ADVANCED CONTROL SYSTEMS

Motion Control - Adaptive Control -

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Outline

Adaptive Control

PROJECT

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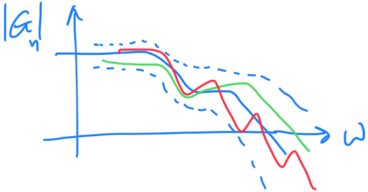
Solution #1. Robust Control: design a controller that stabilizes a *family of plants* around the nominal one.

▷ We need to have a model for the uncertainty not too conservative.



$$\mathcal{G} = \{ G = G_n + \Delta G,\ \Delta G \text{ is distributed some way} \}$$

Problem: How to cope with model uncertainty?

Solution #1. Robust Control: design a controller that stabilizes a *family of plants* around the nominal one.
▷ We need to have a model for the uncertainty not too conservative.

Solution #2. Adaptive Control: *on-line adaptation* of the computational model to the dynamic model.
▷ We need to design a dynamic estimator for the robot parameters. $\hat{\theta}(t)$

1) direct → ~~app~~ adapt the controller

2) indirect → adapt the model of the plant and then modify the controller

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Here we will focus on the adaptive control approach, the design of a robust controller can be found in the textbook.

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When our mathematical model is very close to the real manipulator dynamics, it is safe to resort to the inverse dynamics control scheme.

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Kinds of uncertainties:

- dynamic parameters: masses, center of mass vectors, inertia matrices (constant but unknown)
- friction coefficients (*slowly time-varying*)
- payload at the end-effector in pick-and-place tasks (*poorly known and abruptly change*)

What we assume to know

- kinematic description, i.e. Denavit-Hartenberg parameters

The dynamic model can be written using the regressor matrix $Y(q, \dot{q}, \ddot{q})$

$$\begin{aligned} \boxed{B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q)} &= \tau \\ &= \boxed{Y(q, \dot{q}, \ddot{q})\Theta.} \end{aligned} \tag{1}$$

where Θ is a $n\,p$ vector of constant parameters (p parameters per link) and Y is an $(n \times np)$ matrix.

The regress~~ion~~ or matrix Y depends linearly on $\ddot{q}$, quadratically on $\dot{q}$ (for the terms related to kinetic energy), and trigonometrically on q.

Remark. Knowing Θ is the same of knowing B, C, F, g.

Assumption. The desired trajectory $q_d(t)$ is known and twice-differentiable

Control goal. Design a control law τ such that

$$\boxed{\begin{array}{ccc} \tilde{q} & \rightarrow & 0 \\ \dot{\tilde{q}} & \rightarrow & 0 \end{array}}$$

$\tilde{q}(t) = q_d(t) - q(t)$

$\dot{\tilde{q}}(t) = \dot{q}_d(t) - \dot{q}(t)$

despite uncertainty on the dynamic parameters (unknown, slowly time-varying as friction coefficients, abruptly time-varying as mass and inertia at the end-effector in pich-and-place tasks)

Requirement. We ask for *global stability*: despite how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error

Assumption. The mathematical model is exactly equal to the robot dynamics equation.

Joint Space Inverse Dynamics Control I

$$
\begin{aligned}
\tau &= B(q)y + C(q,\dot q)\dot q + F\dot q + g(q)\\
&= B(q)\left(-K_P q - K_D\dot q + r\right) + C(q,\dot q)\dot q + F\dot q + g(q)\\
&= B(q)\left(-K_P q - K_D\dot q + \ddot q_d + K_D\dot q_d + K_P q_d\right) + C(q,\dot q)\dot q + F\dot q + g(q)\\
&= B(q)\left(\ddot q_d + K_P\tilde q + K_D\dot{\tilde q}\right) + C(q,\dot q)\dot q + F\dot q + g(q)
\end{aligned}
$$

feedforward

PD control law on the position/velocity errors

where $\tilde q = q_d - q$, $\dot{\tilde q} = \dot q_d - \dot q$.

Joint Space Inverse Dynamics Control II

$$
\boxed{\tau = \underbrace{B(q)\ddot q_d + C(q,\dot q)\dot q_d + g(q) + F\dot q_d}_{\text{feed-forward term}} + \underbrace{K_D\dot{\tilde q} + K_P\tilde q}_{\text{PD}}}
$$

In case we have an uncertain knowledge of the robot dynamics, the control laws will look like

Joint Space Inverse Dynamics Control I

$$\tau = \hat{B}(q)\left(\ddot{q}_d + K_P\tilde{q} + K_D\dot{\tilde{q}}\right) + \hat{C}(q,\dot{q})\dot{q} + \hat{F}\dot{q} + \hat{g}(q)$$

Joint Space Inverse Dynamics Control II

$$\tau = \hat{B}(q)\ddot{q}_d + \hat{C}(q,\dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}\dot{q}_d + K_D\dot{\tilde{q}} + K_P\tilde{q}$$

where $\hat{B}(q)$ approximates $B(q)$, $\hat{C}(q,\dot{q})$ approximates $C(q,\dot{q})$, ...

Such control laws are very difficult to be made *adaptive*, unless ...

We introduce the new signal $\dot{q}_r$ (*reference velocity*)

$$\boxed{\dot{q}_r = \dot{q}_d + \Lambda\tilde{q}}$$

which sums the desired velocity $\dot{q}_d$ with a error term related to the displacement between the desired position q_d and the actual position q.

The matrix Λ is a positive definite matrix.

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The matrix Λ is a positive definite matrix.

Λ constant

The time derivative of $\dot{q}_r$ is

$$\ddot{q}_r = \ddot{q}_d + \Lambda \dot{\tilde{q}}$$

We define the new *control law* as

$$\tau = B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \qquad (2)$$

where

$$\boxed{\sigma = \dot{q}_r - \dot{q}} = \dot{\tilde{q}} + \Lambda\tilde{q}$$

$= \dot{q}_d + \Lambda\tilde{q} - \dot{q}$

$\dot{\tilde{q}}$

and K_D is a positive definite matrix.

Remarks.

- The term $K_D\sigma$ in

$$\tau = B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

is equivalent to a PD action on the tracking error

$$\begin{aligned} K_D\sigma &= K_D(\dot{q}_r - \dot{q}) \qquad (*) \\ &= K_D(\dot{q}_d + \Lambda\tilde{q} - \dot{q}) \\ &= K_D\dot{\tilde{q}} + K_D\Lambda\tilde{q} \\ &= \underbrace{K_D(\dot{q}_d - \dot{q})}_{D\text{-action}} + \underbrace{K_D\Lambda(q_d - q)}_{P\text{-action}} \end{aligned}$$

where $K_P := K_D\Lambda$ is the proportional gain.

Remarks.

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where $K_P := K_D\Lambda$ is the proportional gain.

- the control τ be re-written as

$$\begin{aligned} \tau &= B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta + K_D\sigma \end{aligned}$$

Substituting

$$\tau = \underbrace{B(q)\ddot{q}_r} + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

control law

into

$$\underbrace{B(q)\ddot{q}} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau$$

plant

we end up with $(\sigma = \dot{q}_r - \dot{q})$

$$\boxed{B(q)\dot{\sigma} + C(q,\dot{q})\sigma + F\sigma + K_D\sigma = 0.}$$

$$B(q)(\ddot{q}_r - \ddot{q}) = B(q)\dot{\sigma}$$

Substituting

$$\tau = B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

into

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau$$

we end up with ($\sigma = \dot{q}_r - \dot{q}$)

$$B(q)\dot{\sigma} + C(q,\dot{q})\sigma + F\sigma + K_D\sigma = 0.$$

$\sigma, \tilde{q}$ linear combination of $\dot{\tilde{q}}, \tilde{q}$

We consider the following Lyapunov function candidate as a function of $\sigma, \tilde{q}$

$B(q) > 0, \forall q$

$$\boxed{V(\sigma, \tilde{q}) = \frac{1}{2}\sigma^T B(q)\sigma + \frac{1}{2}\tilde{q}^T M\tilde{q}}$$

$\sigma = \dot{q}_r - \dot{q}$
$= \dot{q}_d + \Lambda\tilde{q} - \dot{q}$
$= \dot{\tilde{q}} + \Lambda\tilde{q}$

with M a symmetric positive definite matrix.

$V(\sigma, \tilde{q})$ is a positive definite function (non negative and equal to zero only in $\sigma = 0$ and $\tilde{q} = 0$).

Since $\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q}$, at the equilibrium point $\sigma = 0$ and $\tilde{q} = 0$ we also have

$$\dot{\tilde{q}} = 0.$$

The time derivative of $V(\sigma, \tilde{q})$ along the trajectory of

$$\boxed{B(q)\dot{\sigma} + C(q,\dot{q})\sigma + F\sigma + K_D\sigma = 0.}$$

“$\dot{x} = f(x)$”

is

$$
\begin{aligned}
\dot{V}(\sigma, \tilde{q}) &= \sigma^T B(q)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\
&= \sigma^T\left(-C(q,\dot{q})\sigma - F\sigma - K_D\sigma\right) + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\
&= \sigma^T\left(-F\sigma - K_D\sigma\right) + \frac{1}{2}\sigma^T \underbrace{\left(\dot{B}(q) - 2C(q,\dot{q})\right)}_{\text{skew-symm. matrix}}\sigma + \tilde{q}^T M\dot{\tilde{q}} \\
&= -\sigma^T(F + K_D)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\
&= -\sigma^T F\sigma - \sigma^T K_D\sigma + \tilde{q}^T M\dot{\tilde{q}}
\end{aligned}
$$

$F > 0 \quad K_D > 0 \quad M \geq 0$

$\Lambda > 0, K_D > 0 \Rightarrow M > 0$

Choosing $\boxed{M = 2\Lambda K_D}$

$$\dot{V}(\sigma, \tilde{q}) \quad = \quad -\sigma^T F \sigma - \sigma^T K_D \sigma + 2\tilde{q}^T \Lambda K_D \dot{\tilde{q}}$$

and by substituting

$$\begin{aligned} \sigma &= \dot{\tilde{q}} + \Lambda \tilde{q} \\ \dot{\tilde{q}} &= \sigma - \Lambda \tilde{q} \end{aligned}$$

we get

$$\boxed{\dot{V}(\sigma, \tilde{q}) = -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q},} \tag{3}$$

Therefore, the time derivative is negative definite and vanishes only for $\tilde{q} = 0, \ \dot{\tilde{q}} = 0$.

$\Rightarrow \sigma = 0$

Then the equilibrium point $\boxed{[\tilde{q}^T \ \ \sigma^T]^T = 0}$ is *globally asymptotically stable*.

Lyapunov theorem

We have just proved the following theorem

Theorem

Given the desired trajectory $q_d(t)$ for the robotic arm

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau,$$

the control law

$$\left\{\begin{array}{rcl} \tau & = & B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \\ \dot{q}_r & = & \dot{q}_d + \Lambda\tilde{q}, \qquad \tilde{q} = q_d - q \\ \ddot{q}_r & = & \ddot{q}_d + \Lambda\dot{\tilde{q}}, \qquad \dot{\tilde{q}} = \dot{q}_d - \dot{q} \\ \sigma & = & \dot{q}_r - \dot{q} \end{array}\right.$$

guarantees the global asymptotic stability of $(\tilde{q}(t), \dot{\tilde{q}}(t)) = (0,0)$.

The *nominal* control law

$$\tau_n = B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

becomes

$$\boxed{\tau = \hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma}$$

when we use the estimated matrices $\hat{B}$, $\hat{C}$, $\hat{F}$, $\hat{g}$ based on the available vector $\hat{\Theta}$ of Θ.

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Assumption: $\hat{B}$, $\hat{C}$, $\hat{F}$, and $\hat{g}$ satisfy the same properties of B, C, F, and g.

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In particular

$$\tau_n = \underbrace{B(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + F\dot{q}_r + g(q)}_{Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta} + K_D\sigma$$

$$\tau = \underbrace{\hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q)}_{Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta}} + K_D\sigma$$

$$
\begin{aligned}
\tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma \\
&= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} + K_D\sigma \\
&= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\Theta} + K_D\sigma
\end{aligned}
$$

where $\boxed{\tilde{\Theta} = \hat{\Theta} - \Theta}$ is the error parameter vector.

It is worth highlighting that *the matrix Y depends only on the joint desired accelerations and not on their actual values*, ($\dot{q}_r = \dot{q}_d + \Lambda\tilde{q}$, $\ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}}$).

does not depend on $\ddot{q}$

Using the matrices in the dynamic model

$$
\begin{aligned}
\tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma \\
&= \left(B(q) + \tilde{B}(q)\right)\ddot{q}_r + \left(C(q,\dot{q}) + \tilde{C}(q,\dot{q})\right)\dot{q}_r + \left(F + \tilde{F}\right)\dot{q}_r + \left(g(q) + \tilde{g}(q)\right) + K_D\sigma
\end{aligned}
$$

where

$$
\boxed{\tilde{B} = \hat{B} - B, \quad \tilde{C} = \hat{C} - C, \quad \tilde{F} = \hat{F} - F, \quad \tilde{g} = \hat{g} - g.}
$$

Substituting in the equations of motion

real manipulator

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \dots$$

$$\dots = \left(B(q) + \tilde{B}(q)\right)\ddot{q}_r + \left(C(q,\dot{q}) + \tilde{C}(q,\dot{q})\right)\dot{q}_r + \left(F + \tilde{F}\right)\dot{q}_r + (g(q) + \tilde{g}(q)) + K_D\sigma$$

control law

and remembering that $\sigma = \dot{q}_r - \dot{q}$, $\boxed{\dot{\sigma} = \ddot{q}_r - \ddot{q}}$, we finally have

$\hat{B}_t(q)$

$$(*) \quad \begin{aligned} B(q)\dot{\sigma} + C(q,\dot{q})\sigma + F\sigma + K_D\sigma &= -\tilde{B}(q)\ddot{q}_r - \tilde{C}(q,\dot{q})\dot{q}_r - \tilde{F}\dot{q}_r - \tilde{g}(q) \\ &= -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\Theta} \end{aligned}$$

$\hat{\Theta}(t)$

Since we want to on-line estimate the value of $\hat{\Theta}$, we have to modify the Lyapunov function candidate

$$x = \begin{bmatrix} \sigma \\ \tilde{q} \\ \tilde{\Theta} \end{bmatrix}, \quad x_e = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V(\sigma, \tilde{q}, \cancel{\tilde{\pi}}\,\tilde{\Theta}) = \frac{1}{2}\sigma^T B(q)\sigma + \tilde{q}^T \Lambda K_D \tilde{q} + \boxed{\frac{1}{2}\tilde{\Theta}^T K_\Theta \tilde{\Theta}} \qquad (4)$$

taking into account also the new component for the state $\tilde{\Theta}$, i.e. the *estimation error*.

$K_\Theta \succ 0$

Adaptive Control

If K_Θ is a symmetric positive definite matrix, the Lyapunov function candidate

$$V(\sigma, \tilde{q}, \tilde{\pi}) = \frac{1}{2}\sigma^T B(q)\sigma + \tilde{q}^T \Lambda K_D \tilde{q} + \frac{1}{2}\tilde{\Theta}^T K_\Theta \tilde{\Theta} \qquad (5)$$

is nonnegative and equal to zero only at $\sigma = 0$ (i.e. $\dot{q} = \dot{q}_d$), $\tilde{q} = 0$ (i.e. $q = q_d$), $\tilde{\Theta} = 0$ (i.e. $\hat{\Theta} = \Theta$); this implies that V is a positive definite function.

$$x = \begin{bmatrix} \sigma \\ \tilde{q} \\ \tilde{\Theta} \end{bmatrix}$$

Computing the time derivative

$$\dot{V}(\cdot) = \sigma^T B(q)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T \Lambda K_D \dot{\tilde{q}} + \tilde{\Theta}^T K_\Theta \dot{\tilde{\Theta}}$$

along the trajectories of the system

$$\boxed{B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta}}$$

we have

$$\begin{aligned}
\dot{V}(\cdot) &= \sigma^T \left(-C(q, \dot{q})\sigma - F\sigma - K_D\sigma - Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta}\right) + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T \Lambda K_D \dot{\tilde{q}} + \tilde{\Theta}^T K_\Theta \dot{\tilde{\Theta}} \\
&= -\sigma^T F\sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} - \sigma^T Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta} + \tilde{\Theta}^T K_\Theta \dot{\tilde{\Theta}} \\
&= -\sigma^T F\sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} - \tilde{\Theta}^T Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma + \tilde{\Theta}^T K_\Theta \dot{\tilde{\Theta}}
\end{aligned}$$

$$\begin{aligned}\dot V(\cdot) &= -\sigma^T F\sigma - \dot{\tilde q}^T K_D \dot{\tilde q} - \tilde q^T \Lambda K_D \Lambda \tilde q - \tilde\Theta^T Y^T(q,\dot q,\dot q_r,\ddot q_r)\sigma + \tilde\Theta^T K_\Theta \dot{\tilde\Theta} \\ &= -\sigma^T F\sigma - \dot{\tilde q}^T K_D \dot{\tilde q} - \tilde q^T \Lambda K_D \Lambda \tilde q + \tilde\Theta^T \underbrace{\left(K_\Theta \dot{\tilde\Theta} - Y^T(q,\dot q,\dot q_r,\ddot q_r)\sigma\right)}_{\equiv 0},\end{aligned}$$

If we choose as dynamic equation for the estimation error

$$\dot{\hat\Theta} = K_\Theta^{-1} Y^T(q,\dot q,\dot q_r,\ddot q_r)\sigma,$$

then we also have

$$\boxed{\dot{\tilde\Theta} = K_\Theta^{-1} Y^T(q,\dot q,\dot q_r,\ddot q_r)\sigma,}$$

since the nominal value Θ are assumed constant ($\dot{\hat\Theta} = \frac{d}{dt}\left(\tilde\Theta - \Theta\right) = \dot{\tilde\Theta}$).

Finally

$$\boxed{\dot V(\cdot) = -\sigma^T F\sigma - \dot{\tilde q}^T K_D \dot{\tilde q} - \tilde q^T \Lambda K_D \Lambda \tilde q}$$

$$\dot{V}(\cdot) \quad = \quad -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}$$

is a negative semi-definite function in σ, $\tilde{q}$, $\tilde{\Theta}$.

Since

$$\sigma = \dot{q}_r - \dot{q} = \dot{q}_d + \Lambda \tilde{q} - \dot{q} = \dot{\tilde{q}} + \Lambda \tilde{q} = (\dot{q}_d - \dot{q}) + \Lambda (q_d - q),$$

$$\dot{V}(\cdot) = -\sigma^T F\sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} \quad \lesssim 0$$

is a negative semi-definite function in σ, $\tilde{q}$, $\tilde{\Theta}$.

Since

$$\sigma = \dot{q}_r - \dot{q} = \dot{q}_d + \Lambda\tilde{q} - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q} = (\dot{q}_d - \dot{q}) + \Lambda(q_d - q),$$

the control law and the adaptation law

$\hat{\Theta} \rightarrow \hat{B}, \hat{C}, \hat{F}, \hat{g}$

$$\begin{cases} \tau &= \boxed{\hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q)} + K_D\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} + K_D\sigma \\ \dot{\hat{\Theta}} &= \Gamma Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\sigma, \qquad (\Gamma := K_\Theta^{-1} \succ 0) \end{cases}$$

$\hat{\Theta}(t)$

$(\tilde{q} \to 0, \dot{\tilde{q}} \to 0)$

guarantee the global converge of the closed loop system toward $\sigma = 0$, $\tilde{q} = 0$, which implies *convergence to zero* of $\dot{\tilde{q}}(t)$ and $\tilde{q}(t)$, and *boundness* of $\hat{\Theta}(t)$.

We have just proved the following theorem

$\epsilon\, \mathcal{C}^2$

Theorem

Given the desired trajectory $q_d(t)$ for the robotic arm

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau,$$

the control law

$$\begin{cases} \tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} + K_D\sigma \\ \dot{\hat{\Theta}} &= \Gamma\, Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\sigma, \qquad (\Gamma := K_\Theta^{-1} \succ 0) \end{cases}$$

guarantees that the tracking error along the desired trajectory is globally asymptotically stable, i.e. $(\tilde{q}(t), \dot{\tilde{q}}(t)) \to (0,0)$.

$\forall\ q(t_0),\ \dot{q}(t_0),\ \hat{\Theta}(t_0)$

Remark.
Since

$$\overbrace{B(q)\dot{\sigma} + C(q,\dot{q})\sigma + F\sigma + K_D\sigma}^{\to 0} = -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\Theta}$$

and $\sigma \to 0$, we have that

? $Ax = 0$
$\Leftrightarrow x = 0$

$$\boxed{Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\Theta} \to 0} \quad \text{for } t \to +\infty$$

which does not mean that the estimation error $\tilde{\Theta}$ goes to zero, but only that $\hat{\Theta} - \Theta$ belong to the kernel of $Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$, i.e.

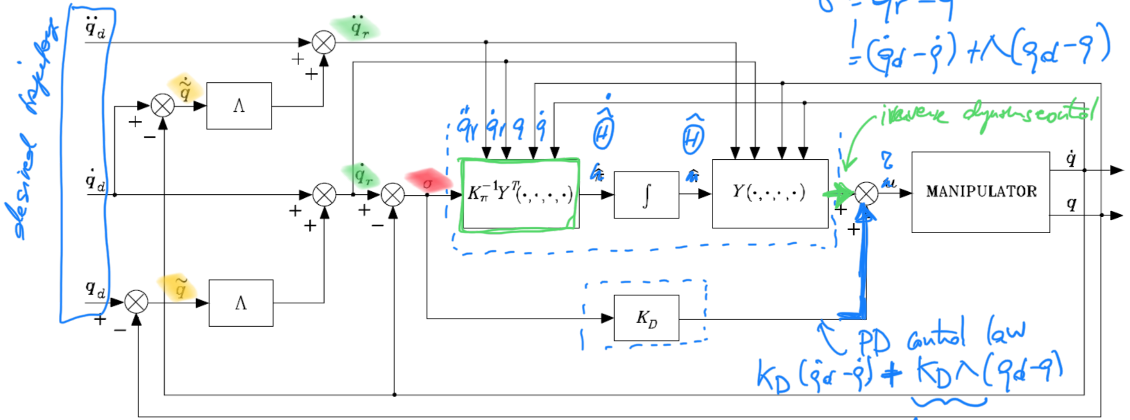
$$\boxed{Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\Theta} \to Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta}$$

even though $\hat{\Theta} \nrightarrow \Theta$.

This adaptive control solves a so-called *direct adaptive control problem*: we find a control law that ensures limited tracking errors; we do not determine the actual parameters of the system (as in an *indirect adaptive control problem*).

Adaptive Control

$$\sigma = \dot{q}_r - \dot{q} = (\dot{q}_d - \dot{q}) + \Lambda(q_d - q)$$



$$K_D(\dot{q}_d - \dot{q}) + \underbrace{K_D \Lambda}_{\triangleq K_P}(q_d - q)$$

$$\{q_d(t), \dot{q}_d(t), \ddot{q}_d(t)\} \quad \forall t \in [t_0, t_f]$$

Figure: Joint Space Adaptive Control block scheme

$$\dot{q}_r = \dot{q}_d + \Lambda(q_d - q)$$
$$\ddot{q}_r = \ddot{q}_d + \Lambda(\dot{q}_d - \dot{q})$$

Final observations:

- $Y\hat{\Theta}$ approximates the inverse dynamics control, i.e. it *approximates the compensation of nonlinear effects and joint decoupling*.
- $K_D\sigma$ stabilizes the tracking error via a *linear PD control action*.
- $\hat{\Theta}$ is updated via an *adaptive law of gradient type*, with a convergence rate depending on K_Θ. $= \Gamma^{-1}$
- Choosing $\sigma = 0$ means adopting a pure inverse dynamics compensation. (Θ)
- The proposed control scheme does not take into account neither external disturbances not unmodelled dynamics. This implies that their effects on the output variables are attributed by the controller to parameter estimate mismatching.


Project

UNIVERSITÀ di VERONA Dipartimento di INFORMATICA

To do

$I\ddot{q} + F_1\dot{q} + \underbrace{F_2\dot{q}^2} + G\sin q = \tau$

- Implement in Simulink the Adaptive Control law for the a 1-DoF link under gravity. [choose the dynamic parameters and their initial estimates, and set the desired trajectory as (1) $q_d(t) = A\sin(\omega t)$, (2) $\dot{q}_d(t)$ a periodic square wave $\pm A$]

Hint.
Plant

$$\boxed{I\ddot{q} + F\dot{q} + \underbrace{mgd}_{G}\sin q = \tau}$$

Linear parameterization

$\ddot{q}_r$?

$$\tau = \underbrace{\begin{bmatrix} \ddot{q} & \dot{q} & \sin q \end{bmatrix}}_{Y(q,\dot{q},\ddot{q})} \underbrace{\begin{bmatrix} I \\ F \\ G \end{bmatrix}}_{\Theta}$$

$Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$

$$\dot{\hat{\Theta}} = \begin{bmatrix} \dot{\hat{I}} \\ \dot{\hat{F}} \\ \dot{\hat{G}} \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{q} \\ \sin q \end{bmatrix} (\dot{q}_r - \dot{q})$$

Reference velocity

$$\dot{q}_r = \dot{q}_d + \lambda\tilde{q} = \dot{q}_d + \frac{k_P}{k_D}(q_q - q)$$