Semantica big-step

$$\textbf{(B-Num)}\frac{-}{\langle n, s\rangle \Downarrow n} \qquad \textbf{(B-Loc)}\frac{-}{\langle l, s\rangle \Downarrow s(l)} \qquad \textbf{(B-Skip)}\frac{-}{\langle \text{skip}, s\rangle \Downarrow s} \qquad \textbf{(B-Add)}\frac{\langle E_1, s\rangle \Downarrow n_1 \quad \langle E_2, s\rangle \Downarrow n_2}{\langle E_1 + E_2, s\rangle \Downarrow n_3} n_3 = \mathtt{add}(n_1, n_2)$$

$$\textbf{(B-Assign)}\frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow s[l \mapsto n]} \qquad \textbf{(B-Assign.s)}\frac{\langle E, s\rangle \Downarrow n}{\langle l := e, s\rangle \Downarrow \langle \text{skip}, s[l \mapsto n]\rangle} \qquad \textbf{(B-Seq)}\frac{\langle C_1, s\rangle \Downarrow s_1 \quad \langle C_2, s_1\rangle \Downarrow s'}{\langle C_1; C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-Seq.s)}\frac{\langle C_1, s\rangle \Downarrow \langle \text{skip}, s_1\rangle \quad \langle C_2, s_1\rangle \Downarrow \langle r, s'\rangle}{\langle C_1; C_2, s\rangle \Downarrow \langle r, s'\rangle}$$

$$\textbf{(B-If.T)}\frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C_1, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-If.T)}\frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C_1, s\rangle \Downarrow \langle r, s'\rangle}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s'\rangle} \qquad \textbf{(B-If.F)}\frac{\langle B, s\rangle \Downarrow \mathtt{false} \quad \langle C_2, s\rangle \Downarrow s'}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow s'} \qquad \textbf{(B-If.F)}\frac{\langle B, s\rangle \Downarrow \mathtt{false} \quad \langle C_2, s\rangle \Downarrow \langle r, s'\rangle}{\langle \text{if } B \text{ then } C_1 \text{ else } C_2, s\rangle \Downarrow \langle r, s\rangle}$$

$$\textbf{(B-While.T)}\frac{\langle B, s\rangle \Downarrow \mathtt{true} \quad \langle C, s\rangle \Downarrow s_1 \quad \langle \text{while } B \text{ do } C, s_1\rangle \Downarrow s'}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow s'} \qquad \textbf{(B-While.F)}\frac{\langle B, s\rangle \Downarrow \mathtt{false}}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow s} \qquad \textbf{(B-Do)}\frac{\langle B, s\rangle \Downarrow v_1 \quad \langle C, s\rangle \Downarrow v_2}{\langle \text{do } E \text{ return } C, s\rangle \Downarrow v_2}$$

$$\textbf{(B-While.UN)}\frac{\langle \text{if } B \text{ then } (C; \text{while } B \text{ do } C) \text{ else skip}, s\rangle \Downarrow r, s'}{\langle \text{while } B \text{ do } C, s\rangle \Downarrow r, s'} \qquad \textbf{(B-Await)}\frac{\langle B, s\rangle \Downarrow \mathtt{true}, s_1 \quad \langle C, s_1\rangle \to^* \langle \text{skip}, s'\rangle}{\langle \text{await } B \text{ protect } C \text{ end}, s\rangle \to \langle \text{skip}, s'\rangle}$$

$$\textbf{(B-Let-CBV)}\frac{P \Downarrow m \quad E\{{}^{m}/_{x}\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Let-CBN)}\frac{E\{{}^{P}/_{x}\} \Downarrow n}{\text{let } x = P \text{ in } E \Downarrow n} \qquad \textbf{(B-Fn)}\frac{-}{\langle \text{fn } x\colon T \;\Rightarrow\; e, s\rangle \to \langle \text{fn } x\colon T \;\Rightarrow\; e, s\rangle}$$

$$\textbf{(B-App-CBV)}\frac{E_1 \Downarrow \text{fn } x\colon T \Rightarrow E \qquad E\{{}^{E_2}/_{x}\} \Downarrow n}{E_1 E_2 \Downarrow n} \qquad \textbf{(B-App-CBN)}\frac{E_1 \Downarrow \text{fn } x\colon T \Rightarrow E \qquad E_2 \Downarrow v \qquad E\{{}^{v}/_{x}\} \Downarrow n}{E_1 E_2 \Downarrow n}$$

Semantica small-step

Grammatica delle espressioni

$$\text{op} \ ::= + \mid \geqslant$$

$$e \in \text{Exp} \ ::= n \mid b \mid e \text{ op } e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{skip} \mid e; e \mid \text{while } e \text{ do } e \mid e_1 := e_2 \mid !e \mid \text{ref } e \mid l \mid \text{fn } x\colon T \Rightarrow e \mid e_1 e_2 \mid \text{let } x\colon T = e \text{ in } e \mid e \oplus e \mid e \| e \mid$$
$$\text{await } e \text{ protect } e \text{ end} \mid \#\text{lab } e$$

Regole per la semantica

$$\textbf{(S-Left)}\ \frac{E_1 \to E_1'}{E_1 + E_2 \to E_1' + E_2} \qquad \textbf{(S-N.Right)}\ \frac{E_2 \to E_2'}{n_1 + E_2 \to n_1 + E_2'} \qquad \textbf{(S-Add)}\ \frac{-}{n_1 + n_2 \to_{ch} n_3}\, n_3 = \mathsf{add}(n_1, n_2)$$

$$\textbf{(S-Left)}\ \frac{E_1 \to_{ch} E_1'}{E_1 + E_2 \to_{ch} E_1' + E_2} \qquad \textbf{(S-Right)}\ \frac{E_2 \to_{ch} E_2'}{E_1 + E_2 \to_{ch} E_1 + E_2'} \qquad \textbf{(op+)}\ \frac{-}{\langle n_1 + n_2, s\rangle \to \langle n, s\rangle}\, n = \mathsf{add}(n_1, n_2)$$

$$\textbf{(op-geq)}\ \frac{-}{\langle n_1 \geqslant n_2, s\rangle \to \langle b, s\rangle}\, b = \mathsf{geq}(n_1, n_2) \qquad \textbf{(op1)}\ \frac{\langle e_1, s\rangle \to \langle e_1', s'\rangle}{\langle e_1 + e_2, s\rangle \to \langle e_1' + e_2, s'\rangle} \qquad \textbf{(op2)}\ \frac{\langle e_2, s\rangle \to \langle e_2', s'\rangle}{\langle v + e_2, s\rangle \to \langle v + e_2', s\rangle}$$

$$\textbf{(op1b)}\ \frac{\langle e_2, s\rangle \to \langle e_2', s'\rangle}{\langle e_1 + e_2, s\rangle \to \langle e_1 + e_2', s'\rangle} \qquad \textbf{(op2b)}\ \frac{\langle e_1, s\rangle \to \langle e_1', s'\rangle}{\langle e_1 + v, s\rangle \to \langle e_1' + v', s'\rangle} \qquad \textbf{(deref1)}\ \frac{-}{\langle !l, s\rangle \to \langle v, s\rangle}\,\text{if } l \in \mathsf{dom}(s) \wedge s(l) = v$$

$$\textbf{(deref2)}\ \frac{\langle e, s\rangle \to \langle e', s'\rangle}{\langle !e, s\rangle \to \langle !e', s'\rangle} \qquad \textbf{(ref1)}\ \frac{-}{\langle \text{ref } v, s\rangle \to \langle l, s[l \mapsto v]\rangle}\, l \notin \mathsf{dom}(s) \qquad \textbf{(ref2)}\ \frac{\langle e, s\rangle \to \langle e', s'\rangle}{\langle \text{ref } e, s\rangle \to \langle \text{ref } e', s\rangle}$$

$$\textbf{(assign1)}\ \frac{-}{\langle l := v, s\rangle \to \langle \text{skip}, s[l \mapsto v]\rangle}\,\text{if } l \in \mathsf{dom}(s) \qquad \textbf{(assign2)}\ \frac{\langle e, s\rangle \to \langle e', s'\rangle}{\langle l := e, s\rangle \to \langle l := e', s\rangle} \qquad \textbf{(assign3)}\ \frac{\langle e_1, s\rangle \to \langle e_1', s'\rangle}{\langle e_1 := e_2, s\rangle \to \langle e_1' := e_2, s'\rangle}$$

$$\textbf{(if-tt)}\ \frac{-}{\langle \text{if } \mathsf{true} \text{ then } e_1 \text{ else } e_2, s\rangle \to \langle e_1, s\rangle} \qquad \textbf{(if-ff)}\ \frac{-}{\langle \text{if } \mathsf{false} \text{ then } e_1 \text{ else } e_2, s\rangle \to \langle e_2, s\rangle} \qquad \textbf{(if)}\ \frac{\langle e, s\rangle \to \langle e', s'\rangle}{\langle \text{if } e \text{ then } e_1 \text{ else } e_2, s\rangle \to \langle \text{if } e' \text{ then } e_1 \text{ else } e_2, s\rangle}$$

$$\textbf{(while)}\ \frac{-}{\langle \text{while } e \text{ do } e_1, s\rangle \to \langle \text{if } e \text{ then } (e_1; \text{while } e \text{ do } e_1) \text{ else skip}, s\rangle} \qquad \textbf{(assign1b)}\ \frac{-}{\langle l := n, s\rangle \to \langle n, s[l \mapsto n]\rangle}\, l \in \mathsf{dom}(s)$$

$$\textbf{(seq.skip)}\ \frac{-}{\langle \mathsf{skip}; e_2, s\rangle \to \langle e_2, s\rangle} \qquad \textbf{(seq)}\ \frac{\langle e_1, s\rangle \to \langle e_1', s\rangle}{\langle e_1; e_2, s\rangle \to \langle e_1'; e_2, s'\rangle} \qquad \textbf{(seq.skipb)}\ \frac{-}{\langle v; e_2, s\rangle \to \langle e_2, s\rangle}$$

$$\textbf{(record1)}\ \frac{\langle e_i, s\rangle \rightharpoonup \langle e_i', s'\rangle}{\langle\{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i, \ldots, \mathrm{lab}_k = e_k\}, s\rangle \rightharpoonup \langle\{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i', \ldots, \mathrm{lab}_k = e_k\}, s'\rangle}$$

$$\textbf{(record2)}\ \frac{-}{\langle \#\mathrm{lab}_i\ \{\mathrm{lab}_1 = v_1, \ldots, \mathrm{lab}_i = e_i, \ldots, \mathrm{lab}_k = e_k\}, s\rangle \rightharpoonup \langle v_i, s\rangle} \qquad \textbf{(record3)}\ \frac{\langle e, s\rangle \rightharpoonup \langle e', s'\rangle}{\langle \#\mathrm{lab}\ e, s\rangle \rightharpoonup \langle \#\mathrm{lab}\ e', s'\rangle}$$

$$\textbf{(par-L)}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 \| e_2, s\rangle \rightharpoonup \langle e_1' \| e_2, s'\rangle} \qquad \textbf{(par-R)}\ \frac{\langle e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}{\langle e_1 \| e_2, s\rangle \rightharpoonup \langle e_1 \| e_2', s'\rangle} \qquad \textbf{(end-L)}\ \frac{-}{\langle \mathrm{skip} \| e, s\rangle \rightharpoonup \langle e, s\rangle} \qquad \textbf{(end-R)}\ \frac{-}{\langle e \| \mathrm{skip}, s\rangle \rightharpoonup \langle e, s\rangle}$$

$$\textbf{(await)}\ \frac{\langle e_1, s\rangle \rightharpoonup^* \langle \mathrm{true}, s'\rangle \qquad \langle e_2, s'\rangle \rightharpoonup^* \langle \mathrm{skip}, s''\rangle}{\langle \mathrm{await}\ e_1\ \mathrm{protect}\ e_2\ \mathrm{end}, s\rangle \rightharpoonup \langle \mathrm{skip}, s''\rangle} \qquad \textbf{(ChoiceL)}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 \oplus e_2, s\rangle \rightharpoonup \langle e_1', s'\rangle} \qquad \textbf{(ChoiceR)}\ \frac{\langle e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}{\langle e_1 \oplus e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}$$

Call-By-Value

$$\textbf{(CBV-app1)}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 e_2, s\rangle \rightharpoonup \langle e_1' e_2, s'\rangle} \qquad \textbf{(CBV- app2)}\ \frac{\langle e_2, s\rangle \rightharpoonup \langle e_2', s'\rangle}{\langle v e_2, s\rangle \rightharpoonup \langle v e_2', s'\rangle} \qquad \textbf{(CBV-fn)}\ \frac{-}{\langle (\mathsf{fn}\ x\colon \mathsf{T} \Rightarrow e) v, s\rangle \rightharpoonup \langle e\{^v/_x\}, s\rangle}$$

$$\textbf{(CBV-let1)}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle \mathrm{let}\ x\colon \mathsf{T} = e_1\ \mathrm{in}\ e_2, s\rangle \rightharpoonup \langle \mathrm{let}\ x\colon \mathsf{T} = e_1'\ \mathrm{in}\ e_2, s'\rangle} \qquad \textbf{(CBV-let2)}\ \frac{-}{\langle \mathrm{let}\ x\colon \mathsf{T} = v\ \mathrm{in}\ e_2, s\rangle \rightharpoonup \langle e_2\{^v/_x\}, s\rangle}$$

$$\textbf{(CBV-fix)}\ \frac{e \equiv\ \mathrm{fn}\ f\colon \mathsf{T}_1 \to \mathsf{T}_2\ \Rightarrow\ e_2}{\mathrm{fix}.e \rightharpoonup e(\mathrm{fn}\ x\colon \mathsf{T}_1 \Rightarrow\ (\mathrm{fix}.e)\ x)}$$

Call-By-Name

$$\textbf{(CBN-app)}\ \frac{\langle e_1, s\rangle \rightharpoonup \langle e_1', s'\rangle}{\langle e_1 e_2, s\rangle \rightharpoonup \langle e_1' e_2, s'\rangle} \qquad \textbf{(CBN-fn)}\ \frac{-}{\langle (\mathsf{fn}\ x\colon \mathsf{T} \Rightarrow e) e_2, s\rangle \rightharpoonup \langle e\{^{e_2}/_x\}, x\rangle}$$

$$\textbf{(CBN-let)}\ \frac{-}{\langle \mathrm{let}\ x\colon \mathsf{T} = e_1\ \mathrm{in}\ e_2, s\rangle \rightharpoonup \langle e_2\{^{e_1}/_x\}, s\rangle}$$

$$\textbf{(CBN-fix)}\ \frac{-}{\mathrm{fix}.e \rightharpoonup e(\mathrm{fix}.e)}$$

Grammatica dei tipi e type system

$$\mathsf{T} ::= \text{int} \mid \text{bool} \mid \text{unit} \mid \mathsf{T}_1 \to \mathsf{T}_2 \mid \mathsf{T}_1 + \mathsf{T}_2 \mid \mathsf{T}_1 * \mathsf{T}_2 \mid \text{ref } \mathsf{T} \mid \{lab_1 \colon \mathsf{T}_1, \dots, lab_k \colon \mathsf{T}_k\} \mid \text{proc}$$

$$\mathsf{T}_{\text{loc}} ::= \text{ref } \mathsf{T}$$

Tipi primitivi e operatori

$$\textbf{(int)}\ \frac{-}{\Gamma \vdash n \colon \text{int}} \text{ for } n \in \mathbb{Z} \qquad \textbf{(bool)}\ \frac{-}{\Gamma \vdash b \colon \text{bool}} \text{ for } b \in \{true, false\} \qquad \textbf{(op +)}\ \frac{\Gamma \vdash e_1 \colon \text{int} \qquad \Gamma \vdash e_2 \colon \text{int}}{\Gamma \vdash e_1 + e_2 \colon \text{int}} \qquad \textbf{(op *)}\ \frac{\Gamma \vdash e_1 \colon \text{int} \qquad \Gamma \vdash e_2 \colon \text{int}}{\Gamma \vdash e_1 * e_2 \colon \text{int}}$$

$$\textbf{(op or)}\ \frac{\Gamma \vdash e_1 \colon \text{bool} \qquad \Gamma \vdash e_2 \colon \text{bool}}{\Gamma \vdash e_1 \ or\ e_2 \colon \text{bool}} \qquad \textbf{(op and)}\ \frac{\Gamma \vdash e_1 \colon \text{bool} \qquad \Gamma \vdash e_2 \colon \text{bool}}{\Gamma \vdash e_1 \ and\ e_2 \colon \text{bool}} \qquad \textbf{(op} \geqslant\textbf{)}\ \frac{\Gamma \vdash e_1 \colon \text{int} \qquad \Gamma \vdash e_2 \colon \text{int}}{\Gamma \vdash e_1 \geqslant e_2 \colon \text{bool}} \qquad \textbf{(skip)}\ \frac{-}{\Gamma \vdash skip \colon unit}$$

$$\textbf{(seq)}\ \frac{\Gamma \vdash e_1 \colon unit \qquad \Gamma \vdash e_2 \colon \mathsf{T}}{\Gamma \vdash e_1 ; e_2 \colon \mathsf{T}} \qquad \textbf{(if)}\ \frac{\Gamma \colon e_1 \colon \text{bool} \quad \Gamma \vdash e_2 \colon \mathsf{T} \quad \Gamma \vdash e_3 \colon \mathsf{T}}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \colon \mathsf{T}} \qquad \textbf{(while)}\ \frac{\Gamma \vdash e_1 \colon \text{bool} \qquad \Gamma \vdash e_2 \colon \mathsf{T}}{\Gamma \vdash \text{while } e_1 \text{ do } e_2 \colon \mathsf{T}} \qquad \textbf{(let)}\ \frac{\Gamma \vdash e_1 \colon \mathsf{T} \qquad \Gamma, x \colon \mathsf{T} \vdash e_2 \colon \mathsf{T}'}{\Gamma \vdash \text{let } x \colon \mathsf{T} = e_1 \text{ in } e_2 \colon \mathsf{T}'}$$

$$\textbf{(T-fix)}\ \frac{\Gamma \vdash e \colon (\mathsf{T}_1 \to \mathsf{T}_2) \to (\mathsf{T}_1 \to \mathsf{T}_2)}{\Gamma \vdash \text{fix}.e \colon \mathsf{T}_1 \to \mathsf{T}_2}$$

Referenze

$$\textbf{(ref)}\ \frac{\Gamma \vdash e \colon \mathsf{T}}{\Gamma \vdash \text{ref } e \colon \text{ref } \mathsf{T}} \qquad \textbf{(deref)}\ \frac{\Gamma \vdash e \colon \text{ref } \mathsf{T}}{\Gamma \vdash !e \colon \mathsf{T}} \qquad \textbf{(assign)}\ \frac{\Gamma \vdash e_1 \colon \text{ref } \mathsf{T} \quad \Gamma \vdash e_2 \colon \mathsf{T}}{\Gamma \vdash (e_1 := e_2) \colon \text{unit}} \qquad \textbf{(loc)}\ \frac{-}{\Gamma \vdash l \colon \text{ref } \mathsf{T}}\ \Gamma(l) = \text{ref } \mathsf{T}$$

Funzioni

$$\textbf{(var)}\ \frac{-}{\Gamma \vdash x \colon \mathsf{T}} \text{ if } \Gamma(x) = \mathsf{T} \qquad \textbf{(fn)}\ \frac{\Gamma, x \colon \mathsf{T} \vdash e \colon \mathsf{T}'}{\Gamma \vdash (fn\ x \colon \mathsf{T} \ \Rightarrow\ e) \colon \mathsf{T} \to \mathsf{T}'} \qquad \textbf{(app)}\ \frac{\Gamma \vdash e_1 \colon \mathsf{T} \to \mathsf{T}' \qquad \Gamma \vdash e_2 \colon \mathsf{T}}{\Gamma \vdash e_1 e_2 \colon \mathsf{T}'}$$

Record

$$\textbf{(record)}\ \frac{\Gamma \vdash e_1 \colon \mathsf{T}_1 \ \dots\ \Gamma \vdash e_k \colon \mathsf{T}_k}{\Gamma \vdash \{\text{lab}_1 = e_1, \dots, \text{lab}_k = e_k\} : \{\text{lab}_1 \colon \mathsf{T}_1, \dots, \text{lab}_k \colon \mathsf{T}_k\}} \qquad \textbf{(recordproj)}\ \frac{\Gamma \vdash e \colon \{\text{lab}_1 \colon \mathsf{T}_1, \dots, \text{lab}_k \colon \mathsf{T}_k\}}{\Gamma \vdash \#\text{lab}_i\ e \colon \mathsf{T}_i}$$

Concorrenza

$$\textbf{(T-sq1)}\ \frac{\Gamma \vdash e_1 \colon \text{unit} \qquad \Gamma \vdash e_2 \colon \text{unit}}{\Gamma \vdash e_1 ; e_2 \colon \text{unit}} \qquad \textbf{(T-sq2)}\ \frac{\Gamma \vdash e_1 \colon \text{proc} \qquad \Gamma \vdash e_2 \colon \text{proc}}{\Gamma \vdash e_1 ; e_2 \colon \text{proc}} \qquad \textbf{(T-par)}\ \frac{\Gamma \vdash e_1 \colon \mathsf{T}_1 \qquad \Gamma \vdash e_2 \colon \mathsf{T}_2}{\Gamma \vdash e_1 \| e_2 \colon \text{proc}}\ \mathsf{T}_1, \mathsf{T}_2 \in \{\text{unit}, \text{proc}\}$$

$$\textbf{(T-await)}\ \frac{\Gamma \vdash e_1 \colon \text{bool} \qquad \Gamma \vdash e_2 \colon \text{unit}}{\Gamma \vdash \text{await } e_1 \text{ protect } e_2 \text{ end} \colon \text{unit}} \qquad \textbf{(T-choice)}\ \frac{\Gamma \vdash e_1 \colon \text{unit} \qquad \Gamma \vdash e_2 \colon \text{unit}}{\Gamma \vdash e_1 \oplus e_2 \colon \text{unit}}$$

Sottotipaggio

$$\textbf{(sub)}\,\frac{\Gamma \vdash e\colon \mathsf{T} \qquad \mathsf{T} <: \mathsf{T}'}{\Gamma \vdash e\colon \mathsf{T}'} \qquad \textbf{(s-refl)}\,\frac{-}{\mathsf{T} <: \mathsf{T}} \qquad \textbf{(s-trans)}\,\frac{\mathsf{T} <: \mathsf{T}' \qquad \mathsf{T}' <: \mathsf{T}''}{\mathsf{T} <: \mathsf{T}''}$$

Sottotipaggio dei record

$$\textbf{(rec-perm)}\,\frac{\pi \text{ una permutazione di } 1, 2, \ldots, k}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\} <: \{p_{\pi(1)}\colon \mathsf{T}_{\pi(1)}, \ldots, p_{\pi(k)}\colon \mathsf{T}_{\pi(k)}\}} \qquad \textbf{(rec-width)}\,\frac{-}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k, p_{k+1}\colon \mathsf{T}_{k+1}, \ldots, p_z\colon \mathsf{T}_z\} <: \{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\}}$$

$$\textbf{(rec-depth)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \ \ldots \ \mathsf{T}_k <: \mathsf{T}_k'}{\{p_1\colon \mathsf{T}_1, \ldots, p_k\colon \mathsf{T}_k\} <: \{p_1\colon \mathsf{T}_1', \ldots, p_k\colon \mathsf{T}_k'\}}$$

Sottotipaggio delle funzioni

$$\textbf{(fun-sub)}\,\frac{\mathsf{T}_1 :\ > \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 \to \mathsf{T}_2 <: \mathsf{T}_1' \to \mathsf{T}_2'}$$

Sottotipaggio somma e prodotto

$$\textbf{(prod-sub)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 * \mathsf{T}_2 <: \mathsf{T}_1' * \mathsf{T}_2'} \qquad \textbf{(sum-sub)}\,\frac{\mathsf{T}_1 <: \mathsf{T}_1' \qquad \mathsf{T}_2 <: \mathsf{T}_2'}{\mathsf{T}_1 + \mathsf{T}_2 <: \mathsf{T}_1' + \mathsf{T}_2'}$$