Nuclear Physics: Extracting the Mass of the Neutron

Antoni Bandachowicz

University of Chicago

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Contents

1 Analysis

The analysis below presents data used to calculate the mass of the neutron (within given uncertainties), and reinforces the suggestion that the full energy peaks measured during the experiments were indeed created by the phenomenon of deuteron formation through the equation below:

$$\mathbf{n} + \mathbf{p} \rightarrow \mathbf{d} + \gamma$$

Figure 1: equation characterizing deuteron formation

Where the mass of the proton from literature was 938.27231MeV and the mass of the deuteron was 1875.61294MeV. our predicted energy for capture gammas was 0.93168Mev.

1.1 Calibration of the PHA

The USX software used had the following settings throughout all spectra measurements:

- The Fine Gain was set to 1.00x
- The Course Gain was set to 1
- The High Voltage (HV) was set to 1000V
- All spectra were gathered within a live time of 60s.

It is important to note that the data was gathered over a period of two separate days. On the second day, the apparatus, including the power supply to the photomultiplier tube (PMT), had to be switched on again and no calibration to the channels was done because we wrongly assumed that no potential drift would occur. This lead to a potential drift of about 5 channels at the extreme for data done on the second day. The relevance and propagation of this uncertainty is discussed in detail later.

The USX software connecting with the PHA plots energy spectra in terms of channels and counts. The relationship between channel number and corresponding energy had to be deduced experimentally. The known gamma sources used for this calibration were Cobalt-60 (Co-60), Sodium-22 (Na-22) and Caesium-137 (Cs-137). Since the energy and shape of the full energy peaks was known, a relationship between channel number and energy (in keV) could be deduced. Table 1 and the plotted spectra (figures 2, 3 and 4) show the identification of the relevant peaks and the values of the centroids.

Peak Identifier	Centroid	Energy (keV)
Cs peak 1	123	662
Co peak 1	220	1173.2
Co peak 2	251	1332
Na peak 1	94	511
Na peak 2	239	1270

Table 1: Energy peaks used for calibration and the corresponding centroids

Note that all of the calibration experiments were surrounded with lead shielding (2 blocks) which adequately shielded the experiments from outside sources of radiation. Figure 5 shows

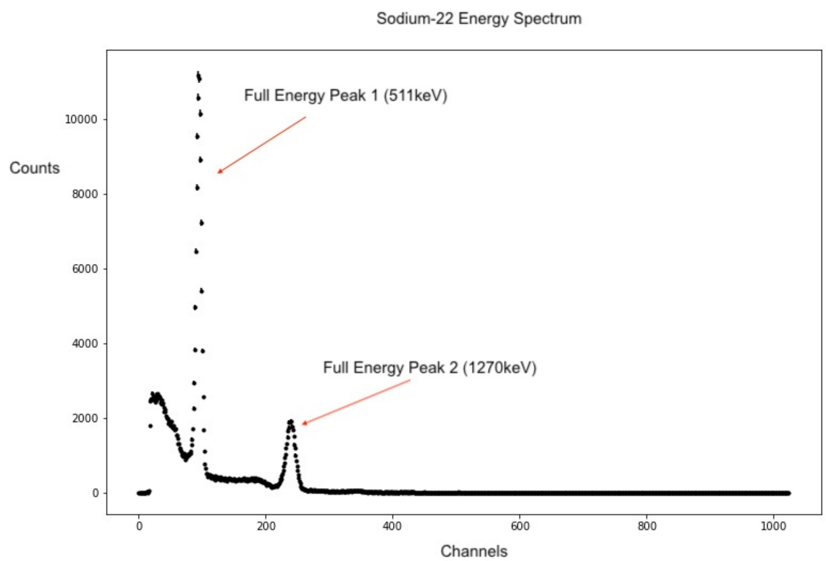


Figure 2: Graph of the Na-22 Spectrum

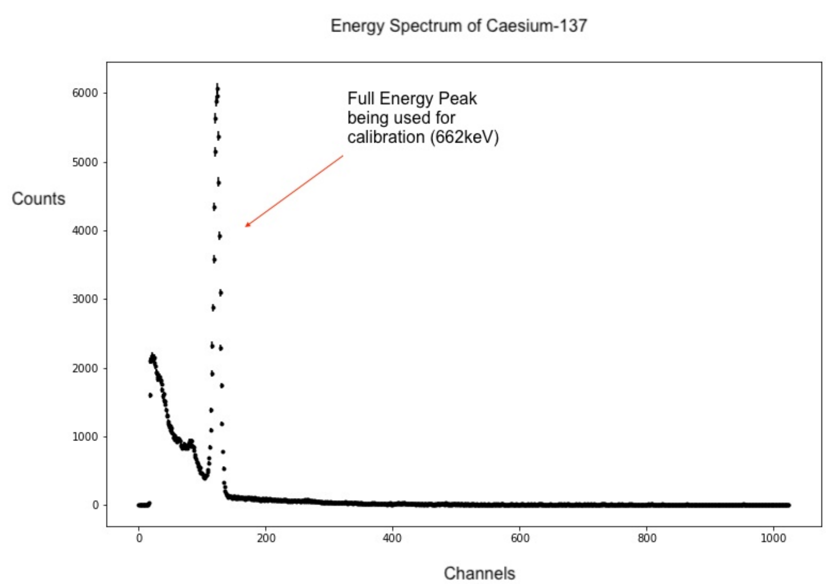


Figure 3: Graph of the Cs-137 Spectrum

an experiment using nothing but shielding to illustrate this point. It is clear that no peaks within the relevant channel ranges are present, suggesting adequate insulation. Note that no third peak for Co-60 was observed for our calibration, and thus could not be used.

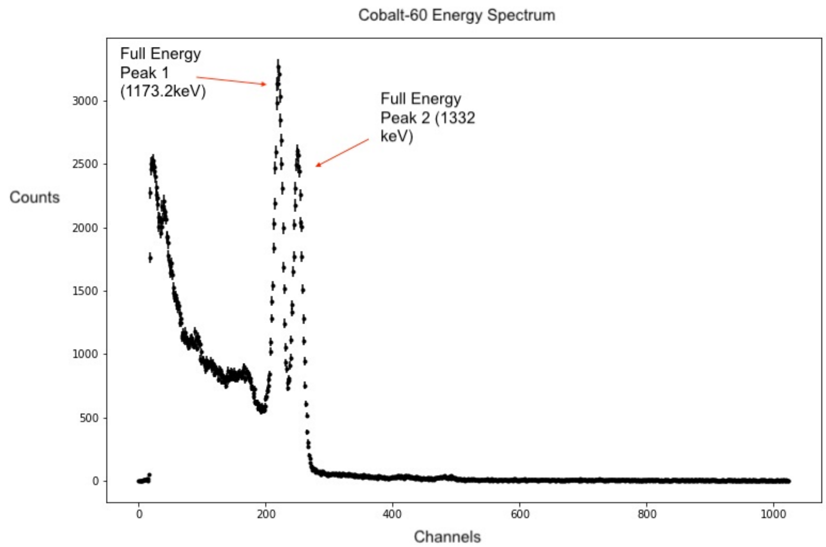


Figure 4: Graph of the Co-60 Spectrum

1.2 Uncertainty in Calibration

The calibration of the channels carries uncertainty with it due to our uncertainty in the centroids of the calibration peaks used. The uncertainties were calculated using a gaussian fit in python, with the standard deviation of the fit being used as a value for the uncertainties. The fit graphs (attached at the end of the analysis for clarity) also demonstrate how well the Gaussian fits the data, through the reduced-χ^2 value. All of the reduced-χ^2 values are below 3, suggesting a relatively good fit to the data.

Peak Identifier	Centroid	Energy (keV)	δCentroid (counts)	reduced-χ^2
Cs peak 1	123	662	4.59	2.07
Co peak 1	220	1173.2	6.27	1.73
Co peak 2	251	1332	6.27	1.26
Na peak 1	94	511	3.84	2.97
Na peak 2	239	1270	6.68	0.66

Table 2: uncertainty in the centroids and reduced-χ^2 values

From this data we can plot a function that relates channel number to energy. Figure 6 illustrates this function. The error bars of the points are the standard deviations about the centroids. This linear fit carries an uncertainty with it as well, which was calculated in Excel using the LINEST function. The R-squared value for the fit is 1. This suggests that the spread of data clusters is linear, but with arguably only two clusters of data points (at the extrema of the fitted line) and only 5 data points, this R-squared value does not carry much information about the accuracy of the fit.

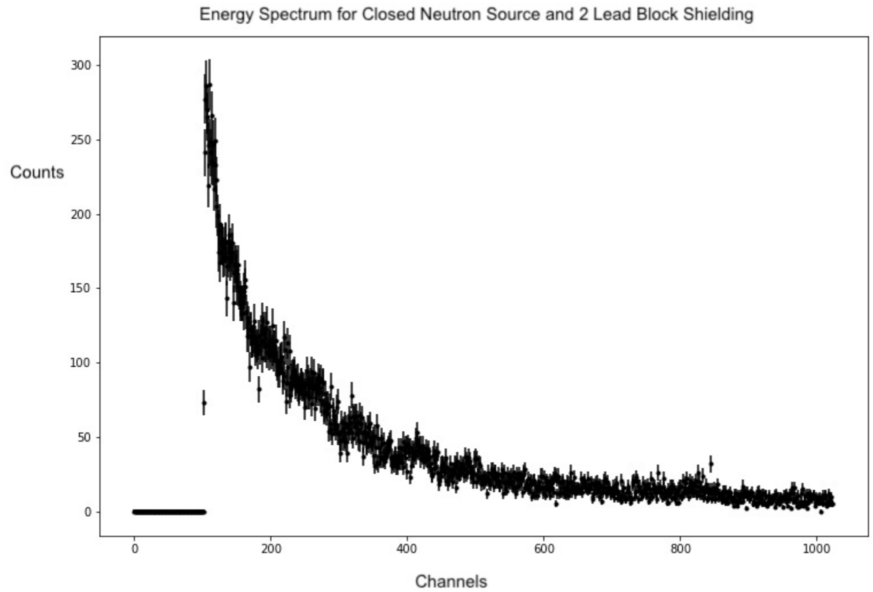


Figure 5: Energy Spectrum with full lead shielding

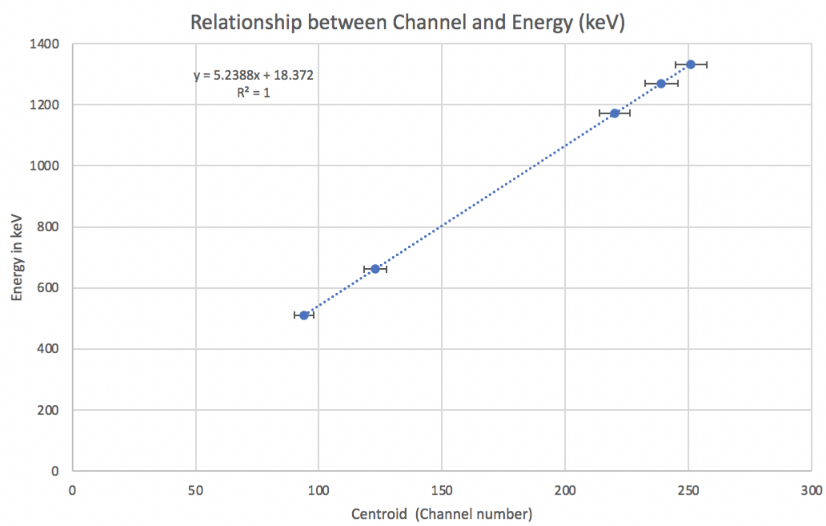


Figure 6: Linear fit of centroid data

From figure 6 we can extrapolate using the fit function to find the energy of a given peak. The uncertainty in the measured energy value therefore comes from two sources:

1. The linear fit parameters (a)
2. The centroid of the measured capture gamma peak (b).

The uncertainties were propagated using the equation:

$$\delta(f) = \sqrt{(\frac{\delta(a)}{a})^2 + (\frac{\delta(b)}{b})^2} \tag{1}$$

The percentage uncertainty was calculated using the equation:

$$\%\delta(f) = \delta(f) \times 100 \tag{2}$$

1.3 Finding the Energy of the Capture Gammas

In order to find the energy of the capture gamma peak we used the experiment setup with 1 block of lead shielding (with a thickness of 5.0±0.1cm) and a block of paraffin (1.9±0.1cm), aimed at by the open howitzer. The experiment gave us the spectrum shown in figure 7.

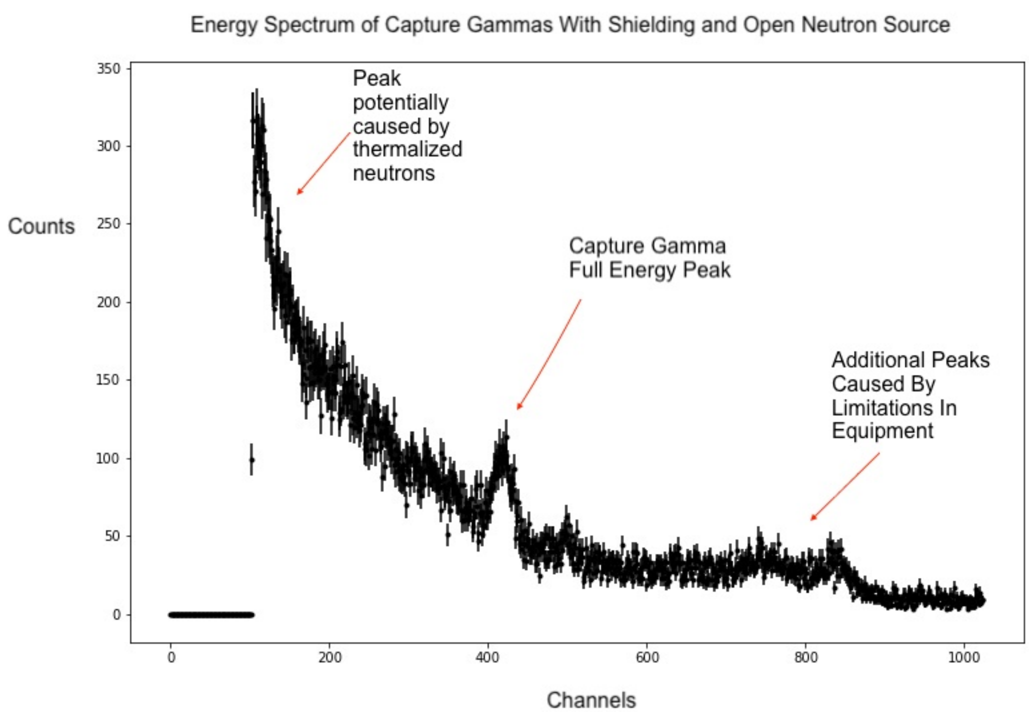


Figure 7: Energy spectrum and capture gamma peak for open source and one block of lead shielding.

We used the central full energy peak as a candidate for the capture gamma peak, due to the location of the centroid (from initial estimates of the energy peak, we found that the energy would be around 1Mev if the neutron was around the same size as the proton. Since we expect the neutron to be larger than the proton from known nuclei mass numbers, we expected to find the peak between the 1-3MeV range). The peaks at the high end of the channel axis are most likely caused by the limitations of the PHA software. In some cases, two incident gammas are so close together that the PHA registers it as one larger peak, and thus attributes the signal to a higher channel. In this case, since the centroid of the capture gamma peak was around 420, we would expect these higher peaks to have a centroid around 840, which was indeed the case. We plotted a Gaussian fit to obtain a more accurate measure of the centroid and the standard deviation (and thus the uncertainty in the centroid). As seen, the centroid of the energy peak

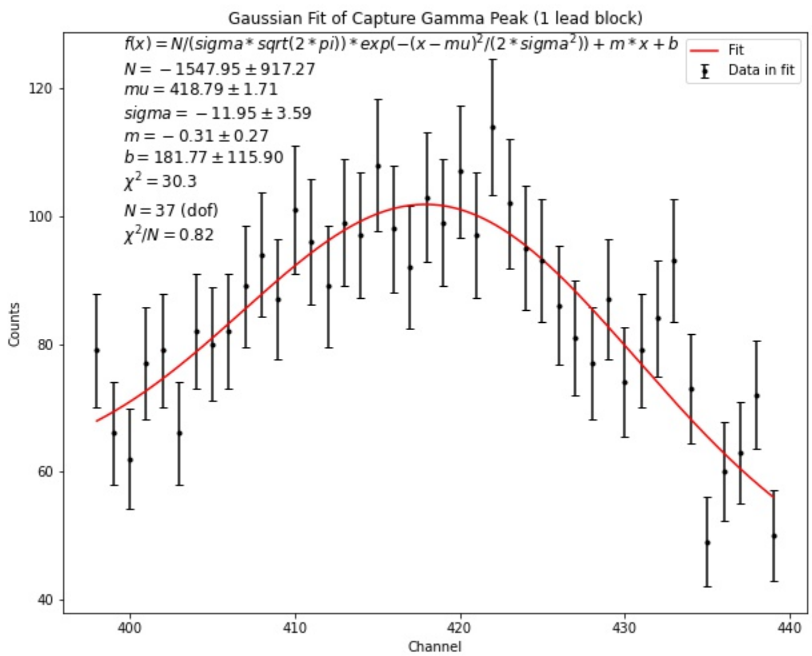


Figure 8: Gaussian fit for the capture gamma peak in the open source experiment discussed above.

is 419 with standard deviation of 12 channels. Using the linear fit function from the calibration we can calculate the energy of the centroid to be 2216keV (2.216MeV). The uncertainty from the linear fit and the centroid value is propagated together to give a value of 3keV. The numbers were rounded to the least significant figures of our data. We could not get more refined data than one channel. For this reason the numbers are rounded to the next full number.

$$\text{Energy} = 5.2388x + 18.372 = 2216 \pm 3 \text{ keV} \tag{3}$$

The slope of the linear fit had uncertainty ± 0.0112 and the y-intercept had uncertainty ± 2.1970.

from table 3 we see that the value found for the capture gamma energy is 99.6 percent

Measured gamma (keV)	Expected gamma (keV)	Percent difference	uncert. in % differnce	measured mass of neutron
2.216	2.224	99.6	0.003	**939.557**

Table 3: Measured mass of the neutron and comparison to literature

similar to literature, with the final mass varrying by only 0.008 MeV. This suggests that our measurement is consistent with literature. The measured mass carries the same uncertainty as the energy, because we assumed that the literature values carry no uncertainty.

2 Verifying the Source of the Peak

We need to be certain that the peak shown by the PHA and USX software indeed corresponds to capture gammas created by the formation of deuterons. Thus, we must discuss possible influences on the capture gamma peak. The following events were considered;

1. The Howitzer itself is a source of capture gammas.
2. Graphite might influence capture gamma attenuation.
3. The lead attenuates neutrons and thus might influence the peak.

2.1 Neutron Attenuation Ratio

We need to reassure ourselves that neutrons are indeed hitting the paraffin and are causing capture gammas, even with lead blocks present, which help attenuate capture gammas from outside sources. In order to find the neutron attenuation ratio a total of four different experiments were performed:

1. Experiment with an open port, no shielding and a block of paraffin
2. Experiment with a closed port, no shielding and no paraffin
3. Experiment with an open port, shielding and a block of paraffin
4. Experiment with a closed port, shielding and no paraffin.

Equation Number	Details	Centroid	Gross Count	count rate (1/s)
1	Open+Paraffin + no shielding (3.0cm)	420	20654	344
2	closed source + no shielding + no paraffin	422	18874	315
3	Open+Paraffin + 2 Blocks (3.0cm)	417	1565	26
4	Closed+no paraffin + 2 Blocks	420	929	15

Uncertainty in count rate	**attenuation rate**	**unct. in attenuation rate**	**attenuation ratio**	**Uncertainty in attenuation ratio**
2	29.7	3.3	0.36	0.25
2	10.6	2.4		
1				
1				

Table 4: Calculations for attenuation ratio and uncertainties

We found the gross count rate of each of these experiments. The difference in the count rate between paraffin and no paraffin when no shielding is present was found to be 29.7. (this was done by subtracting the count rate of experiment 2 from experiment 1). The difference in the count rate between paraffin and no paraffin with shielding was 10.6 (subtracting experiment 3 from experiment 4). The uncertainty in this count rate difference (referred to as the attenuation rate) is found using the equation:

$$\delta(f) = \sqrt{(\delta(a))^2 + (\delta(b))^2} \tag{4}$$

From this point all explicit values for the uncertainties will be listed in data tables.

The uncertainty in the attenuation ratio was found to be ± 0.25 using equation 5. The uncertainties for the count rates were calculated using equation 5, Where G is the gross count, R is the gross count rate, and dR is the uncertainty in R.

$$dR = R\frac{\sqrt{G}}{G} \tag{5}$$

The gross count was found using the USX software. Note that the attenuation ratio was found using 2 lead blocks because of the limited number of experiments we conducted.

2.2 Paraffin and Outside Capture Gamma Sources

We found it necessary to confirm that the paraffin was indeed responsible for creating capture gammas, or more specifically, the hydrogen within the paraffin. In order to do this, we conducted three more series of experiments.

1. The first series of experiments focused on proving that the introduction of paraffin created capture gammas. This could be proven by increasing the thickness of the paraffin. Increasing the thickness would result in an increase in the capture gamma peak.
2. The second series of experiments focused on showing how much of the capture gamma peak was created by the howitzer.
3. The third series of experiments compared paraffin to graphite in order to establish the effect of carbon on capture gammas.

2.2.1 Paraffin Thickness

Table 5 shows the series of experiments conducted in order to measure how paraffin thickness relates to capture gammas. We noticed that increasing the width by 36% ($\pm$5% uncertainty) resulted in a gross count rate increase of 10.6% ($\pm$22% uncertainty). Again, the uncertainties were calculated using equations 1 and 5.

Setup	Centroid	Gross Count	Gross Count Rate (1/s)	Unct. in count rate
Paraffin + 1 Block (1.9cm)	417	3060	51	1
Paraffin + 1 Block (3.0cm)	418	3424	57	1

difference in count rate	unct. In diff.	percentage increase	uncert. in percentage increase
6	1	10.6	22.2

Table 5: Calculations for paraffin thickness and associated uncertainty

This suggests that paraffin is responsible for capture gamma creation. However, the uncertainty in the measurement is very large. This is due to the fact that all of the spectra were gathered over a period of 60 seconds, yielding small numerical values and differences. This is also why all of these comparisons are done in terms of percentages.

2.2.2 Howitzer as a source of capture gammas

In order to account for the howitzer's capture gammas we conducted two experiments: one with the howitzer port open, and the other with the port closed (both used 1 block of lead and a 1.9cm block of paraffin. They were also the same distance apart for each experiment). We

Setup	Gross Count	Gross Count Rate:	Uncert. in count rate	Difference in the rate:
Closed source + 1 Block + paraffin (1.9cm)	2083	34.7	1	16.3
Thin Paraffin + 1 Block (1.9cm)	3060	51.0	1	

unct in diff.	% difference	unct. In % diff.
1	32	8

Table 6: Calculations for howitzer influence and associated uncertainty

noticed a 32% increase in the gross count rate when the howitzer was opened, suggesting that the peak was indeed created by capture gammas from the paraffin. A large percentage was due to the howitzer. Table 6 illustrates this point.

2.2.3 Influence of Graphite

To determine the influence of graphite we conducted an experiment using a 1.9±0.1cm paraffin block of paraffin and a 2.2±0.1cm block of graphite (both with 1 block of lead shielding). The result yielded a gross count rate difference of 28.6% (with 8 percent uncertainty), which is comparable to the difference in the previous experiment with the howitzer. What this suggests is that the graphite peak is caused by the howitzer, and not the graphite. This is supported by theory. The only way carbon interacts with neutrons is through absorption to produce a stable isotope of C13 (and an unstable isotope of C14 in much rarer cases). Thus, we can suggest that graphite had no significant impact on the results.

2.2.4 Overall Comment on the uncertainties of the Experiments Using Gross Counts

The gross counts were calculated using the USX software compatible with the PHA. Using this software has shown that even slightly changing Region of Interest (ROI) ranges would change count values relatively significantly. Additionally, as mentioned above, we experienced potential

Influence of Graphite:	Gross Count	Gross Count Rate:	Uncert. in count rate
Thin Paraffin + 1 Block (1.9cm)	3060	51.0	1
Graphite + 1 Block (2.2cm)	2184	36.4	1
difference	unct. in diff.	pct. difference	Unct. In pct. Difference
14.6	1	28.6	8

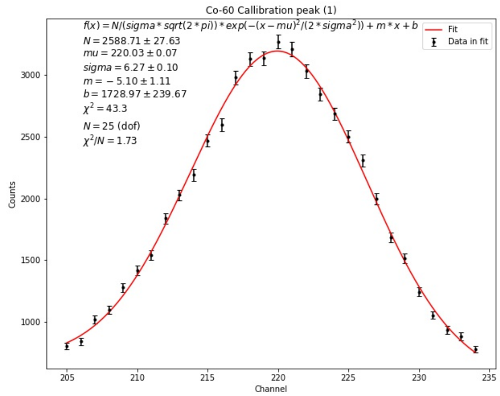
Table 7: Calculations for graphite influence and associated uncertainty

drift in the second day of our data measurements. This lead to an additional uncertainty in the centroid which has been accounted for in this analysis, but it has also made it more difficult to distinguish ROIs, since they have moved. As such, the uncertainty in the gross counts may be underestimated.

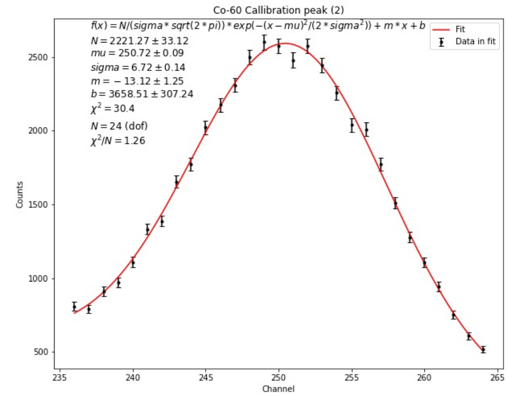
3 Conclusion

Through the above experiments we have proven that neutrons effectively travel through the lead, react significantly with the paraffin and produce capture gammas. Additionally, even with the outside capture gamma source being shielded by lead, we still receive enough neutrons to claim that the peak found in the experiments as the capture gamma peak was indeed significantly created by capture gammas from deuteron formation, and thus this peak can be used reliably to measure the mass of the neutron.

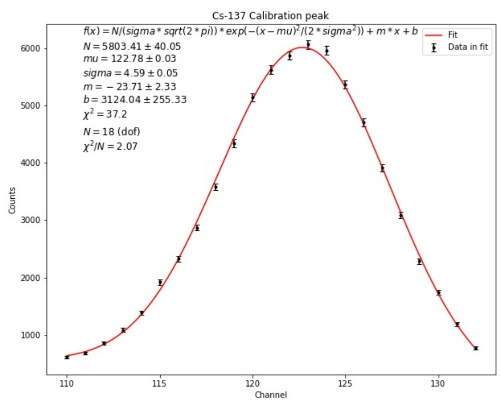
4 Appendix: Gaussian Fits of Calibration Peaks



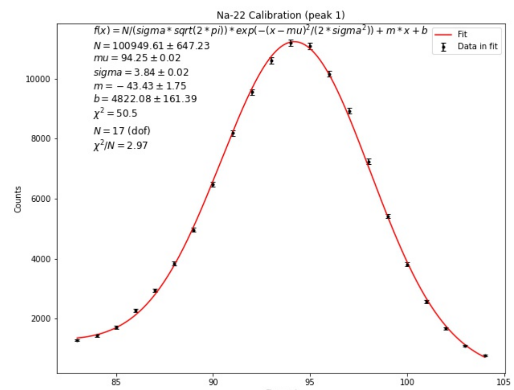
(a) Co-60 Gaussian Fit of first calibration peak.



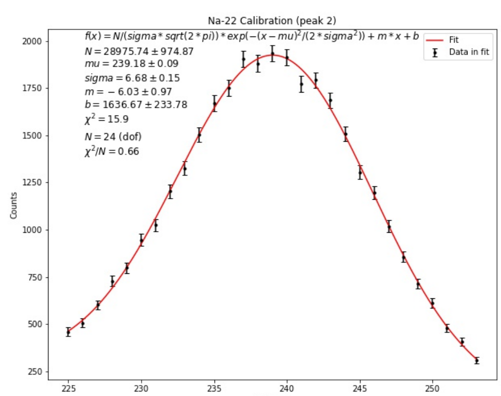
(b) Co-60 Gaussian Fit of second calibration peak.



(c) Cs-137 Gaussian Fit of calibration peak.



(d) Na-22 Gaussian Fit of first calibration peak.



(e) Na-22 Gaussian Fit of second calibration peak.

Figure 9: Fits of calibration peaks used.