0.5) = 0.8607$.