Reference frame conversions

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Contents

1 From Pixels to Meters

1.1 Transformation Matrices

The symbol $\tilde{r}$ expresses vectors in units of pixels. Any other vector r has units of meters.

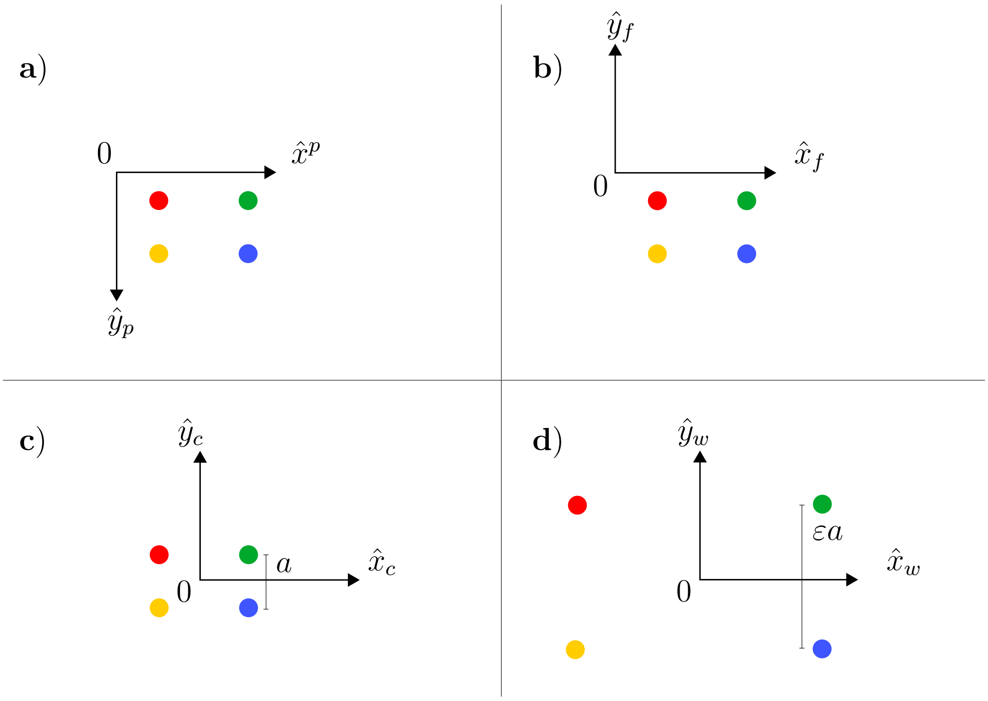

Figure 1: Four coordinates: a) in the camera picture, b) with flipped y axis, c) with the origin at the center of the picture, d) scaled to physical distances.

$$\begin{bmatrix} \tilde{r}^f \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{r}^p \\ 1 \end{bmatrix} = H_p^f \begin{bmatrix} \tilde{r}^p \\ 1 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \tilde{r}^c \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2}\tilde{d}_x \\ 0 & 1 & \frac{1}{2}\tilde{d}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{r}^f \\ 1 \end{bmatrix} = H_f^c \begin{bmatrix} \tilde{r}^f \\ 1 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} r^w \\ 1 \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{r}^c \\ 1 \end{bmatrix} = H_c^w \begin{bmatrix} \tilde{r}^c \\ 1 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} r^w \\ 1 \end{bmatrix} = H_c^w H_f^c H_p^f \begin{bmatrix} \tilde{r}^p \\ 1 \end{bmatrix} \tag{4}$$

The overall transformation from pixels to the real world representation is

$$\begin{bmatrix} r^w \\ 1 \end{bmatrix} = H_p^w \begin{bmatrix} \tilde{r}^p \\ 1 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{r}^w = H_p^w \tilde{\boldsymbol{r}}^p \tag{5}$$

where

$$H_p^w = H_c^w H_f^c H_p^f = \begin{bmatrix} \varepsilon & 0 & -\frac{1}{2}\varepsilon\tilde{d}_x \\ 0 & -\varepsilon & \frac{1}{2}\varepsilon\tilde{d}_y \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

1.2 Scaling Factor

The distance $d_{\text{top/center}}$ in Figure 3 is known, constant, and equivalent for each agent:

$$d_{\text{top/center}} \equiv ||r_{\text{top},i} - r_{\text{center},i}|| \quad \forall\, i \tag{7}$$

Per agent, we can estimate the scaling factor ϵ_i as:

$$\varepsilon_i = \frac{d_{\text{top/center}}}{||\tilde{r}_{\text{top},i} - \tilde{r}_{\text{center},i}||} \quad \text{m/pixel} \tag{8}$$

A more accurate estimate for the overall scaling factor is:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i \tag{9}$$

However, because ε is constant it may be more desirable to calculate it once and use it as a constant parameter. If so, we may divide the arena width by the picture with in pixels:

$$\varepsilon = \frac{d_{\text{field}}}{\tilde{d}_x} \quad \text{m/pixel} \tag{10}$$

2 Local Robot Frames

2.1 From Label Markers to Label Frame

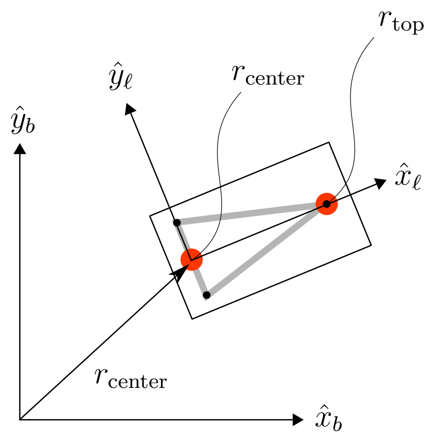

Figure 2: Local vehicle frame definitions

First, transform pixel locations to world coordinates:

$$\boldsymbol{r}^w_{\text{center}} = H^w_p \tilde{\boldsymbol{r}}^p_{\text{center}}, \quad \boldsymbol{r}^w_{\text{top}} = H^w_p \tilde{\boldsymbol{r}}^p_{\text{top}} \tag{11}$$

Obtain the label x and y axes in world coordinates:

$$\hat{x}^w_\ell = \frac{r^w_{\text{top}} - r^w_{\text{center}}}{\left|\left|r^w_{\text{top}} - r^w_{\text{center}}\right|\right|}, \qquad \hat{y}^w_\ell = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \hat{x}_\ell \tag{12}$$

The transformation matrix becomes:

$$\boldsymbol{r}^w = H^w_\ell \boldsymbol{r}^\ell, \qquad H^w_\ell = \begin{bmatrix} \hat{x}^w_\ell & \hat{y}^w_\ell & r^w_{\text{center}} \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Agree on a better name than center. This is confusing

2.2 From Label Frame to Robot Frame

$$\boldsymbol{r}^b = \begin{bmatrix} I_2 & r^b_{\text{center}} \\ 0 & 1 \end{bmatrix} \boldsymbol{r}^\ell \tag{14}$$

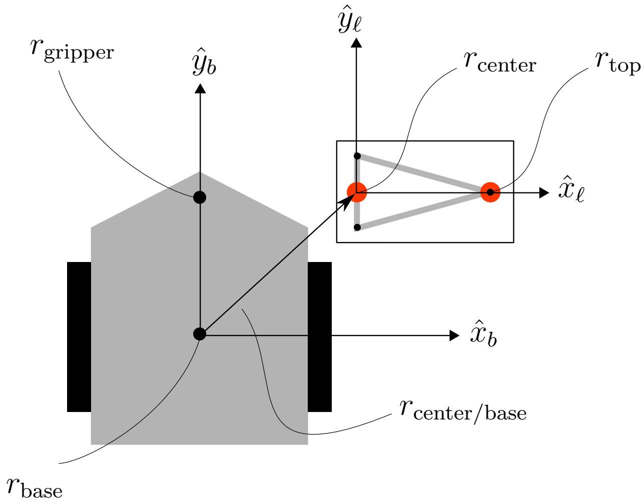

Figure 3: Local vehicle frame definitions

3 Relative Agent Positions and Orientations