

# Structuri pentru mulțimi disjuncte

# Operații cu mulțimi disjuncte

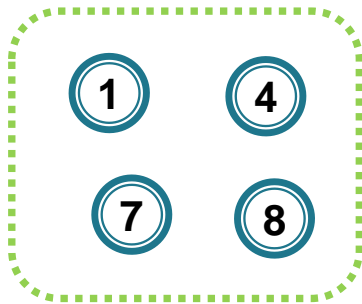
## Problemă

Asupra unei partiții ale mulțimii  $\{1, 2, \dots, n\}$  (în submulțimi disjuncte) se efectuează o succesiune de operații de tip

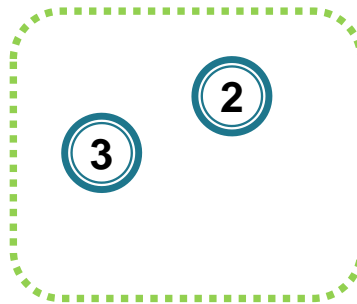
- reuniune
- test de apartenență

Cum putem memora submulțimile astfel încât operațiile să se efectueze "eficient"?

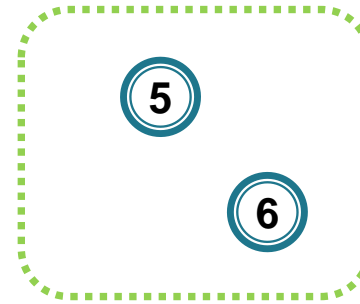
$\{1, 4, 7, 8\}$



$\{2, 3\}$



$\{5, 6\}$



# Operații cu mulțimi disjuncte

## Soluții

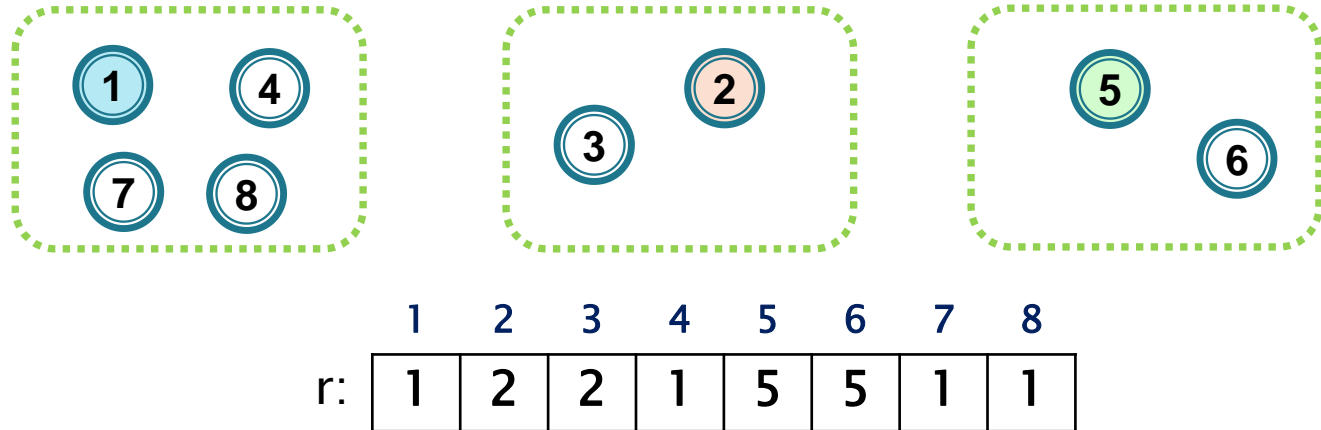
Asociem fiecărei submulțimi un reprezentant (culoare).

Notăm operațiile

- **Initializare**( $u$ ) – creează o mulțime cu un singur element  $u$
- **Reprez**( $u$ ) – returnează reprezentantul mulțimii care conține pe  $u$
- **Reunește**( $u, v$ ) – unește mulțimea care conține  $u$  cu cea care conține  $v$

# Vectori de reprezentanți

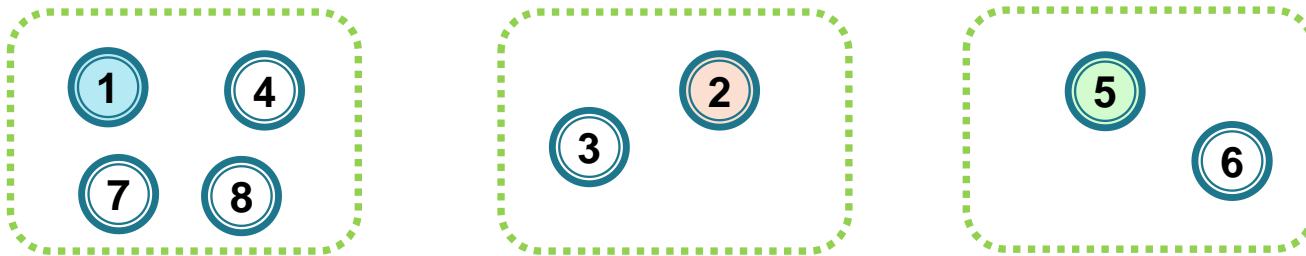
**Varianta 1** – Memorăm într-un vector  $r$  pentru fiecare element  $x$  reprezentantul mulțimii  $r[x]$  – v. Kruskal curs



- **Initializare**( $u$ ) –  $O(1)$
  - **Reprez**( $u$ ) –  $O(1)$
  - **Reuneste**( $u, v$ ) –  $O(n)$
- ```
void Initializare(int u) { r[u]=u; }  
  
int Reprez(int u) { return r[u]; }  
  
void Reuneste(int u, int v) {  
    r1 = Reprez(u); //r1=r[u]  
    r2 = Reprez(v); //r2=r[v]  
    for (k=1; k<=n; k++)  
        if (r[k]==r2)  
            r[k] = r1;  
}
```

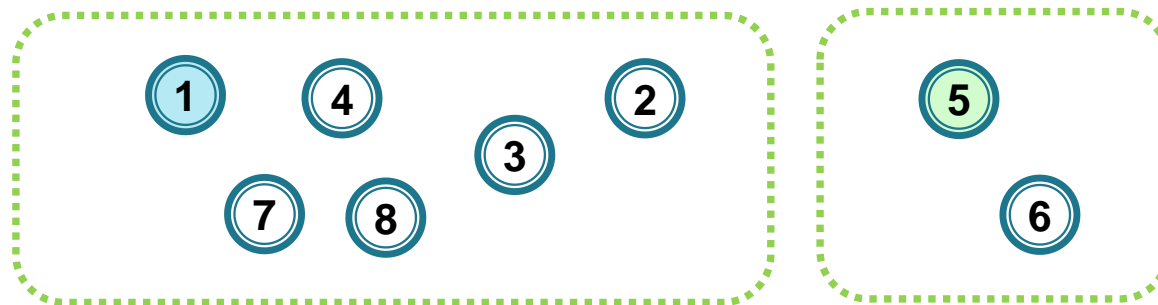
# Vectori de reprezentanți

## Exemplu



|    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----|---|---|---|---|---|---|---|---|
| r: | 1 | 2 | 2 | 1 | 5 | 5 | 1 | 1 |

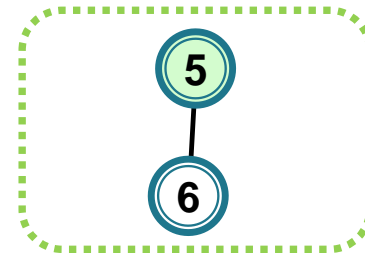
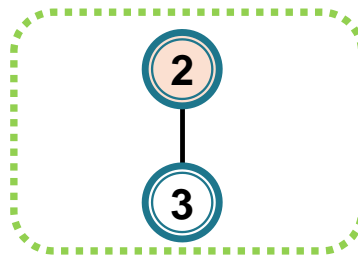
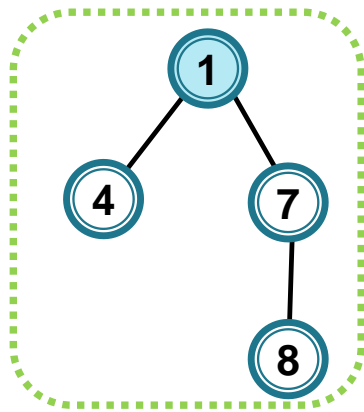
**Reuneste(4, 3)  $\Rightarrow$**



|    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----|---|---|---|---|---|---|---|---|
| r: | 1 | 1 | 1 | 1 | 5 | 5 | 1 | 1 |

# Operații cu mulțimi disjuncte

**Varianta 2** – Memorăm vârfurile fiecărei mulțimi ca un arbore (memorat cu tata), având ca **reprezentant rădăcina**



|       |   |   |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|---|---|
|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| tata: | 0 | 0 | 2 | 1 | 0 | 5 | 1 | 7 |

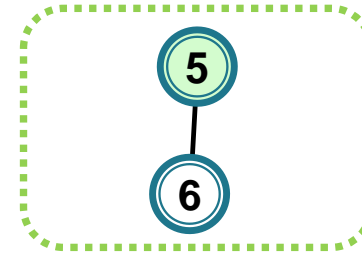
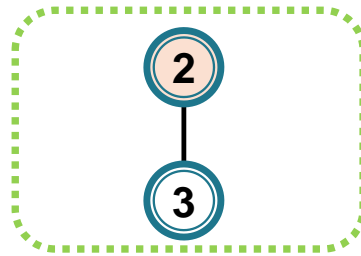
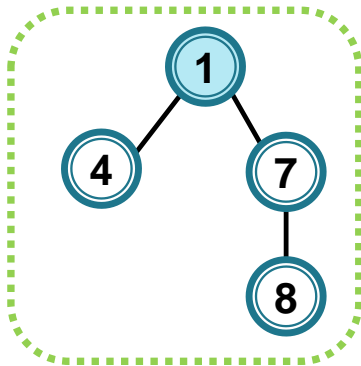
# Păduri de mulțimi disjuncte

**Varianta 2** – Memorăm vârfurile fiecărei mulțimi ca un arbore (memorat cu tata), având ca **reprezentant rădăcina**

- **Initializare(u)** :  $O(1)$       `void Initializare(int u){ tata[u]=h[u]=0;}`
- **Reprez(u)** – determinarea rădăcinii arborelui care conține u  
– liniar în înălțimea arborelui      `int Reprez(int u){  
    while(tata[u]!=0)  
        u=tata[u];  
    return u;  
}`
- **Reuneste(u,v)** – reuniune ponderată **în funcție de înălțimea arborilor**  
–  $O(1)$  după determinarea reprezentanților lui u și v      `void Reuneste(int u,int v){  
    int ru=Reprez(u); int rv=Reprez(v);  
    if (h[ru]>h[rv])  
        tata[rv]=ru;  
    else{ tata[ru]=rv;  
        if (h[ru]==h[rv])  
            h[rv]=h[rv]+1;  
    }  
}`
- **arbori de înălțime logaritmică**

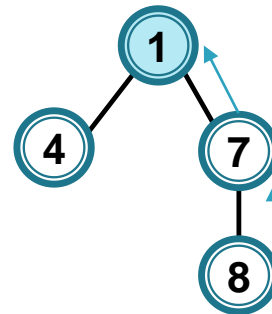
# Păduri de mulțimi disjuncte

Exemplu



|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 0 | 2 | 1 | 0 | 5 | 1 | 7 |
| h:    | 2 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

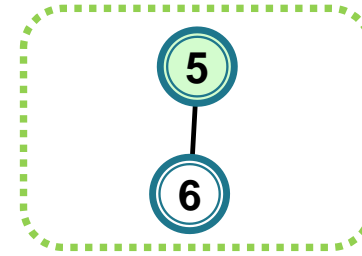
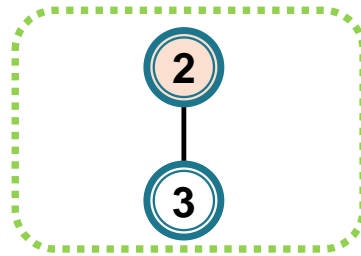
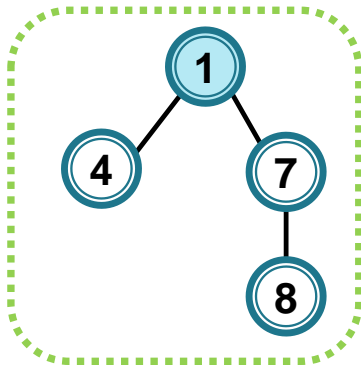
**Reprez(8)**  $\Rightarrow$  returneaza 1



( tata[8] = 7, tata[7] = 1, tata[1] = 0 )

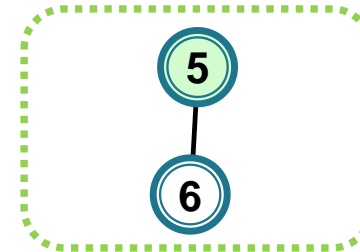
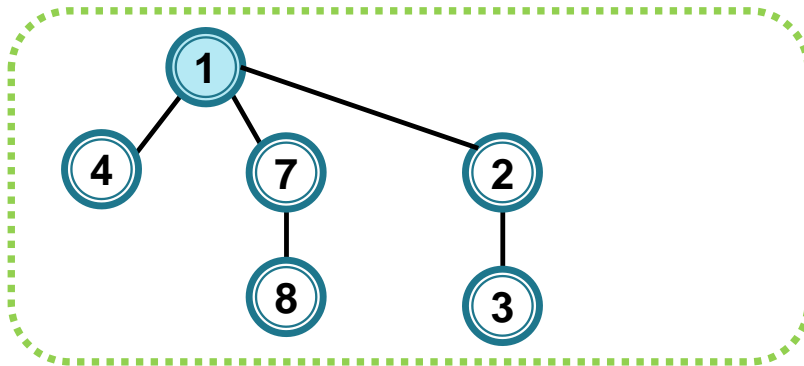


# Păduri de mulțimi disjuncte



|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 0 | 2 | 1 | 0 | 5 | 1 | 7 |
| h:    | 2 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

**Reunește(4, 3)**  $\Rightarrow$  deoarece  $h[1] > h[2]$ , se va seta  $tata[2] = 1$   
(**h nu se modifică**)



|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| tata: | 0 | 1 | 2 | 1 | 0 | 5 | 1 | 7 |

# Păduri de mulțimi disjuncte

**Reprez(u)** Optimizare – **compresie de cale**

– tatăl vârfurilor de pe lanțul de la u la rădăcină se va seta ca fiind rădăcina

(vârfurile de pe acest lanț, parcurs pentru a găsi reprezentantul lui u, vor deveni fii ai rădăcinii, pentru ca reprezentantul lor să fie găsit mai ușor în căutările ulterioare)

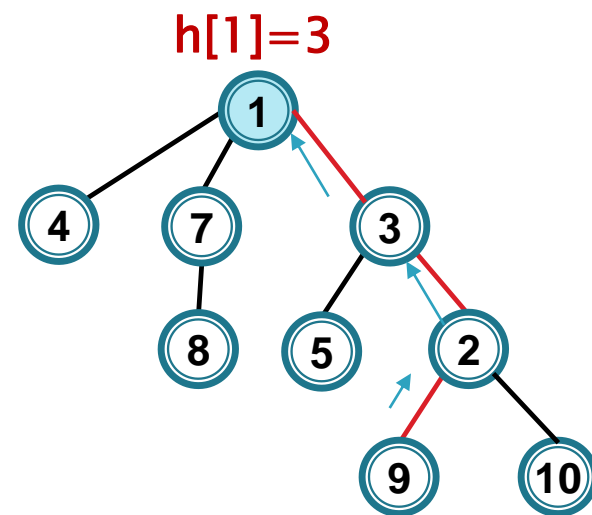
**!! h nu se actualizează**

De exemplu, după apelul **Reprez(9)** pentru arborele

rezultatul va fi 1, iar arborele devine



```
int Reprez(int u){  
    if (tata[u]==0)  
        return u;  
    tata[u]=Reprez(tata[u]);  
    return tata[u];  
}
```



# Păduri de mulțimi disjuncte

**Reprez(u)** Optimizare – **compresie de cale**

– tatăl vârfurilor de pe lanțul de la u la rădăcină se va seta ca fiind rădăcina

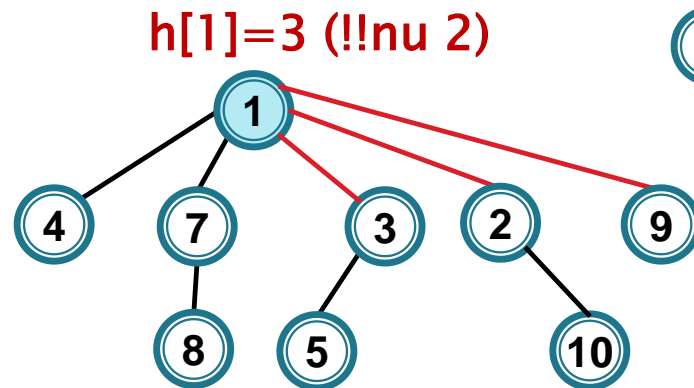
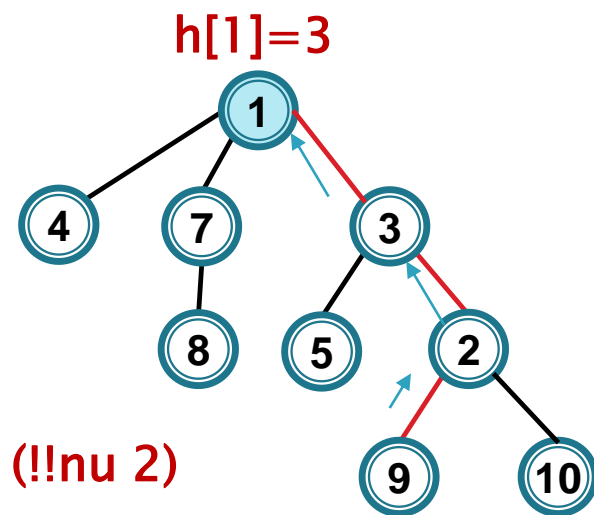
(vârfurile de pe acest lanț, parcurs pentru a găsi reprezentantul lui u, vor deveni fii ai rădăcinii, pentru ca reprezentantul lor să fie găsit mai ușor în căutările ulterioare)

**!! h nu se actualizează**

De exemplu, după apelul **Reprez(9)** pentru arborele

rezultatul va fi 1, iar arborele devine

```
int Reprez(int u){  
    if (tata[u]==0)  
        return u;  
    tata[u]=Reprez(tata[u]);  
    return tata[u];  
}
```



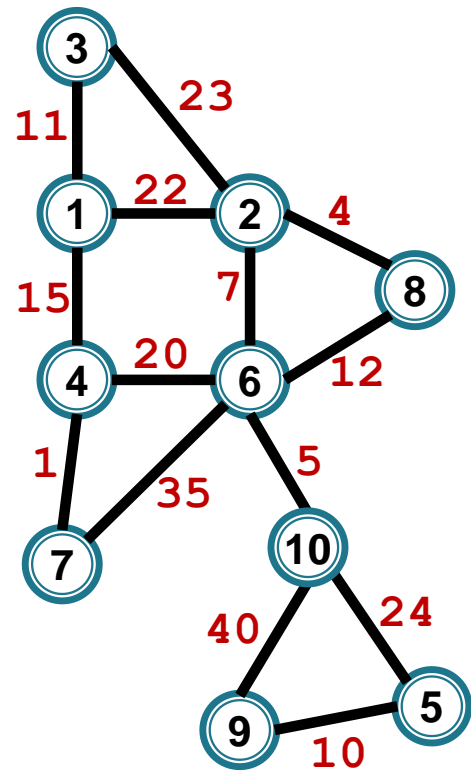
# Algoritmul lui Kruskal

## Implementare cu păduri disjuncte

# Kruskal – Pseudocod

```
sorteaza (E)
for (v=1 ; v<=n ; v++)
    Initializare (v) ;
nrmsel=0
for (uv ∈ E)
    if (Reprez (u) !=Reprez (v) )
    {
        E (T) = E (T) ∪ {uv} ;
        Reuneste (u , v) ;
        nrmsel=nrmsel+1 ;
        if (nrmsel==n-1)
            STOP; //break;
    }
```

# Pădurea de mulțimi disjuncte la pasul curent



## Ordine muchii

(4, 7)      (2, 3)  
 (2, 8)      (5, 10)  
 (6, 10)    (6, 7)  
 (2, 6)      (9, 10)  
 (5, 9)  
 (1, 3)  
 (6, 8)  
 (1, 4)  
 (4, 6)  
 (1, 2)

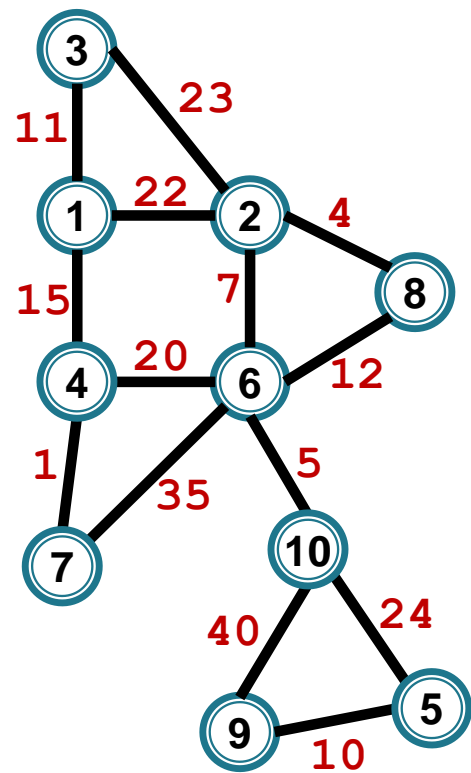
|      | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| tata | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  |
| h    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  |

# Pădurea de mulțimi disjuncte la pasul curent



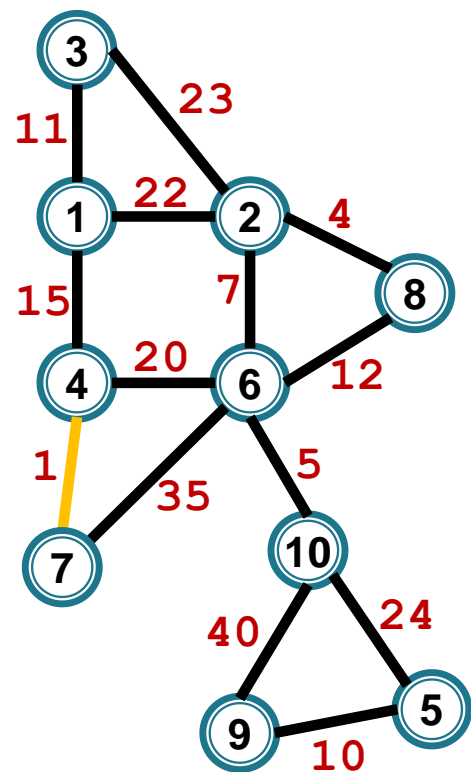
Muchia curentă

(4,7):



Ordine muchii

- (4, 7)
- (2, 3)
- (2, 8)
- (5, 10)
- (6, 10)
- (6, 7)
- (2, 6)
- (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)



Ordine muchii

- (4, 7)** (2, 3)  
 (2, 8) (5, 10)  
 (6, 10) (6, 7)  
 (2, 6) (9, 10)  
 (5, 9)  
 (1, 3)  
 (6, 8)  
 (1, 4)  
 (4, 6)  
 (1, 2)

Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

(4,7):

$\text{Reprez}(4) \neq \text{Reprez}(7)$

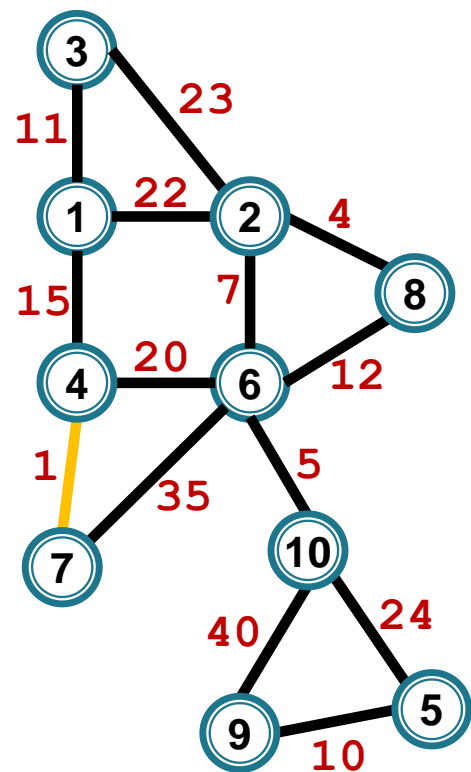


Reunește(4,7)



|      | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| tata | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0  |
| h    | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  |



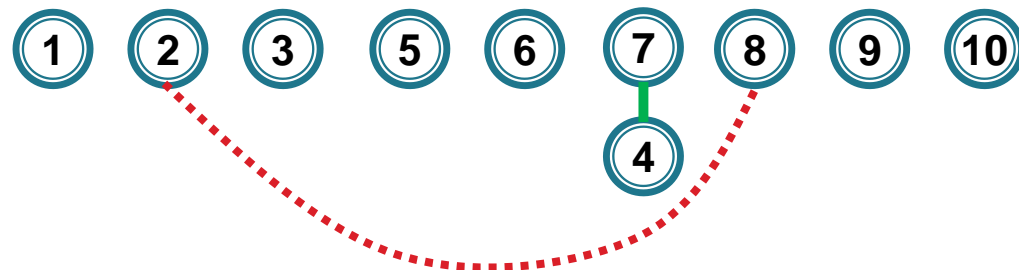


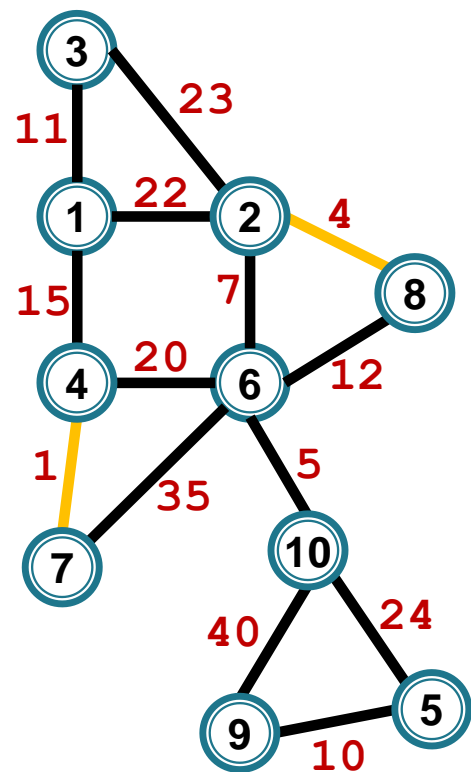
Ordine muchii

- |               |         |
|---------------|---------|
| (4, 7)        | (2, 3)  |
| <b>(2, 8)</b> | (5, 10) |
| (6, 10)       | (6, 7)  |
| (2, 6)        | (9, 10) |
| (5, 9)        |         |
| (1, 3)        |         |
| (6, 8)        |         |
| (1, 4)        |         |
| (4, 6)        |         |
| (1, 2)        |         |

Muchia curentă  
(2,8):

Pădurea de mulțimi disjuncte la pasul curent

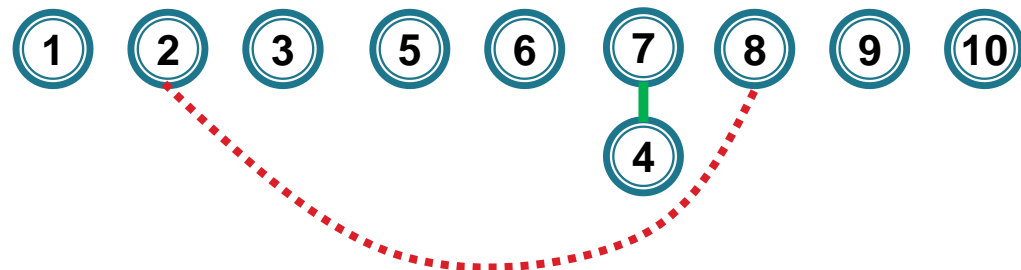




Ordine muchii

- (4, 7)
- (2, 8)**
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

Pădurea de mulțimi disjuncte la pasul curent

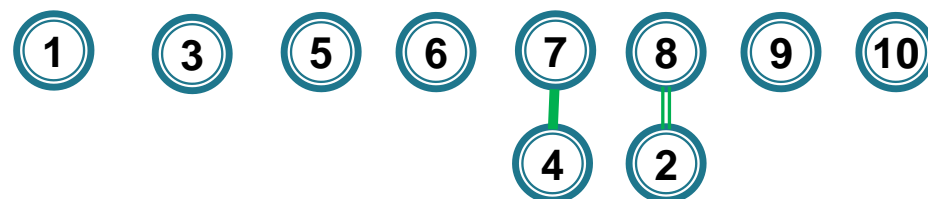


Muchia curentă

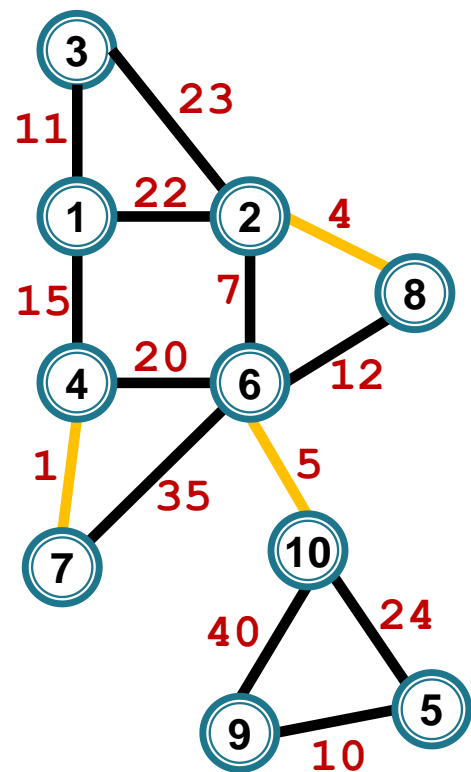
(2,8):

$\text{Reprez}(2) \neq \text{Reprez}(8)$

Reunește(2,8)



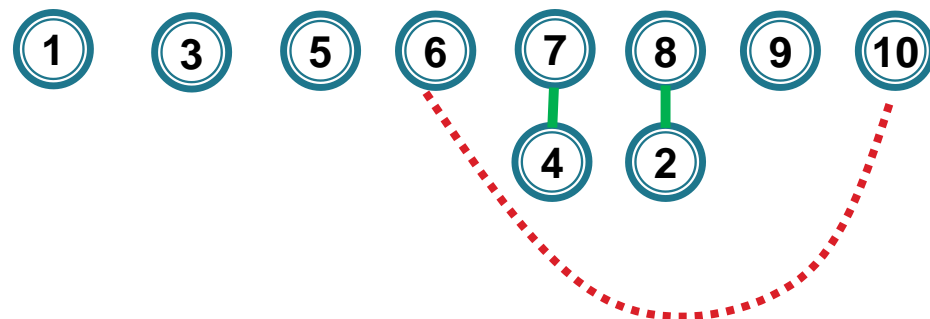
|      | 1 | 2        | 3 | 4 | 5 | 6 | 7 | 8        | 9 | 10 |
|------|---|----------|---|---|---|---|---|----------|---|----|
| tata | 0 | <b>8</b> | 0 | 7 | 0 | 0 | 0 | 0        | 0 | 0  |
| h    | 0 | 0        | 0 | 0 | 0 | 0 | 1 | <b>1</b> | 0 | 0  |



Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)**
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

Pădurea de mulțimi disjuncte la pasul curent

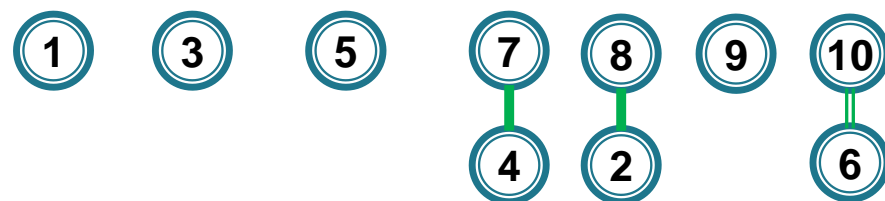


Muchia curentă

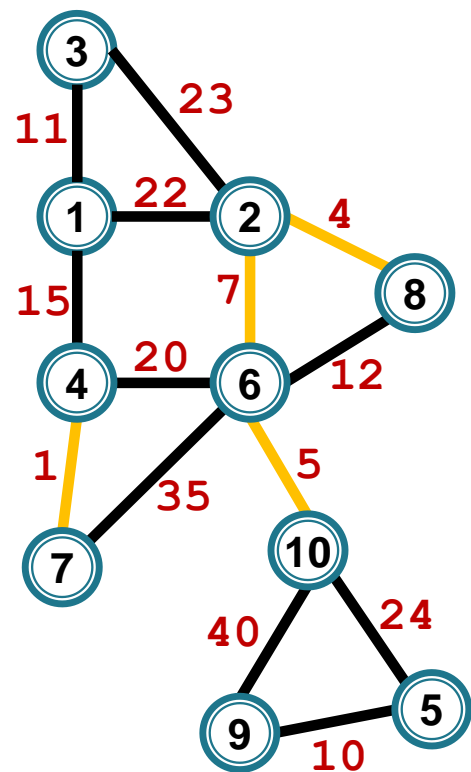
(6,10):

$\text{Reprez}(6) \neq \text{Reprez}(10)$

Reunește(6,10)



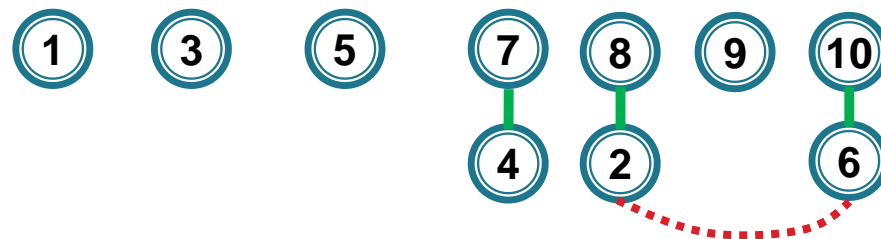
|      | 1 | 2 | 3 | 4 | 5 | 6         | 7 | 8 | 9 | 10       |
|------|---|---|---|---|---|-----------|---|---|---|----------|
| tata | 0 | 8 | 0 | 7 | 0 | <b>10</b> | 0 | 0 | 0 | 0        |
| h    | 0 | 0 | 0 | 0 | 0 | 0         | 1 | 1 | 0 | <b>1</b> |



Ordine muchii

- |               |         |
|---------------|---------|
| (4, 7)        | (2, 3)  |
| (2, 8)        | (5, 10) |
| (6, 10)       | (6, 7)  |
| <b>(2, 6)</b> | (9, 10) |
| (5, 9)        |         |
| (1, 3)        |         |
| (6, 8)        |         |
| (1, 4)        |         |
| (4, 6)        |         |
| (1, 2)        |         |

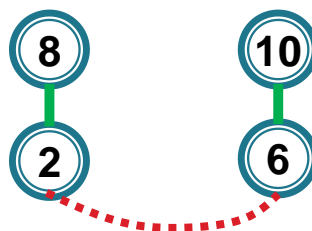
Pădurea de mulțimi disjuncte la pasul curent

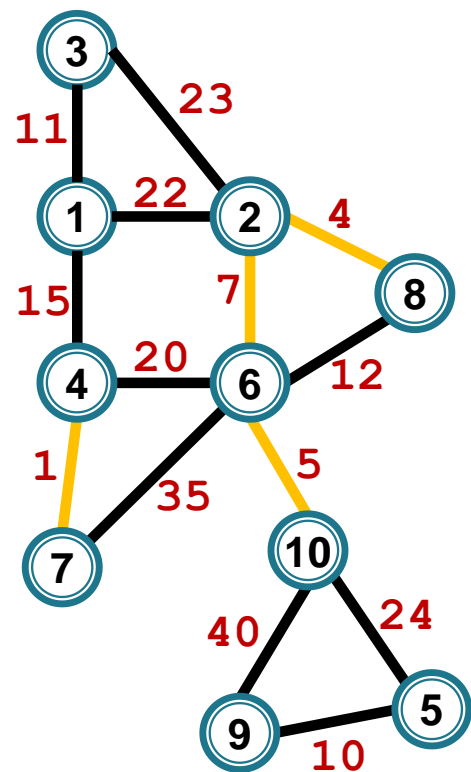


Muchia curentă

(2,6):

$\text{Reprez}(2) \neq \text{Reprez}(6)$

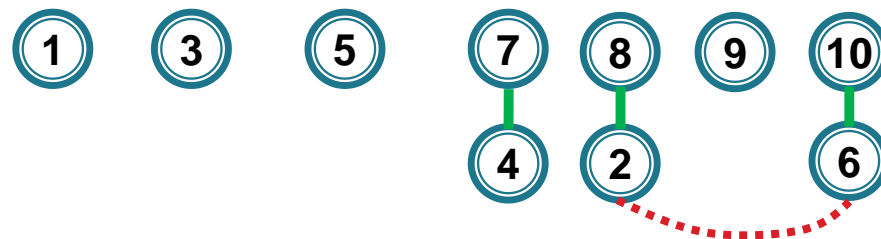




Ordine muchii

- |               |         |
|---------------|---------|
| (4, 7)        | (2, 3)  |
| (2, 8)        | (5, 10) |
| (6, 10)       | (6, 7)  |
| <b>(2, 6)</b> | (9, 10) |
| (5, 9)        |         |
| (1, 3)        |         |
| (6, 8)        |         |
| (1, 4)        |         |
| (4, 6)        |         |
| (1, 2)        |         |

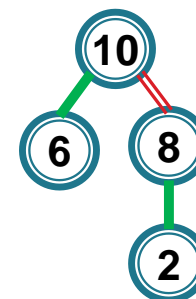
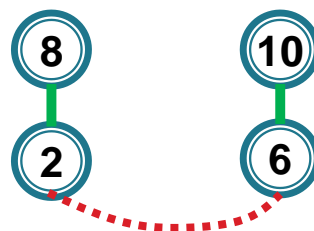
Pădurea de mulțimi disjuncte la pasul curent

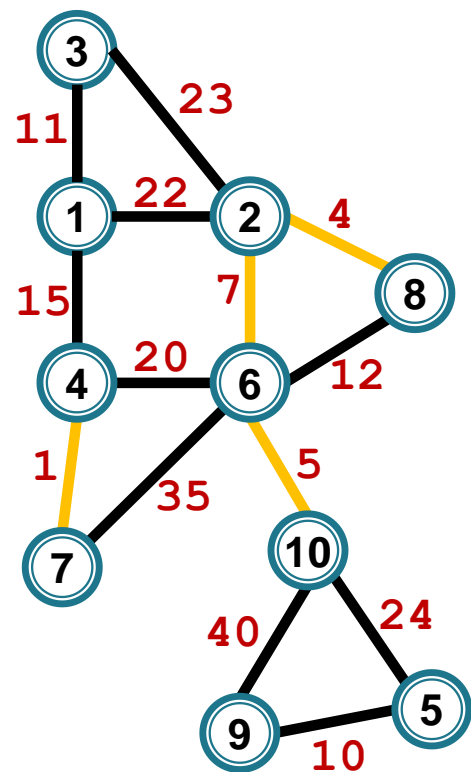


Muchia curentă

(2,6):

$\text{Reprez}(2) \neq \text{Reprez}(6)$

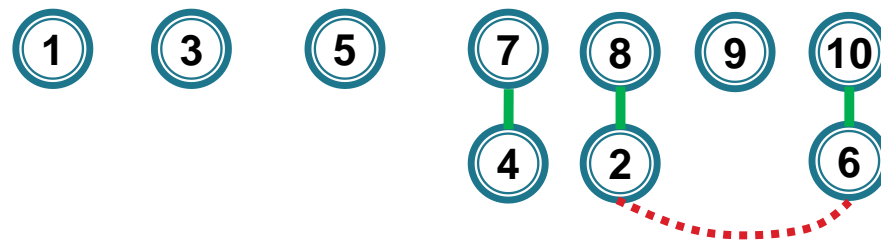




Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)**
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

Pădurea de mulțimi disjuncte la pasul curent

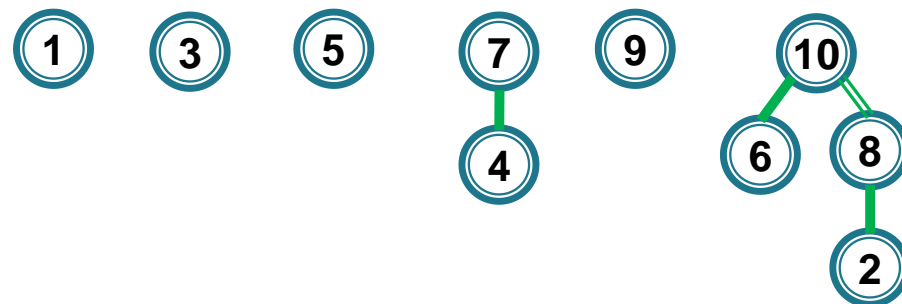


Muchia curentă

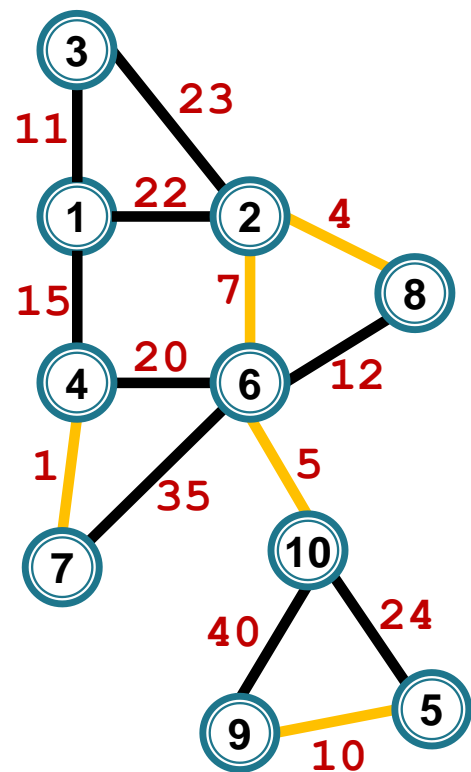
(2,6):

$\text{Reprez}(2) \neq \text{Reprez}(6)$

Reuneste(2, 6)



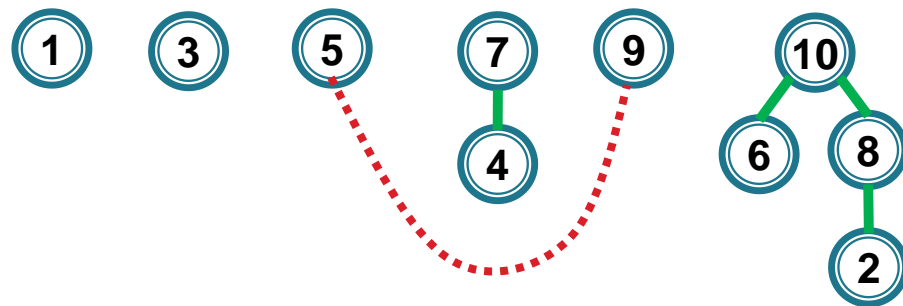
|      | 1 | 2 | 3 | 4 | 5 | 6  | 7 | 8         | 9 | 10       |
|------|---|---|---|---|---|----|---|-----------|---|----------|
| tata | 0 | 8 | 0 | 7 | 0 | 10 | 0 | <b>10</b> | 0 | 0        |
| h    | 0 | 0 | 0 | 0 | 0 | 0  | 1 | 1         | 0 | <b>2</b> |



Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)**
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

Pădurea de mulțimi disjuncte la pasul curent

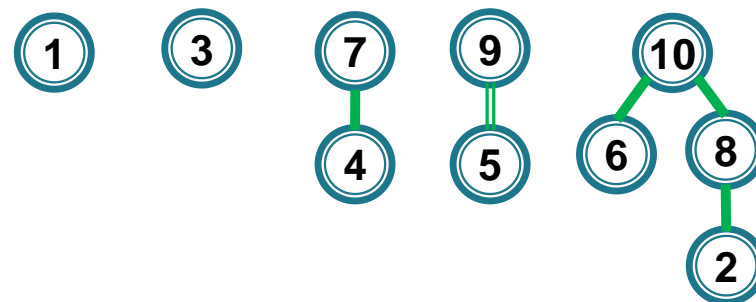


Muchia curentă

(5,9):

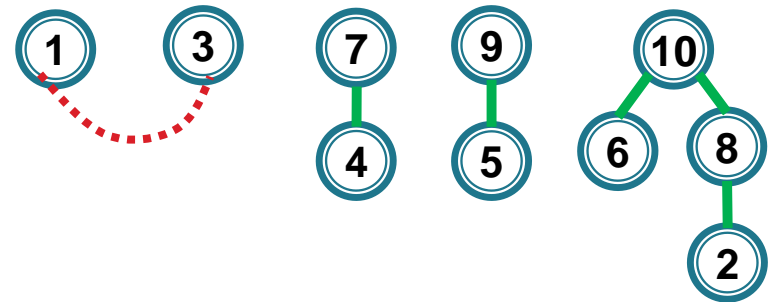
$\text{Reprez}(5) \neq \text{Reprez}(9)$

Reunește(5, 9)



|      | 1 | 2 | 3 | 4 | 5        | 6  | 7 | 8  | 9        | 10 |
|------|---|---|---|---|----------|----|---|----|----------|----|
| tata | 0 | 8 | 0 | 7 | <b>9</b> | 10 | 0 | 10 | 0        | 0  |
| h    | 0 | 0 | 0 | 0 | 0        | 0  | 1 | 1  | <b>1</b> | 2  |

# Pădurea de mulțimi disjuncte la pasul curent

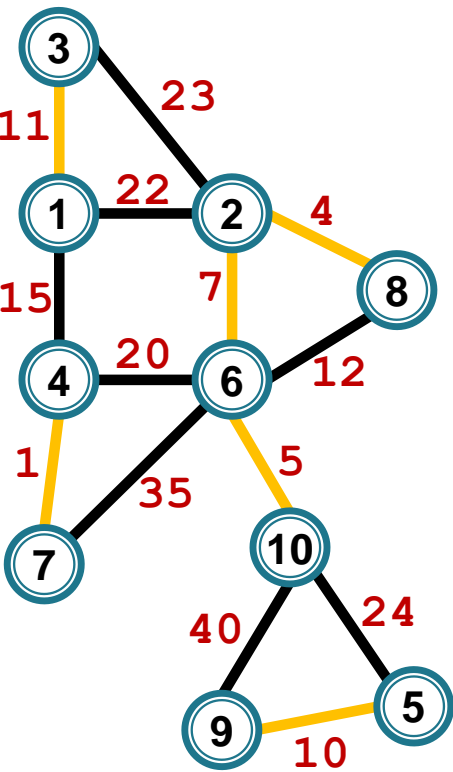
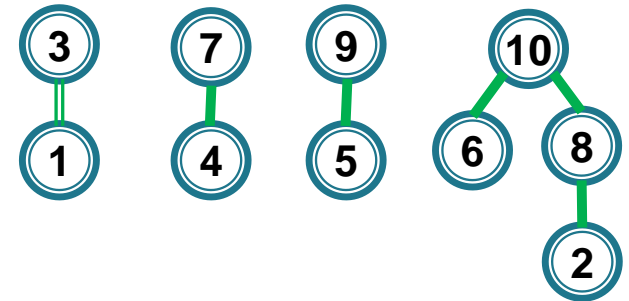


Muchia curentă

(1,3):

$\text{Reprez}(1) \neq \text{Reprez}(3)$

Reunește(1, 3)



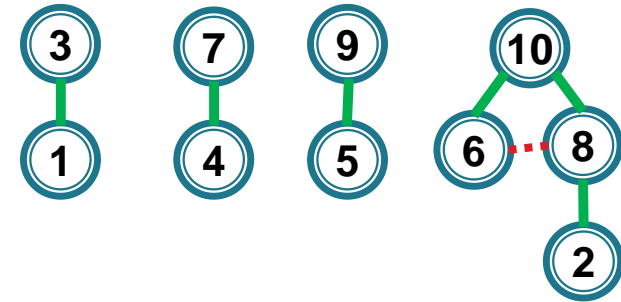
Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)**
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)

|      | 1        | 2 | 3        | 4 | 5 | 6  | 7 | 8  | 9 | 10 |
|------|----------|---|----------|---|---|----|---|----|---|----|
| tata | <b>3</b> | 8 | 0        | 7 | 9 | 10 | 0 | 10 | 0 | 0  |
| h    | 0        | 0 | <b>1</b> | 0 | 0 | 0  | 1 | 1  | 1 | 2  |



# Pădurea de mulțimi disjuncte la pasul curent

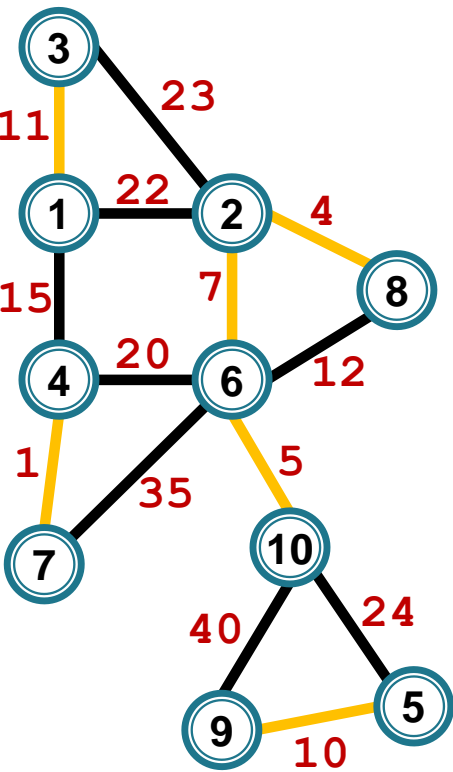


Muchia curentă

(6,8):

$\text{Reprez}(6) = \text{Reprez}(8) \Rightarrow$  nu este selectată

**Observație:** Până acum în funcția Reprez nu a fost modificat vectorul tata prin compresie de cale, deoarece vârfurile erau la distanță cel mult 1 față de rădăcină

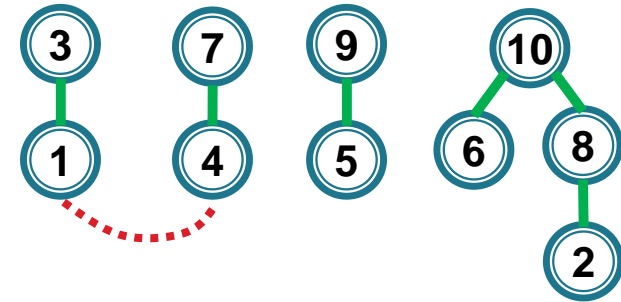


Ordine muchii

- (4, 7)
- (2, 3)
- (2, 8)
- (5, 10)
- (6, 10)
- (6, 7)
- (2, 6)
- (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)**
- (1, 4)
- (4, 6)
- (1, 2)

|      | 1 | 2 | 3 | 4 | 5 | 6  | 7 | 8  | 9 | 10 |
|------|---|---|---|---|---|----|---|----|---|----|
| tata | 3 | 8 | 0 | 7 | 9 | 10 | 0 | 10 | 0 | 0  |
| h    | 0 | 0 | 1 | 0 | 0 | 0  | 1 | 1  | 1 | 2  |

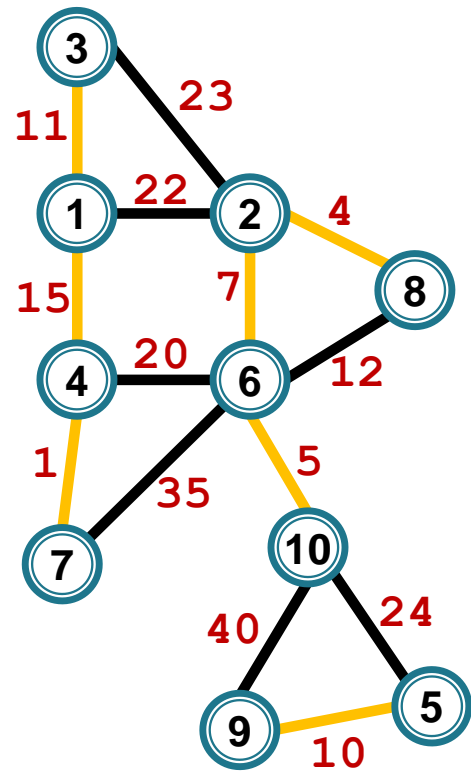
# Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

(1,4):

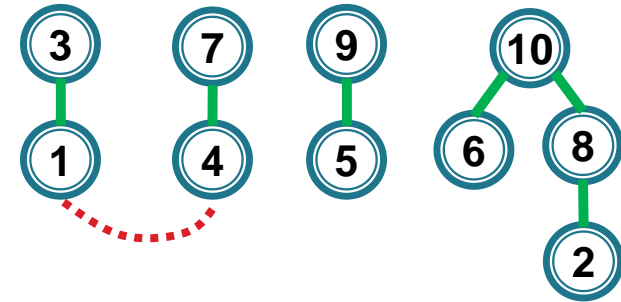
$\text{Reprez}(1) \neq \text{Reprez}(4)$



Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)**
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

# Pădurea de mulțimi disjuncte la pasul curent



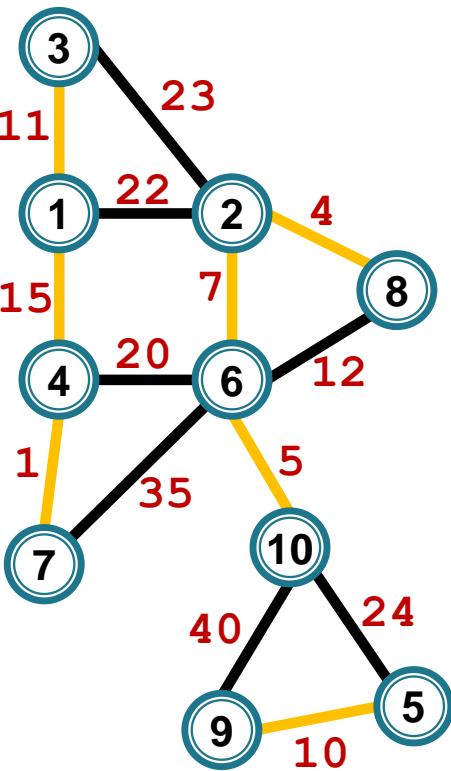
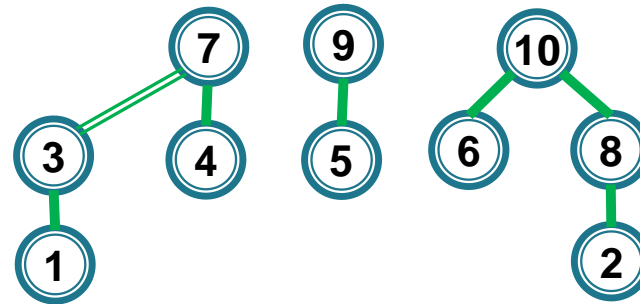
Muchia curentă

(1,4):

$\text{Reprez}(1) \neq \text{Reprez}(4)$



Reuneste(1, 4)

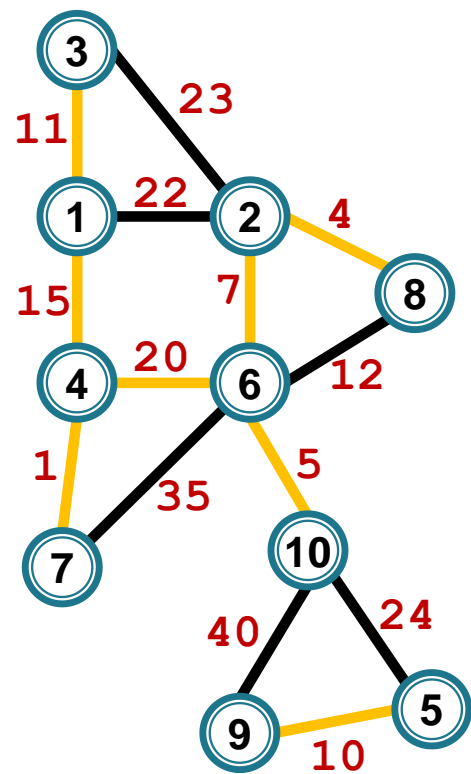


Ordine muchii

- (4, 7)      (2, 3)
- (2, 8)      (5, 10)
- (6, 10)    (6, 7)
- (2, 6)      (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)**
- (4, 6)
- (1, 2)

|      | 1 | 2 | 3        | 4 | 5 | 6  | 7        | 8  | 9 | 10 |
|------|---|---|----------|---|---|----|----------|----|---|----|
| tata | 3 | 8 | <b>7</b> | 7 | 9 | 10 | 0        | 10 | 0 | 0  |
| h    | 0 | 0 | 1        | 0 | 0 | 0  | <b>2</b> | 1  | 1 | 2  |

# Pădurea de mulțimi disjuncte la pasul curent



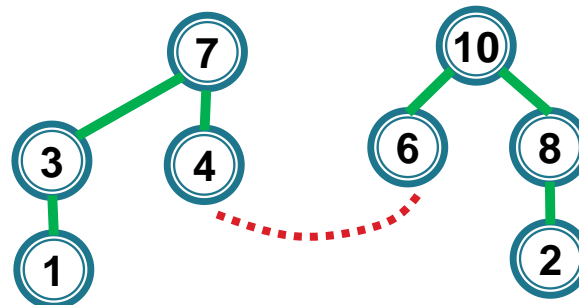
Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)**
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

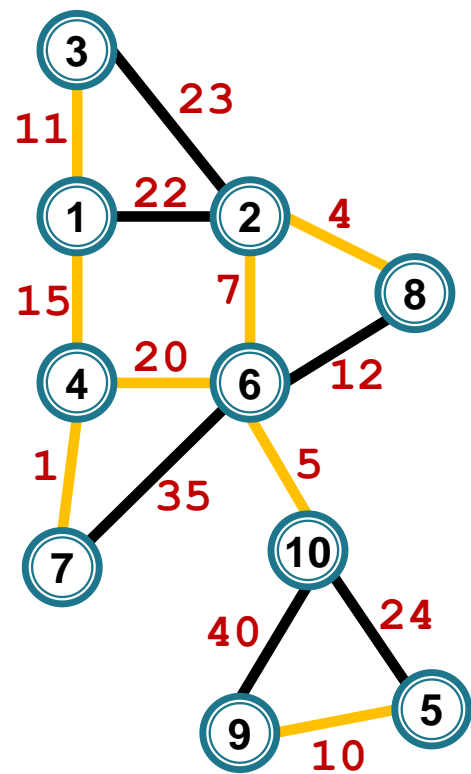
Muchia curentă

(4,6):

$\text{Reprez}(4) \neq \text{Reprez}(6)$



# Pădurea de mulțimi disjuncte la pasul curent



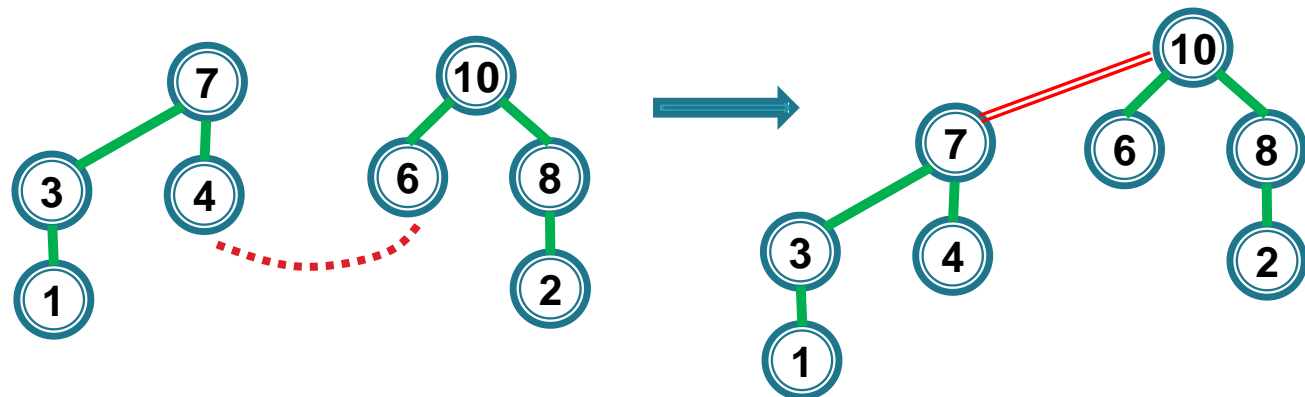
Ordine muchii

- |               |         |
|---------------|---------|
| (4, 7)        | (2, 3)  |
| (2, 8)        | (5, 10) |
| (6, 10)       | (6, 7)  |
| (2, 6)        | (9, 10) |
| (5, 9)        |         |
| (1, 3)        |         |
| (6, 8)        |         |
| (1, 4)        |         |
| <b>(4, 6)</b> |         |
| (1, 2)        |         |

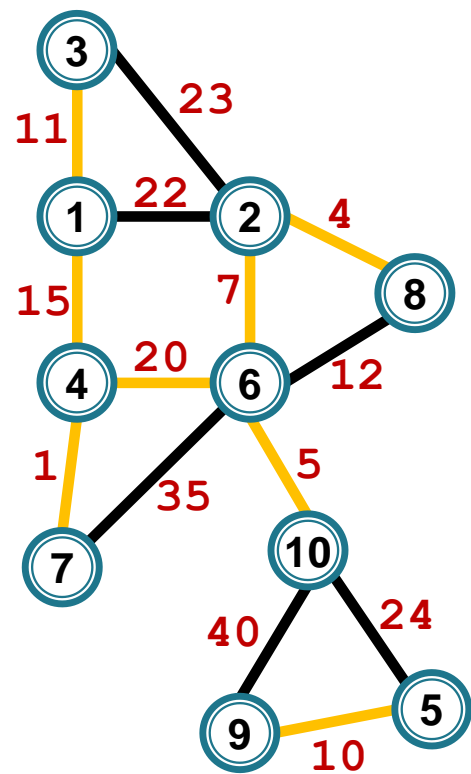
Muchia curentă

(4,6):

$\text{Reprez}(4) \neq \text{Reprez}(6)$



# Pădurea de mulțimi disjuncte la pasul curent



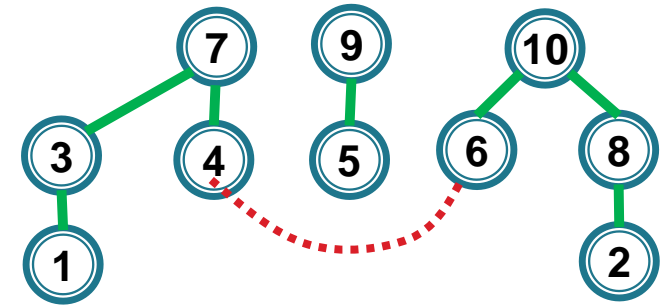
Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)**
- (1, 2)
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

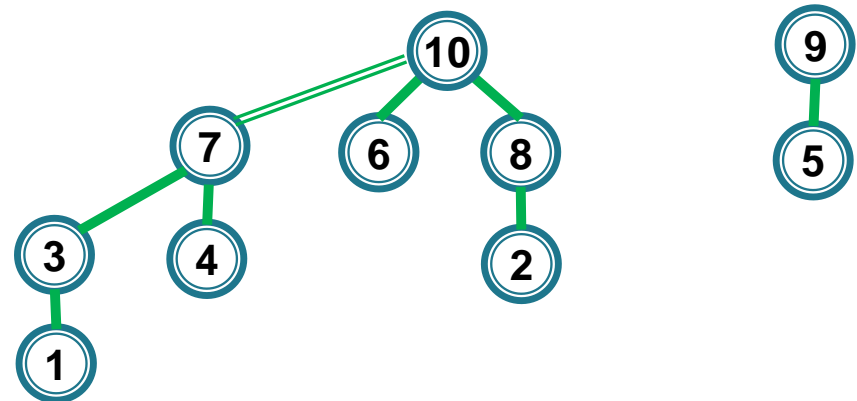
Muchia curentă

(4,6):

$\text{Reprez}(4) \neq \text{Reprez}(6)$

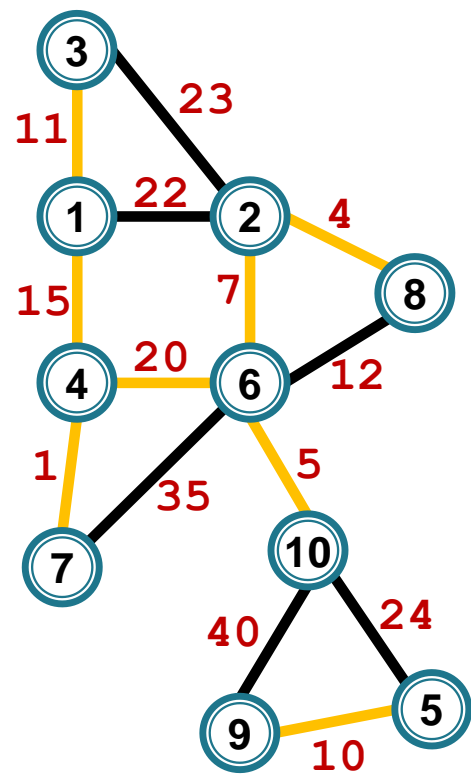


Reunește(4, 6)



|      | 1 | 2 | 3 | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|---|---|---|---|---|----|----|----|---|----|
| tata | 3 | 8 | 7 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0 | 0 | 1 | 0 | 0 | 0  | 2  | 1  | 1 | 3  |

# Pădurea de mulțimi disjuncte la pasul curent



Ordine muchii

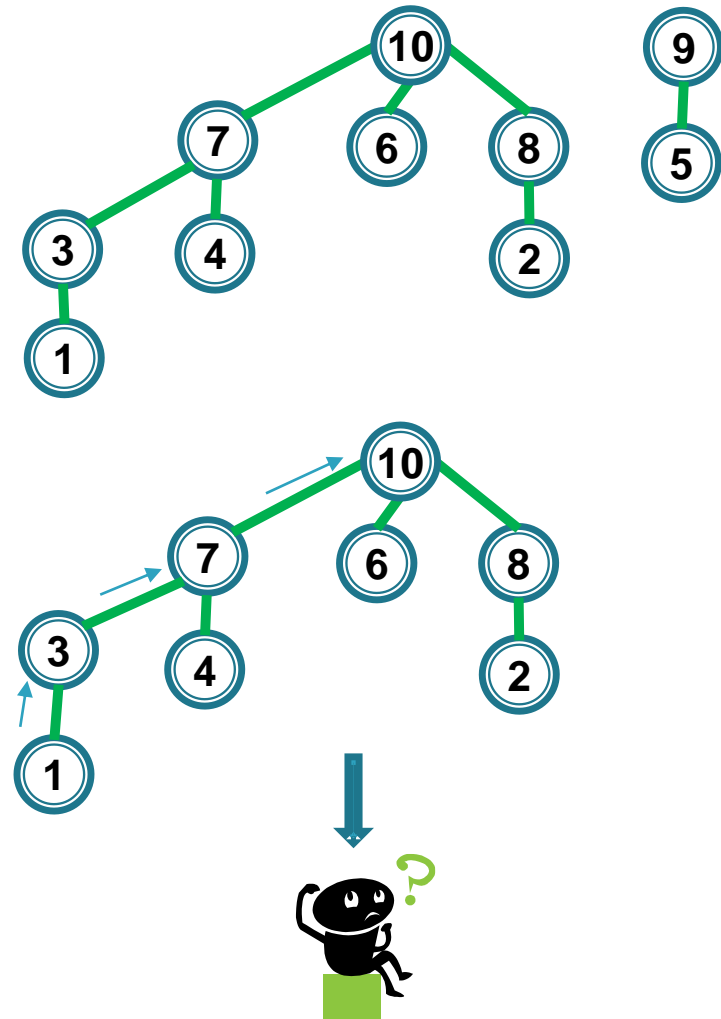
- |         |         |
|---------|---------|
| (4, 7)  | (2, 3)  |
| (2, 8)  | (5, 10) |
| (6, 10) | (6, 7)  |
| (2, 6)  | (9, 10) |
| (5, 9)  |         |
| (1, 3)  |         |
| (6, 8)  |         |
| (1, 4)  |         |
| (4, 6)  |         |
| (1, 2)  |         |

Muchia curentă

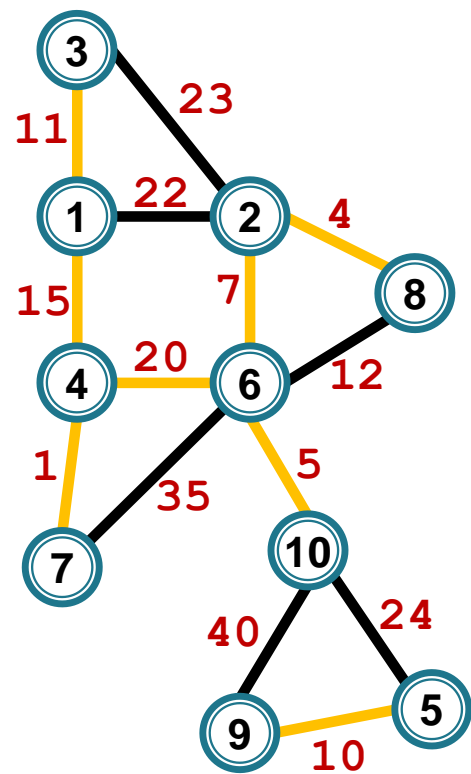
(1,2):

Reprez(1):  $\Rightarrow 10 +$   
compresie de cale

!!h nu se modifica  
(h[7] rămâne 2)



# Pădurea de mulțimi disjuncte la pasul curent



Ordine muchii

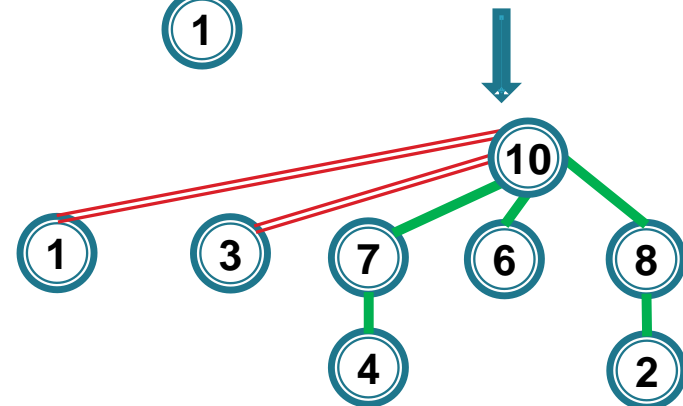
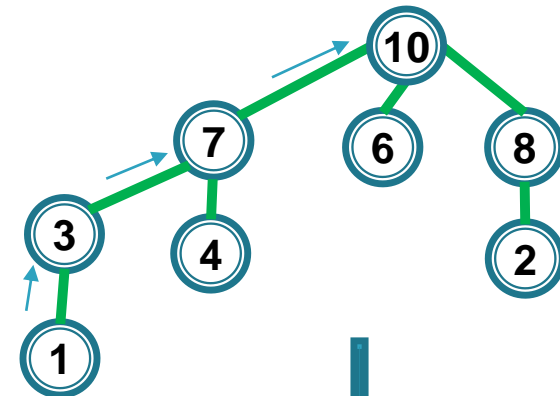
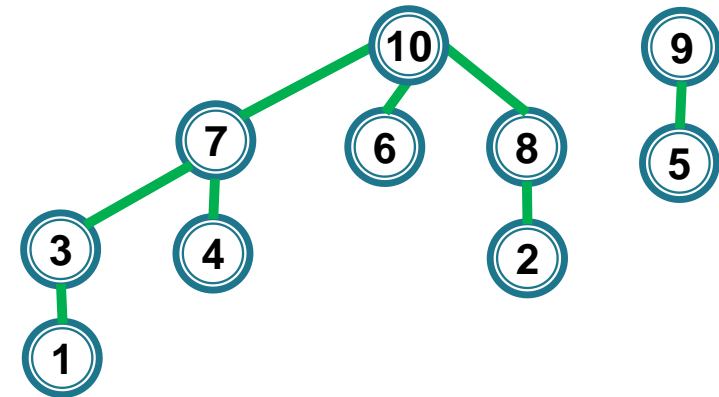
- (4, 7)      (2, 3)
- (2, 8)      (5, 10)
- (6, 10)    (6, 7)
- (2, 6)      (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)**

Muchia curentă

(1,2):

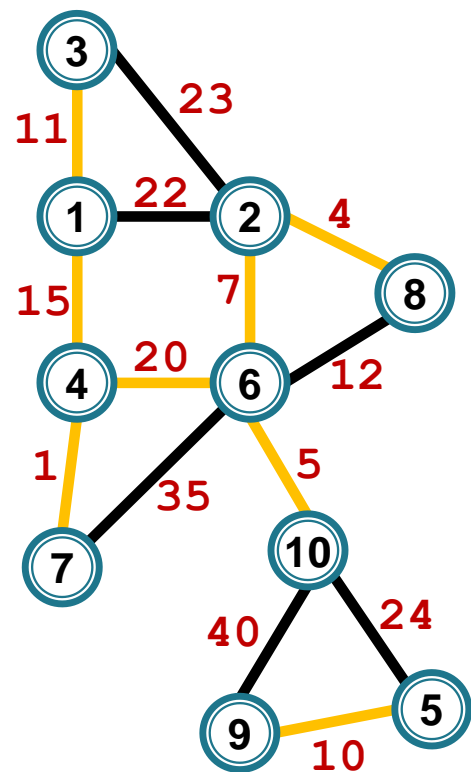
Reprez(1):  $\Rightarrow 10 +$   
compresie de cale

!!h nu se modifica  
(h[7] rămâne 2)



|      | 1  | 2 | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|---|----|---|---|----|----|----|---|----|
| tata | 10 | 8 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0 | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |





Ordine muchii

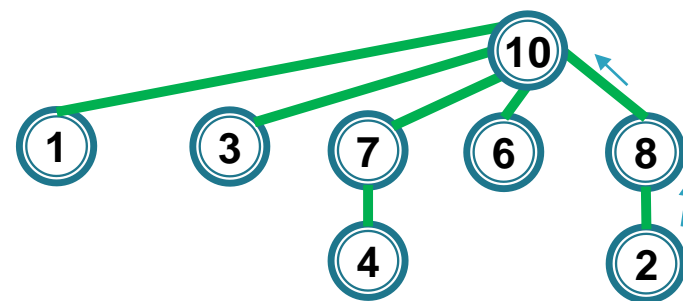
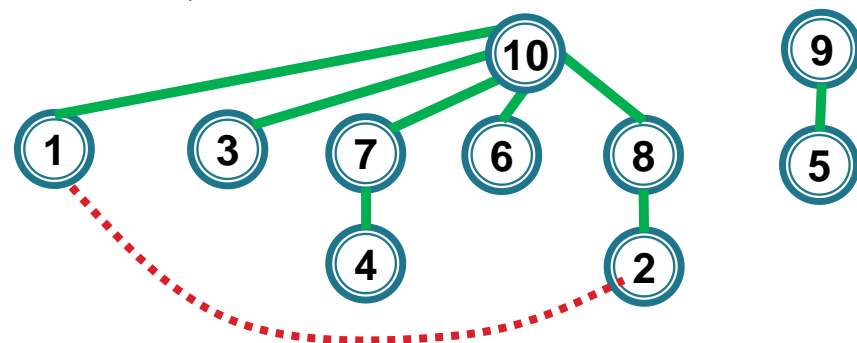
|               |         |
|---------------|---------|
| (4, 7)        | (2, 3)  |
| (2, 8)        | (5, 10) |
| (6, 10)       | (6, 7)  |
| (2, 6)        | (9, 10) |
| (5, 9)        |         |
| (1, 3)        |         |
| (6, 8)        |         |
| (1, 4)        |         |
| (4, 6)        |         |
| <b>(1, 2)</b> |         |

Muchia curentă

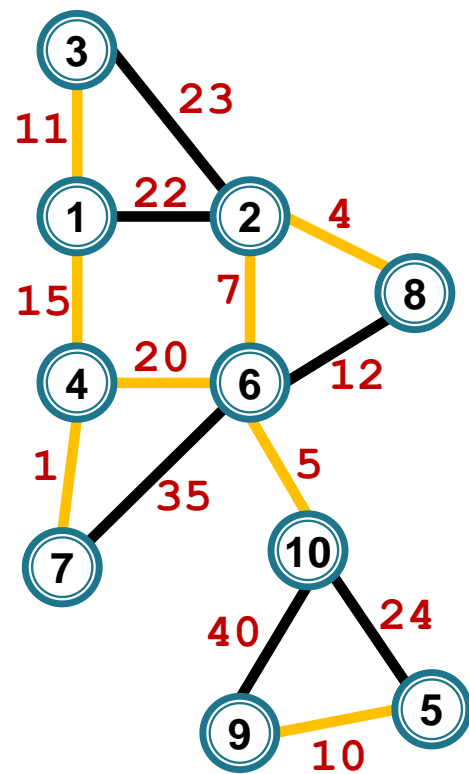
(1,2):

Reprez(2):  $\Rightarrow 10 +$   
**compresie de cale**

Pădurea de mulțimi disjuncte la pasul curent



# Pădurea de mulțimi disjuncte la pasul curent



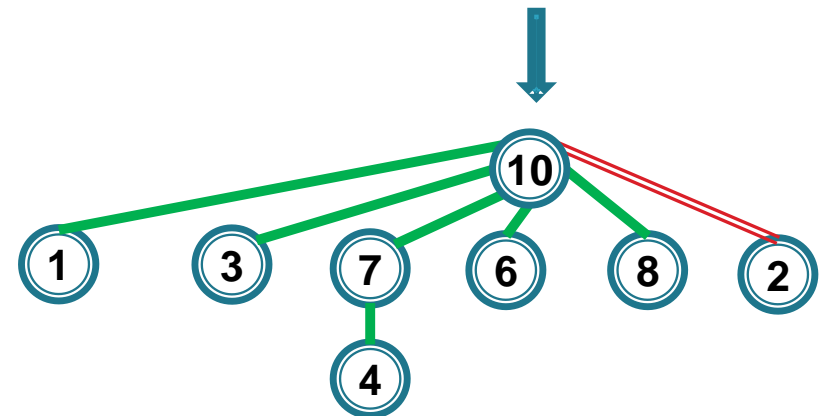
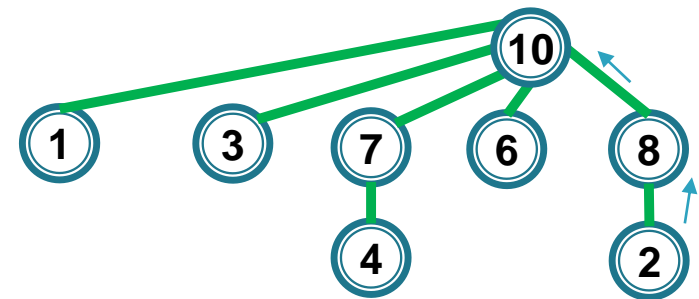
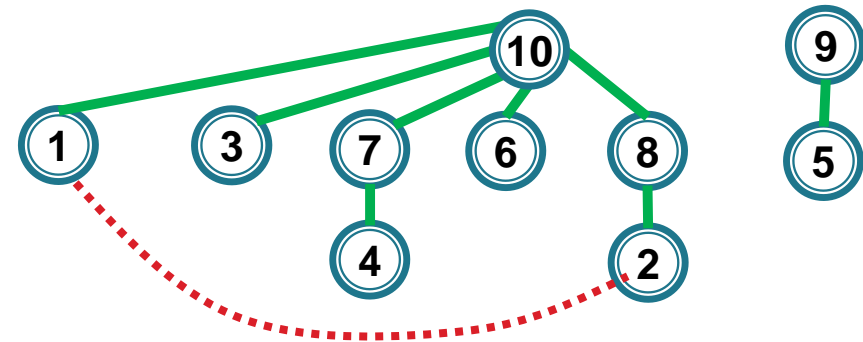
Ordine muchii

- (4, 7)      (2, 3)
- (2, 8)      (5, 10)
- (6, 10)    (6, 7)
- (2, 6)      (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)**

Muchia curentă

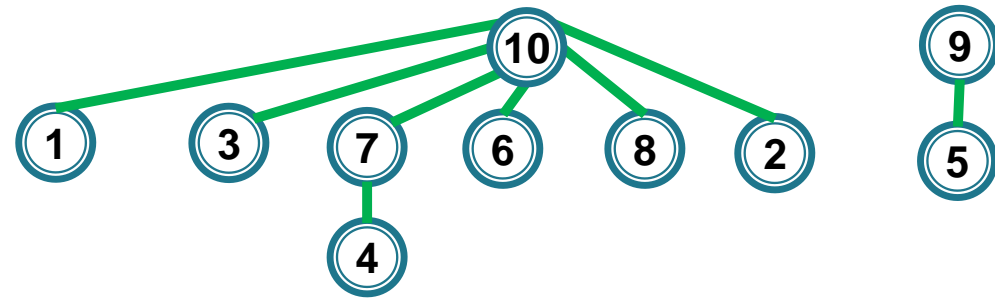
(1,2):

Reprez(2):  $\Rightarrow 10 +$   
**compresie de cale**



|      | 1  | 2         | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|-----------|----|---|---|----|----|----|---|----|
| tata | 10 | <b>10</b> | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0         | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |

# Pădurea de mulțimi disjuncte la pasul curent

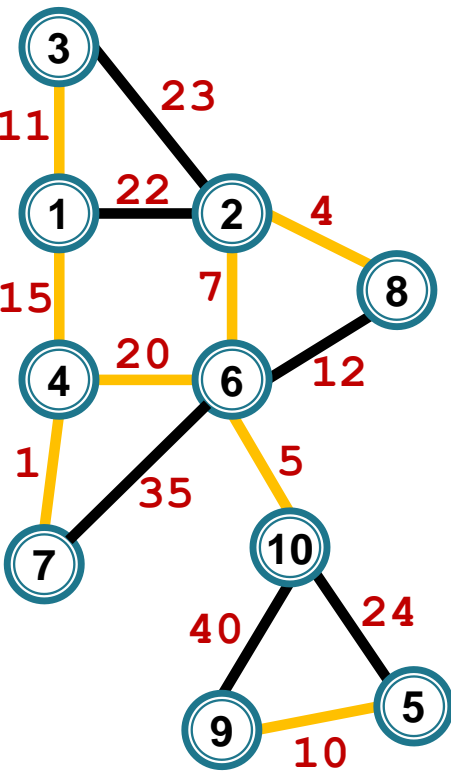


Muchia curentă

(1,2):

Reprez(1) = 10

Reprez(2) = 10  $\Rightarrow$  nu este selectată

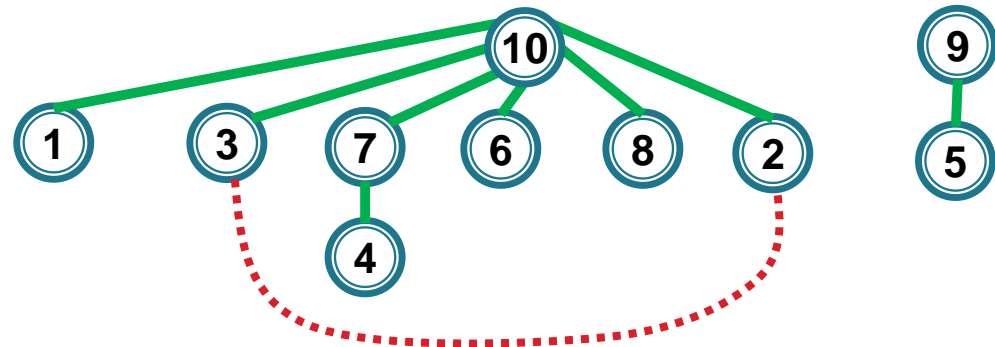


Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)**
- (2, 3)
- (5, 10)
- (6, 7)
- (9, 10)

|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |

# Pădurea de mulțimi disjuncte la pasul curent

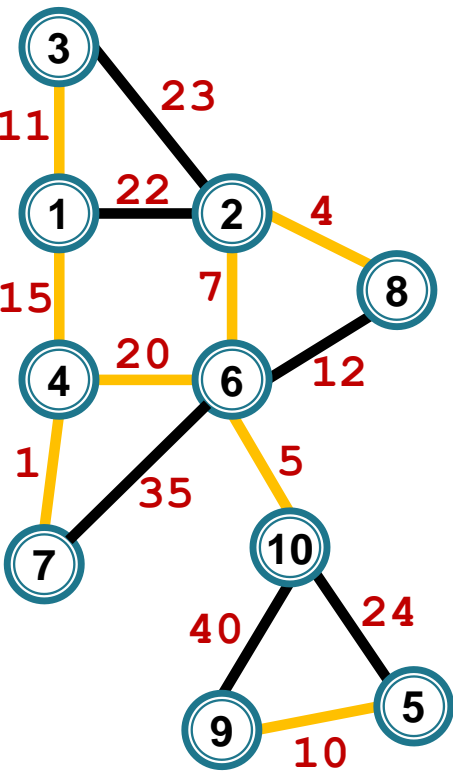


Muchia curentă

(2,3):

$\text{Reprez}(2) = \text{Reprez}(3) \Rightarrow$  nu este selectată

- 2 și 3 sunt fii ai rădăcinii, compresia de cale nu modifică vectorul tata

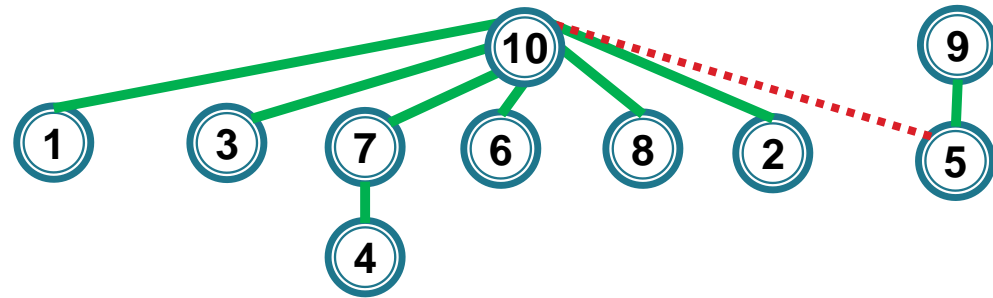


Ordine muchii

(4, 7)      **(2, 3)**  
 (2, 8)      (5, 10)  
 (6, 10)    (6, 7)  
 (2, 6)      (9, 10)  
 (5, 9)  
 (1, 3)  
 (6, 8)  
 (1, 4)  
 (4, 6)  
 (1, 2)

|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |

# Pădurea de mulțimi disjuncte la pasul curent



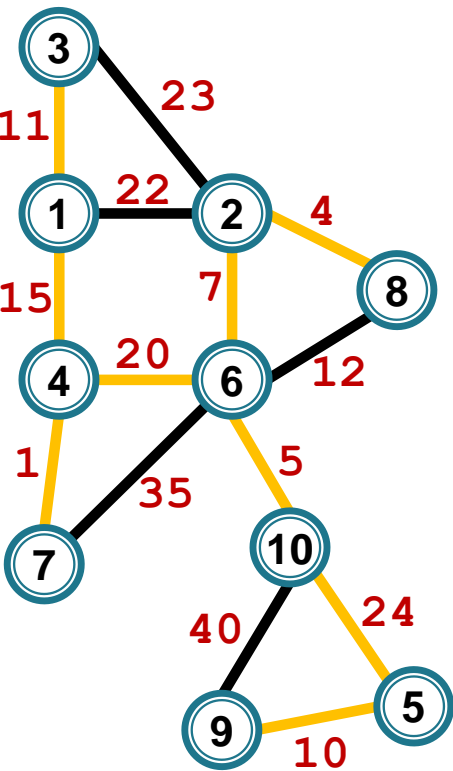
Muchia curentă

(5,10):

$\text{Reprez}(5) \neq \text{Reprez}(10)$

Reuneste(5, 10)

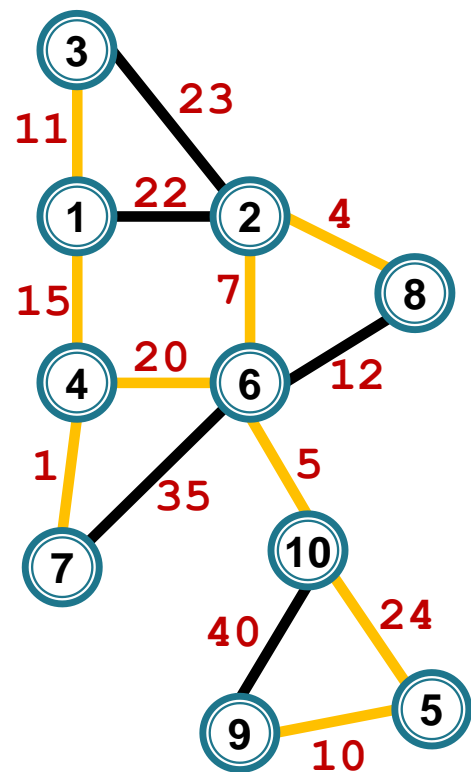
**reuniune ponderată**



Ordine muchii

- (4, 7)
- (2, 8)
- (6, 10)
- (2, 6)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)
- (2, 3)
- (5, 10)**
- (6, 7)
- (9, 10)

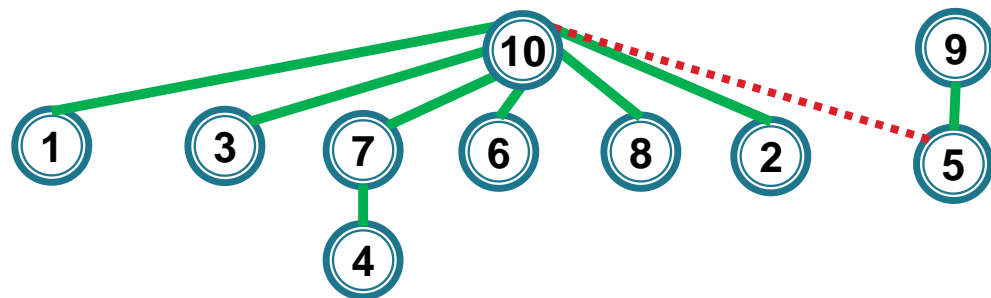
|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9 | 10 |
|------|----|----|----|---|---|----|----|----|---|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | 0 | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1 | 3  |



Ordine muchii

- (4, 7)      (2, 3)
- (2, 8)      **(5, 10)**
- (6, 10)    (6, 7)
- (2, 6)      (9, 10)
- (5, 9)
- (1, 3)
- (6, 8)
- (1, 4)
- (4, 6)
- (1, 2)

Pădurea de mulțimi disjuncte la pasul curent



Muchia curentă

(5, 10):

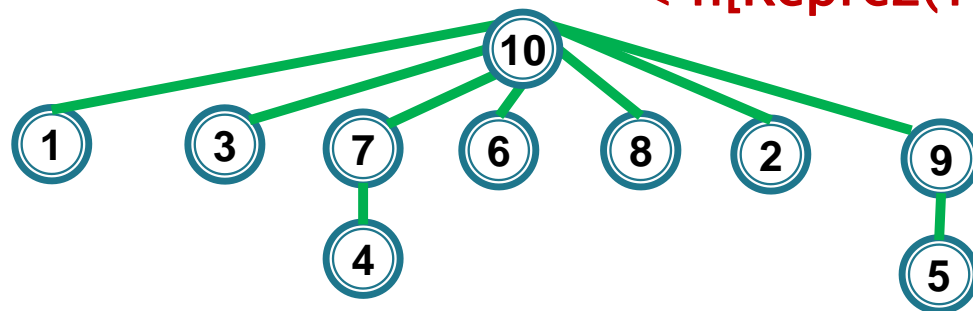
$\text{Reprez}(5) \neq \text{Reprez}(10)$

Reuneste(5, 10)

reuniune ponderată

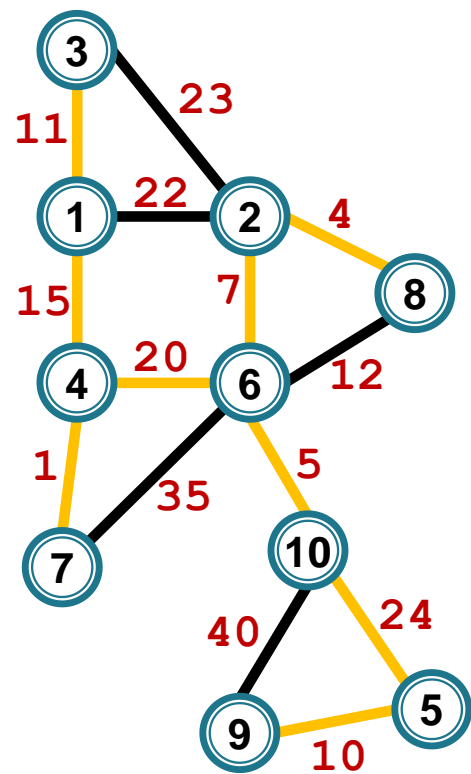
$h[\text{Reprez}(5)] = h[9] = 1$

$< h[\text{Reprez}(10)] = h[10] = 3$



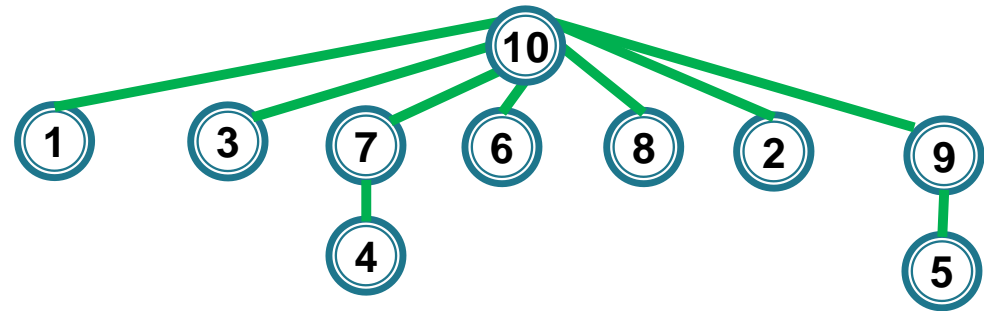
|      | 1  | 2  | 3  | 4 | 5 | 6  | 7  | 8  | 9         | 10 |
|------|----|----|----|---|---|----|----|----|-----------|----|
| tata | 10 | 10 | 10 | 7 | 9 | 10 | 10 | 10 | <b>10</b> | 0  |
| h    | 0  | 0  | 1  | 0 | 0 | 0  | 2  | 1  | 1         | 3  |

# Pădurea de mulțimi disjuncte la pasul curent



Ordine muchii

- |         |         |
|---------|---------|
| (4, 7)  | (2, 3)  |
| (2, 8)  | (5, 10) |
| (6, 10) | (6, 7)  |
| (2, 6)  | (9, 10) |
| (5, 9)  |         |
| (1, 3)  |         |
| (6, 8)  |         |
| (1, 4)  |         |
| (4, 6)  |         |
| (1, 2)  |         |



STOP – au fost selectate  $n-1$  muchii

**Muchii apcm  $\neq$  muchiile din pădurea de mulțimi disjuncte finală (formată dintr-un singur arbore)**

