Centralized and decentralized logistics collaboration

Msc Thesis

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Abstract

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Keywords: Here, go, KEY, words

1. Introduction

During the last decades, logistics collaboration, and specially *horizontal logistics collaboration*, has gained the attention of many researchers, as it has been proofed to be an effective strategy to improve the logistics chain, for example, reducing both ecological and economical costs [1] [2]. By horizontal logistics collaboration we refer to the cooperation between several companies or agents, who playing the same role in the logistics chain, form coalitions or alliances to increase the efficiency of their logistics operations. For example, in the case of the liner hipping industry, by allowing other companies to use part of the capacity of the own ships, increasing the asset utilization ratio [3].

The literature studying the horizontal logistics collaboration between agents can be divided in two main streams depending on whether the collaboration it is centralized or decentralized. In this first case, the common approach is to give all the power to a central planner, who manages all the assets and demands of a coalition of agents and then some allocation rule is used to allocate the benefit generated. In the second one, auctions were agents can exchange demands are frequentlly used. One important difference among this two approaches is that in the decentralized one, agents remain independent decision makers.

In this paper we explore different cooperation mechanisms in the context of the horizontal logistics collaboration between courier companies, modelling the optimization problem they have to solve as a network-design multicommodity flow problem. In section 2 we develop a review on the literature we studied during the development of this work. In section 3 we define the problem and present the model for a single agent. Section 4 contains three different centralized cooperation mechanism. In section 5 we introduce a, to best of our knowledge, novel approach to model a decentralized cooperative mechanism. Section 6 contains the results of the simulations we have done with the models presented in the two previous section. Section 7 is a discussion about the limitations and possible extensions of the models proposed in this paper. We finish with a conclusion in section 8.

2. Literature review

The literature studying the horizontal logistics collaboration between agents can be divided in two main streams depending on whether the collaboration it is centralized or not.

2.1. Centralized cooperation

In the case of centralized systems, agents usually share in a pool all or part of the demands that they have to serve as well as the assets they have available. These assets can be vehicles in the case of transportation carriers, but also inventory levels, employees, etc. Then a central planner, taking into account all the demands and sources shared in the pool by the agents, solves the problem maximizing the total payoff of the coalition. Finally, the benefits of the cooperation are shared among the members of the coalition, usually using some allocation rule which ideally satisfies some desirable conditions in terms of "fairness", as that the resulting allocation is in the *core*, i.e., any subcoalition of the former one could obtain a higher payoff working independently [4]. A well known example of an allocation rule fulfilling this conditions is the *Shapley value* [5].

Some authors have pointed that in the centralized systems, the individual objectives of the agents are often not considered in favour of the coalition objective, what neglects the fact that the agents actually remain independent entities [6]. Therefore, these authors propose different mechanisms where both levels of objectives are considered [7] [8]. Furthermore, it is assumed in the literature studying these type of systems that agents share all their private information with the central planner, what might be unrealistic or problematic in real world cases [9] [10].

2.2. Decentralized cooperation

Some decentralized cooperative mechanism operate based on auctions systems. In that systmem, agents share the part of their demands, an the other members of the coalition bid to be the ones serving that demand, for what the obtain some benefit [11] [12]. Non-auction based decentralized systems have also been stuy [13] [14] . Nevertheless, in this works there is still a central authority which has to make some decisions. We refer the reader to [15] for a survey covering decentralized mechanism based and not based in auctions.

A collaborative system based in a inverse-optimization mechanism is the proposed in [16]. On it, the authors model a multicommodity flow problem, where agents own certain fraction of the capacity of the edges of a network. Thus, the agents can collaborate sharing the capacity of the edges with the members of the coalition, who have to pay some price for using that edges. How much capacity the agents own on each edge of the and which are their demand is assumed to be public information. Using inverse optimization, capacity exchange prices are found, in such a way that if the agents adopt that prices policy, when each agent selfishly optimizes his demands through the shared network, the resulting flow will be the same that if a centralized approach was used. However, we argue that might be difficult or even unrealistic to implement in the real work a cooperation mechanism were a central authority with full information impose the prices at which the agents can exchange assets.

In [17] a novel framework to model decentralized systems is proposed. Based in the concept of *coopetition* [18], the authors proposed a 2-stage mechanism to model the collaboration of agents in decentralized distribution systems. In the first stage, agents individually take strategic decisions, in their case, fixing their inventory levels. In the seconds stage and after all the agents have served their demands in the best way they individually can, they cooperate sharing their residual demands and inventory with the others, obtaining extra profits. They propose sufficient conditions under which a Nash equilibrium in pure strategies exist for the first stage, and for which an allocation rule in the *core* exists for the second stage.

In this work we analyse and compare several centralized systems, giving to central planner different degrees of decision power, and a non-auction-based decentralized system, where we substitute the central planner by a platform where agents can share information, and through an iterative process of optimization and information sharing, similar to what is done in [14] but without leaving any decision power to the platform, agents can achieve an equilibrium point from which no agent would be interested to unilaterally deviate from. The problem we will use as example to implement this models is a multicommodity flow problem, where each agent have to decide which edges to active on the network at certain price, to then find an efficient flow through them which maximizes their payoff.

3. Problem statement: A network design - muldicommodity flow problem

As stated in the previous section, we will use a multicommodity flow problem with a network design stage to illustrate the different cooperation mechanisms we study in this paper. In the problem we propose, a simplified version of the one studied in [3], a set of agents have to first decide which edges they want to activate in a network, with certain costs associated. Then, each have to route their commodities through the edges he has activate. The objective of the agents is to maximize the revenues generated by the served commodities and minimize the costs associated with the activation of the edges. This two subproblems are intrinsically linked: the most efficient flow an agent can find for his commodities depends on which edges he has access too, and to know which edges to activate, the agent need to know how he will route his commodities through that edges.

The agents can collaborate among them sharing the capacity they do not use in their active edges, allowing others to route their commodities through them at some price.

Let $N = \{1, ..., n\}$ be a set of agents. Let $G = (V, E)$ be a directed graph with V and E the sets of nodes and edges respectively. Let Θ be the set of commodities. Each commodity is noted by a tuple, (o, t, i), where $o \in V$ is its origin node, $t \in V$ its destiny node and i is the agent who owns it. A commodity (o, t, i) has a size of $d_{(o,t,i)}$ units and has an associated revenue $r_{(o,t,i)}$. The owner of the commodity (o, t, i) earns $d_(o, t, i) \cdot r_(o, t, i)$ if the commodity is successfully delivered from its origin to its terminal node. The commodities are unsplitable, i.e., can not be divided between different edges. Each edge $e \in E$ will be noted by a tuple, $e = (v, w, i)$, indicating that it connects the node $v \in V$ with the node $w \in V$ and that his owner is the agent $i \in N$. Note that could exist as many edges between a pair of nodes as agents exists. Each edge $e \in E$ has certain units of capacity associated q_e, and a a fixed activation cost c_e, that the owner of the edge have to pay if he wants to use the edge to route any commodity. We will note by $OE(v) \subset E$ the subset of edges whose origin is the node $v \in V$. Similarly, $IE(v) \subset E$ is the subset of edges whose terminal node is $v \in V$.

We will note by $E^i \subset E$ and $\Theta^i \subset \Theta$ the subsets of edges and commodities owned by agent i.

3.1. Single agent model

The objective of an agent $i \in N$ is to maximize his payoff, by routing his commodities, Θ^i, from their origin nodes to their terminal nodes through the edges E^i, he has activated. If the agent does not take part of the collaboration, she will not have any access to other agents edges, neither the other agents

Table 1: Summary of notation

Symbol	Meaning
N	Set of agents.
G	Network.
V	Set of nodes in G.
E	Set of edges in G.
Θ	Sets of commodities.
$\delta^+(v)$	Subset of edges in E which depart from node v.
$\delta^-(v)$	Subset of edges in E which arrive to node v.
Θ^i	Set of commodities owned by agent i.
E^i	Set of edges owned by agent i.
E_A	Subset of edges in E that are active
E_R	Subset of edges in E_A that have residual capacity.
$(o,t,i) \in \Theta$	Commodity with origin node o and terminal node t owned by agent i.
$d_{(o,t,i)}$	Size, on units, of commodity (o,t,i).
$r_{(o,t,i)}$	Revenue associated with serving commodity (o,t,i).
$e = (v,w,i) \in E$	Edge connecting node v with node w, owned by agent i.
c_e	Cost of activate edge e.
q_e	Units of capacity of edge e.
q_e^R	Units of residual capacity of edge e.

can route their commodities through his edges. Thus, the problem which agent $i \in N$ has to solve can be modelled as the following ILP, that we call P_i,

$$P_i: \quad \max \quad \sum_{(o,t,i)\in\Theta^i} \sum_{e\in\delta^+(t)\cap E^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E^i} u_{e'} \cdot c_e \tag{1}$$

subject to:

$$\sum_{e \in \delta^-(t) \cap E^i} f_e^{(o,t,i)} - \sum_{e \in \delta^+(t) \cap E^i} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta^i, \tag{2}$$

$$\sum_{e \in \delta^+(t) \cap E^i} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta^i, \tag{3}$$

$$\sum_{e \in \delta^+(t) \cap E^i} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta^i, \tag{4}$$

$$\sum_{(o,t,i) \in \Theta^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq u_e \cdot q_e, \qquad \forall\ e \in E^i, \tag{5}$$

$$\sum_{\substack{(v,w,k) \in E^i: \\ v,w \in S}} f_{(v,w,i)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta^i, \tag{6}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E^i,\ \forall\ (o,t,i) \in \Theta^i, \tag{7}$$

$$u_e \in \{0,1\}, \qquad \forall\ e \in E^i \tag{8}$$

where, $\forall\ (o,t,i) \in \Theta^i$ and $\forall\ e \in E$,

$$f_e^{(o,t,i)} = \begin{cases} 1 & \text{if } (o,t,i) \text{ is routed through } e, \\ 0 & \text{otherwise} \end{cases}$$

$$u_e = \begin{cases} 1 & \text{if } e \text{ is activated}, \\ 0 & \text{otherwise} \end{cases}$$

The objective of agent i, equation (1), is to maximize the profit generated by the commodities which arrive to their destination nodes while minimizing the cost associated to the activation of the edges. Constraint (2) ensure that, for any commodity, the flow that enters and leaves every transit nodes (neither the origin, neither the terminal node of that commodity) is equal. Constraint (3), together with (7), guarantee that each commodity is send at most only once from its origin node. Constraint (4) ensure that no commodity is send from its terminal node to any other node. Constraint (5) ensures that the capacity of edges is not exceed. Finally, constraint (6) is a subtour elimination constraint [19]. We have included it in our model because, first of all, it makes sense to assume that an agent would not want to create subtours when routing his commodities. Note that subtours could be possible without this constraint because once an edge is activated, there is no extra costs for routing more commodities through it. Furthermore, without the subtour elimination constraints, the other constraints are not sufficient to ensure that all the feasible solution of the ILP are admissible, since the flow of a commodity could contain a subtour disconneted from the origin and terminal nodes of that commodity, without impacting the objective but making the solution not valid for the problem we are modelling.

We will note by P_i^* the optimal solution of P_i which is the maximal payoff agent i can obtain by his own without cooperating with the other agents.

4. The centralized cooperation models

In the previous subsection, we have presented an ILP that models the network design - multicommodity flow problem of an agent $i \in N$ who works alone. In this section, we present different centralized cooperative systems, where the agents in N collaborative in different ways in order to find synergies among them and increase their payoffs.

In all the three different systems we propose, the way in which agents is sharing the capacity of the edges they have activate with other agents, what could allow to some agents to serve commodities in a cheaper way to some agents, and to compensate the cost of the activation of some edges to others. The three systems also have in common the way in which the agents share the benefits generated by the collaboration, what is done as follows

1. The revenues generated by any served commodity are allocated to its owner.
2. The activation cost of any active edge is paid by its owner.
3. If a commodity $(o, t, i) \in \Theta$ is routed through an edge $e = (v, w, j) \in E$ and $j \neq i$, agent i makes a side payment equal to $d_{(o,t,i)}) \cdot \frac{c_e}{q_e}$ to the agent j. In words, for each commodity sent through an edge owned by other agent, the owner of the commodity will pay to the owner of the edge the

fraction of the activation cost of that edge equivalent to the fraction of the total capacity of the edge that his commodity occupies.

In the rest of the section we introduce the three different centralized cooperative systems we propose in this work.

4.1. Full cooperation system

We start introducing the model where more power is given to the central planner. In this model, all the commodities and edges of all the agents are given to the central planner, who then has to solve the following ILP, Γ_F, which is almost identical to P_i, the ILP that an agent $i \in N$ has to solve when no cooperating. Both problems only differ in two aspects:

(a) The central planner manages the commodities and edges of all the agents, Θ and E, while without cooperation an agent i only has access to his own commodities and edges, Θ^i and E^i.

(b) The following two new constraint are added to the problem:

$$\sum_{e \in \delta^-(t) \cap E} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e \in E^j: \\ j \neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e}, \quad \forall\, (o,t,i) \in \Theta, \tag{9}$$

$$\varphi_i(\Gamma_F) \geq P_i^*, \quad \forall\, i \in N; \tag{10}$$

where

$$\begin{aligned} &\varphi_i(\Gamma_F) = \\ &= \sum_{(o,t,i) \in \Theta^i} \left[\sum_{e \in \delta^-(t) \cap E^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e \in \Theta^j: \\ j \neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \\ &+ \sum_{\substack{(o,t,k) \in \Theta: \\ k \neq i}} \left[\sum_{e \in E^i} f_e^{(o,t,k)} \cdot d_{(o,t,k)} \cdot \frac{c_e}{q_e} \right] - \sum_{e \in E^i} u_e \cdot c_e, \end{aligned} \tag{11}$$

is the payoff allocate to the agent $i \in N$ for any feasible solution of Γ_F.

The constraint (9) avoids that the central planner serves a commodity in such a way that the side payments his owner has to make are higher than the revenue he obtains for serving that commodity. Furtheremore, with constraint (10), similarly to what is done in [8], we integrate the final payoffs of the agents in the constraints of the central planner's problem. Thus, these constraints ensure that the final payoff of any agent $i \in N$, φ_i, is greater or equal than what that agent could achieve without cooperation, i.e., P_i^*. Doing so we can say that the final payoff allocation is *individually rational* [4], i.e., no single agent is interested in leaving the coalition unilaterally. [1]

In equation (11) we define how the final payoff of an agent $i \in N$, is computed: the first term is the sum of the profit created by the commodities of the agent, which for each commodity is equal to the revenue it generates when it is served minus the cost the agent has to pay to other agents if the commodity passes through edges which does not belong to him. The second sum is the payments that other agents pays to him when routing their commodities through his edges. The last term is the sum of the costs of the active edges.

4.2. Partial cooperation system

In the partial cooperation system, the central planner has to optimize once again the flow of all the commodities of all the agents through the network, maximizing the total generated revenue. Nevertheless, the decision of which edges to activate remains as an individual decision of each agent.

We propose that, in a first stage, each agent $i \in N$ solves his own P_i problem. Then, in a second stage, the same edges that are active in the optimal solutions

[1] We can extend this idea by forcing the solution resulting from the cooperation to be in the *core*, by adding constraints ensuring that any the sum of the payoffs of the members of any subcoalition is equal or greater than what that agents could obtain by leaving the grand coalition and cooperating only among them. Nevertheless, doing it will increase the complexity of the models and we consider that it does not add value to the paper.

of each agent individual ILP, P_i^*, are the agents that the central planner can use to route the commodities through the network.

Let $E_A \subset E$ be the subset of edges such that $\forall e = (v, w, i) \in E_A$, e is active in P_i^*. Then the ILP, Γ_P, that the central planner has to solve in the partial cooperation system is

$$\Gamma_P : \max \quad \sum_{(o,t,i)\in\Theta} \sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} \tag{12}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E_A} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o, t\},\ \forall\ (o, t, i) \in \Theta, \tag{13}$$

$$\sum_{e\in\delta^+(o)\cap E_A} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o, t, i) \in \Theta, \tag{14}$$

$$\sum_{e\in\delta^+(t)\cap E_A} f_e^{(o,t,i)} = 0, \qquad \forall\ (o, t, i) \in \Theta, \tag{15}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e \qquad \forall\ e \in E_A, \tag{16}$$

$$\sum_{\substack{(v,w,k)\in E_A:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o, t, i) \in \Theta, \tag{17}$$

$$\varphi_i(\Gamma_P) \geq P_i^*, \qquad \forall\ i \in N \tag{18}$$

$$\sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} -$$

$$- \sum_{\substack{e\in E_A^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \geq 0, \qquad \forall\ (o, t, i) \in \Theta, \tag{19}$$

$$f_e^{(o,t,i)} \in \{0, 1\}, \qquad \forall\ e \in E_A, \forall\ (o, t, i) \in \Theta, \tag{20}$$

$$\tag{21}$$

Note that, consequently to the above introduced,this model differs from the previously presented in this work in the absence of decisions variables for the activation of the edges, since this decision is previously make by the agents and not left to the central planner. Also, as in the *full cooperation system*, we include the constraints (18) and (19) which are equivalent to the constraints (9) and (10). In this case, because the absence of the decision variables u_e in the model, we redefine the payoff allocate to each agent $i \in N$ as follows

$$\begin{aligned}
&\varphi_i(\Gamma_P) = \\
&= \sum_{(o,t,i)\in\Theta^i} \left[\sum_{e\in\delta^-(t)\cap E_A} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E^j: \\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \right] + \\
&+ \sum_{\substack{(o,t,k)\in\Theta: \\ k\neq i}} \left[\sum_{e\in E_A^i} f_e^{(o,t,k)} \cdot d_{(o,t,k)} \cdot \frac{c_e}{q_e} \right] - \sum_{e\in E_A^i} c_e.
\end{aligned} \tag{22}$$

4.3. Residual cooperation

Among the three centralized systems we propose in this paper, the *residual cooperation system*, which is inspired in the already mentioned framework presented in [17], is the one where less decision power if left to the central planner.

As in the partial cooperation system, every agent $i \in N$ will individually solve his corresponding P_i problem, deciding which edges to activate and how to route his commodities through that edges. But this time, the agents actually implement this solution, and only inform the central planner about which commodities they have not been able to route in an efficient way, as well as which active edges still have some free capacity. With all this information, the central planner routes in a efficient way the residual commodities of the agents, Θ_R, through the residual capacities of the active edges,E_R. For each edge $e \in E_R$ we will note by q_e^R the residual capacity of that edge, i.e., the units of capacity that the owner of the edge has not used to route his own commodities. The ILP the central planner has to solve in this residual cooperation system, Γ_R, is equivalent to Γ_P, where the commodities and edges the central planner manages are Θ_R and E_R instead of Θ and E_A. Also, the central planner can not route commodities through and edge $e \in E_R$ exceeding the residual capacity of that edge, Q_e^R, so constraint (16) is substituted by

$$\sum_{(o,t,i)\in\Theta_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e^R, \quad \forall\, e \in E_R. \tag{23}$$

In this system, it is guaranteed by design of the collaboration mechanism that each agent end up the cooperation with a payoff greater or equal than the payoff they could obtain by their own, as they always first implement the best solution they can achieve without cooperating, and therefore they can only increase their payoffs with respect to that value. Therefore, constraints equivalent to (19) are not included. Under the residual cooperation system we define the final payoff of an agent $i \in N$ as

$$\begin{aligned}
&\varphi_i(\Gamma_R) = \\
&= \sum_{(o,t,i)\in\Theta_R^i} \left[\sum_{e\in\delta^-(t)\cap E_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{\substack{e\in E_R^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \right] + \\
&+ \sum_{\substack{(o,t,k)\in\Theta_R:\\ k\neq i}} \left[\sum_{e\in E_R^i} f_e^{(o,t,k)} \cdot d_{(o,t,k)} \cdot \frac{c_e}{q_e} \right] - P_i^*
\end{aligned} \tag{24}$$

In table 2 we summarize the differences between the three different centralized cooperative systems we have proposed in this section, indicating which decisions are left to the agents and which are made by the central planner. Also, the complete formulation of the ILPs for the full and residual cooperation systems can be found in the Appendix A.

5. A decentralized cooperation mechanism for two agents

In this section we propose a decentralized cooperation mechanism for two agents, where all the decisions are left to the agents, who make use of an *infor-*

Table 2: Here goes the caption

		Coop. centralized systems		
		Full	Partial	Residual
Agent	Activate edges	No	Yes	Yes
	Route flow	No	No	Yes
Central planner	Activate edges	Yes	No	No
	Route flow	Yes	Yes	Yes*

* Only the residual commodities through the residual capacities of the active edges.

mation platform to interact with each other.

The idea behind this mechanism is that the agents exchange information in an iterative process, informing to the other agent about the residual capacity they have in their own active edges, allowing the other agent to use that residual capacity to route his commodities at some price.

Before go in more detail about how the cooperation mechanism works, we introduce how the information platform works. Each agent $i \in N$ has two different "pools" to share information:

Shared edges: Agent i can share a subset of his active edges which still have some residual capacity, $\widehat{E}^i_R \subseteq E^i_R$. He also has to share how many units of residual capacity has each edge $e \in \widehat{E}^i_R$, q^R_e, and which is the cost of each of that units, $\frac{c_e}{q_e}$

Demanded edges: If agent j has previously shared a set of edges in the platform, agent i can make use that edges when routing his commodities. If agent i actually decides to route a subset of his commodities, $\widehat{\Theta}^i \subset \Theta^i$ through some of that edges, he has to share in the information platform:

1. How much units of capacity he requires (or demands) in total on each edge e shared by agent j, what we note by q^D_e.
2. For each commodity $(o,t,i) \in \widehat{\Theta}^i$, a set $L^{(o,t,i)}$, containing the edges shared by agent j through which the commodity would be routed.

Also which would be the side payment he would make to agent j if finally all the edges e in $L^{(o,t,i)}$ are active and he can make use of q_e^D units of capacity in each of them. That side payment is computed as $p_{L^{(o,t,i)}} = \sum_{e \in L^{(o,t,i)}} d_{(o,t,i)} \cdot \frac{c_e}{q_e}$. We note by $\mathcal{L}^i$ the set containing all $L^{(o,t,i)}$, $\forall\ (o,t,i) \in \widehat{\Theta}^i$.

Making use of the information platform just introduced, both agents iteratively solve an ILP. In each iteration both agents, sequentially and not in parallel, decide which edges to activate, and how to route their commodities through the network, as they do in P_i, but this time, they can route their commodities, through the edges shared in the platform by the other agent with the corresponding costs and respecting the available capacities on that edges. Furthermore, when deciding if activate or not an edge, each agent does not only have to account for the activation cost, but also for the side payments the other agent is willing to make to him if he activates certain combinations of edges and leaves on them certain residual capacities. The ILP, Υ_i, agent $i \in N$ has to solve in each iteration is the following:

$$\begin{aligned}\Upsilon_i: \quad \max \quad & \sum_{(o,t,i)\in\Theta^i} \sum_{e\in\delta^-(t)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E^i} u_e \cdot c_e \quad - \\ & - \sum_{(o,t,i)\in\Theta^i} \sum_{\substack{e\in\widehat{E}_R^j:\\ j\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \\ & + \sum_{\substack{L^{(o,t,j)}\in\mathcal{L}^j:\\ j\neq i}} p_{L^{(o,t,j)}} \cdot t_{L^{(o,t,j)}} \qquad (25)\end{aligned}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} = 0, \quad \forall\ z \in V \setminus \{o,t\}, \quad \forall\ (o,t,i) \in \Theta^i, \qquad (26)$$

$$\sum_{e\in\delta^+(o)\cap(E^i\cup\widehat{E}_R^j)} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta^i, \qquad (27)$$

$$\sum_{e \in \delta^+(t) \cap (E^i \cup \widehat{E}^j_R)} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta^i, \tag{28}$$

$$\sum_{(o,t,i) \in \Theta^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq u_e \cdot q_e, \qquad \forall\ e \in E^i, \tag{29}$$

$$\sum_{(o,t,i) \in \Theta^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e^R, \qquad \forall\ e \in \widehat{E}^j_R, \tag{30}$$

$$\sum_{\substack{(v,w,k) \in E^i \cup \widehat{E}^j_R: \\ v,w \in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta^i, \tag{31}$$

$$\sum_{(o,t,i) \in \Theta^i} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq (q_e - q_e^D) + M(1 - b_e), \qquad \forall e \in L^{(o,t,j)}, \ \forall L^{(o,t,j)} \in \mathcal{L}^j, \tag{32}$$

$$b_e \leq u_e, \qquad \forall e \in L^{(o,t,j)}, \ \forall L^{(o,t,j)} \in \mathcal{L}^j, \tag{33}$$

$$t_{L^{(o,t,j)}} \leq 1 - (|L^{(o,t,j)}| - \sum_{e \in L^{(o,t,j)}} b_e), \qquad \forall L^{(o,t,j)} \in \mathcal{L}^i \tag{34}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E^i,\ \forall\ (o,t,i) \in \Theta^i, \tag{35}$$

$$u_e \in \{0,1\}, \qquad \forall\ e \in E^i, \tag{36}$$

$$b_e \in \{0,1\}, \qquad \forall e \in L^{(o,t,j)}, \ \forall L_{(o,t,j)} \in \mathcal{L}^j, \tag{37}$$

$$t_{L^{(o,t,j)}} \in \{0,1\}, \qquad \forall L^{(o,t,j)} \in \mathcal{L}^j, \tag{38}$$

where M is a very large value.

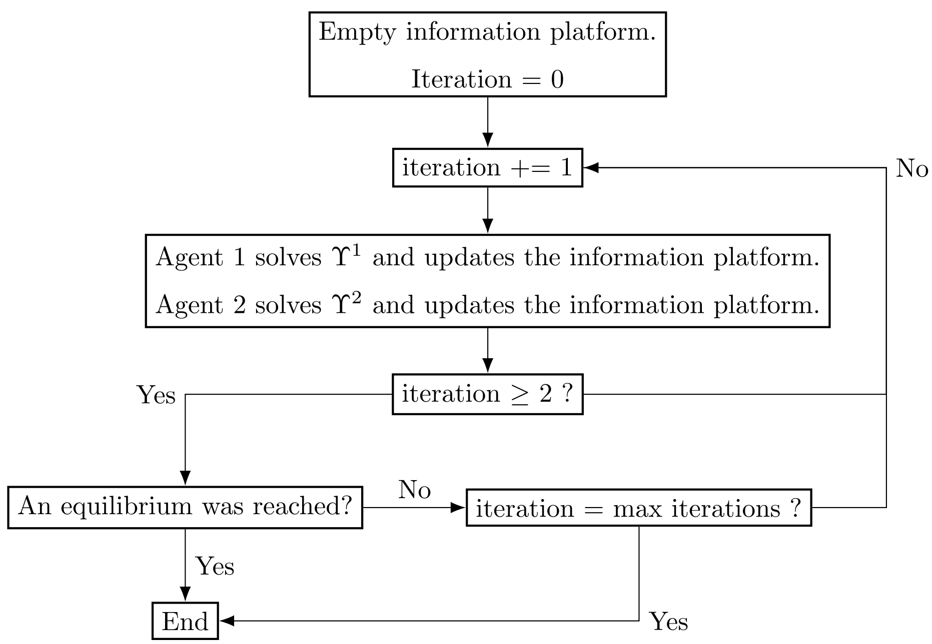
In Υ_i, the first term of the objective function of the agent i, equation (25), is the sum of the revenue generated by the commodities which are served. The second term is the sum of the costs of the edges he has activate. The third term is the sum of the side payments he has to do when routing commodities through the edges shared by the other agent. Finally, the fourth term is the sum of the side payments he receives from the other agent for making use of the edges he has shared in the previous iteration, if they remain available in the conditions the other agent has demanded.

Constraints (26), (27), (28) and (31) are equivalent to constraints (2),(3),(4) and (6) in P_i, with the difference that now the agent i can use the edges shared in the information platform by the agent j, $\widehat{E}^j_R$. Constraint (29) is identical to constraint (5) and constraint (30) is an extension of it, ensuring that the agent does not exceed the residual capacity of the edges shared by agent j when routing his commodities.

Constraints (32), (33) and (34) together with the "technical" decision variables (37) and (38), are related with the side payments agent i receives from agent j. Recall that each $L^{(o,t,j)} \in \mathcal{L}^j$ is a set which contains the edges of agent i that agent j has indicated in the previous iteration he would like to use to route commodity (o, t, j). He has also indicates that if and only if for every $e \in L^{(o,t,j)}$, q^R_e units of capacity are available to him, he would make a side payment to agent i equal to $p_{L^{(o,t,j)}}$. Thus, in order to get the side payment $p_{L^{(o,t,j)}}$, the technical decision variable $t_{L^{(o,t,j)}}$ must be equal to 1. Constraints (34) and (38) ensures that $t_{L^{(o,t,j)}}$ only can be 1 if all the technical decision variables b_e for $e \in L^{(o,t,j)}$ are also equal to 1. Finally, constraints (32) making use of the *big M* coefficient, and (33), ensures that if b_e is equal to one, agent i must leave at least q^D_e units of residual capacity on that edge and the edge e must be active. Note that these constraints are too restrictive. Agent j might be planning to route 2 or more commodities through the edge e, and even if agent i left less than q^R_e units of capacity available, he might be possible to route, not all, but some of that commodities through it. However, these constraint assume that none of the commodities could be routed and therefore, any side payment associated with a commodity passing through that edge would be done. We argue that capture with exactitude this behaviour would make the model too complex, as the problem of deciding which commodities can still be routed and which can not is a *knapsack problem*, which has been proven to be NP-complete [20].

In figure 1 we present a flow chart illustrating the structure of these cooperation mechanism. In the first iteration, as the information platform is empty, agent 1 is in practice solving P^1. Indeed, if the no information was shared in the platform, $\Upsilon^i = P^i$. For the same reason, in the first iteration agent 2 does not expect any side payments from agent 1 (he have not got yet the opportunity of demanding any edges, since no edges were shared for agent 2 in the platform yet). The iterative process continues until an equilibrium is found or the maximum number of iteration allowed is reached or an equilibrium is found. We define an *equilibrium* as the situation were the solutions of Υ^1 and Υ^2 found in the iteration h are exactly the same than the solutions found in the iteration $h-1$. This is equivalent to the definition of the *Nash equilibrum*: given the actions of the others, an agent can not individually improve their payoff by changing his strategy [4]. If the maximum number of iteration is reached without equilibrium, the mechanism considers that no solution was found.

Figure 1: Flowchart of the decentralized cooperation mechanism for two agents



6. Results

In order to analyse the performance of the different models discussed in the work, we tested the 4 different cooperation mechanism over 20 instances of the network design - multicommodity flow problem introduced in section 3, comparing them with the no cooperation system, by simply solving the single agent model (presented also in section 3) and summing the payoff of all the agents.

The models were implemented in Python 3.8 and solved using CPLEX 20.1, by means of the DOCplex library. The tests were performed in an Intel(R) Core(TM) i7-5500U CPU @ 2.40GHz with 8GB of RAM.

Both the instances and the code are available upon request.

6.1. Instance generation

We have created 20 instances for the network design - multicommodity flow problem introduced in section 3. We have decided to randomly generate them and not to use them for two main reasons. First of all, we could not found on the Internet any set of test instances matching exactly our problem characteristics. Secondly, we found sets of test instances for multicommodity flow problems [21] and we considered to adapt them to our problem. Nevertheless, all this instances were too big, and since we are using an exact ILP solver we considered that to use very complex instances was not appropriate for this work.

Therefore, we randomly generate 20 instances, all sharing the following characteristic: a complete and directed graph $G(V, E)$ with 7 nodes, ($|V| = 7$), is generated. For each edge $e \in E$, we randomly select its activation cost, c_e, from a uniform distribution $U[3, 6]$. Also for each commodity $(o, t, i) \in \Theta$ we randomly select its size and revenue per unit from two uniform distributions, $U[0, 5]$ and $U[1, 2]$ respectively. Finally, we differentiate 4 classes among the 20 problem cases, each containing 5 instances, which differ in the number of agents and the capacity of the edges considered. Two of the four classes consider cases with 2 agents, and the other two, with 5. At the same time, among the two instances with the same number of agents, in one class the capacity of the edges is randomly selected from a uniform distribution $U[2, 8]$ (LOW capacity), and for the other class from a uniform distribution $U[5, 12]$ (HIGH capacity). The name of each instance indicates to which class they belong: for example, the instance "`2_LOW_0`" has 2 agents and the capacities of the edges were selected from the $U[2, 8]$ distribution. The last number is just a index to differentiate the five instances inside the same class (from 0 to 4).

6.2. Parameter selection

Only in the decentralize iterative cooperation mechanism there is a parameter, and only one, which is the maximum number of iterations allowed. In our test we set it equal to 100. Furthermore, we have also model this mechanism in our simulations in such a way that the agents always share all their edges with residual capacity in the information platform, i.e., $\widehat{E}^i_R = E^i_R \; \forall \; i \in N$.

6.3. Comparision 3 centralized mechanisms

In table [REF!] we presented the results of the three different centralized models tested over the 10 instances with 5 agents. Each cell represents the sum of the final payoffs (total payoffs) of all the agents. The first column corresponds to the no cooperation system, were the agents do not collaborate at all. Columns 2,4 and 6 corresponds to the total payoffs for the full, partial and residual cooperation systems, and columns 3,5,7 to the percentage of improvement of each of that cooperation systems with respect to the no cooperation case.

Furtheremore, in table [REF!] the running times of each mechanism are reported. We want to remark that this running times must be taken as references and not exactly values. This is because for the 3 centralized systems, the computations of the best solution of each agent when no cooperating were performed sequentially and not in parallel. It could be argued that in a real word case, each agent would make these computations by his own, at least in the residual cooperation system, where the central planner does not need to have access to all the information of each agent, neither to check if the payoff they have reported as the best they can obtain without cooperation is true (as it does not take part of the constraints).

6.4. Comparision centralized mechanisms and iterative one, all for 2 agents

In this subsection we present the results for the instances where only 2 agents are involved.

First of all, in table 3 we compare the total payoffs obtained from the decentralized iterative cooperation mechanism depending on which agent goes first in the iterative process. The first two columns are the total payoff associated to each of the two possible orders, and the third column the percentage of difference between the two. The number of iterations required to reach an equilibrium in each case are presented in columns 3 and 4, and column 5 is the difference on number of iterations required between both orders.

Table 3: Analysis of the impact of the order of the agents in the iterative mechanism

	Order1_payoff	Order2_payoff	Dif%	Order1_it	Order2_it	Dif_it
2_low_0	25.0	25.0	0.00	4.0	3.0	1.0
2_low_1	21.0	21.0	0.00	3.0	3.0	0.0
2_low_2	17.0	17.0	0.00	3.0	3.0	0.0
2_low_3	15.0	15.0	0.00	4.0	4.0	0.0
2_low_4	27.0	26.0	3.70	3.0	3.0	0.0
2_high_0	73.0	72.0	1.37	3.0	3.0	0.0
2_high_1	70.0	69.0	1.49	3.0	3.0	0.0
2_high_2	65.0	63.0	3.08	3.0	3.0	0.0
2_high_3	68.0	67.0	1.47	4.0	4.0	0.0
2_high_4	74.0	73.0	1.35	3.0	3.0	0.0

Tables 4, and 5 are equivalent to tables [REF!], and [REF!] with the difference that the decentralized iterative cooperation mechanism is now included. For this mechanism, we have reported in table 4 the lower total payoff among the two possible ones, depending in the order in which the agents participate on it, i.e., for each instance the minimum among the values in colum 1 and 2 of table 3

Table 4: Comparision of the payoff obtained by each cooperative mechanism for instances with two agents

	No coop	Full	%	Partial	%	Residual	%	Iterative	%
2_low_0	22.0	47.0	113.64	35.0	59.09	22.0	0.00	25.0	13.64
2_low_1	19.0	43.0	126.32	29.0	52.63	20.0	5.26	21.0	10.53
2_low_2	15.0	39.0	160.00	29.0	93.33	17.0	13.33	17.0	13.33
2_low_3	11.0	34.0	209.09	19.0	72.73	11.0	0.00	15.0	36.36
2_low_4	23.0	45.0	95.65	30.0	30.43	25.0	8.70	26.0	13.04
2_high_0	69.0	100.0	44.93	82.0	18.84	69.0	0.00	72.0	4.35
2_high_1	67.0	105.0	56.72	83.0	23.88	67.0	0.00	69.0	2.89
2_high_2	61.0	92.0	50.82	71.0	16.39	63.0	3.28	62.0	1.64
2_high_3	62.0	97.0	56.45	77.0	24.19	63.0	1.61	67.0	8.06
2_high_4	70.0	105.0	50.00	83.0	18.57	72.0	2.86	73.0	4.29

Table 5: Comparision of the running times of each cooperative mechanism for instances with two agents

	Full	Partial	Residual	Iterative
2_low_0	9.85	3.74	2.44	6.12
2_low_1	9.45	2.96	2.45	6.45
2_low_2	6.32	2.49	2.13	3.47
2_low_3	5.16	2.37	2.12	5.37
2_low_4	5.23	3.48	2.85	4.86
2_high_0	1088.24	61.73	60.51	237.08
2_high_1	415.45	46.60	41.79	152.41
2_high_2	221.80	56.95	55.29	137.38
2_high_3	241.28	62.92	60.02	130.08
2_high_4	174.32	49.51	45.84	59.72

7. Discussion

In this section we comment different aspects about the mechanism we have studied in this work as well as the results we have obtained with the simulations, as well as possible extensions for them and open questions for future research.

7.1. Results discussion

Tables 4 show us how the decentralized iterative cooperation mechanism can not compete with the full and partial cooperation systems in terms of total payoffs. Furthermore, the partial cooperation systems is also faster in terms of running time. In the other hand, this mechanism is able to find better solutions than the residual cooperation system in nine of the ten instances in exchange for slightly longer running times. Having into account that the among of information agents require to share in the decentralized mechanism is less than in the centralized residual system, were agents share their residual edges but also their residual commodities and leave the routing decisions to the central planner, we consider argue that if the running times are not a limitation, the decentralized iterative system is superior to the centralized residual cooperation system. [2]

We can observe in table 3 that even if the order of the agents does not appear to affect in general to the number of iteration necessary to reach an equilibrium (with the exception of instance "`2_LOW_0`"), it can have an impact in the total payoff obtained. This seems specially relevant in the instances were the capacity of the edges is higher. A first hypothesis to justify this could be that, as already stated before, a higher capacity in the edges of the graph could mean a bigger number of possibilities to cooperate, making the order in which the agents participate in the process more relevant. Nevertheless, the number of iterations necessary to reach an equilibrium does not grows with the capacity

[2] Of course, the decentralized system is only comparable with the centralized ones when only 2 agents are involved in the problem.

of the edges, and therefore more experiments would be necessary in order to find a reason for this behaviour.

Another interesting results is that an equilibrium was always found in the 10 studied instances, and that the number of iterations required to find is is relatively constant, varying between 3 and 4 iterations in all the cases.

7.2. Possible extensions and future work

Several extensions can be done in order to make this work more complete. Probably the most obvious is to extend the decentralized iterative cooperation mechanism to more than 2 agents. Nevertheless, a direct extension could derive in requiring more iterations in order to reach an equilibrium, or even making that equilibrium unlikely to exist. Also, might be possible that if the number of agents increases, the relevance of the order in which that agents participate in the iterative process increases too. In any case, more work is required in order to confirm or not the above hypothesis.

Another possibility we considered and finally we did not include in this work because of time limitations, is to study a version of the decentralized iterative cooperation mechanism where the agents can not change their decision, both respect to the design of their network neither the flow of their commodities, if they have already share information in the platform related with them. For example, if an agent has shared an edge with residual capacity in the previous iteration and the other agent has requested to use it, he would not be allowed to change his mind and deactivate it in future iterations. We hypothesize that this could reduce the number of iterations required to reach an equilibrium and maybe make the extension to more than 2 agents easier. In the other hand,a system like this could generate less quality solutions from the point of view of the whole coalition and increase the relevance of the order in which the agents participate in the iterative process. Once again, more work and further experiments are necessary to can confirm these hypothesis.

In the other hand, the centralized cooperation systems proposed in this work could be also improved. For example, in the partial cooperation systems, the agents might follow different or even better strategies for deciding which edges to activate than simply solve their individuals optimization problem, P^i for $i \in N$, and choosing the edges that are activate in the respective solutions. Also, more constraints could be added to the three centralized cooperation models, in order to make the solutions not only individually rational, but to force them to be in the core. Other open question is which solution should selected in case that exist multiple optimal solutions for the problem that the central planner has to solve in any of these three mechanism. For instance, multiobjectives approaches could be investigated.

8. Conclusion

In this work we have proposed three difference centralized cooperation systems and a, to the best of our knowledge, novel decentralized cooperation system for two agents. We have model these systems to solve a network design - multicommodity flow problem. Then we have simulate a set of instances to test our models, finding that the decentralized systems is only competitive with the centralized system were the central planner has less decision power, and outperformed by the other two. Nevertheless, the small amount of information agents need to share in the decentralized system might be a reason to dedicate future work to extend it to more than two agents, as well as study different variations of it.

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Appendix A. ILP's

Appendix A.1. ILP to be solved by the central planner in the centralized full cooperation system.

$$\Gamma_F : \max \qquad \sum_{(o,t,i)\in\Theta} \sum_{e\in\delta^-(t)\cap E} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} - \sum_{e\in E} u_e \cdot c_e \tag{A.1}$$

subject to:

$$\sum_{e\in\delta^-(z)\cap E} f_e^{(o,t,i)} - \sum_{e\in\delta^+(z)\cap E} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i)\in\Theta, \tag{A.2}$$

$$\sum_{e\in\delta^+(o)\cap E} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i)\in\Theta, \tag{A.3}$$

$$\sum_{e\in\delta^+(t)\cap E} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i)\in\Theta, \tag{A.4}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq u_e \cdot q_e, \qquad \forall\ e\in E, \tag{A.5}$$

$$\sum_{\substack{(v,w,k)\in E:\\ v,w\in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S\subset V,\ \forall\ (o,t,i)\in\Theta, \tag{A.6}$$

$$\sum_{e\in\delta^-(t)\cap E} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} -$$

$$- \sum_{\substack{e \in E^j: \\ j \neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \geq 0, \qquad \forall\ (o,t,i) \in \Theta, \tag{A.7}$$

$$\varphi_i(\Gamma_F) \geq P_i^*, \qquad \forall\ i \in N, \tag{A.8}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E,\ \forall\ (o,t,i) \in \Theta, \tag{A.9}$$

$$u_e \in \{0,1\}, \qquad \forall\ e \in E. \tag{A.10}$$

Appendix A.2. ILP to be solved by the central planner in the centralized residual cooperation system.

$$\Gamma_R : \max \qquad \sum_{(o,t,i) \in \Theta_R} \sum_{e \in \delta^-(t) \cap E_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} \tag{A.11}$$

subject to:

$$\sum_{e \in \delta^-(z) \cap E_R} f_e^{(o,t,i)} - \sum_{e \in \delta^+(z) \cap E_R} f_e^{(o,t,i)} = 0, \qquad \forall\ z \in V \setminus \{o,t\},\ \forall\ (o,t,i) \in \Theta_R, \tag{A.12}$$

$$\sum_{e \in \delta^+(o) \cap E_R} f_e^{(o,t,i)} \leq 1, \qquad \forall\ (o,t,i) \in \Theta_R, \tag{A.13}$$

$$\sum_{e \in \delta^+(t) \cap E_R} f_e^{(o,t,i)} = 0, \qquad \forall\ (o,t,i) \in \Theta_R, \tag{A.14}$$

$$\sum_{(o,t,i) \in \Theta_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \leq q_e^R \qquad \forall\ e \in E_R, \tag{A.15}$$

$$\sum_{\substack{(v,w,k) \in E_R: \\ v \in S, w \in S}} f_{(v,w,k)}^{(o,t,i)} \leq |S| - 1, \qquad \forall\ S \subset V,\ \forall\ (o,t,i) \in \Theta_R, \tag{A.16}$$

$$\sum_{e \in \delta^-(t) \cap E_R} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r_{(o,t,i)} -$$

$$- \sum_{\substack{e \in E_R^j: \\ j \neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \geq 0, \qquad \forall\ (o,t,i) \in \Theta_R, \tag{A.17}$$

$$f_e^{(o,t,i)} \in \{0,1\}, \qquad \forall\ e \in E_R, \forall\ (o,t,i) \in \Theta_R. \tag{A.18}$$