Semantica big-step

$$\textbf{(B-Num)}\ \frac{-}{\langle n, s\rangle \Downarrow n} \qquad \textbf{(B-Loc)}\ \frac{-}{\langle l, s\rangle \Downarrow s(l)} \qquad \textbf{(B-Skip)}\ \frac{-}{\langle \text{skip}, s\rangle \Downarrow s} \qquad \textbf{(B-Add)}\ \frac{\langle E_1, s\rangle \Downarrow n_1 \quad \langle E_2, s\rangle \Downarrow n_2}{\langle E_1 + E_2, s\rangle \Downarrow n_3}\, n_3 = \mathtt{add}(n_1, n_2)$$

$$\textbf{(B-Assign)}\ \frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow s[l \mapsto n]} \qquad \textbf{(B-Assign.s)}\ \frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow \langle \text{skip}, s[l \mapsto n]\rangle} \qquad \textbf{(B-Seq)}\ \frac{\langle C_1, s\rangle \Downarrow s_1 \quad \langle C_2, s_1\rangle \Downarrow s'}{\langle C_1; C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-Seq.s)}\ \frac{\langle C_1, s\rangle \Downarrow \langle \text{skip}, s_1\rangle \quad \langle C_2, s_1\rangle \Downarrow \langle r, s'\rangle}{\langle C_1; C_2, s\rangle \Downarrow \langle r, s'\rangle}$$

$$\textbf{(B-If.T)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C_1, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-If.T)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C_1, s\rangle \Downarrow \langle r, s'\rangle}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s'\rangle}$$

$$\textbf{(B-If.F)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{false} \quad \langle C_2, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-If.F)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{false} \quad \langle C_2, s\rangle \Downarrow \langle r, s'\rangle}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s\rangle}$$

$$\textbf{(B-While.T)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C, s\rangle \Downarrow s_1 \quad \langle \text{while } B \text{ do } C, s_1\rangle \Downarrow s'}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow s'} \qquad \textbf{(B-While.F)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{false}}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow s} \qquad \textbf{(B-Do)}\ \frac{\langle B, s\rangle \Downarrow v_1 \quad \langle C, s\rangle \Downarrow v_2}{\langle \text{do } E \text{ return } C, s\rangle \Downarrow v_2}$$

$$\textbf{(B-While.UN)}\ \frac{\langle \text{if } B \text{ then } (C; \text{while } B \text{ do } C) \text{ else skip}, s\rangle \Downarrow r, s'}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow r, s'} \qquad \textbf{(B-Await)}\ \frac{\langle B, s\rangle \Downarrow \mathtt{true}, s_1 \quad \langle C, s_1\rangle \rightarrow^* \langle \text{skip}, s'\rangle}{\langle \text{await } B \text{ protect } C \text{ end}, s\rangle \rightarrow \langle \text{skip}, s'\rangle}$$

$$\textbf{(B-Let-CBV)}\ \frac{P \Downarrow m \quad E\{^m/_x\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Let-CBN)}\ \frac{E\{^P/_x\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Fn)}\ \frac{-}{\langle \text{fn } x\colon T \;\Rightarrow\; e, s\rangle \rightarrow \langle \text{fn } x\colon T \;\Rightarrow\; e, s\rangle}$$

$$\textbf{(B-App-CBV)}\ \frac{E_1 \Downarrow \text{fn } x\colon T \Rightarrow E \qquad E\{^{E_2}/_x\} \Downarrow n}{E_1 E_2 \Downarrow n} \qquad \textbf{(B-App-CBN)}\ \frac{E_1 \Downarrow \text{fn } x\colon T \Rightarrow E \qquad E_2 \Downarrow v \qquad E\{^v/_x\} \Downarrow n}{E_1 E_2 \Downarrow n}$$

Semantica small-step

Grammatica delle espressioni

$$op ::= + \mid \geqslant$$

$$e \in \text{Exp} ::= n \mid b \mid e \text{ op } e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{skip} \mid e; e \mid \text{while } e \text{ do } e \mid e_1 := e_2 \mid !e \mid \text{ref } e \mid l \mid \text{fn } x : T \Rightarrow e \mid e_1 e_2 \mid \text{let } x : T = e \text{ in } e \mid e \oplus e \mid e \| e \mid$$
$$\text{await } e \text{ protect } e \text{ end} \mid \#\text{lab } e$$

Regole per la semantica

$$\textbf{(S-Left)}\ \frac{E_1 \twoheadrightarrow E_1'}{E_1 + E_2 \twoheadrightarrow E_1' + E_2} \qquad \textbf{(S-N.Right)}\ \frac{E_2 \twoheadrightarrow E_2'}{n_1 + E_2 \twoheadrightarrow n_1 + E_2'} \qquad \textbf{(S-Add)}\ \frac{-}{n_1 + n_2 \twoheadrightarrow_{ch} n_3}\ n_3 = add(n_1, n_2)$$

$$\textbf{(S-Left)}\ \frac{E_1 \twoheadrightarrow_{ch} E_1'}{E_1 + E_2 \twoheadrightarrow_{ch} E_1' + E_2} \qquad \textbf{(S-Right)}\ \frac{E_2 \twoheadrightarrow_{ch} E_2'}{E_1 + E_2 \twoheadrightarrow_{ch} E_1 + E_2'} \qquad \textbf{(op+)}\ \frac{-}{\langle n_1 + n_2, s\rangle \twoheadrightarrow \langle n, s\rangle}\ n = add(n_1, n_2)$$

$$\textbf{(op-geq)}\ \frac{-}{\langle n_1 \geqslant n_2, s\rangle \twoheadrightarrow \langle b, s\rangle}\ b = geq(n_1, n_2) \qquad \textbf{(op1)}\ \frac{\langle e_1, s\rangle \twoheadrightarrow \langle e_1', s'\rangle}{\langle e_1 + e_2, s\rangle \twoheadrightarrow \langle e_1' + e_2, s'\rangle} \qquad \textbf{(op2)}\ \frac{\langle e_2, s\rangle \twoheadrightarrow \langle e_2', s'\rangle}{\langle v + e_2, s\rangle \twoheadrightarrow \langle v + e_2', s\rangle}$$

$$\textbf{(op1b)}\ \frac{\langle e_2, s\rangle \twoheadrightarrow \langle e_2', s'\rangle}{\langle e_1 + e_2, s\rangle \twoheadrightarrow \langle e_1 + e_2', s'\rangle} \qquad \textbf{(op2b)}\ \frac{\langle e_1, s\rangle \twoheadrightarrow \langle e_1', s'\rangle}{\langle e_1 + v, s\rangle \twoheadrightarrow \langle e_1' + v', s'\rangle} \qquad \textbf{(deref1)}\ \frac{-}{\langle !l, s\rangle \twoheadrightarrow \langle v, s\rangle}\ \text{if } l \in dom(s) \wedge s(l) = v$$

$$\textbf{(deref2)}\ \frac{\langle e, s\rangle \twoheadrightarrow \langle e', s'\rangle}{\langle !e, s\rangle \twoheadrightarrow \langle !e', s'\rangle} \qquad \textbf{(ref1)}\ \frac{-}{\langle \text{ref } v, s\rangle \twoheadrightarrow \langle l, s[l \mapsto v]\rangle}\ l \notin dom(s) \qquad \textbf{(ref2)}\ \frac{\langle e, s\rangle \twoheadrightarrow \langle e', s'\rangle}{\langle \text{ref } e, s\rangle \twoheadrightarrow \langle \text{ref } e', s\rangle}$$

$$\textbf{(assign1)}\ \frac{-}{\langle l := v, s\rangle \twoheadrightarrow \langle \text{skip}, s[l \mapsto v]\rangle}\ \text{if } l \in dom(s) \qquad \textbf{(assign2)}\ \frac{\langle e, s\rangle \twoheadrightarrow \langle e', s'\rangle}{\langle l := e, s\rangle \twoheadrightarrow \langle l := e', s\rangle} \qquad \textbf{(assign3)}\ \frac{\langle e_1, s\rangle \twoheadrightarrow \langle e_1', s'\rangle}{\langle e_1 := e_2, s\rangle \twoheadrightarrow \langle e_1' := e_2, s'\rangle}$$

$$\textbf{(if-tt)}\ \frac{-}{\langle \text{if } true \text{ then } e_1 \text{ else } e_2, s\rangle \twoheadrightarrow \langle e_1, s\rangle} \qquad \textbf{(if-ff)}\ \frac{-}{\langle \text{if } false \text{ then } e_1 \text{ else } e_2, s\rangle \twoheadrightarrow \langle e_2, s\rangle} \qquad \textbf{(if)}\ \frac{\langle e, s\rangle \twoheadrightarrow \langle e', s'\rangle}{\langle \text{if } e \text{ then } e_1 \text{ else } e_2, s\rangle \twoheadrightarrow \langle \text{if } e' \text{ then } e_1 \text{ else } e_2, s\rangle}$$

$$\textbf{(while)}\ \frac{-}{\langle \text{while } e \text{ do } e_1, s\rangle \twoheadrightarrow \langle \text{if } e \text{ then } (e_1; \text{while } e \text{ do } e_1) \text{ else skip}, s\rangle} \qquad \textbf{(assign1b)}\ \frac{-}{\langle l := n, s\rangle \twoheadrightarrow \langle n, s[l \mapsto n]\rangle}\ l \in dom(s)$$

$$\textbf{(seq.skip)}\ \frac{-}{\langle \text{skip}; e_2, s\rangle \twoheadrightarrow \langle e_2, s\rangle} \qquad \textbf{(seq)}\ \frac{\langle e_1, s\rangle \twoheadrightarrow \langle e_1', s\rangle}{\langle e_1; e_2, s\rangle \twoheadrightarrow \langle e_1'; e_2, s'\rangle} \qquad \textbf{(seq.skipb)}\ \frac{-}{\langle v; e_2, s\rangle \twoheadrightarrow \langle e_2, s\rangle}$$

$$\textbf{(record1)}\ \frac{\langle e_i, s\rangle \rightarrow \langle e_i', s'\rangle}{\langle\{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i, \ldots, \mathrm{lab}_k = e_k\}, s\rangle \rightarrow \langle\{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i', \ldots, \mathrm{lab}_k = e_k\}, s'\rangle}$$

$$\textbf{(record2)}\ \frac{-}{\langle \#\mathrm{lab}_i\ \{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i, \ldots, \mathrm{lab}_k = e_k\}, s\rangle \rightarrow \langle v_i, s\rangle} \qquad \textbf{(record3)}\ \frac{\langle e, s\rangle \rightarrow \langle e', s'\rangle}{\langle \#\mathrm{lab}\ e, s\rangle \rightarrow \langle \#\mathrm{lab}\ e', s'\rangle}$$

$$\textbf{(par-L)}\ \frac{\langle e_1, s\rangle \rightarrow \langle e_1', s'\rangle}{\langle e_1 \| e_2, s\rangle \rightarrow \langle e_1' \| e_2, s'\rangle} \qquad \textbf{(par-R)}\ \frac{\langle e_2, s\rangle \rightarrow \langle e_2', s'\rangle}{\langle e_1 \| e_2, s\rangle \rightarrow \langle e_1 \| e_2', s'\rangle} \qquad \textbf{(end-L)}\ \frac{-}{\langle \mathrm{skip} \| e, s\rangle \rightarrow \langle e, s\rangle} \qquad \textbf{(end-R)}\ \frac{-}{\langle e \| \mathrm{skip}, s\rangle \rightarrow \langle e, s\rangle}$$

$$\textbf{(await)}\ \frac{\langle e_1, s\rangle \rightarrow^* \langle \mathrm{true}, s'\rangle \qquad \langle e_2, s'\rangle \rightarrow^* \langle \mathrm{skip}, s''\rangle}{\langle \mathrm{await}\ e_1\ \mathrm{protect}\ e_2\ \mathrm{end}, s\rangle \rightarrow \langle \mathrm{skip}, s''\rangle} \qquad \textbf{(ChoiceL)}\ \frac{\langle e_1, s\rangle \rightarrow \langle e_1', s'\rangle}{\langle e_1 \oplus e_2, s\rangle \rightarrow \langle e_1', s'\rangle} \qquad \textbf{(ChoiceR)}\ \frac{\langle e_2, s\rangle \rightarrow \langle e_2', s'\rangle}{\langle e_1 \oplus e_2, s\rangle \rightarrow \langle e_2', s'\rangle}$$

Call-By-Value

$$\textbf{(CBV-app1)}\ \frac{\langle e_1, s\rangle \rightarrow \langle e_1', s'\rangle}{\langle e_1 e_2, s\rangle \rightarrow \langle e_1' e_2, s'\rangle} \qquad \textbf{(CBV- app2)}\ \frac{\langle e_2, s\rangle \rightarrow \langle e_2', s'\rangle}{\langle v e_2, s\rangle \rightarrow \langle v e_2', s'\rangle} \qquad \textbf{(CBV-fn)}\ \frac{-}{\langle (\mathsf{fn}\ x\colon \mathsf{T} \Rightarrow e) v, s\rangle \rightarrow \langle e\{^{v}/_{x}\}, s\rangle}$$

$$\textbf{(CBV-let1)}\ \frac{\langle e_1, s\rangle \rightarrow \langle e_1', s'\rangle}{\langle \mathrm{let}\ x\colon \mathsf{T} = e_1\ \mathrm{in}\ e_2, s\rangle \rightarrow \langle \mathrm{let}\ x\colon \mathsf{T} = e_1'\ \mathrm{in}\ e_2, s'\rangle} \qquad \textbf{(CBV-let2)}\ \frac{-}{\langle \mathrm{let}\ x\colon \mathsf{T} = v\ \mathrm{in}\ e_2, s\rangle \rightarrow \langle e_2\{^{v}/_{x}\}, s\rangle}$$

$$\textbf{(CBV-fix)}\ \frac{e \equiv\ \mathrm{fn}\ f\colon \mathsf{T}_1 \rightarrow \mathsf{T}_2\ \Rightarrow\ e_2}{\mathrm{fix}.e \rightarrow e(\mathrm{fn}\ x\colon \mathsf{T}_1 \Rightarrow\ (\mathrm{fix}.e)\ x)}$$

Call-By-Name

$$\textbf{(CBN-app)}\ \frac{\langle e_1, s\rangle \rightarrow \langle e_1', s'\rangle}{\langle e_1 e_2, s\rangle \rightarrow \langle e_1' e_2, s'\rangle} \qquad \textbf{(CBN-fn)}\ \frac{-}{\langle (\mathsf{fn}\ x\colon \mathsf{T} \Rightarrow e) e_2, s\rangle \rightarrow \langle e\{^{e_2}/_{x}\}, x\rangle}$$

$$\textbf{(CBN-let)}\ \frac{-}{\langle \mathrm{let}\ x\colon \mathsf{T} = e_1\ \mathrm{in}\ e_2, s\rangle \rightarrow \langle e_2\{^{e_1}/_{x}\}, s\rangle}$$

$$\textbf{(CBN-fix)}\ \frac{-}{\mathrm{fix}.e \rightarrow e(\mathrm{fix}.e)}$$

Grammatica dei tipi e type system

$$T ::= \text{int} \mid \text{bool} \mid \text{unit} \mid T_1 \to T_2 \mid T_1 + T_2 \mid T_1 * T_2 \mid \text{ref } T \mid \{lab_1 \colon T_1, \ldots, lab_k \colon T_k\} \mid \text{proc}$$

$$T_{\text{loc}} ::= \text{ref } T$$

Tipi primitivi e operatori

$$\textbf{(int)}\ \frac{-}{\Gamma \vdash n\colon \text{int}}\ \text{for } n \in \mathbb{Z} \qquad \textbf{(bool)}\ \frac{-}{\Gamma \vdash b\colon \text{bool}}\ \text{for } b \in \{true, false\}$$

$$\textbf{(op +)}\ \frac{\Gamma \vdash e_1\colon \text{int} \qquad \Gamma \vdash e_2\colon \text{int}}{\Gamma \vdash e_1 + e_2\colon \text{int}} \qquad \textbf{(op *)}\ \frac{\Gamma \vdash e_1\colon \text{int} \qquad \Gamma \vdash e_2\colon \text{int}}{\Gamma \vdash e_1 * e_2\colon \text{int}}$$

$$\textbf{(op or)}\ \frac{\Gamma \vdash e_1\colon \text{bool} \qquad \Gamma \vdash e_2\colon \text{bool}}{\Gamma \vdash e_1 \text{ or } e_2\colon \text{bool}} \qquad \textbf{(op and)}\ \frac{\Gamma \vdash e_1\colon \text{bool} \qquad \Gamma \vdash e_2\colon \text{bool}}{\Gamma \vdash e_1 \text{ and } e_2\colon \text{bool}}$$

$$\textbf{(op} \geqslant\textbf{)}\ \frac{\Gamma \vdash e_1\colon \text{int} \qquad \Gamma \vdash e_2\colon \text{int}}{\Gamma \vdash e_1 \geqslant e_2\colon \text{bool}} \qquad \textbf{(skip)}\ \frac{-}{\Gamma \vdash skip\colon unit}$$

$$\textbf{(seq)}\ \frac{\Gamma \vdash e_1\colon unit \qquad \Gamma \vdash e_2\colon T}{\Gamma \vdash e_1; e_2\colon T} \qquad \textbf{(if)}\ \frac{\Gamma\colon e_1\colon \text{bool} \quad \Gamma \vdash e_2\colon T \quad \Gamma \vdash e_3\colon T}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3\colon T}$$

$$\textbf{(while)}\ \frac{\Gamma \vdash e_1\colon \text{bool} \qquad \Gamma \vdash e_2\colon T}{\Gamma \vdash \text{while } e_1 \text{ do } e_2\colon T} \qquad \textbf{(let)}\ \frac{\Gamma \vdash e_1\colon T \qquad \Gamma, x\colon T \vdash e_2\colon T'}{\Gamma \vdash \text{let } x\colon T = e_1 \text{ in } e_2\colon T'}$$

$$\textbf{(T-fix)}\ \frac{\Gamma \vdash e\colon (T_1 \to T_2) \to (T_1 \to T_2)}{\Gamma \vdash \text{fix}.e\colon T_1 \to T_2}$$

Referenze

$$\textbf{(ref)}\ \frac{\Gamma \vdash e\colon T}{\Gamma \vdash \text{ref } e\colon \text{ref } T} \qquad \textbf{(deref)}\ \frac{\Gamma \vdash e\colon \text{ref } T}{\Gamma \vdash !e\colon T} \qquad \textbf{(assign)}\ \frac{\Gamma \vdash e_1\colon \text{ref } T \quad \Gamma \vdash e_2\colon T}{\Gamma \vdash (e_1 := e_2)\colon \text{unit}} \qquad \textbf{(loc)}\ \frac{-}{\Gamma \vdash l\colon \text{ref } T}\ \Gamma(l) = \text{ref } T$$

Funzioni

$$\textbf{(var)}\ \frac{-}{\Gamma \vdash x\colon T}\ \text{if } \Gamma(x) = T \qquad \textbf{(fn)}\ \frac{\Gamma, x\colon T \vdash e\colon T'}{\Gamma \vdash (fn\ x\colon T \ \Rightarrow\ e)\colon T \to T'} \qquad \textbf{(app)}\ \frac{\Gamma \vdash e_1\colon T \to T' \qquad \Gamma \vdash e_2\colon T}{\Gamma \vdash e_1 e_2\colon T'}$$

Record

$$\textbf{(record)}\ \frac{\Gamma \vdash e_1\colon T_1\ \ \ldots\ \ \Gamma \vdash e_k\colon T_k}{\Gamma \vdash \{\text{lab}_1 = e_1, \ldots, \text{lab}_k = e_k\} : \{\text{lab}_1\colon T_1, \ldots, \text{lab}_k\colon T_k\}} \qquad \textbf{(recordproj)}\ \frac{\Gamma \vdash e\colon \{\text{lab}_1\colon T_1, \ldots, \text{lab}_k\colon T_k\}}{\Gamma \vdash \#\text{lab}_i\ e\colon T_i}$$

Concorrenza

$$\textbf{(T-sq1)}\ \frac{\Gamma \vdash e_1\colon \text{unit} \qquad \Gamma \vdash e_2\colon \text{unit}}{\Gamma \vdash e_1; e_2\colon \text{unit}} \qquad \textbf{(T-sq2)}\ \frac{\Gamma \vdash e_1\colon \text{proc} \qquad \Gamma \vdash e_2\colon \text{proc}}{\Gamma \vdash e_1; e_2\colon \text{proc}}$$

$$\textbf{(T-par)}\ \frac{\Gamma \vdash e_1\colon T_1 \qquad \Gamma \vdash e_2\colon T_2}{\Gamma \vdash e_1 \| e_2\colon \text{proc}}\ T_1, T_2 \in \{\text{unit}, \text{proc}\}$$

$$\textbf{(T-await)}\ \frac{\Gamma \vdash e_1\colon \text{bool} \qquad \Gamma \vdash e_2\colon \text{unit}}{\Gamma \vdash \text{await } e_1 \text{ protect } e_2 \text{ end}\colon \text{unit}} \qquad \textbf{(T-choice)}\ \frac{\Gamma \vdash e_1\colon \text{unit} \qquad \Gamma \vdash e_2\colon \text{unit}}{\Gamma \vdash e_1 \oplus e_2\colon \text{unit}}$$

Sottotipaggio

$$\textbf{(sub)}\,\frac{\Gamma \vdash e\colon \mathsf{T} \qquad \mathsf{T} <: \mathsf{T}'}{\Gamma \vdash e\colon \mathsf{T}'} \quad \textbf{(s-refl)}\,\frac{-}{\mathsf{T} <: \mathsf{T}} \qquad \textbf{(s-trans)}\,\frac{\mathsf{T} <: \mathsf{T}' \qquad \mathsf{T}' <: \mathsf{T}''}{\mathsf{T} <: \mathsf{T}''}$$

Sottotipaggio dei record

$$\textbf{(rec-perm)}\,\frac{\pi \text{ una permutazione di } 1, 2, \ldots, k}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\} <: \{p_{\pi(1)}\colon \mathsf{T}_{\pi(1)}, \ldots, p_{\pi(k)}\colon \mathsf{T}_{\pi(k)}\}} \quad \textbf{(rec-width)}\,\frac{-}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k, p_{k+1}\colon \mathsf{T}_{k+1}, \ldots, p_z\colon \mathsf{T}_z\} <: \{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\}}$$

$$\textbf{(rec-depth)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \ \ldots\ \mathsf{T}_k <: \mathsf{T}_k'}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\} <: \{p_1\colon \mathsf{T}_1', \ldots, p_k\colon \mathsf{T}_k'\}}$$

Sottotipaggio delle funzioni

$$\textbf{(fun-sub)}\,\frac{\mathsf{T}_1 :> \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 \to \mathsf{T}_2 <: \mathsf{T}_1' \to \mathsf{T}_2'}$$

Sottotipaggio somma e prodotto

$$\textbf{(prod-sub)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 * \mathsf{T}_2 <: \mathsf{T}_1' * \mathsf{T}_2'} \quad \textbf{(sum-sub)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 + \mathsf{T}_2 <: \mathsf{T}_1' + \mathsf{T}_2'}$$