

**Unplittable demands maximizing revenue associated to delivered demands**

$$\text{maximize} \quad \sum_{(o,t,i) \in \Theta} \left[ \sum_{e \in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r - \sum_{\substack{e=(v,w,j) \in E: \\ j \neq i}} f_{(v,w,j)}^{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \quad (1a)$$

$$\text{subject to} \quad \sum_{e \in IE(z)} f_e^{(o,t,i)} - \sum_{e' \in OE(z)} f_{e'}^{(o,t,i)} = 0, \quad \forall z \in V \setminus \{o, t\}, \forall (o, t, i) \in \Theta \quad (1b)$$

$$\sum_{e \in OE(o)} f_e^{(o,t,i)} \leq d_{(o,t,i)}, \quad \forall (o, t, i) \in \Theta \quad (1c)$$

$$\sum_{e \in OE(t)} f_e^{(o,t,i)} = 0, \quad \forall (o, t, i) \in \Theta \quad (1d)$$

$$\sum_{(o,t,i) \in \Theta} f_e^{(o,t,i)} \leq q_e^*, \quad \forall e \in E \quad (1e)$$

$$\sum_{v \in S} \sum_{w \in S} f_{(v,w)}^d \leq |S| - 1, \quad \forall S \subset V, \forall d \in \Theta \quad (1f)$$

$$f_e^{(o,t,i)} \in \mathbb{N}_0, \quad \forall e \in E, \forall (o, t, i) \in \Theta \quad (1g)$$

Constraint (1e) ensures that the commodities sent through an edge doesn't exceed the total capacity available in that edge (the sum of the capacities each agent has for that edge).

Constraint (1f) is a subtour elimination constraint.

In this model,  $e \in E$  is a tuple  $(v,w,i)$  for  $v, w \in N$  and  $i \in N$  (multiple agents can have an active edge between the same two nodes).

### Unsplittable demand maximizing demands served

$$\begin{aligned}
& \text{maximize} && \sum_{(o,t,i) \in \Theta} \sum_{e \in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r && (2a) \\
& \text{subject to} && \sum_{e \in IE(z)} f_e^{(o,t,i)} - \sum_{e' \in OE(z)} f_{e'}^{(o,t,i)} = 0, && \forall z \in V \setminus \{o, t\}, \forall (o, t, i) \in \Theta \\
& && && (2b) \\
& && \sum_{e \in OE(o)} f_e^{(o,t,i)} \leq d_{(o,t,i)}, && \forall (o, t, i) \in \Theta && (2c) \\
& && \sum_{e \in OE(t)} f_e^{(o,t,i)} = 0, && \forall (o, t, i) \in \Theta && (2d) \\
& && \sum_{(o,t,i) \in \Theta} f_e^{(o,t,i)} \leq q_e^*, && \forall e \in E && (2e) \\
& && \sum_{v \in S} \sum_{w \in S} f_{(v,w)}^d \leq |S| - 1, && \forall S \subset V, \forall d \in \Theta && (2f) \\
& && f_e^{(o,t,i)} \in \mathbb{N}_0, && \forall e \in E, \forall (o, t, i) \in \Theta && (2g)
\end{aligned}$$

An extra constraint might be added to this model:

$$\begin{aligned}
& \sum_{\substack{e=(v,w,j) \\ \in E: z \neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \leq \\
& \leq \sum_{e \in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r + \sum_{\substack{e=(v,w,j) \in E: \\ j \neq i}} f_{(v,w,j)}^{(o,t,i)} \cdot \frac{c_e}{q_e} \quad \forall (o, t, i) \in \Theta
\end{aligned}$$

This will ensure that any agent get loss from the cooperation. Similar effect to maximize the profit from each commodity