Algebră - 1

Def (Inel) $(R,+,\cdot)$ R mult, $+$ și $\cdot$ sunt operații pe R
$\circ: R\times R \to R$ o operație

1) $(R,+)$ grup comutativ — 0 - elem neutru pt +
2) $(R,\cdot)$ monoid — 1 - elem neutru pt $\cdot$
3) $a\cdot(b+c)=a\cdot b+a\cdot c \quad \forall a,b,c\in R$
$(a+b)\cdot c = a\cdot c+b\cdot c$

Reguli de calcul în inel

$0\cdot r = r\cdot 0 = 0$

$0\cdot r = (0+0)\cdot r = 0\cdot r + 0\cdot r$

$\exists r_1 \in R$ a.î. $r_1 + 0\cdot r = 0$

$\underbrace{r_1 + 0\cdot r}_{0} + 0\cdot r = 0 \Rightarrow 0\cdot r = 0$

$(a+b)^2 = (a+b)(a+b) = a^2+ab+ba+b^2$

$\overset{3)}{=} (a+b)\cdot a + (a+b)\cdot b$

Def $(R,+,\cdot)$ s.n. comutativ dacă $a\cdot b = b\cdot a$ $\forall a,b\in R$

$(\mathbb{Q},+,\cdot)$; $(\mathbb{R},+,\cdot)$
$(\mathbb{Z},+,\cdot)$; $\boxed{(\mathbb{C},+,\cdot)}$

Def $(K,+,\cdot)$ se numește corp dacă $(K,+,\cdot)$ este inel și $\forall x\in K$, $x\neq 0 \Rightarrow \exists y\in K$ a.î. $x\cdot y = y\cdot x =$

Ex $(\mathbb{Z}_n,+,\cdot)$ corp $\Leftrightarrow$ n prim

Ex Cele de mai sus

Corpul numerelor complexe

$e^{ix} = \cos x + i\sin x$

$\mathbb{C} = \{(a,b) \mid a,b\in\mathbb{R}\}$

$(a,b)+(c,d) = (a+c, b+d) \quad (a,b)\cdot(c,d) = (ac+bd, ad+bc)$

$a,b,c,d\in\mathbb{R}$

$a+ib = c+id \Leftrightarrow \begin{cases} a=c \\ b=d \end{cases}$

$(a+ib)+(c+id) = a+c+i(b+d)$

$(a+ib)(c+id) = ac+bd+i(ad+bc)$

$z \in \mathbb{C}^* \Rightarrow z = r(\cos\theta + i\sin\theta)$

ai $\begin{cases} \cos\theta = \frac{a}{\sqrt{a^2+b^2}} \\ \sin\theta = \frac{b}{\sqrt{a^2+b^2}} \end{cases}$ $\quad r = |z| = \sqrt{a^2+b^2}$

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$
$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$
$z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

R inel comutativ
pentru orice $(a+b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + \dots + C_n^n b^n$

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$\cos n\theta = \cos^n\theta - C_n^2(\cos\theta)^{n-2}(\sin\theta)^2 + C_n^4(\cos\theta)^{n-4}\sin^4\theta \dots$
$\sin^2\theta = 1 - \cos^2\theta$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$

$f(x) = f'(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$

$g(x) = \frac{f(x)}{e^x} = c \qquad f(x) = c e^x$

$g'(x) = 0, \quad g'(x) = \frac{f'(x)\cdot e^x - f(x)\cdot e^x}{e^{2x}} = \frac{f'(x) - f(x)}{e^x} = 0$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \dots$

$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

$e^{ix} = 1 + \frac{ix}{1!} - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} = \cos x + i\sin x$

$$h(x) = f(x) + i g(x)$$

$$h'(x) = f'(x) + i g'(x)$$

$$= -g(x) + i f(x) = i h(x)$$

$$a(x) = \frac{h(x)}{e^{ix}} = c$$

$$a'(x) = \frac{h'(x) e^{ix} - h(x) e^{ix} \cdot i}{e^{2ix}} = 0$$