POLITECNICO
MILANO 1863

DYNAMICS OF ELECTRICAL MACHINES AND DRIVES

DC MOTOR CONTROL

Antonello Lagalante

Contents

Problem definition

Motor characteristics

A DC self excited motor is used to move an ATM tramway vehicle "Carelli 1928" with the following characteristics:

- Line voltage: 600 V
- Motor rated speed: 314 rad/s
- Efficiency: 0.9
- Armature circuit time costant: 10 ms
- Excitation circuit rated voltage: 120 V
- Excitation circuit rated current: 1A
- Excitation circuit time costant: 1 s

Specifications

The tramway should accelerate from 0 to 60 km/h in 25 s. The tramway mass is 10 T and you should consider 200 people as the tramway trainload with standard weight of 80 kg. The friction force is proportional to the speed and at rated speed is 1/3 of traction force.

Design and simulate speed and current control in order to cover a 10km track considering the following table:

track	slope %	speed
$0-1\,km$	0	$35\ km/h$
$1-3\,km$	0	$60\ km/h$
$3-4\,km$	5%	$60\ km/h$
$4-6\,km$	0	$75\ km/h$
$6-8\,km$	0	$60\ km/h$
$8-9\,km$	-5%	$60\ km/h$
$9-10\ km$	0	$35\ km/h$

Figure 1: Speed profile specification

Parameters identification

Through the problem data, the total mass (tramway + passengers) and the average acceleration can be easily determined:

$$M_{tot} = M + m80 = 26000kg \tag{1}$$

$$a_m = \frac{V_{max}}{t} = 0.6672m/s^2 \tag{2}$$

Through the energy balance it is possible to determine:

- Traction force: $F_{traction} = M_{tot}a_m = 17347N$
- Traction power: $P_{traction} = F_{traction}V_{max} = 289,35\ kW$
- Friction power: $P_{friction} = \frac{1}{3}P_{traction} = 96,45\ kW$
- Rated power: $P_n = P_{traction} + P_{friction} = 385,8\ kW$

Through the classical efficiency formula and neglecting the term referring to the excitation circuit, we can determine the rated current of the armature circuit:

$$I_n = \frac{P_n}{\eta V_n} = 714.4476A \tag{3}$$

The nominal torque and coefficient $K(i_e)$ (assuming it to be a linear function of the excitation current) and the rated e.m.f can now be determined:

$$T_n = \frac{P_n}{\omega_n} = 1228.7\ Nm \tag{4}$$

$$E_n = \eta V_n = 540V \tag{5}$$

Through the energy balance of the armature circuit, the values of the resistances and inductances of the two circuits can be determined. We know that the total power dissipated in the armature circuit is given by the sum of the rated power and the losses due to the Joule effect, or in mathematical terms:

$$V_n I_n = P_n + R_a I_n^2 \tag{6}$$

By isolating the R_a term from the above equation, it is possible to determine the value of the reinforcement resistance and then the value of the remaining parameters:

$$R_a = (1-\eta)\frac{V_n}{I_n} = 0.084\ \Omega \tag{7}$$

$$La = R_a \tau_a = 840\ \mu H \tag{8}$$

$$R_e = \frac{V_{en}}{Ie_n} = 120\ \Omega \tag{9}$$

$$L_e = R_e \tau_e = 120\ H \tag{10}$$

For modeling the mechanical load, it is also necessary to determine the equivalent inertia J_{eq} and the friction coefficient β:

$$J = M\frac{V_{max}^2}{\omega_n^2} = 73.3679\ kgm^2 \tag{11}$$

$$B = \frac{P_{fr}}{\omega_n^2} = 0.9782 \tag{12}$$

Synthesis of controllers

PI controllers were used to control the machine to meet the specifications given in Figure 1. This kind of control scheme is a nested loop controller, so the inner I_a loop need to have a bandwidth higher than the outer Ω_m loop.

In general, based on cascade loop theory, the outer loop must be at least one dedace slower then the inner loop in order to see a unitary closed loop transfer function for the inner loop.

Specifically then, when designing the inner loop, assuming to have a good power supply ($\tau_{PS} < \tau_a$) and sensors ($\tau_{sensor} < \tau_a$) we can consider the simplified I_a loop without considering the power supply and the sensors and, when designing the slower outer Ω_m loop, we can consider the inner loop as unitary gain.

Armature current controller

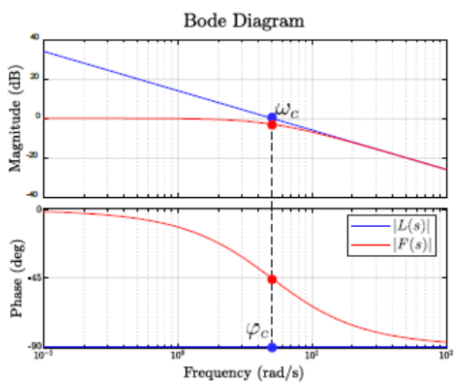
The pole/zero cancellation method is used to calibrate the PI controller referred to the armature circuit and determine the coefficients k_p and k_i. It is possible to cancel the pole of our transfer function $G(s)$ with the zero of our regulator simplifying a lot out open loop transfer function. By calculating the transfer function in open loop inclusive of our controller:

$$\begin{aligned} L(s) = R(s)G(s) &= \Big(k_p + \frac{k_i}{s}\Big)\Big(\frac{1}{R+sL}\Big) = \frac{k_i}{k_p}\frac{1}{s}\Big(1+s\frac{k_i}{k_p}\Big)k_p\Big(\frac{1}{R+sL}\Big) = \\ &= \frac{1}{sT_i}(1+sT_i)k_p\Big(\frac{1}{R+sL}\Big) \end{aligned} \tag{13}$$

Imposing that:

$$T_i = \frac{k_p}{k_i} = \frac{L}{R} \qquad \Rightarrow \qquad L(s) = \frac{k_p}{sRT_i} \tag{14}$$

In this way we obtain an open-loop transfer function equal to a gain and a pure integrator, the bode diagram of which is shown in the figure below:



Now we can compute explicitly the expression of k_p and k_i imposing that the modulus of the open-loop function in our desired cutoff frequency is equal to 1:

$$|L(j\omega_c)| = 1 \quad \Rightarrow \quad \begin{cases} k_p = \omega_c L \\ k_i = \omega_c R \end{cases} \tag{15}$$

Thus having available the values of the inductance and resistance of the armature circuit we can obtain the values of k_p and k_i.

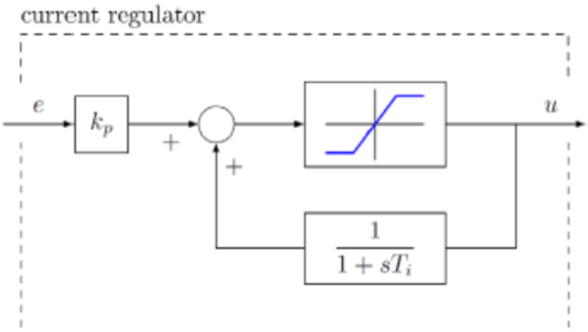
Through these results, as far as the armature current regulator is concerned, by imposing a cutoff frequency related to half time constant of the armature circuit we obtain the following values of k_p and k_i:

$$\omega_c = 200\, rad/s \quad \Rightarrow \quad \begin{cases} k_p = 0.168 \\ k_i = 16.796 \end{cases} \tag{16}$$

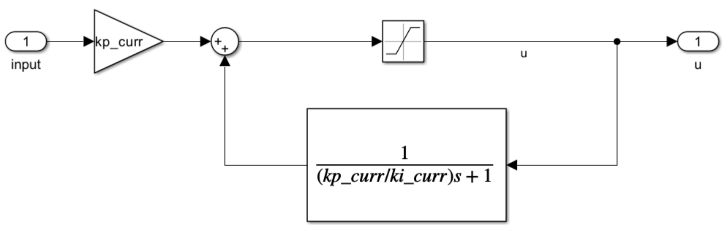
In the following there is the matlab script used to find these values:

```
Ga=1/(Ra+s*La);
tauGa=La/Ra;
TaGa=5*tauGa;

wia=5/(TaGa/2);

kp_curr=wia*La;
ki_curr=wia*Ra;
```

Another important feature to take into account is the nonlinear system effects such as actuator saturation. In particular, we control the armature current by varying the supply voltage. This voltage, however, cannot take any value we want, but is limited, causing the problem of windup effect. To solve this problem, the Back-calculation antiwindup method has been implemented with the following scheme:



The final regulator implemented in simulink is the following one:



Speed controller

Through the same procedure followed for the armature current regulator, the values of k_p and k_i for the speed regulator were determined. It is important to remember that this regulator has a smaller bandwidth than the current regulators for the same reason described at the beginning of this section.

$$\omega_c = 1 \; rad/s \qquad \Rightarrow \qquad \begin{cases} k_p = 73.3679 \\ k_i = 0.9782 \end{cases} \tag{17}$$

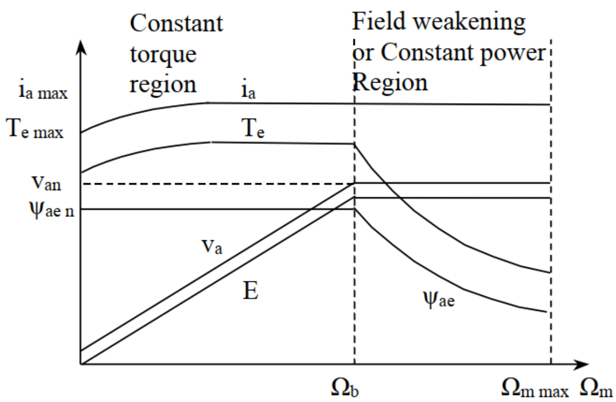
Excitation current controller

Finally, the last pi controller calibrated by the same method is the one inherent in the excitation current. The values determined are the following ones:

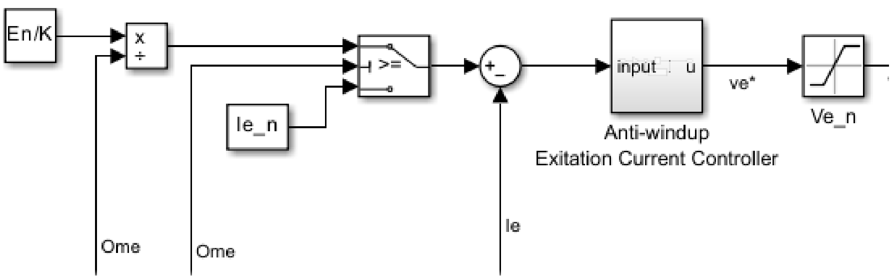
$$\omega_c = 20 \; rad/s \qquad \Rightarrow \qquad \begin{cases} k_p = 2400 \\ k_i = 2400 \end{cases} \tag{18}$$

Flux weakening

Looking at the specifications in Table 1 regarding the speed profile at which the tram must travel, it can be seen that in the section between 4 and 6 km it must travel at a speed of 75 km/h, which is higher than the nominal speed (60km/h = 314 rad/s). Having to maintain this speed for an extended period of time, it is necessary to adopt the technique of flux weakening. In particular, observing the operating regions of the DC machine, it is possible to see that in order to reach higher speeds than the base one, having to keep the emf constant and equal to its nominal value, to increase the mechanical speed of the rotor it is necessary to act on the excitation current to reduce the flux.



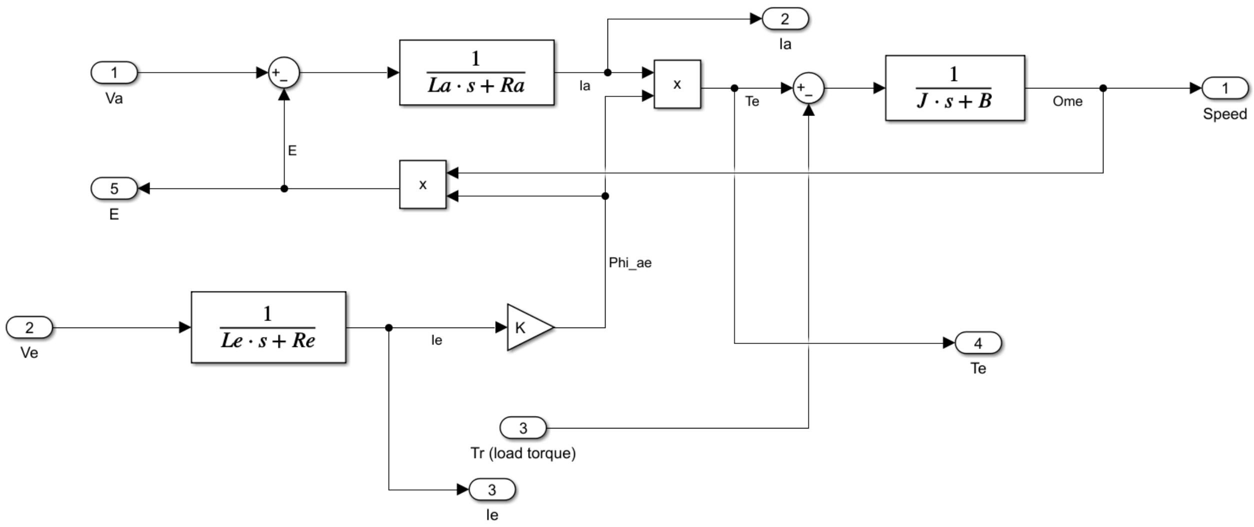
We will therefore keep the excitation current constant at its rated value, but when the mechanical speed of the rotor exceeds its rated value (314 rad/s), with a switch we will set its value equal to $E_n/K\Omega_m$. This has been implemented in matlab through the following scheme:



Simulink simulation

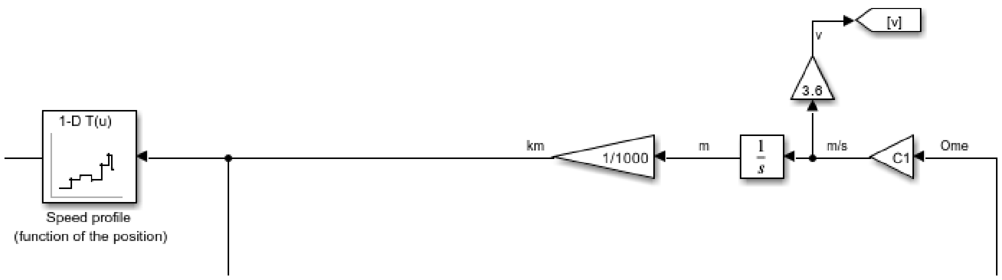
DE SE Motor scheme

Through the theory and parameters determined in section 1 of the report, the DC machine has been implemented in simulink with the following scheme:



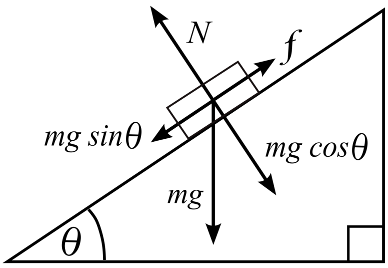
Speed reference

The reference speed is calculated in real time using the position of the tram instant by instant and the speed profile required by the specifications. This has been implemented through a 1-D Lookup Table:



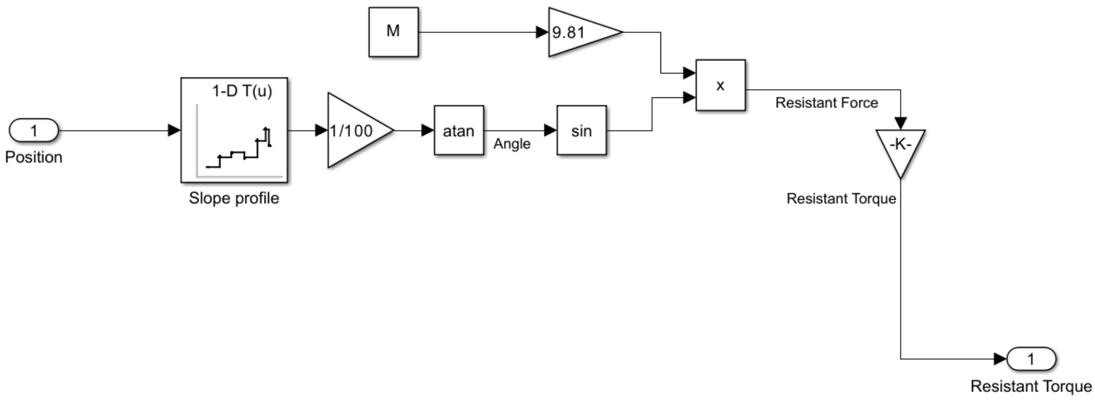
Resistance torque profile

Whenever the tram faces sections of its route with a slope, an external torque acts on the system which must be taken into account in the control of the machine. This external torque is given by the weight force of the tram as it crosses the sloping sections. We can schematize the situation of the tram in the sloping areas as below:

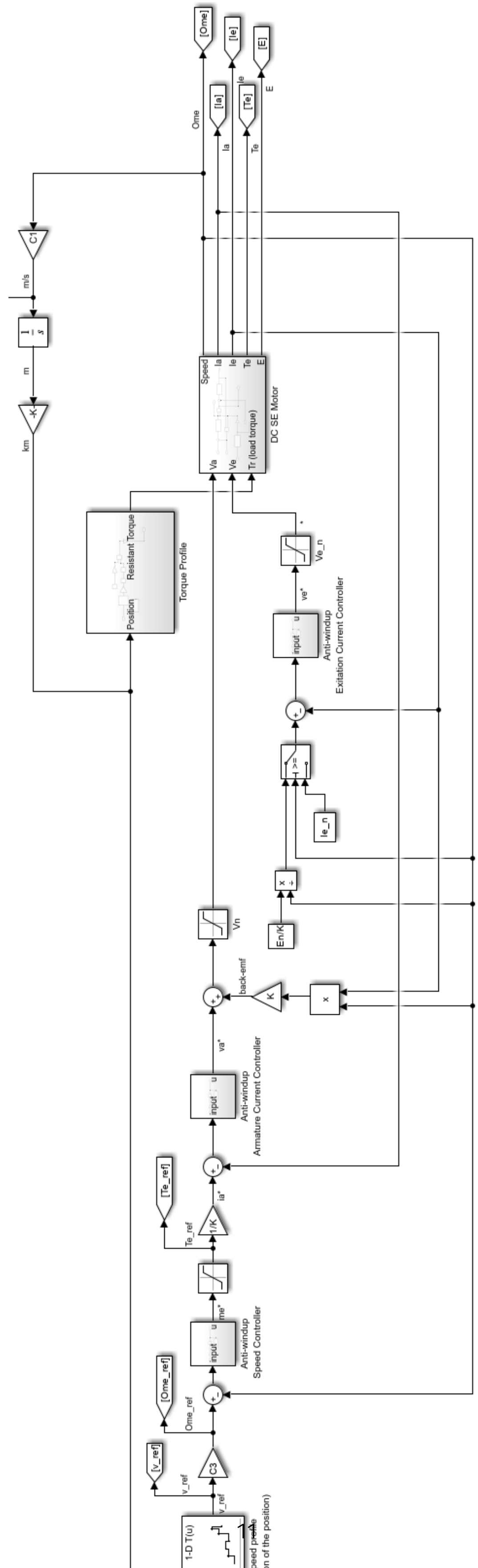


$$\begin{cases} \theta = arctg\left(\frac{slope\%}{100}\right) \\ F_p = Mgsin\theta \end{cases} \tag{19}$$

The weight force theat acts against or in favor of the motion is then converted into a resistant torque, as can be seen in the following diagram in simulink:



Complete scheme



Results

The results obtained through the simulation in simulink are reported below.

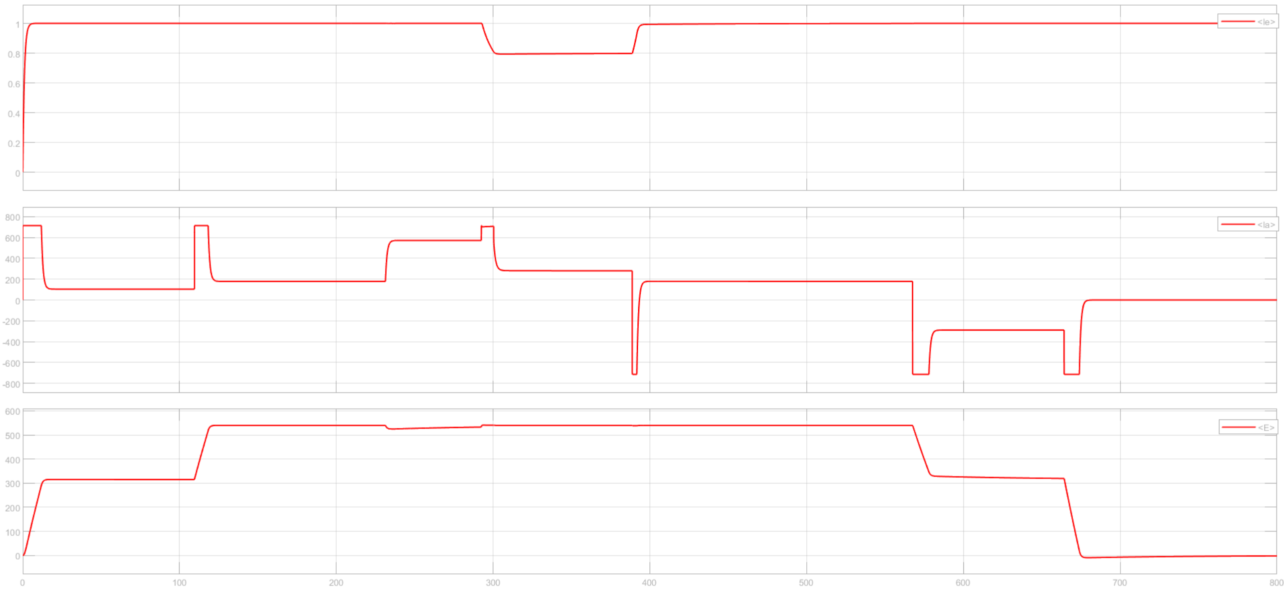


Figure 2: (1) Excitation current, (2) Armature current, (3) Back-emf

The figure above shows the trend of the excitation and armature currents and of the back-emf.

Starting from the first trend it is possible to see how the **excitation current** remains constant and equal to its rated value (1 A) for the entire duration of the simulation except in the flux-weakening area where is required a tram speed equal to 75km/h, higher than the base speed of the DC Motor (60 km/h). In particular the following law is followed:

$$\begin{cases} I_e = I_{en} & \omega \leq \omega_b \\ I_e = \dfrac{E_n}{K\omega} & \omega > \omega_b \end{cases} \tag{20}$$

The second trend in Figure 2 refers to the **armature current**. The peaks are equal to the rated current (714.4 A) and are in correspondence with the areas of the route where the tram is required to increase its speed. Whenever the tram is in the constant speed zones, the armature current always assumes different values since the motor must supply a torque capable of compensating the losses due to friction proportional to the speed.

The trend of the **back-emf** shown in the Figure 2 is consistent with the theoretical equations and further confirms that in the flux-weakening area it remains constant and equal to its nominal value (540 V).

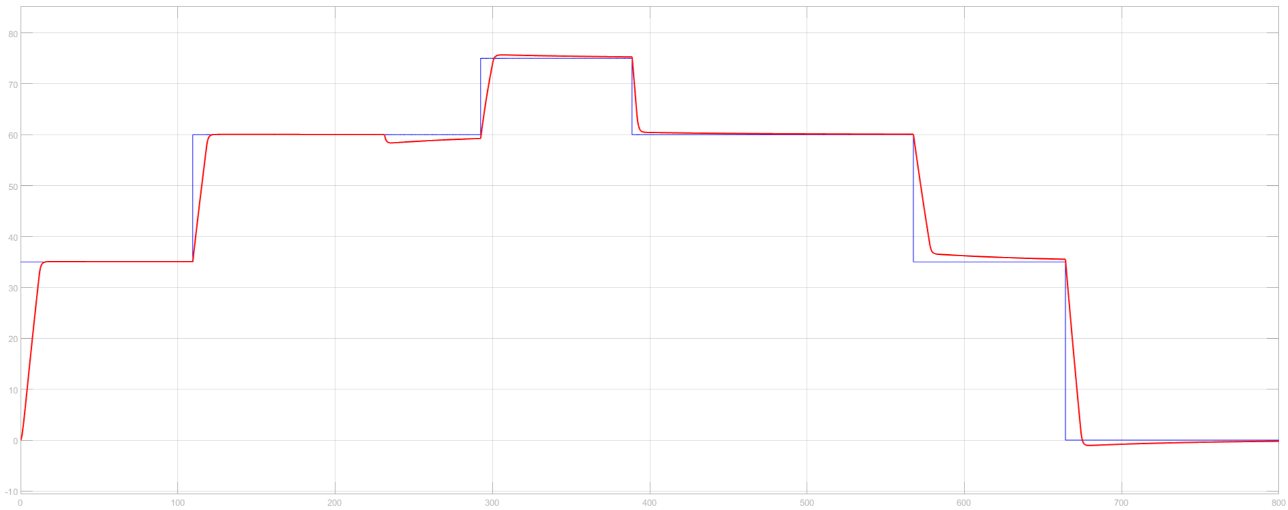


Figure 3: Mechanical speed

The figure above shows the machine **mechanical speed** trend in km/s for easier interpretation of results and comparison with specifications.

Through the realized control scheme, it is possible to observe how there is good tracking of the reference speed and compliance with the required settling time. Furthermore, it can be observed that the section where a speed of 75 km/h is required in Figure 3 (between about 280 s and 380 s in the simulation) corresponds exactly to the flux-weakening zone visible in Figure 2 described above.

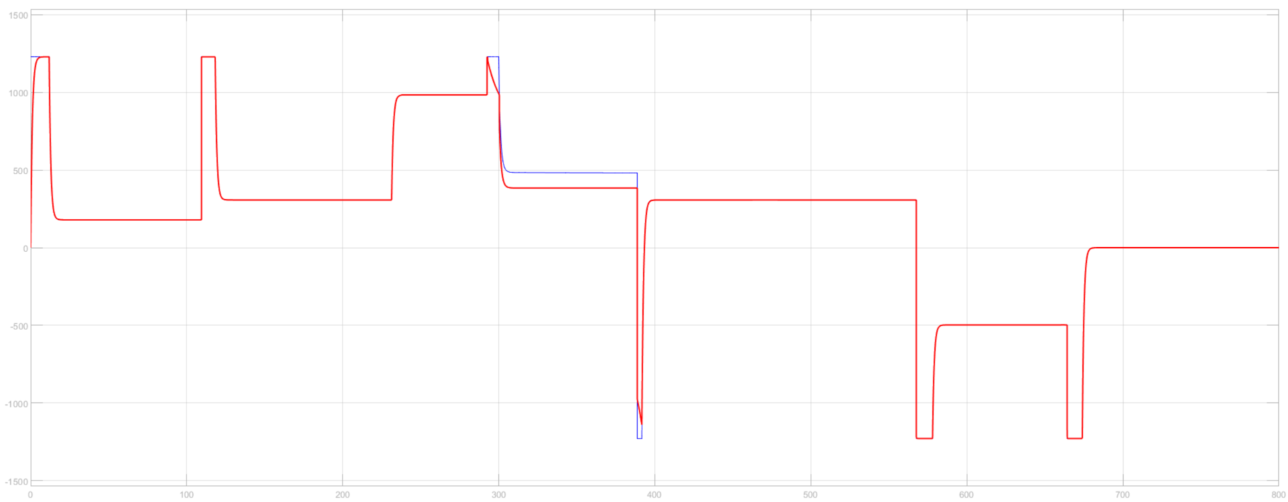


Figure 4: Torque

Finally, the last figure shows the trend of **torque** delivered by the machine.

The peaks visible in the graph correspond to the area where the tram is required to accelerate (positive peaks) or decelerate (negative peaks) corresponding to the path sections where there is a change in speed. In the downhill area with negative slope, the torque remains constantly negative, as it is the torque required to brake the tram to maintain a constant speed.