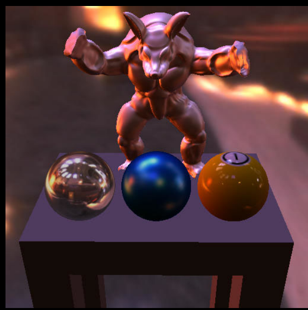
Spherical Harmonic Lighting



70001 – Advanced Computer Graphics: Photographic Image Synthesis

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Lecture 11, Feb. 13th 2024

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Image Based Lighting

- Ray-tracing
- Requires sampling of lights in Environment maps
  - Monte Carlo
  - Deterministic
- Spherical Harmonics basis function
  - efficient encoding
  - efficient evaluation (no ray-tracing!)
  - ideal for real time rendering with GPUs

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Spherical Harmonics

- Frequency decomposition of a 3D function
- 3D analog of Fourier basis functions
  - Representing increasing frequencies of a function with higher order polynomials
- Ideal for representing functions defined over a sphere of directions
  - EMs
  - BRDFs

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Spherical Harmonics

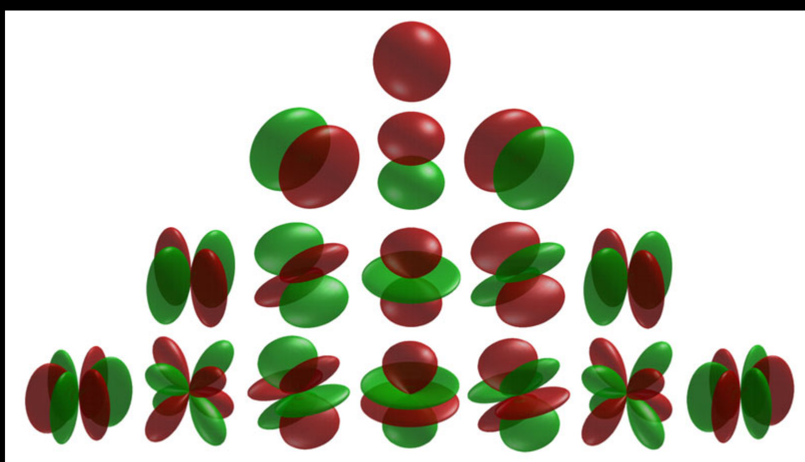
l – order, m – degree, red – positive, green - negative

l = 0

l = 1

l = 2

l = 3



-l >= m <= l

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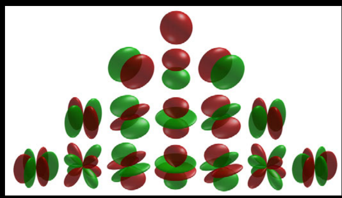
Spherical Harmonics

$$(x, y, z) = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$$

$$Y_l^m(\theta,\varphi) = K_l^m \; e^{im\varphi} P_l^{|m|}(\cos\theta), \; l \in \mathbf{N}, \; -l \le m \le l$$

$$K_l^m = \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}}$$



$$y_l^m = \begin{cases} \sqrt{2}\,\mathrm{Re}(Y_l^m), & m > 0 \\ \sqrt{2}\,\mathrm{Im}(Y_l^m), & m < 0 \\ Y_l^0, & m = 0 \end{cases} = \begin{cases} \sqrt{2}\,K_l^m \cos(m\varphi) P_l^m(\cos\theta), & m > 0 \\ \sqrt{2}\,K_l^m \sin(-m\varphi) P_l^{-m}(\cos\theta), & m < 0 \\ K_l^0 P_l^0(\cos\theta), & m = 0 \end{cases}$$

P_l^m – Legendre polynomial of order *l* and degree *m*

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SH Representation of EM

$$L_{lm} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} L(\theta,\phi) Y_{lm}(\theta,\phi) \sin\theta \, d\theta d\phi$$

- L_{lm} – SH coefficient (order *l* and degree *m*) of Lighting (EM)
- Y_{lm} – SH basis function of order *l* and degree *m*

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SH Reconstruction of EM

$$L(\theta, \phi) = \sum_{l,m} L_{lm} Y_{lm}(\theta, \phi)$$

- L_{lm} – SH coefficient (order l and degree m) of Lighting (EM)
- Y_{lm} – SH basis function of order l and degree m

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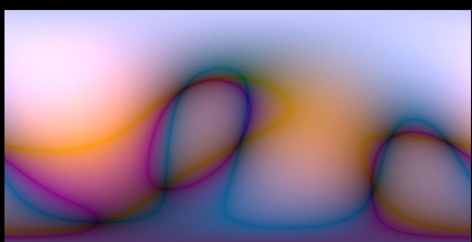
SH Reconstruction of EM

$$L(\theta, \phi) = \sum_{l,m} L_{lm} Y_{lm}(\theta, \phi)$$



2nd order reconstruction

l = 2

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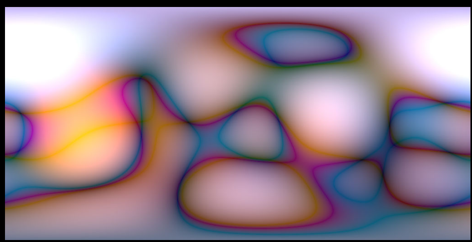
SH Reconstruction of EM

$$L(\theta, \phi) = \sum_{l,m} L_{lm} Y_{lm}(\theta, \phi)$$



4th order reconstruction

l = 4

Note the much stronger ringing artifacts!

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The general case …

$$c_l^m = \int_S f(s) y_l^m(s) ds$$

SH projection

$$\tilde{f}(s) = \sum_{l=0}^{n-1} \sum_{m=-l}^{l} c_l^m y_l^m(s) = \sum_{i=0}^{n^2} c_i y_i(s)$$

SH reconstruction

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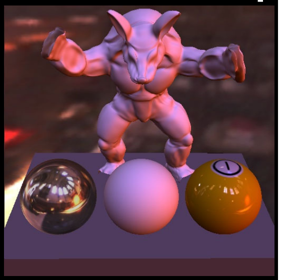
Irradiance

[Ramamoorthi & Hanrahan 2001]

$$E(\mathbf{n}) = \int_{\Omega(\mathbf{n})} L(\omega)(\mathbf{n}\cdot\omega)\, d\omega$$



$$E(\theta,\phi) \quad = \quad \sum_{l,m} E_{lm} Y_{lm}(\theta,\phi)$$

SH reconstruction

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Irradiance

[Ramamoorthi & Hanrahan 2001]

$$E(\theta,\phi) \quad = \quad \sum_{l,m} E_{lm} Y_{lm}(\theta,\phi)$$

SH reconstruction

$$A = (\mathbf{n}\cdot\omega) \; = \max\,[\cos\theta, 0] = \sum_{l} A_l Y_{l0}(\theta)$$

A_l is a 1D function, no dependence on m

$$E_{lm} = \sqrt{\frac{4\pi}{2l+1}} A_l L_{lm}$$

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Irradiance

[Ramamoorthi & Hanrahan 2001]

$$E_{lm} = \sqrt{\frac{4\pi}{2l+1}} A_l L_{lm}$$

Let $\hat{A}_l = \sqrt{\frac{4\pi}{2l+1}} A_l$

SH coefficient of irradiance

$$E(\theta,\phi) = \sum_{l,m} E_{lm} Y_{lm}(\theta,\phi)$$

$$= \sum_{l,m} \hat{A}_l L_{lm} Y_{lm}(\theta,\phi)$$

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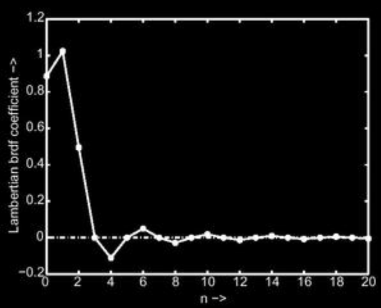
Irradiance

[Ramamoorthi & Hanrahan 2001]

$$l = 1 \quad \hat{A}_1 = \frac{2\pi}{3}$$

$$l > 1, odd \quad \hat{A}_l = 0$$

$$l \ \ even \quad \hat{A}_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[\frac{l!}{2^l (\frac{l}{2}!)^2} \right]$$



$$\hat{A}_0 = 3.141593 \quad \hat{A}_1 = 2.094395 \quad \hat{A}_2 = 0.785398$$

$$\hat{A}_3 = 0 \quad \hat{A}_4 = -0.130900 \quad \hat{A}_5 = 0 \quad \hat{A}_6 = 0.049087$$

- Irradiance is well approximated by only the **first 9 SH** coefficients (l <= 2)!

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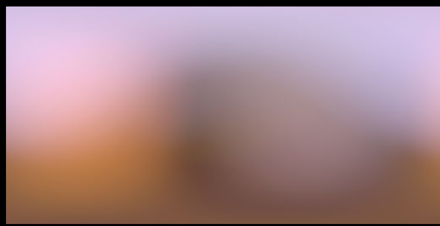
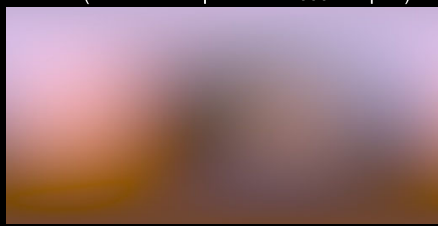
Irradiance



Diffuse convolution

(brute force expensive – 1000 samples)



9 term SH approximation –
fast and efficient

- Irradiance is well approximated by only the **first 9 SH** coefficients ($l <= 2$)!

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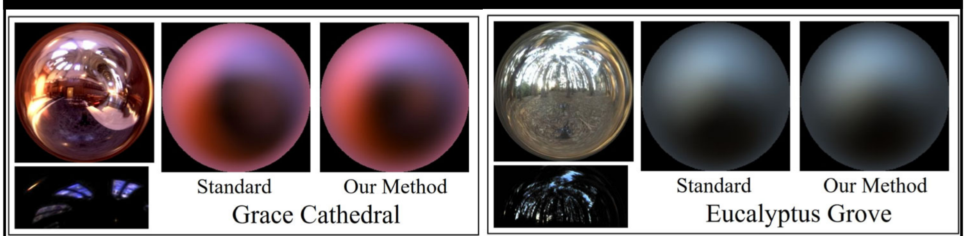
[Ramamoorthi & Hanrahan 2001]

EM coefficients for irradiance approx.

	Grace Cathedral			Eucalyptus Grove			St. Peters Basilica		
L_{00}	.79	.44	.54	.38	.43	.45	.36	.26	.23
L_{1-1}	.39	.35	.60	.29	.36	.41	.18	.14	.13
L_{10}	-.34	-.18	-.27	.04	.03	.01	-.02	-.01	-.00
L_{11}	-.29	-.06	.01	-.10	-.10	-.09	.03	.02	.01
L_{2-2}	-.11	-.05	-.12	-.06	-.06	-.04	.02	.01	.00
L_{2-1}	-.26	-.22	-.47	.01	-.01	-.05	-.05	-.03	-.01
L_{20}	-.16	-.09	-.15	-.09	-.13	-.15	-.09	-.08	-.07
L_{21}	.56	.21	.14	-.06	-.05	-.04	.01	.00	.00
L_{22}	.21	-.05	-.30	.02	-.00	-.05	-.08	-.06	.00

RGB values of 9 SH coeffs.



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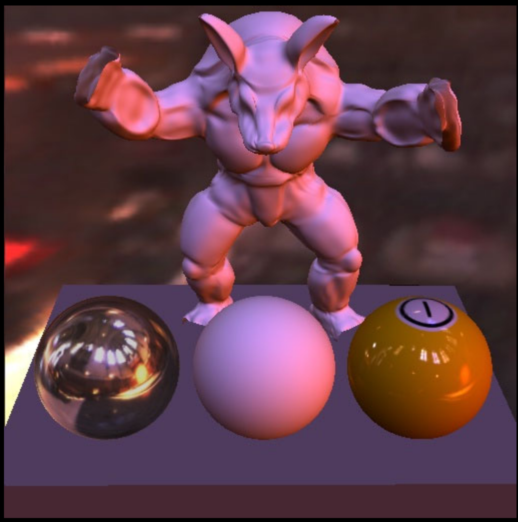
Irradiance

- 9 SH coefficients
- 9 Basis functions
    - Stored as textures in GPU



9 term SH approximation of diffuse reflectance

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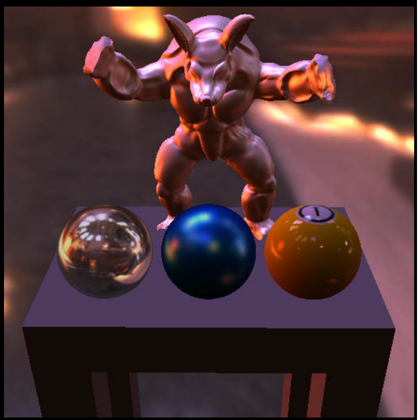
Glossy BRDFs

[Ramamoorthi & Hanrahan 2002]

- Spherical Harmonics Reflection Maps
- Glossy BRDFs require higher order SH
- SH coefficients of order $l >= 4$



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Reflected Radiance

$$B(\vec{N};\vec{\omega}_o) = \int_\Omega L(\vec{N};\vec{\omega}_i)\rho(\vec{\omega}_i,\vec{\omega}_o)(\vec{\omega}_i \cdot \vec{z})\, d\omega_i$$

$$B(\alpha,\beta;\theta_o,\phi_o) = \int_\Omega L\left(R_{\alpha,\beta}\left(\theta_i,\phi_i\right)\right) \hat{\rho}(\theta_i,\theta_o, |\ \phi_o - \phi_i\ |)\, d\omega_i$$

Isotropic BRDF

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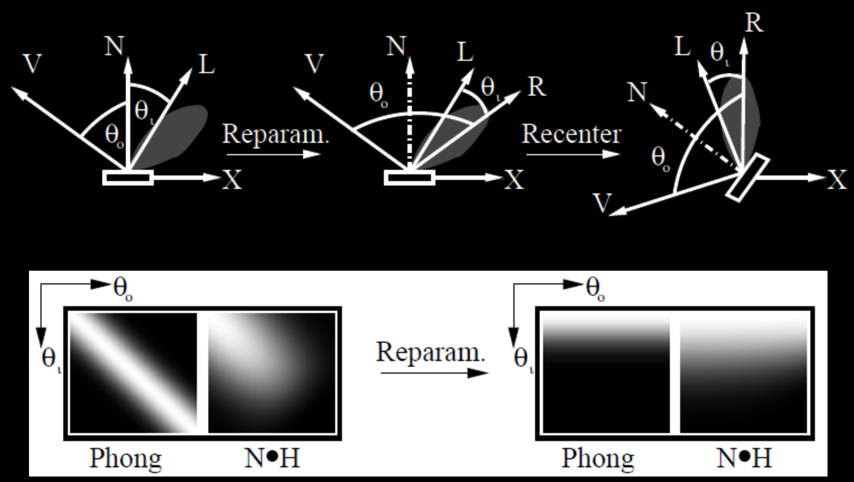
Reflection Reparameterization



Reparameterization makes the BRDF simpler, low rank 1D function!

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Reflected Radiance in SH basis

$$B(\alpha, \beta, \theta_o, \phi_o) = \sum_a \sum_b c_{ab} d_b(\alpha, \beta) h_a(\theta_o, \phi_o)$$

c_{ab} is the product of 1D BRDF and 2D lighting coefficient

$$g_a(\alpha, \beta) = \sum_b c_{ab} d_b(\alpha, \beta)$$

Precompute SH coefficient maps!

$$B(\alpha, \beta, \theta_o, \phi_o) = \sum_a g_a(\alpha, \beta) h_a(\theta_o, \phi_o)$$

Run-time lookups into SH coefficient and basis maps!

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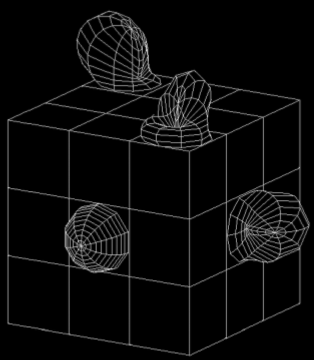
Reflected Radiance in SH basis

$$B(\alpha, \beta, \theta_o, \phi_o) = \sum_{p=0}^{P_B} \sum_{q=-p}^{p} B_{pq}(\alpha, \beta) Y_{pq}(\theta_o, \phi_o)$$

coefficient map basis map



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SH Reflection Map Renderings

$$B(\alpha, \beta, \theta_o, \phi_o) = \sum_{p=0}^{P_B} \sum_{q=-p}^{p} \underbrace{B_{pq}}_{\text{coefficient map}}(\alpha, \beta) \underbrace{Y_{pq}}_{\text{basis map}}(\theta_o, \phi_o)$$

Order 4 – 8 SH typically used
for rendering glossy BRDFs

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Precomputed Radiance Transfer

unshadowed irradiance

Precomputed
radiance transfer

[Sloan et al. 2002]

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Precomputed Radiance Transfer

- Precomputation – A global illumination simulator computes how a model shadows and scatters light onto itself.
    - Diffuse materials – result recorded as transfer vector of SH coefficients per vertex (BDRF * ambient occlusion!)
    - Glossy materials – result recorded as a transfer matrix of SH coefficients per vertex

- Run time – Incident EM illumination is projected into SH basis
    - Diffuse materials – model's transfer vector is dotted with lighting coefficients
    - Glossy materials – transfer matrix is applied to lighting coefficients followed by BRDF convolution

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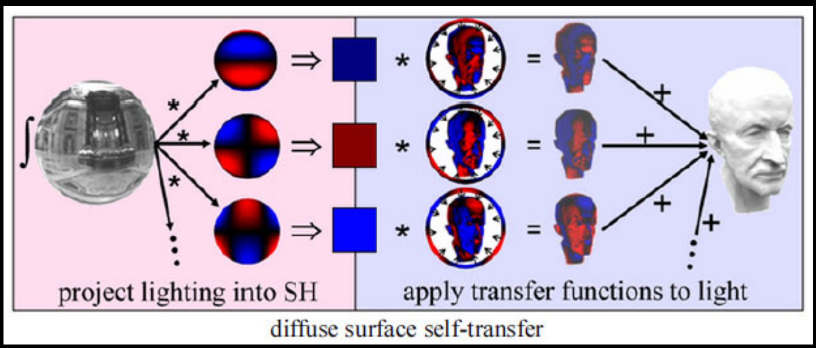
Precomputed Diffuse Radiance Transfer



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Precomputed Diffuse Radiance Transfer

unshadowed

shadows + interreflections

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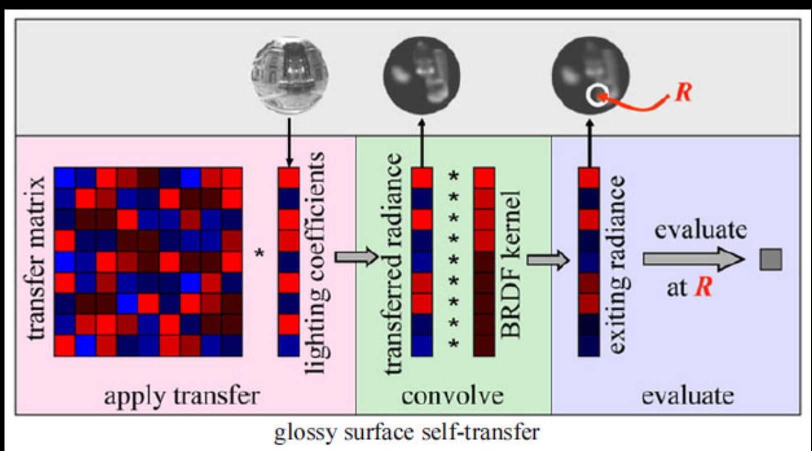
Precomputed Glossy Radiance Transfer



For glossy materials, the transfer function becomes a directionally-dependent matrix instead of a vector for the diffuse case. Each row of the matrix encodes transfer in a different viewing direction.

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Precomputed Glossy Radiance Transfer

unshadowed

shadows + interreflections

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