Regole di inferenza

Semantica big-step

$$\textbf{B-Num}\frac{-}{\langle n, s\rangle \Downarrow n}$$

$$\textbf{B-Loc}\frac{-}{\langle l, s\rangle \Downarrow s(l)}$$

$$\textbf{B-Skip}\frac{-}{\langle skip, s\rangle \Downarrow s}$$

$$\textbf{B-Add}\frac{\langle E_1, s\rangle \Downarrow n_1 \quad \langle E_2, s\rangle \Downarrow n_2}{\langle E_1 + E_2\rangle \Downarrow n_3} n_3 = add(n_1, n_2)$$

$$\textbf{B-Assign}\frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow s[l \mapsto n]}$$

$$\textbf{B-Assign.s}\frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow \langle skip, s[l \mapsto n]\rangle}$$

$$\textbf{B-Seq}\frac{\langle C_1, s\rangle \Downarrow s_1 \quad \langle C_2, s_1\rangle \Downarrow s'}{\langle C_1; C_2, s\rangle \Downarrow s'}$$

$$\textbf{B-Seq.s}\frac{\langle C_1, s\rangle \Downarrow \langle skip, s_1\rangle \quad \langle C_2, s_1\rangle \Downarrow \langle r, s'\rangle}{\langle C_1; C_2, s\rangle \Downarrow \langle r, s'\rangle}$$

$$\textbf{B-If.T}\frac{\langle B, s\rangle \Downarrow true \quad \langle C_1, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s\rangle}$$

$$\textbf{B-If.T}\frac{\langle B, s\rangle \Downarrow false \quad \langle C_2, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s\rangle}$$

Semantica small-step

$$\textbf{S-Left}\ \frac{E_1 \rightharpoonup E_1'}{E_1 + E_2 \rightharpoonup E_1' + E_2}$$

$$\textbf{S-N.Right}\ \frac{E_2 \rightharpoonup E_2'}{n_1 + E_2 \rightharpoonup n_1 + E_2'}$$

$$\textbf{S-Add}\ \frac{-}{n_1 + n_2 \rightharpoonup n_3}\ n_3 = add(n_1, n_2)$$

$$\textbf{S-Left}\ \frac{E_1 \rightharpoonup_{ch} E_1'}{E_1 + E_2 \rightharpoonup_{ch} E_1' + E_2}$$

$$\textbf{S-Right}\ \frac{E_2 \rightharpoonup_{ch} E_2'}{E_1 + E_2 \rightharpoonup_{ch} E_1 + E_2'}$$

$$\textbf{op+}\ \frac{-}{\langle n_1 + n_2, s\rangle \rightharpoonup \langle n, s\rangle}\ n = add(n_1, n_2)$$

$$\textbf{op-geq}\ \frac{-}{\langle n_1 \geq n_2, s\rangle \rightharpoonup \langle b, s\rangle}\ b = geq(n_1, n_2)$$

$$\textbf{op1}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 + e_2, s\rangle \rightharpoonup \langle e_1' + e_2, s'\rangle}$$

$$\textbf{op2}\ \frac{\langle e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}{\langle v + e_2, s\rangle \rightharpoonup \langle v + e_2', s'\rangle}$$

$$\textbf{op1b}\ \frac{\langle e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}{\langle e_1 + e_2, s\rangle \rightharpoonup \langle e_1 + e_2', s'\rangle}$$

$$\textbf{op2b}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 + v, s\rangle \rightharpoonup \langle e_1' + v', s'\rangle}$$

$$\textbf{deref1}\ \frac{-}{\langle !l, s\rangle \rightharpoonup \langle v, s\rangle}\ \text{if } l \in dom(s) \wedge s(l) = v$$

$$\textbf{deref2}\ \frac{\langle e, s\rangle \rightharpoonup \langle e', s'\rangle}{\langle !e, s\rangle \rightharpoonup \langle !e', s'\rangle}$$

$$\textbf{ref1}\ \frac{-}{\langle \text{ref } v, s\rangle \rightharpoonup \langle l, s[l \mapsto v]\rangle}\ l \notin dom(s)$$

$$\textbf{ref2}\ \frac{\langle e, s\rangle \rightharpoonup \langle e', s'\rangle}{\langle \text{ref } e, s\rangle \rightharpoonup \langle \text{ref } e', s\rangle}$$

$$\textbf{assign1}\ \frac{-}{\langle l := v, s\rangle \rightharpoonup \langle \text{skip}, s[l \mapsto v]\rangle}\ \text{if } l \in dom(s)$$

$$\textbf{assign2}\ \frac{\langle e, s\rangle \rightharpoonup \langle e', s'\rangle}{\langle l := e, s\rangle \rightharpoonup \langle l := e', s\rangle}$$

$$\textbf{assign3}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 := e_2, s\rangle \rightharpoonup \langle e_1' := e_2, s'\rangle}$$

$$\textbf{if-tt}\ \frac{-}{\langle \text{if } true \text{ then } e_1 \text{ else } e_2, s\rangle \rightharpoonup \langle e_1, s\rangle}$$

$$\textbf{if-ff}\ \frac{-}{\langle \text{if} false \text{then} e_1 \text{else} e_2, s\rangle \rightharpoonup \langle e_2, s\rangle}$$

$$\textbf{if}\ \frac{\langle e, s\rangle \rightharpoonup \langle e', s'\rangle}{\langle \text{if } e \text{ then } e_1 \text{ else } e_2, s\rangle \rightharpoonup \langle \text{if } e' \text{ then } e_1 \text{ else } e_2, s\rangle}$$

$$\textbf{while}\ \frac{-}{\langle \text{while } e \text{ do } e_1, s\rangle \rightharpoonup \langle \text{if } e \text{ then } (e_1; \text{while } e \text{ do } e_1) \text{ else } skip, s\rangle}$$

$$\textbf{assign1b}\ \frac{-}{\langle l := n, s\rangle \rightharpoonup \langle n, s[l \mapsto n]\rangle}\ l \in dom(s)$$

$$\textbf{seq.skip}\ \frac{-}{\langle skip; e_2, s\rangle \rightharpoonup \langle e_2, s\rangle}$$

$$\textbf{seq}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s\rangle}{\langle e_1; e_2, s\rangle \rightharpoonup \langle e_1'; e_2, s'\rangle}$$

$$\textbf{seq.skipb}\ \frac{-}{\langle v; e_2, s\rangle \rightharpoonup \langle e_2, s\rangle}$$

Grammatica dei tipi

$$T ::= \text{int} \mid \text{bool} \mid \text{unit} \mid T_1 \to T_2 \mid T_1 + T_2 \mid T_1 * T_2 \mid \text{ref } T \mid \{lab_1 : T_1, \ldots, lab_k : T_k\}$$

Regole per il Tipaggio

Tipi primitivi e operatori

$$\textbf{(int)}\frac{-}{\Gamma \vdash n : \text{int}} \text{ for } n \in \mathbb{Z}$$

$$\textbf{(bool)}\frac{-}{\Gamma \vdash b : \text{bool}} \text{ for } n \in \{true, false\}$$

$$\textbf{(op +)}\frac{\Gamma \vdash e_1 : \text{int} \qquad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

$$\textbf{(op *)}\frac{\Gamma \vdash e_1 : \text{int} \qquad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 * e_2 : \text{int}}$$

$$\textbf{(op or)}\frac{\Gamma \vdash e_1 : \text{bool} \qquad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \ or \ e_2 : \text{bool}}$$

$$\textbf{(op and)}\frac{\Gamma \vdash e_1 : \text{bool} \qquad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \ and \ e_2 : \text{bool}}$$

$$\textbf{(op} \geq\textbf{)}\frac{\Gamma \vdash e_1 : \text{int} \qquad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 \geq e_2 : \text{bool}}$$

$$\textbf{(assign)}\frac{\Gamma \vdash e : \text{int}}{\Gamma \vdash l := e : \text{unit}} \text{ if } \Gamma(l) = \text{intref}$$

$$\textbf{(skip)}\frac{-}{\Gamma \vdash skip : unit}$$

$$\textbf{(seq)}\frac{\Gamma \vdash e_1 : unit \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1; e_2 : T}$$

$$\textbf{(if)}\frac{\Gamma : e_1 : \text{bool} \quad \Gamma \vdash e_2 : T \quad \Gamma \vdash e_3 : T}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : T}$$

$$\textbf{(while)}\frac{\Gamma \vdash e_1 : \text{bool} \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash \text{while } e_1 \text{ do } e_2 : T}$$

$$\textbf{(let)}\frac{\Gamma \vdash e_1 : T \qquad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \text{let } x : T = e_1 \text{ in } e_2 : T'}$$

Funzioni

$$\textbf{(var)}\frac{-}{\Gamma \vdash : T} \text{ if } \Gamma(x) = T$$

$$\textbf{(fn)}\frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash (fn \ x : T \ \Rightarrow \ e) : T \to T'}$$

$$\textbf{(app)}\frac{\Gamma \vdash e_1 : T \to T' \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 e_2 : T'}$$

Sottotipaggio

$$\textbf{(sub)}\frac{\Gamma \vdash e : T \qquad T <: T'}{\Gamma \vdash e : T'}$$

$$\textbf{(s-refl)}\frac{-}{T <: T}$$

$$\textbf{(s-trans)}\frac{T <: T' \qquad T' <: T''}{T <: T''}$$

Sottotipaggio dei record

$$\textbf{(rec-perm)}\frac{\pi \text{ una permutazione di } 1, 2, \ldots, k}{\{p_1 : T_1, \ldots, p_k : T_k\} <: \{p_{\pi(1)} : T_{\pi(1)}, \ldots, p_{\pi(k)} : T_{\pi(k)}\}}$$

$$\textbf{(rec-width)}\frac{-}{\{p_1 : T_1, \ldots, p_k : T_k, p_{k+1} : T_{k+1}, \ldots, p_z : T_z\} <: \{p_1 : T_1, \ldots, p_k : T_k\}}$$

$$\textbf{(rec-depth)}\frac{T_1 <: T_1' \ \ldots \ T_k <: T_k'}{\{p_1 : T_1, \ldots, p_k : T_k\} <: \{p_1 : T_1', \ldots, p_k : T_k'\}}$$

Sottotipaggio delle funzioni

$$\textbf{(fun-sub)}\frac{T_1 :> T_1' \qquad T_2 <: T_2'}{T_1 \to T_2 <: T_1' \to T_2'}$$

Sottotipaggio somma e prodotto

$$\textbf{(prod-sub)}\,\frac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 * T_2 <: T_1' * T_2'} \qquad\qquad \textbf{(sum-sub)}\,\frac{T_1 <: T_1' \qquad T_2 <: T_2'}{T_1 + T_2 <: T_1' + T_2'}$$