Reference frame conversions

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Contents

1 Preliminaries

1.1 Points and Vectors

$$r^c_{b/a} = p^c_b - p^c_a \tag{1}$$

add picture

1.2 Transformation

$$\begin{bmatrix} p^i \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} R^i_j & r^i_{j/i} \\ 0 & 1 \end{bmatrix}}_{H^i_j} \begin{bmatrix} p^j \\ 1 \end{bmatrix} \tag{2}$$

1.3 Inverse Transformation

$$\begin{bmatrix} p^j \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} (R^i_j)^T & -(R^i_j)^T r^i_{j/i} \\ 0 & 1 \end{bmatrix}}_{H^i_j} \begin{bmatrix} p^j \\ 1 \end{bmatrix} \tag{3}$$

2 From Pixels to Meters

2.1 Transformation Matrices

The symbol $\tilde{p}$ expresses points in units of pixels. Any other point p has position units of meters.

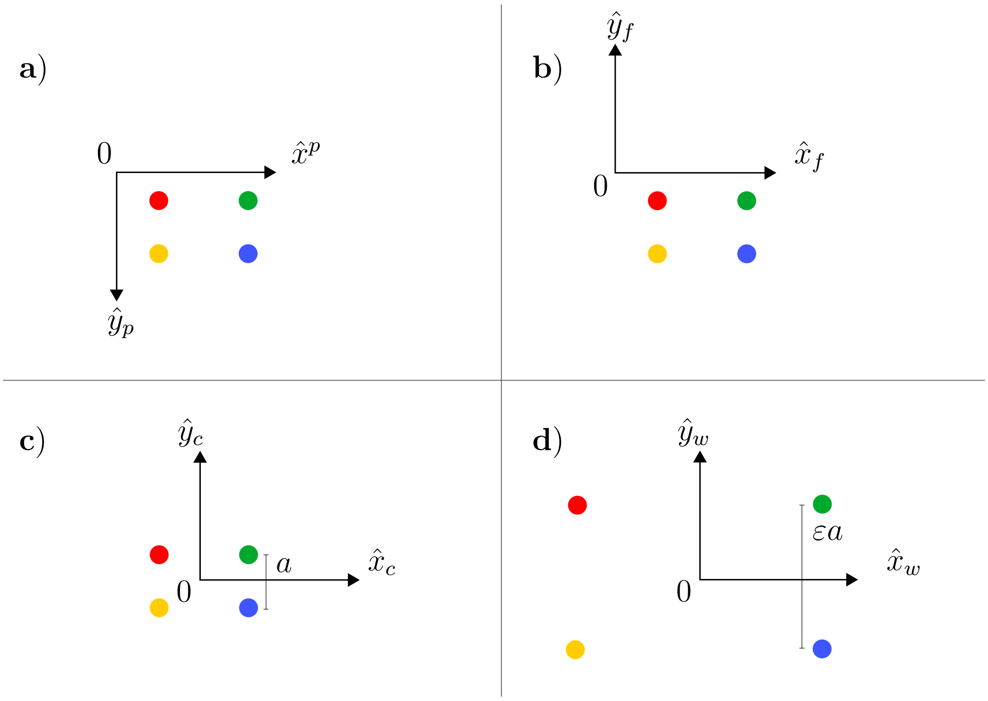

Figure 1: Four coordinates: a) in the camera picture, b) with flipped y axis, c) with the origin at the midbase of the picture, d) scaled to physical distances.

$$\begin{bmatrix} \tilde{p}^f \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{p}^p \\ 1 \end{bmatrix} = H_p^f \begin{bmatrix} \tilde{p}^p \\ 1 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \tilde{p}^c \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2}\tilde{d}_x \\ 0 & 1 & \frac{1}{2}\tilde{d}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{p}^f \\ 1 \end{bmatrix} = H_f^c \begin{bmatrix} \tilde{p}^f \\ 1 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} p^w \\ 1 \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{p}^c \\ 1 \end{bmatrix} = H_c^w \begin{bmatrix} \tilde{p}^c \\ 1 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} p^w \\ 1 \end{bmatrix} = H_c^w H_f^c H_p^f \begin{bmatrix} \tilde{p}^p \\ 1 \end{bmatrix} \tag{7}$$

The overall transformation from pixels to the real world representation is

$$\begin{bmatrix} p^w \\ 1 \end{bmatrix} = H_p^w \begin{bmatrix} \tilde{p}^p \\ 1 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{p}^w = H_p^w \tilde{\boldsymbol{p}}^p \tag{8}$$

where

$$H_p^w = H_c^w H_f^c H_p^f = \begin{bmatrix} \varepsilon & 0 & -\frac{1}{2}\varepsilon\tilde{d}_x \\ 0 & -\varepsilon & \frac{1}{2}\varepsilon\tilde{d}_y \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

2.2 Scaling Factor

The distance $d_{\text{apex/midbase}}$ in Figure **??** is known, constant, and equivalent for each agent:

$$d_{\text{apex/midbase}} \equiv ||p_{\text{apex},i} - p_{\text{midbase},i}|| \quad \forall\, i \tag{10}$$

Per agent, we can estimate the scaling factor ϵ_i as:

$$\varepsilon_i = \frac{d_{\text{apex/midbase}}}{||\tilde{p}_{\text{apex},i} - \tilde{p}_{\text{midbase},i}||} \quad \text{m/pixel} \tag{11}$$

A more accurate estimate for the overall scaling factor is:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i \tag{12}$$

However, because ε is constant it may be more desirable to calculate it once and use it as a constant parameter. If so, we may divide the arena width by the picture with in pixels:

$$\varepsilon = \frac{d_{\text{field}}}{\tilde{d}_x} \quad \text{m/pixel} \tag{13}$$

3 Local Robot Frames

cross check p's and then define vector as difference between two points; and then vectors cannot be transformed; just rotated

3.1 From Label Markers to Label Frame

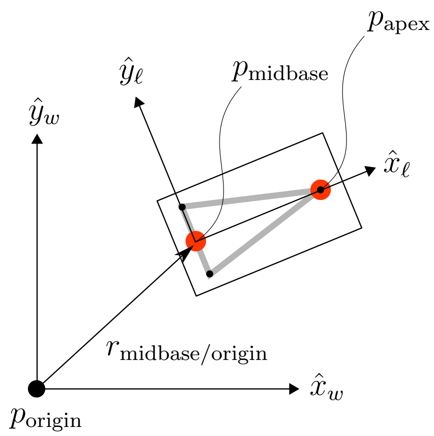

Figure 2: Local vehicle frame definitions

First, transform pixel locations to world coordinates:

$$\boldsymbol{p}^w_{\text{midbase}} = H^w_p \tilde{\boldsymbol{p}}^p_{\text{midbase}}, \quad \boldsymbol{p}^w_{\text{apex}} = H^w_p \tilde{\boldsymbol{p}}^p_{\text{apex}} \tag{14}$$

Obtain the label x and y axes in world coordinates:

$$\hat{x}^w_\ell = \frac{p^w_{\text{apex}} - p^w_{\text{midbase}}}{||p^w_{\text{apex}} - p^w_{\text{midbase}}||}, \qquad \hat{y}^w_\ell = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \hat{x}_\ell \tag{15}$$

The transformation matrix becomes:

$$\boldsymbol{p}^w = H^w_\ell \boldsymbol{p}^\ell, \qquad H^w_\ell = \begin{bmatrix} \hat{x}^w_\ell & \hat{y}^w_\ell & r^w_{\text{midbase/origin}} \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

Agree on a better name than center. This is confusing

replace terminology: top = apex, center = midbase

FIX: Inside H matrix are always VECTORS. not p's.

3.2 From Label Frame to Robot Frame

distinguish between points and vectors: Then vectors suddenly make sense: points in a frame are always defined with respect to the origin of that frame

$$\boldsymbol{p}^b = \begin{bmatrix} I_2 & r^b_{\text{midbase/robot}} \\ 0 & 1 \end{bmatrix} \boldsymbol{p}^\ell \tag{17}$$

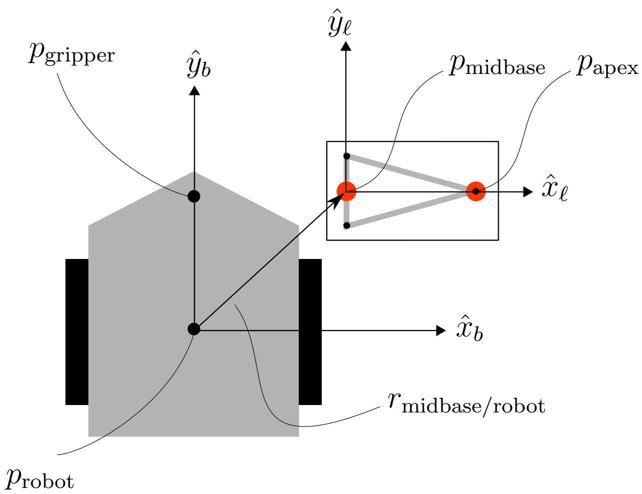

Figure 3: Local vehicle frame definitions

4 Relative Agent Positions and Orientations