**Unplitable demands maximizing revenue associated to delivered demands**

$$\text{maximize} \quad \sum_{(o,t,i)\in\Theta} \left[ \sum_{e\in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r - \sum_{\substack{e=(v,w,j)\in E: \\ j\neq i}} f_{(v,w,j)}^{(o,t,i)} \cdot \frac{c_e}{q_e} \right] \tag{1a}$$

$$\text{subject to} \quad \sum_{e\in IE(z)} f_e^{(o,t,i)} - \sum_{e'\in OE(z)} f_{e'}^{(o,t,i)} = 0, \qquad \forall\, z \in V \setminus \{o,t\},\ \forall (o,t,i)\in\Theta \tag{1b}$$

$$\sum_{e\in OE(o)} f_e^{(o,t,i)} \le d_{(o,t,i)}, \quad \forall\, (o,t,i)\in\Theta \tag{1c}$$

$$\sum_{e\in OE(t)} f_e^{(o,t,i)} = 0, \qquad \forall\, (o,t,i)\in\Theta \tag{1d}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \le q_e^*, \qquad \forall\, e\in E \tag{1e}$$

$$\sum_{v\in S}\sum_{w\in S} f_{(v,w)}^d \le |S|-1, \quad \forall\, S\subset V,\ \forall\, d\in\Theta \tag{1f}$$

$$f_e^{(o,t,i)} \in \mathbb{N}_0, \qquad \forall\, e\in E,\ \forall (o,t,i)\in\Theta \tag{1g}$$

$$Partial - 1 : \max \quad \sum_{(o,t,i)\in\Theta} \left[ \sum_{e\in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r \right] \tag{2a}$$

(2b)

$$
\begin{aligned}
\text{subject to} \sum_{e \in IE(z)} f_e^{(o,t,i)} - \sum_{e' \in OE(z)} f_{e'}^{(o,t,i)} &= 0, && \forall\ z \in V \setminus \{o,t\}, \\
&&& \forall\ (o,t,i) \in \Theta, \quad (2c) \\
\sum_{e \in OE(o)} f_e^{(o,t,i)} &\leq 1, && \forall\ (o,t,i) \in \Theta, \quad (2d) \\
\sum_{e \in OE(t)} f_e^{(o,t,i)} &= 0, && \forall\ (o,t,i) \in \Theta, \quad (2e) \\
\sum_{(o,t,i) \in \Theta} f_e^{(o,t,i)} \cdot d_{(o,t,i)} &\leq u_e \cdot q_{(o,t,i)}, && \forall\ e \in E, \quad (2f) \\
\sum_{v \in S} \sum_{w \in S} f_{(v,w)}^{(o,t,i)} &\leq |S| - 1, && \forall\ S \subset V, \\
&&& \forall\ (o,t,i) \in \Theta, \quad (2g) \\
f_e^{(o,t,i)} &\in \{0,1\}, && \forall\ e \in E, \\
&&& \forall\ (o,t,i) \in \Theta, \quad (2h) \\
&&& (2i)
\end{aligned}
$$

Constraint (1e) ensures that the commoditiese sent through an edge doesn't exceed the total capacity available in that edge (the sum of the capacities each agent has for that edge).

Constraint (1f) is a subtour elimination constraint.

In this model, $e \in E$ is a tuple (v,w,i) for $v, w \in N$ and $i \in N$ (multiple agents can have an active edge between the same two nodes).

**Unsplitable demand maximizing demands served**

$$\text{maximize} \quad \sum_{(o,t,i)\in\Theta} \sum_{e\in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r \tag{3a}$$

$$\text{subject to} \quad \sum_{e\in IE(z)} f_e^{(o,t,i)} - \sum_{e'\in OE(z)} f_{e'}^{(o,t,i)} = 0, \qquad \forall\, z \in V \setminus \{o,t\},\ \forall (o,t,i) \in \Theta \tag{3b}$$

$$\sum_{e\in OE(o)} f_e^{(o,t,i)} \le d_{(o,t,i)}, \qquad \forall\, (o,t,i) \in \Theta \tag{3c}$$

$$\sum_{e\in OE(t)} f_e^{(o,t,i)} = 0, \qquad \forall\, (o,t,i) \in \Theta \tag{3d}$$

$$\sum_{(o,t,i)\in\Theta} f_e^{(o,t,i)} \le q_e^*, \qquad \forall\, e \in E \tag{3e}$$

$$\sum_{v\in S} \sum_{w\in S} f_{(v,w)}^d \le |S| - 1, \qquad \forall\, S \subset V,\ \forall\, d \in \Theta \tag{3f}$$

$$f_e^{(o,t,i)} \in \mathbb{N}_0, \qquad \forall\, e \in E,\ \forall (o,t,i) \in \Theta \tag{3g}$$

An extra contraint might be added to this model:

$$\sum_{\substack{e=(v,w,j)\\ \in E:\ z\neq i}} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot \frac{c_e}{q_e} \le$$

$$\le \sum_{e\in OE(o)} f_e^{(o,t,i)} \cdot d_{(o,t,i)} \cdot r + \sum_{\substack{e=(v,w,j)\in E:\\ j\neq i}} f_{(v,w,j)}^{(o,t,i)} \cdot \frac{c_e}{q_e} \quad \forall\, (o,t,i) \in \Theta$$

This will ensure that any agent get loss from the cooperation. Similar effect to maximize the profit from each commodity