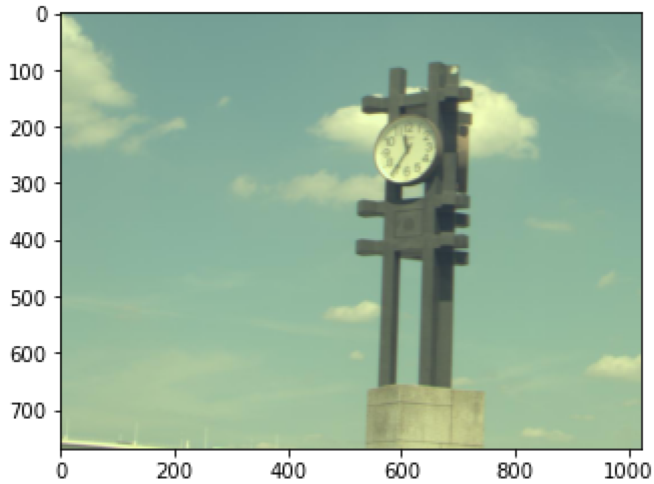
Criteria for Truncated SVD: Python codes

In [48]:

```
#Necessary libraries to be imported:
import numpy as np
import scipy.linalg as spl
from scipy.interpolate import interp1d
import scipy.integrate as spi
from KDEpy import FFTKDE
from sklearn.ensemble import IsolationForest
from sklearn.cluster import KMeans
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
import time

```

In [49]:

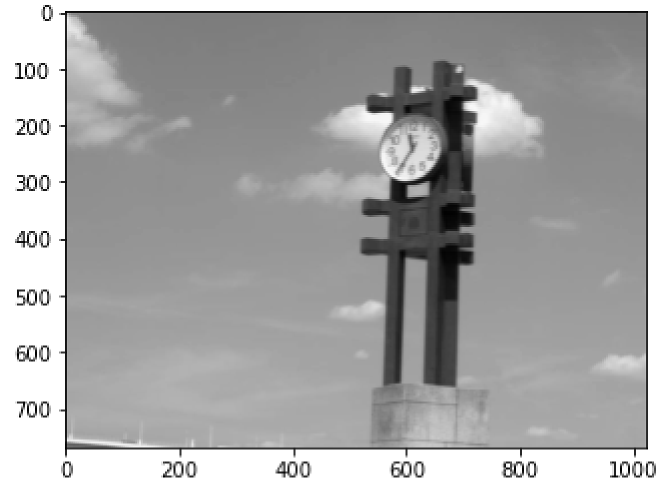
```
#Loading and plotting the firts image:
img = mpimg.imread('0002.jpg')
imgplot = plt.imshow(img)
```



In [50]:

```
#Gray-scale transformation
print('shape of original image',img.shape)
rgb_weights = [0.2989, 0.5870, 0.1140]
imgg=np.dot(img,rgb_weights)
print('shape gray scale',imgg.shape)
imgplot = plt.imshow(imgg,cmap=plt.get_cmap("gray"))
```

```
shape of original image (768, 1024, 3)
shape gray scale (768, 1024)
```



In [51]:

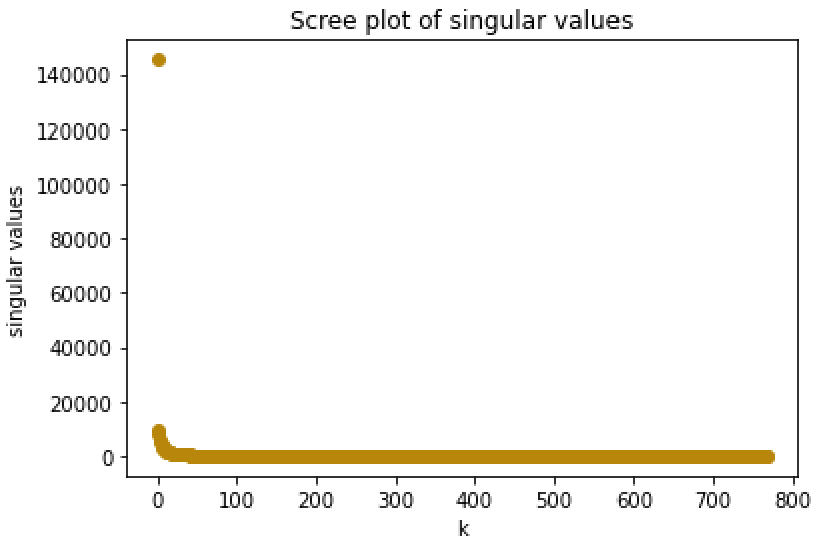
```
#SVD computation
Uimg, sigm, Vimg = spl.svd(imgg, full_matrices=False)
r = np.linalg.matrix_rank(imgg)
```

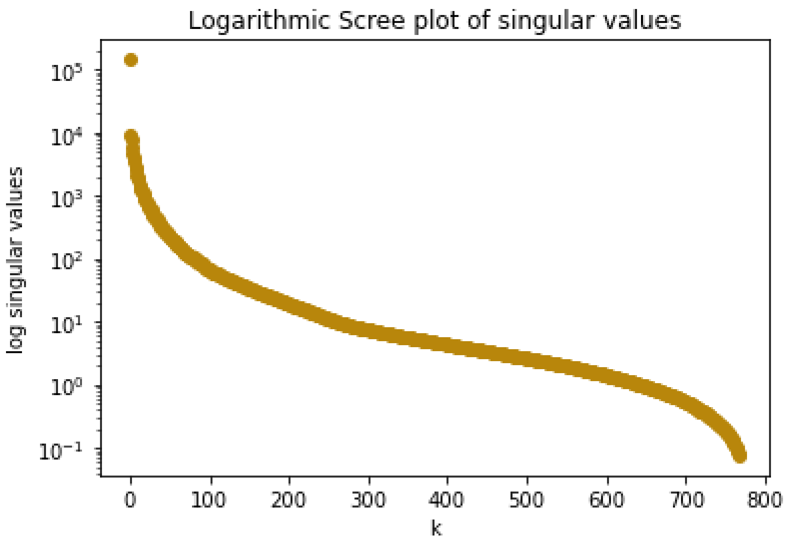
Scree plot

In [52]:

```python
#Scree plot of the singular values
t = time.time()
plt.figure()
plt.plot(np.linspace(0,len(sigm),len(sigm)),sigm,'o', color='darkgoldenrod')
plt.title('Scree plot of singular values')
plt.xlabel('k')
plt.ylabel('singular values')
plt.figure()
plt.semilogy(np.linspace(0,len(sigm),len(sigm)),sigm,'o', color = 'darkgoldenrod')
plt.title('Logarithmic Scree plot of singular values')
plt.xlabel('k')
plt.ylabel('log singular values')
elapsed = time.time() - t
print('\n Used time  in seconds to do the Scree plot:', elapsed)
```

```
 Used time  in seconds to do the Scree plot: 0.10290122032165527
```





We use the logarithmic Scree plot as for the standard Scree plot is not easy to see the carachteristic "elbow" shape

In [53]:

```
Sigma_scree= sigm[0:300]*np.eye(300,300)
Rec_scree = np.dot(Uimg[:,0:300], np.dot(Sigma_scree,Vimg[0:300,:]))

print('\n Relative error with logarithmic Scree plot: ', np.linalg.norm(imgg-Rec_scree)
```

```
 Relative error with logarithmic Scree plot:  0.00048448281963696854
```

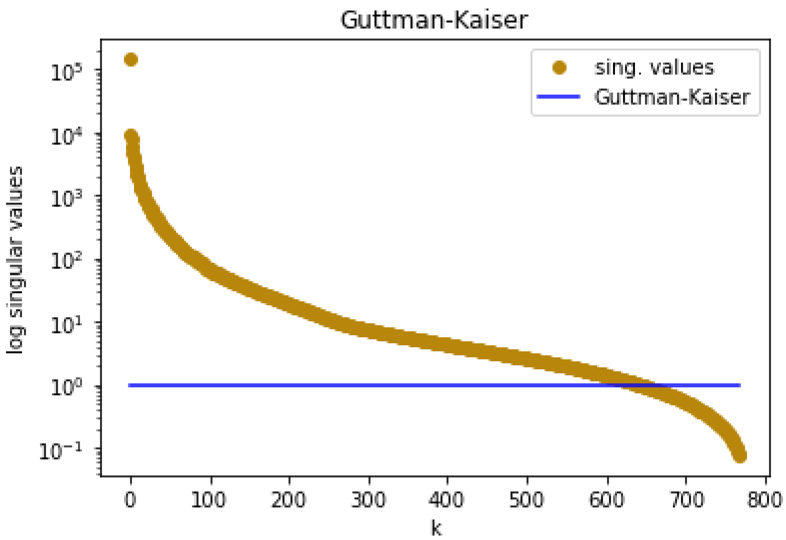
Guttman-Kaiser criterion

Choose the first k singular values such that: $\forall i > k, \sigma_i < 1$.

In [54]:

```
t = time.time()
plt.figure()
plt.semilogy(np.linspace(0,len(sigm),len(sigm)),sigm,'o', color='darkgoldenrod',label =
plt.plot(np.linspace(0,len(sigm),len(sigm)), np.ones([len(sigm),1]),'-b', label = 'Gutt
plt.title('Guttman-Kaiser')
plt.xlabel('k')
plt.ylabel('log singular values')
plt.legend()
elapsed = time.time() - t
print('\n Used time  in seconds for the Guttman-Keiser plot:', elapsed)
```

```
 Used time  in seconds for the Guttman-Keiser plot: 0.024935007095336914
```



In [55]:

```
Sigma_GK= sigm[0:600]*np.eye(600,600)
Rec_GK = np.dot(Uimg[:,0:600], np.dot(Sigma_GK,Vimg[0:600,:]))

print('\n Relative error with Gutman-Kaiser criterion: ', np.linalg.norm(imgg-Rec_GK)/s
```

```
 Relative error with Gutman-Kaiser criterion:  6.905372329328467e-05
```

Broken-stick

In [56]:

```
def brokenStick(s):
    l = len(s)
    b = np.zeros([1,l])

    for i in range(len(s)):

        b[0,i] = np.sum(np.linspace(i,l,l-i))

    return b/l
```

In [57]:

```
t = time.time()
br = brokenStick(sigm)
elapsed = time.time() - t
print('\n Broken-stick criterion computation in seconds', elapsed)
```

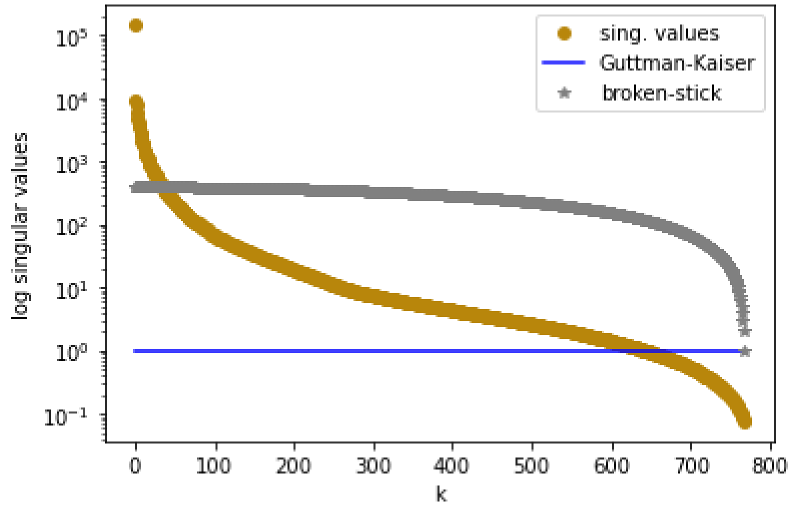
```
 Broken-stick criterion computation in seconds 0.046874046325683594
```

In [58]:

```
plt.figure()
plt.semilogy(np.linspace(0,len(sigm),len(sigm)),sigm,'o', color='darkgoldenrod',label =
plt.plot(np.linspace(0,len(sigm),len(sigm)), np.ones([len(sigm),1]),'-b', label = 'Gutt
plt.semilogy(np.linspace(0,len(sigm),len(sigm)), br[0,:], '*', color ='gray', label = '
#plt.title('')
plt.xlabel('k')
plt.ylabel('log singular values')
plt.legend()
```

Out[58]:

```
<matplotlib.legend.Legend at 0x2255f550048>
```



In [59]:

```
Sigma_bstick= sigm[0:60]*np.eye(60,60)
Rec_bstick = np.dot(Uimg[:,0:60], np.dot(Sigma_bstick,Vimg[0:60,:]))

print('\n Relative error with broken-stick: ', np.linalg.norm(imgg-Rec_bstick)/sigm[0])
```

```
 Relative error with broken-stick:  0.005577044096581399
```

Hard Thresholding Method

The code for the omega coefficient computation can be found in Matlab here:

D. L. Donoho and M. Gavish. (2014, Mar. 27). Code supplement to ‘the optimal hard threshold for singular values is $4/\sqrt{3}$ [Online]. Available: [http://purl.stanford.edu/vg705qn9070 (http://purl.stanford.edu/vg705qn9070)](http://purl.stanford.edu/vg705qn9070)

In [60]:

```
t = time.time()
ymedian = np.median(sigm)
omega = 2.502697248849711 # Matlab
tau_star = omega*ymedian
s_tau = sigm> tau_star
elapsed = time.time() - t
print('\n Time in seconds  %5 for the HT method ', elapsed)
print('\n indices of retained singular values\n',   np.nonzero(s_tau))
```

```
 Time in seconds  %5 for the HT method  0.0009994506835937 5

 indices of retained singular values
 (array([  0,   1,   2,   3,   4,   5,   6,   7,   8,   9,  10,  11,  12,
         13,  14,  15,  16,  17,  18,  19,  20,  21,  22,  23,  24,  25,
         26,  27,  28,  29,  30,  31,  32,  33,  34,  35,  36,  37,  38,
         39,  40,  41,  42,  43,  44,  45,  46,  47,  48,  49,  50,  51,
         52,  53,  54,  55,  56,  57,  58,  59,  60,  61,  62,  63,  64,
         65,  66,  67,  68,  69,  70,  71,  72,  73,  74,  75,  76,  77,
         78,  79,  80,  81,  82,  83,  84,  85,  86,  87,  88,  89,  90,
         91,  92,  93,  94,  95,  96,  97,  98,  99, 100, 101, 102, 103,
        104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116,
        117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129,
        130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142,
        143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155,
        156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168,
        169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181,
        182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194,
        195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207,
        208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220,
        221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233,
        234, 235, 236, 237, 238, 239, 240, 241, 242, 243], dtype=int64),)
```

In [61]:

```
Sigma_tau= sigm[0:244]*np.eye(244,244)
Rec_tau = np.dot(Uimg[:,0:244], np.dot(Sigma_tau,Vimg[0:244,:]))

print('\n Relative error with Tresholding method: ', np.linalg.norm(imgg-Rec_tau)/sigm
```

```
 Relative error with Tresholding method:  0.0006802486974626732
```

Random-projection range finder

In [62]:

```python
#Choose a tolerance:
tol = 1e-3

def range_finder(A,tol,max_it):

#Generate a list for the approximation error
    err = []
    err.append(1)
#Size of the input matrix
    [m,n] = A.shape

    l = 1
    it = 0
    while((err[-1]>tol) & (it<max_it)):

#1. Draw an n × l Gaussian random matrix Ω
        Omega = np.random.randn(n,l)

#2. Form the m × l sample matrix Y = AΩ.
        Y = np.dot(A,Omega)

#3. Form an m × l orthonormal matrix Q such that Y = QR.
        [Q,R]=spl.qr(Y,pivoting=False)

#Add the error for this iteration
        err.append(np.linalg.norm(A- np.dot(Q[:,0:l],np.dot((Q[:,0:l]).T,A))))
        it = it+1
        l = l+1
    l = l-1
#4. Form the l × n matrix B = Q_l*A.
    B = np.dot((Q[:,0:l]).T, A)

#5. Compute the SVD of the small matrix B: B = \hat{U}ΣV*.
    [Uhat, Sigma, V] = spl.svd(B)

#6. Form the matrix U = Q_l(\hat{U}).
    U = np.dot(Q[:,0:l],Uhat)

    return l,err,it,U,Sigma,V
```

In [63]:

```python
t = time.time()
k,error,n_it,Ur,Sigmar,Vr = range_finder(imgg,tol,100)
elapsed = time.time() - t
print('\n Used time in seconds for the range finder criterion ', elapsed)
```

```
Used time in seconds for the range finder criterion  3.9062225818634033
```

In [64]:

```
print("\n Number of singular values ", k)
print("\n error at the last iteration ", error[-1])
print("\n Number of required iterations ", n_it)
```

```
 Number of singular values  100

 error at the last iteration  858.332379629877

 Number of required iterations  100
```

In [65]:

```
#Reconstructed image
Sigmarr = Sigmar[0:k]*np.eye(k,k)
Recon = np.dot(Ur, np.dot(Sigmarr,Vr[0:k,:]))
```

In [66]:

```
print("\n approximation error ", np.linalg.norm(imgg-Recon))
print ("\n sigma_{k+1} ", sigm[k+1])

print("\n Relative error ", np.linalg.norm(imgg-Recon)/sigm[0])

```

```
 approximation error  858.3323796298777

 sigma_{k+1}  65.49845842917813

 Relative error  0.005900957359829836
```

In this case we do not have the control of the error as the algoirthm exits due to maximal number of iterations reached rather than set accuracy reached.

Entropy

In [67]:

```
t = time.time()
f = sigm**2/np.sum(sigm**2)
E = -1/np.log(r)*np.sum(f*np.log(f))
elapsed = time.time() - t
print('\n Used time in second for total entropy computation ', elapsed)
print('\n Entropy ', E)
```

```
 Used time in second for total entropy computation  0.0009989738464355469

 Entropy  0.015062674090159548
```

In [68]:

```
E_p70 = int(r*0.7*E) # 70% of total Entropy
print('\n number of singular values to retain to keep 70% ', E_p70)
E_p90 = int(r*0.9*E)
print('\n number of singular values to retain to keep 90% ', E_p90)

E_p100 = int(r*E)
print('\n number of singular values to retain to keep 100% ', E_p100)
```

```
 number of singular values to retain to keep 70%  8

 number of singular values to retain to keep 90%  10

 number of singular values to retain to keep 100%  11
```

In [69]:

```
t = time.time()
f_SVD = sigm/np.sum(sigm)
E_SVD = -1/np.log(r)*np.sum(f_SVD*np.log(f_SVD))
elapsed = time.time() - t
print('\n Time in second for the computation of the total Entropy_SVD criterion ', elap
print('\n Entropy ', E_SVD)
```

```
 Time in second for the computation of the total Entropy_SVD criterion  0.00
09653568267822266

 Entropy  0.317994382677596
```

In [70]:

```
E_SVD_p70 = int(r*0.7*E_SVD)
print('\n number of singular values to retain to keep E_SVD 70% ', E_SVD_p70)
E_SVD_p90 = int(r*0.9*E_SVD)
print('\n number of singular values to retain to keep E_SVD 90% ', E_SVD_p90)
E_SVD_p100 = int(r*E_SVD)
print('\n number of singular values to retain to keep E_SVD 100% ', E_SVD_p100)
```

```
 number of singular values to retain to keep E_SVD 70%  170

 number of singular values to retain to keep E_SVD 90%  219

 number of singular values to retain to keep E_SVD 100%  244
```

In [71]:

```
#Reconstructed image
Sigma_Ep70 = sigm[0:E_p70]*np.eye(E_p70,E_p70)
Sigma_Ep90 = sigm[0:E_p90]*np.eye(E_p90,E_p90)
Sigma_Ep100 = sigm[0:E_p100]*np.eye(E_p100,E_p100)

Sigma_SVD_Ep70 = sigm[0:E_SVD_p70]*np.eye(E_SVD_p70,E_SVD_p70)
Sigma_SVD_Ep90 = sigm[0:E_SVD_p90]*np.eye(E_SVD_p90,E_SVD_p90)
Sigma_SVD_Ep100 = sigm[0:E_SVD_p100]*np.eye(E_SVD_p100,E_SVD_p100)


Rec_Ep70 = np.dot(Uimg[:,0:E_p70], np.dot(Sigma_Ep70,Vimg[0:E_p70,:]))
Rec_Ep90 = np.dot(Uimg[:,0:E_p90], np.dot(Sigma_Ep90,Vimg[0:E_p90,:]))
Rec_Ep100 = np.dot(Uimg[:,0:E_p100], np.dot(Sigma_Ep100,Vimg[0:E_p100,:]))
Rec_SVD_Ep70 = np.dot(Uimg[:,0:E_SVD_p70], np.dot(Sigma_SVD_Ep70,Vimg[0:E_SVD_p70,:]))
Rec_SVD_Ep90 = np.dot(Uimg[:,0:E_SVD_p90], np.dot(Sigma_SVD_Ep90,Vimg[0:E_SVD_p90,:]))
Rec_SVD_Ep100 = np.dot(Uimg[:,0:E_SVD_p100], np.dot(Sigma_SVD_Ep100,Vimg[0:E_SVD_p100,:


print('\n Relative error with 70% ', np.linalg.norm(imgg-Rec_Ep70)/sigm[0])
print('\n Relative error with 90% ', np.linalg.norm(imgg-Rec_Ep90)/sigm[0])
print('\n Relative error with 100% ', np.linalg.norm(imgg-Rec_Ep100)/sigm[0])

print('\n Relative error with SVD 70% ', np.linalg.norm(imgg-Rec_SVD_Ep70)/sigm[0])
print('\n Relative error with SVD 90% ', np.linalg.norm(imgg-Rec_SVD_Ep90)/sigm[0])
print('\n Relative error with SVD 100% ', np.linalg.norm(imgg-Rec_SVD_Ep100)/sigm[0])
```

```
 Relative error with 70%  0.04216170665296159

 Relative error with 90%  0.0355028418238158

 Relative error with 100%  0.03281326312826331

 Relative error with SVD 70%  0.0013030580101773082

 Relative error with SVD 90%  0.0008266608790944184

 Relative error with SVD 100%  0.0006802486974626732
```

Total variance

In [77]:

```
t = time.time()
tot_var = np.sum(sigm**2)
elapsed = time.time() - t
print('\n Time in seconds  for the total variance ', elapsed)
print('\n total variance as by the singular values', tot_var)
print('\n 10% of total variance as by the singular values ', 0.1*tot_var )
```

```
 Time in seconds  for the total variance  0.0

 total variance as by the singular values 21441023776.228905

 10% of total variance as by the singular values  2144102377.6228905
```

In [78]:

```
Corr_imgg = np.dot(imgg,imgg.T)
rC,cC = Corr_imgg.shape

for i in range(cC):
    Corr_imgg[:,i] = Corr_imgg[:,i]-np.mean(Corr_imgg[:,i])/np.std(Corr_imgg[:,i])
```

In [79]:

```
L, E = np.linalg.eig(Corr_imgg)
```

In [80]:

```
Tot_var_Corr = np.sum(L)
print('\n total variance correlation matrix ', Tot_var_Corr)
print('\n percentaige of variance explained by the first eigenvalue ', 100*L[0]/Tot_var
print('\n percentaige of variance explained by the second eigenvalue ', 100*L[1]/Tot_va

index = L>0.05*Tot_var_Corr
print('\n number of eigenvalues explaining the 5% of total variance ', np.where(index==
```

```
 total variance correlation matrix  21441010803.761154

 percentaige of variance explained by the first eigenvalue  98.6780408680959
6

 percentaige of variance explained by the second eigenvalue  0.4087990162446
963

 number of eigenvalues explaining the 5% of total variance  (array([0], dtyp
e=int64),)
```

In [81]:

```
#Reconstructed image
Sigma_var = sigm[0]*np.eye(1,1)
U_var = Uimg[:,0].reshape(768,1)
Recon_var = np.dot(U_var, np.dot(Sigma_var,Vimg[0,:].reshape(1,1024)))
print('\n Relative error Total Variance ', np.linalg.norm(imgg-Recon_var)/sigm[0])

```

```
 Relative error Total Variance  0.11574404746321416
```

Percentage cumulative variance

In [82]:

```
t = time.time()
val = np.cumsum(L)/np.sum(L)
elapsed = time.time() - t
print('\n Time in seconds for the percentage cumulative varicance ', elapsed)
print('\n min value of val ', np.min(val))

```

```
 Time in seconds for the percentage cumulative varicance  0.0

 min value of val  0.9867804086809597
```

Since the first eigenvalue already hits 98% of the total variance, then by adding the contribution of the additional eigenvalue we exceed the set tolerance value by far.

Kullback-Leibler divergence method

In [83]:

```
def divergence_KL(f,r,a,b,n_pts):
    grid = np.linspace(a,b,n_pts)
    T = []
    kde = FFTKDE( kernel='tri')
    x,y= kde.fit(f).evaluate()

# Mirror the data about the domain boundary
    low_bound = 0
    data = np.concatenate((f, 2 * low_bound - f))

# Compute KDE using the bandwidth found, i.e. the parameter h, and twice as many grid p
    x, y = FFTKDE(bw=kde.bw, kernel='tri').fit(data).evaluate()
    y[x<=low_bound] = 0  # Set the KDE to zero outside of the domain
    y = y * 2  # Double the y-values to get integral of ~1

    kde_t = FFTKDE( kernel='tri')
    for k in range(r):
        x_t,y_t= kde_t.fit(f[0:k+1]).evaluate()
# Mirror the data about the domain boundary
        low_bound = 0
        data_t = np.concatenate((f[0:k+1], 2 * low_bound - f[0:k+1]))

# Compute KDE using the bandwidth found, and twice as many grid points
        x_t, y_t = FFTKDE(bw=kde_t.bw, kernel='tri').fit(data_t).evaluate()
        y_t[x_t<=low_bound] = 0  # Set the KDE to zero outside of the domain
        y_t = y_t * 2  # Double the y-values to get integral of ~1

        #Linear spline necessary only if evaluation is done at non-uniform grid
        f_linear = interp1d(x, y, kind="linear", assume_sorted=True, fill_value="extrap
        f_t_linear =interp1d(x_t, y_t, kind="linear", assume_sorted=True, fill_value="e

        Eval_f = lambda x_grid: f_linear(x_grid)*np.log(f_linear(x_grid)/f_t_linear(x_g

        T.append(spi.trapz(Eval_f(grid),grid))

    return T
```

In [84]:

```
tol = 1e-3
t = time.time()
divergence = divergence_KL(f,r, 1e-13,3,30)
tupla = np.where(np.array(divergence)<tol)[0]
elapsed = time.time() - t
print('\n Time in seconds for the KL method ', elapsed)
print("\n Numer of singular values to retain ", tupla[0])
```

```
 Time in seconds for the KL method  1.486753225326538

 Numer of singular values to retain  58
```

In [85]:

```
Sigma_KL= sigm[0:tupla[0]]*np.eye(tupla[0],tupla[0])

Rec_KL = np.dot(Uimg[:,0:tupla[0]], np.dot(Sigma_KL,Vimg[0:tupla[0],:]))

print('\n Relative error with KL-method ', np.linalg.norm(imgg-Rec_KL)/sigm[0])
```

```
 Relative error with KL-method  0.00583031025952867
```

Unsupervised anomaly detection based methods

In [86]:

```
t = time.time()
clf = IsolationForest(random_state=0, contamination = 0.05).fit_predict(sigm.reshape(-1
index_s = np.where(clf==-1)
index_s = index_s[0].astype(int)

Sigma_an1= sigm[index_s]*np.eye(len(index_s),len(index_s))
Rec_an1 = np.dot(Uimg[:,index_s], np.dot(Sigma_an1,Vimg[index_s,:]))
elapsed = time.time() - t
print('\n Time in seconds for the KIF method ', elapsed)
print(index_s)
print('\n Relative error with anomaly detection, first method: ', np.linalg.norm(imgg-R
```

```
 Time in seconds for the KIF method  0.2833232879638672
[ 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38]

 Relative error with anomaly detection, first method:  0.009468801588735927
```

In [87]:

```
t = time.time()
kmeans = KMeans(n_clusters=2, random_state=0).fit(np.log(sigm.reshape(-1,1)))
lab = kmeans.labels_

Cl0 = np.where(lab==0)
Cl1 = np.where(lab==1)

l0 = len(Cl0[0])
l1 = len(Cl1[0])

if 0 in  Cl0[0]:
    clf2 = IsolationForest(random_state=0).fit_predict(np.log(sigm[Cl0[0]].reshape(-1,1
    #print("Cl0 ", Cl0)
else:
    clf2 = IsolationForest(random_state=0).fit_predict(np.log(sigm[Cl1[0]].reshape(-1,1
    #print("Cl1 ", Cl1)
indice = np.where(clf2==-1)
indice = indice[0].astype(int)
Last = np.where(np.diff(indice)>1)[0]
Last = Last[0]
elapsed = time.time() - t
print('\n Time in seconds for the KMIF method ', elapsed)
print("\n number of singular values to retain ", Last)
```

```
 Time in seconds for the KMIF method  0.2879488468170166

 number of singular values to retain  30
```

In [88]:

```
Sigma_an2= sigm[0:Last]*np.eye(Last,Last)
Rec_an2 = np.dot(Uimg[:,0:Last], np.dot(Sigma_an2,Vimg[0:Last,:]))

print('\n Relative error with anomaly detection, second method: ', np.linalg.norm(imgg-
```

```
 Relative error with anomaly detection, second method:  0.012825831283319914
```

In [89]:

```
Sigma_an2_full= sigm[indice]*np.eye(len(indice),len(indice))
Rec_an2_full = np.dot(Uimg[:,indice], np.dot(Sigma_an2_full,Vimg[indice,:]))

print('\n Relative error with anomaly detection, second method full: ', np.linalg.norm(
```

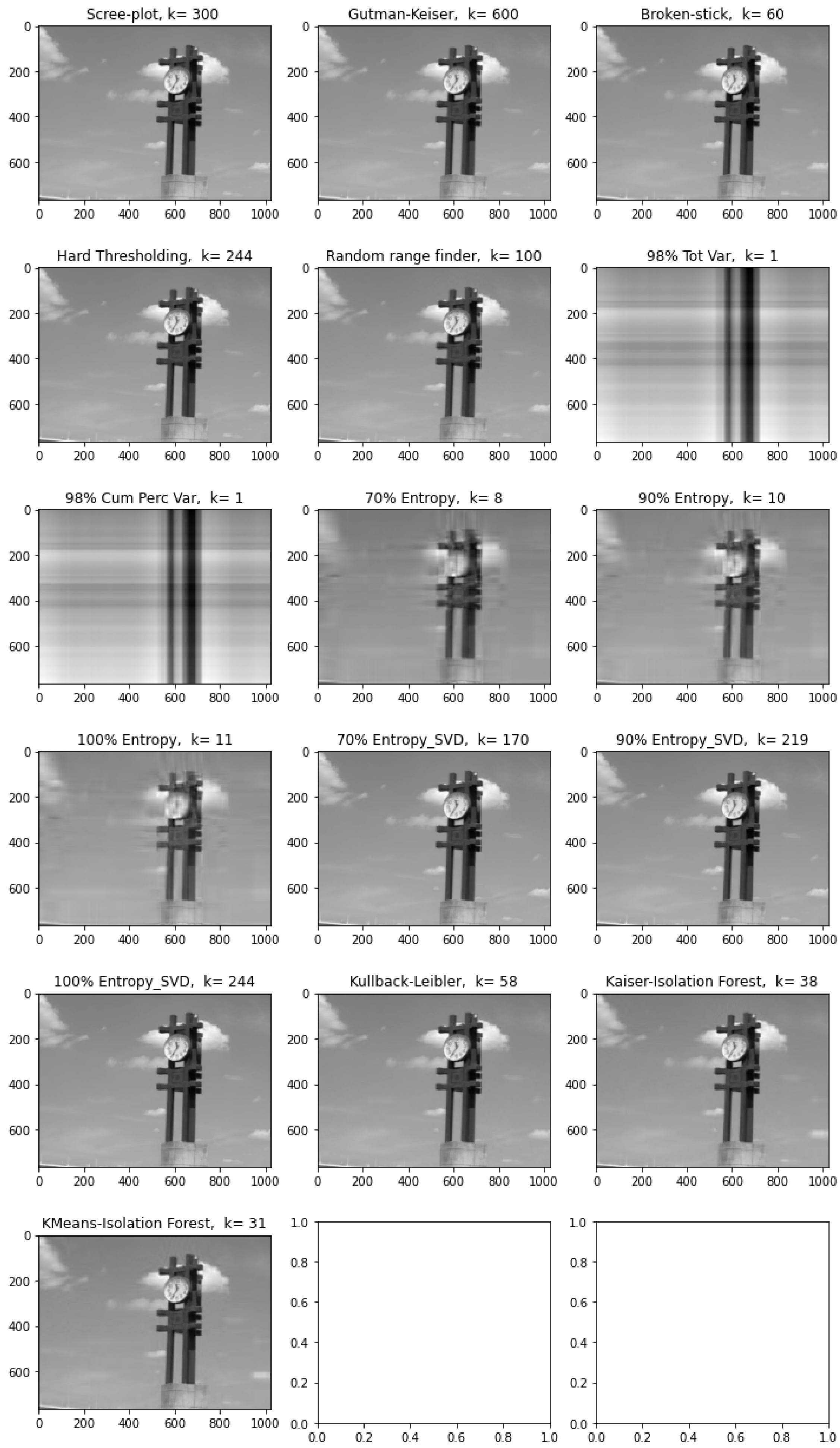
```
 Relative error with anomaly detection, second method full:  0.0121477323117
53973
```

In [45]:

```
fig, axarr=plt.subplots(nrows=6, ncols=3, figsize=(12, 22))

axarr[0,0].imshow(Rec_scree, cmap='gray')
axarr[0,0].set_title("Scree-plot, k= 300 ")
axarr[0,1].imshow(Rec_GK, cmap='gray')
axarr[0,1].set_title("Gutman-Keiser,  k= 600")
axarr[0,2].imshow(Rec_bstick, cmap='gray')
axarr[0,2].set_title("Broken-stick,  k= 60")


axarr[1,0].imshow(Rec_tau, cmap = 'gray')
axarr[1,0].set_title("Hard Thresholding,  k= 244")


axarr[1,1].imshow(Recon, cmap='gray')
axarr[1,1].set_title("Random range finder,  k= 100")

axarr[1,2].imshow(Recon_var, cmap='gray')
axarr[1,2].set_title("98% Tot Var,  k= 1")

axarr[2,0].imshow(Recon_var, cmap='gray')
axarr[2,0].set_title("98% Cum Perc Var,  k= 1")

axarr[2,1].imshow(Rec_Ep70, cmap='gray')
axarr[2,1].set_title("70% Entropy,  k= 8")
axarr[2,2].imshow(Rec_Ep90, cmap='gray')
axarr[2,2].set_title("90% Entropy,  k= 10")

axarr[3,0].imshow(Rec_Ep100, cmap='gray')
axarr[3,0].set_title("100% Entropy,  k= 11")

axarr[3,1].imshow(Rec_SVD_Ep70, cmap='gray')
axarr[3,1].set_title("70% Entropy_SVD,  k= 170")
axarr[3,2].imshow(Rec_SVD_Ep90, cmap='gray')
axarr[3,2].set_title("90% Entropy_SVD,  k= 219")
axarr[4,0].imshow(Rec_SVD_Ep100, cmap='gray')
axarr[4,0].set_title("100% Entropy_SVD,  k= 244")

axarr[4,1].imshow(Rec_KL, cmap='gray')
axarr[4,1].set_title("Kullback-Leibler,  k= 58")
axarr[4,2].imshow(Rec_an1, cmap='gray')
axarr[4,2].set_title("Kaiser-Isolation Forest,  k= 38")

axarr[5,0].imshow(Rec_an2, cmap='gray')
axarr[5,0].set_title("KMeans-Isolation Forest,  k= 31")
#axarr[1,4].imshow(Rec_an2_full, cmap='gray')
#axarr[1,4].set_title("Anomaly detection 2, full")
```

Out[45]:

```
Text(0.5, 1.0, 'KMeans-Isolation Forest,  k= 31')
```


Scree-plot, k= 300
Gutman-Keiser, k= 600
Broken-stick, k= 60
Hard Thresholding, k= 244
Random range finder, k= 100
98% Tot Var, k= 1
98% Cum Perc Var, k= 1
70% Entropy, k= 8
90% Entropy, k= 10
100% Entropy, k= 11
70% Entropy_SVD, k= 170
90% Entropy_SVD, k= 219
100% Entropy_SVD, k= 244
Kullback-Leibler, k= 58
Kaiser-Isolation Forest, k= 38
KMeans-Isolation Forest, k= 31