Semantica big-step

$$\textbf{(B-Num)} \; \frac{-}{\langle n, s \rangle \Downarrow n} \qquad \textbf{(B-Loc)} \; \frac{-}{\langle l, s \rangle \Downarrow s(l)} \qquad \textbf{(B-Skip)} \; \frac{-}{\langle \text{skip}, s \rangle \Downarrow s} \qquad \textbf{(B-Add)} \; \frac{\langle E_1, s \rangle \Downarrow n_1 \quad \langle E_2, s \rangle \Downarrow n_2}{\langle E_1 + E_2 \rangle \Downarrow n_3} \; n_3 = add(n_1, n_2)$$

$$\textbf{(B-Assign)} \; \frac{\langle E, s \rangle \Downarrow n}{\langle l := e, s \rangle \Downarrow s[l \mapsto n]} \qquad \textbf{(B-Assign.s)} \; \frac{\langle E, s \rangle \Downarrow n}{\langle l := e, s \rangle \Downarrow \langle \text{skip}, s[l \mapsto n] \rangle} \qquad \textbf{(B-Seq)} \; \frac{\langle C_1, s \rangle \Downarrow s_1 \quad \langle C_2, s_1 \rangle \Downarrow s'}{\langle C_1; C_2, s \rangle \Downarrow s'} \qquad \textbf{(B-Seq.s)} \; \frac{\langle C_1, s \rangle \Downarrow \langle \text{skip}, s_1 \rangle \quad \langle C_2, s_1 \rangle \Downarrow \langle r, s' \rangle}{\langle C_1; C_2, s \rangle \Downarrow \langle r, s' \rangle}$$

$$\textbf{(B-If.T)} \; \frac{\langle B, s \rangle \Downarrow \text{true} \quad \langle C_1, s \rangle \Downarrow s'}{\langle \text{if B then } C_1 \text{ else } C_2, s \rangle \Downarrow s'} \qquad \textbf{(B-If.T)} \; \frac{\langle B, s \rangle \Downarrow \text{true} \quad \langle C_1, s \rangle \Downarrow \langle r, s' \rangle}{\langle \text{if B then } C_1 \text{ else } C_2, s \rangle \Downarrow \langle r, s' \rangle} \qquad \textbf{(B-If.F)} \; \frac{\langle B, s \rangle \Downarrow \text{false} \quad \langle C_2, s \rangle \Downarrow s'}{\langle \text{if B then } C_1 \text{ else } C_2, s \rangle \Downarrow s'} \qquad \textbf{(B-If.F)} \; \frac{\langle B, s \rangle \Downarrow \text{false} \quad \langle C_2, s \rangle \Downarrow \langle r, s' \rangle}{\langle \text{if B then } C_1 \text{ else } C_2, s \rangle \Downarrow \langle r, s \rangle}$$

$$\textbf{(B-While.T)} \; \frac{\langle B, s \rangle \Downarrow \text{true} \quad \langle C, s \rangle \Downarrow s_1 \quad \langle \text{while B do } C, s_1 \rangle \Downarrow s'}{\langle \text{while B do } C, s \rangle \Downarrow s'} \qquad \textbf{(B-While.F)} \; \frac{\langle B, s \rangle \Downarrow \text{false}}{\langle \text{while B do } C, s \rangle \Downarrow s} \qquad \textbf{(B-Do)} \; \frac{\langle B, s \rangle \Downarrow v_1 \quad \langle C, s \rangle \Downarrow v_2}{\langle \text{do E return } C, s \rangle \Downarrow v_2}$$

$$\textbf{(B-While.UN)} \; \frac{\langle \text{if B then } (C; \text{while B do } C) \text{ else skip}, s \rangle \Downarrow r, s'}{\langle \text{while B do } C, s \rangle \Downarrow r, s'} \qquad \textbf{(B-Await)} \; \frac{\langle B, s \rangle \Downarrow \text{true}, s_1 \quad \langle C, s_1 \rangle \twoheadrightarrow^* \langle \text{skip}, s' \rangle}{\langle \text{await B protect } C \text{ end}, s \rangle \twoheadrightarrow \langle \text{skip}, s' \rangle}$$

$$\textbf{(B-Let-CBV)} \; \frac{P \Downarrow m \quad E\{m/x\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Let-CBN)} \; \frac{E\{P/x\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Fn)} \; \frac{-}{\langle \text{fn } x : T \Rightarrow e, s \rangle \twoheadrightarrow \langle \text{fn } x : T \Rightarrow e, s \rangle}$$

$$\textbf{(B-App-CBV)} \; \frac{E_1 \Downarrow \text{fn } x : T \Rightarrow E \quad E\{E_2/x\} \Downarrow n}{E_1 E_2 \Downarrow n} \qquad \textbf{(B-App-CBN)} \; \frac{E_1 \Downarrow \text{fn } x : T \Rightarrow E \quad E_2 \Downarrow v \quad E\{v/x\} \Downarrow n}{E_1 E_2 \Downarrow n}$$

Semantica small-step

Grammatica delle espressioni

$$\text{op} ::= + \mid \geqslant$$

$$e \in \text{Exp} ::= n \mid b \mid e \text{ op } e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{skip} \mid e; e \mid \text{while } e \text{ do } e \mid e_1 := e_2 \mid !e \mid \text{ref } e \mid l \mid \text{fn } x : T \Rightarrow e \mid e_1 e_2 \mid \text{let } x : T = e \text{ in } e \mid e \oplus e \mid e \| e \mid$$
$$\text{await } e \text{ protect } e \text{ end} \mid \#\text{lab } e$$

Regole per la semantica

$$\textbf{(S-Left)} \frac{E_1 \twoheadrightarrow E_1'}{E_1 + E_2 \twoheadrightarrow E_1' + E_2} \qquad \textbf{(S-N.Right)} \frac{E_2 \twoheadrightarrow E_2'}{n_1 + E_2 \twoheadrightarrow n_1 + E_2'} \qquad \textbf{(S-Add)} \frac{-}{n_1 + n_2 \twoheadrightarrow n_3} \, n_3 = \text{add}(n_1, n_2)$$

$$\textbf{(S-Left)} \frac{E_1 \twoheadrightarrow_{ch} E_1'}{E_1 + E_2 \twoheadrightarrow_{ch} E_1' + E_2} \qquad \textbf{(S-Right)} \frac{E_2 \twoheadrightarrow_{ch} E_2'}{E_1 + E_2 \twoheadrightarrow_{ch} E_1 + E_2'} \qquad \textbf{(op+)} \frac{-}{\langle n_1 + n_2, s \rangle \twoheadrightarrow \langle n, s \rangle} \, n = \text{add}(n_1, n_2)$$

$$\textbf{(op-geq)} \frac{-}{\langle n_1 \geqslant n_2, s \rangle \twoheadrightarrow \langle b, s \rangle} \, b = geq(n_1, n_2) \qquad \textbf{(op1)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 + e_2, s \rangle \twoheadrightarrow \langle e_1' + e_2, s' \rangle} \qquad \textbf{(op2)} \frac{\langle e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}{\langle v + e_2, s \rangle \twoheadrightarrow \langle v + e_2', s \rangle}$$

$$\textbf{(op1b)} \frac{\langle e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}{\langle e_1 + e_2, s \rangle \twoheadrightarrow \langle e_1 + e_2', s' \rangle} \qquad \textbf{(op2b)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 + v, s \rangle \twoheadrightarrow \langle e_1' + v', s' \rangle} \qquad \textbf{(deref1)} \frac{-}{\langle !l, s \rangle \twoheadrightarrow \langle v, s \rangle} \, \text{if } l \in \text{dom}(s) \wedge s(l) = v$$

$$\textbf{(deref2)} \frac{\langle e, s \rangle \twoheadrightarrow \langle e', s' \rangle}{\langle !e, s \rangle \twoheadrightarrow \langle !e', s' \rangle} \qquad \textbf{(ref1)} \frac{-}{\langle \text{ref } v, s \rangle \twoheadrightarrow \langle l, s[l \mapsto v] \rangle} \, l \notin \text{dom}(s) \qquad \textbf{(ref2)} \frac{\langle e, s \rangle \twoheadrightarrow \langle e', s' \rangle}{\langle \text{ref } e, s \rangle \twoheadrightarrow \langle \text{ref } e', s \rangle}$$

$$\textbf{(assign1)} \frac{-}{\langle l := v, s \rangle \twoheadrightarrow \langle \text{skip}, s[l \mapsto v] \rangle} \, \text{if } l \in \text{dom}(s) \qquad \textbf{(assign2)} \frac{\langle e, s \rangle \twoheadrightarrow \langle e', s' \rangle}{\langle l := e, s \rangle \twoheadrightarrow \langle l := e', s \rangle} \qquad \textbf{(assign3)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 := e_2, s \rangle \twoheadrightarrow \langle e_1' := e_2, s' \rangle}$$

$$\textbf{(if-tt)} \frac{-}{\langle \text{if true then } e_1 \text{ else } e_2, s \rangle \twoheadrightarrow \langle e_1, s \rangle} \qquad \textbf{(if-ff)} \frac{-}{\langle \text{if false then } e_1 \text{ else } e_2, s \rangle \twoheadrightarrow \langle e_2, s \rangle} \qquad \textbf{(if)} \frac{\langle e, s \rangle \twoheadrightarrow \langle e', s' \rangle}{\langle \text{if } e \text{ then } e_1 \text{ else } e_2, s \rangle \twoheadrightarrow \langle \text{if } e' \text{ then } e_1 \text{ else } e_2, s \rangle}$$

$$\textbf{(while)} \frac{-}{\langle \text{while } e \text{ do } e_1, s \rangle \twoheadrightarrow \langle \text{if } e \text{ then } (e_1; \text{while } e \text{ do } e_1) \text{ else skip}, s \rangle} \qquad \textbf{(assign1b)} \frac{-}{\langle l := n, s \rangle \twoheadrightarrow \langle n, s[l \mapsto n] \rangle} \, l \in \text{dom}(s)$$

$$\textbf{(seq.skip)} \frac{-}{\langle \text{skip}; e_2, s \rangle \twoheadrightarrow \langle e_2, s \rangle} \qquad \textbf{(seq)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s \rangle}{\langle e_1; e_2, s \rangle \twoheadrightarrow \langle e_1'; e_2, s' \rangle} \qquad \textbf{(seq.skipb)} \frac{-}{\langle v; e_2, s \rangle \twoheadrightarrow \langle e_2, s \rangle}$$

$$\textbf{(record1)} \frac{\langle e_i, s \rangle \twoheadrightarrow \langle e_i', s' \rangle}{\langle \{lab_1 = v_1, \ldots, lab_i = e_i, \ldots, lab_k = e_k\}, s \rangle \twoheadrightarrow \langle \{lab_1 = v_1, \ldots, lab_i = e_i', \ldots, lab_k = e_k\}, s' \rangle}$$

$$\textbf{(record2)} \frac{-}{\langle \#lab_i \ \{lab_1 = v_1, \ldots, lab_i = e_i, \ldots, lab_k = e_k\}, s \rangle \twoheadrightarrow \langle v_i, s \rangle} \qquad \textbf{(record3)} \frac{\langle e, s \rangle \twoheadrightarrow \langle e', s' \rangle}{\langle \#lab \ e, s \rangle \twoheadrightarrow \langle \#lab \ e', s' \rangle}$$

$$\textbf{(par-L)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 \| e_2, s \rangle \twoheadrightarrow \langle e_1' \| e_2, s' \rangle} \qquad \textbf{(par-R)} \frac{\langle e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}{\langle e_1 \| e_2, s \rangle \twoheadrightarrow \langle e_1 \| e_2', s' \rangle} \qquad \textbf{(end-L)} \frac{-}{\langle skip \| e, s \rangle \twoheadrightarrow \langle e, s \rangle} \qquad \textbf{(end-R)} \frac{-}{\langle e \| skip, s \rangle \twoheadrightarrow \langle e, s \rangle}$$

$$\textbf{(await)} \frac{\langle e_1, s \rangle \twoheadrightarrow^* \langle true, s' \rangle \qquad \langle e_2, s' \rangle \twoheadrightarrow^* \langle skip, s'' \rangle}{\langle await \ e_1 \ protect \ e_2 \ end, s \rangle \twoheadrightarrow \langle skip, s'' \rangle} \qquad \textbf{(ChoiceL)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 \oplus e_2, s \rangle \twoheadrightarrow \langle e_1', s' \rangle} \qquad \textbf{(ChoiceR)} \frac{\langle e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}{\langle e_1 \oplus e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}$$

Call-By-Value

$$\textbf{(CBV-app1)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 e_2, s \rangle \twoheadrightarrow \langle e_1' e_2, s' \rangle} \qquad \textbf{(CBV- app2)} \frac{\langle e_2, s \rangle \twoheadrightarrow \langle e_2', s' \rangle}{\langle v e_2, s \rangle \twoheadrightarrow \langle v e_2', s' \rangle} \qquad \textbf{(CBV-fn)} \frac{-}{\langle (fn \ x \colon T \Rightarrow e)v, s \rangle \twoheadrightarrow \langle e\{v/x\}, s \rangle}$$

$$\textbf{(CBV-let1)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle let \ x \colon T = e_1 \ in \ e_2, s \rangle \twoheadrightarrow \langle let \ x \colon T = e_1' \ in \ e_2, s' \rangle} \qquad \textbf{(CBV-let2)} \frac{-}{\langle let \ x \colon T = v \ in \ e_2, s \rangle \twoheadrightarrow \langle e_2\{v/x\}, s \rangle}$$

$$\textbf{(CBV-fix)} \frac{e \equiv fn \ f \colon T_1 \to T_2 \ \Rightarrow \ e_2}{fix.e \twoheadrightarrow e(fn \ x \colon T_1 \Rightarrow \ (fix.e) \ x)}$$

Call-By-Name

$$\textbf{(CBN-app)} \frac{\langle e_1, s \rangle \twoheadrightarrow \langle e_1', s' \rangle}{\langle e_1 e_2, s \rangle \twoheadrightarrow \langle e_1' e_2, s' \rangle} \qquad \textbf{(CBN-fn)} \frac{-}{\langle (fn \ x \colon T \Rightarrow e)e_2, s \rangle \twoheadrightarrow \langle e\{e_2/x\}, x \rangle}$$

$$\textbf{(CBN-let)} \frac{-}{\langle let \ x \colon T = e_1 \ in \ e_2, s \rangle \twoheadrightarrow \langle e_2\{e_1/x\}, s \rangle}$$

$$\textbf{(CBN-fix)} \frac{-}{fix.e \twoheadrightarrow e(fix.e)}$$

Grammatica dei tipi e type system

$$T ::= \text{int} \mid \text{bool} \mid \text{unit} \mid T_1 \to T_2 \mid T_1 + T_2 \mid T_1 * T_2 \mid \text{ref } T \mid \{\text{lab}_1 : T_1, \ldots, \text{lab}_k : T_k\} \mid \text{proc}$$

$$T_{\text{loc}} ::= \text{ref } T$$

Tipi primitivi e operatori

$$\textbf{(int)} \frac{-}{\Gamma \vdash n : \text{int}} \text{ for } n \in \mathbb{Z} \qquad \textbf{(bool)} \frac{-}{\Gamma \vdash b : \text{bool}} \text{ for } n \in \{\text{true}, \text{false}\} \qquad \textbf{(op +)} \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \qquad \textbf{(op *)} \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 * e_2 : \text{int}}$$

$$\textbf{(op or)} \frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \text{ or } e_2 : \text{bool}} \qquad \textbf{(op and)} \frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \text{ and } e_2 : \text{bool}} \qquad \textbf{(op} \geqslant \textbf{)} \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 \geqslant e_2 : \text{bool}} \qquad \textbf{(skip)} \frac{-}{\Gamma \vdash \text{skip} : \text{unit}}$$

$$\textbf{(seq)} \frac{\Gamma \vdash e_1 : \text{unit} \quad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 ; e_2 : T} \qquad \textbf{(if)} \frac{\Gamma : e_1 : \text{bool} \quad \Gamma \vdash e_2 : T \quad \Gamma \vdash e_3 : T}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : T} \qquad \textbf{(while)} \frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : T}{\Gamma \vdash \text{while } e_1 \text{ do } e_2 : T} \qquad \textbf{(let)} \frac{\Gamma \vdash e_1 : T \quad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \text{let } x : T = e_1 \text{ in } e_2 : T'}$$

$$\textbf{(T-fix)} \frac{\Gamma \vdash e : (T_1 \to T_2) \to (T_1 \to T_2)}{\Gamma \vdash \text{fix}.e : T_1 \to T_2}$$

Referenze

$$\textbf{(ref)} \frac{\Gamma \vdash e : T}{\Gamma \vdash \text{ref } e : \text{ref } T} \qquad \textbf{(deref)} \frac{\Gamma \vdash e : \text{ref } T}{\Gamma \vdash !e : T} \qquad \textbf{(assign)} \frac{\Gamma \vdash e_1 : \text{ref } T \quad \Gamma \vdash e_2 : T}{\Gamma \vdash (e_1 := e_2) : \text{unit}} \qquad \textbf{(loc)} \frac{-}{\Gamma \vdash l : \text{ref } T} \; \Gamma(l) = \text{ref } T$$

Funzioni

$$\textbf{(var)} \frac{-}{\Gamma \vdash : T} \text{ if } \Gamma(x) = T \qquad \textbf{(fn)} \frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash (\text{fn } x : T \Rightarrow e) : T \to T'} \qquad \textbf{(app)} \frac{\Gamma \vdash e_1 : T \to T' \quad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 e_2 : T'}$$

Record

$$\textbf{(record)} \frac{\Gamma \vdash e_1 : T_1 \; \ldots \; \Gamma \vdash e_k : T_k}{\Gamma \vdash \{\text{lab}_1 = e_1, \ldots, \text{lab}_k = e_k\} : \{\text{lab}_1 : T_1, \ldots, \text{lab}_k : T_k\}} \qquad \textbf{(recordproj)} \frac{\Gamma \vdash e : \{\text{lab}_1 : T_1, \ldots, \text{lab}_k : T_k\}}{\Gamma \vdash \#\text{lab}_i \; e : T_i}$$

Concorrenza

$$\textbf{(T-sq1)} \frac{\Gamma \vdash e_1 : \text{unit} \quad \Gamma \vdash e_2 : \text{unit}}{\Gamma \vdash e_1 ; e_2 : \text{unit}} \qquad \textbf{(T-sq2)} \frac{\Gamma \vdash e_1 : \text{proc} \quad \Gamma \vdash e_2 : \text{proc}}{\Gamma \vdash e_1 ; e_2 : \text{proc}} \qquad \textbf{(T-par)} \frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 \| e_2 : \text{proc}} \; T_1, T_2 \in \{\text{unit}, \text{proc}\}$$

$$\textbf{(T-await)} \frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{unit}}{\Gamma \vdash \text{await } e_1 \text{ protect } e_2 \text{ end} : \text{unit}} \qquad \textbf{(T-choice)} \frac{\Gamma \vdash e_1 : \text{unit} \quad \Gamma \vdash e_2 : \text{unit}}{\Gamma \vdash e_1 \oplus e_2 : \text{unit}}$$

Sottotipaggio

$$\text{(sub)}\ \dfrac{\Gamma \vdash e : T \qquad T <: T'}{\Gamma \vdash e : T'} \qquad \text{(s-refl)}\ \dfrac{-}{T <: T} \qquad\qquad \text{(s-trans)}\ \dfrac{T <: T' \qquad T' <: T''}{T <: T''}$$

Sottotipaggio dei record

$$\text{(rec-perm)}\ \dfrac{\pi \text{ una permutazione di } 1, 2, \ldots, k}{\{p_1 : T_1, \ldots, p_k : T_k\} <: \{p_{\pi(1)} : T_{\pi(1)}, \ldots, p_{\pi(k)} : T_{\pi(k)}\}} \qquad \text{(rec-width)}\ \dfrac{-}{\{p_1 : T_1, \ldots, p_k : T_k, p_{k+1} : T_{k+1}, \ldots, p_z : T_z\} <: \{p_1 : T_1, \ldots, p_k : T_k\}}$$

$$\text{(rec-depth)}\ \dfrac{T_1 <: T_1' \ \ldots \ T_k <: T_k'}{\{p_1 : T_1, \ldots, p_k : T_k\} <: \{p_1 : T_1', \ldots, p_k : T_k'\}}$$

Sottotipaggio delle funzioni

$$\text{(fun-sub)}\ \dfrac{T_1 :> T_1' \qquad T_2 <: T_2'}{T_1 \to T_2 <: T_1' \to T_2'}$$

Sottotipaggio somma e prodotto

$$\text{(prod-sub)}\ \dfrac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 * T_2 <: T_1' * T_2'} \qquad \text{(sum-sub)}\ \dfrac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 + T_2 <: T_1' + T_2'}$$