Math Quiz for STAT 131

1 Introduction

- Familiarity to the following concepts is essential for your success in this course:

  Function, limit, continuity, differentiation (e.g. product rule, quotient rule and chain rule), integration, integration techniques such as integration by parts and by substitution, function of two variables, double integrals, infinite series, maxima and minima of functions.

- Take this quiz by yourself, preferably before classes start and no later than by our second class to help you determine whether you have the necessary mathematical background. It should take no more than an hour.

2 Questions

1. (Square completion) Express $4x^2+8x+7$ in the form $a(x+b)^2+c$, where a, b, c are constants.

2. (Operation of log) Express the following expressions in terms of $\log(x)$ and $\log(y)$:

   (a) $\log(x^2)$

   (b) $\log(x/y)$

3. (Geometric series) Evaluate

   (a) $\sum_{i=0}^{n} \frac{1}{3^i}$

   (b) $\sum_{i=0}^{\infty} \frac{1}{3^i}$

4. Use integration by parts to evaluate $\int_0^1 xe^x dx$. $\int u\, dv = uv - \int v\, du$

5. Use integration by substitution to evaluate $\int_0^1 xe^{x^2} dx$.

6. Evaluate $\int\int_{\{(x,y):x^2+y^2\leq 1\}} dxdy$.

7. Differentiate with respect to x

   (a) e^{-x^2}

   (b) $\log(x^5)$

   (c) x^2e^{-x} $g(t)=\int_0^t ye^{-y}dy = G(t)-G(0)$, $G'(x)=xe^{-x}$

   (d) $\int_0^{x^2} \frac{y}{e^y} dy$ for some $x > 0$. $= g(x^2)$

8. Sketch rough plots for the following functions on a $x-y$ plane,

   (a) $f(x) = e^{-x}$, $x \in \mathbb{R}$

   (b) $f(x) = x^2$, $x \in \mathbb{R}$

   (c) $f(x) = e^{-x^2}$, $x \in \mathbb{R}$

9. Evaluate the determinant and find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

1. (Square completion) Express $4x^2+8x+7$ in the form $a(x+b)^2+c$, where a, b, c are constants.

$$4x^2+8x+7$$
$$\longleftrightarrow 4[x^2+2x]+7$$
$$\longleftrightarrow 4[x^2+2x+1]+7-4$$
$$\longleftrightarrow 4[x+1]^2+3$$

2. (Operation of log) Express the following expressions in terms of $\log(x)$ and $\log(y)$:

(a) $\log(x^2) = 2\log(x)$

(b) $\log(x/y) = \log(x\,y^{-1}) = \log(x) + \log(y^{-1}) = \log(x) - \log(y)$

3. (Geometric series) Evaluate

(a) $\sum_{i=0}^{n} \frac{1}{3^i} = \sum_{i=0}^{n} \left(\frac{1}{3}\right)^i = \frac{1-(1/3)^{n-1}}{1-(1/3)}$

(b) $\sum_{i=0}^{\infty} \frac{1}{3^i} = \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i = \frac{1}{1-(1/3)}$

c) $\sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i = \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i - 1 = \frac{1}{1-(1/2)} - 1$

$$S_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$\sum_{i=1}^{17} [e^{i\pi} \cdot \cos(i)]^i = \sum_{i=0}^{17} [e^{i\pi} \cdot \cos(i)]^i - 1$$

$$\sum_{i=1}^{n} (1/2)^i = \sum_{i=0}^{n} (1/2)^i - 1$$

4. Use integration by parts to evaluate $\int_0^1 xe^x dx$.

$$\int u\, dv = uv - \int v\, du$$

$$u = x \rightarrow du = 1$$
$$v = e^x \leftarrow dv = e^x$$

$$\int_0^1 xe^x = uv\Big|_0^1 - \int_0^1 v\,du$$
$$= xe^x\Big|_0^1 - \int_0^1 e^x \cdot 1\, dx$$
$$= (e-0) - (e-1)$$
$$= 1$$

5. Use integration by substitution to evaluate $\int_0^1 xe^{x^2} dx$.

$$u = x^2$$
$$du = 2x\,dx \leftrightarrow \tfrac{1}{2}du = x\,dx$$

$$\int_0^1 xe^{x^2} dx = \int_0^1 e^{x^2} x\, dx = \int_0^1 e^u \tfrac{1}{2} du = \tfrac{1}{2} e^u\Big|_0^1 = \tfrac{1}{2}[e-1]$$

6. Evaluate $\cancel{\int\int_{\{(x,y):x^2+y^2\leq 1\}} dxdy}$.

$$\int_{\{(x,y):x^2+y^2\leq 1\}} dx = \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx = x\Big|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} = 2\sqrt{1-y^2}$$

$$x^2 \leq 1-y^2$$
$$-\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}$$

7. Differentiate with respect to x

(a) e^{-x^2} $= \exp(-x^2) \longrightarrow \frac{d}{dx}\exp(-x^2) = \exp(-x^2)(-2x) = -e^{-x^2}2x$

(b) $\log(x^5)$ $\longrightarrow \frac{d}{dx}\log(x^5) = \frac{1}{x^5}\cdot 5x^4 = 5\frac{1}{x}$

(c) x^2e^{-x} $\rightarrow 2xe^{-x} + x(-e^{x})$

(d) $\int_0^{x^2} \frac{y}{e^y}dy$ for some $x > 0$.

$$\int_0^{x^2} ye^{-y}dy = \int_0^{x^2} g(y)dy = G(y)\Big|_0^{x^2} = G(x^2) - G(0)$$

$$\frac{d}{dx}\int_0^{x^2} ye^{-y}dy = \frac{d}{dx}G(x^2) - G(0) = \frac{d}{dx}G(x^2) = g(x^2)\cdot 2x = x^2e^{-x^2}\cdot 2x$$

$$\frac{d}{dx}\int_a^{h(x)} g(y)dy = g(h(x))\cdot h'(x)$$

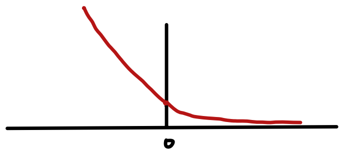
8. Sketch rough plots for the following functions on a $x - y$ plane,

(a) $f(x) = e^{-x}$, $x \in \mathbb{R}$

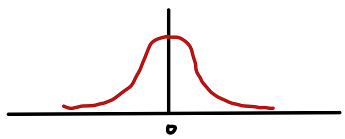
(b) $f(x) = x^2$, $x \in \mathbb{R}$

(c) $f(x) = e^{-x^2}$, $x \in \mathbb{R}$

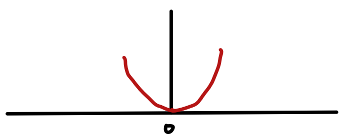
a)



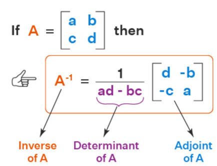
c)



b)



9. Evaluate the determinant and find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$



$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{1}{1\cdot 4 - 2\cdot 3}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= -0.5\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$