Gramiano di Controllabilità : $\Gamma(t) = \int_{0}^{t} e^{A\tau} B B^{T} e^{A^{T} \tau} d\tau$ non singolare $(\det \neq 0)$

Matrice di Controllabilità : $\mathfrak{T} = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$ rank $(\mathfrak{T}) = \mathfrak{n}$

$$\mathfrak{I}^{-1} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_{n-1} & 1 \\ \alpha_2 & \alpha_3 & \vdots & \ddots & 1 & 0 \\ \alpha_3 & \vdots & \alpha_{n-1} & \ddots & 0 & 0 \\ \vdots & \alpha_{n-1} & 1 & \cdots & 0 & 0 \\ \alpha_{n-1} & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{A}_{c} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \frac{-\alpha_{0}}{\alpha_{n}} & \frac{-\alpha_{1}}{\alpha_{n}} & \frac{-\alpha_{2}}{\alpha_{n}} & \cdots & \frac{-\alpha_{n-1}}{\alpha_{n}} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \boldsymbol{B}_{c}$$

$$\boldsymbol{C}_{c} = \begin{bmatrix} \beta_{0} & \beta_{1} & \beta_{2} & \beta_{3} & \beta_{4} & \cdots & \beta_{n-1} \end{bmatrix} \begin{bmatrix} \beta_{n} \end{bmatrix} = \boldsymbol{D}_{c}$$

$$oldsymbol{F} = oldsymbol{J} oldsymbol{J}$$
 $oldsymbol{A}_{
m c} = oldsymbol{P}^{-1}oldsymbol{AP} oldsymbol{B}_{
m c} = oldsymbol{P}^{-1}oldsymbol{B}$ $oldsymbol{C}_{
m c} = oldsymbol{CP} oldsymbol{D}_{
m c} = oldsymbol{D}$

$$W(s) = \frac{\beta_m s^m + \beta_{m-1} s^{m-1} + \dots + \beta_1 s + \beta_0}{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \dots + \alpha_1 s + \alpha_0}$$
$$= C(s\mathbb{I} - A)^{-1} B + D$$

$$\begin{split} \mathsf{P}(s) &= \det(s\mathbb{I} - \boldsymbol{A}) \quad \to \ \alpha \\ \mathsf{P}_c(s) &= (s - \overline{\lambda}_1) \cdots (s - \overline{\lambda}_n) \quad \to \ \overline{\alpha} \end{split}$$

$$m{K}_{c} = egin{bmatrix} \overline{lpha}_{0} - lpha_{0} & \overline{lpha}_{1} - lpha_{1} & \cdots & \overline{lpha}_{n-1} - lpha_{n-1} \end{bmatrix}$$
 $m{K} = m{K}_{c} m{P}^{-1}$

Osservabilità

Gramiano di Osservabilità : $O(t) = \int_0^t e^{A^T \tau} C^T C e^{A \tau} d\tau$ non singolare $(\det \neq 0)$

Matrice di Osservabilità : $0 = \begin{bmatrix} C & CA & CA^2 & \cdots & CA^{n-1} \end{bmatrix}^{\mathsf{T}} \operatorname{rank}(0) = \mathsf{n}$

$$\mathbf{A}_{\mathbf{o}} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_{0}/\alpha_{n} \\ 1 & 0 & \cdots & 0 & -\alpha_{1}/\alpha_{n} \\ 0 & 1 & \cdots & 0 & -\alpha_{2}/\alpha_{n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\alpha_{n-1}/\alpha_{n} \end{bmatrix} \begin{bmatrix} \beta_{0} \\ \beta_{1} \\ \beta_{2} \\ \vdots \\ \beta_{n-1} \end{bmatrix} = \mathbf{B}_{\mathbf{o}}$$

Osservatore di Luenberger:

$$egin{aligned} oldsymbol{P}_{
m o} = \mathbb{O}^{-1}\mathbb{O} & oldsymbol{L}_{
m o} = egin{bmatrix} \overline{lpha}_{
m o} - lpha_{
m o} \ \overline{lpha}_{
m i} - lpha_{
m i} \ \overline{lpha}_{
m n-1} - lpha_{
m n-1} \end{bmatrix} \ oldsymbol{A}_{
m o} = oldsymbol{A} - oldsymbol{L}_{
m o} & oldsymbol{B}_{
m o} = igg[oldsymbol{B} & oldsymbol{L}_{
m o}igg] & oldsymbol{C}_{
m o} = oldsymbol{C} & oldsymbol{D}_{
m o} = oldsymbol{D} \end{aligned}$$

$$egin{aligned} m{A}_{
m o} &= m{A} - m{L}_{
m o} & m{B}_{
m o} &= m{ar{B}} & m{L}_{
m o} \end{bmatrix} & m{C}_{
m o} &= m{C} & m{D}_{
m o} &= m{L} \ m{L} &= m{P}_{
m o}^{-1} m{L}_{
m o} & m{\overline{u}}({
m t}) &= egin{bmatrix} m{u}({
m t}) \ m{y}() \end{bmatrix} \end{aligned}$$

Sylvester per Matrice di transizione di stato

$$\begin{split} e^{\mathbf{A}t} &= \sum_{i=0}^{n-1} \beta_i(t) \mathbf{A}^i = \beta_0(t) \mathbf{I} + \beta_1(t) \mathbf{A} + \dots + \beta_{n-1}(t) \mathbf{A}^{n-1} \\ \mu_\alpha &= 1 \\ \begin{pmatrix} \beta_0(t) + \lambda_1 \beta_1(t) + \lambda_1^2 \beta_2(t) + \dots + \lambda_1^{n-1} \beta_{n-1}(t) = e^{\lambda_1 t} \\ \beta_0(t) + \lambda_2 \beta_1(t) + \lambda_2^2 \beta_2(t) + \dots + \lambda_2^{n-1} \beta_{n-1}(t) = e^{\lambda_2 t} \\ \vdots &\vdots \\ \beta_0(t) + \lambda_n \beta_1(t) + \lambda_n^2 \beta_2(t) + \dots + \lambda_n^{n-1} \beta_{n-1}(t) = e^{\lambda_n t} \end{pmatrix} \\ \begin{pmatrix} \beta_0(t) + \lambda \beta_1(t) + \dots + \lambda^{n-1} \beta_{n-1}(t) = e^{\lambda t} \\ \frac{d}{d\lambda} \left(\beta_0(t) + \lambda \beta_1(t) + \dots + \lambda^{n-1} \beta_{n-1}(t) \right) = \frac{d}{d\lambda} e^{\lambda t} \\ \vdots &\vdots \\ \frac{d^{\nu-1}}{d\lambda^{\nu-1}} \left(\beta_0(t) + \lambda \beta_1(t) + \dots + \lambda^{n-1} \beta_{n-1}(t) \right) = \frac{d^{\nu-1}}{d\lambda^{\nu-1}} e^{\lambda t} \end{split}$$

$$\left\{\frac{d^{\nu-1}}{d\lambda^{\nu-1}}\left(\beta_0(t)+\lambda\beta_1(t)+\cdots+\lambda^{n-1}\beta_{n-1}(t)\right)=\frac{d^{\nu-1}}{d\lambda^{\nu-1}}e^{\lambda t}\right\}$$

Autovalori complessi $\lambda, \lambda' = \alpha \pm j\omega$:

$$\begin{cases} \beta_0(t) + \text{Re}(\lambda)\beta_1(t) + \text{Re}(\lambda^2)\beta_2(t) + \dots + \text{Re}(\lambda^{n-1})\beta_{n-1}(t) = e^{\lambda_1 t} \text{cos}(\omega t) \\ \text{Im}(\lambda)\beta_1(t) + \text{Im}(\lambda)^2\beta_2(t) + \dots + \text{Im}(\lambda^{n-1})\beta_{n-1}(t) = e^{\lambda t} \text{sin}(\omega t) \end{cases}$$

dove $Re(\lambda) = \alpha$ e $Im(\lambda) = \omega$.