Linear Models in R (M1–MIDO)
Lab Session 3 — Solutions

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Dataset Overview: *data_pokemon.csv*

This dataset is adapted from a popular Kaggle Pokémon dataset.
Even if you are not familiar with Pokémon, the data is straightforward:
it combines numeric statistics with categorical attributes, making it well-suited for applying **Ordinary Least Squares (OLS)** in R.

What it contains

- Unique identifiers and names for each Pokémon
- Battle statistics (health, attack, defense, special attack, special defense, speed)
- Categorical features (primary/secondary type, generation, legendary flag)

Fields (Codebook)

- `id`: Unique Pokémon ID
- `name`: Pokémon name
- `type_1`: Primary type (e.g., Water, Fire)
- `type_2`: Secondary type (optional)
- `hp`: Hit points (overall health)
- `attack`: Physical attack strength (**we will use this as** y in most regressions)
- `defense`: Physical defense strength
- `sp_attack`: Special (non-physical) attack strength
- `sp_defense`: Special defense strength
- `speed`: Speed / turn order
- `generation`: Game generation label
- `legendary`: Indicator for legendary status (TRUE/FALSE)

Note on notation

- We treat `attack` as the outcome variable Y.
- Predictor variables (e.g., `defense`, `speed`) will be denoted as $x_1, x_2, \ldots$.
- Factors like `type_1` or `legendary` will be included as categorical predictors.

Setup

To keep numbers readable and reproducible, we set display options:

```
options(scipen = 999, digits = 5)
```

We also load the packages used during this session.

> ⚠ Warning
>
> Don't worry if you don't know them all — we'll introduce functions as we need them. Some provide regression tools, others are for data visualization or diagnostics.

```
library(broom)
library(performance)
library(parameters)
library(datawizard)
library(see)
library(effectsize)
library(insight)
library(correlation)
library(modelbased)
library(glue)
library(scales)
library(GGally)
library(ggpubr)
library(car)
library(lmtest)
library(multcomp)
library(rstatix)
library(matrixTests)
library(ggfortify)
library(qqplotr)
library(patchwork)
library(gtsummary)
library(kableExtra)
library(openxlsx)
library(janitor)
library(collapse)
library(tidyverse)
```

```
source("helper_functions3.R")
```

Question 1. Loading dataset

Import the dataset `data_pokemon.csv` with **`read_csv()`** and save it in an object called `pok`.
Using **`select()`**, keep only the variables `id`, `name`, `attack`, `speed`, `defense`, `hp`, `sp_attack`, and `sp_def`.
Display the first 10 rows of `pok` using **`head()`** or **`slice()`**.

Solutions

- Loading `data_pokemon.csv`

```
pok <- read_csv("data_pokemon.csv", show_col_types = FALSE)
```

- **`head()`** on `pok`

```
head(pok, n = 10)
```

```
# A tibble: 10 x 12
     id name     type_1 type_2   hp attack defense sp_attack sp_def speed generation legendary
   <dbl> <chr>     <chr>  <chr>  <dbl>  <dbl>   <dbl>     <dbl>  <dbl> <dbl>      <dbl> <chr>
 1     1 Bulbasaur Grass  Poison    45     49      49        65     65    45          1 No
 2     2 Ivysaur   Grass  Poison    60     62      63        80     80    60          1 No
 3     3 Venusaur  Grass  Poison    80     82      83       100    100    80          1 No
 4     4 Mega Ven~ Grass  Poison    80    100     123       122    120    80          1 No
 5     5 Charmand~ Fire   None      39     52      43        60     50    65          1 No
 6     6 Charmele~ Fire   None      58     64      58        80     65    80          1 No
 7     7 Charizard Fire   Flying    78     84      78       109     85   100          1 No
 8     8 Mega Cha~ Fire   Dragon    78    130     111       130     85   100          1 No
 9     9 Mega Cha~ Fire   Flying    78    104      78       159    115   100          1 No
10    10 Squirtle  Water  None      44     48      65        50     64    43          1 No
```

Question 2: Exploring categorical variables

The Pokémon dataset contains 4 categorical variables

- `type_1` : the primary type (always present)
- `type_2` : the secondary type (may be missing)
- `legendary`: if the pokemon is legendary of not.
- `genration`: the pokemon generation.

1. Create frequency tables for `type_1`, `type_2`, `legendary` and `genration`. Display both the counts and the relative proportions.

2. Produce bar plots for the distributions of `type_1` and `type_2`.
   - Make one bar plot for type_1 and one for type_2.
   - Ensure that categories on the x-axis are readable (e.g., rotate labels if necessary).

Solutions

- Frequency table for `type_1` with `count()` from `{dplyr}`

```
table_type1_prop <- mutate(pok, type_1 = fct_infreq(type_1)) |>
  count(type_1) |>
  mutate(pct = 100 * n / sum(n))

table_type1_prop
```

```
# A tibble: 18 x 3
   type_1       n   pct
   <fct>    <int> <dbl>
 1 Water      112 14
 2 Normal      98 12.2
 3 Grass       70  8.75
 4 Bug         69  8.62
 5 Psychic     57  7.12
 6 Fire        52  6.5
 7 Electric    44  5.5
 8 Rock        44  5.5
 9 Dragon      32  4
10 Ghost       32  4
11 Ground      32  4
12 Dark        31  3.88
13 Poison      28  3.5
14 Fighting    27  3.38
15 Steel       27  3.38
16 Ice         24  3
17 Fairy       17  2.12
18 Flying       4  0.5
```

- Frequency table for `type_2` with `tabyl()` from `{janitor}`

```
table_type2_prop <- tabyl(pok, type_2) |>
  adorn_pct_formatting(digits = 2) |>
  arrange(desc(n))

table_type2_prop
```

```
   type_2   n percent
     None 386  48.25%
   Flying  97  12.12%
   Ground  35   4.38%
   Poison  34   4.25%
  Psychic  33   4.12%
 Fighting  26   3.25%
    Grass  25   3.12%
    Fairy  23   2.88%
    Steel  22   2.75%
     Dark  20   2.50%
   Dragon  18   2.25%
    Ghost  14   1.75%
      Ice  14   1.75%
     Rock  14   1.75%
    Water  14   1.75%
     Fire  12   1.50%
 Electric   6   0.75%
   Normal   4   0.50%
      Bug   3   0.38%
```

- Frequency table for `legendary` and `generation` with `tabyl()`

```
tabyl(pok, legendary) |>
  adorn_pct_formatting(digits = 2)
```

```
 legendary   n percent
        No 735  91.88%
       Yes  65   8.12%
```

```
tabyl(pok, generation) |>
  adorn_pct_formatting(digits = 2)
```

```
 generation   n percent
          1 166  20.75%
          2 106  13.25%
          3 160  20.00%
          4 121  15.12%
          5 165  20.62%
          6  82  10.25%
```

- Frequency tables for `type_1` and `type_2` with `tbl_summary()` from `{gtsummary}`

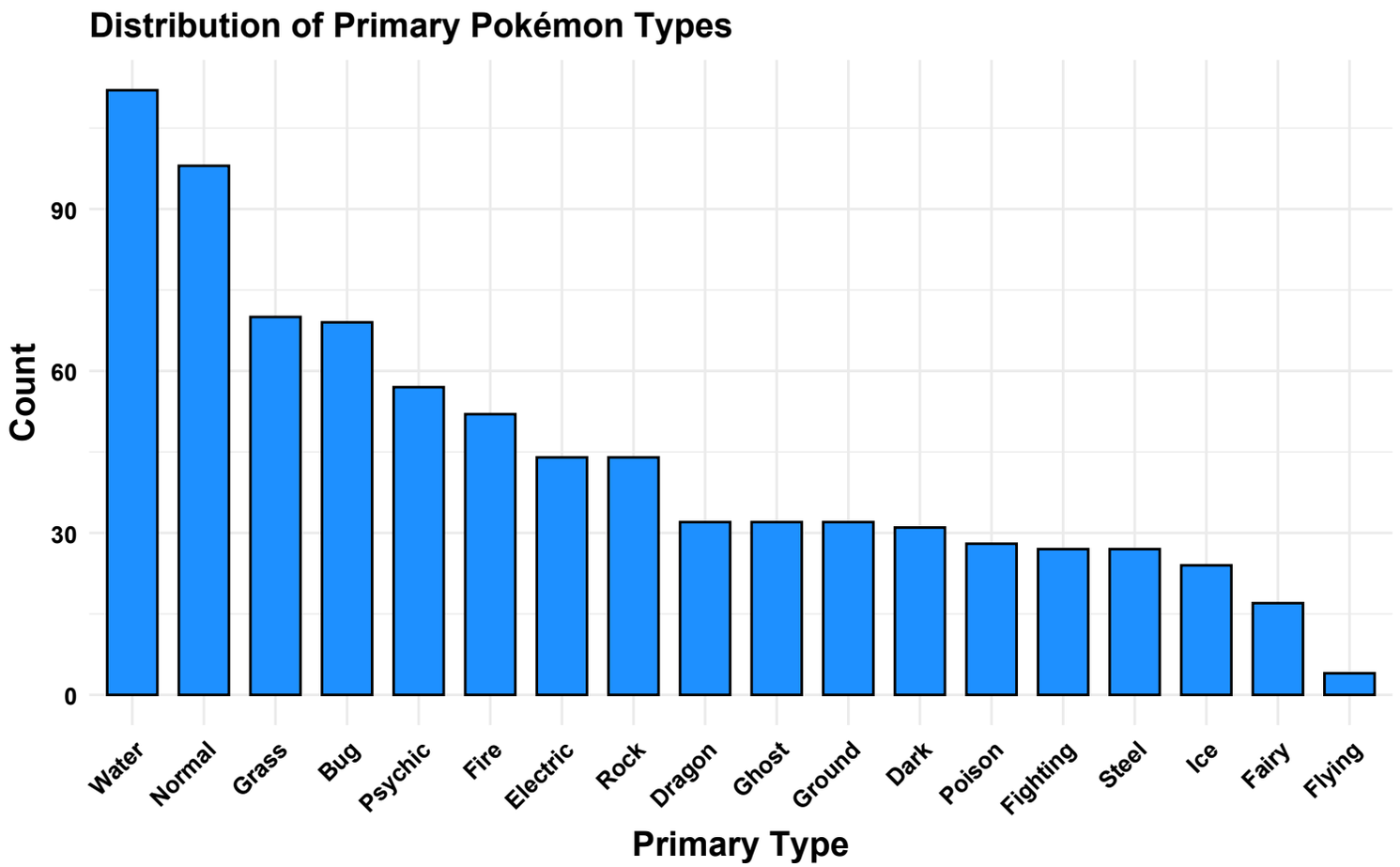
```
select(pok, type_1) |>
  mutate(type_1 = fct_infreq(type_1)) |>
  tbl_summary(label = list(type_1 = "Pokemon primary type")) |>
  modify_header(all_stat_cols() ~ "**{level} (n={n})**") |>
  bold_labels() |>
  remove_footnote_header()

select(pok, type_2) |>
  mutate(type_2 = fct_infreq(type_2)) |>
  tbl_summary(label = list(type_2 = "Pokemon secondary type")) |>
  modify_header(all_stat_cols() ~ "**{level} (n={n})**") |>
  bold_labels() |>
  remove_footnote_header()
```

Characteristic	Overall (n=800)
Pokemon primary type	
Water	112 (14%)
Normal	98 (12%)
Grass	70 (8.8%)
Bug	69 (8.6%)
Psychic	57 (7.1%)
Fire	52 (6.5%)
Electric	44 (5.5%)
Rock	44 (5.5%)
Dragon	32 (4.0%)
Ghost	32 (4.0%)
Ground	32 (4.0%)
Dark	31 (3.9%)
Poison	28 (3.5%)
Fighting	27 (3.4%)
Steel	27 (3.4%)
Ice	24 (3.0%)
Fairy	17 (2.1%)
Flying	4 (0.5%)

Characteristic	Overall (n=800)
Pokemon secondary type	
None	386 (48%)
Flying	97 (12%)
Ground	35 (4.4%)
Poison	34 (4.3%)
Psychic	33 (4.1%)
Fighting	26 (3.3%)
Grass	25 (3.1%)
Fairy	23 (2.9%)
Steel	22 (2.8%)
Dark	20 (2.5%)
Dragon	18 (2.3%)
Ghost	14 (1.8%)
Ice	14 (1.8%)
Rock	14 (1.8%)
Water	14 (1.8%)
Fire	12 (1.5%)
Electric	6 (0.8%)
Normal	4 (0.5%)
Bug	3 (0.4%)

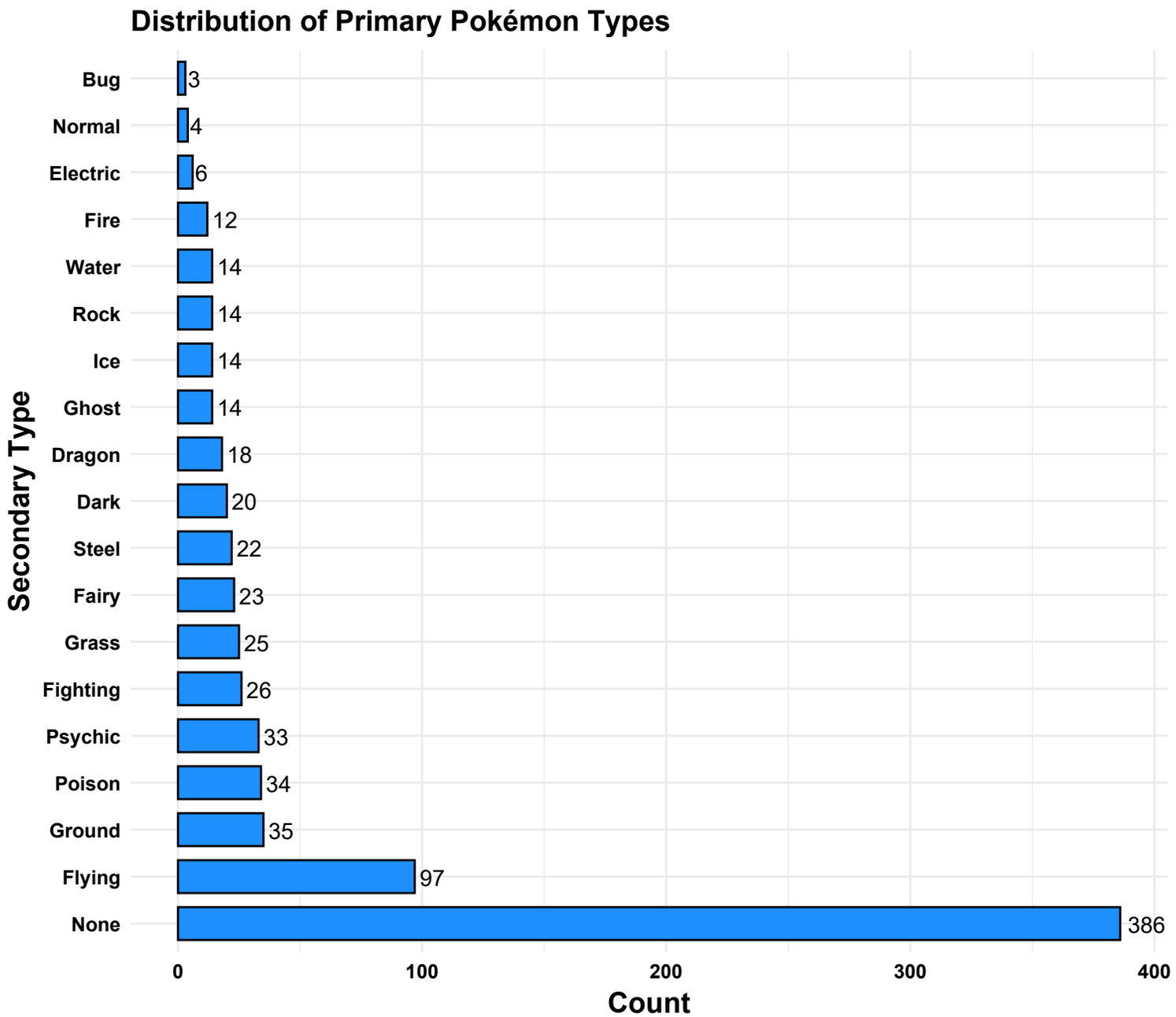
- Bar plots for the distribution of `type_1` with `geom_bar()`

```
pok |>
  mutate(type_1 = fct_infreq(type_1)) |>    # order by descending frequency
  ggplot(aes(x = type_1)) +
  geom_bar(fill = "dodgerblue", color = "black", width = 0.7) +
  labs(
    title = "Distribution of Primary Pokémon Types",
    x = "Primary Type",  y = "Count"
  ) +
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, hjust = 1)) +
  labs_pubr()
```



- Bar plots for the distribution of `type_1` with `geom_col()`

```
pok |>
  mutate(type_2 = fct_infreq(type_2)) |>    # order by descending frequency
  count(type_2) |>  # get the count
  ggplot(aes(y = type_2, x = n, label = n)) +
  geom_col(fill = "dodgerblue", color = "black", width = 0.7) +
  geom_text(hjust = -0.2) +
  labs(
    title = "Distribution of Primary Pokémon Types",
    x = "Count",  y = "Secondary Type"
  ) +
  theme_minimal() +
  labs_pubr()
```



Question 3: Data management, variable creation

The variable (`type_1`), has 18 levels, which can be too many to include directly in an OLS regression. To simplify the analysis, we want to group these 18 types into 3 broader, meaningful groups:

- Physical/Material: Bug, Fighting, Ground, Rock, Steel, Normal
- Elemental/Environmental: Fire, Water, Grass, Electric, Ice, Flying, Poison
- Mystical/Supernatural: Psychic, Ghost, Dragon, Fairy, Dark

1. In the `pok` dataframe, create a new `factor` variable called `type_group3` that assigns each Pokémon to one of the 3 groups above based on its primary type (`type_1`).

2. Create a new binary variable called `has_secondary_type` defined as follows:
   - "Yes" if the Pokémon has a secondary type (`type_2` is not "None")
   - "No" if the Pokémon does not have a secondary type (`type_2` is "None")

3. Transform the variables `legendary` and `generation` into `factors`.

4. Verify that the new variables are well created

Solutions

Creation of the variable `type_group3` and `has_secondary_type`

- Define the 3-level grouping map

```
type_map3 <- list(
  physical_material = c("Bug", "Fighting", "Ground", "Rock", "Steel", "Normal"),
  elemental_env = c("Fire", "Water", "Grass", "Electric", "Ice", "Flying", "Poison"),
  mystical_supernatural = c("Psychic", "Ghost", "Dragon", "Fairy", "Dark")
)
```

- Create variables with `mutate()` from `{dplyr}`

```
pok <- mutate(
  pok,
  type_group3 = case_when(
    type_1 %in% type_map3$physical_material ~ "Physical/Material",
    type_1 %in% type_map3$elemental_env ~ "Elemental/Environmental",
    type_1 %in% type_map3$mystical_supernatural ~ "Mystical/Supernatural",
    .default = NA_character_
  ),
  has_secondary_type = ifelse(type_2 == "None", 0, 1) |> factor(labels = c("No", "Yes"))
) |>
  mutate(type_group3 = fct_infreq(type_group3), legendary = factor(legendary)) |>
  mutate(generation = factor(generation, labels = paste0("G", 1:6))) |>
  relabel(
    type_group3 = "Primary Type", has_secondary_type = "Has a secondary type",
    legendary = "Legendary", generation = "Pokemon generation"
  )
```

- `type_group3`: variable checking

```
tabyl(pok, type_group3) |> as_tibble()
```

```
# A tibble: 3 x 3
  type_group3                n percent
  <fct>                  <int>   <dbl>
1 Elemental/Environmental   334   0.418
2 Physical/Material         297   0.371
3 Mystical/Supernatural     169   0.211
```

```
tabyl(pok, type_1, type_group3) |> as_tibble()
```

```
# A tibble: 18 x 4
   type_1   `Elemental/Environmental` `Physical/Material` `Mystical/Supernatural`
   <chr>                        <dbl>                <dbl>                    <dbl>
 1 Bug                              0                   69                        0
 2 Dark                             0                    0                       31
 3 Dragon                           0                    0                       32
 4 Electric                        44                    0                        0
 5 Fairy                            0                    0                       17
 6 Fighting                         0                   27                        0
 7 Fire                            52                    0                        0
 8 Flying                           4                    0                        0
 9 Ghost                            0                    0                       32
10 Grass                           70                    0                        0
11 Ground                          0                   32                        0
12 Ice                            24                    0                        0
13 Normal                          0                   98                        0
14 Poison                         28                    0                        0
15 Psychic                         0                    0                       57
16 Rock                            0                   44                        0
17 Steel                           0                   27                        0
18 Water                         112                    0                        0
```

- `has_secondary_type`: variable checking

```
tabyl(pok, has_secondary_type) |> as_tibble()
```

```
# A tibble: 2 x 3
  has_secondary_type     n percent
  <fct>              <int>   <dbl>
1 No                   386   0.482
2 Yes                  414   0.518
```

```
tabyl(pok, type_2, has_secondary_type) |> as_tibble()
```

```
# A tibble: 19 x 3
   type_2      No   Yes
   <chr>    <dbl> <dbl>
 1 Bug          0     3
 2 Dark         0    20
 3 Dragon       0    18
 4 Electric     0     6
 5 Fairy        0    23
 6 Fighting     0    26
 7 Fire         0    12
 8 Flying       0    97
 9 Ghost        0    14
10 Grass        0    25
11 Ground       0    35
12 Ice          0    14
13 None       386     0
14 Normal       0     4
15 Poison       0    34
16 Psychic      0    33
17 Rock         0    14
18 Steel        0    22
19 Water        0    14
```

- Cross tabulation between `type_group3` and `has_secondary_type`

```
tabyl(pok, type_group3, has_secondary_type) |>
  adorn_totals(where = c("row", "col")) |>
  adorn_percentages(denominator = "row") |>
  adorn_pct_formatting(digits = 1) |>
  adorn_ns(position = "front") |>
  as_tibble() |>
  mutate(across(everything(), str_squish))
```

```
# A tibble: 4 x 4
  type_group3              No          Yes         Total
  <chr>                    <chr>       <chr>       <chr>
1 Elemental/Environmental 177 (53.0%) 157 (47.0%) 334 (100.0%)
2 Physical/Material       125 (42.1%) 172 (57.9%) 297 (100.0%)
3 Mystical/Supernatural   84 (49.7%)  85 (50.3%)  169 (100.0%)
4 Total                   386 (48.2%) 414 (51.7%) 800 (100.0%)
```

- `legendary` and `generation` : variable checking

```
count(pok, legendary)
```

```
# A tibble: 2 x 2
  legendary     n
  <fct>     <int>
1 No          735
2 Yes          65
```

```
count(pok, generation)
```

```
# A tibble: 6 x 2
  generation     n
  <fct>      <int>
1 G1           166
2 G2           106
3 G3           160
4 G4           121
5 G5           165
6 G6            82
```

Question 4: Box plot

We now want to explore how the Pokémon attack distribution varies across several categorical variables in the dataset. Boxplots are useful for comparing the distribution of a numeric variable across groups.

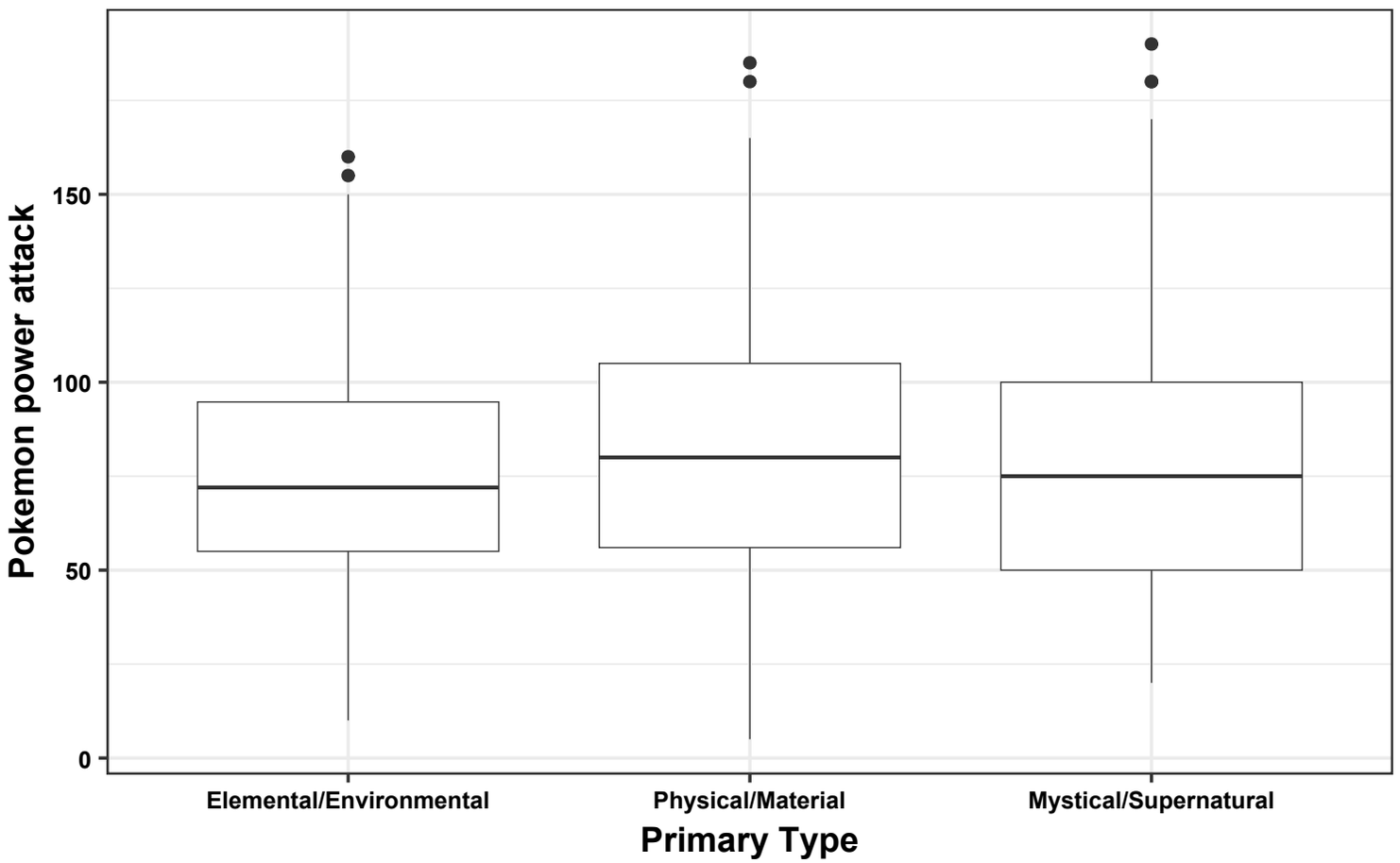
Using the Pokémon dataset, create four separate boxplots where the response variable is `attack`, and the grouping variables are `type_group3` , `has_secondary_type`, `legendary`, `generation`.

Produce the four boxplots either separately or arranged in a multi-panel layout (your choice).

Solutions

- Boxplots of `attack` by `type_group3`

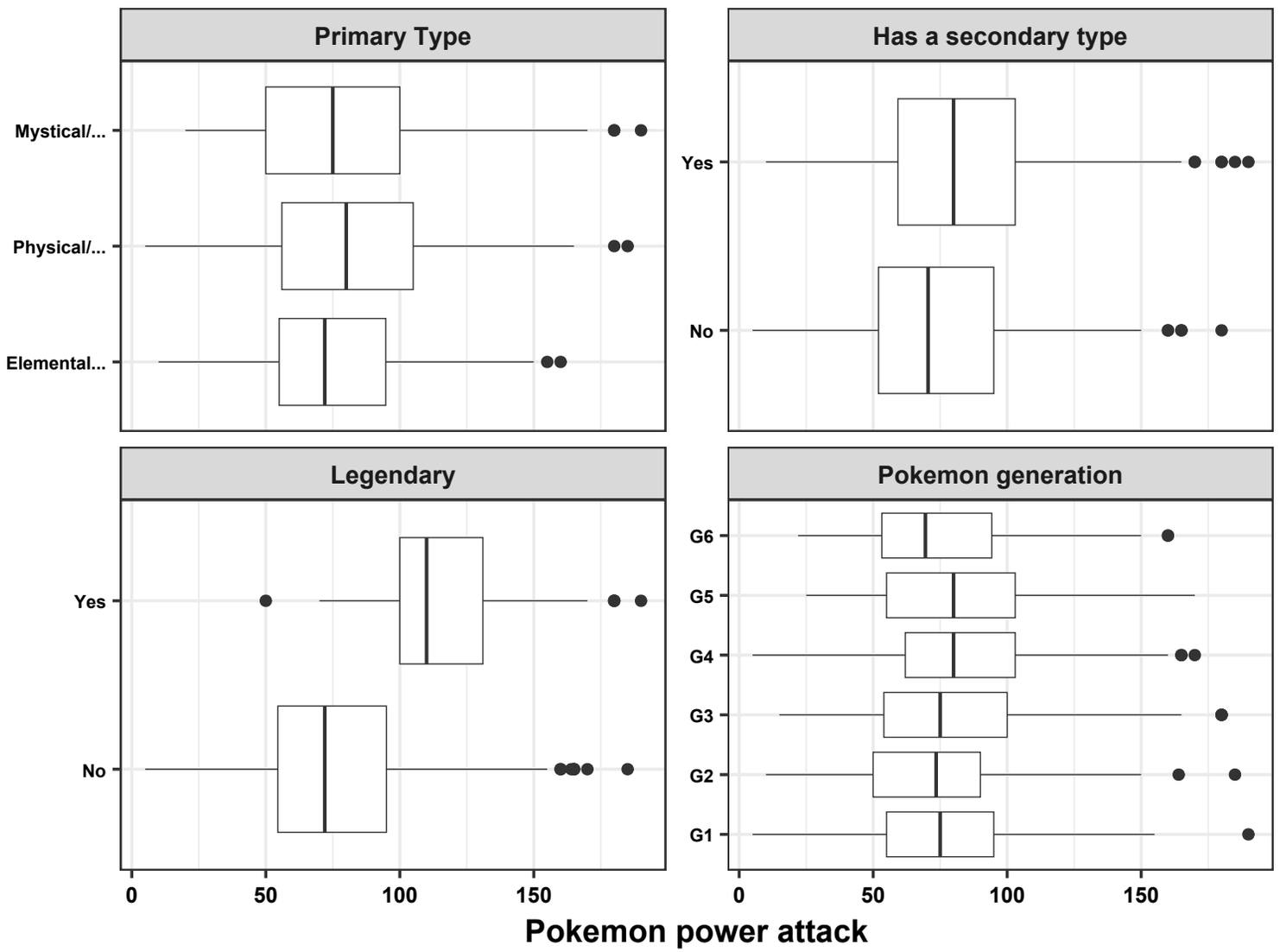
```
ggplot(pok, aes(x = type_group3, y = attack)) +
  geom_boxplot(linewidth = 0.25, median.linewidth = 0.75) +
  labs(y = "Pokemon power attack", x = vlabels(pok$type_group3)) +
  theme_bw(base_size = 14) +
  labs_pubr()
```



- Boxplots in a multi-panel layout

```
varcats <- c("type_group3", "has_secondary_type", "legendary", "generation")
```

```
boxplots <- select(pok, id, attack, all_of(varcats)) |>
  pivot_longer(all_of(varcats), names_to = "var") |>
  mutate(var = factor(var, levels = varcats, labels = vlabels(pok[, varcats]))) |>
  mutate(value = fct_relabel(value, \(x) str_trunc(x, 12))) |>
  ggplot(aes(x = attack, y = value)) +
  geom_boxplot(linewidth = 0.25, median.linewidth = 0.75) +
  facet_wrap(vars(var), scales = "free_y") +
  labs(x = "Pokemon power attack", y = NULL) +
  theme_bw(base_size = 14) +
  labs_pubr() +
  theme(
    strip.text = element_text(size = 11, face = "bold"),
    axis.text.y = element_text(size = 8, face = "bold")
  )

boxplots
```



Question 5: Attack mean over group variables

You have explored the distribution of the variable `attack` using boxplots. We now want to summarize these differences numerically by computing the mean Attack score for several grouping variables.

Using the updated Pokémon dataset, compute the mean value of `attack` for each level of the following categorical variables:

- `type_group3` (3-level grouped primary type)
- `has_secondary_type` ("No" / "Yes")
- `legendary` ("No" / "Yes")
- `generation` ("G1" to "G6")

For each variable, produce a summary table showing at least:

- the group name
- the mean of `attack`
- the standard deviation of `attack`
- the number of observations in each group

Hint: Use `mean_by_group()` from `helper_functions3.R`

Solutions

- We use `mean_by_group()`

```
mean_by_group(data = pok, x = "attack", by = "type_group3")
```

```
# A tibble: 4 x 3
  Variable                    N `Mean (SD)`
  <chr>                   <int> <chr>
1 Primary Type               NA <NA>
2 Elemental/Environmental   334 74.9 (26.8)
3 Physical/Material         297 82.0 (34.4)
4 Mystical/Supernatural     169 81.7 (38.0)
```

```
mean_by_group(pok, "attack", "has_secondary_type")
```

```
# A tibble: 3 x 3
  Variable                 N `Mean (SD)`
  <chr>                <int> <chr>
1 Has a secondary type    NA <NA>
2 No                     386 74.5 (30.5)
3 Yes                    414 83.2 (33.7)
```

```r
mean_by_group(pok, "attack", "legendary")
```

```
# A tibble: 3 x 3
  Variable      N `Mean (SD)`
  <chr>     <int> <chr>
1 Legendary    NA <NA>
2 No          735 75.7 (30.5)
3 Yes          65 116.7 (30.3)
```

```r
mean_by_group(pok, "attack", "generation", digits = 2)
```

```
# A tibble: 7 x 3
  Variable               N `Mean (SD)`
  <chr>              <int> <chr>
1 Pokemon generation    NA <NA>
2 G1                   166 76.64 (30.74)
3 G2                   106 72.03 (32.71)
4 G3                   160 81.63 (36.59)
5 G4                   121 82.87 (32.78)
6 G5                   165 82.07 (30.37)
7 G6                    82 75.80 (29.18)
```

- We can use `mean_by_group()` on different groups with `split()` from Base R and `map()` from `{purrr}`

```r
split(pok, ~legendary) |>
  map(\(d) mean_by_group(d, x = "attack", by = "type_group3", digits = 2)) |>
  list_rbind(names_to = "Legendary Pokemon")
```

```
# A tibble: 8 x 4
  `Legendary Pokemon` Variable                   N `Mean (SD)`
  <chr>               <chr>                  <int> <chr>
1 No                  Primary Type              NA <NA>
2 No                  Elemental/Environmental  314 73.25 (26.25)
3 No                  Physical/Material        283 79.89 (33.20)
4 No                  Mystical/Supernatural    138 72.51 (32.82)
5 Yes                 Primary Type              NA <NA>
6 Yes                 Elemental/Environmental   20 101.40 (21.08)
7 Yes                 Physical/Material         14 125.29 (30.59)
8 Yes                 Mystical/Supernatural     31 122.65 (32.42)
```

- All in one table using `map()` from `{purrr}`

```
tabmeans <- map(varcats, \(by) mean_by_group(pok, "attack", by)) |>
  list_rbind() |>
  mutate(Variable = ifelse(is.na(N), Variable, paste0("\U2000", Variable)))

tabmeans |>
  kable(align = "lcc", padding = 2) |>
  row_spec(c(1, 5, 8, 11), bold = TRUE, color = "white", background = "black")
```

Variable	N	Mean (SD)
Primary Type		
Elemental/Environmental	334	74.9 (26.8)
Physical/Material	297	82.0 (34.4)
Mystical/Supernatural	169	81.7 (38.0)
Has a secondary type		
No	386	74.5 (30.5)
Yes	414	83.2 (33.7)
Legendary		
No	735	75.7 (30.5)
Yes	65	116.7 (30.3)
Pokemon generation		
G1	166	76.6 (30.7)
G2	106	72.0 (32.7)
G3	160	81.6 (36.6)
G4	121	82.9 (32.8)
G5	165	82.1 (30.4)
G6	82	75.8 (29.2)

- To export the table in Excel format, you can use `write.xlsx()` from `{openxlsx}`

```
write.xlsx(tabmeans, "tabmeans.xlsx")
```

Question 6: Dummy variables creation

In the Pokémon dataset create these variables

$$\texttt{legend1} = \begin{cases} 1 & \text{if } \texttt{legendary = "Yes"} \\ 0 & \text{otherwise} \end{cases} \qquad \texttt{legend0} = \begin{cases} 1 & \text{if } \texttt{legendary = "No"} \\ 0 & \text{otherwise} \end{cases}$$

$$\texttt{typeg1} = \begin{cases} 1 & \text{if } \texttt{type\_group3 = "Elemental/Environmental"} \\ 0 & \text{otherwise} \end{cases}$$

$$\texttt{typeg2} = \begin{cases} 1 & \text{if } \texttt{type\_group3 = "Physical/Material"} \\ 0 & \text{otherwise} \end{cases}$$

$$\texttt{typeg3} = \begin{cases} 1 & \text{if } \texttt{type\_group3 = "Mystical/Supernatural"} \\ 0 & \text{otherwise} \end{cases}$$

Hint: use `ifelse()` or any other functions/methods

Solutions

- We create the requested variables

```
pok <- pok |>
  mutate(
    legend1 = ifelse(legendary == "Yes", 1, 0), legend0 = ifelse(legendary == "No", 1, 0)
  ) |>
  mutate(typeg1 = 1 * (type_group3 == "Elemental/Environmental")) |>
  mutate(typeg2 = 1 * (type_group3 == levels(type_group3)[2])) |>
  mutate(typeg3 = 1 * (type_group3 == levels(type_group3)[3]))
```

- Quick check with `count()` from `{dplyr}`

```
count(pok, legendary, legend0, legend1) |> as.data.frame()
```

```
  legendary legend0 legend1   n
1        No       1       0 735
2       Yes       0       1  65
```

```
count(pok, type_group3, typeg1, typeg2, typeg3) |> as.data.frame()
```

```
              type_group3 typeg1 typeg2 typeg3   n
1 Elemental/Environmental      1      0      0 334
2       Physical/Material      0      1      0 297
3   Mystical/Supernatural      0      0      1 169
```

Question 7: Using dummy variables in OLS regression

You have created the dummy variables `legend1`, `legend0`, `typeg1`, `typeg2`, and `typeg3`, which encode information about whether a Pokémon is Legendary and which primary type-group it belongs to.

We now want to explore how these characteristics relate to the `attack` variable using simple and multiple linear regressions.

Using the Pokémon dataset and the dummy variables you created, estimate the following **three OLS regression models**, each with `attack` as the dependent variable:

$$\texttt{attack} = \beta_0 + \beta_1 \texttt{legend1} + \varepsilon \quad \text{(Model 1)}$$

$$\texttt{attack} = \beta_0 + \beta_1 \texttt{typeg2} + \beta_2 \texttt{typeg3} + \varepsilon \quad \text{(Model 2)}$$

$$\texttt{attack} = \beta_0 + \beta_1 \texttt{legend1} + \beta_2 \texttt{typeg2} + \beta_3 \texttt{typeg3} + \varepsilon \quad \text{(Model 3)}$$

1. For each model, report the regression output and interpret the coefficients.
2. Compare the 3 models with `compare_performance()` from `{performance}`.
3. Refit the 3 models using the `factor` variables `legendary` and `type_group3`.

Solutions

- We fit the 3 models

```
mod1 <- lm(attack ~ legend1, data = pok)
```

- This specification implies that non-legendary Pokémon is the reference group

```
mod2 <- lm(attack ~ typeg2 + typeg3, data = pok)
```

- This specification implies that `Elemental/Environmental` Pokémon is the reference group

```
mod3 <- lm(attack ~ legend1 + typeg2 + typeg3, data = pok)
```

- This specification implies that non-legendary Pokémon and `Elemental/Environmental` are the reference groups

Model 1 with `model_parameters()` from `{parameters}`

```
model_parameters(mod1, ci_method = "residual", digits = 1)
```

```
Parameter   | Coefficient |  SE |       95% CI | t(798) |      p
-----------------------------------------------------------------
(Intercept) |        75.7 | 1.1 | [73.5, 77.9] |   67.3 | < .001
legend1     |        41.0 | 3.9 | [33.3, 48.7] |   10.4 | < .001
```

$\texttt{attack} = \beta_0 + \beta_1 \texttt{legend1} + \varepsilon$	
Profile	$\mathbb{E}(\texttt{attack} \mid \cdot)$
Non-legendary Pokémon	β_0
Legendary Pokémon	$\beta_0 + \beta_1$

- `Intercept`: $\hat{\beta}_0 = 75.7$: mean `attack` for non-legendary Pokémon.
- `legend1`: $\hat{\beta}_1 = 41.0$: difference in mean `attack` between legendary and non-legendary Pokémon is on average equal to 41.0 $(p < 0.001)$.

Model 2

```
model_parameters(mod2, ci_method = "residual", digits = 1)
```

```
Parameter   | Coefficient |  SE |       95% CI | t(797) |      p
-----------------------------------------------------------------
(Intercept) |        74.9 | 1.8 | [71.5, 78.4] |   42.4 | < .001
typeg2      |         7.1 | 2.6 | [ 2.0, 12.1] |    2.8 | 0.006
typeg3      |         6.8 | 3.1 | [ 0.8, 12.8] |    2.2 | 0.027
```

$\texttt{attack} = \beta_0 + \beta_1 \texttt{typeg2} + \beta_2 \texttt{typeg3} + \varepsilon$	
Profile	$\mathbb{E}(\texttt{attack} \mid \cdot)$
Elemental/Environmental	β_0
Physical/Material	$\beta_0 + \beta_1$
Mystical/Supernatural	$\beta_0 + \beta_2$

- `Intercept`: $\hat{\beta}_0 = 74.9$: mean `attack` for Pokémon in the `Elemental/Environmental` group (reference).
- `typeg2`: $\hat{\beta}_1 = 7.1$: difference in mean `attack` between the `Physical/Material` group and the `Elemental/Environmental` group is on average equal to 7.1 $(p = 0.006)$.
- `typeg3`: $\hat{\beta}_2 = 6.8$: difference in mean `attack` between the `Mystical/Supernatural` group and the `Elemental/Environmental` group is on average equal to 6.8 $(p = 0.027)$.

Model 3

```
model_parameters(mod3, ci_method = "residual", digits = 1)
```

```
Parameter   | Coefficient |  SE |       95% CI | t(796) |      p
-------------------------------------------------------------------
(Intercept) |        72.4 | 1.7 | [69.1, 75.7] |   43.2 | < .001
legend1     |        41.8 | 4.0 | [34.0, 49.7] |   10.5 | < .001
typeg2      |         7.6 | 2.4 | [ 2.9, 12.4] |    3.2 | 0.002
typeg3      |         1.6 | 2.9 | [-4.1,  7.3] |    0.5 | 0.583
```

$\texttt{attack} = \beta_0 + \beta_1 \texttt{legend1} + \beta_2 \texttt{typeg2} + \beta_3 \texttt{typeg3} + \varepsilon$	
Profile	$\mathbb{E}(\texttt{attack} \mid \cdot)$
Non-legendary, Elemental/Environmental	β_0
Non-legendary, Physical/Material	$\beta_0 + \beta_2$
Non-legendary, Mystical/Supernatural	$\beta_0 + \beta_3$
Legendary, Elemental/Environmental	$\beta_0 + \beta_1$
Legendary, Physical/Material	$\beta_0 + \beta_1 + \beta_2$
Legendary, Mystical/Supernatural	$\beta_0 + \beta_1 + \beta_3$

- `Intercept`: $\hat{\beta}_0 = 72.4$: mean `attack` for non-legendary Pokémon in the `Elemental/Environmental` group (reference category).
- `legend1`: $\hat{\beta}_1 = 41.8$: difference in mean `attack` between the legendary and non-legendary Pokémon is on average equal to 41.8 ($p < 0.001$), **for all primary type**.
- `typeg2`: $\hat{\beta}_2 = 7.6$: difference in mean `attack` between the `Physical/Material` group and the `Elemental/Environmental` group is on is on average equal to 7.6 ($p = 0.002$), **for all legendary status**.
- `typeg3`: $\hat{\beta}_3 = 1.6$: difference in mean `attack` between the `Mystical/Supernatural` group and the `Elemental/Environmental` group is on is on average equal to 1.6 ($p = 0.583$), **for all legendary status**.

- We compare the 3 models

```
compare_performance(
  mod1, mod2, mod3,
  metrics = c("AIC", "BIC", "R2", "R2_adj", "SIGMA", "RMSE")
)
```

```
# Comparison of Model Performance Indices

Name | Model |  AIC (weights) |  BIC (weights) |    R2 | R2 (adj.) |   RMSE |  Sigma
-------------------------------------------------------------------------------------
mod1 |    lm | 7741.5 (0.038) | 7755.6 (0.811) | 0.119 |     0.118 | 30.441 | 30.479
mod2 |    lm | 7836.1 (<.001) | 7854.9 (<.001) | 0.011 |     0.009 | 32.254 | 32.315
mod3 |    lm | 7735.1 (0.962) | 7758.5 (0.189) | 0.131 |     0.127 | 30.242 | 30.318
```

- We perform nested F tests

```
anova(mod1, mod3)
```

```
linearHypothesis(mod3, c("typeg2", "typeg3"))
```

```
Linear hypothesis test:
typeg2 = 0
typeg3 = 0

Model 1: restricted model
Model 2: attack ~ legend1 + typeg2 + typeg3

  Res.Df    RSS Df Sum of Sq    F Pr(>F)
1    798 741307
2    796 731682  2      9624 5.24 0.0055 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(mod2, mod3)
```

```
Analysis of Variance Table

Model 1: attack ~ typeg2 + typeg3
Model 2: attack ~ legend1 + typeg2 + typeg3
  Res.Df    RSS Df Sum of Sq   F               Pr(>F)
1    797 832263
2    796 731682  1    100580 109 <0.0000000000000002 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- We refit the 3 models using the `factor` variables `legendary` and `type_group3`

```
mod1bis <- lm(attack ~ legendary, data = pok)
model_parameters(mod1bis, digits = 1, include_reference = TRUE)
```

```
Parameter       | Coefficient |  SE |       95% CI | t(798) |      p
-------------------------------------------------------------------
(Intercept)     |        75.7 | 1.1 | [73.5, 77.9] |   67.3 | < .001
legendary [No]  |         0.0 |     |              |        |
legendary [Yes] |        41.0 | 3.9 | [33.3, 48.7] |   10.4 | < .001
```

```
mod2bis <- lm(attack ~ type_group3, data = pok)
model_parameters(mod2bis, digits = 1, include_reference = TRUE)
```

```
Parameter                                | Coefficient |  SE |       95% CI | t(797) |      p
-------------------------------------------------------------------------------------------
(Intercept)                              |        74.9 | 1.8 | [71.5, 78.4] |   42.4 | < .001
type group3 [Elemental/Environmental]    |         0.0 |     |              |        |
type group3 [Physical/Material]          |         7.1 | 2.6 | [ 2.0, 12.1] |    2.8 | 0.006
type group3 [Mystical/Supernatural]      |         6.8 | 3.1 | [ 0.8, 12.8] |    2.2 | 0.027
```

```
mod3bis <- lm(attack ~ legendary + type_group3, data = pok)
model_parameters(mod3bis, digits = 1, include_reference = TRUE)
```

```
Parameter                                | Coefficient |  SE |       95% CI | t(796) |      p
-------------------------------------------------------------------------------------------
(Intercept)                              |        72.4 | 1.7 | [69.1, 75.7] |   43.2 | < .001
legendary [No]                           |         0.0 |     |              |        |
legendary [Yes]                          |        41.8 | 4.0 | [34.0, 49.7] |   10.5 | < .001
type group3 [Elemental/Environmental]    |         0.0 |     |              |        |
type group3 [Physical/Material]          |         7.6 | 2.4 | [ 2.9, 12.4] |    3.2 | 0.002
type group3 [Mystical/Supernatural]      |         1.6 | 2.9 | [-4.1,  7.3] |    0.5 | 0.583
```

- With this parametrization, the function `Anova` from `{car}` is very usefull to test the significance of every variable in the model

```
Anova(mod3bis, type = 3)
```

```
Anova Table (Type III tests)

Response: attack
             Sum Sq  Df F value               Pr(>F)
(Intercept) 1716647   1 1867.55 <0.0000000000000002 ***
legendary    100580   1  109.42 <0.0000000000000002 ***
type_group3    9624   2    5.24               0.0055 **
Residuals    731682 796
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- You can change the reference group with `relevel()` or with `C()`. For example, if you want `Mystical/Supernatural` as the reference for primary type (`type_group3`)

```
lm(attack ~ legendary + relevel(type_group3, ref = 3), data = pok) |>
  model_parameters() |>
  format_table(select = "{estimate} [{ci}]|{p}", digits = 1)
```

```
                                          Parameter  Coefficient [CI]      p
1                                       (Intercept) 74.0 [69.2, 78.8] <0.001
2                                   legendary [Yes] 41.8 [34.0, 49.7] <0.001
3 relevel(type group3, ref = 3)Elemental/Environmental -1.6 [-7.3,  4.1]  0.583
4       relevel(type group3, ref = 3)Physical/Material  6.0 [ 0.2, 11.9]  0.043
```

```
lm(attack ~ legendary + C(type_group3, base = 3), data = pok) |>
  model_parameters() |>
  format_table(select = "{estimate} [{ci}]|{p}", digits = 1)
```

```
                                        Parameter  Coefficient [CI]      p
1                                     (Intercept) 74.0 [69.2, 78.8] <0.001
2                                 legendary [Yes] 41.8 [34.0, 49.7] <0.001
3 C(type group3, base = 3) [Elemental/Environmental] -1.6 [-7.3,  4.1]  0.583
4       C(type group3, base = 3) [Physical/Material]  6.0 [ 0.2, 11.9]  0.043
```

- We can also use the constraint $\sum \alpha_i = 0$ (course notation).

```
lm(attack ~ legendary + C(type_group3, contr = sum), data = pok) |>
  model_parameters() |>
  format_table(select = "{estimate} [{ci}]|{p}", digits = 1)
```

```
                       Parameter  Coefficient [CI]      p
1                    (Intercept) 75.5 [73.2, 77.8] <0.001
2                legendary [Yes] 41.8 [34.0, 49.7] <0.001
3 C(type group3, contr = sum) [1] -3.1 [-6.0, -0.2]  0.038
4 C(type group3, contr = sum) [2]  4.6 [ 1.6,  7.5]  0.003
```

- See the course for the interpretation of coefficients. Some statisticians dislike this parameterization.

Question 8: Testing equality of coefficients

Consider OLS model (`mod3`):

$$\widehat{\texttt{attack}} = 72.435 + 41.84 \times \texttt{legend1} + 7.623 \times \texttt{typeg2} + 1.595 \times \texttt{typeg3}$$

```
Parameter   | Coefficient |    SE |          95% CI | t(796) |      p
-------------------------------------------------------------------
(Intercept) |      72.435 | 1.676 | [69.145, 75.725] | 43.215 | < .001
legend1     |      41.840 | 4.000 | [33.988, 49.691] | 10.460 | < .001
typeg2      |       7.623 | 2.419 | [ 2.876, 12.371] |  3.152 | 0.002
typeg3      |       1.595 | 2.904 | [-4.106,  7.296] |  0.549 | 0.583
```

Perform the following test

$$H_0 : \beta_2 = \beta_3 \qquad \text{versus} \qquad H_1 : \beta_2 \neq \beta_3.$$

Solutions

- We can perform the test with `linearHypothesis()` from `{car}`

```
linearHypothesis(mod3, c("typeg2 = typeg3")) |> as_tibble()
```

```
# A tibble: 2 x 6
  Res.Df     RSS    Df `Sum of Sq`     F `Pr(>F)`
   <dbl>   <dbl> <dbl>       <dbl> <dbl>    <dbl>
1    797 735465.    NA          NA  NA     NA
2    796 731682.     1       3783.  4.12   0.0428
```

- **Conclusion**: β_2 is significantly different from β_3.
  There is difference in power between `Physical/Material` and `Mystical/Supernatural`.

- We can perform the test with `glht()` from `{multcomp}`. We also get the estimated difference and CI

```
glht(mod3, linfct = c("typeg2 - typeg3 = 0")) |>
  model_parameters(digits = 3, verbose = FALSE)
```

```
# Fixed Effects

Parameter            | Coefficient |    SE |          95% CI | t(796) |     p
-------------------------------------------------------------------------------
typeg2 - typeg3 == 0 |       6.029 | 2.972 | [0.195, 11.862] |  2.029 | 0.043
```

- What if we use `factor` variables in the model ?

```
model_parameters(mod3bis, digits = 1, verbose = FALSE)
```

```
Parameter                          | Coefficient |  SE |       95% CI | t(796) |      p
-------------------------------------------------------------------------------------
(Intercept)                        |        72.4 | 1.7 | [69.1, 75.7] |   43.2 | < .001
legendary [Yes]                    |        41.8 | 4.0 | [34.0, 49.7] |   10.5 | < .001
type group3 [Physical/Material]    |         7.6 | 2.4 | [ 2.9, 12.4] |    3.2 | 0.002
type group3 [Mystical/Supernatural] |         1.6 | 2.9 | [-4.1,  7.3] |    0.5 | 0.583
```

- Name of the model coefficients with `find_parameters()` from `{insight}`

```
name_param <- find_parameters(mod3bis, flatten = TRUE)
name_param
```

```
[1] "(Intercept)"                      "legendaryYes"
[3] "type_group3Physical/Material"     "type_group3Mystical/Supernatural"
```

- Test with `linearHypothesis()`

```
linearHypothesis(mod3bis, glue("{name_param[3]} = {name_param[4]}")) |>
  as.data.frame()
```

```
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)
1    797 735465 NA        NA     NA       NA
2    796 731682  1      3783 4.1155 0.042824
```

- Test with `glht()`

```
C <- rbind("beta2 - beta3" = c(0, 0, 1, -1))
C
```

```
             [,1] [,2] [,3] [,4]
beta2 - beta3    0    0    1   -1
```

```
glht(mod3bis, linfct = C) |>
  model_parameters(digits = 3, verbose = FALSE)
```

```
# Fixed Effects

Parameter          | Coefficient |    SE |          95% CI | t(796) |     p
-------------------------------------------------------------------------
beta2 - beta3 == 0 |       6.029 | 2.972 | [0.195, 11.862] |  2.029 | 0.043
```

Question 9: Predicting mean `attack` for all category combinations

Consider the following regression model estimated using the Pokémon dataset (`mod3bis`):

$$\texttt{attack} = \beta_0 + \beta_1 \texttt{legendary}_{\texttt{Yes}} + \beta_2 \texttt{type\_group3}_{\texttt{Physical/Material}} + \beta_3 \texttt{type\_group3}_{\texttt{Mystical/Supernatural}} + \varepsilon$$

Parameter	Coefficient	SE	95% CI	t(796)	p
(Intercept)	72.4	1.7	[69.1, 75.7]	43.2	< .001
legendary [Yes]	41.8	4.0	[34.0, 49.7]	10.5	< .001
type group3 [Physical/Material]	7.6	2.4	[2.9, 12.4]	3.2	0.002
type group3 [Mystical/Supernatural]	1.6	2.9	[-4.1, 7.3]	0.5	0.583

We want to compute the **predicted mean Attack** for every combination of these two variables.

1. Create a prediction grid containing all $2 \times 3 = 6$ combinations of
   - `legendary` $\in$ {Yes, No}
   - `type_group3` $\in$ {Elemental/Environmental, Physical/Material, Mystical/Supernatural}
2. Compute the predicted mean of `attack` for each of the six combinations.
3. Present the results in a table showing:
   - the combination of categories
   - the predicted mean Attack with standard error or 95% confidence interval

Solutions

- Create all $2 \times 3 = 6$ combinations of `legendary` and `type_group3`

```
newdata_grid <- expand_grid(
  legendary = fct_unique(pok$legendary),
  type_group3 = fct_unique(pok$type_group3)
) |>
  arrange(legendary, type_group3)

newdata_grid
```

```
# A tibble: 6 x 2
  legendary type_group3
  <fct>     <fct>
1 No        Elemental/Environmental
2 No        Physical/Material
3 No        Mystical/Supernatural
4 Yes       Elemental/Environmental
5 Yes       Physical/Material
6 Yes       Mystical/Supernatural
```

- Get predictions with standard errors and CI

```
predictions <- predict(
  mod3bis,
  newdata = newdata_grid, se.fit = TRUE, interval = "confidence"
)

pred_means_tidy <- bind_cols(newdata_grid, predictions[["fit"]]) |>
  mutate(across(is.numeric, \(x) style_number(x, digits = 1))) |>
  mutate("Mean [95% CI]" = glue("{fit} [{lwr}, {upr}]")) |>
  select(Legendary = legendary, "Primary Type" = type_group3, "Mean [95% CI]")
```

```
pred_means_tidy
```

```
# A tibble: 6 x 3
  Legendary `Primary Type`           `Mean [95% CI]`
  <fct>     <fct>                    <glue>
1 No        Elemental/Environmental 72.4 [69.1, 75.7]
2 No        Physical/Material       80.1 [76.6, 83.5]
3 No        Mystical/Supernatural   74.0 [69.2, 78.8]
4 Yes       Elemental/Environmental 114.3 [106.2, 122.3]
5 Yes       Physical/Material       121.9 [113.7, 130.1]
6 Yes       Mystical/Supernatural   115.9 [108.0, 123.7]
```

- With `estimate_expectation()` from `{modelbased}`

```
estimate_expectation(mod3bis, data = newdata_grid, ci = 0.95) |>
  rename(Legendary = 1, "Primary Type" = type_group3)
```

```
means <- estimate_expectation(mod3bis, by = c("type_group3", "legendary"), ci = 0.95) |>
  rename(Legendary = 1, "Primary Type" = type_group3)

means
```

```
Model-based Predictions

Primary Type            | legendary | Predicted |   SE |           95% CI
-------------------------------------------------------------------------
Elemental/Environmental | No        |     72.43 | 1.68 | [ 69.14,  75.72]
Physical/Material       | No        |     80.06 | 1.77 | [ 76.58,  83.53]
Mystical/Supernatural   | No        |     74.03 | 2.44 | [ 69.23,  78.83]
Elemental/Environmental | Yes       |    114.27 | 4.11 | [106.21, 122.34]
Physical/Material       | Yes       |    121.90 | 4.20 | [113.66, 130.14]
Mystical/Supernatural   | Yes       |    115.87 | 4.01 | [107.99, 123.75]

Variable predicted: attack
Predictors modulated: type_group3, legendary
```

- Contrast analysis can be achieved through `estimate_contrasts()` from `{modelbased}`

```
predictors <- c("legendary", "type_group3")
```

```
mod3bis |>
  estimate_contrasts(contrast = predictors, comparison = "pairwise", p_adjust = "bonferroni") |>
  format_table(select = "{estimate} [{ci}]|{p}", digits = 1) |>
  kable(format = "pipe", align = "l")
```

Level1	Level2	Difference [CI]	p
No, Physical/Material	No, Elemental/Environmental	7.6 [2.9, 12.4]	0.025
No, Mystical/Supernatural	No, Elemental/Environmental	1.6 [-4.1, 7.3]	> .999
Yes, Elemental/Environmental	No, Elemental/Environmental	41.8 [34.0, 49.7]	<0.001
Yes, Physical/Material	No, Elemental/Environmental	49.5 [40.2, 58.7]	<0.001
Yes, Mystical/Supernatural	No, Elemental/Environmental	43.4 [34.6, 52.3]	<0.001
No, Mystical/Supernatural	No, Physical/Material	-6.0 [-11.9, -0.2]	0.642
Yes, Elemental/Environmental	No, Physical/Material	34.2 [25.1, 43.3]	<0.001
Yes, Physical/Material	No, Physical/Material	41.8 [34.0, 49.7]	<0.001
Yes, Mystical/Supernatural	No, Physical/Material	35.8 [26.9, 44.7]	<0.001
Yes, Elemental/Environmental	No, Mystical/Supernatural	40.2 [29.8, 50.7]	<0.001
Yes, Physical/Material	No, Mystical/Supernatural	47.9 [37.3, 58.5]	<0.001
Yes, Mystical/Supernatural	No, Mystical/Supernatural	41.8 [34.0, 49.7]	<0.001
Yes, Physical/Material	Yes, Elemental/Environmental	7.6 [2.9, 12.4]	0.025
Yes, Mystical/Supernatural	Yes, Elemental/Environmental	1.6 [-4.1, 7.3]	> .999
Yes, Mystical/Supernatural	Yes, Physical/Material	-6.0 [-11.9, -0.2]	0.642

Question 10. Residual diagnostics

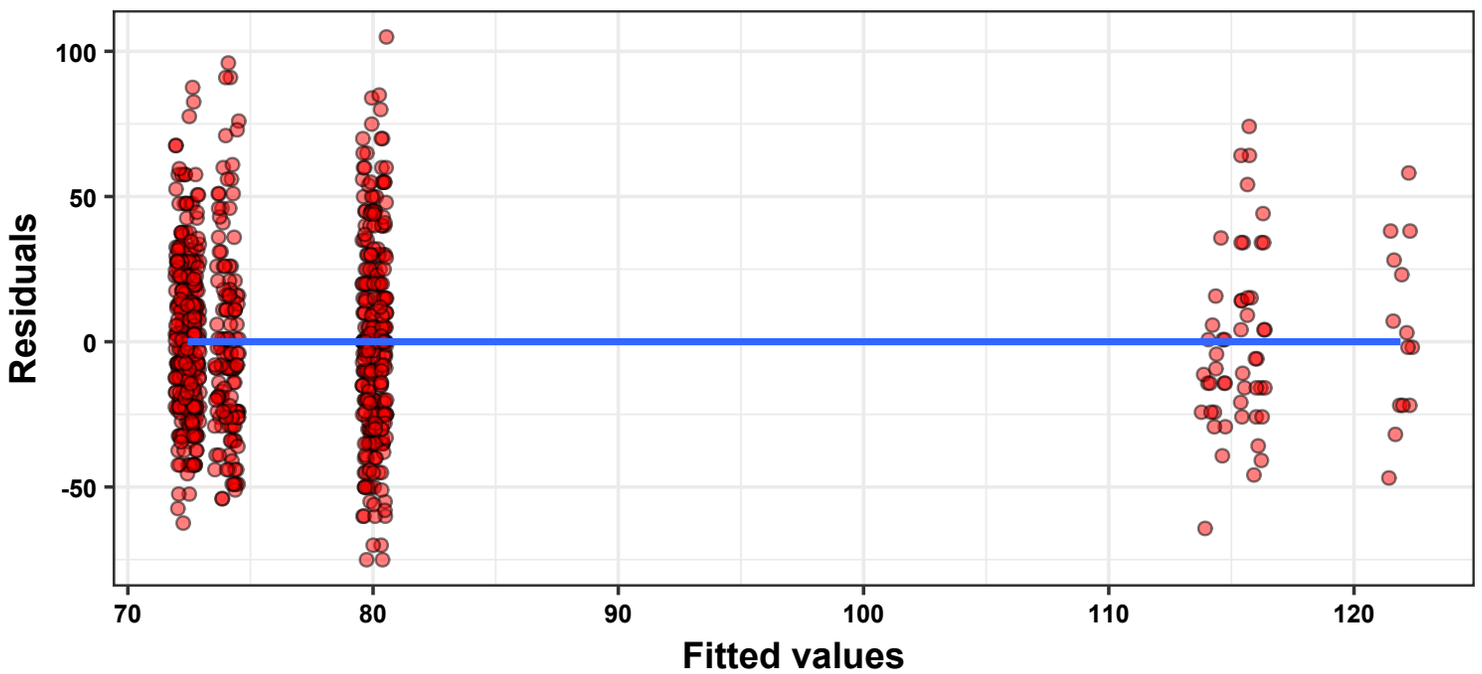
Using `mod3bis` and functions from the file `helper_functions3.R`:

1. Plot residuals vs fitted values and vs each predictor.
2. Plot $\sqrt{|\text{Standardized residuals}|}$ vs fitted values and vs each predictor.
3. Plot studentized residuals vs fitted values and vs each predictor.
4. Plot residuals vs order of observation.
5. Plot a histogram and normal Q-Q plot of the standardized residuals.

Solutions

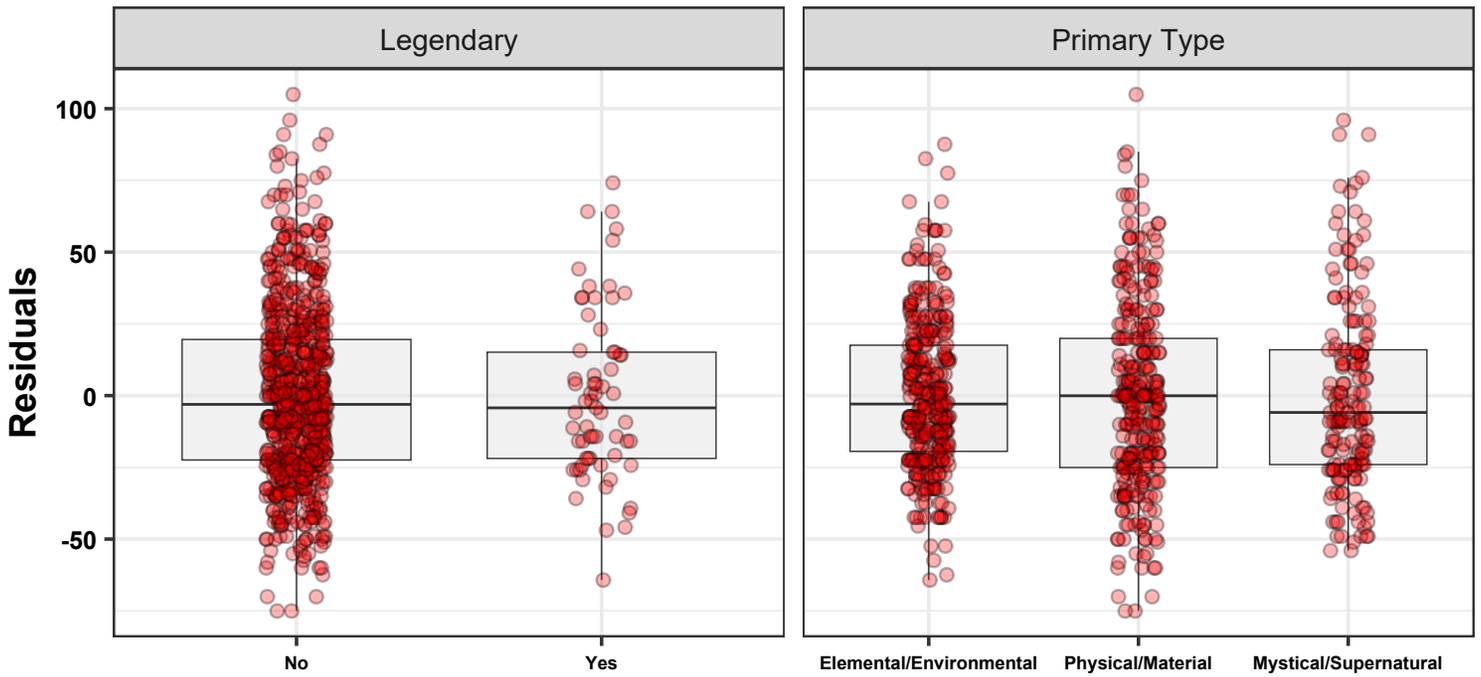
- Residuals vs fitted

```
resid_vs_fit(mod3bis, which = "res", jitter = TRUE)
```



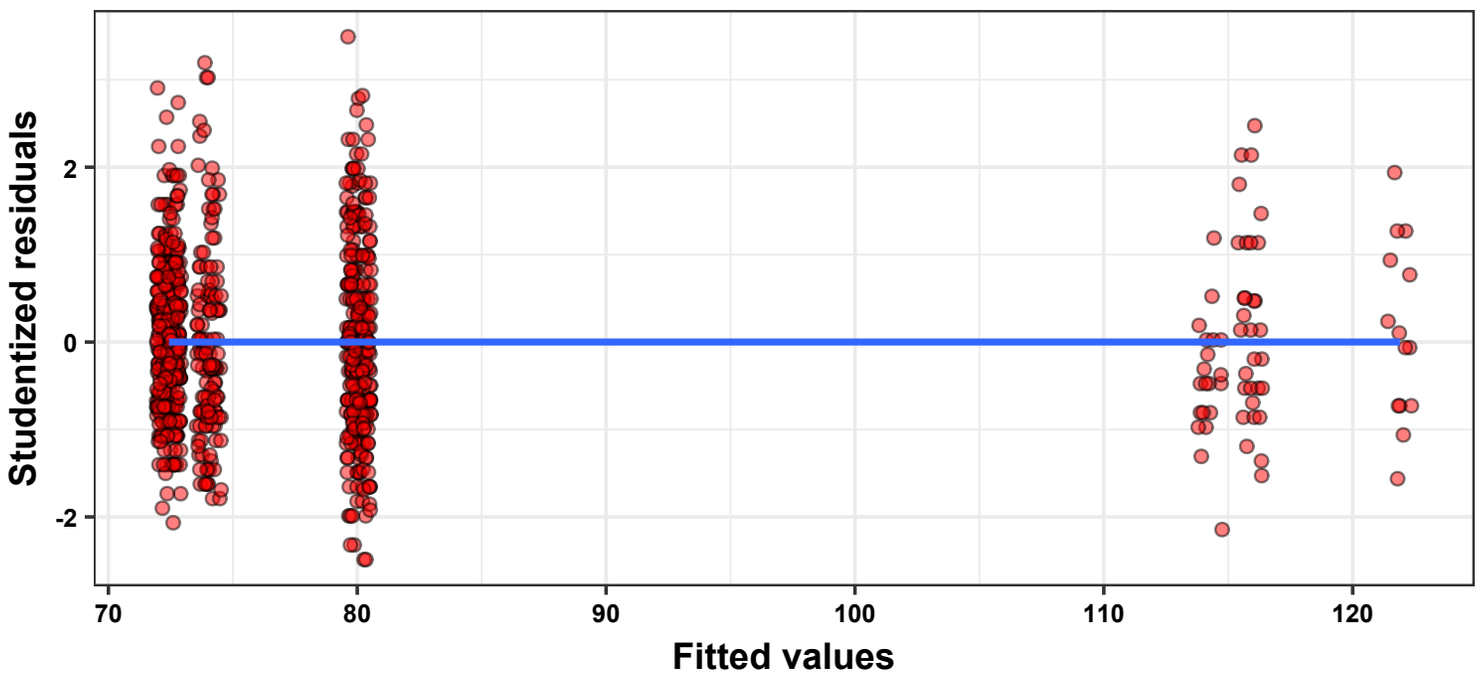
- Residuals vs predictors

```
resid_vs_var_factor(mod3bis, predictors, which = "res", fill = "red") +
  theme(axis.text.x = element_text(size = 7))
```



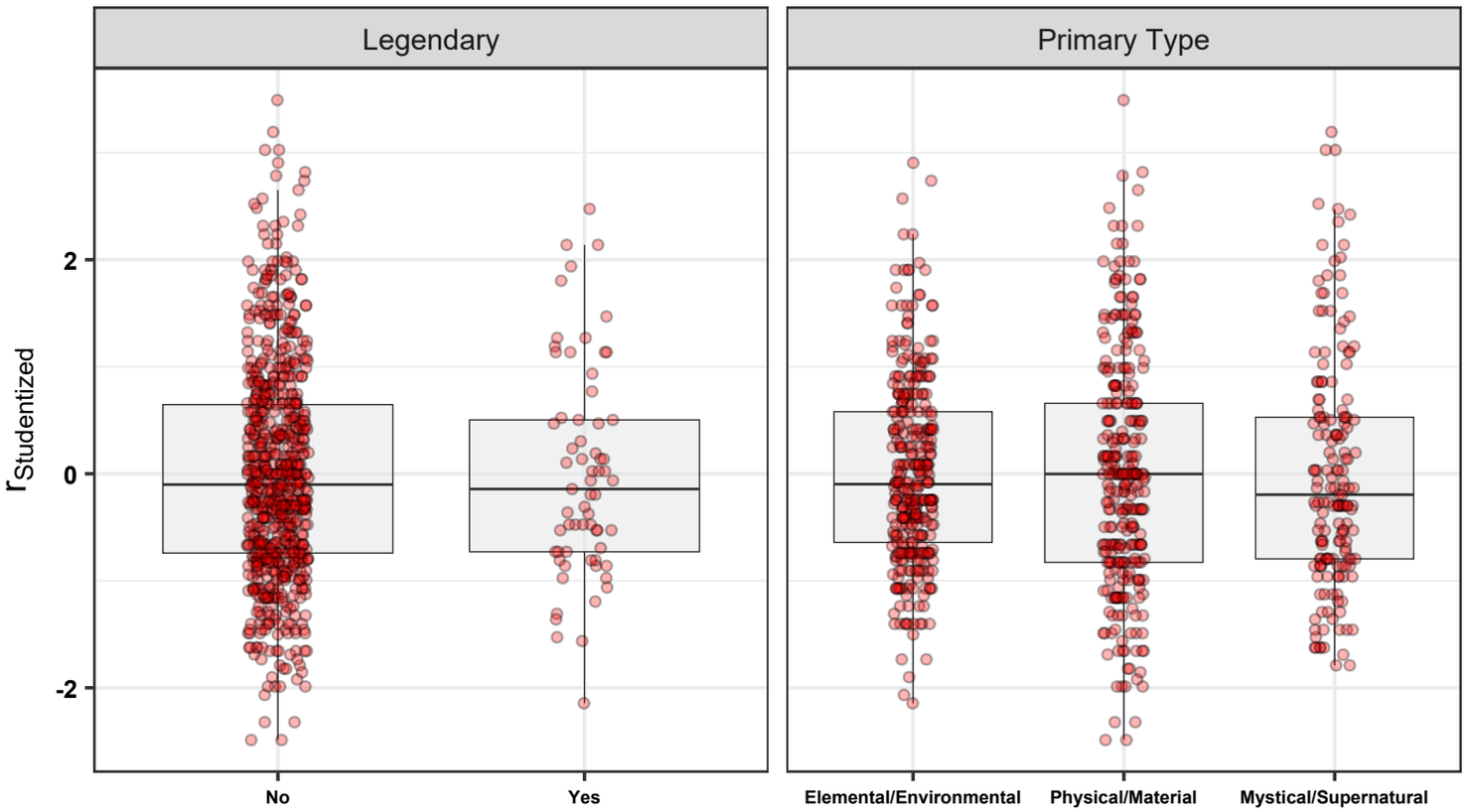
- Studentized residuals vs fitted values

```
resid_vs_fit(mod3bis, which = "rstud", jitter = TRUE)
```



- Residuals vs predictors

```
resid_vs_var_factor(mod3bis, predictors, which = "rstud", fill = "red", size = 1.5) +
  theme(axis.text.x = element_text(size = 7))
```

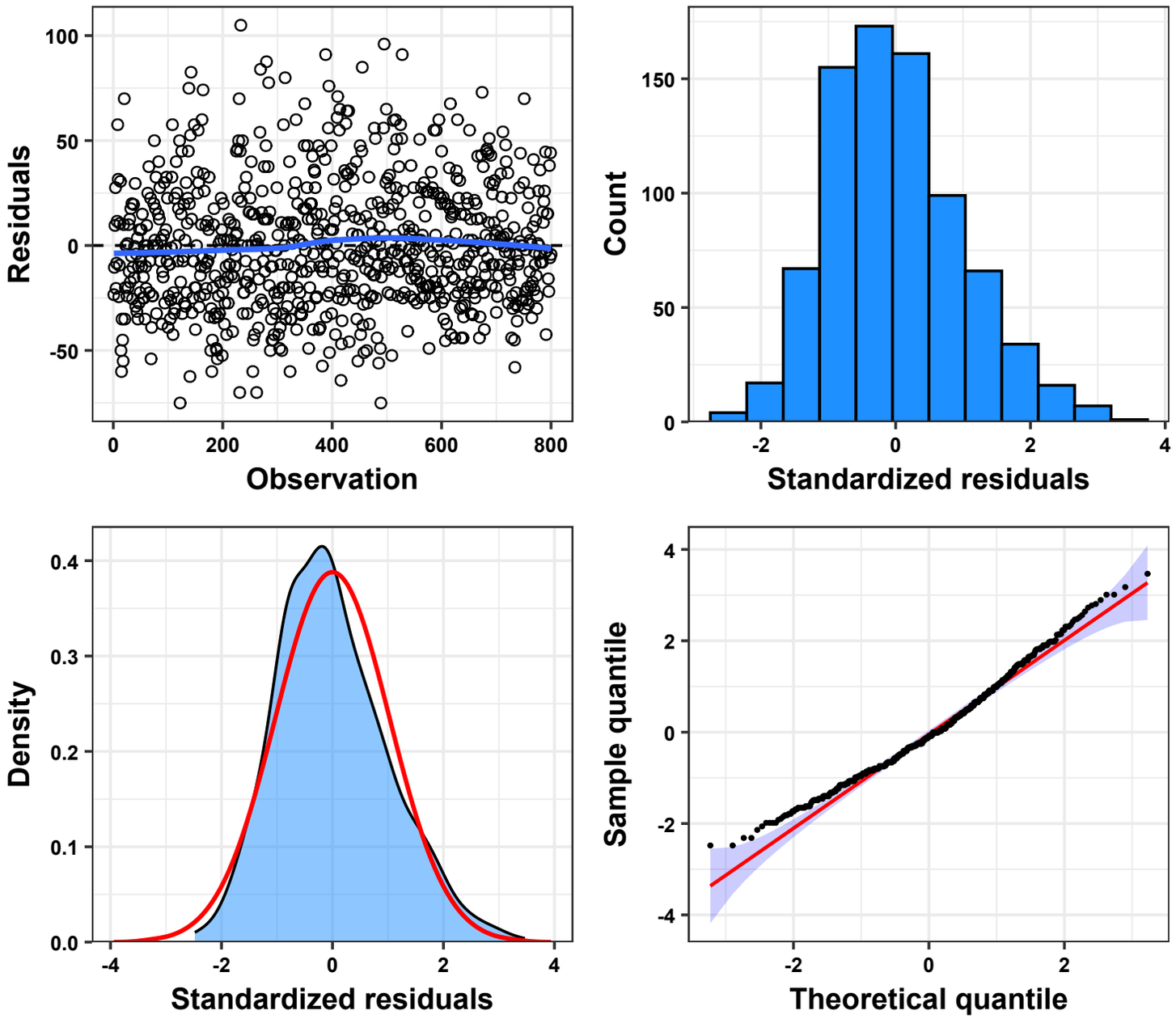


- Residuals vs observation order. Histogram, Density Q–Q plot of standardized residuals

```
p1 <- resid_vs_order(mod3bis)
p2 <- resid_stand_hist(mod3bis)
p3 <- resid_stand_dens(mod3bis)
p4 <- resid_stand_qq(mod3bis)
```

- We combine the 4 plots with the help of `+` from `{patchwork}`

```
p1 + p2 + p3 + p4
```



Session Info

Package	Version
broom	1.0.10
car	3.1-3
collapse	2.1.4
correlation	0.8.8
datawizard	1.3.0
effectsize	1.0.1
GGally	2.4.0
ggfortify	0.4.19
ggpubr	0.6.2
glue	1.8.0
gtsummary	2.4.0
insight	1.4.2
janitor	2.2.1
kableExtra	1.4.0
lmtest	0.9-40
matrixTests	0.2.3.1
modelbased	0.13.0
multcomp	1.4-29
openxlsx	4.2.8
parameters	0.28.2
patchwork	1.3.2
performance	0.15.2
qqplotr	0.0.7
rstatix	0.7.3
scales	1.4.0
see	0.12.0
tidyverse	2.0.0