Intertemporal logistics collaboration

Msc Thesis

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Abstract

Here goes the abstract

Keywords: Here, go, KEY, words

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1. Introduction

During the last decades, logistics collaboration, and specially *horizontal logistics collaboration*, has gained the attention of many researchers, as it has been proofed to be an effective strategy to improve the logistics chain, for example, reducing both ecological and economical costs [1] [2]. By horizontal logistics collaboration we refer to the cooperation between several companies or agents, who playing the same role in the logistics chain, form coalitions or alliances to increase the efficiency of their logistics operations. For example, in the case of the liner hipping industry, by allowing other companies to use part of the capacity of the own ships, increasing the asset utilization ratio [3]. In this paper, we use the terms "collaboration" and "cooperation" interchangeably.

The literature studying the horizontal logistics collaboration between agents can be divided in two main streams depending on whether the collaboration it is centralized or not.

In the case of centralized systems, agents usually share in a pool all or part of the demands that they have to serve as well as the assets they have available. These assets can be vehicles in the case of transportation carriers, but also inventory levels, employees, etc. Then a central planner, taking into account all the demands and sources shared in the pool by the agents, solves the problem maximizing the total payoff of the coalition. Finally, the benefits of the cooperation are shared among the members of the coalition, usually using some allocation rule which ideally satisfies some desirable conditions in terms of "fairness", as that the resulting allocation is in the *core*, i.e., any subcoalition of the former one could obtain a higher payoff working independently [4]. A well known example of an allocation rule fulfilling this conditions is the *Shapley value* [5].

Some authors have argued in the centralized systems, the individual objectives of the agents are not considered in favour of the coalition objective, while actually the agents remain independent entities [6]. Therefore, these authors propose different mechanisms where both levels of objectives are considered [7] [8]. Furthermore, it is assumed in the literature studying these type of systems that agents share all their private information with the central planner, what might be unrealistic or problematic in real world cases [9] [10].

In the other hand, in the decentralized systems there is not a central planner who carries out the optimization and then allocates the benefits, but the agents remain independent decision makers during the whole collaboration process. Decentralized systems commonly operate based on auctions systems or inverse-optimization techniques [11]. In the first, agents share the part of their demands, an the other members of the coalition bid to be the ones serving that demand on exchange of a recompense from the original owner of the order. In the other hand, an example of a collaborative system based in a inverse-optimization mechanism is the proposed in [12]. On it, the authors model a multicommodity flow problem, where agents own certain fraction of the capacity of the edges of a network. Thus, the agents can collaborate sharing the capacity of the edges with the members of the coalition, who have to pay some price for using that edges. How much capacity the agents own on each edge of the and which are their demand is assumed to be public information. Using inverse optimization, capacity exchange prices are found, in such a way that if the agents adopt that prices policy, when each agent selfishly optimizes his demands through the shared network, the resulting flow will be the same that if a centralized approach was used.

Some authors have argued that in the centralized systems, the individual objectives of the agents are not considered in favour of the coalition objective, while the agents actually remain independent entities [6]. Therefore, these authors propose different mechanisms where both levels of objectives are considered through multi-objective approaches [7] or by integrating the objectives of the agents as constraints to the coalition problem [8].

In [13] a novel framework to model decentralized systems is proposed. Based in the concept of *coopetition* [14], the authors proposed a 2-stage mechanism to model the collaboration of agents in decentralized distribution systems. In the first stage, agents individually take strategic decisions, in their case, fixing their inventory levels. In the seconds stage an allocation of the assets is obtained through cooperation, were agents share their residual demands and inventory with the others, obtaining extra profits. They propose sufficient conditions under which a Nash equilibrium in pure strategies exist for the first stage, and for which allocation rule in the *core* exists for the second stage.

Most of the current literature studies the horizontal logistics collaboration from a static point of view [15]

In this paper we will compare the efficiency between a centralized system, giving to central planner different degrees of decision power, and a one, where we substitute the central planner by a platform where agents can share information, and through an iterative process of optimization and information sharing, agents can achieve an equilibrium point from which any of the agents would be interested to deviate from. The problem we will use as example to implement this models is a multicommodity flow problem, where each agent have to decide which edges to active on the network at certain price, to then find an efficient flow through them which maximizes their payoff.

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2. Problem statement: A network design - muldicommodity flow problem

As stated in the previous section, we will use a multicommodity flow problem with a network design stage to illustrate the different cooperation mechanisms we study in this paper. In the problem we propose, a simplified version of the one studied in [3], a set of agents have to first decide which edges they want to activate in a network, with certain costs associated. Then, each have to route their commodities through the edges he has activate. The objective of the agents is to maximize the revenues generated by the served commodities and minimize the costs associated with the activation of the edges. This two subproblems are intrinsically linked: the most efficient flow an agent can find for his commodities depends on which edges he has access too, and to know which edges to activate, the agent need to know how he will route his commodities through that edges.

The agents can collaborate among them sharing the capacity they do not use in their active edges, allowing others to route their commodities through them at some price.

Let $N = \{1, ..., n\}$ be a set of agents. Let $G = (V, E)$ be a directed graph with V and E the sets of nodes and edges respectively. Let Θ be the set of commodities. Each commodity is noted by a tuple, (o, t, i), where $o \in V$ is its origin node, $t \in V$ its destiny node and i is the agent who owns it. A commodity (o, t, i) has a size of $d_{(o,t,i)}$ units and has an associated revenue $r_{(o,t,i)}$. The owner of the commodity (o, t, i) earns $d_(o, t, i) \cdot r_(o, t, i)$ if the commodity is successfully delivered from its origin to its terminal node. The commodities are unsplitable, i.e., can not be divided between different edges. Each edge $e \in E$ will be noted by a tuple, $e = (v, w, i)$, indicating that it connects the node $v \in V$ with the node $w \in V$ and that his owner is the agent $i \in N$. Note that could exist as many edges between a pair of nodes as agents exists. Each edge $e \in E$ has certain units of capacity associated q_e, and a a fixed activation cost c_e, that the owner of the edge have to pay if he wants to use the edge to route any commodity. We will note by $OE(v) \subset E$ the subset of edges whose origin is the node $v \in V$. Similarly, $IE(v) \subset E$ is the subset of edges whose terminal node is $v \in V$.

We will note by $E^i \subset E$ and $\Theta^i \subset \Theta$ the subsets of edges and commodities owned by agent i.

Table 1: Summary of notation

Symbol	Meaning
N	Set of agents.
G	Network.
V	Set of nodes in G.
E	Set of edges in G.
Θ	Sets of commodities.
$OE(v)$	Subset of edges in E which depart from node v.
$IE(v)$	Subset of edges in E which arrive to node v.
Θ^i	Set of commodities owned by agent i.
E^i	Set of edges owned by agent i.
E_A	Subset of edges in E that are active
E_R	Subset of edges in E_A that have residual capacity.
$(o, t, i) \in \Theta$	Commodity with origin node o and terminal node t owned by agent i.
$d_{(o,t,i)}$	Size, on units, of commodity (o, t, i).
$r_{(o,t,i)}$	Revenue associated with serving commodity (o, t, i).
$e = (v, w, i) \in E$	Edge connecting node v with node w, owned by agent i.
c_e	Cost of activate edge e.
q_e	Units of capacity of edge e.
q_e^R	Units of residual capacity of edge e.

2.1. Single agent model

The objective of an agent $i \in N$ is to maximize his payoff, by routing his commodities, Θ^i, from their origin nodes to their terminal nodes through the edges E^i, he has activated. If the agent does not take part of the collaboration, she will not have any access to other agents edges, neither the other agents can route their commodities through his edges. Thus, the problem which agent $i \in N$ has to solve can be modelled as the following ILP, that we call P_i,

$$P_i: \quad \max \sum_{(o,t,i)\in\Theta^i} \sum_{e\in\delta^+(t)\cap E^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E^i} u_{e'} \cdot c_e \tag{1}$$

subject to:

$$\sum_{e\in\delta^-(t)\cap E^i} f_e^{(o,t,i)} - \sum_{e\in\delta^+(t)\cap E^i} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta^i, \tag{2}$$

$$\sum_{e\in\delta^+(t)\cap E^i} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta^i, \tag{3}$$

$$\sum_{e\in\delta^+(t)\cap E^i} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta^i, \tag{4}$$

$$\sum_{(o,t,i)\in\Theta^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq u_e \cdot q_e, \qquad \forall\ e \in E^i, \tag{5}$$

$$\sum_{\substack{(v,w,k)\in E^i:\\ v,w\in S}} f_{(v,w,i)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta^i, \tag{6}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E^i,\ \forall\ (o,t,i) \in \Theta^i, \tag{7}$$

$$u_e \in \{0,1\}, \qquad \forall\ e \in E^i \tag{8}$$

where, $\forall\ (o,t,i) \in \Theta^i$ and $\forall\ e \in E$,

$$f_e^{(o,t,i)} = \begin{cases} 1 & \text{if } (o,t,i) \text{ is routed through } e, \\ 0 & \text{otherwise} \end{cases}$$

$$u_e = \begin{cases} 1 & \text{if } e \text{ is activated}, \\ 0 & \text{otherwise} \end{cases}$$

The objective of agent i, equation (1), is to maximize the profit generated by the commodities which arrive to their destination nodes while minimizing the cost associated to the activation of the edges. Constraints (2) ensure that, for any commodity, the flow that enters and leaves every transit nodes (neither the origin, neither the terminal node of that commodity) is equal. Constraints (3), together with (7), guarantee that each commodity is send at most only once from its origin node. Constraints (4) ensure that no commodity is send from its terminal node to any other node. Constraint (5) ensures that the capacity of edges is not exceed. Finally, constraints (6) are subtour elimination constraints [16]. We have included these constraints in our model because, first of all, it makes sense to assume that an agent would not want to create subtours when routing his commodities, as they would mean that some commodities would occupy unnecessary units of capacity in some edges. Note that his can be possible because once an edge is activated, there is no extra costs for routing more commodities through it. Furthermore, without the subtour elimination constraints, the other constraints are not sufficient to ensure that all the feasible solution of the ILP are admissible, since the flow of a commodity could contain a subtour disconneted from the origin and terminal nodes of that commodity, as we illustrate in Figure [Include figure!!!!!]. [Or maybe include cost for routing commodities through edges?]

We will note by P_i^* the optimal solution of P_i which is the maximal payoff agent i can obtain by his own without cooperating with the other agents.

3. The centralized cooperation models

In the previous subsection, we have presented an ILP that models the network design - multicommodity flow problem of an agent $i \in N$ who works alone. In this section, we present different centralized cooperative scenarios, where the agents in N collaborative in different ways in order to find synergies among them and increase their payoffs.

In all the three different scenarios we propose, the way in which agents is sharing the capacity of the edges they have activate with other agents, what could allow to some agents to serve commodities in a cheaper way to some agents, and to compensate the cost of the activation of some edges to others. The three scenarios also have in common the way in which the agents share the benefits generated by the collaboration, what is done as follows

1. The revenues generated by any served commodity are allocated to its owner.
2. The activation cost of any active edge is paid by its owner.
3. If a commodity $(o,t,i) \in \Theta$ is routed through an edge $e = (v,w,j) \in E$ and $j \neq i$, agent i makes a side payment equal to $d_{(o,t,i)}) \cdot \frac{c_e}{q_e}$ to the agent j. In words, for each commodity sent through an edge owned by other agent, the owner of the commodity will pay to the owner of the edge the fraction of the activation cost of that edge equivalent to the fraction of the total capacity of the edge that his commodity occupies.

In the rest of the section we introduce the three different centralized cooperative scenarios we propose in this work.

3.1. Full cooperation scenario

We start introducing the model where more power is given to the central planner. In this model, all the commodities and edges of all the agents are given to the central planner, who then has to solve the following ILP, Γ_F, which is almost identical to P_i, the ILP that an agent $i \in N$ has to solve when no cooperating. Both problems only differ in two aspects:

(a) The central planner manages the commodities and edges of all the agents, Θ and E, while without cooperation an agent i only has access to his own commodities and edges, Θ^i and E^i.

(b) The following new constraint is added to the problem:

$$\varphi_i(\Gamma_F) \geq P_i^*, \quad \forall\, i \in N; \tag{9}$$

where

$$\varphi_i(\Gamma_F) =$$

$$= \sum_{(o,t,i)\in\Theta^i} \left[\sum_{e\in\delta^-(t)\cap E^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in\Theta^j: \\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \right] +$$

$$+ \sum_{\substack{(o,t,k)\in\Theta: \\ k\neq i}} \left[\sum_{e\in E^i} f_e^{(o,t,k)} \cdot d_{(o,t,k)} \cdot \frac{c_e}{q_e} \right] - \sum_{e\in E^i} u_e \cdot c_e, \tag{10}$$

is the payoff allocate to the agent $i \in N$ for any feasible solution of Γ_F. Similarly to what is done in [8], we integrate the final payoffs of the agents in the constraints of the central planner's problem. Thus, constraints (9) ensure that the final payoff of any agent $i \in N$, φ_i, is greater or equal than what that agent could achieve without cooperation, i.e., P_i^*. Doing so we can say that the final payoff allocation is *individually rational* [4], i.e., no single agent is interested in leaving the coalition unilaterally. [1]

In equation (10) we define how the final payoff of an agent $i \in N$, is computed: the first term is the sum of the profit created by the commodities of the agent, which for each commodity is equal to the revenue it generates when it is served minus the cost the agent has to pay to other agents if the commodity passes through edges which does not belong to him. The second sum is the payments that other agents pays to him when routing their commodities through his edges. The last term is the sum of the costs of the active edges.

[1] We can extend this idea by forcing the solution resulting from the cooperation to be in the *core*, by adding constraints ensuring that any the sum of the payoffs of the members of any subcoalition is equal or greater than what that agents could obtain by leaving the grand coalition and cooperating only among them. Nevertheless, doing it will increase the complexity of the models and we consider that it does not add value to the paper.

3.2. Partial cooperation scenario

In the partial cooperation scenario, the central planner has to optimize once again the flow of all the commodities of all the agents through the network, maximizing the total generated revenue. Nevertheless, the decision of which edges to activate remains as an individual decision of each agent.

We propose that, in a first stage, each agent $i \in N$ solves his own P_i problem. Then, in a second stage, the same edges that are active in the optimal solutions of each agent individual ILP, P_i^*, are the agents that the central planner can use to route the commodities through the network.

Let $E_A \subset E$ be the subset of edges such that $\forall e = (v, w, i) \in E_A$, e is active in P_i^*. Then the ILP, Γ_P, that the central planner has to solve in the partial cooperation scenario is

$$\Gamma_P : \max \quad \sum_{(o,t,i)\in\Theta} \left[\sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E_A^j: \\ j\neq i}} f_e^{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \tag{11}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E_A} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\, z \in V \setminus \{o,t\},\ \forall\, (o,t,i) \in \Theta, \tag{12}$$

$$\sum_{e\in\delta^+(o)\cap E_A} f_e^{(o,t,i)} \leq 1, \qquad \forall\, (o,t,i) \in \Theta, \tag{13}$$

$$\sum_{e\in\delta^+(t)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\, (o,t,i) \in \Theta, \tag{14}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e \qquad \forall\, e \in E_A, \tag{15}$$

$$\sum_{\substack{(v,w,k)\in E_A:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S|-1, \qquad \forall\; S\subset V,\; \forall\;(o,t,i)\in\Theta, \quad (16)$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\; e\in E_A, \forall\;(o,t,i)\in\Theta, \quad (17)$$

$$\varphi_i(\Gamma_P) \geq P_i^*, \qquad \forall\; i\in N \quad (18)$$

Note that, consequently to the above introduced,this model differs from the previously presented in this work in the absence of decisions variables for the activation of the edges, since this decision is previously make by the agents and not left to the central planner. Also we include the constraints (A.17 which are equivalent to the constraints (9) included in the full cooperation scenario problem, to again ensure that any feasible solution of the problem is individually rational. In this case, because the absence of the decision variables u_e in the model, we redefine the payoff allocate to each agent $i\in N$ as follows

$$\begin{aligned}
&\varphi_i(\Gamma_P) = \\
&= \sum_{(o,t,i)\in\Theta^i}\left[\sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)}\cdot d_{(o,t,i)}\cdot r_{(o,t,i)} - \sum_{\substack{e\in E^j:\\ j\neq i}} f_e^{(o,t,i)}\cdot d_{(o,t,i)}\cdot\frac{c_e}{q_e}\right] + \\
&+ \sum_{\substack{(o,t,k)\in\Theta:\\ k\neq i}}\left[\sum_{e\in E_A^i} f_e^{(o,t,k)}\cdot d_{(o,t,k)}\cdot\frac{c_e}{q_e}\right] - \sum_{e\in E_A^i} c_e.
\end{aligned} \quad (19)$$

3.3. Residual cooperation

Among the three centralized systems we propose in this paper, the *residual cooperation scenario*, which is inspired in the already mentioned framework presented in [13], is the one where less decision power if left to the central planner.

As in the partial cooperation scenario, every agent $i\in N$ will individually solve his corresponding P_i problem, deciding which edges to activate and how to route his commodities through that edges. But this time, the agents actually implement this solution, and only inform the central planner about which commodities they have not been able to route in an efficient way, as well as which active edges still have some free capacity. With all this information, the central planner routes in a efficient way the residual commodities of the agents, Θ_R, through the residual capacities of the active edges,E_R. For each edge $e \in E_R$ we will note by q_e^R the residual capacity of that edge, i.e., the units of capacity that the owner of the edge has not used to route his own commodities. The ILP the central planner has to solve in this residual cooperation scenario, Γ_R, is equivalent to Γ_P, where the commodities and edges the central planner manages are Θ_R and E_R instead of Θ and E_A. Also, the central planner can not route commodities through and edge $e \in E_R$ exceeding the residual capacity of that edge, Q_e^R, so constraint (15) is substituted by

$$\sum_{(o,t,i)\in\Theta_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e^R, \quad \forall\, e \in E_R. \tag{20}$$

In this scenario, it is guaranteed by design of the collaboration mechanism that each agent end up the cooperation with a payoff greater or equal than the payoff they could obtain by their own, as they always first implement the best solution they can achieve without cooperating, and therefore they can only increase their payoffs with respect to that value. Therefore, constraints equivalent to (A.17) are not included. Under the residual cooperation scenario we define the final payoff of an agent $i \in N$ as

$$\begin{aligned}
&\varphi_i(\Gamma_R) = \\
&= \sum_{(o,t,i)\in\Theta_R^i} \left[\sum_{e\in\delta^-(t)\cap E_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E_R^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \right] + \\
&+ \sum_{\substack{(o,t,k)\in\Theta_R:\\ k\neq i}} \left[\sum_{e\in E_R^i} f_e^{(o,t,k)} \cdot d_{(o,t,k)} \cdot \frac{c_e}{q_e} \right] - P_i^*
\end{aligned} \tag{21}$$

In table 2 we summarize the differences between the three different centralized cooperative scenarios we have proposed in this section, indicating which decisions are left to the agents and which are made by the central planner.

Table 2: Here goes the caption

		Coop. centralized scenarios		
		Full	Partial	Residual
Agent	Activate edges	No	Yes	Yes
	Route flow	No	No	Yes
Central planner	Activate edges	Yes	No	No
	Route flow	Yes	Yes	Yes*

* Only the residual commodities through the residual capacities of the active edges.

4. A decentralized cooperation mechanism for two agents

In this section we propose a decentralized cooperation mechanism for two agents, where all the decisions are left to the agents, who make use of an *information platform* to interact with each other.

The idea behind this mechanism is that the agents exchange information in an iterative process, informing to the other agent about the residual capacity they have in their own active edges, allowing the other agent to use that residual capacity to route his commodities at some price.

Before go in more detail about how the cooperation mechanism works, we introduce how the information platform works. Each agent $i \in N$ has two different pools where he can share information:

Shared edges: Agent i can share a subset of his active edges which still have some residual capacity, $\widehat{E}^i_R \subset E^i_R$. He also has to share how many units of residual capacity has each edge $e \in E^i_R$, q^R_e, and which is the cost of each of that units, $\frac{c_e}{q_e}$

Demanded edges: If agent j has previously share a set of edges in the platform, agent i can use that edges when routing his commodities. If agent i actually decides to route a subset of his commodities, $\widehat{\Theta}^i \subset \Theta^i$ through some of that edges, he has to share in the information platform:

1. How much units of capacity he requires (or demands) on each edge e shared by agent j, what we note by q_e^D.
2. For each commodity $(o,t,i) \in \widehat{\Theta}^i$, a set $L^{(o,t,i)}$, containing the edges that commodity would pass through, as well as which would be the side payment he would do to agent j if finally he can have access to all the edges in $L^{(o,t,i)}$, $p_{L^{(o,t,i)}} = \sum_{e \in L^{(o,t,i)}} d_{(o,t,i)} \cdot \frac{c_e}{q_e}$. We note by $\mathcal{L}^i$ the set containing all $L^{(o,t,i)}$, $\forall\ (o,t,i) \in \widehat{\Theta}^i$.

Making use of the information platform just introduced, both agents iteratively solve an ILP. In each iteration both agents, sequentially and not in parallel, decide which edges to activate, and how to route their commodities through the network, as they do in P_i, but this time, they can route their commodities, through the edges shared in the platform by the other agent with the corresponding costs and respecting the available capacities on that edges. Furthermore, when deciding if activate or not an edge, each agent does not only have to account for the activation cost, but also for the side payments the other agent is willing to make to him if he activates certain combinations of edges and leaves on them certain residual capacities. The ILP, Υ_i, agent $i \in N$ has to solve in each iteration is the following:

$$
\begin{aligned}
\Upsilon_i: \quad \max \quad & \sum_{(o,t,i)\in\Theta^i} \sum_{e\in\delta^-(t)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E^i} u_e \cdot c_e \quad - \\
& - \sum_{(o,t,i)\in\Theta^i} \sum_{\substack{e\in\widehat{E}_R^j: \\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \\
& + \sum_{\substack{L^{(o,t,j)}\in\mathcal{L}^j: \\ j\neq i}} p_{L^{(o,t,j)}} \cdot t_{L^{(o,t,j)}}
\end{aligned} \tag{22}
$$

subject to:

$$\sum_{e\in\delta^-(z)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} = 0, \quad \forall\ z\in V\setminus\{o,t\}, \quad \forall\ (o,t,i)\in\Theta^i, \tag{23}$$

$$\sum_{e\in\delta^+(o)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} \le 1, \quad \forall\ (o,t,i)\in\Theta^i, \tag{24}$$

$$\sum_{e\in\delta^+(t)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} = 0, \quad \forall\ (o,t,i)\in\Theta^i, \tag{25}$$

$$\sum_{(o,t,i)\in\Theta^i} f_e^{(o,t,i)}\cdot d_{(o,t,i)} \le u_e\cdot q_e, \quad \forall\ e\in E^i, \tag{26}$$

$$\sum_{(o,t,i)\in\Theta^i} f_e^{(o,t,i)}\cdot d_{(o,t,i)} \le q_e^R, \quad \forall\ e\in \widehat{E}_R^j, \tag{27}$$

$$\sum_{\substack{(v,w,k)\in E^i\cup\widehat{E}_R^j:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \le |S|-1, \quad \forall\ S\subset V,\ \forall\ (o,t,i)\in\Theta^i, \tag{28}$$

$$\sum_{(o,t,i)\in\Theta^i} f_e^{(o,t,i)}\cdot d_{(o,t,i)} \le (q_e - q_e^D) + M(1-b_e), \quad \forall e\in L^{(o,t,j)},\ \forall L^{(o,t,j)}\in\mathcal{L}^j \tag{29}$$

$$t_{L^{(o,t,j)}} \le 1 - (|L^{(o,t,j)}| - \sum_{e\in L^{(o,t,j)}} b_e), \quad \forall L^{(o,t,j)}\in\mathcal{L}^i \tag{30}$$

$$f_e^{(o,t,i)} \in\{0,1\}, \quad \forall\ e\in E^i,\ \forall\ (o,t,i)\in\Theta^i, \tag{31}$$

$$u_e\in\{0,1\}, \quad \forall\ e\in E^i \tag{32}$$

$$b_e\in\{0,1\}, \quad \forall e\in L^{(o,t,j)},\ \forall L_{(o,t,j)}\in\mathcal{L}^j \tag{33}$$

$$t_{L^{(o,t,j)}}\in\{0,1\}, \quad \forall L^{(o,t,j)}\in\mathcal{L}^j \tag{34}$$

where M is a very large value.

In this ILP, the objective function of the agent i, equation (22), the first term is the sum of the revenue generated by the commodities he has served and the side payments the other agent makes to him when routing commodities through his edges. The second term are the costs of the edges he has activate. The third and term are the side payments he has to do when routing commodities through the edges shared by the other agent. In the other hand, the fourth term are the side payments he receives from the other agent for making use of the edges he shared in the previous iteration and remain available in the same conditions to him in the current one.

Constraints (23), (24), (25) and (28) are equivalent to constraints (2),(3),(4) and (6) in P_i, with the difference that now the agent i can use the edges shared in the information platform by the agent j, $\widehat{E}_R^j$. Constraint (26) is identical to constraint (5) and constraint (27) is an extension of it, ensuring that the agent does not exceed the residual capacity of the edges shared by agent j when routing his commodities.

Constraints (29) and (30), together with the "technical" decision variables (33) and (34), are related with the side payments agent i receives from agent j. Recall that each $L^{(o,t,j)} \in |$ is a set which contains the edges agent j has indicated in the previous iteration he would like to use to route commodity (o, t, j). He has also indicate that if for every $e \in L^{(o,t,j)}$, he requires q_e^R units of capacity, he would make a side payment to agent i equal to $p_{L^{(o,t,j)}}$. Thus, in order to get the side payment $p_{L^{(o,t,j)}}$, the technical decision variable $t_{L^{(o,t,j)}}$ must be equal to 1. Constraints (30) and (34) ensures that only can happen if all the technical decision variables b_e for $e \in L^{(o,t,j)}$ are also equal to 1. Finally, constraint (29), making use of the *big M* method, ensures that if b_e is equal to one, agent i must leave at least q_e^D units of residual capacity on that edge. Note that this constraints are too restrictive, since agent j might be planning to route 2 or more commodities through the edge e, and that agent i left less than q_e^R units of capacity available does not necessarily mean that none of that commodities can be route through it, but that not all of them. Nevertheless, we argue that capture that particularities would make the model to complex, as the problem of deciding which commodities can still be routed and which can not is a version of the well known *knapsack problem*, which has been proven to be a NP-complete problem [17].

5. Results

5.1. Comparision 3 centralized mechanisms

5.2. Comparision centralized mechanisms and iterative one, all for 2 agents

6. Conclusion

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Appendix A. ILP's

Appendix A.1. ILP to be solved by the central planner in the centralized full cooperation scenario.

$$\Gamma_F : \max \quad \sum_{(o,t,i)\in\Theta} \sum_{e\in\delta^-(t)\cap E} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E} u_e \cdot c_e \tag{A.1}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta, \tag{A.2}$$

$$\sum_{e\in\delta^+(o)\cap E} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta, \tag{A.3}$$

$$\sum_{e\in\delta^+(t)\cap E} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta, \tag{A.4}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq u_e \cdot q_e, \qquad \forall\ e \in E, \tag{A.5}$$

$$\sum_{\substack{(v,w,k)\in E:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta, \tag{A.6}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E,\ \forall\ (o,t,i) \in \Theta, \tag{A.7}$$

$$u_e \in \{0,1\}, \qquad \forall\ e \in E, \tag{A.8}$$

$$\varphi_i(\Gamma_F) \geq P_i^*, \qquad \forall\ i \in N; \tag{A.9}$$

Appendix A.2. ILP to be solved by the central planner in the centralized partial cooperation scenario.

$$\Gamma_P : \max \quad \sum_{(o,t,i)\in\Theta} \left[\sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E_A^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \tag{A.10}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E_A} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta, \tag{A.11}$$

$$\sum_{e\in\delta^+(o)\cap E_A} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta, \tag{A.12}$$

$$\sum_{e\in\delta^+(t)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta, \tag{A.13}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e \qquad \forall\ e \in E_A, \tag{A.14}$$

$$\sum_{\substack{(v,w,k)\in E_A:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta, \tag{A.15}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E_A, \forall\ (o,t,i) \in \Theta, \tag{A.16}$$

$$\varphi_i(\Gamma_P) \geq P_i^*, \qquad \forall\ i \in N \tag{A.17}$$

Appendix A.3. ILP to be solved by the central planner in the centralized residual cooperation scenario.

$$\Gamma_R : \max \quad \sum_{(o,t,i)\in\Theta_R} \left[\sum_{e\in\delta^-(t)\cap E_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E_R^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \tag{A.18}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E_R} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E_R} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta_R, \tag{A.19}$$

$$\sum_{e\in\delta^+(o)\cap E_R} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta_R, \tag{A.20}$$

$$\sum_{e\in\delta^+(t)\cap E_R} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta_R, \tag{A.21}$$

$$\sum_{(o,t,i)\in\Theta_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e^R \qquad \forall\ e \in E_R, \tag{A.22}$$

$$\sum_{\substack{(v,w,k)\in E_R:\\ v\in S, w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta_R, \tag{A.23}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E_R, \forall\ (o,t,i) \in \Theta_R. \tag{A.24}$$