Lista 8 de Cálculo I
Data da entrega: 03/12/2019

Exercício 1 (Substituição trigonométrica) Calcule:

1. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$

3. $\displaystyle\int \frac{dx}{x\sqrt{x^2+4}}$

5. $\displaystyle\int \frac{dx}{x\sqrt{25-x^2}}$

7. $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}}$

11. $\displaystyle\int \frac{dx}{(4x^2-9)^{3/2}}$

13. $\displaystyle\int \frac{2\,dt}{t\sqrt{t^4+25}}$

Respostas:

1. $-\dfrac{\sqrt{4-x^2}}{4x}+C$ **3.** $\frac{1}{2}\ln\left|\dfrac{\sqrt{x^2+4}-2}{x}\right|+C$ **5.** $\frac{1}{5}\ln\left|\dfrac{5-\sqrt{25-x^2}}{x}\right|+C$ **7.** $\ln|x+\sqrt{x^2-a^2}|+C$

9. $\frac{1}{4}\operatorname{tg}^{-1}\frac{1}{2}x-\dfrac{x}{2(x^2+4)}+C$ **11.** $-\dfrac{x}{9\sqrt{4x^2-9}}+C$ **13.** $\frac{1}{5}\ln\left(\dfrac{\sqrt{t^4+25}-5}{t^2}\right)+C$ **15.** $\ln|x+2+\sqrt{4x+x^2}|+C$

17. $\dfrac{x+2}{9\sqrt{5-4x-x^2}}+C$ **19.** $\dfrac{\operatorname{tg} x}{4\sqrt{4-\operatorname{tg}^2 x}}+C$ **21.** $\frac{1}{3}\sqrt{\ln^2 w-4}(8+\ln^2 w)+C$ **23.** $-\dfrac{e^t+4}{9\sqrt{e^{2t}+8e^t+7}}+C$

Exercício 2 (Integral definida) Calcule:

25. $\displaystyle\int_0^{\pi/2}\cos^3 x\,dx$

27. $\displaystyle\int_0^{1}\operatorname{sen}^4 \tfrac{1}{2}\pi x\,dx$

29. $\displaystyle\int_0^{2} xe^{2x}\,dx$

31. $\displaystyle\int_0^{\pi/4} e^{3x}\operatorname{sen} 4x\,dx$

Respostas:

25. $\frac{2}{3}$ **27.** $\frac{3}{8}$ **29.** $\frac{1}{4}(3e^4+1)$ **31.** $\frac{4}{25}(e^{3\pi/4}+1)$

Exercício 3 Nos exercícios a seguir, desenhe o conjunto A e dado e calcule a área.

1. A é o conjunto do plano limitado pelas retas $x=1$, $x=3$, pelo eixo $0x$ e pelo gráfico de $y=x^3$.
3. A é o conjunto de todos (x, y) tais que $x^2-1\leqslant y\leqslant 0$.
7. A é o conjunto do plano limitado pela reta $y=0$ e pelo gráfico de $y=3-2x-x^2$, com $-1\leqslant x\leqslant 2$.
9. A é o conjunto do plano limitado pelo eixo $0x$, pelo gráfico de $y=x^3-x$, $-1\leqslant x\leqslant 1$.
12. A é o conjunto de todos (x, y) tais que $x\geqslant 0$ e $x^3\leqslant y\leqslant x$.
13. A é o conjunto do plano limitado pela reta $y=x$, pelo gráfico de $y=x^3$, com $-1\leqslant x\leqslant 1$.
15. A é o conjunto do plano limitado pelas retas $x=0$, $x=\dfrac{\pi}{2}$ e pelos gráficos de $y=\operatorname{sen} x$ e $y=\cos x$.
17. A é o conjunto de todos os pontos (x, y) tais que $x^2-1\leqslant y\leqslant x+1$.

18. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \frac{\pi}{2}$ e pelos gráficos de $y = \cos x$ e $y = 1 - \cos x$.

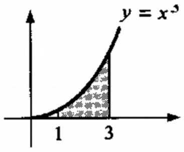
19. $A = \{ (x, y) \in \mathbb{R}^2 \mid x \geqslant 0 \text{ e } x^3 - x \leqslant y \leqslant -x^2 + 5x \}$.

21. A é o conjunto de todos os pontos (x, y) tais que $x \geqslant 0$ e $-x \leqslant y \leqslant x - x^2$

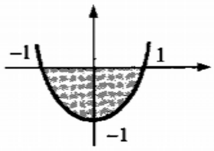
22. A é o conjunto de todos (x, y) tais que $x > 0$ e $\frac{1}{x^2} \leqslant y \leqslant 5 - 4x^2$.

Respostas:

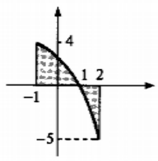
1. Área $= 20$



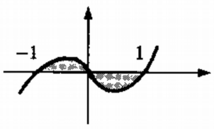
3. Área $= \frac{4}{3}$



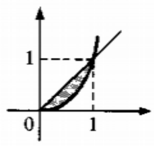
7. Área $= \frac{23}{3}$



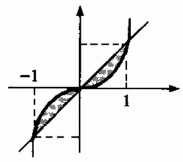
9. Área $= \frac{1}{2}$



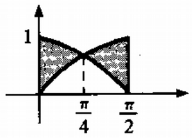
12. Área $= \frac{1}{4}$



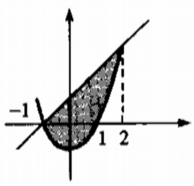
13. Área $= \frac{1}{2}$



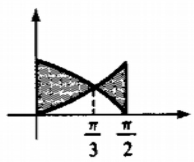
15. Área $= 2(\sqrt{2} - 1)$



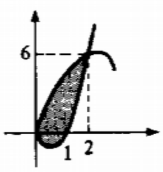
17. Área $= \frac{9}{2}$



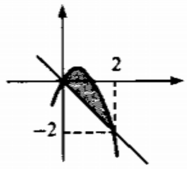
18. Área $= \frac{1}{6}(12\sqrt{3} - \pi - 12)$



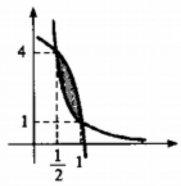
19. Área $= \frac{16}{3}$



21. Área $= \frac{4}{3}$



22. Área $= \frac{1}{3}$



Exercício 4 (Integração por frações parciais) Calcule:

1. $\int \frac{dx}{x^2 - 4}$

3. $\int \frac{5x - 2}{x^2 - 4}\, dx$

5. $\int \frac{4w - 11}{2w^2 + 7w - 4}\, dw$

7. $\int \frac{6x^2 - 2x - 1}{4x^3 - x}\, dx$

9. $\int \frac{dx}{x^3 + 3x^2}$

11. $\int \frac{dx}{x^2(x + 1)^2}$

13. $\int \frac{x^2 - 3x - 7}{(2x + 3)(x + 1)^2}\, dx$

15. $\int \frac{3z + 1}{(z^2 - 4)^2}\, dz$

Respostas:

1. $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$ **3.** $\ln|C(x-2)^2(x+2)^3|$ **5.** $\ln\left|\frac{C(w+4)^3}{2w-1}\right|$ **7.** $\frac{1}{4}\ln\left|\frac{Cx^4(2x+1)^3}{2x-1}\right|$ **9.** $\frac{1}{9}\ln\left|\frac{x+3}{x}\right| - \frac{1}{3x} + C$

11. $2\ln\left|\frac{x+1}{x}\right| - \frac{1}{x} - \frac{1}{x+1} + C$ **13.** $\frac{3}{x+1} + \ln|x+1| - \frac{1}{2}\ln|2x+3| + C$

15. $\frac{5}{16(z+2)} - \frac{7}{16(z-2)} + \frac{1}{32}\ln\left|\frac{z+2}{z-2}\right| + C$ **17.** $\frac{1}{2}x^2 + 2x - \frac{3}{x-1} - \ln|x^2+2x-3| + C$

Exercício 5 (Integral definida) Calcule:

63. $\int_0^{\pi} \sqrt{2 + 2\cos x}\,dx$

65. $\int_1^2 \frac{2x^2+x+4}{x^3+4x^2}\,dx$

67. $\int_0^2 \frac{t^3\,dt}{\sqrt{4+t^2}}$

69. $\int_{-2}^{2\sqrt{3}} \frac{x^2\,dx}{(16-x^2)^{3/2}}$

71. $\int_0^{\pi/4} \sec^4 x\,dx$

81. $\int_1^2 \frac{x+2}{(x+1)^2}\,dx$

83. $\int_0^{\pi} |\cos^3 x|\,dx$

87. $\int_0^{1/2} \frac{x\,dx}{\sqrt{1-4x^4}}$

Respostas:

61. $\begin{cases} \frac{x^{n+1}\ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C & \text{se } n \neq -1 \\ \frac{1}{2}\ln^2 x + C & \text{se } n = -1 \end{cases}$ **63.** 4 **65.** $\frac{1}{2} + 2\ln\frac{6}{5}$ **67.** $\frac{16}{3} - \frac{8}{3}\sqrt{2}$ **69.** $\frac{4}{3}\sqrt{3} - \frac{1}{2}\pi$ **71.** $\frac{4}{3}$

73. $\frac{1}{2} - \frac{1}{4}\ln 2$ **75.** $-a^2(\frac{9}{8}\sqrt{3} - \frac{1}{2}\pi)$ **77.** $\frac{1}{2}\ln\frac{9}{2} - \frac{1}{6}\pi$ **79.** 5 **81.** $\frac{1}{6} + \ln\frac{3}{2}$ **83.** $\frac{4}{3}$ **85.** $1 - \frac{1}{2}\ln 3$ **87.** $\frac{1}{24}\pi$

89. $\sqrt{3} - \frac{1}{2}\ln(2+\sqrt{3})$ **91.** $\frac{1}{5}\ln\frac{3}{2}$ **93.** $\frac{256}{15}$ **95.** $\frac{1}{3}k(1 - e^{-9})$ kg; $\frac{e^9 - 10}{3(e^9 - 1)}$ m de um extremo

Exercício 6: Nos itens 1, 2,3 e 4 a seguir, esboce o gráfico da função f a partir das informações dadas.

1. (a) $D(f) = [2,+\infty) \cup (-\infty,-2]$
   (b) $f(2) = f(-2) = 0$
   (c) $f'(x) > 0$ para $x \in (2,+\infty)$
   (d) $f'(x) < 0$ para $x \in (-2,-\infty)$
   (e) $f''(x) < 0$ para $x \in (2,+\infty)$
   (f) $f''(x) < 0$ para $x \in (-2,-\infty)$
   (g) $\lim_{x\to+\infty}(f(x) - 5x) = 0$
   (h) $\lim_{x\to-\infty}(f(x) + x) = 0$

2. (a) $D(f) = \mathbb{R} - \{-1\}$
   (b) $f(0) = 0, f'(-2) = 0, f'(0) = 0$ e $f(-2) = -5$
   (c) $f'(x) > 0 \quad \forall x \in (-\infty,-2) \cup (0,+\infty)$
   (d) $f'(x) < 0 \quad \forall x \in (-2,-1) \cup (-1,0)$
   (e) $f''(x) < 0$ para $x \in (-\infty,-1)$
   (f) $f''(x) > 0$ para $x \in (-1,+\infty)$
   (g) $\lim_{x\to-1^-} f(x) = -\infty$
   (h) $\lim_{x\to-1^+} f(x) = +\infty$
   (i) $\lim_{x\to+\infty}[f(x) - (x-1)] = 0$
   (j) $\lim_{x\to-\infty}[f(x) - (x-1)] = 0$

3. (a) $D(f) = \mathbb{R} - \{0\}$

   (b) $f(-2) = 0, f'(1) = 0, f''(3) = 0, f(1) = 1$ e $f(3) = 2$

   (c) $f'(x) > 0 \quad \forall x \in (-\infty, 0) \cup (1, +\infty)$

   (d) $f'(x) < 0 \quad \forall x \in (0, 1)$

   (e) $f''(x) < 0$ para $x \in (3, +\infty)$

   (f) $f''(x) > 0$ para $x \in (-\infty, 0) \cup (0, 3)$

   (g) $\lim\limits_{x \to 0} f(x) = +\infty$

   (h) $\lim\limits_{x \to +\infty} [f(x) - x] = 0$

   (i) $\lim\limits_{x \to -\infty} [f(x) - x] = 0$

4. (a) $D(f) = \{x \in \mathbb{R} : x \neq -0.5$ e $x \neq 2\}$

   (b) $f(0) = 0, f'(-4) = 0, f'(0) = 0, f(1) = 1$ e $f(-4) = 0.5$

   (c) $f'(x) > 0 \quad \forall x \in (-4, -0.5) \cup (-0.5, 0)$

   (d) $f'(x) < 0 \quad \forall x \in (-\infty, -4) \cup (0, 2) \cup (2, +\infty)$

   (e) $\lim\limits_{x \to -1^-} f(x) = +\infty$

   (f) $\lim\limits_{x \to -1^+} f(x) = -\infty$

   (g) $\lim\limits_{x \to 2^-} f(x) = -\infty$

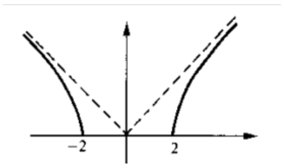
   (h) $\lim\limits_{x \to 2^+} f(x) = +\infty$

   (i) $\lim\limits_{x \to +\infty} f(x) = 1$

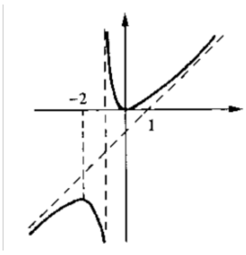
   (j) $\lim\limits_{x \to -\infty} f(x) = 1$

Respostas:

1.



2.



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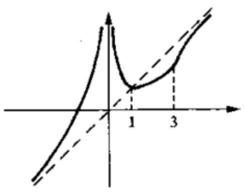

4.