# How has VIX's current level historically affected next month's returns for 9 different Cboe benchmarks of S&P 500-bullish options strategies?

## Contents




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# Useful concepts

I wrote this non-essential chapter as well as some further explanations in the appendix to clarify certain finance-specific concepts in order to make my research more understandable and readable. <u>If the reader already knows about options, option strategies, and option benchmarks this chapter can be skipped.</u> Full understanding of this chapter is unnecessary, but I thought that having a summary of the argument treated might be helpful.

**Argument-specific knowledge:** Options are a type of derivative: a financial instrument that derives its value from an underlying asset. Specifically, options are contracts giving the right, but not the obligation to either buy (call options) or sell (put options) the underlying asset at a predetermined price (called strike price), until a certain date (called expiry date). The option buyer pays a premium to the option seller for this right. American-style options allow the option buyer to exercise the right at any given time before the expiration date, while European-style only at the expiry date. This research focuses on American options, more specifically options on the S&P 500 Index.

Call [put] options with strike price above [below] the underlying asset price are said to be OTM (out of the money). If they expire, they are not exercised and become worthless. Options near the underlying price are ATM (at the money) and otherwise are ITM (in the money). Conventionally, 1 stock-option contract represents 100 shares of the underlying company.

The premium paid depends upon several factors (such as the underlying asset price, the time to expiration, if the option is OTM/ATM/ITM, etc..). It also depends on how volatile the market thinks the underlying asset will be. In other words, (market) implied volatility. At a theoretical level, the premium paid is determined by the Black-Scholes formula[1]. In reality, premiums are determined by the market, and used as input in the Black-Scholes formula to find instead the theoretical (or implied) volatility. Such value is based on expectations and often diverges from real volatility (called also historical volatility)[2]. Is also important to understand that since volatility is the only variable calculated and not given by the market, it also becomes a “catch-all” variable for all the factors not correctly or fully described by the Back-Scholes formula (for example, increase in option supply or demand).

There exist several option strategies. The simplest is just buying a call or put option. If held until expiration, it will be exercised if its ability is convenient to the investor (buying at price lower than market price, selling above), if not it will expire worthless. Some more advanced strategies involved in this study are:

**Covered call [put]:** Selling a call [put] while owning [shorting] 100 shares of the underlying.

**Naked call [put]:** Selling a call [put] without owning [shorting] 100 shares of the underlying.

**Cash-covered put:** Selling a put while keeping cash & equivalents as collateral to actually buy the underlying shares at the strike price (therefore not just pay the loss/difference) if put expires ITM.

**Research-specific knowledge**: There exist several benchmarks replicating returns of different strategies on specific underlying assets. This study focuses on S&P 500 option strategies benchmarks and the correlation of their returns to their implied volatility. Those studied benchmarks are among the most popular and are created by the CBOE (the major stock option exchange).

There is another index, called the VIX, which serves as benchmark for the average implied volatility of S&P 500 option. Since the analysed strategy benchmarks buy/sell options expiring after one month, and hold them until expiration, VIX index values are relevant for benchmark returns 1 month in advance. For example, the VIX level in June 1986 is relevant for the monthly benchmarks returns in July 1986, because the July returns refer to profits of options bought/sold in June.

**Other technical terms that need to be explained are:**

**S&P 500:** US stock market index composed by 500 of the larges United States publicly listed companies

**Market bullish strategies:** Positively correlated; profits when market price increases. Bearish means the opposite

**Delta:** Ratio that compares the change in the price of the underlying to the corresponding change in the price of its option. Greater the delta, more ITM an option is. 50% delta means the option is ATM.

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[1] Appendix 1
[2] Long-term, historical volatility is usually lower than implied volatility. More information why in Appendix 2

**BuyWrite:** Buying stocks of a security and then selling covered calls (1 for every 100 stocks owned)

**PutWrite:** Selling cash-secured puts (cash is usually held in short term US bonds)

**Collar:** Position with capped losses and capped gains. Investor owns shares, and for every 100 shares he/she sells an OTM call and buys an OTM put

**Put-protection:** Investors own shares and buys 1 put for every 100 shares to protect in cases of losses

**Indexes replicating specific strategies on the S&P 500:**

*Cboe S&P 500 BuyWrite* **(BXM):** BuyWrite an ATM call

*Cboe S&P 500 PutWrite Index* **(PUT):** PutWrite an ATM put

*Cboe S&P 500 95-110 Collar Index* **(CLL)**: Collar made by buying a 5% OTM put expiring long term and selling 10% OTM calls expiring more short term

*Cboe S&P 500 2% OTM BuyWrite* **(BXY):** BuyWrite a 2% OTM call

*Cboe S&P 500 30-Delta BuyWrite Index* **(BXMD):** BuyWrite a 30% OTM call

*Cboe S&P 500 Iron Butterfly Index* **(BFLY)**: Capped profit and capped losses strategy that gains if underlying prices stay within a certain range, i.e., implied volatility is less than realized

*Cboe S&P 500 Zero-Cost Put Spread Collar* **(CLLZ):** Collar made by buying a put and selling a call of equal cost

*Cboe S&P 500 Iron Condor Index* **(CNDR):** Similar to Iron Butterfly but the range is larger and maximum profits are smaller

*Cboe S&P 500 5% Put Protection Index* **(PPUT)**: Put-protection with a 5% OTM put

# How has VIX's current level historically affected next month's returns for 9 different Cboe benchmarks of S&P 500-bullish options strategies?

**ABSTRACT:** Implied volatility (IV) is a fundamental concept for option investors. For buyers, it tells how expensive option are; for sellers, how risky. The higher the VIX, the higher the premium, and the higher the expected move in the underlying.

So… How should investors behave? Conservatives usually avoid high IV while others, like Tom Sosnoff from Tastytrade, purposefully look for it with instruments such as IV rank and IV percentile screeners. What should you do? It is well known that bullish S&P 500 strategies have performed well historically (substantially outperforming bearish strategies), but did they always perform well regardless of implied volatility?

**BACKGROUND:** There is a strong correlation between the VIX monthly variation (next month's – today's) and the next month options returns. Indeed, if the VIX rises, sellers of S&P 500 options generally lose money. This because when volatility increases the same occurs to the option probability to not expire worthless, meaning*, ceteris paribus*, that sellers sold a "right" that is now more valuable.

Unfortunately, future VIX levels are not easy to predict. If this would not be the case, no study would be needed! The VIX will rise? Just buy! Will decrease? Sell! What if you just know today's VIX? This study aims to analyse how well S&P 500 bullish options strategies have historically performed compared to VIX levels, and how much of that performance solely depends on the VIX or, whether alternatively further factors play a significant role.

**DATA AND UNIVARIATE ANALYSIS:** Data used are the official CBOE monthly prices from 31 Jan 1990 to 28 Jun 2019 of 9 S&P 500-bullish CBOE indexes (BXM,PUT,CLL,BXY,BXMD,BFLY,CLLZ,CNDR,PPUT) as well as the corresponding VIX level[3], for a total of 354 prices for each index (3186 in total) and 355 VIX levels. I used those to calculate the monthly returns[4], from which I performed the univariate analysis.

*Fig. 1 and 2* **Univariate Analysis Data**. *Monthly returns are returns on investment and, as such, have to be intended as percentages*

| | aritm_means | medians | variances | mins | maxs | ranges | standard_devs | quart_1 | quart_3 | interq_ranges | coef_variations |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **BXM.Monthly.returns** | 0.007280124 | 0.011165161 | 8.458281e-04 | -0.15130786 | 0.10014628 | 0.2514541 | 0.02908312 | -0.0021588073 | 0.02050875 | 0.02266756 | 3.9948667 |
| **PUT.monthly.returns** | 0.008010280 | 0.011492981 | 7.811157e-04 | -0.17650160 | 0.08977969 | 0.2662813 | 0.02794845 | 0.0014953271 | 0.02006382 | 0.01856849 | 3.4890723 |
| **CLL.monthly.returns** | 0.005879925 | 0.007052701 | 8.789355e-04 | -0.07946556 | 0.08272488 | 0.1621904 | 0.02964685 | -0.0153007796 | 0.02748949 | 0.04279027 | 5.0420449 |
| **BXY.monthly.returns** | 0.008432961 | 0.013200701 | 1.171488e-03 | -0.15699768 | 0.11423979 | 0.2712375 | 0.03422701 | -0.0078187330 | 0.02713277 | 0.03495151 | 4.0587176 |
| **BXMD.monthly.returns** | 0.008745409 | 0.013350852 | 1.260534e-03 | -0.15599085 | 0.11279108 | 0.2687819 | 0.03550400 | -0.0078662622 | 0.02728744 | 0.03515370 | 4.0597300 |
| **BFLY.monthly.returns** | 0.004318530 | 0.004100338 | 9.722464e-04 | -0.08576023 | 0.09966652 | 0.1854267 | 0.03118087 | -0.0176564675 | 0.02766000 | 0.04531647 | 7.2202493 |
| **CLLZ.monthly.returns** | 0.005994506 | 0.008429472 | 1.011836e-03 | -0.15197049 | 0.08278548 | 0.2347560 | 0.03180936 | -0.0109910156 | 0.02553833 | 0.03652934 | 5.3064191 |
| **CNDR.monthly.returns** | 0.004905871 | 0.010295404 | 4.012217e-04 | -0.08400237 | 0.04889386 | 0.1328962 | 0.02003052 | -0.0007459903 | 0.01627662 | 0.01702261 | 4.0829688 |
| **PPUT.monthly.returns** | 0.006020947 | 0.008416412 | 1.131140e-03 | -0.09802333 | 0.10401891 | 0.2020422 | 0.03363243 | -0.0173926827 | 0.02903021 | 0.04642289 | 5.5859035 |
| **VIX.Level** | 19.226968839 | 17.190000000 | 5.478774e+01 | 9.51000000 | 59.89000000 | 50.3800000 | 7.40187393 | 13.6800000000 | 23.26000000 | 9.58000000 | 0.3849735 |

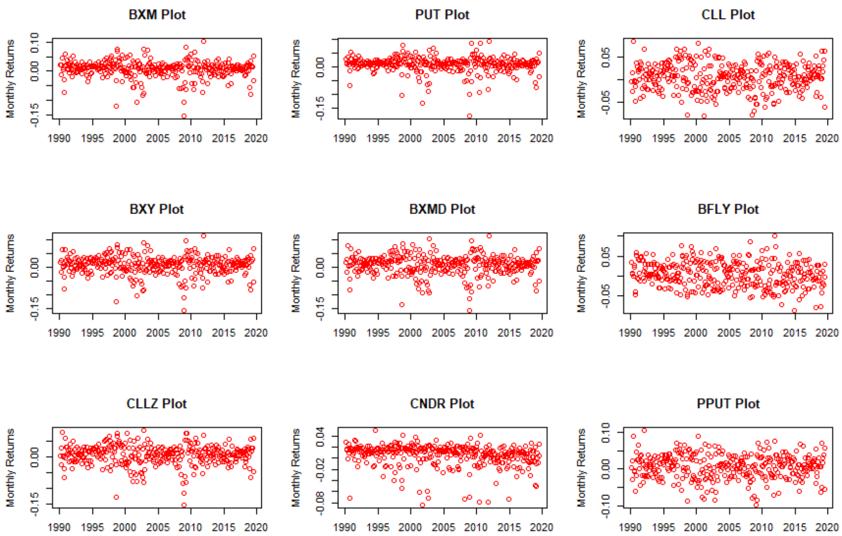
| Date | BXM Monthly returns | PUT monthly returns | CLL monthly returns | BXY monthly returns | BXMD monthly returns | BFLY monthly returns | CLLZ monthly returns | CNDR monthly returns | PPUT monthly returns |
|---|---|---|---|---|---|---|---|---|---|
| Cumulative Return | 1013.01% | 1352.15% | 579.31% | 1474.77% | 1629.78% | 286.24% | 588.85% | 423.67% | 582.44% |
| Annual Avg Return | 8.54% | 9.53% | 6.73% | 9.83% | 10.18% | 4.70% | 6.78% | 5.79% | 6.75% |
| Monthly Avg Return | 0.69% | 0.76% | 0.54% | 0.78% | 0.81% | 0.38% | 0.55% | 0.47% | 0.55% |

First thing investors look at are average returns[5]. In the last 30 years, the best performing index was BXMD, followed by BXY and the well-known PUT. Iron butterfly's BFLY remarkably underperformed, even with the no-commission advantage.[6] Except for PPUT, CLL and BFLY (3 relatively protective strategies), the biggest losses were greater than the largest gains, and the range of returns is particularly high, especially for monthly returns. As a consequence, except for BFLY, arithmetic means are well below medians. However, they both were positive for all strategies. Standard deviations between indexes vary but not as remarkably as returns. It is particularly impressive how >75% of PUT

[3] Original data available here: https://ww2.cboe.com/micro/buywrite/monthendpricehistory.xls. The attached excel file (at page 11) is edited.
[4] Unfortunately, 1997 and 2020's VIX levels (two remarkably high VIX periods) where not present. Still, there are several other valid periods, more remarkably 2008-2009, but also 1990, 1997-1998, 2002-2003, 2010-2011 etc. Indeed, there are 55 months recorded with VIX above 25 and 24 months with VIX above 30.
[5] Average returns are geometrical means. Data were monthly prices, so the monthly average return is the geometrical sample mean.
[6] Indexes do not consider commissions. Iron butterflies are one of the most commission-expensive options strategies.

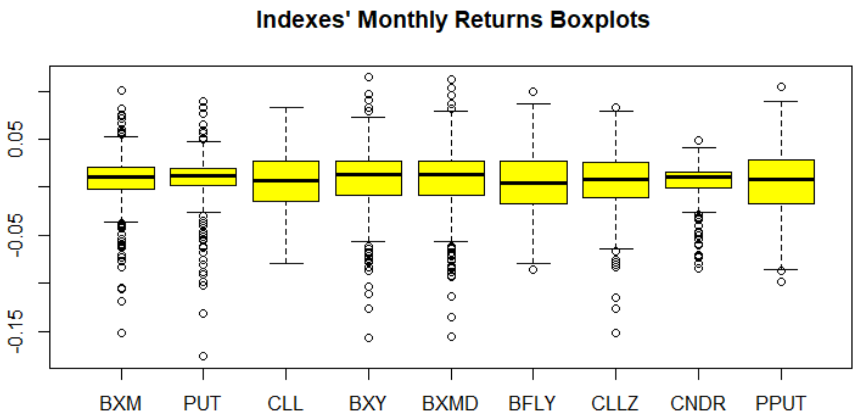
returns were profitable and >75% of CNDR were below 1.65%. Interquartile ranges vary substantially, as well as coefficient of variations. They all have a CV higher than their underlying (S&P 500's CV is historically around 2.6). Unintuitively, SD and CV were not at all higher for more profitable (and less conservative) strategies. For instance, BFLY has a much higher CV than every other index; PPUT, probably the most conservative strategy analysed, had one of the highest SD. Regarding the VIX, it has an arithmetic mean higher than its median, and this because of the extremely high maximums it reaches during market crashes. Consequently, VIX has also a really high range. Variance, standard deviation, and interquartile range are instead low. However, we have to remember that the analysed period (1990-2019) is a relatively low VIX period. It excludes two of the highest VIX periods ever (1987 and 2020), and includes two long, steady bull markets (from 2009 to 2019 and from 1990 to 2000), where the VIX is usually low. VIX's CV is not really high, since the majority of the values do not disperse too much away from the mean (although when they do, they do by much). It is also lower than I expected because SD is lower.

*Fig. 3* **Monthly returns over time**



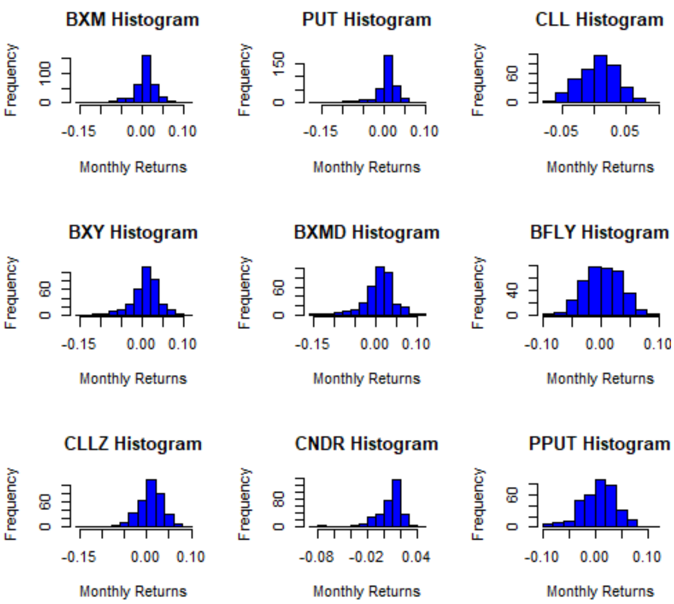

The following graphs help interpreting the univariate index data previously found. It can be seen in Fig.3 how certain strategies (expecially PUT and BXM) have a dispersion of returns much lower than others (expecially the third column). This doesn't translate in an absence of large losses during crashes. For Fig.3 , regression lines are inappropriate: changes in returns in a 20-years span are more likely to be related to the ciclycal nature of markets than to actual permanent changes.

*Fig. 4*




Outliers are clearly visualized with boxplots. The best performing indexes (BXM, PUT, BXY, BXMD) had also the worst losses. Despite contradictory SDs, the old mantra "bigger risk = bigger reward" might still be valid. Conservative strategies had extremely few outliers (for example CLL, because it has capped returns, it did not have any in 30 years!), while best performing had lots, especially in the negative territory.

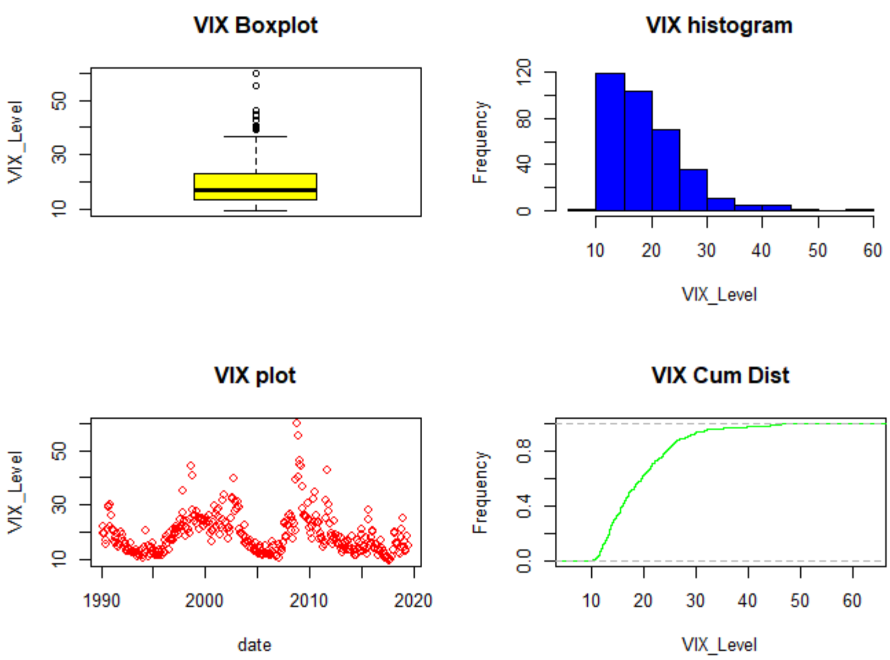
*Fig. 5* ***Histograms***




Less conservative strategies are more negatively skewed than conservative ones. In Fig.5, CNDR is visibly negatively skewed, but it is important to note that its x-axis has a greater zoom since its interquartile range is much smaller. In general, bullish option strategies are known for constantly generating small gains and they sporadically incur in substantial losses. This is especially true for strategies involving selling naked put options, like PUT does. (PUT actually sells cash-covered puts, which means that losses are always coverable. Nonetheless, this does not make the strategy less exposed to drawbacks, it just eliminates the risks of a margin call.)

We can now see how positively skewed VIX levels are. Mean, first quartile, and minimum are really near. It is notable how VIX values never went under 10 except on September 2017 (9.51). In my opinion, no matter how low expected price movements are, there will always be some "natural" option demand, keeping IV above a certain threshold. In contrast, there are several outliers, with a peak at 60 VIX (2008 crisis, when the VIX actually peaked above 80 intramonth). The bottom-left plot shows the cyclicality of the VIX; having "waves" peaking at financial crisis and lowering during steady periods, but never going below the "10-points threshold". The last graph shows again the majority of points lying within a small range.

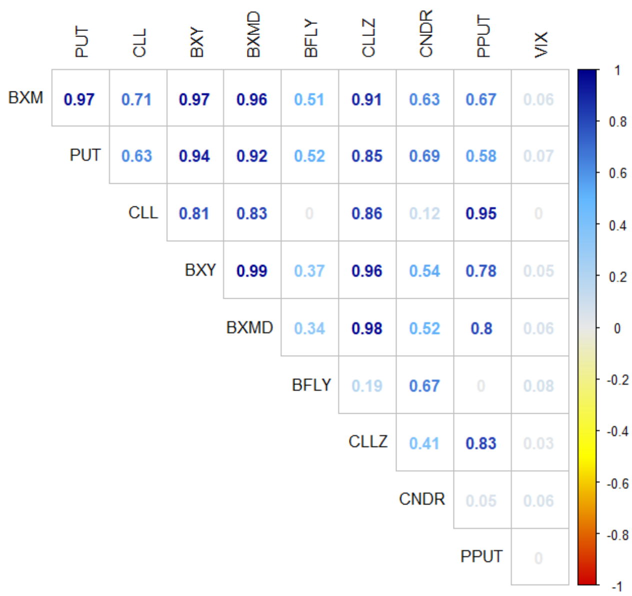
*Fig. 6* **VIX Univariate Analysis**




**BIVARIATE ANALYSIS:**

The following graph shows correlations within our dataset. Firstly, PUT, BXY, BXMD, CLL and CLLZ are all extremely correlated to each other. This because they are bullish: they all appreciate when the S&P 500 increases in price. In contrast, CNDR and BFLY, two relatively more market-neutral strategies, were less correlated to the remaining dataset, a useful information for investors looking to diversify returns and decrease overall portfolio volatility.

*Fig.7* **Correlations**




The most important correlation is, however, the one with the last month's VIX. Theories suggest no correlation, and indeed collars (CLL and CLLZ), and protective puts (PPUT) were linearly non-correlated to VIX. In this case, unless an unlikely non-linear correlation exists, VIX levels are useless at predicting future returns. Regarding the remaining strategies, they had similar slightly positive VIX correlations, between 5-8%. This result neither confirms or contradicts IV rank/percentile options strategies.[7] The higher the VIX has been, the higher the average returns, even if slightly, but a one-digit correlation is often considered negligible. However, because of heteroscedasticity, an 8% is still somewhat relevant (more on next page's scatter plots). Despite such relevance, *the intelligent investor* must listen to what the data is saying: "Do NOT base your option strategy just upon the VIX level!" Higher IV may slightly increase average returns, but the most evident effect is the substantial increase in the variance of returns.

| | B0(VIX) | B1(VIX) | R^2(VIX) | Cov(VIX) | Cor(VIX) |
|---|---|---|---|---|---|
| **BXM** | 0.002641860 | 2.408726e-04 | 3.762086e-03 | 0.0132106285 | 0.0613358451 |
| **PUT** | 0.003241682 | 2.476410e-04 | 4.305921e-03 | 0.0135818455 | 0.0656195166 |
| **CLL** | 0.005999262 | -6.197341e-06 | 2.396569e-06 | -0.0003398925 | -0.0015480857 |
| **BXY** | 0.003775435 | 2.418729e-04 | 2.738877e-03 | 0.0132654906 | 0.0523342846 |
| **BXMD** | 0.002751173 | 3.112904e-04 | 4.216120e-03 | 0.0170726854 | 0.0649316584 |
| **BFLY** | -0.002041632 | 3.302935e-04 | 6.154035e-03 | 0.0181149131 | 0.0784476599 |
| **CLLZ** | 0.003675573 | 1.204260e-04 | 7.860797e-04 | 0.0066047493 | 0.0280371133 |
| **CNDR** | 0.001696163 | 1.666853e-04 | 3.797927e-03 | 0.0091418388 | 0.0616273205 |
| **PPUT** | 0.005947055 | 3.837334e-06 | 7.139683e-07 | 0.0002104582 | 0.0008449665 |

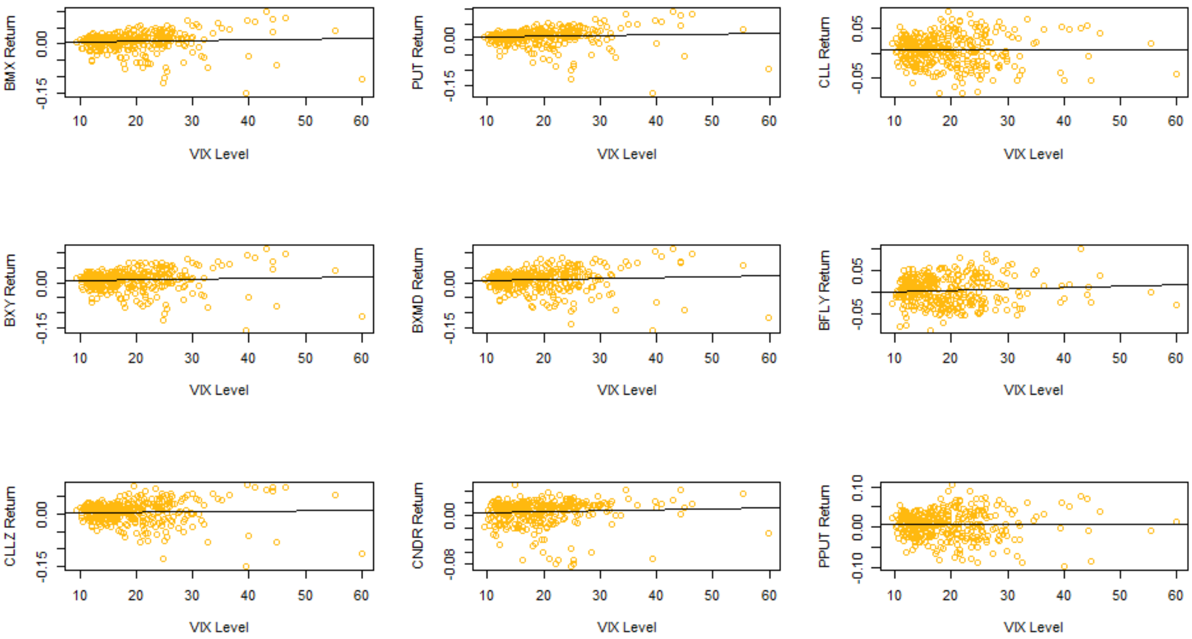
Summarizing, all correlations to the VIX are small but positive, except for CLL, which is slightly negative, in a negligible way. $R^2$ values are extremely small, meaning that the linear regression expressed in the following table (y = B0 + B1x) explains data slightly better than just the mean. This is expected, since correlation is low. This does not necessarily mean that the linear regression itself is a bad model, but (more likely) that since average returns do not change drastically as VIX changes, much of the relation is already explained by the mean, leaving little room for any regression model.

[7] Those strategies state that IV is higher than realized volatility, and that over time IV regresses to the mean. Investors should sell options (put options since we have bullish assumptions) only when IV is high, to capture greater premiums. Therefore: high IV = higher profits for sellers

## $R^2$ VALUES AND HETEROSCEDASTICITY REPERCUSSIONS

It can be noticed that R and $R^2$ values are low. This intuitively leads to the conclusion that probably VIX levels would not impact next month's return in any significant way. Such intuition is relevant and insightful, but to measure the impact of IV on option returns it is important to also fully understand its implications. In fact, the study discusses a phenomenon that in theory should not exist (which is the high correlation between VIX levels and next month's returns), and its existence in any point in time is in itself self-destructing. This because, since its existence can be a source of profit (which is the reason why this study was done) investors will try to exploit it, by either buying or selling options. This leads price to adjust, narrowing the profit that can be made and therefore reducing the phenomenon itself. Therefore, it is unlikely to ever reach a remarkably high correlation over long periods of time (30 years). Despite that, lower levels of correlations are still relevant. For instance, a 12% yearly return increased by 8% means 13% return, which thanks to compound interest, is the difference between $1.000.000 becoming $40.000.000 instead of $30.000.000 in a 30 years' time period.

*Fig.8* **Scatter plots and regression lines**




Furthermore, it is important to recognize the heteroscedasticity of the data studied. By nature, options are leveraged products, meaning that they offer more exposure for the same amount invested than their underlying. This means that they naturally have very wide ranges of returns. Moreover, the bigger is the IV of an option, the bigger is the premium, and therefore the bigger is the quantity invested, either as direct investment for buyers or as collateral required for sellers. This leads options to have a wider and wider range of returns as implied volatility increases. Option investors will often win big and lose big, and higher the IV, bigger will be their profits and their losses. Supposing that the relationship studied actually follows a linear regression, the aforementioned phenomenon leads datapoints to be further and further away from any regression line, drastically reducing $R^2$.

*Fig.9* **Snapshot of Benchmarks' returns (edited)**

| 1 | Date | BXM Monthly returns | PUT monthly returns | CLL monthly returns | BXY monthly returns | BXMD monthly returns | BFLY monthly returns | CLLZ monthly returns | CNDR monthly returns | PPUT monthly returns | VIX Level 1 MONTH EARLIER |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 337 | 31-Jan-2018 | 0.95% | 0.90% | 5.21% | 2.49% | 1.60% | -7.94% | 2.36% | -3.09% | 5.52% | 11.04 |
| 338 | 28-Feb-2018 | -1.42% | -2.16% | -3.62% | -2.84% | -2.42% | -0.57% | -3.01% | 1.50% | -3.84% | 13.54 |
| 339 | 29-Mar-2018 | -1.09% | -1.34% | -2.82% | -1.93% | -1.92% | 1.23% | -1.60% | -0.60% | -2.03% | 19.85 |
| 340 | 30-Apr-2018 | 1.33% | 2.05% | 0.58% | 0.79% | 0.89% | 2.82% | -0.47% | 2.68% | -1.11% | 19.97 |
| 341 | 31-May-2018 | 2.09% | 1.99% | 1.10% | 2.74% | 2.89% | 2.60% | 2.16% | 1.33% | 2.02% | 15.93 |
| 342 | 29-Jun-2018 | -0.06% | 0.16% | 0.95% | 0.68% | 0.24% | -1.28% | 0.74% | 0.17% | 0.64% | 15.43 |
| 343 | 31-Jul-2018 | 2.93% | 2.60% | 2.44% | 3.81% | 3.88% | 2.60% | 3.20% | 1.34% | 3.14% | 16.09 |
| 344 | 31-Aug-2018 | 1.90% | 1.54% | 2.93% | 2.83% | 2.70% | -0.20% | 2.58% | 0.20% | 2.93% | 12.83 |
| 345 | 28-Sep-2018 | 0.03% | 0.22% | 0.62% | 0.11% | 0.03% | -1.22% | 0.14% | -0.62% | 0.48% | 12.86 |
| 346 | 31-Oct-2018 | -5.46% | -5.59% | -3.97% | -6.37% | -6.24% | -7.58% | -3.65% | -4.83% | -6.35% | 12.12 |
| 347 | 30-Nov-2018 | 2.24% | 1.68% | 0.11% | 1.99% | 2.03% | 0.73% | 1.20% | 1.90% | 0.58% | 21.23 |
| 348 | 31-Dec-2018 | -7.73% | -7.56% | 0.13% | -8.32% | -8.39% | -4.13% | -6.64% | -5.01% | -5.00% | 18.07 |
| 349 | 31-Jan-2019 | 3.43% | 2.77% | 6.18% | 4.42% | 5.21% | -2.73% | 5.65% | -1.02% | 7.08% | 25.42 |
| 350 | 28-Feb-2019 | 1.43% | 1.40% | 3.07% | 2.36% | 2.28% | -1.68% | 2.74% | 0.10% | 2.90% | 16.57 |
| 351 | 29-Mar-2019 | 1.76% | 1.21% | 1.77% | 2.31% | 2.45% | 1.15% | 1.89% | 0.99% | 1.68% | 14.78 |
| 352 | 30-Apr-2019 | 1.57% | 1.58% | 4.25% | 2.94% | 2.48% | -2.33% | 2.84% | -0.77% | 3.79% | 13.71 |
| 353 | 31-May-2019 | -3.21% | -3.75% | -5.94% | -5.27% | -4.82% | -0.34% | -4.71% | 0.30% | -5.67% | 13.12 |
| 354 | 28-Jun-2019 | 5.06% | 4.77% | 6.30% | 6.83% | 6.67% | 2.85% | 5.95% | 2.54% | 5.60% | 18.71 |
| 355 | | | | | | | | | | | |
| 356 | Cumulative | 1013.01% | 1352.15% | 579.31% | 1474.77% | 1629.78% | 286.24% | 588.85% | 423.67% | 582.44% | |
| 357 | Annual Avg Return | 8.54% | 9.53% | 6.73% | 9.83% | 10.18% | 4.70% | 6.78% | 5.79% | 6.75% | |
| 358 | Monthly Avg Return | 0.69% | 0.76% | 0.54% | 0.78% | 0.81% | 0.38% | 0.55% | 0.47% | 0.55% | |

# Appendix 1

*Fig. 10* **Black-Scholes formula:**

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

$C$ = call option price

$N$ = CDF of the normal distribution

$S_t$ = spot price of an asset

$K$ = strike price

$r$ = risk-free interest rate

$t$ = time to maturity

$\sigma$ = volatility of the asset

# Appendix 2

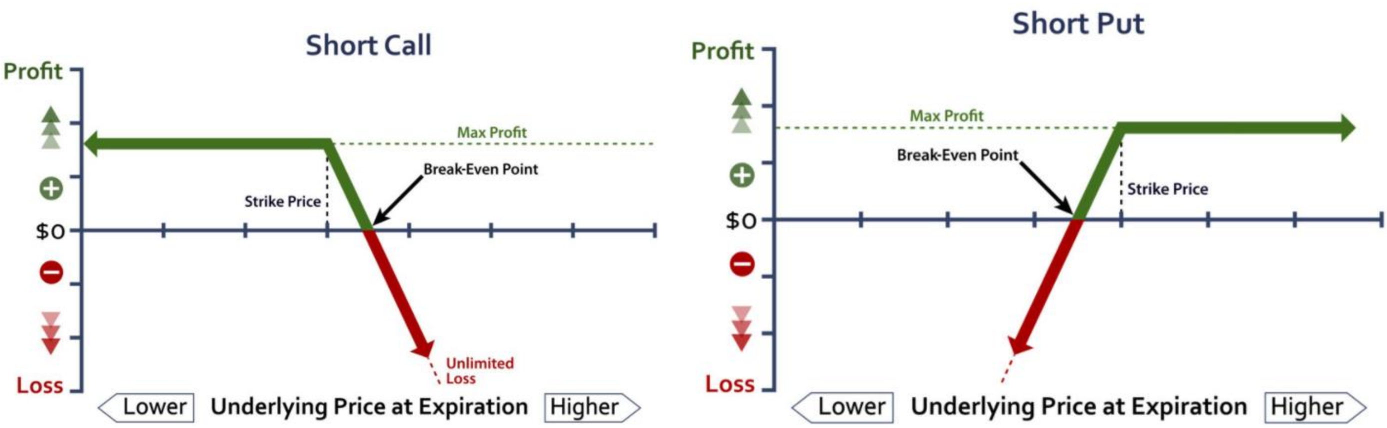
Options, as derivatives, are a zero-sum game, meaning that if one side is profiting, the other side is certainly losing money-wise. The only situation where both seller and buyer can be satisfied about an option outcome is when the buyer uses options as an insurance.

For example, a pension fund might accept a negative expected value for long-term returns in exchange of a lower portfolio volatility and protection during market crashes. It might buy OTM put options (with a market index as underlying) that expire worthless the majority of the time, but become highly profitable once the market crashes, reducing overall losses during the crash in the remaining beta-positive (i.e. market correlated) portfolio.

The use of options as insurance (in particular after the 1987 crash) and the human tendency to overestimate price changes are the main reason why realized volatility is often lower than implied (theoretical) volatility long term.

Is also important to recognize how selling naked options (options without protection, i.e. without a corresponding position on the underlying, or without being long on similar option contracts to limit the downside) offers asymmetrical returns, as shown below, with limited profit (the premium received) and unlimited/substantial loss (difference between strike price and underlying price when exercised). This makes option-selling less attractive to investors, diminishing option supply and therefore further increasing the “catch-all” variable implied volatility compared to realized volatility.

*Fig. 11* **Returns for selling European-style options**



# Appendix 3 – R code

```
library(readxl)
library(stats)
library(ggplot2)
library(corrplot)
Monthly_prices <- read_excel("EXCEL FILE AT PAGE 11",
      sheet = "Only monthly VIX and returns",
      col_types = c("date", "numeric", "numeric",
               "numeric", "numeric", "numeric",
               "numeric", "numeric", "numeric",
               "numeric", "numeric"),
      range = "A1:K354")
dataopt <- data.frame(Monthly_prices)
aritm_means <- NULL
medians <- NULL
variances <- NULL
mins <- NULL
maxs <- NULL
ranges <- NULL
standard_devs <- NULL
quart_1 <- NULL
quart_3 <- NULL
interq_ranges <- NULL
coef_variations <- NULL
for (column in 2:11){
date <- dataopt[, 1]
Month_return <- dataopt[, column]
  temp_aritm_means <- mean(Month_return)
  temp_medians <- median(Month_return)
  temp_variances <- var(Month_return)
  temp_mins <- min(Month_return)
  temp_maxs <- max(Month_return)
  temp_ranges <- temp_maxs - temp_mins
  temp_standard_devs <- sd(Month_return)
  temp_quart_1 <- quantile(Month_return,0.25)
  temp_quart_3 <- quantile(Month_return,0.75)
  temp_interq_ranges <- temp_quart_3 - temp_quart_1
  temp_coef_variations <- temp_standard_devs/temp_aritm_means
 aritm_means <- c(aritm_means,temp_aritm_means)
 medians <- c(medians,temp_medians)
 variances <- c(variances,temp_variances)
 mins <- c(mins,temp_mins)
 maxs <- c(maxs,temp_maxs)
 ranges <- c(ranges,temp_ranges)
 standard_devs<- c(standard_devs,temp_standard_devs)
 quart_1 <- c(quart_1,temp_quart_1)
 quart_3 <- c(quart_3,temp_quart_3)
 interq_ranges <- c(interq_ranges,temp_interq_ranges)
 coef_variations <- c(coef_variations,temp_coef_variations)
}
par(mfrow=c(3,3))
hist(dataopt[,2], col="blue", main="BXM Histogram", xlab="Monthly Returns")
hist(dataopt[,3], col="blue", main="PUT Histogram", xlab="Monthly Returns")
hist(dataopt[,4], col="blue", main="CLL Histogram", xlab="Monthly Returns")
hist(dataopt[,5], col="blue", main="BXY Histogram", xlab="Monthly Returns")
hist(dataopt[,6], col="blue", main="BXMD Histogram", xlab="Monthly Returns")
hist(dataopt[,7], col="blue", main="BFLY Histogram", xlab="Monthly Returns")
hist(dataopt[,8], col="blue", main="CLLZ Histogram", xlab="Monthly Returns")
```

```r
hist(dataopt[,9], col="blue", main="CNDR Histogram", xlab="Monthly Returns")
hist(dataopt[,10], col="blue", main="PPUT Histogram", xlab="Monthly Returns")
par(mfrow=c(3,3))
plot(dataopt[,2], col="red", x = date, main="BXM Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,3], col="red", x = date, main="PUT Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,4], col="red", x = date, main="CLL Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,5], col="red", x = date, main="BXY Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,6], col="red", x = date, main="BXMD Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,7], col="red", x = date, main="BFLY Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,8], col="red", x = date, main="CLLZ Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,9], col="red", x = date, main="CNDR Plot", ylab="Monthly Returns", xlab = "")
plot(dataopt[,10], col="red", x = date, main="PPUT Plot", ylab="Monthly Returns", xlab = "")
par(mfrow=c(3,3))
plot(ecdf(dataopt[,2]), col="green", main="BXM Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,3]), col="green", main="PUT Cum Distr",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,4]), col="green", main="CLL Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,5]), col="green", main="BXY Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,6]), col="green", main="BXMD Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,7]), col="green", main="BFLY Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,8]), col="green", main="CLLZ Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,9]), col="green", main="CNDR Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
plot(ecdf(dataopt[,10]), col="green", main="PPUT Cum Dist",cex=0,xlab= "Month_returns", ylab="Frequency")
par(mfrow=c(1,1))
boxplot(dataopt[2:10], col="yellow", main=" Indexes' Monthly Returns Boxplots", names =
c("BXM","PUT","CLL","BXY","BXMD","BFLY","CLLZ","CNDR","PPUT"))
par(mfrow=c(2,2))
boxplot(dataopt[,11], col="yellow", main="VIX Boxplot",ylab="VIX_Level")
hist(dataopt[,11], col="blue", main="VIX histogram", xlab="VIX_Level")
plot(dataopt[,11], col="red", x = date, main= "VIX plot", ylab="VIX_Level")
plot(ecdf(dataopt[,11]), col="green", main="VIX Cum Dist",cex=0,xlab= "VIX_Level", ylab="Frequency")
par(mfrow=c(1,1))
df_final <-
data.frame(aritm_means,medians,variances,mins,maxs,ranges,standard_devs,quart_1,quart_3,interq_ranges,coef_v
ariations)
rownames(df_final) <- names(dataopt)[-1]
View(df_final)
indexes_returns_2 <- data.frame(read_excel("EXCEL FILE AT PAGE 11",
                               sheet = "Only monthly VIX and returns",
                               range = "A356:J358", col_names = c("returns_data",names(dataopt)[2:10])))
rownames(indexes_returns_2) <- data.frame(indexes_returns_2)[1:3,1]
indexes_returns <- indexes_returns_2[,-1]
rm(indexes_returns_2)
View(indexes_returns)
dataopt_simple_names <- dataopt
names(dataopt_simple_names) = c("Date","BXM","PUT","CLL","BXY","BXMD","BFLY","CLLZ","CNDR","PPUT","VIX")
correlations <- cor(dataopt_simple_names[,2:11])
covariances <- cov(dataopt_simple_names[,2:11])
View(correlations)
View(covariances)
par(mfrow=c(1,1))
coul <- c("red3","yellow","grey91","steelblue1","blue4")
coul <- colorRampPalette(coul)(500)
corrplot(correlations, method="number", type = "upper", col= coul, tl.col ="black", diag = FALSE)
B_0 <- NULL
B_1 <- NULL
R2 <- NULL
for (i in 2:10){
  date <- dataopt[, 1]
  Month_return <- dataopt[, column]
  temp_B_1 <- cov(dataopt[,11],dataopt[,i])/var(dataopt[,11])
```

```
    temp_B_0 <- mean(dataopt[,i])-temp_B_1*mean(dataopt[,11])
    temp_R2 <- summary(lm(dataopt[,11]~dataopt[,i]))$r.squared
    B_0 <- c(B_0,temp_B_0)
    B_1 <- c(B_1,temp_B_1)
    R2 <- c(R2, temp_R2)
}
  Regression_lines_cov_and_cor <- data.frame(B_0,B_1,R2,covariances[1:9,10],correlations[1:9,10])
  rownames(Regression_lines_cov_and_cor) <- c("BXM","PUT","CLL","BXY","BXMD","BFLY","CLLZ","CNDR","PPUT")
  colnames(Regression_lines_cov_and_cor) <- c("B0(VIX)","B1(VIX)","R^2(VIX)","Cov(VIX)","Cor(VIX)")
  View(Regression_lines_cov_and_cor)
par(mfrow=c(3,3))
plot(dataopt[,11],dataopt[,2], col="darkgoldenrod1", ylab = "BMX Return", xlab = "VIX Level")
abline(B_0[1],B_1[1])
plot(dataopt[,11],dataopt[,3],col="darkgoldenrod1", ylab = "PUT Return", xlab = "VIX Level")
abline(B_0[2],B_1[2])
plot(dataopt[,11],dataopt[,4],col="darkgoldenrod1", ylab = "CLL Return", xlab = "VIX Level")
abline(B_0[3],B_1[3])
plot(dataopt[,11],dataopt[,5],col="darkgoldenrod1", ylab = "BXY Return", xlab = "VIX Level")
abline(B_0[4],B_1[4])
plot(dataopt[,11],dataopt[,6],col="darkgoldenrod1", ylab = "BXMD Return", xlab = "VIX Level")
abline(B_0[5],B_1[5])
plot(dataopt[,11],dataopt[,7],col="darkgoldenrod1", ylab = "BFLY Return", xlab = "VIX Level")
abline(B_0[6],B_1[6])
plot(dataopt[,11],dataopt[,8],col="darkgoldenrod1", ylab = "CLLZ Return", xlab = "VIX Level")
abline(B_0[7],B_1[7])
plot(dataopt[,11],dataopt[,9],col="darkgoldenrod1", ylab = "CNDR Return", xlab = "VIX Level")
abline(B_0[8],B_1[8])
plot(dataopt[,11],dataopt[,10],col="darkgoldenrod1", ylab = "PPUT Return", xlab = "VIX Level")
abline(B_0[9],B_1[9])
```

# Appendix 4 – Data Used

Monthly_prices_S&P500_Options_Benchmarks

Click to view.