GABARITO DA P3

① (a) $\int 3x\,(2x^2-4)^4\,dx = \frac{3}{4}\int u^4\,du = \frac{3}{4}\frac{u^5}{5}+C = \frac{3}{20}(2x^2-4)^5+C$

$\begin{cases} u=2x^2-4 \\ du = 4x\,dx \end{cases} \rightarrow x\,dx = \frac{1}{4}du$

(b) $\int \underset{f}{x}\,\underset{g'}{e^{2x}}\,dx = x\cdot\frac{e^{2x}}{2} - \int 1\cdot\frac{e^{2x}}{2}dx = \frac{x\,e^{2x}}{2} - \frac{e^{2x}}{4} + C = \frac{e^{2x}}{4}(2x-1)+C$ //

$f(x)=x \qquad g'(x)=e^{2x}$

$f'(x)=1 \qquad g(x)=\frac{e^{2x}}{2}$

(c) $\int \underset{f}{\cos(\pi x)}\,\underset{g'}{e^x}\,dx = \cos\pi x\; e^x + \int \pi\,\operatorname{sen}\pi x\; e^x\,dx$

$= \cos\pi x\cdot e^x + \pi\left[\operatorname{sen}\pi x\, e^x - \int \pi\cdot\cos(\pi x)\,e^x\,dx\right]$

$= \cos\pi x\; e^x + \pi\,\operatorname{sen}\pi x\; e^x - \pi^2\int\cos(\pi x)\,e^x\,dx$

$\Rightarrow (1+\pi^2)\int\cos\pi x\; e^x\,dx = (\cos\pi x + \pi\,\operatorname{sen}\pi x)\,e^x$

$\Rightarrow \int\cos\pi x\; e^x\,dx = \frac{1}{1+\pi^2}(\cos\pi x + \pi\,\operatorname{sen}\pi x)\,e^x$

(d) $\int \frac{3x+1}{4+x^2}dx = \int\frac{3x}{4+x^2}dx + \int\frac{dx}{4+x^2}$

$\begin{cases} u=4+x^2 \\ du=2x\,dx \end{cases}$ $\qquad\qquad$ $\begin{cases} v = x/2 \\ dv = \frac{1}{2}dx \end{cases} \rightarrow dx = 2dv$

$= \frac{3}{2}\int\frac{1}{u}du + \int\frac{dx}{4(1+(x/2)^2)}$

$= \frac{3}{2}\ln|u| + \frac{1}{4}\int\frac{2\,dv}{1+v^2}$

$= \frac{3}{2}\ln(4+x^2) + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+C$

aqui usamos que $\int\frac{dv}{1+v^2} = \operatorname{arctg}(v)+C$

(2) Sabemos que se $G(x) = \int_{g(x)}^{f(x)} h(t)\,dt \Rightarrow G'(x) = h(f(x)) \cdot f'(x) - h(g(x))g'(x)$

(a) $F'(x) = \operatorname{sen}^6 x \; f(\operatorname{sen}^3 x) \cdot 3\operatorname{sen}^2 x \cdot \cos x - x^2 f(x)$.

(b) $\circledast \lim_{x \to 0} \dfrac{\int_0^{x^2} e^{-t^2} dt}{x^2}$ é do tipo "$\frac{0}{0}$" $\therefore$ Pode-se aplicar

a regra de L'Hôspital

$$\circledast = \lim_{x \to 0} \frac{e^{-x^4} \cdot \cancel{2x}}{\cancel{2x}} = \lim_{x \to 0} e^{-x^4} = 1$$

(3) (a) $\int_{-\pi}^{\pi} \dfrac{\operatorname{sen} x}{3 + x^2} dx = 0$ pois a função $f(x) = \dfrac{\operatorname{sen}(x)}{x^2 + 3}$

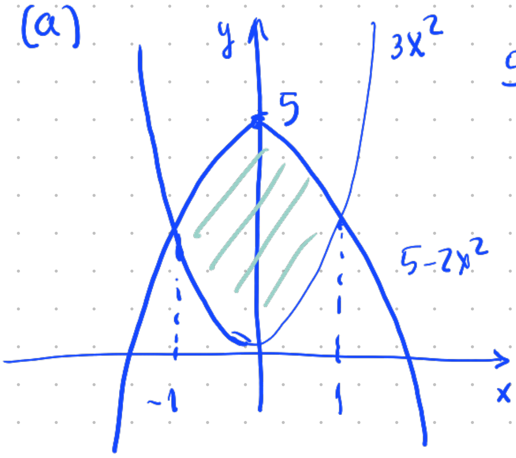
é ímpar, isto é: $f(-x) = -f(x)$ e o intervalo é simétrico

(b) (i) $-2\int_0^1 f(x)\,dx + 4\int_0^1 g(x)\,dx = (-2) \cdot 5 + 4 \cdot 3 = 2.$

(ii) $\int_0^1 x\, f(1 - x^2)\,dx = -\int_1^0 f(u) \dfrac{du}{2} = \dfrac{1}{2}\int_0^1 f(u)\,du = \dfrac{5}{2}.$

$$\begin{cases} u = 1 - x^2 \\ du = -2x\,dx \end{cases}$$

(4) (a)



$5 - 2x^2 = 3x^2$

$5x^2 = 5$

$x^2 = 1$

$x = \pm 1$

$$A = \int_{-1}^{1} 5 - 2x^2 - 3x^2\,dx$$

$$= \int_{-1}^{1} 5 - 5x^2\,dx$$

$$= 2\int_0^1 5 - 5x^2\,dx$$

$$= 2\left(5x - \frac{5}{3}x^3\right)\Big|_0^1 = 2\left(5 - \frac{5}{3}\right)$$

$$= \frac{20}{3}$$

(b) $\text{Area} = \int_0^1 x^2+1 \, dx + \int_1^2 \frac{2}{x} - (x-1) \, dx$

$$= \frac{x^3}{3} + x \Big|_0^1 + \left(2 \ln x - \frac{x^2}{2} + x\right)\Big|_1^2$$

$$= \frac{1}{3} + 1 + \left(2 \ln 2 - \overset{-2}{\frac{\cancel{2^2}}{\cancel{2}}} + 2 - \underbrace{2 \ln 1}_{=0} - \frac{1}{2} \cancel{+1}\right)$$

$$= \frac{4}{3} + 2 \ln 2 - \frac{1}{2} = \frac{5}{6} + 2 \ln 2 \quad //$$