

$$\text{minimize} \quad \sum_{(i,j) \in \Theta} \sum_{v \in V \setminus \{i\}} f_{(i,v)}^{(i,j)} \cdot d_{(i,j)} \cdot r - \sum_{e \in E} u_e \cdot c_e \quad (1a)$$

$$\text{subject to} \quad \sum_{v \in V \setminus \{z\}} f_{(v,z)}^{(i,j)} - \sum_{w \in V \setminus \{z\}} f_{(z,w)}^{(i,j)} = 0, \quad \forall z \in V \setminus \{i, j\}, \forall (i, j) \in \Theta, \quad (1b)$$

$$\sum_{v \in V \setminus \{i\}} f_{(i,v)}^{(i,j)} \leq 1, \quad \forall (i, j) \in \Theta, \quad (1c)$$

$$\sum_{v \in V \setminus \{j\}} f_{(j,v)}^{(i,j)} = 0, \quad \forall (i, j) \in \Theta, \quad (1d)$$

$$\sum_{(i,j) \in \Theta} f_e^{(i,j)} \cdot d_{(i,j)} \leq u_e \cdot q, \quad \forall e \in E, \quad (1e)$$

$$f_e^{(i,j)} \in \{0, 1\}, \quad \forall e \in E, \forall (i, j) \in \Theta, \quad (1f)$$

$$u_e \in \{0, 1\}, \quad \forall e \in E \quad (1g)$$

Constraint (1b) ensures that, for any commodity, the flow that enters and leaves transit nodes (neither the origin neither the terminal) is equal.

Constraint (1c), together with (1f), ensures that any commodity, in case of be send, is completely send and only through and edge from its origin node. It could be substituted by:

$$\sum_{v \in V \setminus \{i\}} f_{(z,v)}^{(i,j)} \leq 1, \quad \forall z \in V \setminus \{j\}, \forall (i, j) \in \Theta$$

To do it would avoid possible sub-tours. This sub-tours could not affect the objective value in any case, but to avoid them seems reasonable.

Constraint (1d) ensures that none commodity is send from its terminal node. Constraint (1e) ensures that the capacity of any edge is exceed.