Sistemas de Controle de Processos Contínuos
2023

Sistemas de 2ª ordem - Resposta Transitória × em Frequência

Engenharia de Automação Industrial

CEFET-MG / Campus Araxá
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Sistemas de 2ª ordem

Função de transferência

$$G = \frac{c}{as^2 + bs + c}$$

Pólos:

$$p_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} \quad ; \Delta = b^2 - 4ac$$

Δ>0 → Sistema sobreamortecido

$$G = \frac{p_1 p_2}{(s - p_1)(s - p_2)}$$

Δ=0 → Sistema amortecimento crítico

$$G = \frac{p^2}{(s - p)^2}$$

Δ<0 → Sistema subamortecido

$$G = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}$$

Sistemas de 2ª ordem

Função de transferência

$$G = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Pólos:

$$p_{1,2} = -\delta \pm \sqrt{\Delta} \qquad \begin{cases} \delta = \zeta\omega_n \\ \Delta = {\omega_n}^2\left(\zeta^2 - 1\right) \end{cases}$$

Δ>0 → ζ>1→ Sistema sobreamortecido $p_1 \neq p_2$

Δ=0 → ζ=1→ Amortecimento crítico $p_1 = p_2$

Δ<0 → ζ<1→ Sistema subamortecido $p_{1,2} = -\delta \pm j\omega_d$

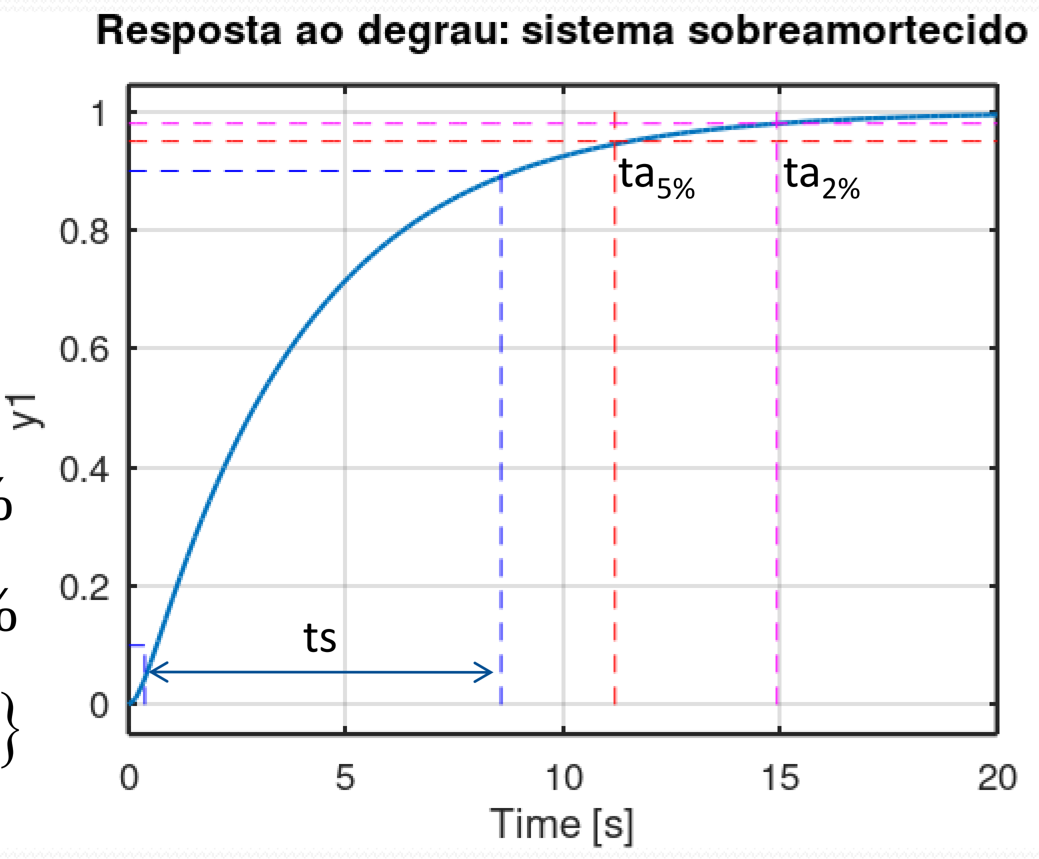
$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

Sistemas de 2ª ordem

Sistema sobreamortecido (Δ>0) (ζ>1)

$$G = \frac{p_1 p_2}{(s-p_1)(s-p_2)} \qquad G = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$\tau_1 = \frac{1}{|p_1|} \qquad \tau_2 = \frac{1}{|p_2|}$$



$$t_a = 3\,\tau_{maior} \quad \textit{critério de } 5\%$$

$$t_a = 4\,\tau_{maior} \quad \textit{critério de } 2\%$$

$$t_s = 2{,}2\,\tau_{maior} \quad \{10\% \ a \ 90\%\}$$

Sistemas de 2ª ordem

Sistema subamortecido (Δ<0) **(ζ<1)**

$$G=\frac{\omega_n{}^2}{s^2+2\zeta\omega_n s+\omega_n{}^2}$$

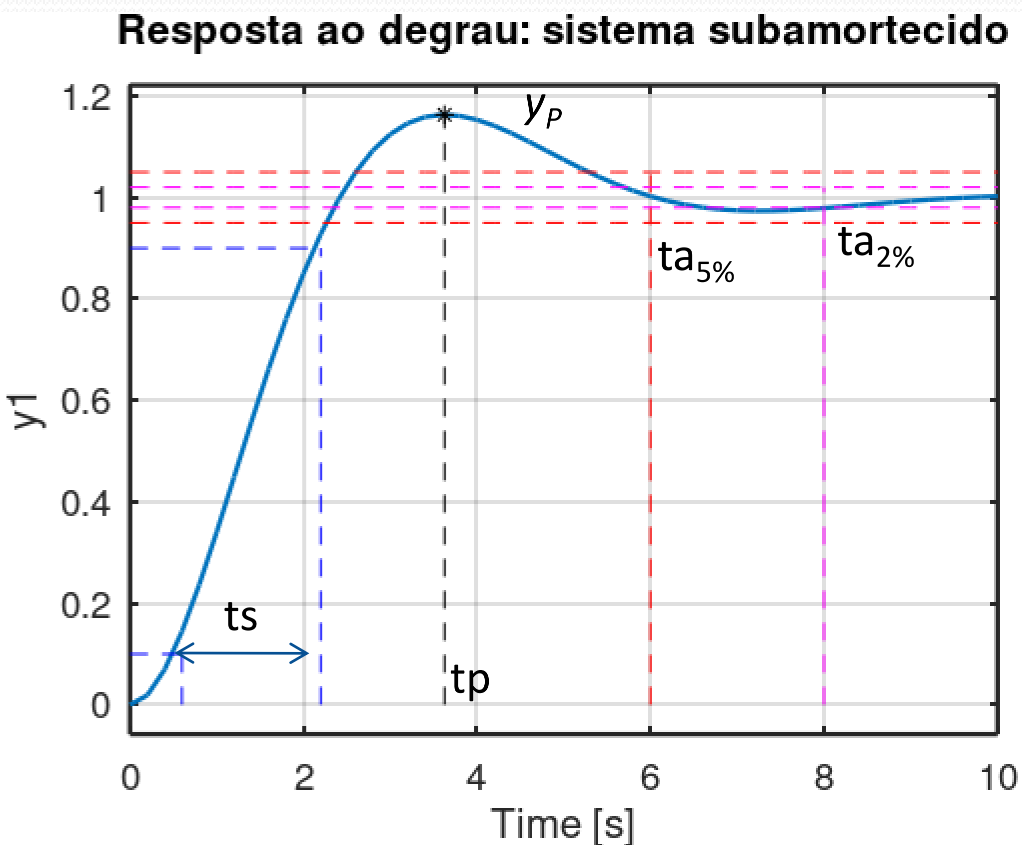
$$p_{1,2}=-\delta\pm j\omega_d \quad \begin{cases}\delta=\zeta\omega_n\\ \omega_d=\omega_n\sqrt{1-\zeta^2}\end{cases}$$

$$M_p=e^{\frac{-\pi\delta}{\omega_d}}$$

$$t_p=\frac{\pi}{\omega_d} \qquad y_p=1+M_p$$

$$t_a=\frac{3}{\delta} \quad \textit{critério de } 5\%$$

$$t_a=\frac{4}{\delta} \quad \textit{critério de } 2\%$$



Sistemas de 2ª ordem

Sistema subamortecido (Δ<0) **(ζ<1)**

$$|G| = \frac{\omega_n^{\ 2}}{\sqrt{\left(\omega_n^{\ 2} - \omega^2\right)^2 + \left(2\zeta\omega_n\right)^2}}$$

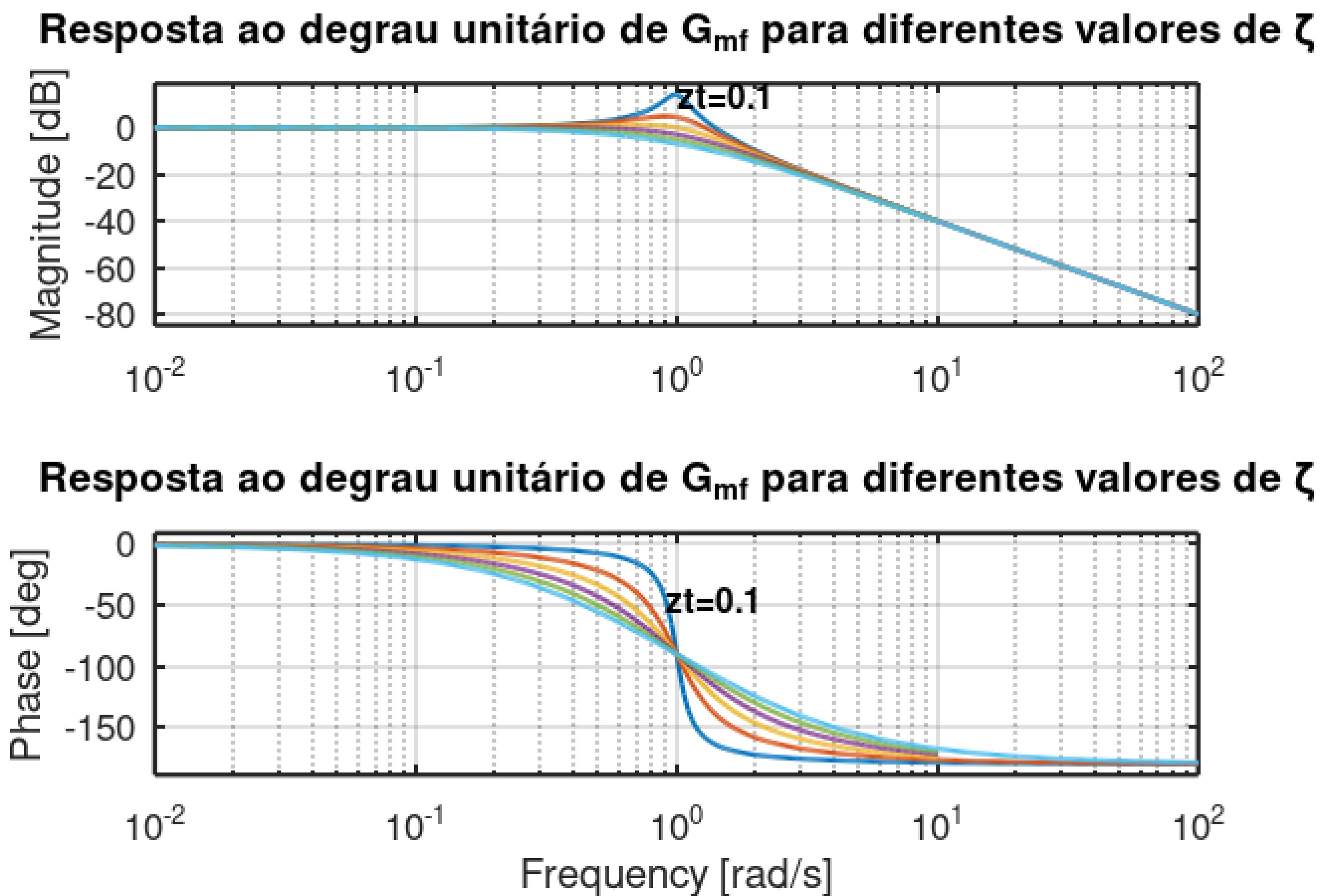
Existe pico p/ $\zeta < \frac{1}{\sqrt{2}}$

$$M_{pico} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$|G|_{pico} = 20\log M_{pico}$$

$$\omega_{pico} = \omega_n\sqrt{1-2\zeta^2}$$

Obs.: $M_{pico} \neq M_p$



Sistemas de 2ª ordem

Sistema subamortecido (Δ<0) (ζ<1)

<u>Largura de Banda versus Velocidade</u>

$$\omega_{BW} = \omega_n \sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}} \qquad \omega_n = \frac{4}{t_s\zeta} \qquad \omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

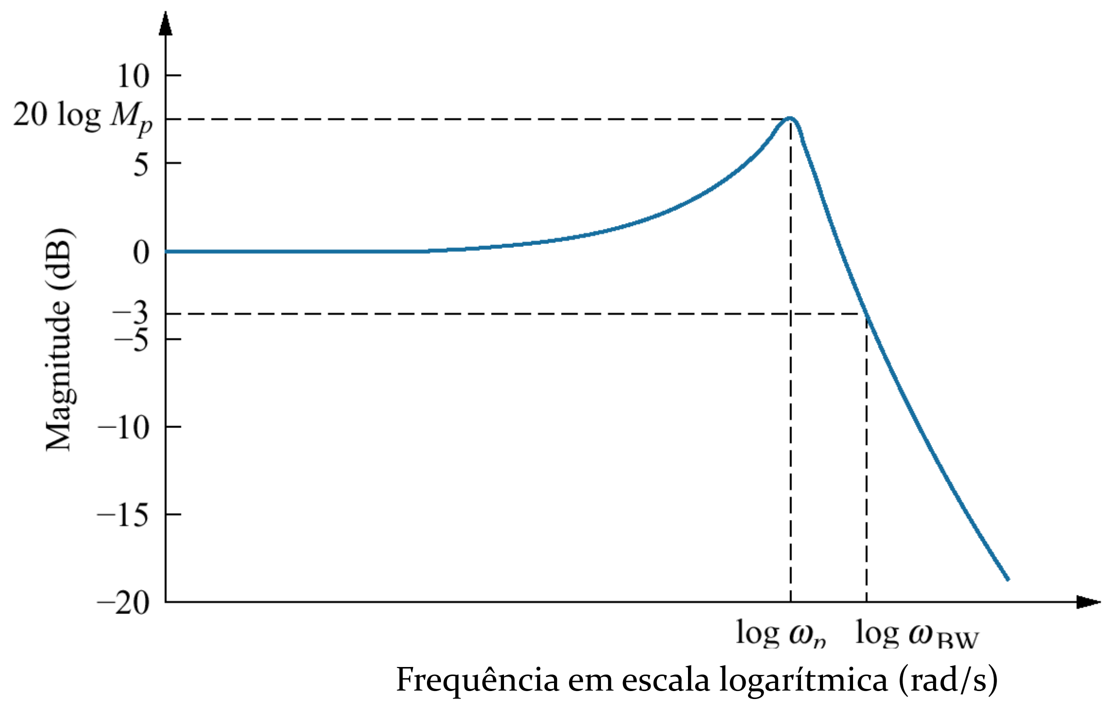
Existe M_{pico} para $\zeta < \frac{1}{\sqrt{2}}$

$$M_{pico} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$|G|_{pico} = 20\log M_{pico}$$

$$\omega_{pico} = \omega_n\sqrt{1-2\zeta^2}$$

Obs.: $M_{pico} \neq M_p$



Sistemas de 2ª ordem

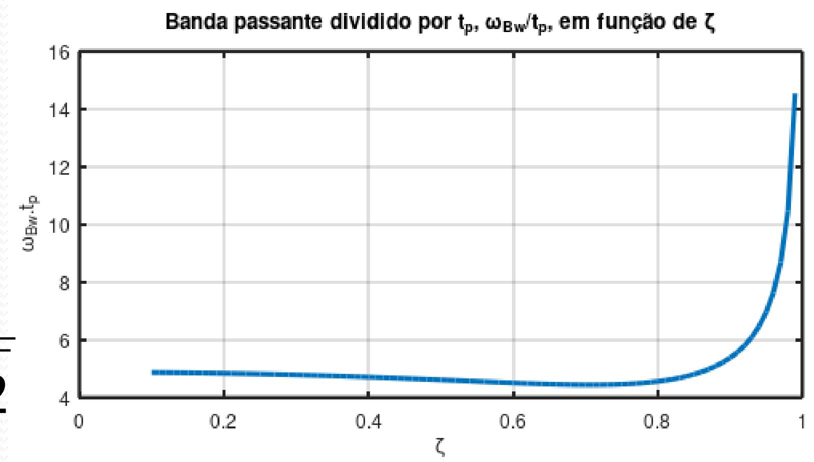
Sistema subamortecido (Δ<0) (ζ<1)

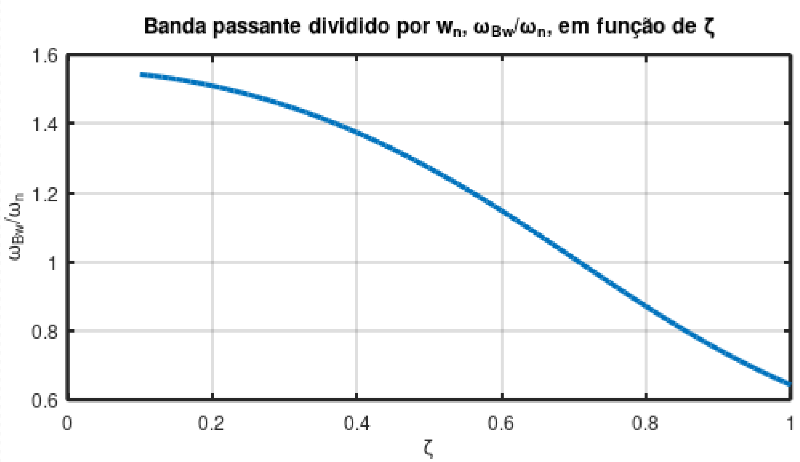
<u>Largura de Banda versus Velocidade</u>

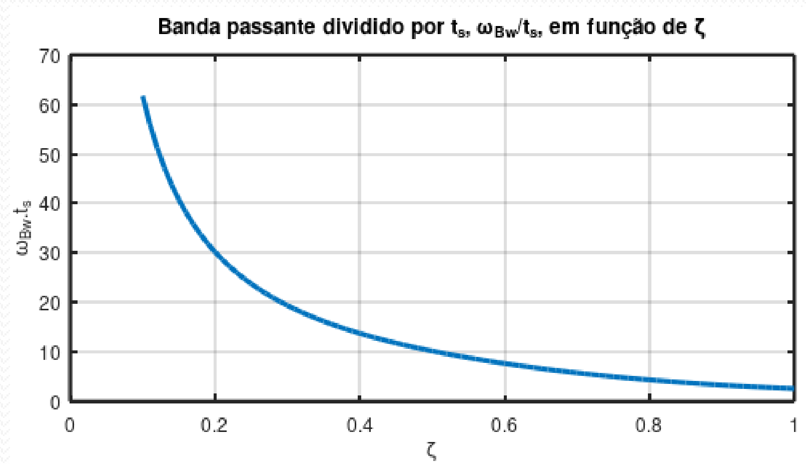
$$\frac{\omega_{BW}}{\omega_n} = \sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}}$$

$$\omega_{BW} t_s = \frac{4}{\zeta}\sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}}$$

$$\omega_{BW} t_p = \frac{\pi}{\sqrt{1-\zeta^2}}\sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}}$$







Sistemas de 2ª ordem

Sistema subamortecido (Δ<0) (ζ<1)

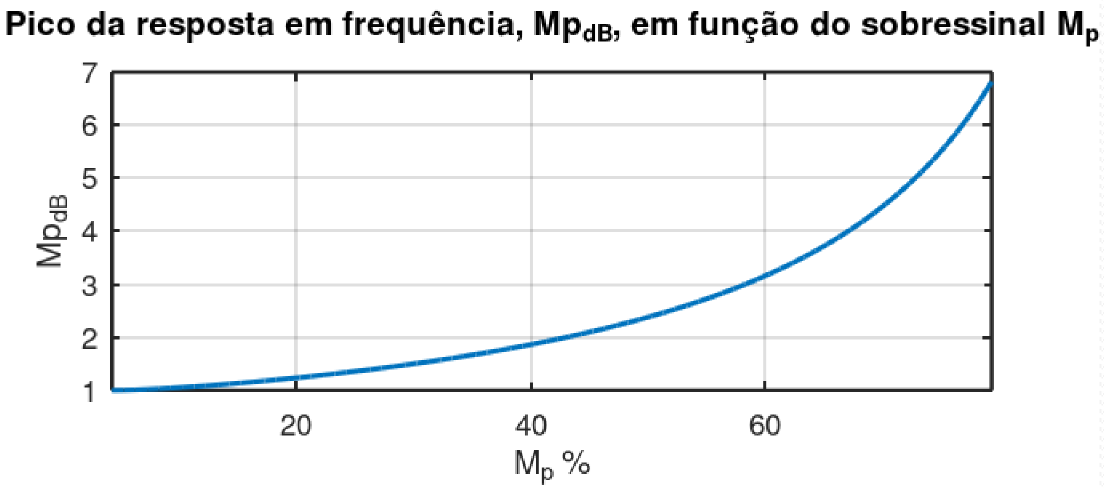
Existe pico p/ $\zeta < \frac{1}{\sqrt{2}}$

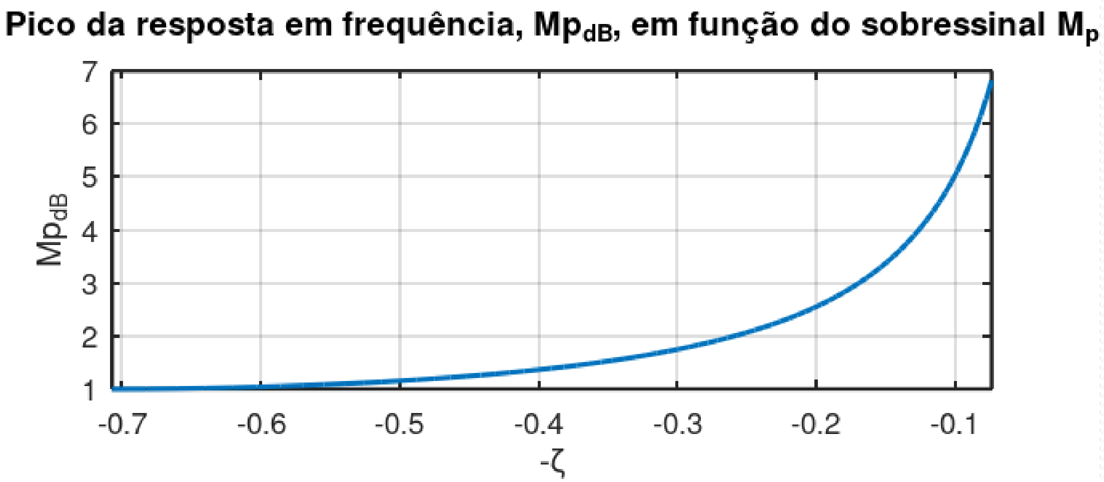
$$M_{pico} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$|G|_{pico} = 20\log M_{pico}$$

$$\omega_{pico} = \omega_n\sqrt{1-2\zeta^2}$$

Obs.: $M_{pico} \neq M_p$

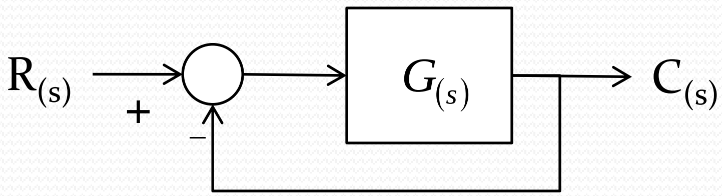




Sistemas de 2ª ordem

Sistema subamortecido (Δ<0) **(ζ<1)**

Relação de amortecimento a partir da margem de fase:



$$G = \frac{\omega_n^2}{s\left(s + 2\zeta\omega_n\right)}$$

$$G = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\left|G_{(j\omega)}\right| = \frac{\omega_n^2}{\left|-\omega^2 + j\omega 2\zeta\omega_n\right|} = 1 \quad \rightarrow \quad \omega_1 = \omega_n\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}$$

$$\rightarrow \quad \angle G_{(j\omega)} = -90 - tg^{-1}\left(\frac{\omega_1}{2\zeta\omega_n}\right) \quad \rightarrow \quad \angle G_{(j\omega)} = -90 - tg^{-1}\left(\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta}\right)$$

$$\Phi_M = \angle G_{(j\omega)} - 180 = 90 - tg^{-1}\left(\frac{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}{2\zeta}\right) \quad \rightarrow \quad \boxed{\Phi_M = tg^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}\right)}$$