How to draw a $K(n,2)$ Kneser graph?†

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Take a 2-page drawing $D(K_{2\lceil \frac{n}{2}\rceil -1})$ of the complete graph $K_{2\lceil \frac{n}{2}\rceil -1}$ from algorithm (de Klerk, Pasechnik and Salazar (2013)) (a), (b) and (c). Replace each vertex of $K_{2\lceil \frac{n}{2}\rceil -1}$ by $q = \lceil \frac{n-1}{2} \rceil$ vertices corresponding to clique $C_i, i \in \{1,2,\ldots,2\lceil \frac{n}{2}\rceil -1\}$ with the order of the Hamiltonian cycle from algorithm (Berge (1973)). Add the edges between the pair of vertices of each 2 cliques according to the geometric position of the $D(K_{2\lceil \frac{n}{2}\rceil -1})$ edges. Place the 1-page drawing of $K_{\lceil \frac{n-1}{2}\rceil}$ from (de Klerk, Pasechnik and Salazar (2013)) for each clique C_i on the half-plane with the fewest outgoing edges of the vertex C_i of $D(K_{2\lceil \frac{n}{2}\rceil -1})$ (d).

Let $\nu(G)$ and $\nu_2(G)$ be the minimum number of crossings for a drawing $D(G)$ of G, respectively, in the plane, and into a 2-page drawing, we prove that $\frac{n^8}{2^{13}} - 9\frac{n^7}{2^{13}} - \frac{n^6}{2^{10}} - \frac{n^4}{2^7} - \frac{n^3}{2^9} \le \nu(K(n,2)) \le \nu_2(K(n,2)) \le \frac{n^8}{2^{10}} - \frac{3n^7}{2^8} + \frac{31n^6}{2^8 3} + \frac{7n^5}{2^6} - \frac{563n^4}{2^7 3} + \frac{517n^3}{2^5 3} - \frac{267n^2}{2^5} + \frac{107n}{2^3 3}$.

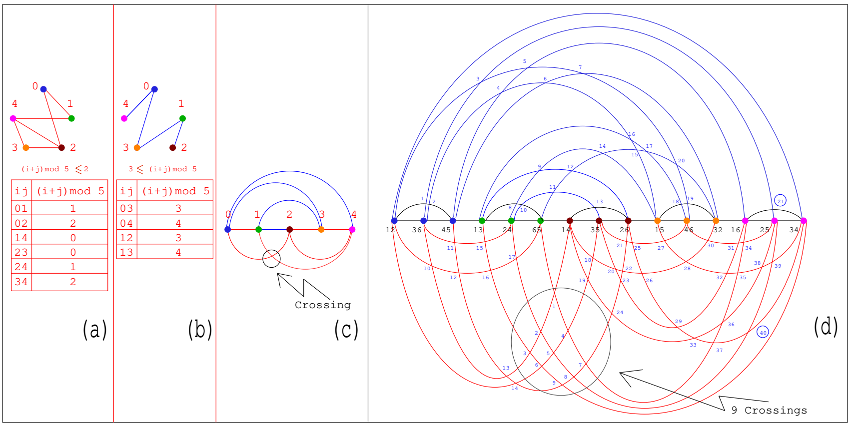

Figure 1 2-page drawing construction of K_5 in (a) and (b), and 2-page drawings of K_5 in (c) and $K(6,2)$ in (d).

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