ADVANCED CONTROL SYSTEMS

Force Control

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Outline

Force control

PROJECT

Parallel Force/Position Control

PROJECT

Force control

Direct Force Control

Indirect force control: the interaction force h_e can be indirectly controlled by acting on the desired pose of the end-effector assigned to the motion control system.
E.g. compliance, impedance, admittance control.

Direct force control: the interaction force h_e can be directly controlled by specifying the desired force in a force feedback loop.
E.g. stabilizing PD control action on the force error + the nonlinear compensation actions.

A force control system typically consists of a control law based on both force measurements and position/velocity measurements ($\rightarrow$ nested loops).

Assumptions:

- we will develop control schemes on the operational space
- we assume to know only the position $x_e \in \mathbb{R}^3$, where $\Sigma_e = \{o_e; x_e y_e z_e\}$ is the *end-effector frame*, $x_e = o_e$
- the control schemes are based on an inverse dynamic control position
- the environment is modeled as an elastic system

$$f_e = K(x_e - x_r)$$

where $\Sigma_r = \{o_r; x_r y_r z_r\}$ is the *environment rest frame*, $x_r = o_r$. (no torques!)

- the axes of the frame attached to the environment rest position Σ_t are parallel to the axes of the base frame Σ_b

Force Control with Inner Position Loop

Inverse dynamic control:

$$y = J^{-1}(q)M_d^{-1}\big(- K_D\dot{x}_e + K_P(x_F - x_e) - M_d\dot{J}(q,\dot{q})\dot{q}\big), \tag{1}$$

where x_F is a suitable *reference position* to be related to a *force error*.

There are no control action using $\dot{x}_F$ (D-action) or $\ddot{x}_F$ (feedforward)

Since we are *not* considering the orientation in the operational space, then $J(q) = J_A(q)$.

The system

$$\begin{cases} B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau - J^T(q)h_e \\ \tau = B(q)y + n(q,\dot{q}) + J^T(q)h_e \\ y = J^{-1}(q)M_d^{-1}\big(- K_D\dot{x}_e + K_P(x_F - x_e) - M_d\dot{J}(q,\dot{q})\dot{q}\big) \end{cases}$$

ends up with

$$M_d\ddot{x}_e + K_D\dot{x}_e + K_P x_e = K_P x_F, \tag{2}$$

This is a position-position system mapping x_F (reference) into x_e (actual).

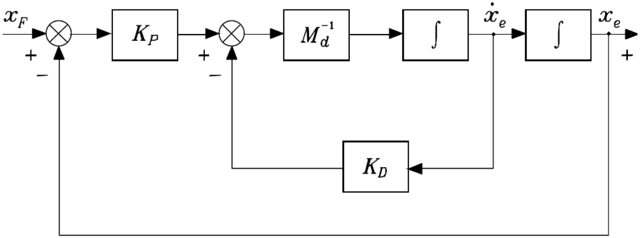

Figure: Equivalent system

Force Control with Inner Position Loop

Let f_d be the *desired constant force* reference; we can define a diagonal matrix C_F playing the role of a *compliance matrix* (mapping force into position) such that

$$x_F = C_F(f_d - f_e), \tag{3}$$

where f_e is the measured interaction force.

Using the expression for the spring-like interaction model

$$f_e = K(x_e - x_r)$$

we get

$$M_d\ddot{x}_e + K_D\dot{x}_e + K_P(I + C_F K)x_e = K_P C_F(Kx_r + f_d) \tag{4}$$

Force Control with Inner Position Loop

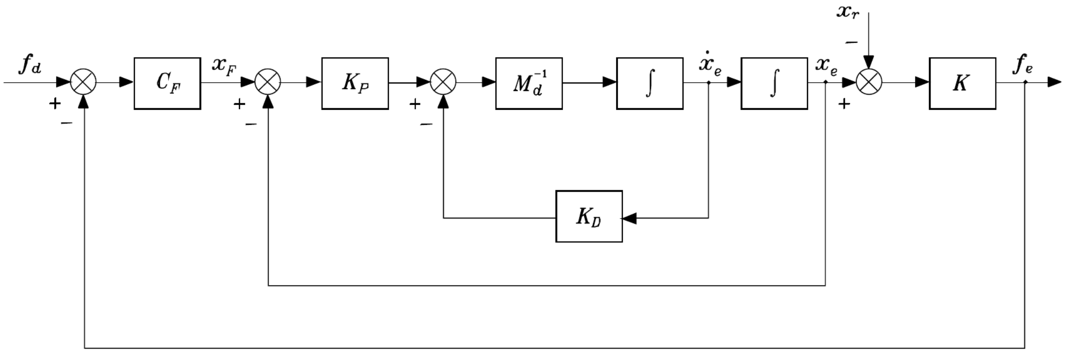

Figure: Block scheme of force control with inner position loop.

If C_F has a purely proportional control action (i.e. the 'controller' C_F is just the matrix C_F), then f_e cannot reach f_d (no zero steady-state error).

The position of the environment x_r affects the interaction force also at steady state (i.e. the amount of the steady-state error).

If C_F is a PI controller

$$C_F(s) = K_F + K_I \frac{1}{s}$$

then we can reach zero stady-state error

$$f_e = f_d$$

at the position

$$Kx_e = Kx_r + f_d.$$

Force Control with Inner Velocity Loop

Suppose that we have only velocity measurements available, $\dot{x}_e$.

The modified inverse dynamic control will be

$$y = J^{-1}(q)M_d^{-1}\left(- K_D\dot{x}_e + K_P x_F - M_d\dot{J}(q,\dot{q})\dot{q}\right), \tag{5}$$

where x_F is again a suitable *reference position* to be related to a *force error*.

The inner controller+robot system is equivalent to

$$M_d\ddot{x}_e + K_D\dot{x}_e = K_P x_F, \tag{6}$$

where we considered that $J_A(q) = J(q)$ because the operational space is defined only by position variables.

If $C_F(s) = K_F$ (P-action) the outer force loop computes

$$x_F = K_F(f_d - f_e), \tag{7}$$

As before K_F is a diagonal matrix and has a meaning of *compliance*.

Force Control with Inner Velocity Loop

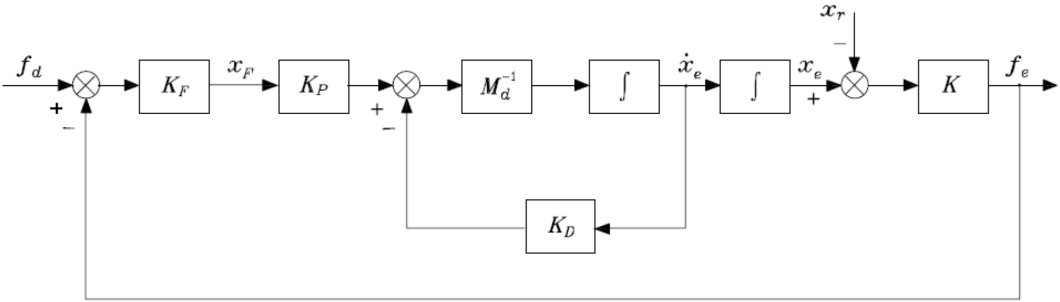

Figure: Block scheme of force control with inner velocity loop.

Force Control with Inner Velocity Loop

Under the assumption of elastically compliant environment, $f_e = K(x_e - x_r)$, the overall system dynamics is described by

$$M_d \ddot{x}_e + K_D \dot{x}_e + K_P K_F K x_e = K_P K_F (K x_r + f_d) \tag{8}$$

The relationship between position and contact force at the equilibrium is

$$K x_e = K x_r + f_d \tag{9}$$

(since K_P and K_F are non singular.)

Remarks:

- Since K_P and K_F are multiplies, then we can design only a single matrix $K_F' = K_P K_F$
- The absence of an integral operator does not ensure $f_e = f_d$ in steady-state.

Both Force Control with Inner Position/Velocity Loops have a *drawback*: if f_d has components outside $Image(K)$, they cause a drift of the end-effector position.


Project

To do

- Implement the Force Control with Inner Position Loop.

Parallel Force/Position Control

The *Parallel Force/Position Control* is a control scheme where both the *desired reference force* f_d and the *desired reference position* x_d are provided.

The control y is changed as

$$\begin{aligned} y &= J^{-1}(q)M_d^{-1}\left(-K_D\dot{x}_e + K_P(x_F - x_e + x_d) - M_d\dot{J}_A(q,\dot{q})\dot{q}\right) \\ &= J^{-1}(q)M_d^{-1}\left(-K_D\dot{x}_e + K_P(x_F + \tilde{x}) - M_d\dot{J}_A(q,\dot{q})\dot{q}\right), \end{aligned}$$

where $\tilde{x} = x_d - x_e$.

- position control action $K_P\tilde{x}$ (inner loop)
- force control action $K_P C_F(f_d - f_e)$ where C_F is a constant K_F or a PI controller $C_F = K_F + K_I\frac{1}{s}$ (outer loop)

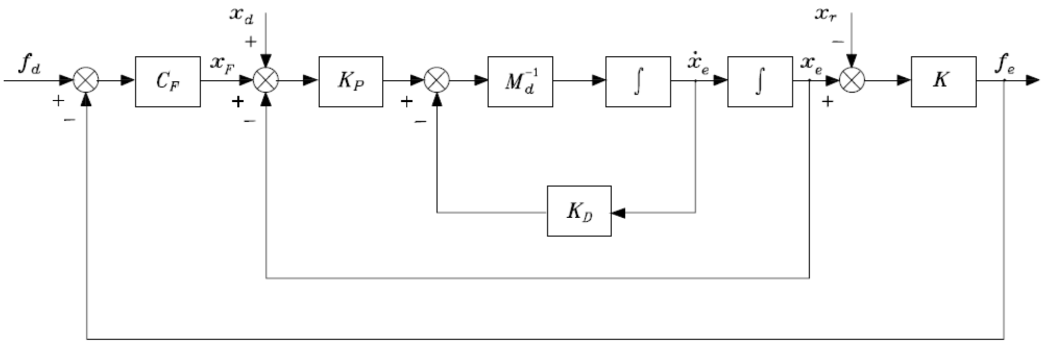

Figure: Block scheme of parallel force/position control.

The equilibrium position satisfies

$$x_e = x_d + C_F\big(K(x_r - x_e) + f_d\big). \tag{10}$$

- Along directions outside *Image*(K) (i.e. *unconstrained motion*): x_d is reached by x_e;
- Along directions belonging to *Image*(K) (i.e. *constrained motion*): x_d acts like an additional disturbance.

  With an integral action in C_F, the desired force f_d is reached by f_e at steady state

  $x_e \neq x_d$: the displacement is related to the environment compliance (i.e. K)


Project

To do

- Implement the Parallel Force/Position Control.


That's all Folks!