Positron Emission Tomography and β_+ Decay Source Localization

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Contents

1 Introduction

Positron Emission Tomography (PET) is a functional imagining technique in medicine. It is used in both medical and research settings to analyse bodily functions and perform diagnoses in a non-invasive manner. In medicine, radiotracers (radiation sources) are introduced into the body and the incident radiation from these sources is captured by a PET scanner. The series of experiments described in this report utilize a simplified version of this technology for the purposes of instruction on topics such as tomography, radiation, and data analysis.

The objective of this series of experiments is to determine, as accurately as possible, the location and intensity of β_+ sources within an area using PET. These experiments first aim to determine the resolution of the apparatus used in terms of a source's location, intensity, and distance from other sources. Resolution will allow us to ultimately locate an unknown number of sources in unknown locations. The PET data will also allow us to determine the relative intensity of the unknown sources.

2 Calibration and Resolution

2.1 Experimental Setup

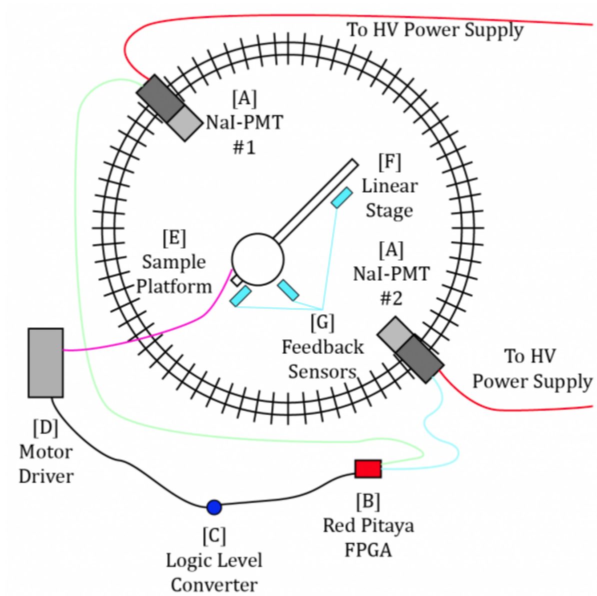


Figure 1: Diagram of the experimental setup. [A] are the PMT+NaI detectors. The line intersecting these two detectors forms the Line of Response (LoR). [B] and [C] are electronics executing the code which controls the motor driver [D]. Radiation sources are placed on the sample platform [E] and moved through the LoR by the linear stage [F].

The apparatus is constructed such that a rotating plastic disc moves one dimensionally forward, in a direction perpendicular to the Line of Response (LoR) of the PMT+NaI detectors. Sources are placed on the rotating disc. In order to determine the resolution of the apparatus, a single source was placed in the centre of the rotating disc. "A run" here signifies an entire movement of the disc across the LoR. The data extracted from a run is in the form of a one-dimensional numpy array, which we call a "strip". Each datapoint within the numpy array corresponds to the number of counts detected by the PMT+NaI detectors. We call these counts the "intensity" of incident positrons.

In order to determine the spatial resolution of the apparatus, three runs were done with different step size settings. Insodoing, we aimed to determine two things:

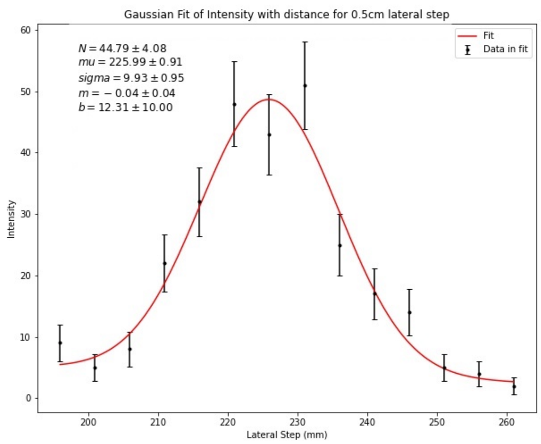
1. The behaviour of coincidence count as the source moves through the LoR.
2. How close two sources can come together before they are no longer distinct.

The step sizes were 0.5 cm, 0.1 cm, and 2.05 cm. All other parameters were held constant. The parameters were held constant at the following values: The time per slice was 15 seconds and the number of angles was two (since the pitaya did not allow for single angle inputs). The threshold was held at 0.10-0.50 (meaning at a 409V-1023V trigger threshold).

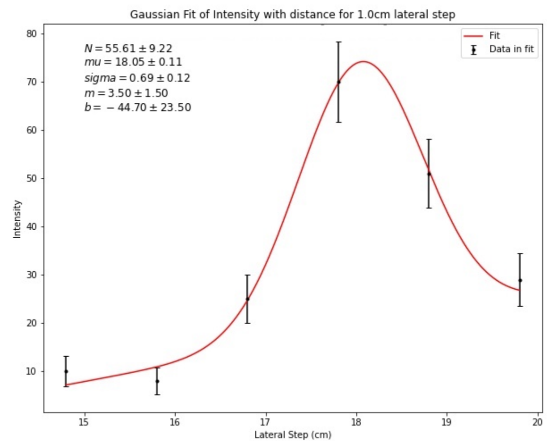
2.2 Intensity Peak Fitting with FWHM

Below (figure 2) are graphs of intensity versus distance as the source moves from one side of the LoR to the other (note that the first graph demonstrates step sizes in millimeters). The graphs were fitted with a Gaussian function. It is expected for the data to follow a Gaussian due to an influence of several random variables, including the subtle randomness of the pulse values created by the PMT+NaI detector. Other randomness may be due to differences in illumination intensity of the NaI crystal as it emits photons within a range of energies.

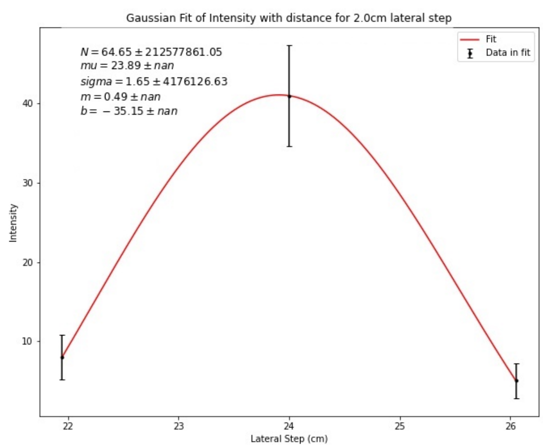
Note that the 2.05 cm fit only has three values. This makes Gaussian fitting of the intensity values difficult, since the lateral step basically jumps over the detection area of the detectors. The surface area of the detectors is a circle with a diameter of 6.0±0.1cm, measured with a meter rule. This means that any step size larger than 6.0cm will not show any Gaussian fit, and anything higher than its radius will result in not enough data to fit properly. Thus, all measurements must be done with a lateral step size of 1.0cm or smaller.



(a) Gaussian fit of Intensity with 0.5 cm lateral step.



(b) Gaussian fit of Intensity with 1.0 cm lateral step.



(c) Gaussian fit of Intensity with 2.05 cm lateral step.

Figure 2: Resolution peaks.

A table (table 1) of full width at half maximum (FWHM) values for each of the three fits is shown. We see that as the lateral step size decreases, the FWHM values also decrease. This suggests higher resolution at smaller lateral step values. We take FWHM as a measure of resolution due to the Rayleigh Criterion, which states that if two intensities following a normal distribution are a distance equal to the FWHM or smaller, then they can no longer be resolved.

Step size (cm)	**FWHM (cm)**	**Uncertainty (cm)**
0.5	1.6	0.2
1.0	2.3	0.3
2.1	3.9	0.3

Table 1: FWHM values for each Gaussian fit of intensity values.

The FWHM were calculated using the equation: FWHM $= 2.355 * \sigma$ where σ denotes the standard deviation of the Gaussion fit. Standard error of the standard deviation was used as a measure of uncertainty. This was done by taking the Jacobian matrix of the fit variables, and transposing it in order to find the covariance matrix of the fit variables. The diagonal of this matrix was taken and square rooted in order to find the standard deviation values for each parameter, including σ. Note that this assumes that the fit variables also follow a Gaussian distribution, which is reasonable due to the influences on the experiment mentioned above. All values in this lab are rounded to the precision of the standard uncertainties.

Thus, it has been determined from resolution that the two source PET scan would be done at a 1.0 cm, and kept at a distance of at least 2.6 cm (taking into account upper bound of uncertainty value for FHWM). At a lateral step of 1.0 cm, the spatial resolution is 2.3 ± 0.3 cm.

3 Two source PET scan

3.1 Parameter Setup and Anomaly Correction

A PET scan of two β_+ sources was used in order to determine the spatial resolution between two sources, as well as the relative intensity. The distance between the sources was measured with a meter rule. This measurement was used to determine the accuracy of the PET scan.

Since two sources were used, a lower bound of 3 angles is needed in order to be able to distinguish real sources from ghost peaks. For the sake of time and resolution, a total of 10 angles were taken over a sample of 180°, thus leading to an angle step size of 20. Time per slice was 15 seconds and lateral step size was 1.0 cm (again chosen due to time conservation, at the expense of a greater resolution at 0.5 cm)

After the runs were completed for the two sources, the data had to be converted to a coordinate grid with intensities plotted over, as well as a sinogram. Sinograms were made by combining 1D strips of intensity values for each angle measured. The coordinate grid of intensity values was done by expanding the 1D strips to 2D arrays using the numpy.outer() function. This takes the outer product of the 1D strips. The 2D arrays are then superimposed over each other, while also rotating each array by the angle it represented. For example, with two 2D arrays, one with the disc and source oriented at 0°, and the other at 45°, the two arrays would be superimposed with a 45° rotation on the second array.

Due to uncontrollable interference with the PMT+NaI detectors, which could occur due to the presence of other β_+ sources in the room, among other factors, anomaly correction must be done on the arrays. This was done by taking the average intensity of the 4 nearest intensity squares in the array, and replacing the value with the anomalous intensity. A simple set of Python functions was written in order to accelerate the process of anomaly correction.

```
def multi_error_correc(data,error_array): #inputs a list of anomaly values
    for error in error_array:
        anomaly_correction(data,error)
    return data

def anomaly_correction(data,point):
    for j,row in enumerate(data):
        for i,col in enumerate(row):
            if col == point:
                averaged=(row[i-1]+row[i+1]+data[j-1][i] + data[j+1][i])/4
                row[i]=averaged
```

Listing 1: Python code used for anomaly correction.

The code works by inputting a list of known anomalies within a larger array of floats representing recorded intensities. It then locates values within the intensity array that match the values within the list, and takes the average of the four nearest values to the anomaly in question, and replaces the anomaly with the calculated average. Anomalies are generally distinguished by judging by eye how much a given intensity varies from the values around it. All anomaly values were at least 10 times larger than their surroundings, and thus were easy to distinguish. This also means that it is likely that anomalies with lower intensity values were not accounted for.

3.2 Intensity and Position Data Extraction with Uncertainties

After anomaly correction, the 2D position-intensity array was used to get a rough location of the sources. One dimensional strips of intensity values were extracted from the 2D array in order to fit Gaussian distributions, and receive more accurate values for intensity and location of the source. This means strings where the x-value in position is held constant, or the y-value. This essentially divides the sources into grids. Between 3 and 4 strips were taken for each axis, for each source. This means between 6-7 Gaussian fits were performed on each source, for a total of 13 fits. Below is a table (figure 2) showing values retrieved from the fits. The yellow data rows in figure 2 demonstrate fits with the highest intensity and thus most likely to have the source located within the corresponding strip.

3.2.1 Determining The Position Of The Sources

In order to find the position of the source, the position of peaks within each fit was added together and averaged. This means that the peak *position* value of each strip in figure 2 was added together and divided by the number of strips for that axis. For example, For x-strips for source point 1, the average position of point source 1 on the axis was found through the following manipulation:

$$\begin{aligned} \text{Average position on x-axis} &= \frac{\text{Peak Position}_{x=6} + \text{Peak Position}_{x=7} + \text{Peak Position}_{x=8}}{3} \\ &= \frac{15.8 + 16.5 + 15.8}{3} \\ &= 16.0 \end{aligned} \tag{1}$$

with the appropriate standard error propagation:

$$\Delta x = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2} \tag{2}$$

equation 2 demonstrates error propagation for the standard error in the fit, meaning

$$\text{STD}(\mu) = \sigma/\sqrt{N}.$$

However, there is also the uncertainty in the fit, which for the x values was never higher than ± 0.08cm and the uncertainty in the lateral step made by the pitaya, which is ± 0.01cm (this was a form of systemic bias, which affected all measurements in the same manner, and could not be improved with the apparatus at hand). Thus, it is clear that the most dominant source of error was the standard error in the fit, and so in all other uncertainty calculations only the standard error in the fit is used. Examples of the Gaussian fits are shown in figure 2.

The difference in distance was then found using the Pythagorean Theorem. The distance between x-values and y-values were found, and then the hypotenuse between those two differences was found. The uncertainty in the value for the distance between the points was expressed as

$$\delta\text{Distance} = \sqrt{\left(\frac{2x}{2 \cdot \sqrt{x^2 + y^2}} \cdot \delta x\right)^2 + \left(\frac{2y}{2 \cdot \sqrt{x^2 + y^2}} \cdot \delta y\right)^2}. \tag{3}$$

3.2.2 Determining The Intensity Of The Sources

The intensity of each source was found by averaging the intensity values of the yellow rows for the x and y directions. This means looking at the *intensity* values for each yellow strip for each source, and then averaging the result. For example, for source point 1, the following manipulation was done:

$$\begin{aligned} \text{Intensity}_{avg.} &= \frac{\text{Intensity}_{x=7} + \text{Intensity}_{y=15}}{2} \\ &= \frac{1247 + 709}{2} \\ &= 978 \text{ counts} \end{aligned} \tag{4}$$

The standard error was propagated in the exact same manner as shown in equation 2. As discussed previously, the standard error in intensity was found through Jacobian Matrix manipulation, which gives the standard deviation of the intensity, assuming the intensity follows a Gaussian distribution. After the standard deviation was found, the standard error was calculated through the equation SE(Intensity)=$\sigma/\sqrt{n}$, where n is the number of data points within the sampling distribution.

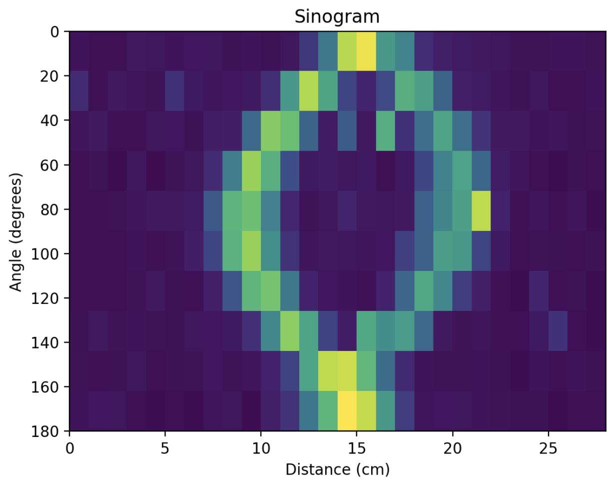
The uncertainty in the intensity ratio between the sources was found using the equation

$$\delta \text{ in intensity ratio} = \sqrt{\left(\frac{1}{S_1} \cdot \delta S_1\right)^2 + \left(\frac{-S_1}{S_2^2} \cdot \delta S_2\right)^2}, \tag{5}$$

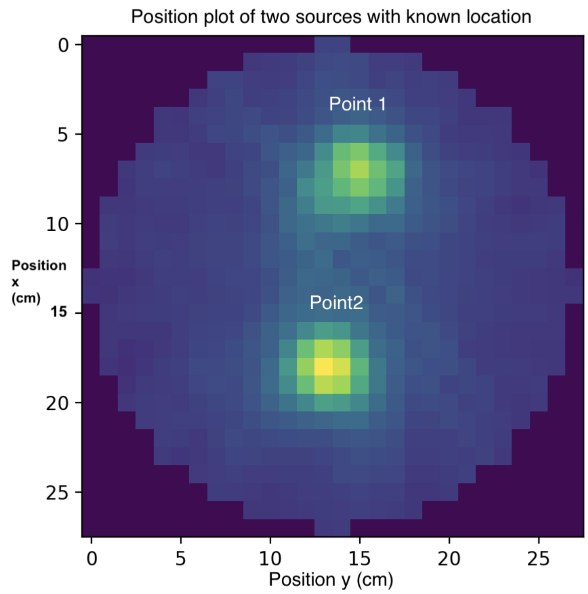
where S_1 and S_2 are intensity uncertainties for source 1 and 2.

3.3 Data of Two Source PET scan

As seen in figure 4, the intensity values between strip x17 and x19 varied very slightly, suggesting that the strips are almost equidistant from the actual location of the source. Additionally, the N-value depicted in the graphs and the perceived heights of the peaks are different. This is because the fitting code starts counting height from the initial point in the fit, and not zero. The intensity error bars for all fits were square rooted values of the counts.



(a) Sinogram of two sources of known location. Squares colored in bright-yellow contain the largest intensity values. Squares in dark-blue have negligible intensity values.



(b) Two dimensional intensity array of 2 sources placed on outskirts of the circular disc. A strip tag of x=6 cm means a 1D array of intensity values extracted horizontally along the y-axis at an x-position of 6 cm.

Figure 3: Graphs used to find the location of the two sources.

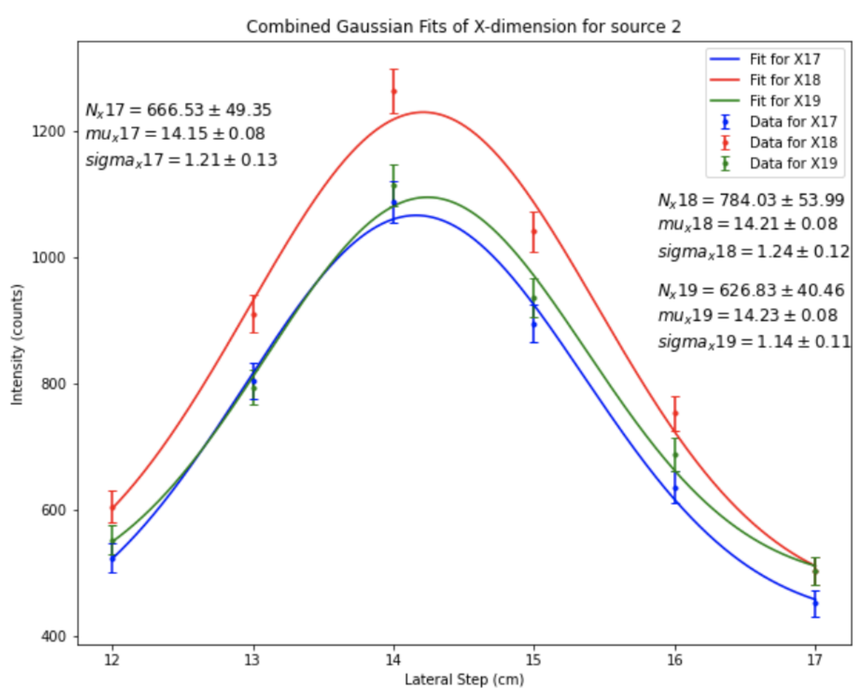


Figure 4: Combined Gaussian fits for strips of intensity values where x=17,18,19. Comparison of peak values was used to determine where the source was located. Here, x=18 has the highest intensity value.

Strip tag (cm)	peak position (cm)	intensity (counts)	δ position (cm)	δ intensity (counts)
		Source Point 1 Strip Data		
x=6	15.8	840	0.8	80
x=7	16.5	1250	1.1	80
x=8	15.8	680	0.7	10
y=13	8.7	480	0.7	20
y=14	7.9	640	0.7	20
y=15	8.1	710	0.6	10
y=16	8.3	640	0.5	10
		Source Point 2 Strip Data		
x=17	14.2	670	0.5	20
x=18	14.2	780	0.5	20
x=19	14.2	630	0.4	20
y=12	19.0	630	0.6	10
y=13	19.1	970	0.5	10
y=14	19.1	790	0.6	50

Table 2: Fit values for the known two-source sample. Rows highlighted in yellow signify the strip with the highest peak intensity. Strips with the highest intensity are nearest to a given source.

Source	Intensity (counts)	δ Intensity (counts)
1	980	81
2	875	23

Table 3: Intensity values for the two known sources, with uncertainty.

Difference in x distance	2±1 cm
Difference in y distance	11±1 cm
Distance between points	11±2 cm

Table 4: Distance between the two known sources, with uncertainty.

The distance between the points was calculated to be 11±2 cm. The measured distance with a meter rule was 10.6±0.1 cm. Thus, the PET scan provides an accurate depiction of the distance between sources, provided they are far enough to be resolved.

Additionally, the two sources used were of identical intensity. The first source had a measured intensity of 980±80 counts, while the second source had an intensity of 880±20 counts. The relative intensity of the second source with regards to the first was 0.90±0.08. This suggests some inaccuracy within intensity measurements. This is likely due to some anomalies being unaccounted for in correction, resulting in sources appearing as more/less intense than expected.

4 unknown configuration PET scan

For the unknown configuration, an unknown amount of β_+ sources was placed in a styrofoam box in unknown locations. A PET scan was performed, with the following parameters:

1. Angle step size: 20 degrees
2. Number of angles: 10
3. Maximum angle: 180 degrees
4. Lateral step: 0.5cm
5. Lateral distance: 0-28.0cm
6. Threshold: 204V-1023V

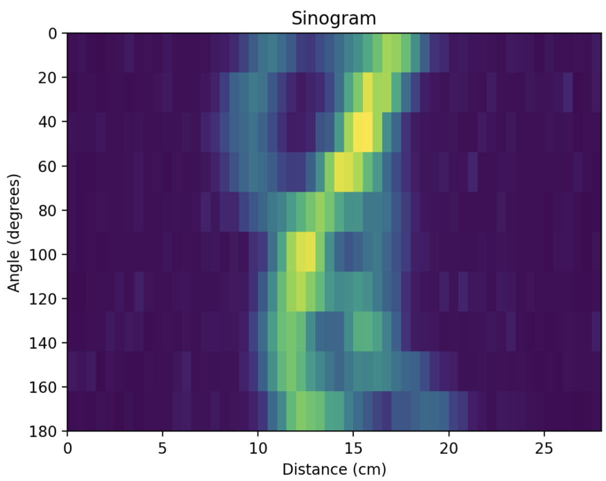
Note that only ten angles at 0.5 cm lateral step were chosen due to fears that the equipment would fail a second time during the overnight run. The 0.5 cm proved too large of a step size to adequately distinguish between two sources within the configuration, as is elaborated below. 10 angles were taken in order to thoroughly get rid of any potential ghost peaks.

The same approach used for 1 and 2 source configurations was used to find the location, intensity and distance between the sources. Thus, a full elaboration of the techniques used to find distance and relative intensity with uncertainty can be found in the *Two Source PET scan* section. From the sinogram below, we see that three sources are present. The sinograms were created in the exact same manner as described in the two source section, with the same method for anomaly correction. Between 6-8 Gaussian fits were performed on each source, for a total of 19 fits.

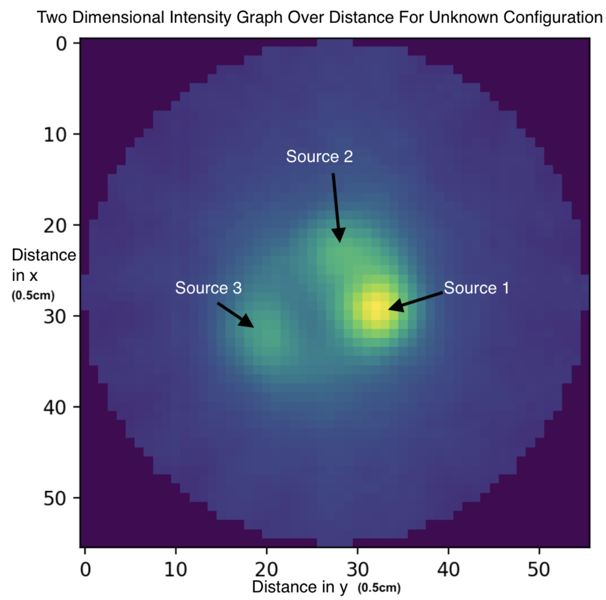
The distance between values was found by constructing triangles with differences in x and y values, and computing the hypotenuse. Uncertainty in these values was found by using the equation denoted in equation 3. The uncertainty in relative intensity was found using the relation in equation 5.

Note that the distance between sources 1 and 2 is 2±1cm. This makes it difficult to resolve the sources, since previously it was calculated that two sources become unresolved at a distance of 1.6±0.2cm for a lateral step size of 0.5cm. This was clear even when plotting gaussian fits for the first source. The two sources were close enough for the intensities to superimpose and distort the Gaussian of source 1 to the point where the distribution was clearly not Gaussian anymore.

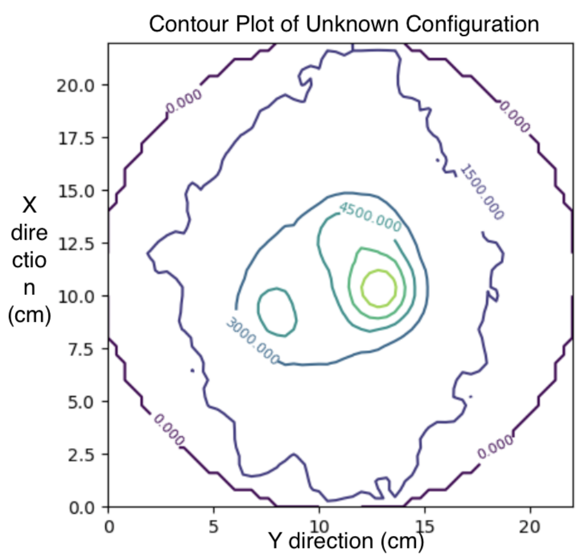
This introduces error greater than that demonstrated by the uncertainties. An immediate and simple improvement to the experiment would thus be to lower the step size. An example of a distorted distribution is found in the appendix in figure 6.



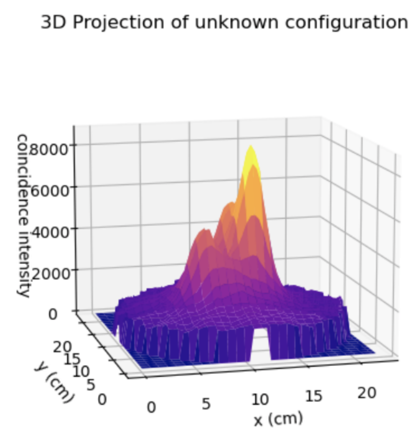
(a) Sinogram of unknown source configuration. Three sources are present.



(b) 2D position array of the unknown configuration. Source 1 is the most intense source of β_+ decay.



(c) A contour plot of the unknown configuration gives a clearer picture of differences in intensity in the 2D grid.



(d) A 3D projection of the unknown configuration helps accentuate the difference in intensity of the weaker sources.

Figure 5: Graphs used to find the locations and intensities of the sources.

Strip tag	peak position (cm)	intensity (counts)	δ position (cm)	δ intensity (counts)
		Source Point 1 Strip Data		
Y=31	14,7	6000	0,5	10
Y=32	14,8	6176	0,5	9
Y=33	15,1	5960	0,4	10
X=28	16,6	5710	0,4	20
X=29	16,6	6160	0,4	20
X=30	16,6	6010	0,4	20
		Source Point 2 Strip Data		
x=15	16,3	800	0,7	100
x=16	16,4	1000	0,6	40
x=17	15,9	950	0,5	30
Y=27	12,6	4150	0,7	30
y=28	13,2	4890	0,7	30
y=29	12,8	4340	0,7	40
		Source Point 3 Strip Data		
Y=18	16,3	2870	0,5	20
Y=19	16,5	3330	0,6	30
Y=20	16,7	3530	0,6	50
Y=21	16,7	2240	0,4	20
X=30	10,1	2400	0,5	200
X=31	10,3	2400	0,4	200
X=32	10,4	2300	0,5	100

Table 5: Gaussian fit values of location and intensity of the sources of the unknown configuration.

Source	Position of source (cm,cm)	uncert. in position (cm,cm)	Intensity of source (counts)	uncert. in intensity (counts)	relative intensity (to source 1)	Uncert in ratio:
1	(16.6,14.8)	(0.8,0.7)	6170	20	1.00	0
2	(16,13)	(1,1)	2850	60	0.46	0.01
3	(16.6,10.3)	(0.9,0.8)	3000	200	0.49	0.03

Table 6: Information about the position and relative intensities of the unknown sources, with uncertainty.

Difference in distance	Distance (cm)	Uncertainty in distance (+/-cm)
between 1 and 2	2	1
between 1 and 3	5	1
between 2 and 3	3	1

Table 7: Distance between sources, with uncertainty.

4.1 Conclusion

From table 7 we see that sources 1 and 3 are furthest away from each other, with a distance of 5±1cm. Additionally, sources 2 and 3 also interfere with each other's intensities, making it harder to distinguish between them. This is due to their distance being close to the resolution distance at this lateral step size. As mentioned above, the resolution at a step size of 0.5cm is 1.6±0.2cm, while the distance between sources 2 and 3 is 3±1cm.

In terms of intensities, the most radioactive source was source 1, with an intensity of 6170±20 counts. The relative intensities of sources 2 and 3 compared to source 1 are 0.46±0.01 and 0.49±0.03 respectfully. Thus, it is likely that these sources could have had the same intensity, since the value of source 2 is within the uncertainty of source 3. However, as seen with the two source PET scan, the equipment is not accurate enough to fully claim that the sources are the same.

5 Appendix

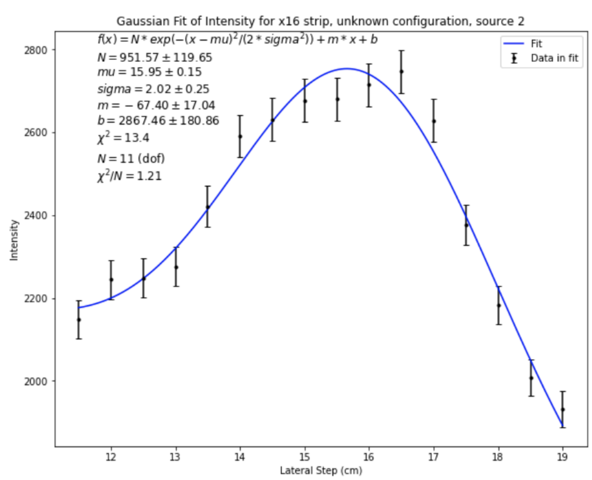


Figure 6: Gaussian fit of strip x=16 for source 3. We note that the distribution fails to be Gaussian due to interference caused by source 1.