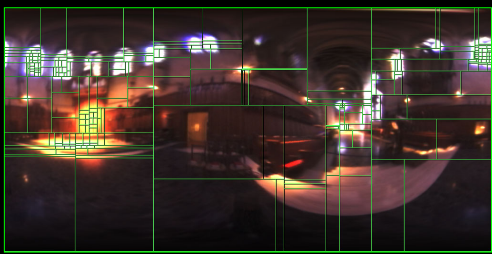
Sampling Direct Illumination



70001 – Advanced Computer Graphics: Photographic Image Synthesis

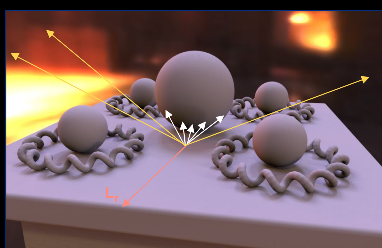
Abhijeet Ghosh

Lecture 10, Feb. 09th 2024

1

Rendering Equation [Kajiya 86]

- $L_r(x, \omega_r) = \int_\Omega f_r(x, \omega_r, \omega_i)\, L_i(x, \omega_i) \cos\theta_i\, V(\omega_i)\, d\omega_i$.
  - direct illumination integral
- Brute force solution too expensive!
  - especially under EM illumination
- Monte Carlo integration
  - stochastic approximation



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Monte Carlo Recap …

- Stochastic sampling
  - Integral computes Expected value or average
  - Diminishing returns due to variance ~ $1/N^2$
- Importance Sampling
  - PDF & CDF Inversion
  - BRDF sampling (analytic)
  - EM sampling (numeric)

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Importance Sampling

- $I(f) = \int_S f(x)\, dx$
- Unbiased MC estimate of $I(f)$

$$I(f) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

  - $p(x_i)$ is a PDF, also called a proposal distribution

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BRDF Models

- Several BRDF models can be sampled analytically
  - cosine lobe, Gaussian lobe, GGX
- Sampling a Phong lobe

  $$p(\theta, \varphi) = \frac{(n + 1)\cos^n\theta}{2\pi}$$

- $(\theta, \varphi) = (\arccos((1 - u_1)^{1/(n+1)}), 2\pi u_2)$

Phong lobe

- $(\theta, \varphi) = (\arccos((1 - u_1)^{1/(n+1)}), 2\pi u_2)$
  - sample direction distributed about local +Z
  - need to rotate sample to be about global reflection vector ω_r!
- Half-vector parameterization $p(\omega_h)$
  - Microfacet models
  - samples generated about ω_h

Isotropic Gaussian

- Ward BRDF $p(\omega_h) \sim \exp-(\tan^2\theta_h/\alpha^2)$ (proportionality implies CDF!)
- Sample θ_h :

  $\theta_h = \arctan(\alpha \sqrt{-\log u_1})$
- Sample φ_h :

  $\varphi_h = 2\pi u_2$

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Anisotropic Gaussian

- Ward BRDF $p(\omega_h) \sim \exp-(\tan^2 \theta_h(\cos^2\varphi_h/\alpha_x^2 + \sin^2\varphi_h/\alpha_y^2)$
- Sample θ_h :

  $\theta_h = \arctan(\sqrt{-\log u_1 / (\cos^2\varphi_h/\alpha_x^2 + \sin^2\varphi_h/\alpha_y^2)})$
- Sample φ_h :

  $\varphi_h = \arctan((\alpha_y/\alpha_x)\tan(2\pi u_2))$

First sample φ_h using u_2, then sample θ_h using sampled φ_h and u_1

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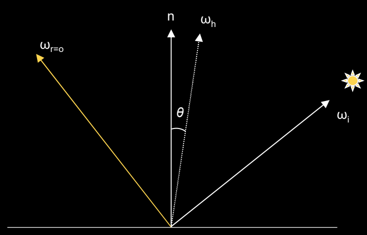
Microfacet lobe

- Analytic inversion of PDF $p(\omega_h)$
- Need for reflection of ω_r about ω_h

  $\omega_i = 2(\omega_h \cdot \omega_r)\, \omega_h - \omega_r$

  ray-tracing done for sampled ω_i



- Final step conversion of PDF $p(\omega_h)$ to $p(\omega_r)$

  $$p(\omega_r) = \frac{p(\omega_h)}{4(\omega_h \cdot \omega_i)}$$

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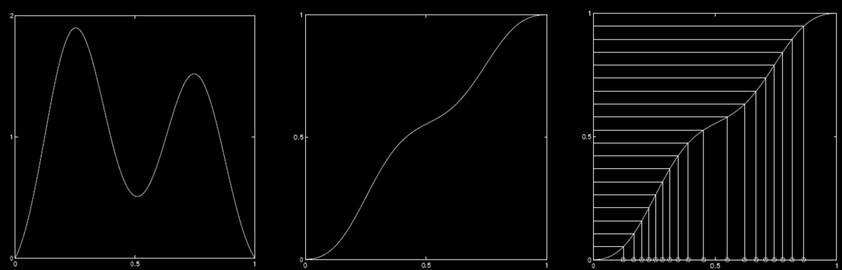
Numerical CDF Inversion



- Numerical integration of PDF
- Uniform samples on Y-axis → $x \sim p(x)$ on X-axis
- Useful for sampling from EMs!

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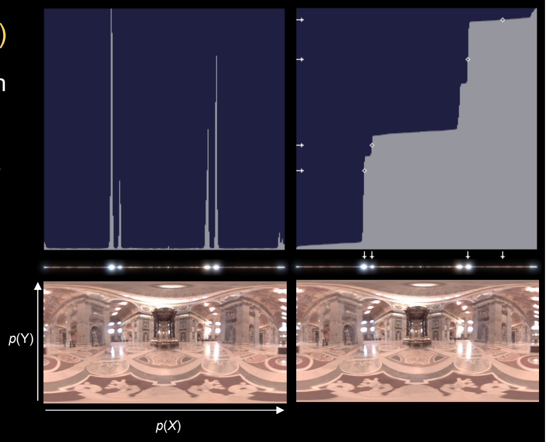
Sampling from EMs

- First sample along rows $p(Y)$
  - based on average energy in a scan line
- Then sample along columns $p(X)$ of the selected row



- Cumulative PDF

  $p(X, Y) = p(Y) \cdot p(X)$

- Need to account for $d\omega$!
  - spherical to lat-long param.
  - weighting by $\sin\theta$

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Direct Illumination Integral

- $L_r(\omega_r) = \int_\Omega f_r(\omega_r, \omega_i) \cos\theta_i L_i(\omega_i) V(\omega_i) d\omega_i$.

- MC estimate:

$$L_{r,N}(\omega_r) = \frac{1}{N} \sum_{j=1}^{N} \frac{f_r(\omega_r, \omega_{i,j}) \cos\theta_{i,j} L_i(\omega_{i,j}) V(\omega_{i,j})}{q(\omega_{i,j})}$$

  - q is the proposal distribution for importance sampling
  - EM or BRDF importance

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Importance Sampling from EM

- $q_L(\omega_i) = \dfrac{L_i(\omega_i)}{\int_\Omega L_i(\omega_i)\, d\omega_i}$.

- MC estimate:

$$L_{r,N}(\omega_r) = \frac{1}{N} \sum_{j=1}^{N} \frac{f_r(\omega_r, \omega_{i,j}) \cos\theta_{i,j}\, L_i(\omega_{i,j})\, V(\omega_{i,j})}{q_L(\omega_{i,j})}$$

$$= \frac{\int_\Omega L_i(\omega_i)\, d\omega_i}{N} \sum_{j=1}^{N} f_r(\omega_r, \omega_{i,j}) \cos\theta_{i,j}\, V(\omega_{i,j})$$

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Importance Sampling from EM

- $q_L(\omega_i) = \dfrac{L_i(\omega_i)}{\int_\Omega L_i(\omega_i)\, d\omega_i}$.

- MC variance:

$$\mathrm{var}(L_{r,N}(\omega_r)) = \frac{\int L_i^2}{N}\ \mathrm{var}(\, f_r(\omega_r, \omega_{i,j}) \cos\theta_{i,j}\, V(\omega_{i,j}))$$

  – variance of EM sampling proportional to variance in BRDF!

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Multiple Importance Sampling

EM
100 samples

BRDF
75 samples

MIS

Veach and Guibas, SIGGRAPH 1995

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Multiple Importance Sampling

- MIS estimate:

$$I_{MIS} = \frac{1}{M+N} \left(\sum_{i=1}^{M} \frac{f(x_i)\, w_m(x_i)}{q_m(x_i)} + \sum_{j=1}^{N} \frac{f(x_j)\, w_n(x_j)}{q_n(x_j)} \right)$$

  - q_m and q_n are the proposal distributions
  - w_m and w_n are the weighting functions

- Default in ray tracers like PBRT for direct illumination!

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Bidirectional Importance

- $L_r(\omega_r) = \int_\Omega f_r(\omega_i \rightarrow \omega_r) \cos\theta_i \, L_i(\omega_i) V(\omega_i) d\omega_i \,,$ (1)
- Target distribution *p* for direct illumination:

$$p(\omega_i) := \frac{f_r(\omega_i \rightarrow \omega_r)\cos\theta_i \, L_i(\omega_i)}{\int_\Omega f_r(\omega_i \rightarrow \omega_r)\cos\theta_i \, L_i(\omega_i)\, d\omega_i} \,, \qquad (2)$$

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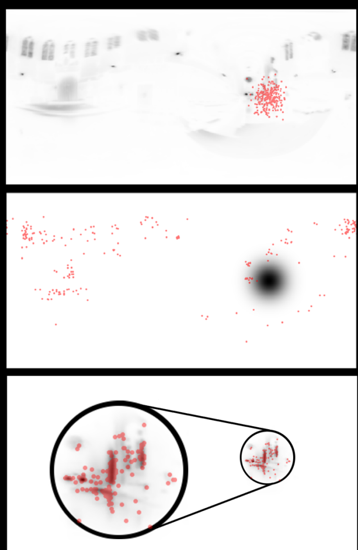
Bidirectional Importance Sampling

Importance function of Grace Cathedral EM

Samples according to BRDF

Samples according to EM

Importance function of Phong BRDF (s = 50)



Burke, Ghosh, and Heidrich EGSR 2005

Samples according to bidirectional importance

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Bidirectional Importance

EM sampling

Bidirectional sampling

Burke, Ghosh, and Heidrich
EGSR 2005

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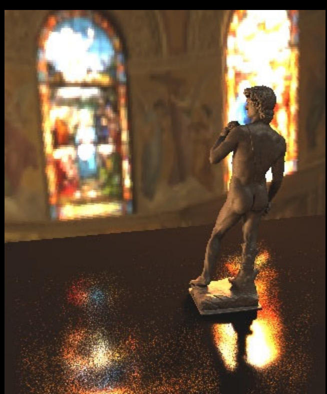
Bidirectional Importance



EM sampling

Bidirectional sampling

Burke, Ghosh, and Heidrich
EGSR 2005

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Bidirectional Importance

- If $\omega_i \sim p(\omega_i)$, Equation 1 can be estimated as:

$$L_{N,p}(\omega_r) = \frac{1}{N} \sum_{j=1}^{N} \frac{f_r(\omega_{i,j} \rightarrow \omega_r) \cos\theta_{i,j}\, L_i(\omega_{i,j}) V(\omega_{i,j})}{p(\omega_{i,j})}$$

$$= \frac{L_{ns}}{N} \sum_{j=1}^{N} V(\omega_{i,j}) \qquad (3)$$

where L_{ns} ("no-shadows") $:= \int_\Omega f_r(\omega_i \rightarrow \omega_r) \cos\theta_i\, L_i(\omega_i)\, d\omega_i$

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Bidirectional Importance

- Variance of bidirectional estimator:

$$\omega_{i,j} \sim p(\omega_i) \longrightarrow \mathrm{var}(L_{N,p}) = \frac{L_{ns}^2}{N} \mathrm{var}(V(\omega_i))$$

where L_{ns} ("no-shadows") $:= \int_\Omega f_r(\omega_i \rightarrow \omega_r) \cos\theta_i\, L_i(\omega_i)\, d\omega_I$

- variance of bidirectional sampling proportional only to variance in visibility
  - Visibility evaluated using ray-tracing
- Not dependent on BRDF or EM! (Optimal)

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Realizing Bidirectional Sampling

- 2 step approach
  - generate samples from one distribution
  - adjust samples to be proportional to the product *p*
- 2 Monte Carlo techniques for redistribution
  - rejection sampling
  - sampling importance re-sampling (SIR)

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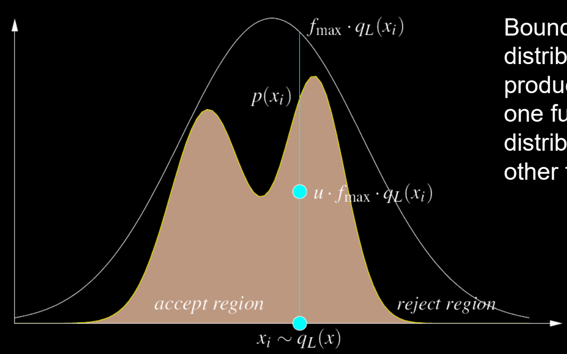
Rejection Sampling



Bound the product distribution with the product of max of one function with the distribution of the other function!

- Accept $\omega_{i,j}$ with probability:

$$\frac{p(\omega_{i,j})}{f_{max} \cdot q_L(\omega_{i,j})} = \frac{f_r(\omega_{i,j} \rightarrow \omega_r) \cos\theta_{i,j} \int_\Omega L_i(\omega_i) d\omega_i}{f_{max} \cdot L_{ns}}$$

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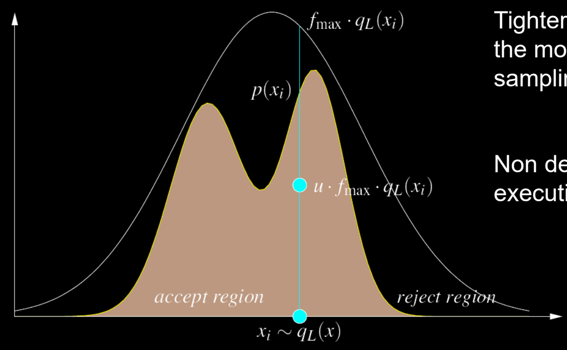
Rejection Sampling



Tighter the bound, the more efficient the sampling!

Non deterministic execution

- Accept $\omega_{i,j}$ with probability:

$$\frac{p(\omega_{i,j})}{f_{max} \cdot q_L(\omega_{i,j})} = \frac{f_r(\omega_{i,j} \rightarrow \omega_r)\cos\theta_{i,j} \int_\Omega L_i(\omega_i) d\omega_i}{f_{max} \cdot L_{ns}}$$

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Sampling Importance Resampling

Stage 1

$\omega_{i,j} \sim f(\omega_{i,j})$	$L(\omega_{i,j})$
$\omega_{i,1}$	$L(\omega_{i,1})$
$\omega_{i,2}$	$L(\omega_{i,2})$
. . .	. . .
$\omega_{i,M}$	$L(\omega_{i,M})$

Stage 2

$\omega_{i,j} \sim L_M(\omega_{i,j})$
$\omega_{i,1}$
$\omega_{i,2}$
. . .
$\omega_{i,N}$

M ≈ 100 · N

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Sampling Importance Resampling

Deterministic execution, no need to choose bounding function

Stage 1

$\omega_{i,j} \sim f(\omega_{i,j})$	$L(\omega_{i,j})$
$\omega_{i,1}$	$L(\omega_{i,1})$
$\omega_{i,2}$	$L(\omega_{i,2})$
. . .	. . .
$\omega_{i,M}$	$L(\omega_{i,M})$

Stage 2

$M \approx 100 \cdot N$

$\omega_{i,j} \sim L_M(\omega_{i,j})$
$\omega_{i,1}$
$\omega_{i,2}$
. . .
$\omega_{i,N}$

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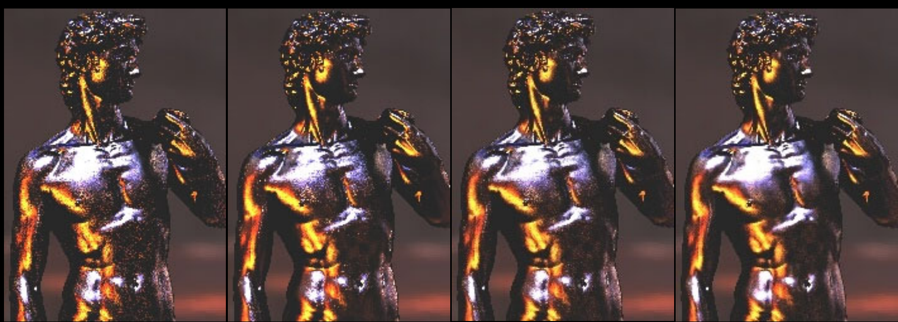
Phong BRDF ($s = 50$, $k_s = 1.0$)



EM
100 samples

BRDF
75 samples

MIS

SIR
15/800
(N/M)
samples

- Render time 13 seconds for 176 × 248 image
- Visibility traced only for N=15 samples with SIR!

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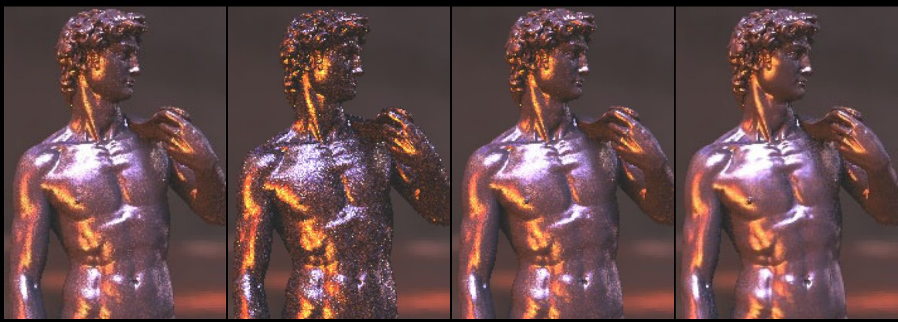
Phong BRDF ($s = 50$, $k_s = 1.0$, $k_d = 0.5$)



EM
100 samples

BRDF
75 samples

MIS

SIR
15/800
(N/M)
samples

- Render time 13 seconds for 176 × 248 image

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Bias Vs Variance

- Monte Carlo sampling – correct solution
  - drawback variance
- Deterministic sampling – consistent solution
  - drawback bias

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Deterministic Approach - Median Cut

1. Add the entire light probe image to the region list as a single region
2. For each region, subdivide along the longest dimension such that its light energy is divided evenly
3. If the number of iterations is less than *n*, return to step 2.
4. Place a light at the centroid of each region with its color and energy.

Paul Debevec, SIGGRAPH 2005 Poster

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Median Cut Algorithm

1 region

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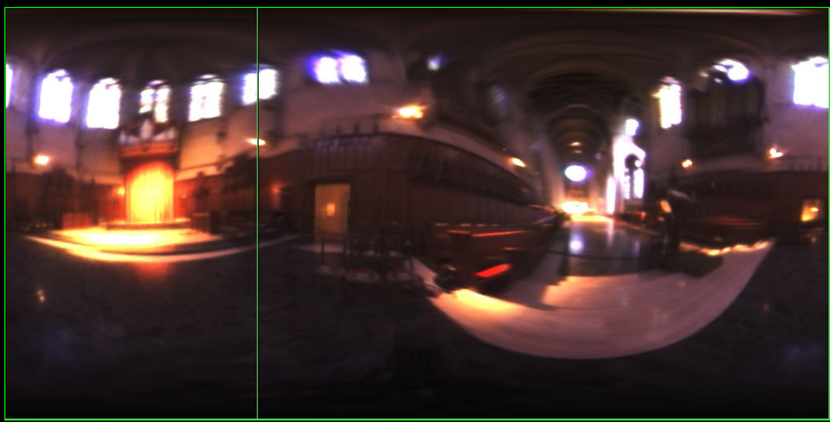
Median Cut Algorithm



2 regions

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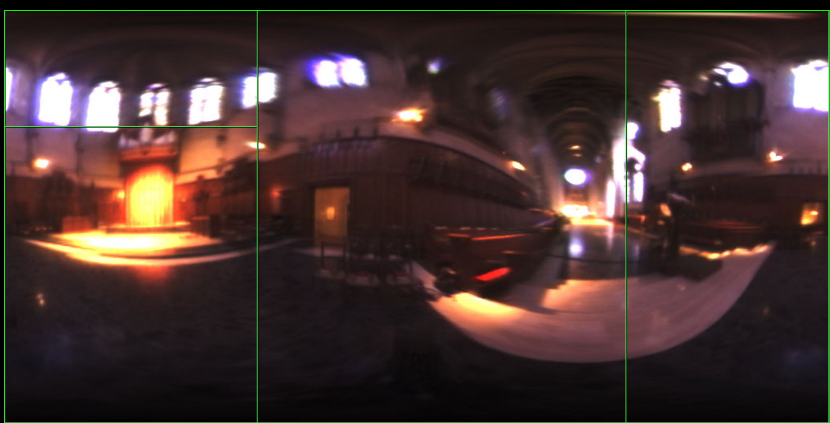
Median Cut Algorithm



4 regions

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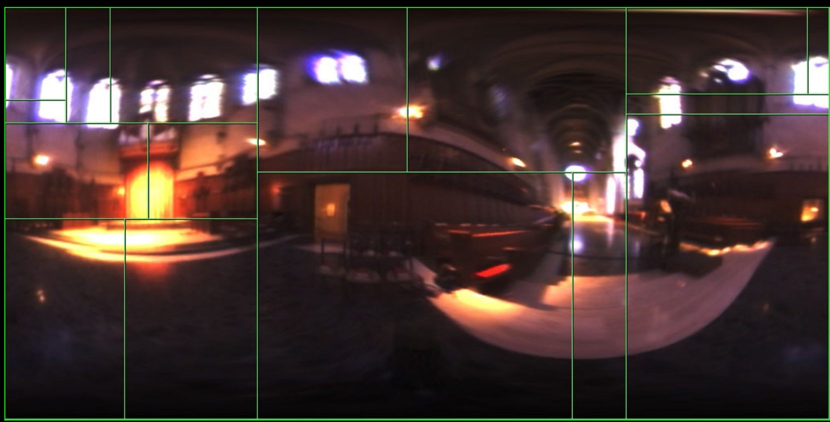
Median Cut Algorithm



16 regions

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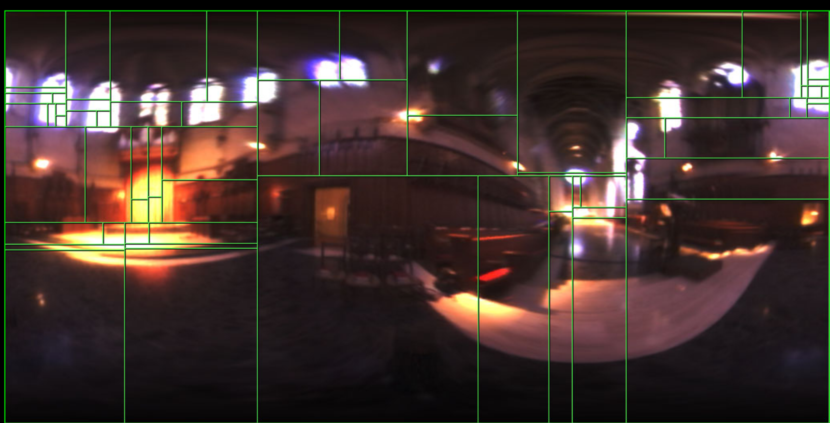
Median Cut Algorithm



64 regions

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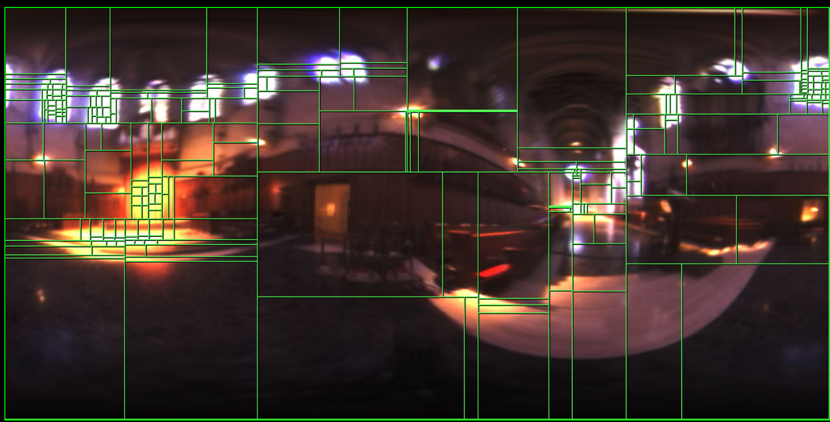
Median Cut Algorithm



256 regions

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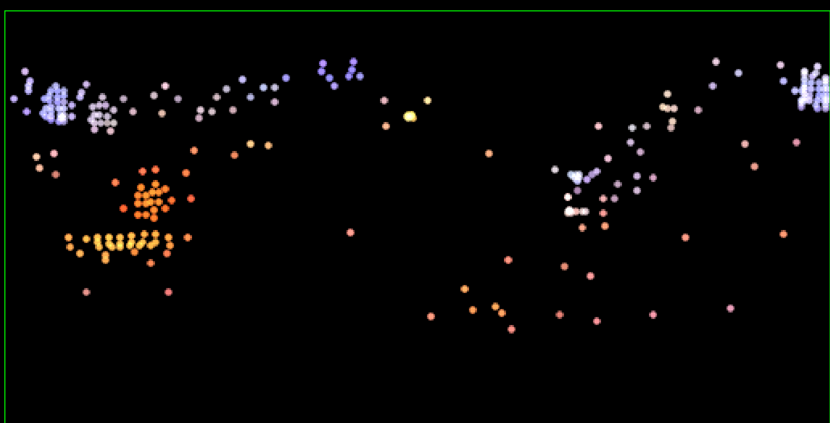
Final Light Sources



256 lights

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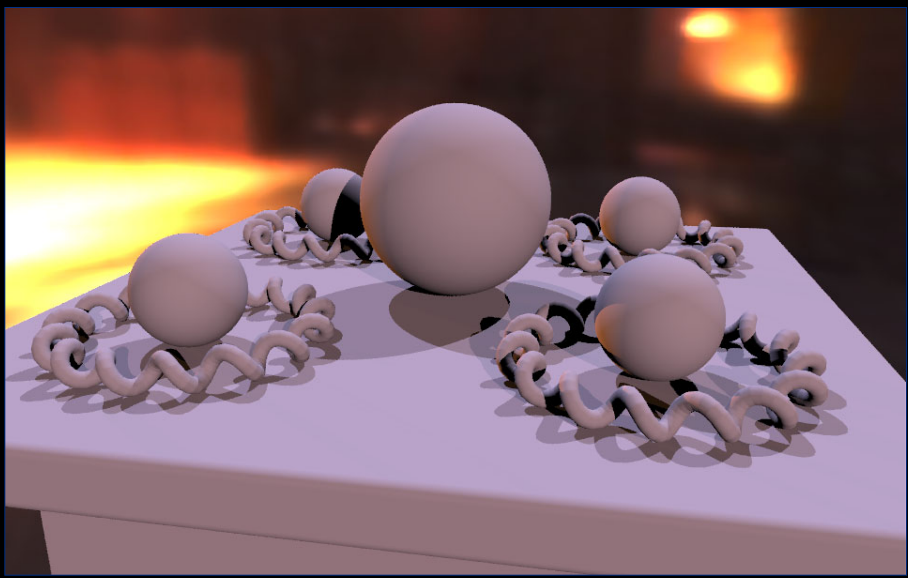

4 lights

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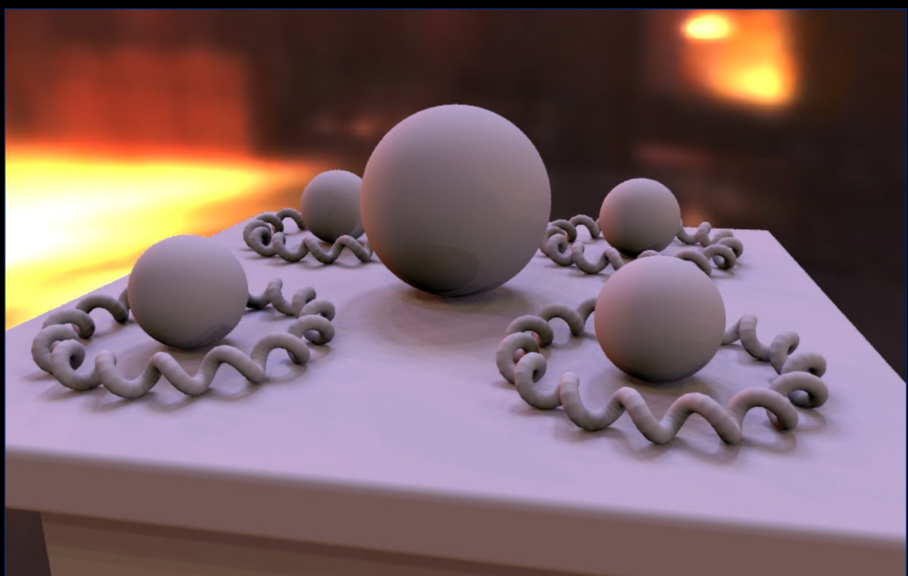

64 lights

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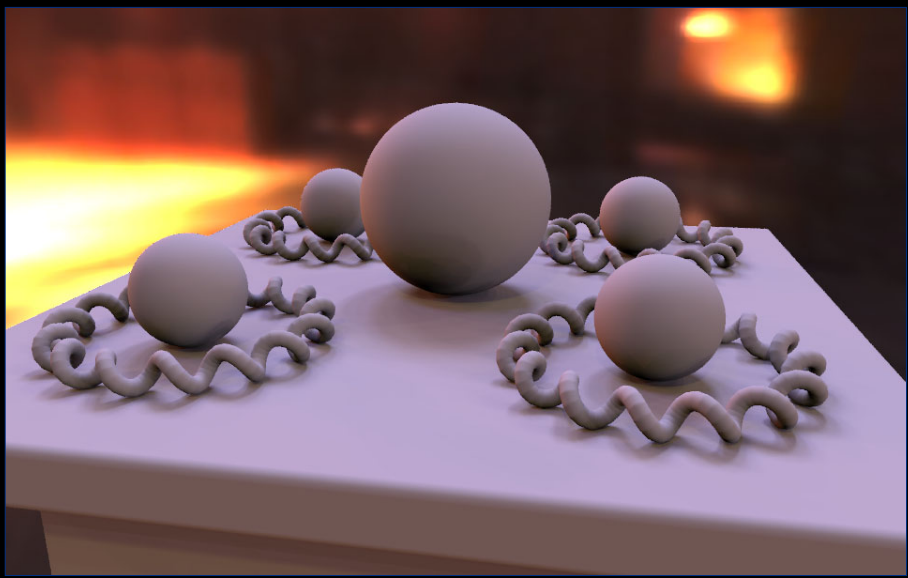

256 lights

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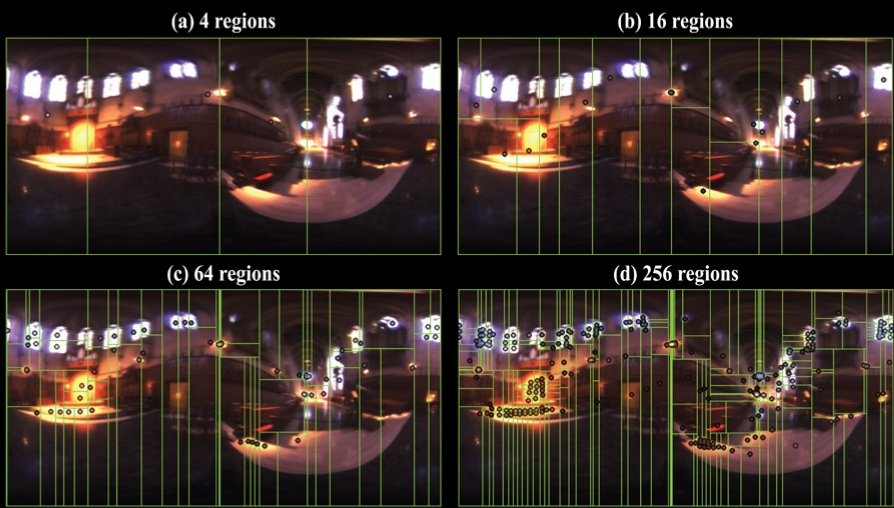
Variance Minimization



- Kuntee Viriyothai and Paul Debevec, SIGGRAPH 2009 poster

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Variance Minimization

Algorithm:

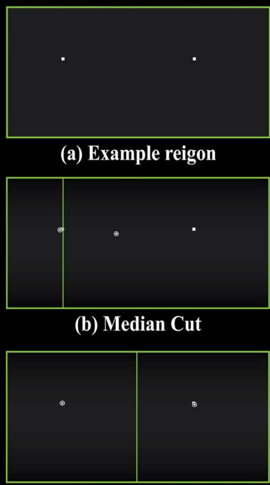
The algorithm recursively divides the entire light probe image into 2^n regions as follows:

1. Add the entire light probe image to the region list as a single region.

2. For each region in the list, subdivide such that the maximum of the two sub-regions' variances is minimized.

3. If the number of iterations is less than ***n***, return to step 2.

4. Place a light source at the centroid of each region, and set the light source color to the sum of the pixel values within the region.



(a) Example reigon

(b) Median Cut

(c) Variance Minimization

- Kuntee Viriyothai and Paul Debevec, SIGGRAPH 2009 poster

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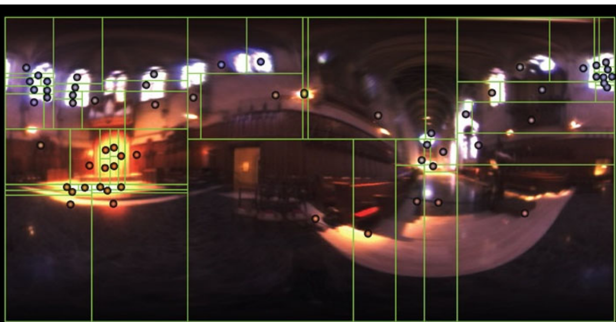

(a) Median Cut 64 lights

(b) Variance Minimization 64 lights

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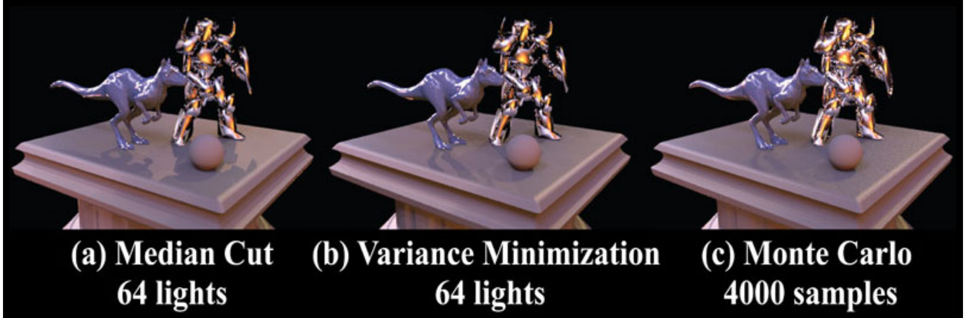
Variance Minimization



- Kuntee Viriyothai and Paul Debevec, SIGGRAPH 2009 poster

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