Gramiano di Controllabilità : $\Gamma(t) = \int_0^t e^{A\tau} B B^{\mathsf{T}} e^{A^{\mathsf{T}} \tau} d\tau$ non singolare (det $\neq 0$)

Matrice di Controllabilità : $\mathfrak{T} = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$ rank $(\mathfrak{T}) = \mathfrak{n}$

$$\mathfrak{I}^{-1} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_{n-1} & 1 \\ \alpha_2 & \alpha_3 & \vdots & \ddots & 1 & 0 \\ \alpha_3 & \vdots & \alpha_{n-1} & \ddots & 0 & 0 \\ \vdots & \alpha_{n-1} & 1 & \cdots & 0 & 0 \\ \alpha_{n-1} & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{A}_{c} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \frac{-\alpha_{0}}{\alpha_{n}} & \frac{-\alpha_{1}}{\alpha_{n}} & \frac{-\alpha_{2}}{\alpha_{n}} & \cdots & \frac{-\alpha_{n-1}}{\alpha_{n}} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \boldsymbol{B}_{c}$$

$$\boldsymbol{C}_{c} = \begin{bmatrix} \beta_{0} & \beta_{1} & \beta_{2} & \beta_{3} & \beta_{4} & \cdots & \beta_{n-1} \end{bmatrix} \begin{bmatrix} \beta_{n} \end{bmatrix} = \boldsymbol{D}_{c}$$

$$oldsymbol{A}_{ ext{c}} = oldsymbol{P}^{-1}oldsymbol{A}oldsymbol{P} \quad oldsymbol{B}_{ ext{c}} = oldsymbol{P}^{-1}oldsymbol{B}$$
 $oldsymbol{C}_{ ext{c}} = oldsymbol{C}oldsymbol{P} \quad oldsymbol{D}_{ ext{c}} = oldsymbol{D}$

$$W(s) = \frac{\beta_{m} s^{m} + \beta_{m-1} s^{m-1} + \dots + \beta_{1} s + \beta_{0}}{\alpha_{n} s^{n} + \alpha_{n-1} s^{n-1} + \dots + \alpha_{1} s + \alpha_{0}}$$
$$= C(s\mathbb{I} - A)^{-1} B + D$$

$$\begin{split} \mathsf{P}(s) &= \det(s\mathbb{I} - \boldsymbol{A}) \quad \to \ \alpha \\ \mathsf{P}_c(s) &= (s - \overline{\lambda}_1) \cdots (s - \overline{\lambda}_n) \quad \to \ \overline{\alpha} \end{split}$$

$$m{K}_{c} = egin{bmatrix} \overline{lpha}_{0} - lpha_{0} & \overline{lpha}_{1} - lpha_{1} & \cdots & \overline{lpha}_{n-1} - lpha_{n-1} \end{bmatrix}$$
 $m{K} = m{K}_{c} m{P}^{-1}$

Osservabilità

Gramiano di Osservabilità : $O(t) = \int_0^t e^{A^T \tau} C^T C e^{A \tau} d\tau$ non singolare $(\det \neq 0)$

Matrice di Osservabilità : $0 = \begin{bmatrix} C & CA & CA^2 & \cdots & CA^{n-1} \end{bmatrix}^\mathsf{T} \quad \text{rank}(0) = \mathsf{n}$

$$O^{-1} = \mathfrak{T}^{-1} \qquad \qquad \text{Osservatore di Luenberger:} \\ A_{o} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_{0}/\alpha_{n} \\ 1 & 0 & \cdots & 0 & -\alpha_{1}/\alpha_{n} \\ 0 & 1 & \cdots & 0 & -\alpha_{2}/\alpha_{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\alpha_{n-1}/\alpha_{n} \end{bmatrix} \begin{bmatrix} \beta_{0} \\ \beta_{1} \\ \beta_{2} \\ \vdots \\ \beta_{n-1} \end{bmatrix} = B_{o} \qquad \qquad P_{o} = 0^{-1}0 \qquad L_{o} = \begin{bmatrix} \overline{\alpha}_{0} - \alpha_{0} \\ \overline{\alpha}_{1} - \alpha_{1} \\ \vdots \\ \overline{\alpha}_{n-1} - \alpha_{n-1} \end{bmatrix} \\ C_{o} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{n} \end{bmatrix} = D_{o} \qquad \qquad A_{o} = A - L_{o}C \quad B_{o} = \begin{bmatrix} B & L_{o} \end{bmatrix} \quad C_{o} = C \quad D_{o} = D$$

Osservatore di Luenberger:

$$m{P}_{o} = \mathfrak{O}^{-1}\mathfrak{O} \qquad m{L}_{o} = egin{bmatrix} \overline{lpha}_{0} - lpha_{0} \\ \overline{lpha}_{1} - lpha_{1} \\ \vdots \\ \overline{lpha}_{n-1} - lpha_{n-1} \end{bmatrix}$$

$$egin{aligned} m{A}_{
m o} &= m{A} - m{L}_{
m o}m{C} & m{B}_{
m o} &= m{m{B}} & m{L}_{
m o} \end{bmatrix} & m{C}_{
m o} &= m{C} & m{D}_{
m o} &= m{L} \ m{L} &= m{P}_{
m o}^{-1}m{L}_{
m o} & m{ar{u}}({
m t}) &= egin{bmatrix} m{u}({
m t}) \ m{y}({
m t}) \end{bmatrix} \end{aligned}$$

Sylvester per Matrice di transizione di stato

$$\begin{split} e^{\boldsymbol{A}t} &= \sum_{i=0}^{n-1} \beta_i(t) \boldsymbol{A}^i = \beta_0(t) \boldsymbol{I} + \beta_1(t) \boldsymbol{A} + \dots + \beta_{n-1}(t) \boldsymbol{A}^{n-1} \\ \mu_\alpha &= 1 \\ \begin{cases} \beta_0(t) + \lambda_1 \beta_1(t) + \lambda_1^2 \beta_2(t) + \dots + \lambda_1^{n-1} \beta_{n-1}(t) = e^{\lambda_1 t} \\ \beta_0(t) + \lambda_2 \beta_1(t) + \lambda_2^2 \beta_2(t) + \dots + \lambda_2^{n-1} \beta_{n-1}(t) = e^{\lambda_2 t} \\ \vdots & \vdots \\ \beta_0(t) + \lambda_n \beta_1(t) + \lambda_n^2 \beta_2(t) + \dots + \lambda_n^{n-1} \beta_{n-1}(t) = e^{\lambda_n t} \end{cases} \end{split}$$

$$\begin{cases} \beta_0(t) + \lambda_1 \beta_1(t) + \lambda_1 \beta_1(t) + \dots + \lambda_1^{n-1} \beta_{n-1}(t) = e^{\lambda t} \\ \frac{d}{d\lambda} \left(\beta_0(t) + \lambda \beta_1(t) + \dots + \lambda^{n-1} \beta_{n-1}(t) \right) = \frac{d}{d\lambda} e^{\lambda t} \\ \vdots & \vdots \\ \frac{d^{\nu-1}}{d\lambda^{\nu-1}} \left(\beta_0(t) + \lambda \beta_1(t) + \dots + \lambda^{n-1} \beta_{n-1}(t) \right) = \frac{d^{\nu-1}}{d\lambda^{\nu-1}} e^{\lambda t} \end{split}$$

Autovalori complessi $\lambda, \lambda' = \alpha \pm j\omega$:

$$\begin{cases} \beta_0(t) + \text{Re}(\lambda)\beta_1(t) + \text{Re}(\lambda^2)\beta_2(t) + \dots + \text{Re}(\lambda^{n-1})\beta_{n-1}(t) = e^{\lambda_1 t} \text{cos}(\omega t) \\ \text{Im}(\lambda)\beta_1(t) + \text{Im}(\lambda)^2\beta_2(t) + \dots + \text{Im}(\lambda^{n-1})\beta_{n-1}(t) = e^{\lambda t} \text{sin}(\omega t) \end{cases}$$

dove $Re(\lambda) = \alpha$ e $Im(\lambda) = \omega$.