POLITECNICO
MILANO 1863

Politecnico di Milano
Automation and Control Engineering

YEARWORK REPORT

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Contents

Q1. Geometrical analysis of the structure

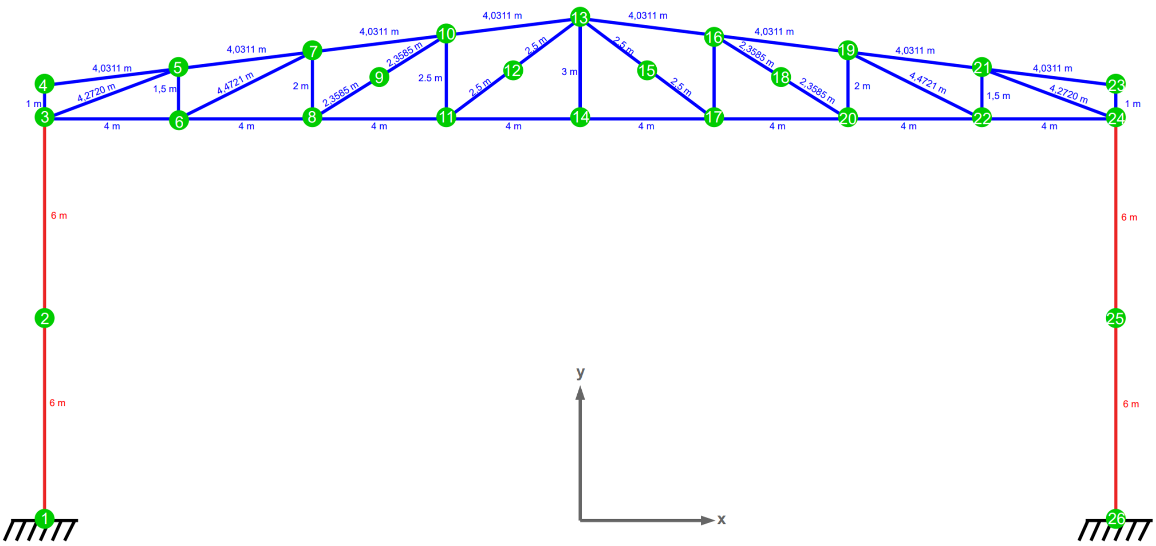
1.1 Maximum length

We can calculate the maximum length for each type of beam knowing that

- the maximum frequency of the analysis $f_{max} = 20Hz$
- assuming the safety coefficient $c = 2$

$$L_{max,red} = 6,40 \ \ m \tag{1.1}$$

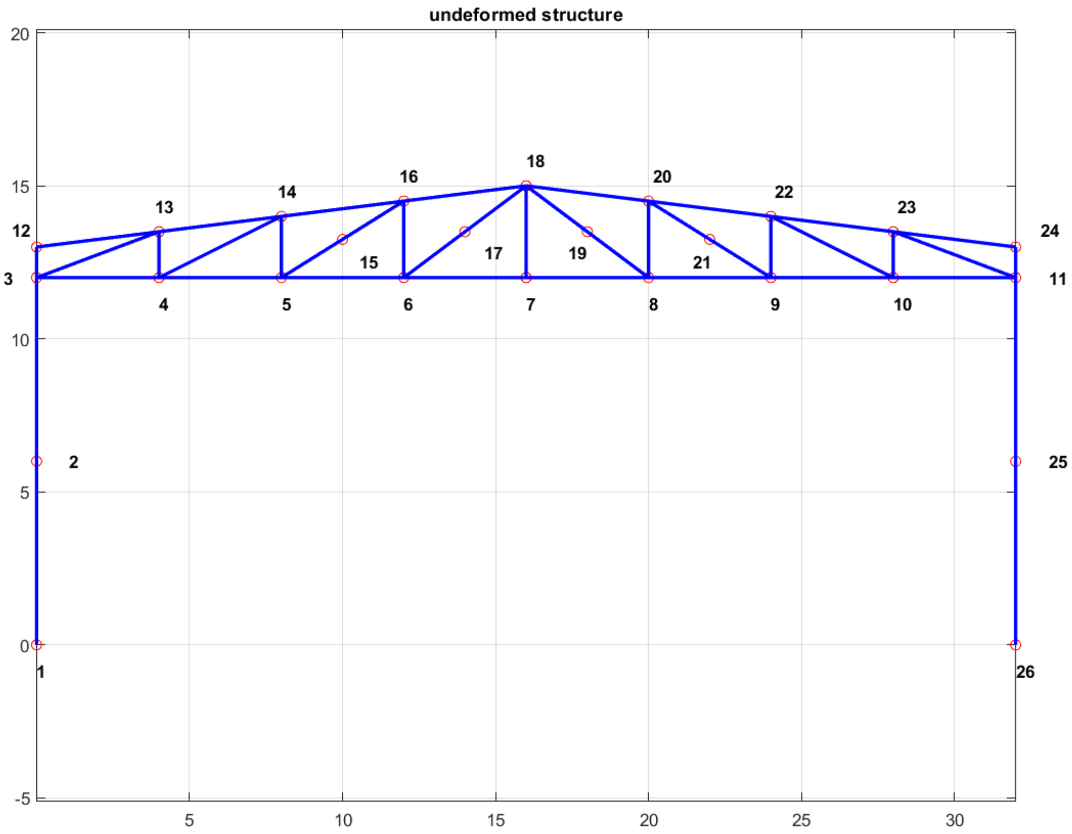
$$L_{max,blue} = 4,48 \ \ m \tag{1.2}$$



In this case, the FE model of the structure contains:

- **26 nodes**
- **41 nodal sections**

1.2 Matlab reconstruced undeformed structure



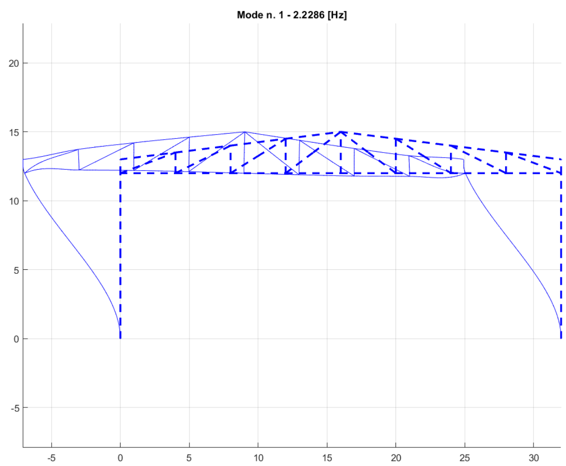
- Total nodes number: 26
- Number of d.o.f.: 72
- Number of beam elements: 41
- Total mass: **5804,66 Kg**

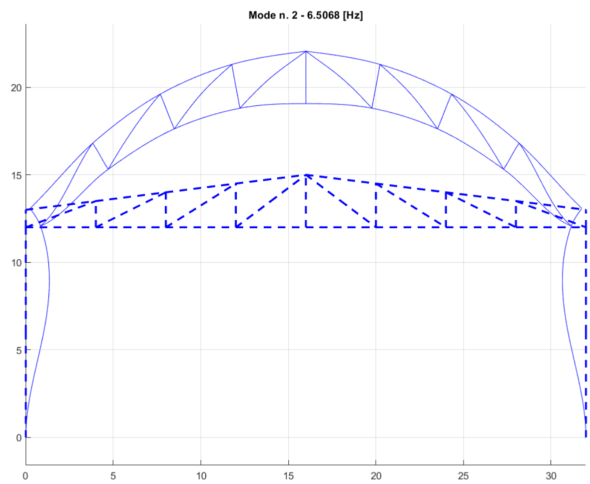
Q2. Natural freq. and modes of vibration

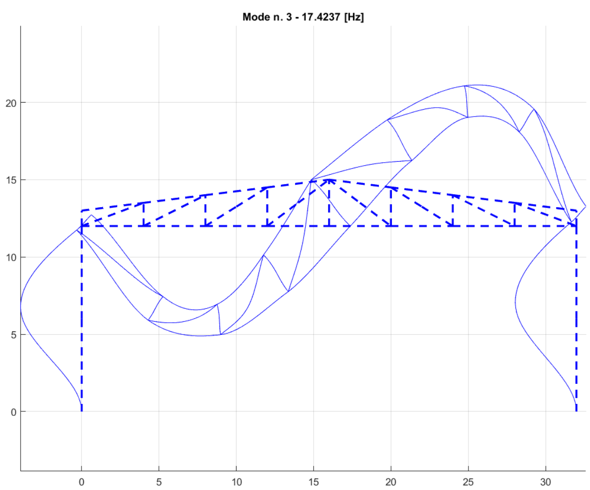
The first **4 natural frequencies of the structure** are:

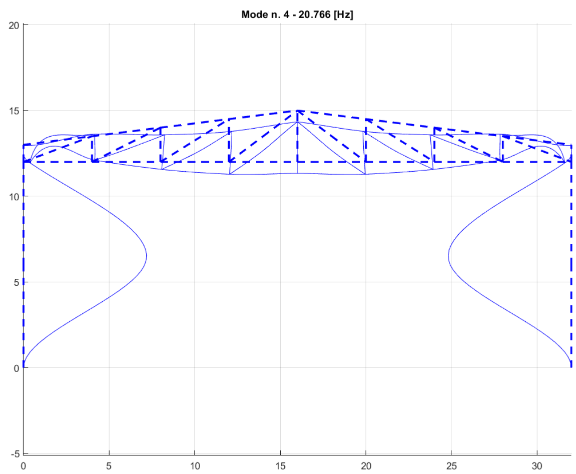
- $f_1 = 2,2286\ \ Hz$
- $f_2 = 6,5068\ \ Hz$
- $f_3 = 17,4237\ \ Hz$
- $f_4 = 20,766\ \ Hz$

And the related **modal shapes** are:



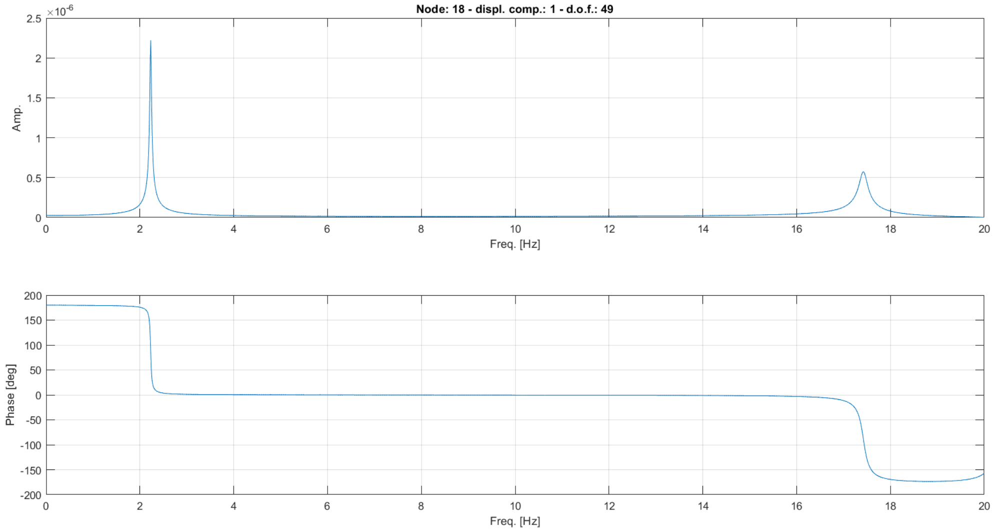




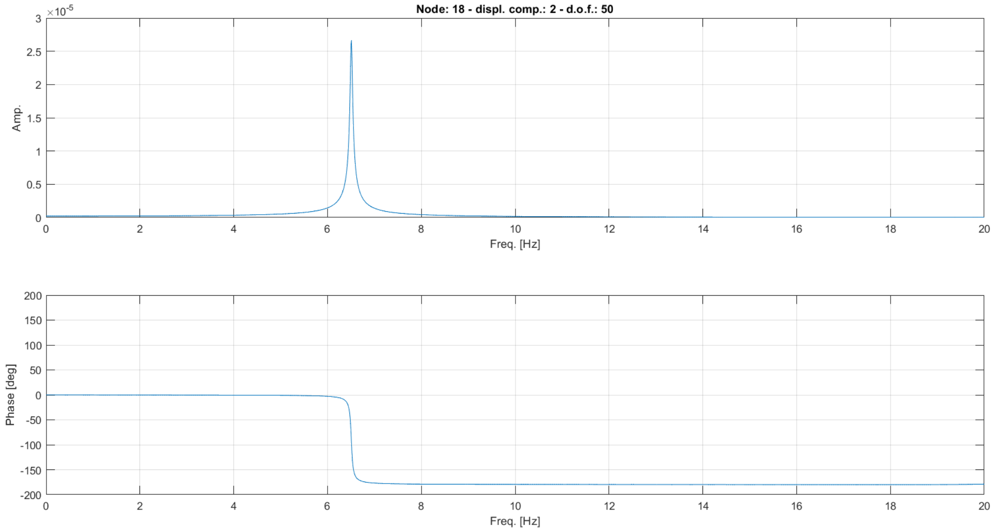


Q3. Frequency response functions (Point B)

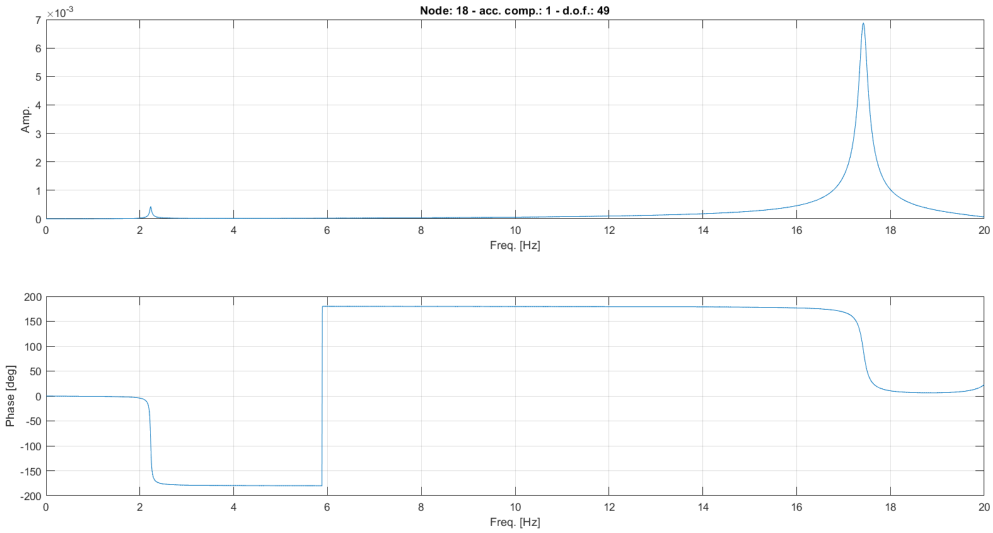
1. **Horizontal displacement of point B**



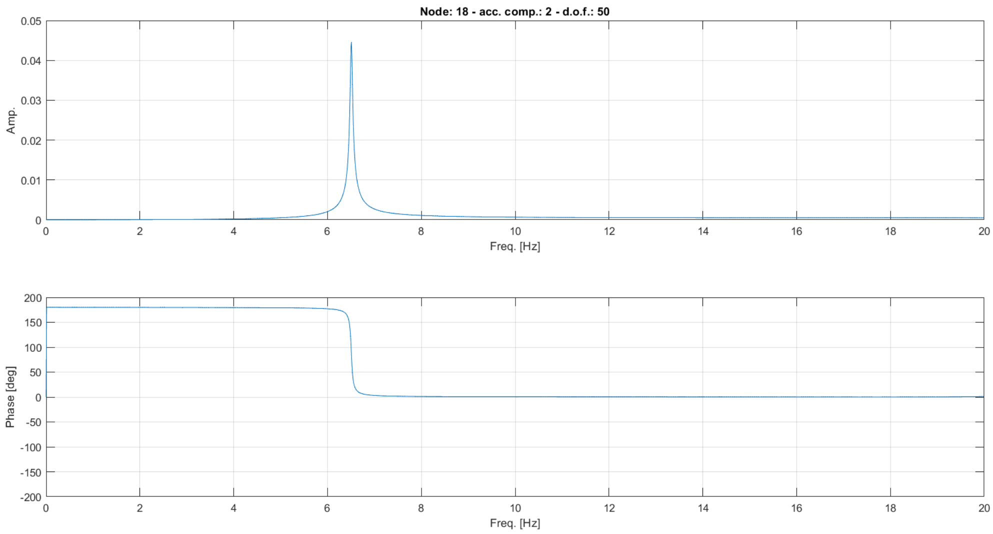
2. **Vertical displacement of point B**



3. Horizontal acceleration of point B



4. Vertical acceleration of point B



Q4. Frequency response functions (Pillar)

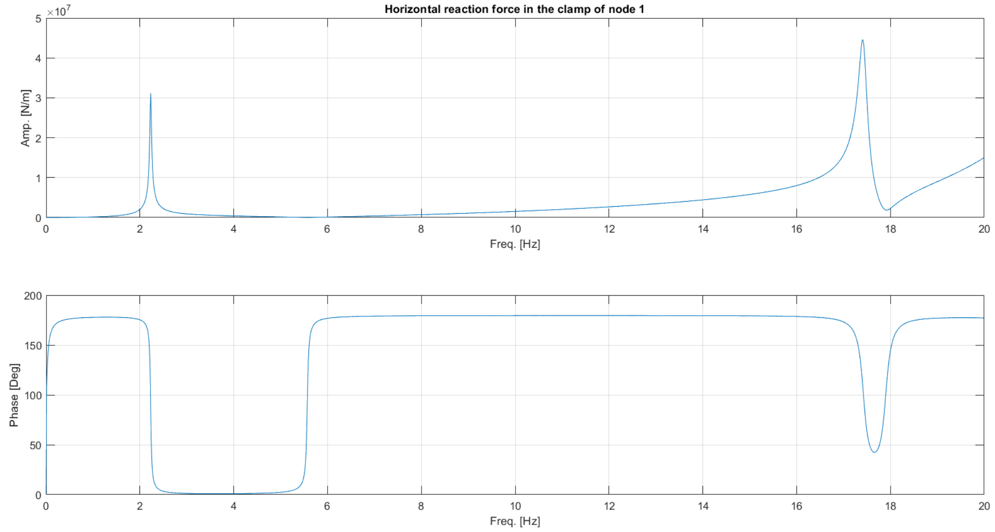
4.1 Script

```

ndof=72;
ndoc=6;
ntot=ndof+ndoc;

MFF=M(1:ndof,1:ndof);
CFF=R(1:ndof,1:ndof);
KFF=K(1:ndof,1:ndof);

MFC=M(1:ndof,ndof+1:ntot);
CFC=R(1:ndof,ndof+1:ntot);
KFC=K(1:ndof,ndof+1:ntot);

MCF=M(ndof+1:ntot,1:ndof);
CCF=R(ndof+1:ntot,1:ndof);
KCF=K(ndof+1:ntot,1:ndof);

MCC=M(ndof+1:ntot,ndof+1:ntot);
CCC=R(ndof+1:ntot,ndof+1:ntot);
KCC=K(ndof+1:ntot,ndof+1:ntot);

% Frequency response function

xC0=zeros(ndoc,1);
doc_x1=idb(1,1);
doc_x26=idb(26,1);
xC0(1)=1;
xC0(4)=1;

i=sqrt(-1);
vect_f=0:0.01:20;

for j=1:length(vect_f)
    ome=2*pi*vect_f(j);
    A=-ome^2*MFF+i*ome*CFF+KFF;
    B=-ome^2*MFC+i*ome*CFC+KFC;
    vect_x0=-inv(A)*B*xC0;

    C=-ome^2*MCF+i*ome*CCF+KCF;
    D=-ome^2*MCC+i*ome*CCC+KCC;

    QC=0;

    vect_R=C*vect_x0+D*xC0-QC;

    H=vect_R(1);
    M=vect_R(3);
```

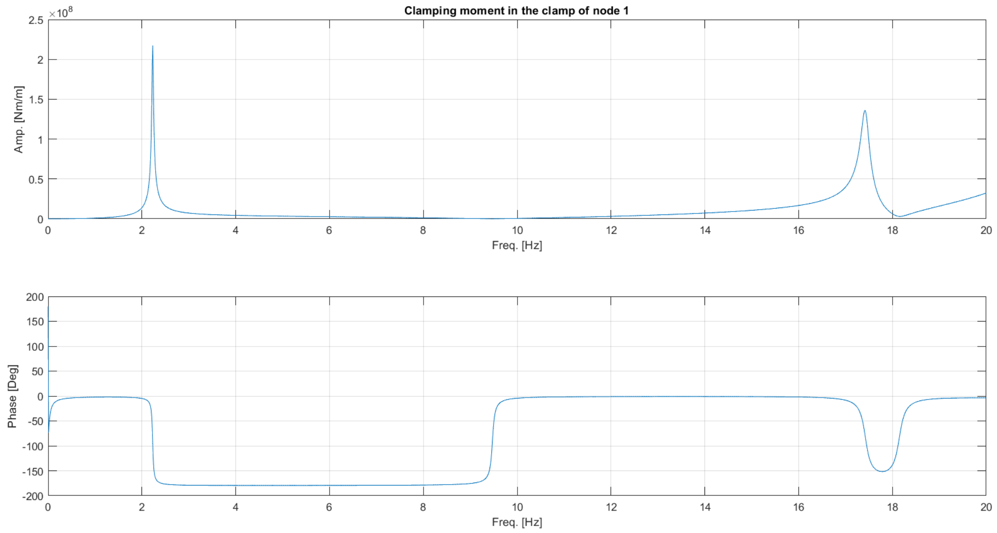
```
    mod1(j)=abs(H);
    phase1(j)=angle(H);

    mod2(j)=abs(M);
    phase2(j)=angle(M);
end
```

4.2 Results

1. **Horizontal reaction in the left pillar**



2. **Clamping moment in the left pillar**



Q5. Time history

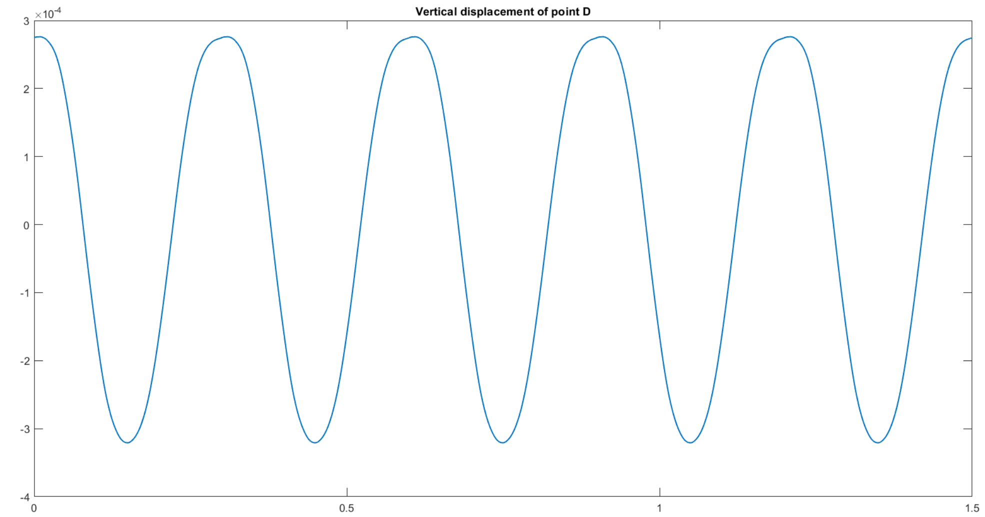
5.1 Script

```

% Triangular waveform
T = 0.3; % period
A = 2000; % amplitude
t = 0:1e-3:1.5; % time vector
ty=0:1e-3:T;
y = zeros(1,length(ty));
y = A*(2/T)*abs(mod(ty-T/4,T) - T/2) - A/2;

figure()
plot(ty, y);
xlabel('Time (seconds)');
ylabel('Amplitude');


%Fourier transform
fftout=fft(y);
N=length(y);
df=1/T;
fmax=(N/2-1)*df;
vett_freq=0:df:fmax;
modf(1)=1/N*abs(fftout(1));
modf(2:N/2)=2/N*abs(fftout(2:N/2));
fasf(1:N/2)=angle(fftout(1:N/2));

figure
subplot 211;bar(vett_freq,modf);
subplot 212;plot(vett_freq,fasf);

dof_yD=idb(7,2); %vertical displacement for point D --> node 7

ome0=2*pi/T; %foundamental frequency
output_yD=zeros(1,length(t));
output_yDdd=zeros(1,length(t));

vect_force=zeros(ndof,1);
vect_force(dof_yD,1)=1;

i=sqrt(-1); %the imaginary operator i has to be defined in the script

for k = 1:N/2
   omega(k)=2*pi*vett_freq(k);
   if omega(k)<=(pi*fmax*2)
    A=-omega(k)^2*MFF+i*omega(k)*CFF+KFF;
    x0 = A\(modf(k)*vect_force);
    yD=x0(dof_yD);

```

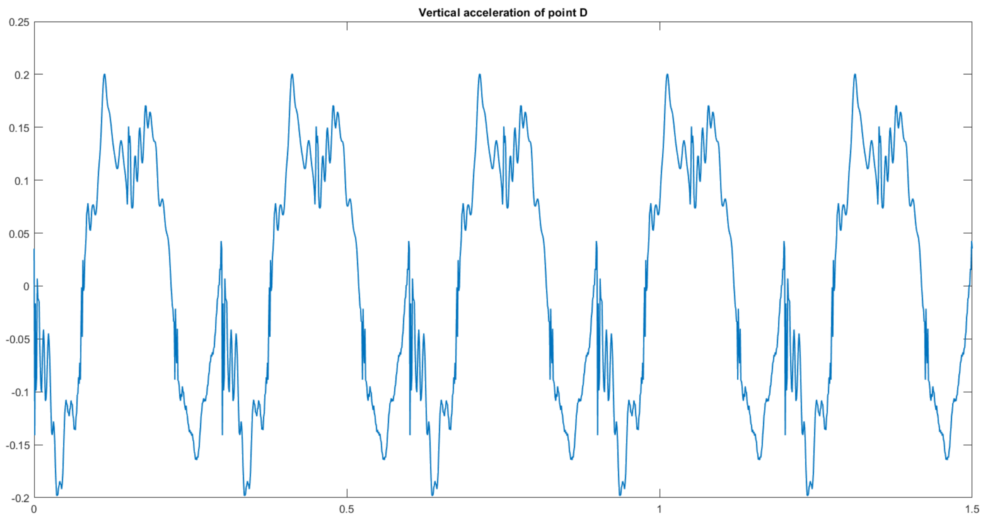
```
    yDdd=-omega(k)^2*yD;
    output_yD=output_yD+abs(yD)*cos(omega(k)*t+angle(yD));
    output_yDdd=output_yDdd+abs(yDdd)*cos(omega(k)*t+angle(yDdd));
  end
end
```

5.2 Results

1. **Time history of the vertical displacement of point D**



2. **Time history of the vertical acceleration of point D**



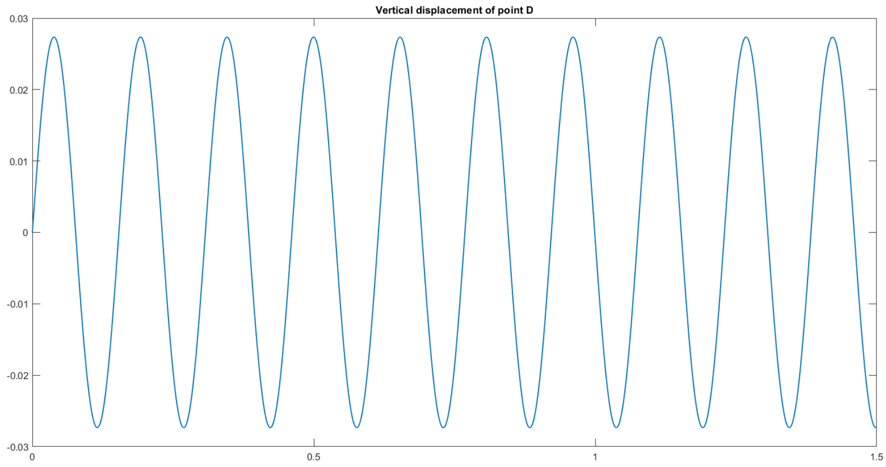
5.3 Resonance

Resonance of a mechanical system occurs when the value of the frequency of the input force is the same as one of the natural frequencies of the system itself. Considering the natural frequencies determined in point 2, values of the period T between 0.1s and 0.5s for which resonance occurs are:

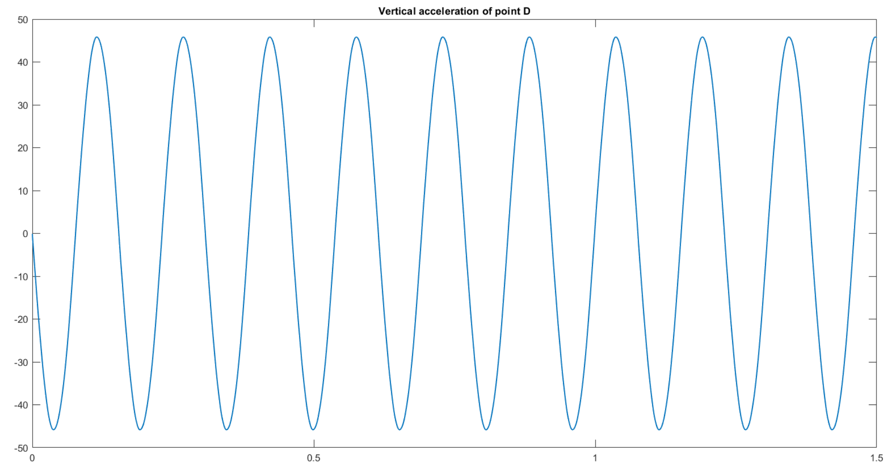
$$T_1 = \frac{1}{f_1} = 0,4487s; \qquad T_2 = \frac{1}{f_2} = 0,1537s$$

Indeed, we can see for example that for $T_2 = 0,1537s$, the vertical displacement and acceleration of point D produced by a vertical force with period T_2 applied at D are:

- **Time history of the vertical displacement of point D**



- **Time history of the vertical acceleration of point D**



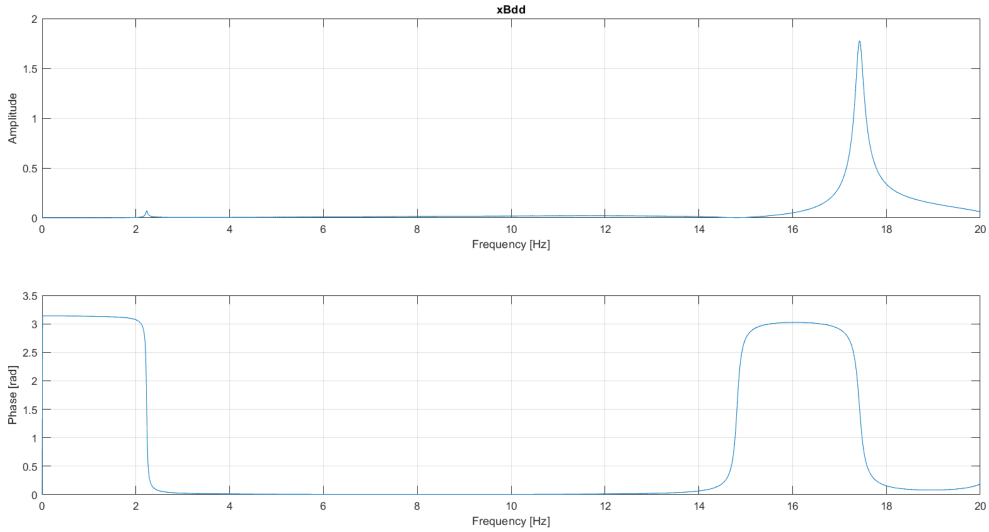
Q6. Unbalanced mass

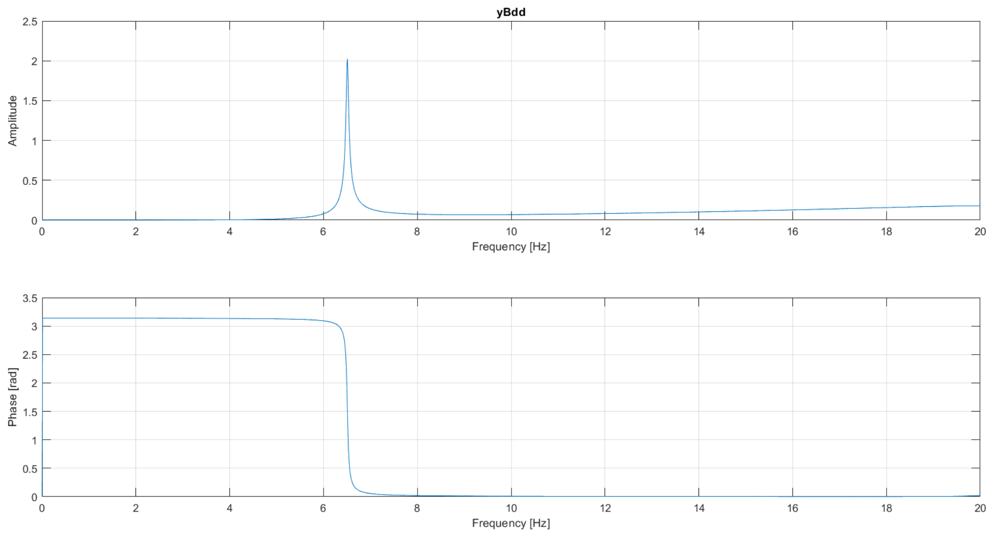
6.1 Script

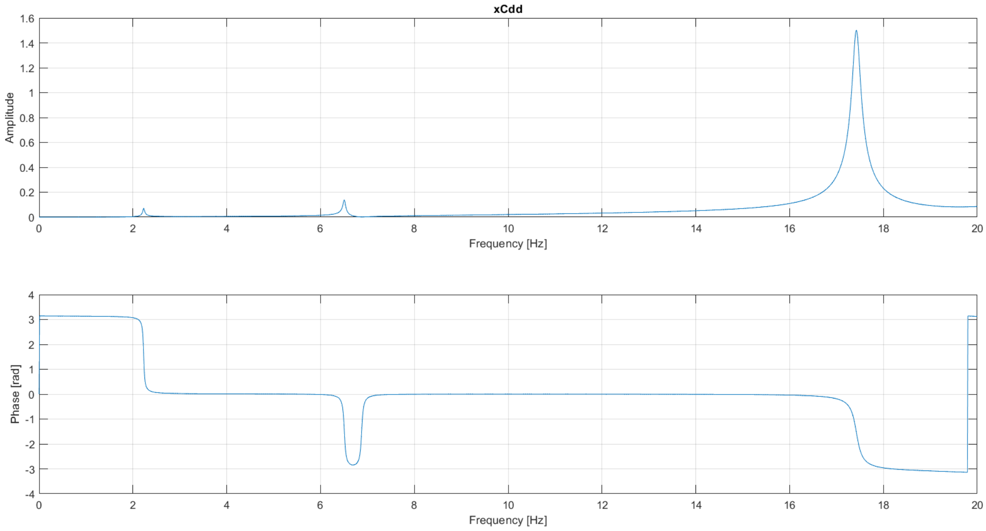
The procedure for determining the FRF required in point 6 is the same as that adopted for point 4. The only difference is that in this case the amplitude of the excitation force is no longer constant but proportional to ω^2, specifically it is proportional to $me\omega^2$.

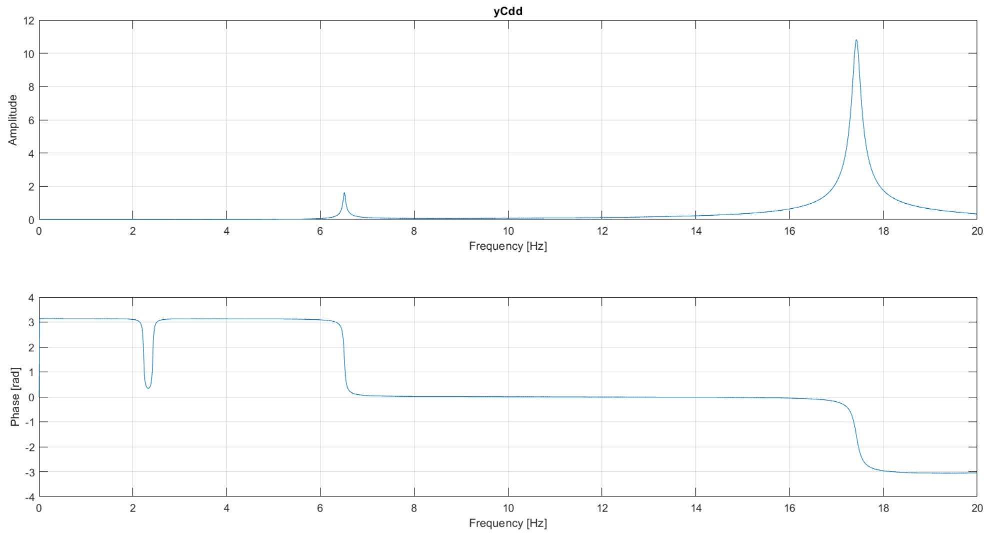
```
dof_xB=idb(18,1);
dof_yB=idb(18,2);

dof_xC=idb(9,1);
dof_yC=idb(9,2);

dof_xA=idb(5,1);
dof_yA=idb(5,2);

epsilon=0.005;
ms=5;

vect_force=zeros(ndof,1);
vect_force(dof_yA,1)=1;
vect_force(dof_xA,1)=1;

vect_f=0:0.01:20;

for k=1:length(vect_f)
    omega=vect_f(k)*2*pi;
    A=-omega^2*MFF+i*omega*CFF+KFF;
    vect_f0=vect_force*ms*epsilon*omega^2;
    x0=A\vect_f0;

    xB=x0(dof_xB);
    xBdd=-omega^2*xB;
    yB=x0(dof_yB);
    yBdd=-omega^2*yB;
    xC=x0(dof_xC);
    xCdd=-omega^2*xC;
    yC=x0(dof_yC);
    yCdd=-omega^2*yC;

    mod1(k)=abs(xBdd);
    phase1(k)=angle(xBdd);
    mod2(k)=abs(yBdd);
    phase2(k)=angle(yBdd);
    mod3(k)=abs(xCdd);
    phase3(k)=angle(xCdd);
    mod4(k)=abs(yCdd);
    phase4(k)=angle(yCdd);

end
```

6.2 Results


xBdd
Amplitude
Frequency [Hz]
Phase [rad]
Frequency [Hz]

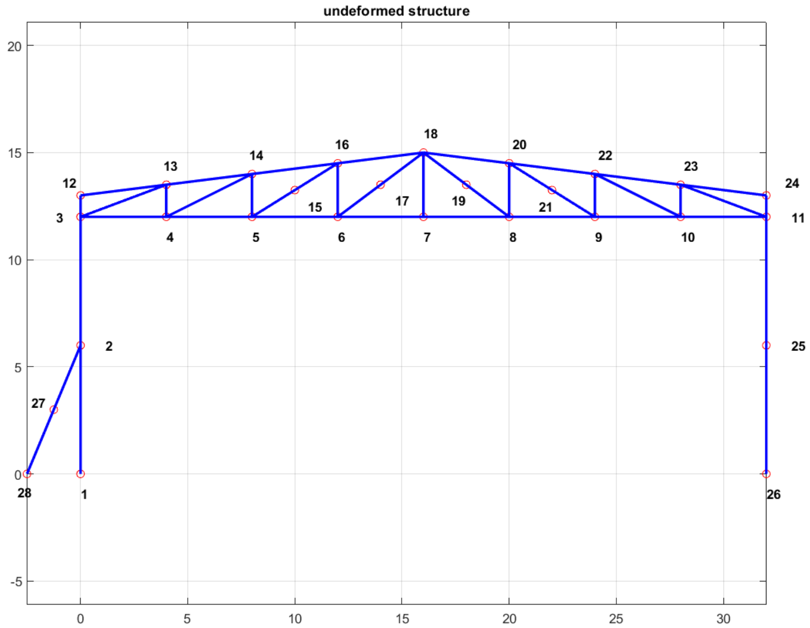

yBdd
Amplitude
Frequency [Hz]
Phase [rad]
Frequency [Hz]


xCdd
Amplitude
Frequency [Hz]
Phase [rad]
Frequency [Hz]

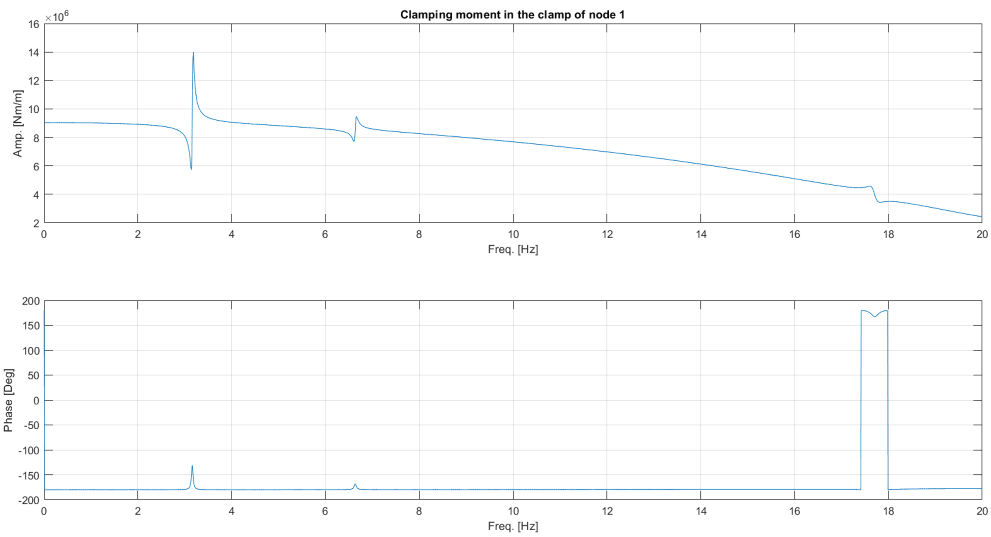

yCdd
Amplitude
Frequency [Hz]
Phase [rad]
Frequency [Hz]

Q7. Modified design of the structure

In order not to exceed the limit imposed on the total mass of the system and at the same time reduce the clamping moment in the left pillar an IPE240 type beam was added to support the left pillar of the structure as shown in the following figure:



Plotting the same FRF as in point 4, we can see that with this solution the clamping moment has been reduced more than 50% as requested:



In addition, in the initial structure we had a total mass equal to 5804,66 Kg, so the maximum mass allowed for the modified structure: **6094,90 Kg** (+5%). With the new structure we have a total mass of 6004,21 Kg.