70001 - Sample example questions with solutions

Q1) A camera pixel measures the following LDR values at 1-stop apart: 32, 39, 52, 79, 132, 239, 255. The camera response function is known to be the function: **f(x) = 4/3x + 0.1**.

i) Estimate the linear values [0, 1] for these exposures after inverting the camera response function.
ii) Combine the above obtained linear values into an HDR value [0, 1] with uniform weighting.

Solution:
Camera response function f(x) = 4/3x + 0.1. Inverse function $f^{-1}(x) = (y - 0.1)*0.75$.

i) First we compute the values in [0, 1] from original [0, 255]: 32 → 0.1098, 39 → 0.12157, 52 → 0.15294, 79 → 0.211765, 132 → 0.3294, 239 → 0.5647. The last value 255 is ignored as it is saturated.

Transforming these values by the inverse response function $f^{-1}(x)$ linearizes them to the following values 1-stop apart: 0.01911765, 0.039706, 0.07794, 0.157353, 0.313235, 0.62794.

ii) Given uniform weights for all values, we combine these 6 values into the final HDR value as:

I = ((0.01911765)/1 *1/6) + ((0.039706)/2 *1/6) + ((0.07794)/4 * 1/6) + ((0.157353)/8 * 1/6) + ((0.313235)/16 * 1/6) + ((0.62794)/32 * 1/6) = 0.0031863 + 0.0033088 + 0.0032475 + 0.00327818 + 0.00326286 + 0.0032705 = 0.019554.

Q2) Assume a completely uniform grey sky illuminates a diffuse surface (**ρ_d = 0.4**) on the ground. A camera looking at the diffuse surface records a reflected radiance of **0.3** W/(m^2sr). What is the incident radiance on the surface from all directions of the grey sky?

Solution: Total reflected radiance towards camera $L(\omega_r) = 0.3$. The incident constant illumination from all directions $L(\omega_i)$ can be computed using for the following irradiance equation:

$$L(\omega_r) = 0.3 = \rho_d/\pi * \int_\Omega L(\omega_i) \cos\theta_i \, d\omega.$$

Since $L(\omega_i)$ is constant from all directions in the upper hemisphere, the equation simplifies to:

$$0.3 = (0.4/\pi) * L(\omega_i) * \int_\Omega \cos\theta_i \, d\omega.$$

$\int_\Omega \cos\theta_i \, d\omega$ over the upper hemisphere = π. Thus, the incident constant radiance $L(\omega_i)$ is give as:

$$L(\omega_i) = 0.3/0.4 = 0.75 \text{ W/(m}^2\text{sr)}.$$

Q3)) If you construct a light-field camera with the main sensor resolution of 4000×4000, and a micro-lens array of resolution 200×200, what is the resolution of the viewing (u,v) plane of the light-field, and what is the resolution of the focal (s,t) plane of the light-field?

Solution: With such a light-field camera, we can create images of a scene with 200×200 resolution (equal to the number of micro-lenses) which is the resolution of the (s,t) focal plane. However, each micro-lens collects 20×20 rays of the scene, making the resolution of the viewing plane (u,v) to be 20×20.

Q4) Given index of refraction $\boldsymbol{\eta}$ = 1.65, estimate the reflectance at normal incidence $\mathbf{R_0}$. Assuming this is a dielectric surface, compute the unpolarized reflectance **R** at an angle of incidence $\boldsymbol{\theta_i}$ = 45° using the Schlick approximation formula. Also compute the angle of transmittance $\boldsymbol{\theta_t}$ across the interface for light incident at $\boldsymbol{\theta_i}$.

Solution: Given index of refraction η = 1.65, the reflectance at normal incidence R_0 can be estimated using the following: $R_0 = (1 - \eta)^2/(1 + \eta)^2 = (1 - 1.65)^2/(1 + 1.65)^2 = 0.4225/7.0225 = 0.06016$.

Employing the Schlick approximation formula, the unpolarized reflectance $R = R_0 + (1 - R_0)(1 - \cos\theta)^5 = 0.06016 + 0.93984*(1 - 0.707107)^5 = 0.06016 + 0.93984 * 0.002155 = 0.062185$.

The angle of transmittance θ_t for light incident at θ_i = 45° is given by Snell's law: $\eta_i \sin\theta_i = \eta_t \sin\theta_t$.

Thus $\theta_t = \sin^{-1}(\sin\theta_i/\eta_t)$, since $\eta_i = 1.0$. This give $\theta_t = \sin^{-1}(0.70717/1.65) = 25.3755°$.

Q5) Compute the reflected radiance on a diffuse surface with albedo $\rho_d = 0.5$ for the following two cases:

i) A light probe with constant radiance of π.

ii) An area light source of surface area 1.5 m^2 emitting unit radiance towards the surface. The area source is at a distance of 4m from the surface and incident at an angle θ_i = 30° from the surface normal. The area light is itself oriented at an angle θ_o = 20° with the incident direction.

Solution:

i) We need to compute the following integral: $I = \int_\Omega f_r(x, \omega_i, \omega_r)\, L_i(\omega_i) \cos\theta_i\, d\omega_i$, where Ω is the upper hemisphere. For a diffuse BRDF $f_r(x, \omega_i, \omega_r) = \rho_d/\pi$, and the incident illumination has a constant value of π. Thus, the integral simplifies to:
$I = \int_\Omega (0.5/\pi) * \pi * \cos\theta_i\, d\omega_i = 0.5 \int_{\varphi=0\to2\pi} \int_{\theta=0\to\pi/2} \cos\theta_i * \sin\theta_i\, d\theta_i\, d\varphi_i$.
$= 0.5 * 2\pi * \int_{\theta=0\to\pi/2} \cos\theta_i * \sin\theta_i\, d\theta_i = \pi * ½ = \pi/2$.

ii) We need to convert the angular integral $I = \int_\Omega f_r(x, \omega_i, \omega_r)\, L_i(\omega_i) \cos\theta_i\, d\omega_i$ into an area integral over the light source as $I_A = \int_A f_r(x, \omega_i, \omega_r)\, L_i(\omega_i) \cos\theta_i\, (\cos\theta_o/d^2)\, dA$, where d is the distance to the area light source.

The above simplifies to $I_A = (0.5/\pi) * 1.0 * \cos(30°) * \cos(20°)/(4*4) * 1.5 = 0.01492 * 0.866 * 0.93969 = 0.01214$ W/(m^2 sr).

Q6) Sample $\boldsymbol{\omega_h}$ from the isotropic Ward distribution: $\boldsymbol{p(\omega_h) \sim \exp\!-(\tan^2\theta_h/\alpha^2)}$, given $\boldsymbol{\alpha} = 0.2$ and using two random variables $\boldsymbol{\mu_1 = 0.3}$, and $\boldsymbol{\mu_2 = 0.5}$. (Hint: μ_1, μ_2 control sampling of $\boldsymbol{\theta_h}$, $\boldsymbol{\varphi_h}$, resp.)

Solution: The isotropic Ward distribution can be sampled by inverting the given CDF and separating the two variables θ and φ. Then the sampling of each variable requires drawing a random number *u* in [0, 1] and mapping it through the analytic inverse CDF as follows:

$$\theta_h = \arctan(\alpha \sqrt{-\log u_1})$$

$$\varphi_h = 2\pi\, u_2.$$

Given $\alpha = 0.2$, and two random variables $u_1 = 0.3$, and $u_2 = 0.5$, we first u_1 to sample θ_h as:

$\theta_h = \arctan(0.2 * \sqrt{-\log 0.3}) = \arctan(0.2 * 1.09725) = \arctan(0.21945) = 0.2160269 = 12.377357°$.

We then employ u_2 to sample φ_h as: $\varphi_h = 2\pi*0.5 = \pi = 180.0°$.

Q7) i) Given a two layered diffusion medium with known reflectance $\mathbf{R_1} = 0.25$ and transmittance $\mathbf{T_1} = 0.65$ of the top layer, and a measured total reflectance due to the two layers $\mathbf{R_{12}} = 0.35$, compute the reflectance of the second layer $\mathbf{R_2}$.

ii) Also, given the measured total transmittance across both layers $\mathbf{T_{12}} = 0.45$, compute the transmittance of just second layer $\mathbf{T_2}$.

Solution:

i) Applying the Kubelka-Munk formula: $R_{12} = R_1 + T_1*R_2*T_1/(1- R_1*R_2)$

Therefore, $(R_{12} - R_1)(1 - R_1*R_2) = T_1*R_2*T_1$. Substituting the values: $(0.35 – 0.25)(1 – 0.25*R_2) = 0.65*0.65* R_2$. This becomes: $0.1 – 0.025*R_2 = 0.4225*R_2$.

Thus $R_2 = 0.1/(0.4225 + 0.025) = 0.1/0.4475 = 0.22346$

ii) Now applying the formula: $T_{12} = T_1*T_2/(1- R_1*R_2)$

Therefore: $0.45 = 0.65*T_2 /(1 – 0.25 * 0.22345)$, which leads to $T_2 = 0.45/0.65 * 0.9441375 = 0.65363365$.