ADVANCED CONTROL SYSTEMS

Compliance & Impedance Control

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Outline

Problem statement

Why Force Control?

Anytime the robot interacts with/manipulates the environment, it is necessary to control the *interaction force/torque*.

In such scenarios, the motion control architectures cannot be used as they are because they do not take into account explicitly the external wrench.

- *Contact force/torque measurements/estimations* are needed to describe the state of interaction; ← math model
- Force control allows to avoid *high values* of the interaction force (*safety*);
- Key concepts: *mechanical compliance* and *mechanical impedance/admittance*;
- Since contact forces are naturally expressed in the operational space, all the schemes are expressed in the *operational space*.
- Constraints: *natural constraints* set by the task geometry and environment, and *artificial constraints* set by the control strategy (e.g. virtual wall, repulsive potential fields, ...)

Why Force Control?

Architectures:

- *indirect force control methods*: force control via motion control without explicit closure of a force feedback loop; e.g.
  - compliance control
  - impedance control

- *direct force control methods*: control the contact force to a desired value, thanks to the closure of a force feedback loop; e.g.
  - force control
  - hybrid force/motion control

Why Force Control?

During the interaction, the environment sets constraints on the geometric paths that can be followed by the end-effector

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau - J^T(q)h_e \tag{1}$$

due to h_e, i.e. the manipulator's end effector contact forces vector exerted on the environment.

This situation is generally referred to as *constrained motion*.

When $h_e = 0$, the manipulator moves in free motion and it is possible to steer the end-effector toward the desired position x_d.

The same thing could in theory happen if the task planning can relay on a very accurate model of the manipulator and, more challenging, on a very accurate model of the environment or the mechanical parts we need to assemble.

▷ such accuracy must be of an order of magnitude greater than mechanical tolerance. In most of the cases this is impossible to guarantee due to uncertainty.

Compliance control

Compliance control

What happens to the operational space PD controller + gravity compensation when $h_e \neq 0$?

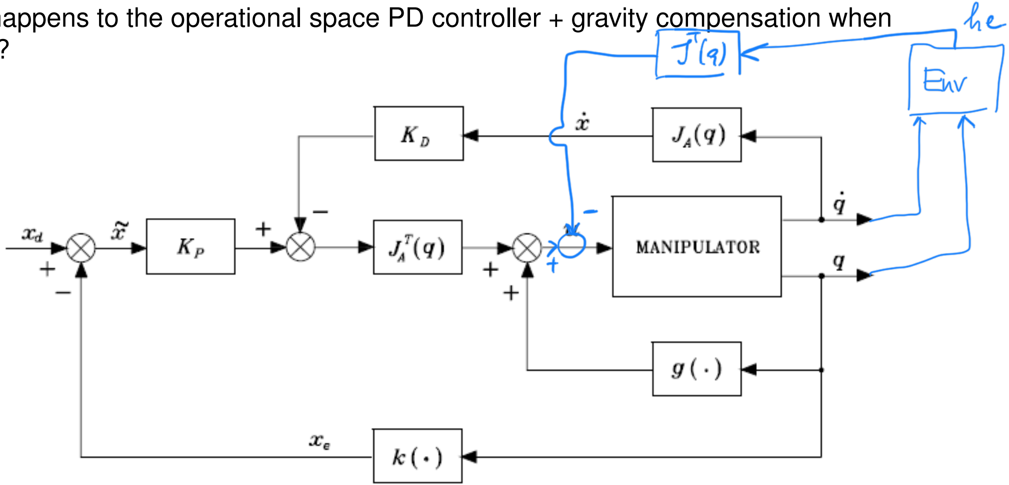


Figure: Operational Space PD control with gravity compensation block scheme.

Compliance control

Reg. problem x_d const

Since

$$
\begin{aligned}
J(q) &= T_A(\phi)J_A(q)\\
\tilde{x}(t) &= x_d - x(t)\\
\text{control command} \quad \tau &= g(q) + \underbrace{J_A^T(q)K_P\tilde{x} - J_A^T(q)K_DJ_A(q)\dot{q}}_{\text{PD control law}}
\end{aligned}
$$

at the equilibrium posture ($\dot{q} = 0, \ddot{q} = 0$), we have

$$
\begin{aligned}
B(q)\underbrace{\cancel{\ddot{q}}}_{=0} + C(q,\dot{q})\underbrace{\cancel{\dot{q}}}_{=0} + F\underbrace{\cancel{\dot{q}}}_{=0} + \cancel{g(q)} &= \cancel{g(q)} + J_A^T(q)K_P\tilde{x} - J_A^T(q)K_DJ_A(q)\overbrace{\cancel{\dot{q}}}^{=0} - J^T(q)h_e\\
0 &= J_A^T(q)K_P\tilde{x} - J^T(q)h_e
\end{aligned}
$$

Controlled system (robot + controller)

i.e.

$$
\boxed{J_A^T(q)K_P\tilde{x} = J^T(q)h_e.}
$$

Hp. Assuming a full-rank Jacobian, from

$$J_A^T(q)K_P\tilde{x} = J^T(q)h_e$$

we get

error of the equilibrium

$$\begin{aligned}\tilde{x} &= \left(J_A^T(q)K_P\right)^{-1} J^T(q)h_e \\ &= K_P^{-1}\boxed{\left(J_A^T(q)\right)^{-1} J^T(q)}h_e \\ &= K_P^{-1}T_A^T(q)h_e \\ &= K_P^{-1}T_A^T(x)h_e \\ &= K_P^{-1}h_A\end{aligned}$$

$x = k(q)$ direct. kin.

$$x = \begin{bmatrix} x \\ y \\ z \\ \psi \\ \theta \\ \varphi \end{bmatrix}$$

where h_A is the vector of equivalent forces in the operational space and K_P^{-1} is the *compliance matrix*, i.e. K_P is the *stiffness matrix* ($h_A = K_P\tilde{x}$).

Compliance control

Observations:

- If K_P is diagonal, and recalling the expression of the transformation matrix

$$T_A(\phi) = \begin{bmatrix} I & O \\ O & T(\phi) \end{bmatrix}$$

  we can observe that the *linear compliance due to force components* is independent of the configuration, whereas *torsional compliance due to moment components* does depend on the current end-effector orientation through the matrix $T(\phi)$.

- If $h_e \in \ker(J^T)$, then

$$\tilde{x} = 0, \qquad \text{even though} \qquad h_e \neq 0,$$

  this means that contact forces are completely balanced by manipulator mechanical structure.

- the compliant behaviour of the manipulator is achieved by control and is called *active compliance*, whereas the term *passive compliance* denotes mechanical systems with an elastic-like dynamics (e.g. serial elastic actuators).

Compliance control - Active compliance

Active compliance: the compliance can be modified acting on the control software to satisfy the requirements of different interaction tasks.

Goal: To derive an equilibrium equation

$\Sigma_e = \{O_e; x_e y_e z_e\}$ *end-effector frame* with corresponding transformation matrix

$$\boxed{T_e = \begin{bmatrix} R_e & o_e \\ 0^T & 1 \end{bmatrix}}$$

$\Sigma_d = \{O_d; x_d y_d z_d\}$ *desired frame ~~frame~~* with corresponding transformation matrix

$$\boxed{T_d = \begin{bmatrix} R_d & o_d \\ 0^T & 1 \end{bmatrix}.} \quad \leftrightarrow \quad x_d$$

We can express the end-effector orientation and displacement w.r.t. the desired frame as

$$\boxed{T_e^d = (T_d)^{-1} T_e} = \begin{bmatrix} R_e^d & o_{e,d}^d \\ 0^T & 1 \end{bmatrix} = \boxed{\begin{bmatrix} R_d^T R_e & R_d^T(o_e - o_d) \\ 0^T & 1 \end{bmatrix}} \quad (2)$$

The *error vector* is

$$\tilde{x} = x_d - x_e = -\begin{bmatrix} o^d_{d,e} \\ \phi_{d,e} \end{bmatrix} \tag{3}$$

where $\phi_{d,e}$ is the vector of Euler angles extracted from the rotation matrix R^d_e.

Assumptions: o_d and R_d are constant. (Regulation problem $(\star)$)

Then

$$o^d_{e,d} = R_d^T(o_e - o_d)$$

$$\begin{aligned}
\dot{\tilde{x}} &= -\begin{bmatrix} \dot{o}^d_{d,e} \\ \dot{\phi}_{d,e} \end{bmatrix} \\
&= -\begin{bmatrix} R_d^T(\dot{o}_e - \dot{o}_d) - S(\omega^d_d)R_d^T(o_e - o_d) \\ T^{-1}(\phi_{d,e})\omega^d_{d,e} \end{bmatrix} \\
&= -\begin{bmatrix} R_d^T(\dot{o}_e - \dot{o}_d) - S(\omega^d_d)R_d^T(o_e - o_d) \\ T^{-1}(\phi_{d,e})R_d^T(\omega_e - \omega_d) \end{bmatrix} \\
&\overset{(\star)}{=} -\begin{bmatrix} R_d^T\dot{o}_e \\ T^{-1}(\phi_{d,e})R_d^T\omega_e \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
\dot{\tilde{x}} &= -\begin{bmatrix} R_d^T \dot{o}_e \\ T^{-1}(\phi_{d,e}) R_d^T \omega_e \end{bmatrix} \\
&= -\underbrace{\begin{bmatrix} I & 0 \\ 0 & T^{-1}(\phi_{d,e}) \end{bmatrix}}\begin{bmatrix} R_d^T & 0 \\ 0 & R_d^T \end{bmatrix} \begin{bmatrix} \dot{o}_e \\ \omega_e \end{bmatrix} \\
&= -T_A^{-1}(\phi_{d,e}) \begin{bmatrix} R_d^T & O \\ O & R_d^T \end{bmatrix} v_e
\end{aligned}
$$

Since the vector of linear and angular velocity of the end-effector is

$$
v_e = \begin{bmatrix} \dot{o}_e \\ \omega_e \end{bmatrix} = J(q)\dot{q}
$$

we finally get

$$
\begin{aligned}
\dot{\tilde{x}} &= -T_A^{-1}(\phi_{d,e}) \begin{bmatrix} R_d^T & O \\ O & R_d^T \end{bmatrix} J(q)\dot{q} \\
&= -J_{A_d}(q, \tilde{x})\dot{q}
\end{aligned}
$$

The matrix $J_{A_d}(q, \tilde{x})$ is the analytic Jacobian corresponding to the error $\tilde{x}$ in the operational space.

The PD control with gravity compensation with the new error notation in the operational space is given by

$$\boxed{\tau = g(q) + J^T_{A_d}(q, \tilde{x})\left(K_P\tilde{x} - K_D J_{A_d}(q, \tilde{x})\dot{q}\right).} \tag{4}$$

Remark. The position error is referred to the desired frame and not to the base frame as before.

- If there is no interaction ($h^d_e = 0$) then we can prove using the same Lyapunov argument the asymptotic stability to the equilibrium pose $\tilde{x} = 0$;
- If there is interaction with the environment ($h^d_e \neq 0$) the equilibrium will be at the pose

$$\underbrace{J^T_{A_d}(q)K_P\tilde{x}}_{\Sigma_d} = \underbrace{J^T(q)h_e^{d}}_{\Sigma_d}.$$

$$\underbrace{J_A^T(q)K_P\tilde{x}}_{\Sigma_b} = \underbrace{J^T(q)h_e}_{\Sigma_b}$$

Assuming a full-rank Jacobian, it holds

$$\boxed{h_e^d = T_A^{-T}(\phi_{d,e}) K_P \tilde{x},} \tag{5}$$

where K_P has the meaning of an *active stiffness* (generalized spring acting between the end-effector frame and the desired frame) and, consequently, K_P^{-1} in the inverse formulation has the meaning of an *active compliance*.

By properly choosing the Euler angle corresponding to the rotation matrix and re-writing the above expression in terms of *elementary displacements* we get

$$\begin{aligned} h_e &= K_P \, dx_{e,d}, && \text{(spring)} \\ dx_{e,d} &= K_P^{-1} \, h_e, && \text{(compliance)} \end{aligned}$$

Remark. The choice of the elements of matrix $K_P \succ 0$ is driven by the geometry and mechanical features of the environment. (e.g. different values along different directions.)

Compliance control - Active compliance

Interaction with the environment modeled as a generalized spring

Let $\Sigma_r = \{O_r; x_r y_r z_r\}$ be the *reference frame attached to the environment in its rest position*

The interaction force between the end-effector and the environment is

$$\boxed{h_e = K\, dx_{r,e}}$$

where $K \succeq 0$ is the environment stiffness matrix, and $dx_{r,e}$ is the elementary displacement between Σ_r and Σ_e

Since

$$\boxed{dx_{r,e} = dx_{r,d} - dx_{e,d}}$$

we have the *equilibrium force*

$$\boxed{h_e = \left(I + K K_P^{-1}\right)^{-1} K\, dx_{r,d}}$$

and the corresponding *pose error* (again at the equilibrium)

$$dx_{e,d} = K_P^{-1}\left(I + K K_P^{-1}\right)^{-1} K\, dx_{r,d}$$

Σ_d $dx_{r,d}$ Σ_r

$dx_{e,d}$ $dx_{r,e}$

Σ_e

$h_e = K_P\, dx_{e,d}$

$h_e = K dx_{r,e}$
$= K(dx_{r,d} - dx_{e,d})$
$\overset{!}{=} K(dx_{r,d}) - K_P^{-1} h_e)$

Σ_e

Σ_r Σ_d

- The vectors h_e and $dx_{e,d}$ can be referred, equivalently, to the end-effector frame Σ_e, to the desired frame Σ_d, or to the frame attached to the environment rest position Σ_r;
- Σ_e, Σ_d and Σ_r coincide at the equilibrium;
- The end-effector pose error at the equilibrium depends on the environment rest position, on the desired pose imposed by the control system of the manipulator, as well as on the compliance matrices.

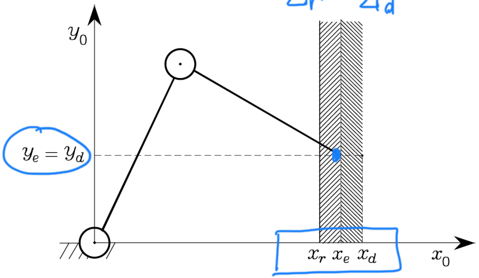


Figure: Two-link planar arm in contact with an elastically compliant plane.

(2) $K_x \gg K_{P_x} \Rightarrow x_r \simeq x_e$

$h_e = K_{P_x} dx_{e,d}$

(1) $K_{P_x} \gg K_x \Rightarrow x_e \simeq x_d$

$h_e \simeq K_x dx_{r,e}$

$dx_{r,e} \simeq dx_{r,d}$

Compliance control - Active compliance

$k_{P,i} >> k_i$

1. ▶ along the directions where the manipulator stiffness is much higher than the environment stiffness, the intensity of the elastic force mainly depends on the stiffness of the environment and on the displacement $dx_{r,e}$ between the equilibrium pose of the end-effector (which practically coincides with the desired pose) and the rest pose of the environment;

$k_i >> k_{P,i}$

2. ▶ along the directions where the environment stiffness is much higher than the manipulator stiffness, the intensity of the elastic force mainly depends on the manipulator stiffness and on the displacement $dx_{e,d}$ between the desired pose and the equilibrium pose of the end-effector (which practically coincides with the rest pose of the environment).

$h_e \simeq k_{P,i}\, dx_{e,d}$


Project

To do

- Study the compliance control.

- simulate the previous two "extreme" cases when the EE is tracking the environment (≡ planar surface) h_e ? one of the coordinates of FORCE

- K_p in the same range of K, h_e = ?

Impedance control

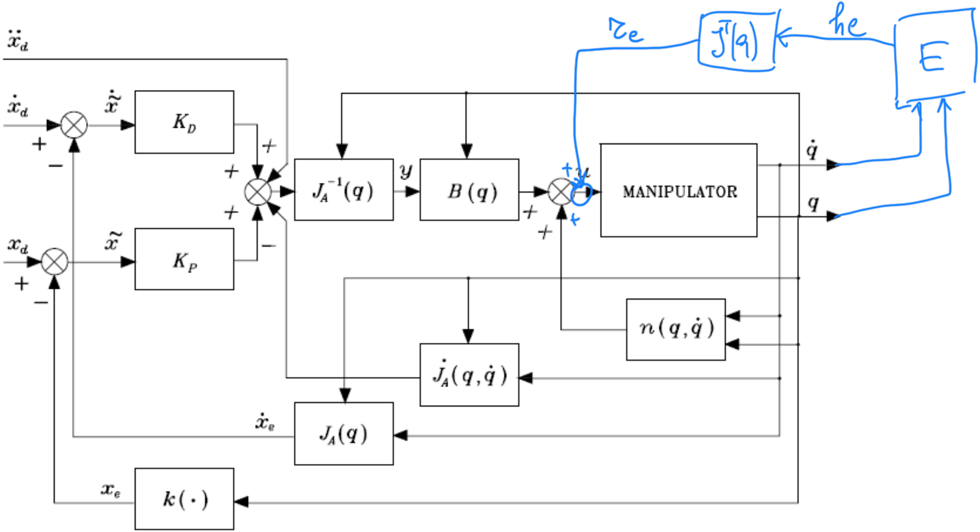


Figure: Operational Space Inverse Dynamics block scheme

Operational Space Inverse Dynamics Control + h_e

The *Operation Space Inverse Dynamics Control* consists of two parts:

1. a *nonlinear state feedback* able to make an *exact linearization* of the nonlinear system dynamics
2. a *stabilizing linear controller*

What happens when there is an external force h_e?

$$B(q)\ddot{q} + \underbrace{C(q,\dot{q})\dot{q} + F\dot{q} + g(q)}_{\triangleq n(q,\dot{q})} = \tau - J^T(q)h_e$$

The nonlinear state feedback

$$\boxed{\tau = B(q)y + n(q,\dot{q}),} \qquad n(q,\dot{q}) = C(q,\dot{q})\dot{q} + F\dot{q} + g(q).$$

brings to

$$\boxed{B(q)\ddot{q} = B(q)y - J^T(q)h_e}$$

$$\cancel{\ddot{q} = y}$$

Since $B(q)$ is nonsingular, it leads to the double integrator dynamics driven by a nonlinear coupling term due to contact forces

$$\ddot{q} = y - \underbrace{B^{-1}(q)J^T(q)}h_e \qquad (*)$$

In the Operational Space Inverse Dynamics Control in free motion we chose

$$\boxed{y = J_A^{-1}(q)\left(\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - \dot{J}_A(q,\dot{q})\dot{q}\right)}$$

where

$$\boxed{\begin{array}{rcl} \tilde{x} & = & x_d - x \\ \dot{\tilde{x}} & = & \dot{x}_d - \dot{x} \end{array}}$$

$x \equiv x_e$

and K_D, K_P are diagonal positive definite matrices.

Hp. We also assumed J_A is nonsingular (non-redundant manipulators).

When the end-effector is in contact with the environment we resort to

$$\boxed{y = J_A^{-1}(q)M_d^{-1}\left(M_d\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_A(q,\dot{q})\dot{q}\right),} \tag{6}$$

where M_d is a positive definite diagonal matrix. We end up with

$K'_D = M_d^{-1} K_D$

$$\underbrace{M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x}} = \underbrace{M_d B_A^{-1}(q)h_A} \tag{7}$$

$K'_P = M_d^{-1} K_P$

where

$$\begin{aligned} B_A(q) &= J_A^{-T}(q)B(q)J_A^{-1}(q), \qquad [J_A \text{ full rank and } x = \kappa(q)] \\ \boxed{h_A} &\boxed{= T_A^T(x)h_e} \end{aligned}$$

(see equations of motions in the operational space).

The equation (7) is a *mechanical impedance* in the operational space mapping the force $M_d B_A^{-1}(q)h_A$ into the displacement $\tilde{x}$.

Impedance control

The impedance is characterized my an *equivalent mass matrix* M_d, an *equivalent damping matrix* K_D and an *equivalent stiffness matrix* K_P.

The matrices M_d, K_D and K_P should be chosen in order to have the expected dynamic behaviour along the different position and orientation directions.

Unfortunately, the matrix $B_A^{-1}(q)$ coupled the effect of $h_A = T_A^T(x)h_e$ along the different directions.

To obtain a linear and decoupled equivalent system when the end-effector is interacting with the environment, we need to measure the contact force h_e.

Knowing h_e, and so $\boxed{h_A = T_A^T(x)h_e}$, we can define the following nonlinear state feedback command

$$\tau = B(q)y + n(q,\dot{q}) + J^T(q)h_e \tag{8}$$

$B(q)\ddot{q} = B(q)y$

$\ddot{q} = y$

and stabilizing linear control law

$$y = J_A^{-1}(q)M_d^{-1}\left(M_d\ddot{x}_d + K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_A(q,\dot{q})\dot{q} - h_A\right), \tag{9}$$

The equivalent *linear and decoupled* system is

$$\boxed{M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x} = h_A.} \tag{10}$$

i-th component of h_A effects only the dynamics of i-th component of $\tilde{x}$

Remarks:

1. the term $J^T(q)h_e$ in (8) cancels out the effect of the contact force;
2. the term h_A in (9) is needed to have a compliant behavior;
3. since eq (10) depends on the actual rotation matrix of the end-effector, it is difficult to relate the matrices M_d, K_D, K_P with the actual Cartesian motion and rotation;
4. problems when the robot is close to a singular configuration.

Impedance Control

The solution consists of designing the control law for y as a function of the *error vector*

$$\tilde{x} = x_d - x_e = -\begin{bmatrix} o^d_{d,e} \\ \phi_{d,e} \end{bmatrix}$$

Σ_d Σ_e

where $\phi_{d,e}$ is the vector of Euler angles extracted from the rotation matrix R^d_e.

If $\Sigma_d = \{O_d - x_d y_d z_d\}$ is time-varying, the time derivative of $\tilde{x}$ is

$-o_{d,e}$

$$(*) \quad \begin{aligned} \dot{\tilde{x}} &= -\begin{bmatrix} R_d^T(\dot{o}_e - \dot{o}_d) - S(\omega^d_d)\boxed{R_d^T}(o_e - o_d) \\ T^{-1}(\phi_{d,e})\omega^d_{d,e} \end{bmatrix} \\ &= -J_{A_d}(q,\tilde{x})\dot{q} + b(\tilde{x}, R_d, \dot{o}_d, \omega_d) \end{aligned}$$

where

$$J_{A_d}(q,\tilde{x}) = T_A^{-1}(\phi_{d,e})\begin{bmatrix} R_d^T & O \\ O & R_d^T \end{bmatrix} J(q), \qquad b(\tilde{x}, R_d, \dot{o}_d, \omega_d) = \begin{bmatrix} R_d^T\dot{o}_d + S(\omega^d_d)o^d_{d,e} \\ T^{-1}(\phi_{d,e})\omega^d_d \end{bmatrix}$$

Computing the second time derivative of the error, we end up with

$= \frac{d}{dt}(*)$

$$\ddot{\tilde{x}} = -J_{A_d}\ddot{q} - \dot{J}_{A_d}\dot{q} + \dot{b}; \qquad (11)$$

The new vector y can be chosen as

$$y = J_A^{-1}(q)M_d^{-1}\left(K_D\dot{\tilde{x}} + K_P\tilde{x} - M_d\dot{J}_{A_d}(q,\dot{q})\dot{q} - M_d b(\tilde{x}, R_d, \dot{o}_d, \omega_d) - h_e^d\right), \qquad (12)$$

and the linear and decoupled system looks like

$$\boxed{M_d\ddot{\tilde{x}} + K_D\dot{\tilde{x}} + K_P\tilde{x} = h_e^d} \qquad \tilde{x} \rightarrow \tilde{x}^d \qquad (13)$$

where each vector is referred to the desired frame.

This linear mechanical impedance is independent of the manipulator configuration q.

Passive impedance: The overall behavior is the result of the parallel interconnection between the robot+impedance controller and the environment impedance.

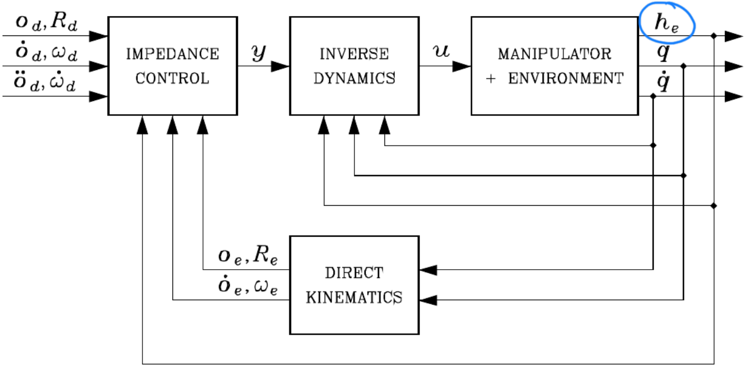


Figure: Block scheme of impedance control.

Impedance Control

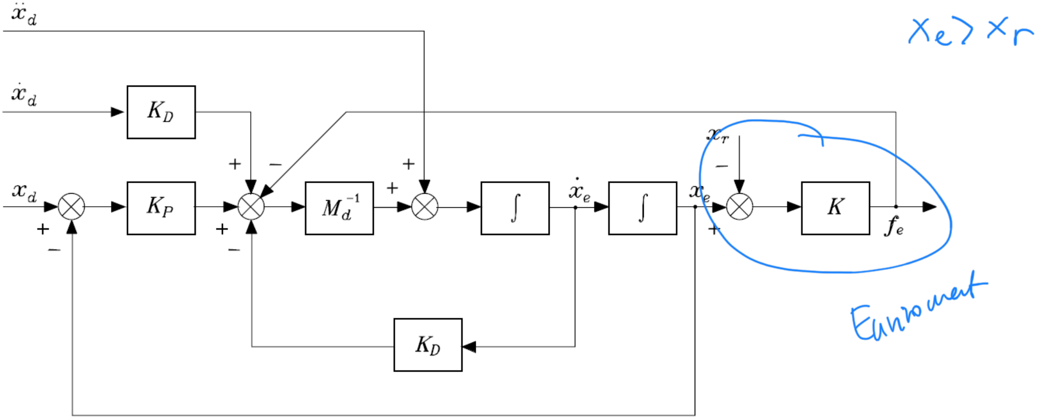


Figure: Equivalent block scheme of a planar manipulator in contact with an elastic environment under impedance control.

Final remarks:

- The execution of a complex task, involving different types of interaction in different directions, requires different values for the impedance parameters.
- The impedance control is equivalent to an inverse dynamics position control in free motion.
- With the impedance control we do not control neither the motion nor the interaction force, but we ‘force’ the port behavior of the robot+controller system.
- The closed-loop dynamics along the free motion directions is different from the closed-loop dynamics along the constrained directions (there is the environment...).


Project

PROJECT – Assignment # 12

To do

- Implement the impedance control in the operational space.

(check eq. (13))

impedance vel $\rightarrow$ force
admittance force $\rightarrow$ vel

Admittance control

Observations:

1. the lower the values for the elements of K_P, the more compliant the impedance control is;
2. the higher the values for the elements of K_P, the better the rejection factor of the disturbance due to model uncertainty/approximation.

A possible solution is to separate the *motion control problem* from the *impedance control problem*.

The idea is to introduce a suitable *compliant frame* to describe the ideal end-effector behavior under impedance control

$$\Sigma_t = \{O_t - x_t y_t z_t\} \qquad \Rightarrow \qquad \{o_t, R_t\} \tag{14}$$

We then integrate the impedance equations in the *mechanical admittance* equation

$$M_t\ddot{\tilde{z}} + K_{Dt}\dot{\tilde{z}} + K_{Pt}\tilde{z} = h_e^d$$

(*)

where M_t, K_{Dt}, K_{Pt} are the parameters of a mechanical impedance, h_e^d is the measured interaction force in Σ_d, and $\tilde{z}$ is the operational space error between the desired frame Σ_d and the compliant frame Σ_t

$$\tilde{z} = x_d - x_t = -\begin{bmatrix} o_{d,t}^d \\ \phi_{d,t} \end{bmatrix}$$

(*) input: h_e^d
output: $\tilde{z}$

where $\phi_{d,t}$ is the vector of Euler angles extracted from the rotation matrix R_t^d. $\Rightarrow x_t$

The kinematic variable of the complaint frame x_t are sent as reference input to the inverse dynamics motion control.

The relationship $M_t\ddot{\tilde{z}} + K_{Dt}\dot{\tilde{z}} + K_{Pt}\tilde{z} = h_e^d$ is used as an admittance: the input is a force and the output is a position/velocity.

$x_t \rightarrow \dot{x}_t, \ddot{x}_t$

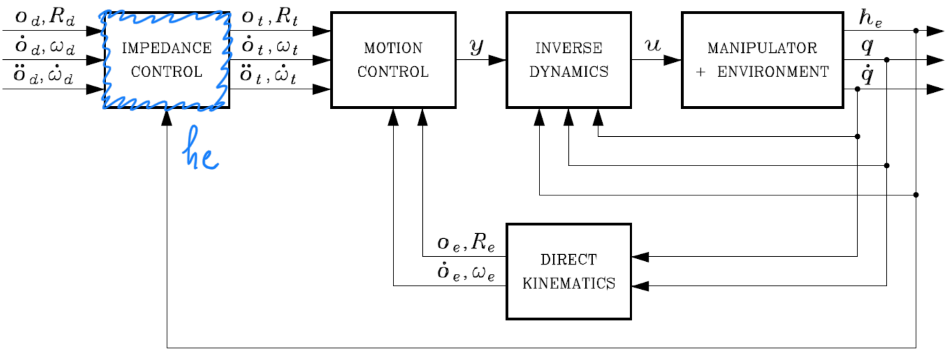


Figure: Block scheme of admittance control.

$\tilde{z} = x_d - x_t$

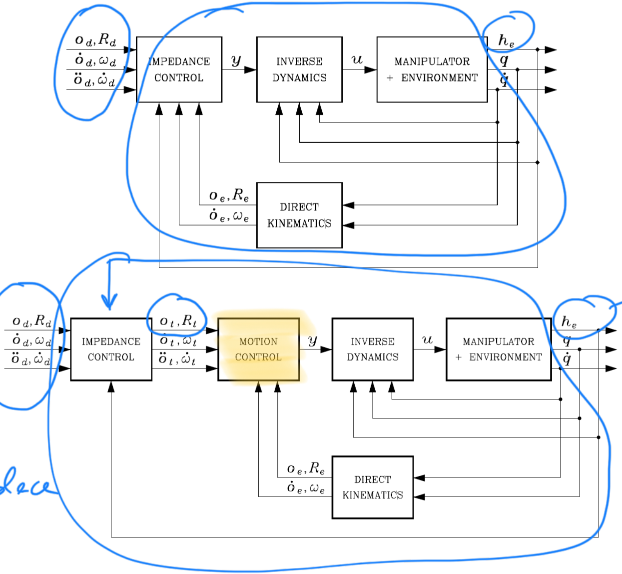
$(x_e \simeq x_d)$

Final remarks:

1
- the gains of the motion control law can be tuned to guarantee a high value of the disturbance rejection factor;

2
- the gains of the impedance control law $M_t\ddot{\tilde{z}} + K_{Dt}\dot{\tilde{z}} + K_{Pt}\tilde{z} = h_e^d$ can be set to guarantee satisfactory behavior during the interaction with the environment;
- the equivalent bandwidth of the motion control loop should be larger than the equivalent bandwidth of the admittance control loop (*inner loop faster than outer loop*);
- the overall control law is mapping motion variables (input) into equivalent end-effector forces (output), i.e. it is behaving as a *mechanical impedance*;
- admittance control should be used anytime there is an embedded position or velocity control loop provided by the robot company that cannot be by-passed (i.e. it is not possible to send torque commands).

Admittance VS Impedance Control



impedance control

output

input

admittance control

~ impedance


Project

To do

- Implement the admittance control in the operational space.