Networked Control

Project Presentation (Siljac 01)

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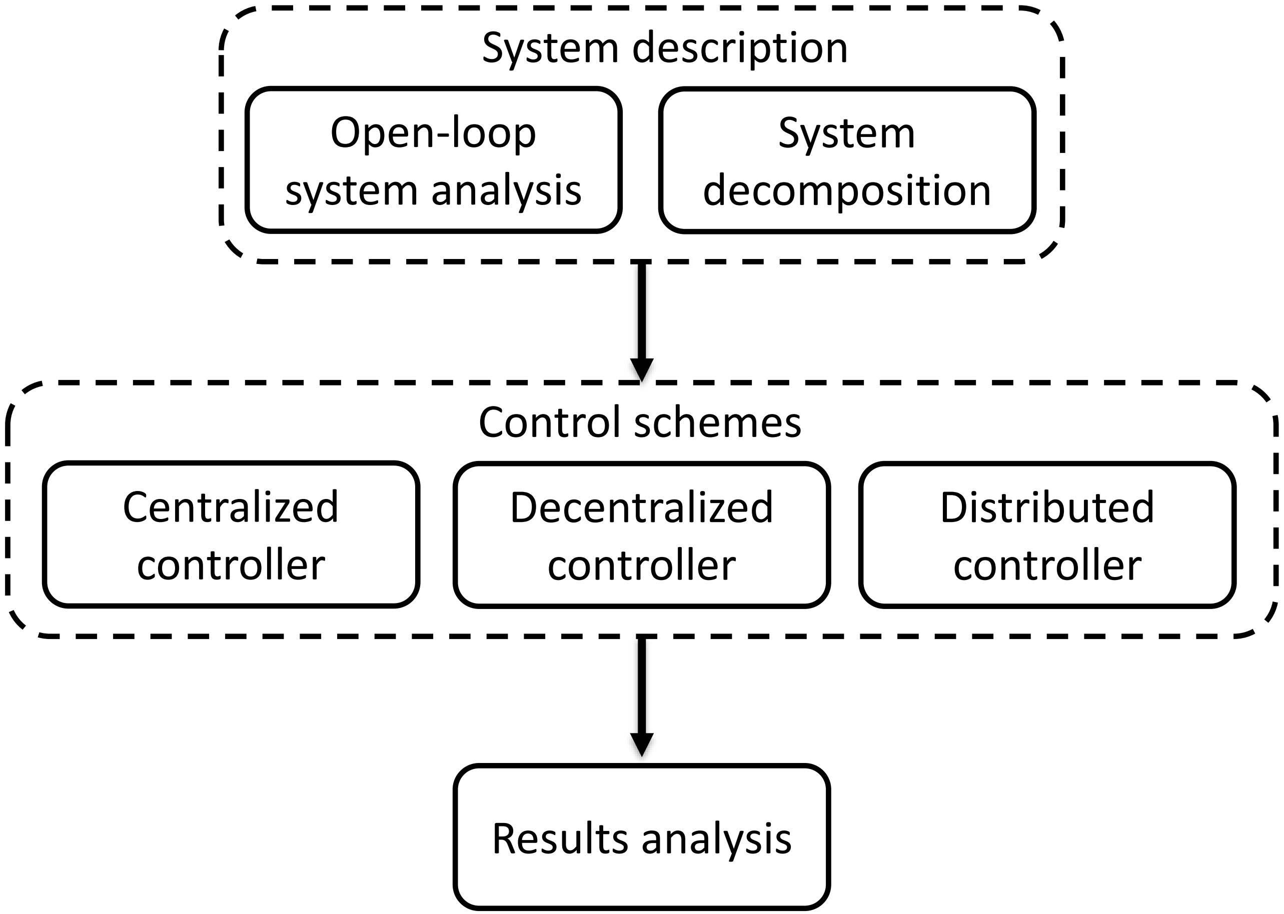
Presentation outline



Open-loop system analysis – Continuous-time system

The state-space representation of the system under control is:

2 inputs acting on x_3 and x_5

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\\\dot{x}_5\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 3.2 & 1.98\\0 & 0 & 1 & -14.72 & 0.49\\-8.86 & 8.50 & 9.39 & -7.92 & 36.01\\1.69 & 1.26 & 0.08 & 0 & 1\\-7.52 & -5.23 & 0.49 & 32.32 & -1.36\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 0\\1 & 0\\0 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

5 states

$$\begin{bmatrix}y_1\\y_2\\y_3\\y_4\\y_5\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}$$

All states are measurable

MIMO, 5th order (n=5), linear, time-invariant, strictly-proper system.

Open-loop system analysis – Stability

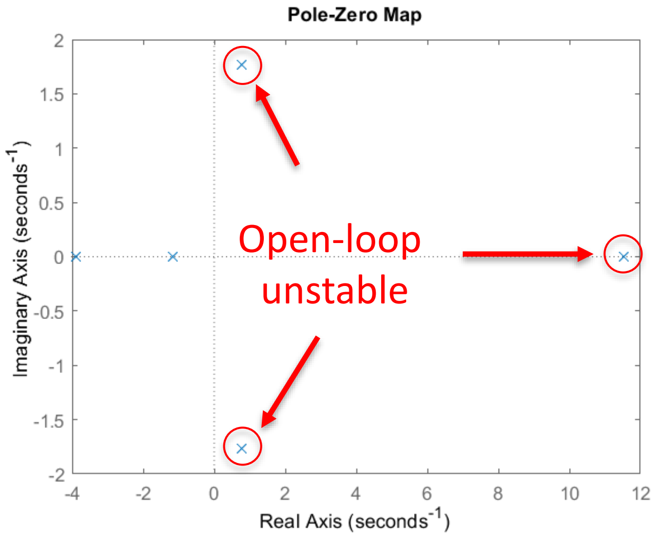
The **open-loop poles** of the continuous-time and discrete-time systems are:

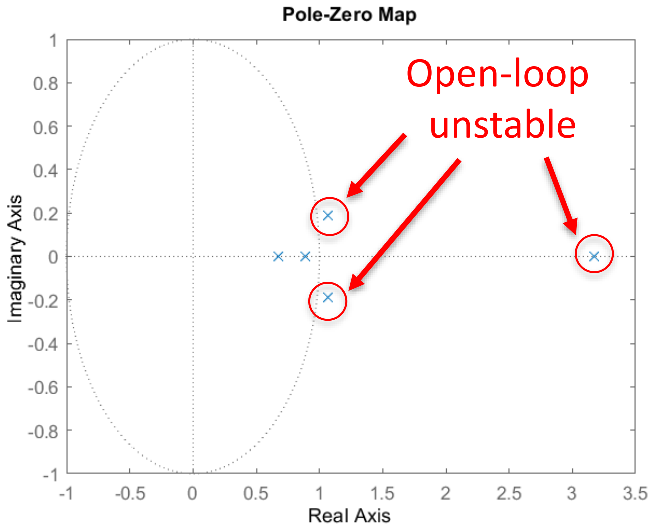
Spectral abscissa

$$eig(A) = \begin{bmatrix} \boxed{11.5} \\ -3.9 \\ 0.78 + 1.77i \\ 0.78 - 1.77i \\ -1.15 \end{bmatrix}$$

Spectral radius

$$eig(F) = \begin{bmatrix} \boxed{3.17} \\ 1.06 + 0.19i \\ 1.06 - 0.19i \\ 0.68 \\ 0.89 \end{bmatrix}$$





Open-loop system analysis – Decomposition

The **decomposed system matrices** are the following ones:

$$A = \begin{bmatrix} 0 & 1 & 0 & 3.2 & 1.98 \\ 0 & 0 & 1 & -14.72 & 0.49 \\ -8.86 & 8.50 & 9.39 & -7.92 & 36.01 \\ 1.69 & 1.26 & 0.08 & 0 & 1 \\ -7.52 & -5.23 & 0.49 & 32.32 & -1.36 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Physical coupling between states (neither block diagonal nor banded)

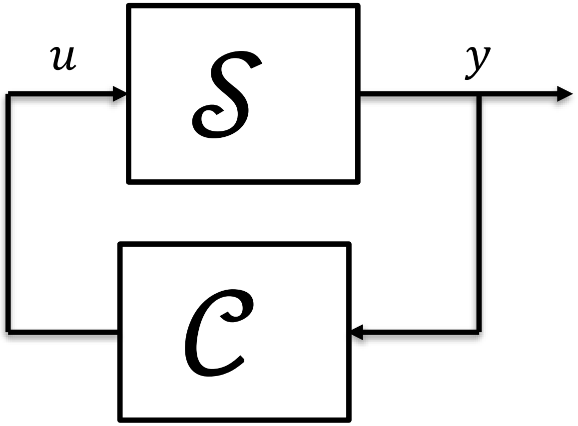
Sub-systems input decoupled

So, we have **two channels**:

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad y = \begin{bmatrix} y_{1,3} \\ y_{4,5} \end{bmatrix}$$

Same considerations apply to the discrete-time system.

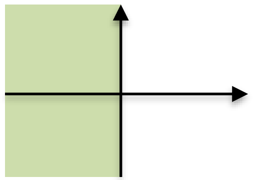
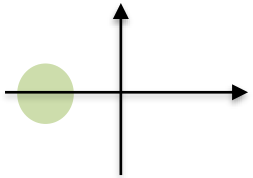
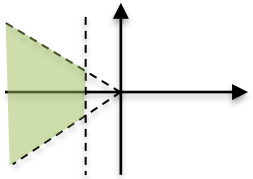
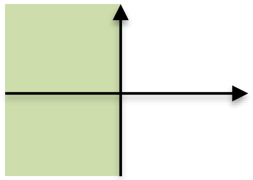
Centralized control – Fixed modes



The system has **no continuous-time centralized fixed modes**.

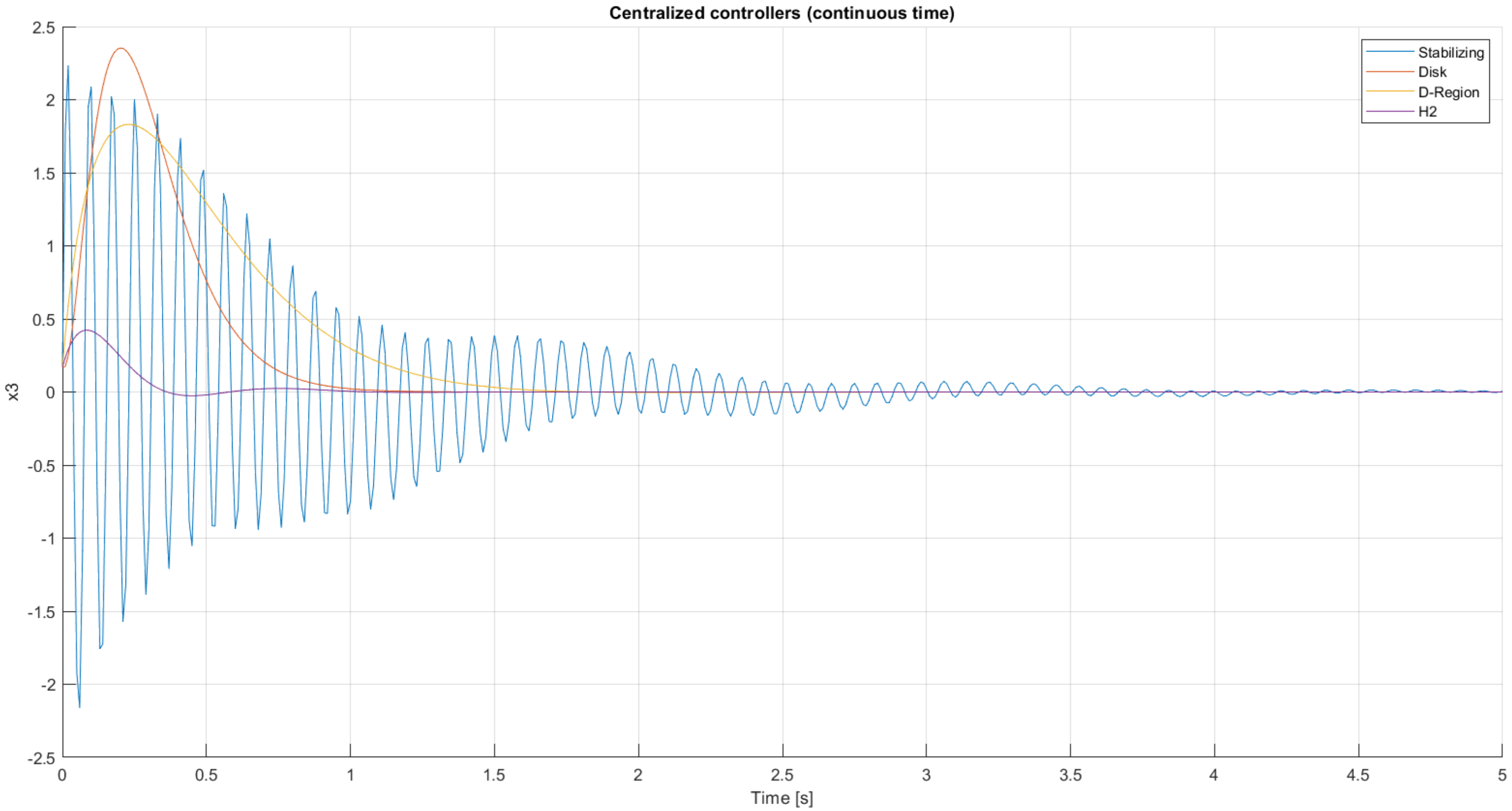
The system has **no discrete-time centralized fixed modes**.

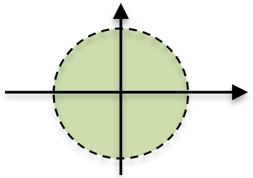
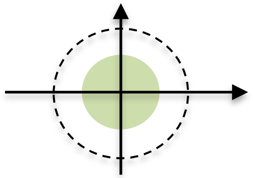
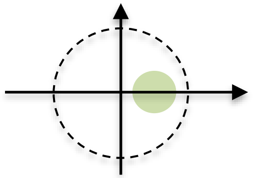
Centralized control – Continuous time

	Poles region	Parameters	Feasibility	Spectral abscissa	Closed-loop poles
Stabilizing		-	Yes	-0.92	$\begin{bmatrix} -1.2+80.5i \\ -1.2-80.5i \\ -0.92+4.4i \\ -0.92-4.4i \\ -1.6 \end{bmatrix}$
Disk		$\alpha=-10, \rho=1$	Yes	-9.6	$\begin{bmatrix} -10.2 \\ -9.88+0.05i \\ -9.88-0.05i \\ -9.7 \\ -9.6 \end{bmatrix}$
D-Region (+speed)		$\theta=\pi/4$ $\alpha=-1$	Yes	-7.1	$\begin{bmatrix} -38.3 \\ -7.1+2.9i \\ -7.1-2.9i \\ -7.7 \\ -9.3 \end{bmatrix}$
H2 (LQR)		$Q=diag(10)$ $R=diag(0.01)$	Yes	-3.4	$\begin{bmatrix} -14.98 \\ -4.6+8.3i \\ -4.6-8.3i \\ -3.4+2.1i \\ -3.4-2.1i \end{bmatrix}$

Centralized control – Continuous time

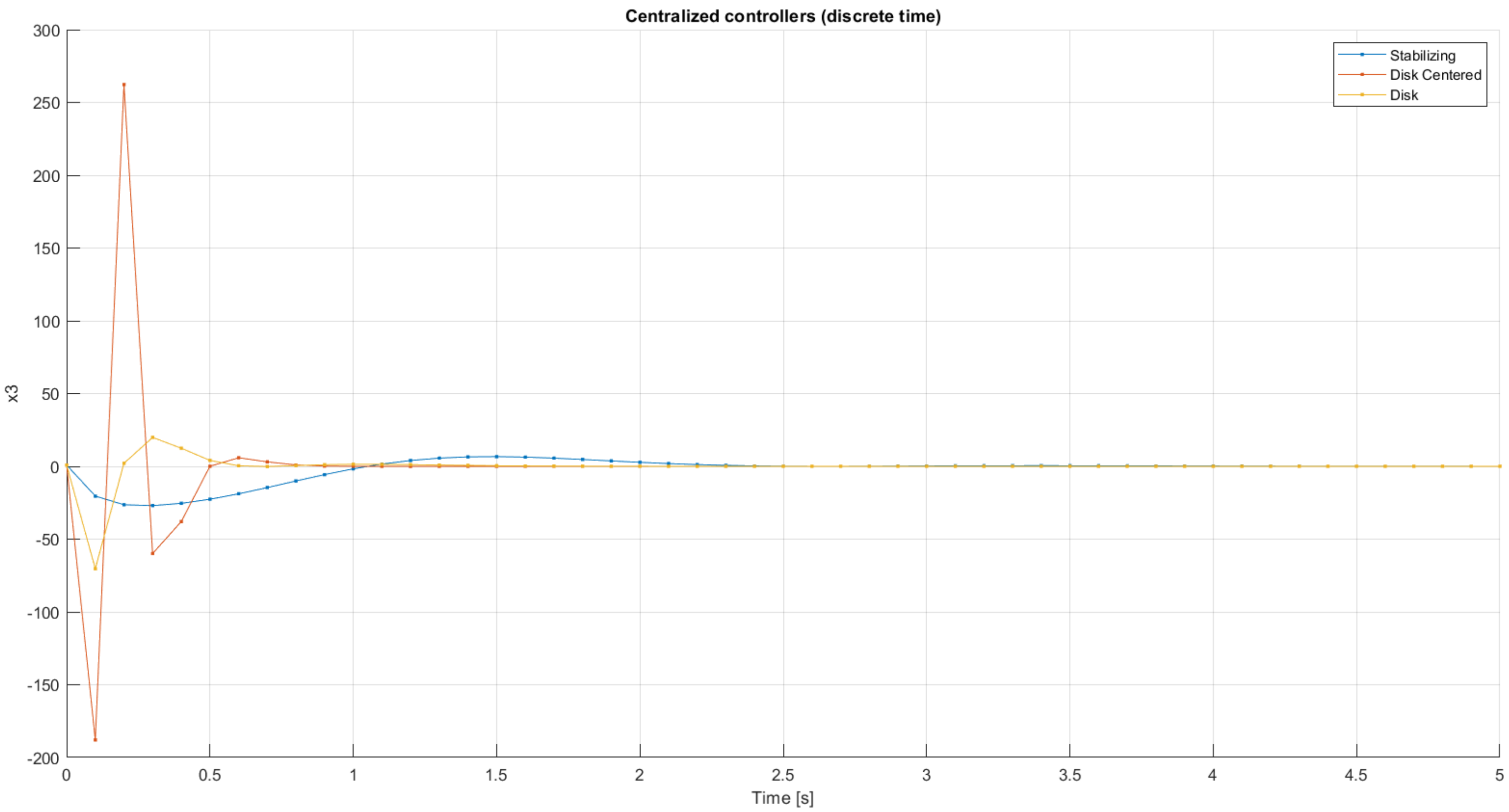
The comparison between the various continuous-time centralized controllers is reported below:



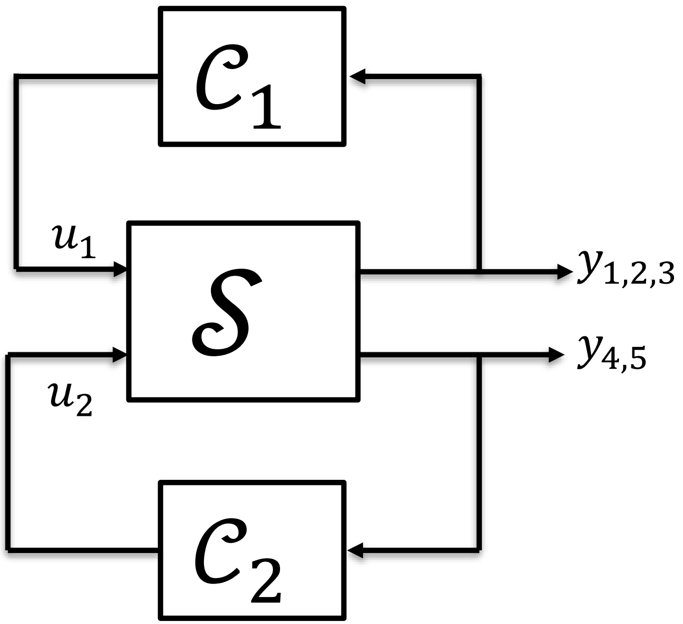
	Poles region	Parameters	Feasibility	Spectral radius	Closed-loop poles
Stabilizing		-	Yes	0.89	$\begin{bmatrix} 0.81 + 0.26i \\ 0.81 - 0.26i \\ 0.89 \\ 0.40 \\ 0.65 \end{bmatrix}$
Centred Disk		$\rho = 0.6$	Yes	0.51	$\begin{bmatrix} -0.06 \\ 0.5 + 0.09\mathrm{i} \\ 0.5 - 0.09\mathrm{i} \\ 0.4 + 0.23\mathrm{i} \\ 0.4 - 0.23\mathrm{i} \end{bmatrix}$
Disk		$\alpha = -0.4, \rho = 0.2$	Yes	0.57	$\begin{bmatrix} 0.37 + 0.06\mathrm{i} \\ 0.37 - 0.06\mathrm{i} \\ 0.47 + 0.07\mathrm{i} \\ 0.47 - 0.07\mathrm{i} \\ 0.57 \end{bmatrix}$

Centralized control – Discrete time

The comparison between the various discrete-time centralized controllers is reported below:



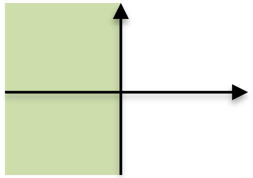
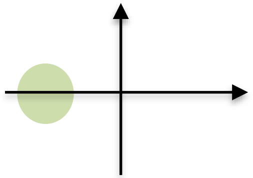
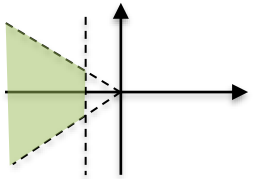
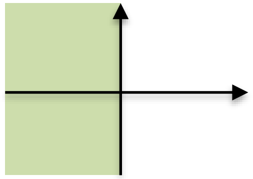
Decentralized control – Fixed modes



The system has **no continuous-time decentralized fixed modes**.

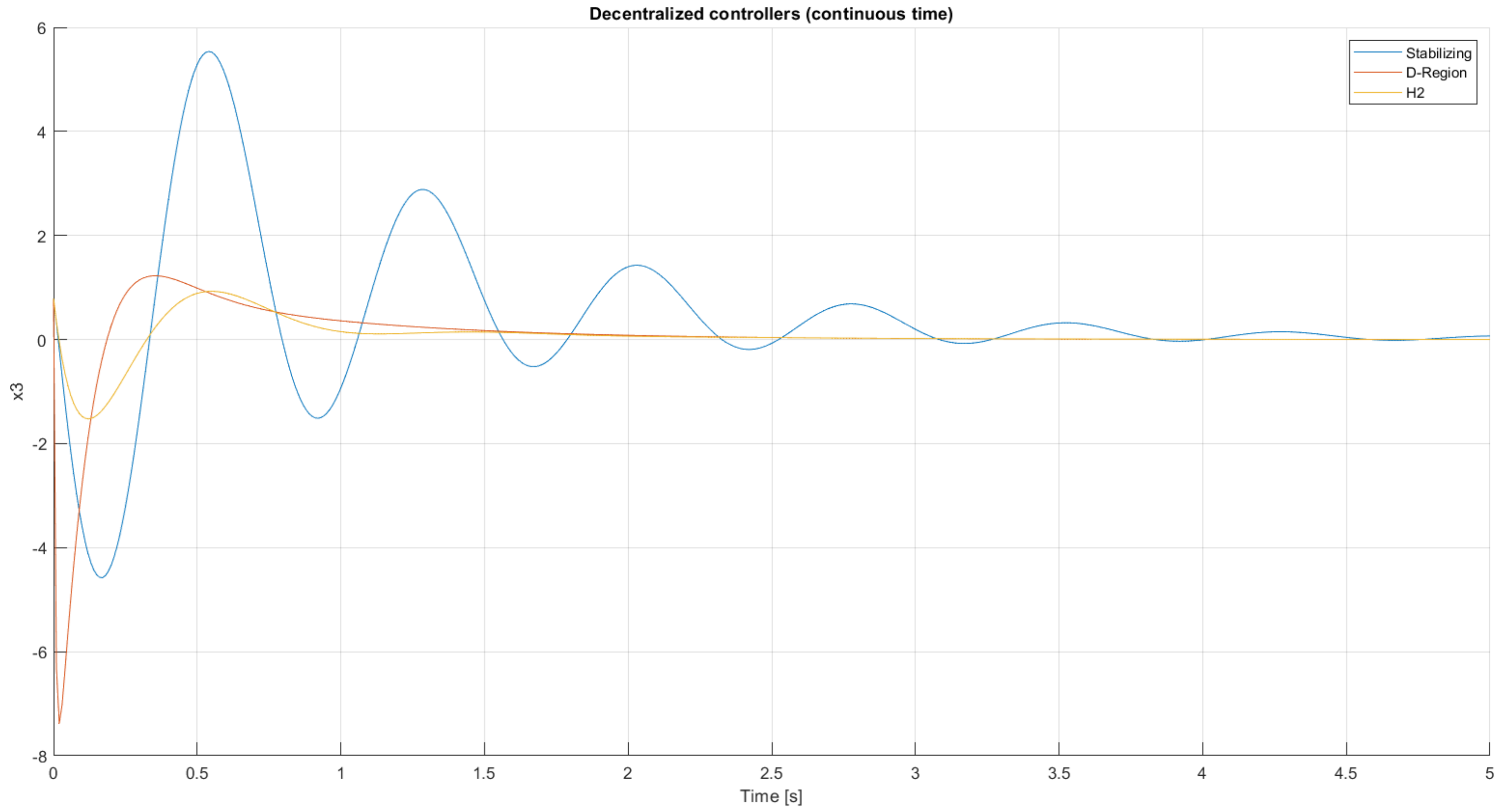
The system has **no discrete-time decentralized fixed modes**.

Decentralized control – Continuous time

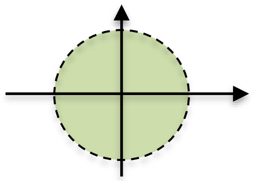
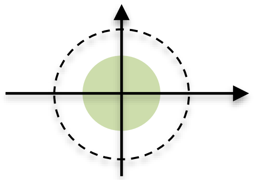
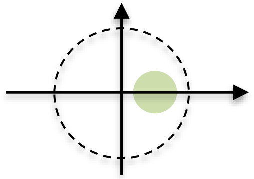
	Poles region	Parameters	Feasibility	Spectral abscissa	Closed-loop poles
Stabilizing		-	Yes	-1.03	$\begin{bmatrix} -81.32 \\ -1.03+8.4i \\ -1.03-8.4i \\ -1.47 \\ -1.13 \end{bmatrix}$
Disk		$\alpha=-10, \rho=1$	No	$+8.68$	$\begin{bmatrix} -9.7 \\ -0.07+5.7i \\ -0.07-5.7i \\ -0.83 \\ +8.68 \end{bmatrix}$
D-Region (+speed)		$\theta=\pi/4$ $\alpha=-1$	Yes	-1.49	$\begin{bmatrix} -186.2 \\ -143.5 \\ -1.49 \\ -7.6+4.7i \\ -7.6-4.7\mathrm{i} \end{bmatrix}$
H2 (LQR)		$Q=diag(10)$ $R=diag(0.01)$	Yes	-1.24	$\begin{bmatrix} -33.1 \\ -3.3+6.2i \\ -3.3-6.2i \\ -1.24 \\ -18.6 \end{bmatrix}$

Decentralized control – Continuous time

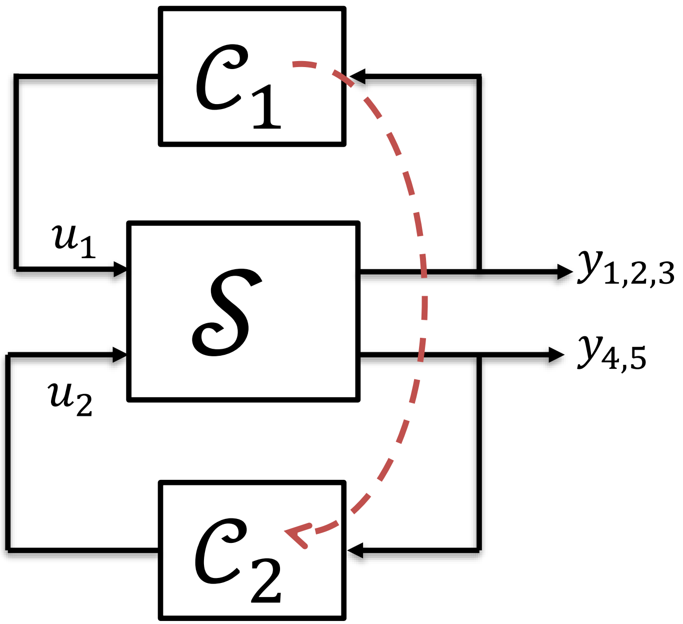
The comparison between the various continuous-time decentralized controllers is reported below:



Decentralized control – Discrete time

	Poles region	Parameters	Feasibility	Spectral radius	Closed-loop poles
Stabilizing		-	No	$+1.72$	$-$
Centred Disk		$\rho = 0.6$	No	$+1.8$	$-$
Disk		$\alpha = -0.4, \rho = 0.2$	No	$+1.9$	$-$

Distributed control (1→2) – Fixed modes



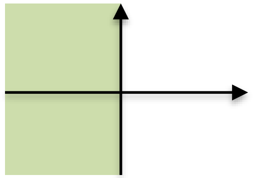
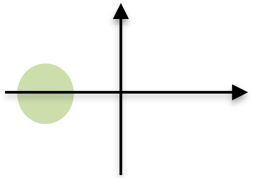
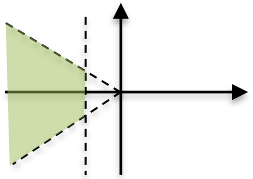
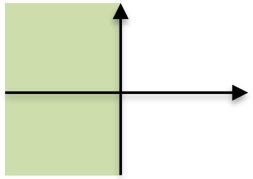
The system has **no continuous-time distributed fixed modes**.

The system has **no discrete-time distributed fixed modes**.

Information structure constraint set:

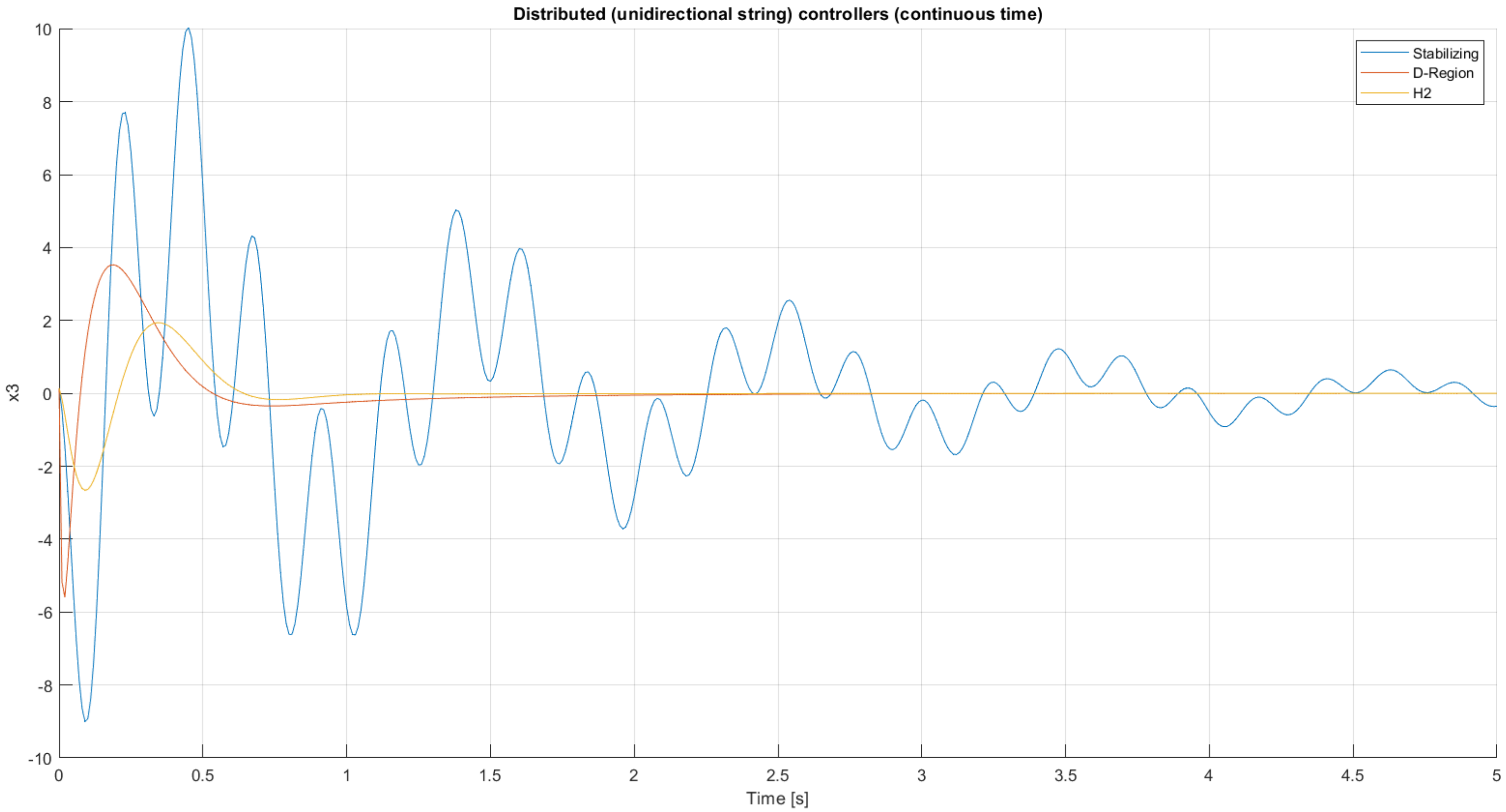
$$\boldsymbol{\mathcal{N}} = \{\{\}, \{\mathbf{1}\}\}$$

Distributed control (1→2) – Continuous time

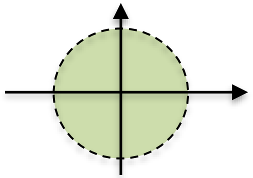
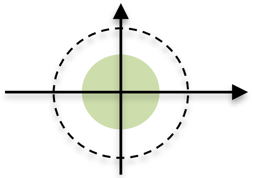
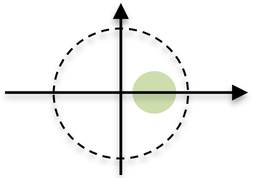
	Poles region	Parameters	Feasibility	Spectral abscissa	Closed-loop poles
Stabilizing		-	Yes	-0.63	$\begin{bmatrix} -0.7+27i \\ -0.7-27i \\ -1.62 \\ -0.63+5.97i \\ -0.63-5.97i \end{bmatrix}$
Disk		$\alpha = -10, \rho = 1$	No	$+2.6$	$-$
D-Region (+speed)		$\theta = \pi/4$ $\alpha = -1$	Yes	-1.49	$\begin{bmatrix} -124.6+25i \\ -124.6-25i \\ -1.49 \\ -7.3+4.8i \\ -7.3-4.8i \end{bmatrix}$
H2 (LQR)		$Q = diag(10)$ $R = diag(0.01)$	Yes	-1.28	$\begin{bmatrix} -30.94 \\ -1.28 \\ -6.2+8.1i \\ -6.2-8.1i \\ -10.84 \end{bmatrix}$

Distributed control (1→2) – Continuous time

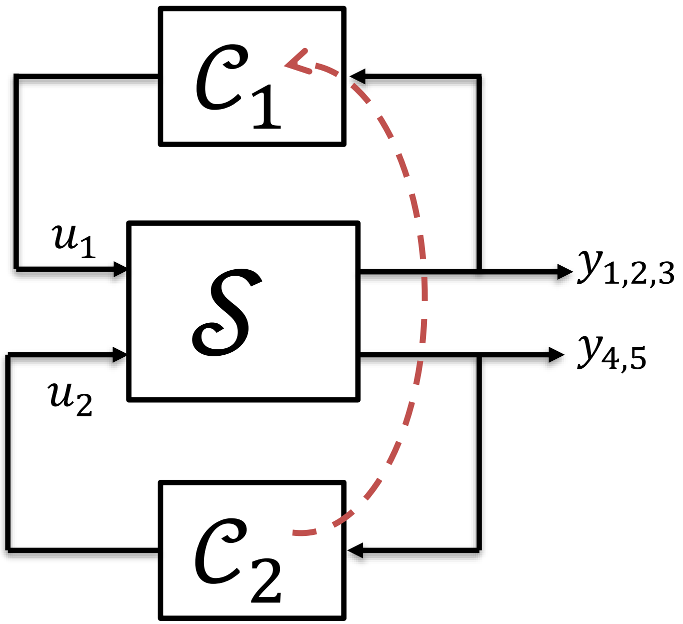
The comparison between the various continuous-time distributed (1→2) controllers is reported below:



Distributed control (1→2) – Discrete time

	Poles region	Parameters	Feasibility	Spectral radius	Closed-loop poles
Stabilizing		-	No	$+1.71$	$-$
Centred Disk		$\rho = 0.6$	No	$+1.8$	$-$
Disk		$\alpha = -0.4, \rho = 0.2$	No	$+1.9$	$-$

Distributed control (2→1) – Fixed modes



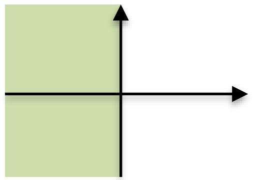
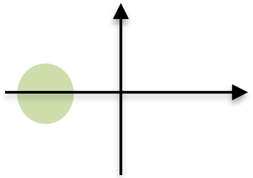
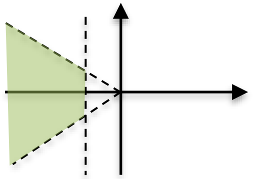
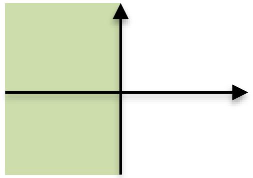
The system has **no continuous-time distributed fixed modes**.

The system has **no discrete-time distributed fixed modes**.

Information structure constraint set:

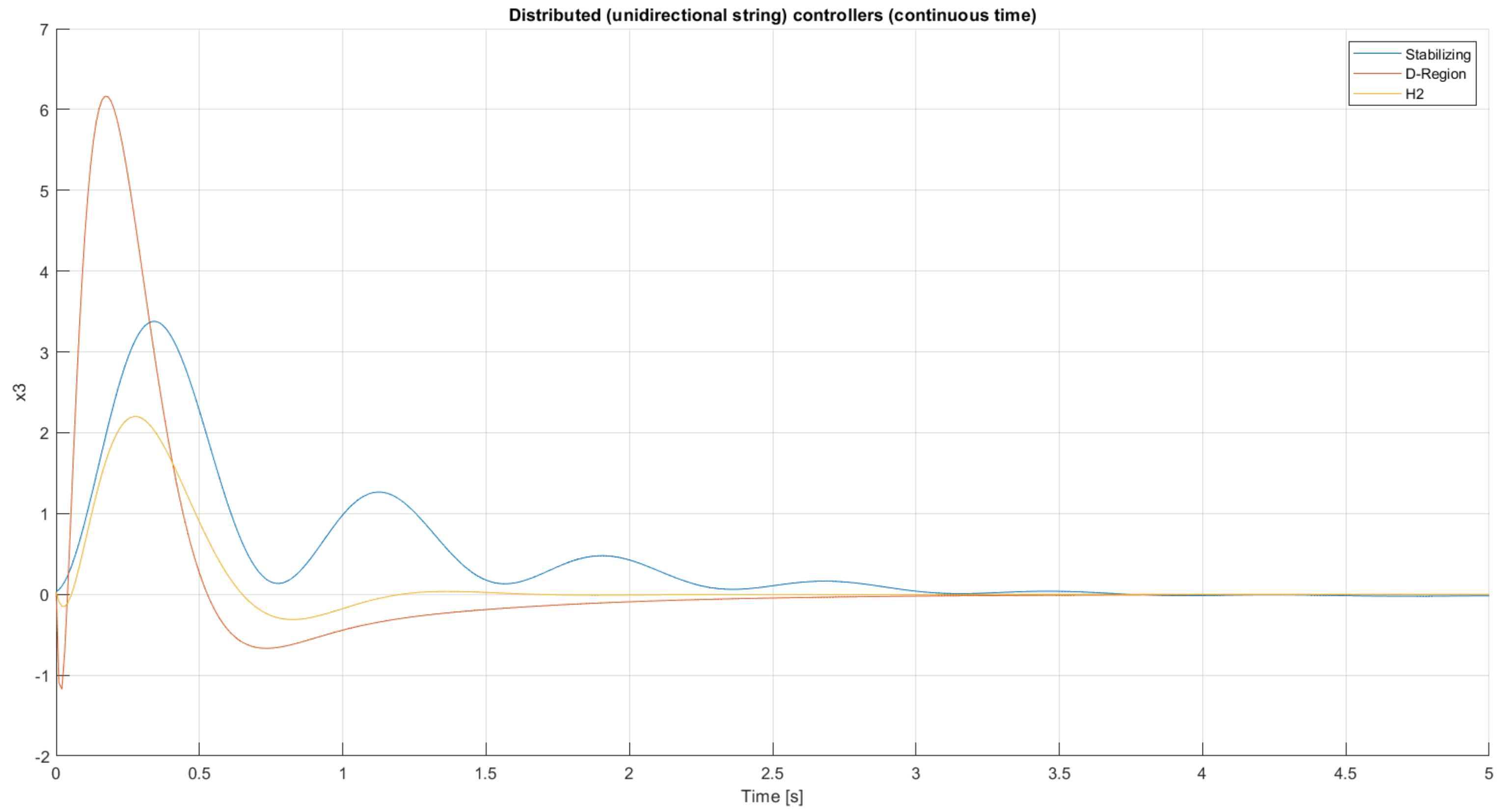
$$\mathcal{N} = \{\{\mathbf{2}\}, \{\}\}$$

Distributed control (2→1) – Continuous time

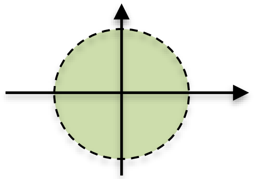
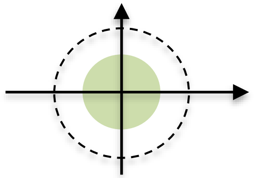
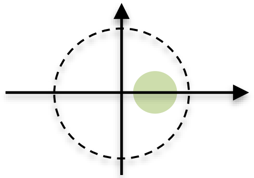
	Poles region	Parameters	Feasibility	Spectral abscissa	Closed-loop poles
Stabilizing		-	Yes	-0.72	$\begin{bmatrix} -1.38+7.99i \\ -1.38-7.99i \\ -2.69 \\ -0.72+0.28i \\ -0.72-0.28i \end{bmatrix}$
Disk		$\alpha = -10, \rho = 1$	No	$+2.54$	$-$
D-Region (+speed)		$\theta = \pi/4$ $\alpha = -1$	Yes	-1.44	$\begin{bmatrix} -68.92 \\ -34.56 \\ -1.44 \\ -6.6+5i \\ -6.6-5i \end{bmatrix}$
H2 (LQR)		$Q = diag(10)$ $R = diag(0.01)$	Yes	-1.01	$\begin{bmatrix} -1.01 \\ -3.65+5.8i \\ -3.65-5.8i \\ -18.95 \\ -12.79 \end{bmatrix}$

Distributed control (2→1) – Continuous time

The comparison between the various continuous-time distributed (2→1) controllers is reported below:



Distributed control (2→1) – Discrete time

	Poles region	Parameters	Feasibility	Spectral radius	Closed-loop poles
Stabilizing		-	No	$+1.71$	$-$
Centred Disk		$\rho = 0.6$	No	$+1.8$	$-$
Disk		$\alpha = -0.4, \rho = 0.2$	No	$+1.9$	$-$

Results

In conclusion we can state that:

1. As expected, since the **system is controllable**, the centralized controller is feasible both in CT and DT.

2. It is not possible to determine a controller with state feedback capable of stabilizing the system in the case of discrete-time decentralized and distributed schemes. One possible solution would be to **reduce the sampling time**.

3. The best controller seems to be the **centralized H2 (LQ)** one, which has at the same time the smallest settling time and overshoot due to the minimum-phase zero.

4. Since our system is composed of only two sub-systems, other types of distributed schemes lead to the same result of the string (unidirectional or bidirectional) structure.

5. If we adopt the **all-to-all distributed** control scheme, we got the exact same result of the centralized case having at the same time more robustness.