Seminar 5

$f(x,y,z) = (x-y)^2 (x-z)^2 (y-z)^2$

Scrieți f în fct. de pol. simetrice fundamentale s_1, s_2, s_3.

Atunci am găsit

$$\left[\, f = a s_1^2 s_2^2 + b s_1^3 s_3 + c s_2^3 + d s_1 s_2 s_3 + e s_3^2 \right.$$

Pas I $T(f) = x^2 \cdot x^2 y^2 = x^4 y^2$ (in ordine lexicografică lui f)

$T(f) = x^4 y^2 z^0 \rightsquigarrow$

$$x^{a_1} y^{a_2} z^{a_3} \rightsquigarrow \boxed{s_1^{a_1-a_2} \quad s_2^{a_2-a_3} \quad s_3^{a_3}}$$

$T(f) = s_1^{4-2} s_2^{2-0} s_3^0 = s_1^2 s_2^2$

Pas II Caut monoamele de același grad ca termenul dominant dar mai mici lexico-grafic.

$(4,2,0) \to s_1^2 \cdot s_2^2$

$(4,1,1) \to x^4 y z \rightsquigarrow s_1^{4-2} s_2^{1-1} s_3^1 = s_1^2 s_3$

$(4,0,2) \to s_1^4 s_2^{-2} s_3^2 \to \underline{\underline{NU}}$

$(3,3,0) \to x^3 y^3 \to s_1^{3-3} s_2^{3-0} s_3^0 = s_2^3$

$(3,2,1) \to x^3 y^2 z^1 \to s_1^{3-2} s_2^{2-1} s_3^1 = s_1 s_2 s_3$

$(2,2,2) \to x^2 y^2 z^2 \to s_1^{2-2} s_2^{2-2} s_3^2 = s_3^2$

$f(x,y,z) = a \cdot s_1^2 s_2^2 + b \cdot s_1^3 s_3 + c s_2^3 + d s_1 s_2 s_3 + e s_3^2$

$a, b, c, d = ?$

$a =$ coef. termenului dominant $=$ coef $T(f) = \boxed{1}$

x	y	z	s_1	s_2	s_3	$f(x,y,z)$
1	1	0	2	1	0	$f(1,1,0) = (1-1)^2 \cdot (1-0) \cdot (0-1)^2 = 0$

aleg convenabil

$\boxed{\begin{array}{l} s_2 = xy + yz + xz \\ s_3 = x \cdot y \cdot z. \\ s_1 = x+y+z. \end{array}}$

$2^2 \cdot 1 + c \cdot 1 = 0 \Rightarrow 4 + c = 0 \Rightarrow \boxed{c = -4}$

x	y	z	s_1	s_2	s_3	$f(x,y,z)$
-1	1	0	0	-1	0	$f(-1,1,0)$
-1	-1	2	0	-3	2	0

$x=2$
~~$z=2$~~
$z=-1$
$y=0$

$(-4)(-3)^3 + e \cdot 2^2 = \boxed{e = -27}$

x	y	z	s_1	s_2	s_3	$f(x,y,z)$
1	1	1	3	3	1	0
1	1	-1	1	-1	-1	0

$81 + b\cdot 27 - 4\cdot 27 + d\cdot 9 - 27 = 0$ ~~f(1,1,1)~~

$\boxed{27b + 9d = 54}$

$1 + b(-1) + (-4)\cdot(-1) + d - 27 = 0$

$1 - b + 4 + d - 27 = 0$

$\boxed{-b + d = 22}$

$\begin{cases} -b + d = 22 \\ 27b + 9d = 54 \end{cases}$; $b = -4$, $d = 18$ $\longrightarrow f(x,y,z) = s_1^2 s_2^2 - 4 s_1^3 s_3 - 4 s_2^3 + 18 s_1 s_2 s_3 - 27 s_3^2$

Pt. $n = 2$: $x_1 x_2$

$a x^2 + b x + c = 0$

$a = 0$, $a, b, c \in \mathbb{C}$

are răd distincte $\Longleftrightarrow \Delta \neq 0$.

$\Delta = b^2 - 4ac$

$\boxed{s_1 = x_1 + x_2 = \frac{-b}{a} \\ s_2 = x_1 \cdot x_2 = \frac{c}{a}}$ | Viette

$\Delta = b^2 - 4ac = a^2\left(\frac{b^2}{a^2} - \frac{4c}{a}\right) = a^2\left(s_1^2 - 4 s_2\right)$

$s_1^2 - 4 s_2 = (x_1 + x_2)^2 - 4(x_1 x_2) = (x_1 - x_2)^2$

$\Delta = a^2 (x_1 - x_2)^2 \neq 0 \Longleftrightarrow x_1 \neq x_2$

Pb:

Dat un polinom: $a_n x^n + \dots + a_1 x + a_0$, $a_n \neq 0$.

Det daca ~~care~~ toate răd distincte

$\Delta = \prod (x_i - x_j)^2$, ~~$x_i \neq x_j$~~ $i \neq j$ $= g(s_1, \dots, s_n)$

$s_1 = \frac{-a_{n-1}}{a_n}$

$s_2 = \frac{a_{n-2}}{a_n}$

$s_3 = \frac{-a_{n-3}}{a_n}$

$\vdots$

$\Delta \neq 0 \Longleftrightarrow$ toate răd sunt distincte

② Fie $f(t)=\hat{2}t^3-\hat{5}t^2+\hat{4}t+\hat{1} \in \mathbb{Z}_{11}[t]$
Det. dacă f are răd distincte două câte două.

$s_1=-(-\hat{5})(\hat{2})^{-1}=\hat{5}\cdot\hat{6}=\hat{30}=\hat{8}$

$s_2=\hat{4}\cdot(\hat{2})^{-1}=\hat{4}\cdot\hat{6}=\hat{24}=\hat{2}$

$s_3=(-\hat{1})\cdot(\hat{2})^{-1}=\hat{10}\cdot\hat{6}=\hat{60}=\hat{5}$

$\Delta=(x-y)^2(x-z)^2(y-z)^2=s_1^2 s_2^2-4s_1^3 s_3-4s_2^3+18s_1s_2s_3-27s_3^2$

$\Delta=236-\ldots$

$\Delta\neq 0 \rightarrow$ $\boxed{f(t) \text{ are toate răd dist}}$

③ Fie x_1, x_2, x_3 răd pol $2x^3+5x+6$.
Constr. un pol. cu răd y_1, y_2, y_3 unde:

a) $y_i=x_i+2$

b) $y_i=\frac{1}{x_i+1}$

a) $S_{1y}=y_1+y_2+y_3=S_{1x}+6$

$S_{1x}=0$

$S_{2x}=\frac{5}{2}$

$S_3=-3$

$S_{2y}=y_1y_2+y_1y_3+y_2y_3=(x_1+2)(x_2+2)+(x_1+2)+(x_2+2)(x_3+2)$

$=x_1x_2+2x_2+4+x_1\cdot x_3+2x_1+4+x_2x_3+2x_3+2x_2+4$

$=S_{2x}+12+4x_1+4x_2+4x_3=12+S_{2x}+4S_1=12+\frac{5}{2}+0=\frac{19}{2}$

$S_{3y} = y_1 \cdot y_2 \cdot y_3 = (x_1+2)(x_2+2)(x_3+2)$

$= (x_1x_2 + 2x_1 + 2x_2 + 4)(x_3+2)$ ~~$x_1x_2x_3 + 2x_1x_3 + 2x_2x_3 + 8 = S_{3x}$~~

$= x_1x_2x_3 + 2x_1x_3 + 2x_2x_3 + 4x_3 + 2x_1x_2 + 4x_1 + 4x_2 + 8 =$

$= S_{3x} + 2S_{2x} + 4S_{1x} + 8$

$= -3 + \cancel{2} \cdot \frac{5}{\cancel{2}} + 4 \cdot 0 + 8 = 10$

$P(y) = y^3 - S_{1y}\, y^2 + S_{2y}\, y - S_{3y} = y^3 - 6y^2 + \frac{29}{2} y - 10.$

b) $S_{1y} = y_1 + y_2 + y_3 = \frac{1}{x_1+1} + \frac{1}{x_2+1} + \frac{1}{x_3+1} =$

$= \frac{(x_2+1)(x_3+1) + (x_1+1)(x_3+1) + (x_1+1)(x_2+1)}{(x_1+1)(x_2+1)(x_3+1)}$

$= \frac{x_2x_3 + x + x_3 + 1 + x_1x_3 + x_1 + x_3 + 1 + x_1x_2 + x_1 + x_2 + 1}{(x_1+1)(x_2+1)(x_3+1}$

$= \frac{S_{2x} + 2S_{1x} + 3}{(x_1+1)(x_2+1)(x_3+1)}$

(4) Calculați $x^5 y^5 z^5$, unde x, y, z sunt răd ec. $t^3 + t + 1 = 0$.

$\Delta 1 = 0$
$\Delta 2 = 1$
$\Delta 3 = -1$

x, y, z sunt răd ale ec: $\begin{cases} x^3 + x + 1 = 0 \mid x^2 \\ y^3 + y + 1 = 0 \mid y^2 \\ z^3 + z + 1 = 0 \mid z^2 \end{cases}$ (+)

$x^3 + y^3 + z^3 = -(x+y+z) - 3 \Rightarrow x^3 + y^3 + z^3 = -3$

$(=) \begin{cases} x^5 + x^3 + x^2 = 0 \\ y^5 + y^3 + y^2 = 0 \\ z^5 + y^3 + y^2 = 0 \end{cases}$

$x^5 + y^5 + z^5 = -(\underbrace{x^3 + y^3 + z^3}_{-3}) - (\underbrace{x^2 + y^2 + z^2}_{s_1^2 - 2s_2 = -2}) = +3 + 2 = 5.$

(5) Calc. $x_1^{2018} + x_2^{2018} + x_3^{2018} \pmod{11}$, unde x_1, x_2, x_3 sunt răd ec $x^3 + 2x + 3 = 0$.

$\begin{cases} x_1^3 + 2x_1 + 3 = 0 \\ x_2^3 + 2x_2 + 3 = 0 \\ x_3^3 + 2x_3 + 3 = 0 \end{cases}$ (+)

$x_1^3 + x_2^3 + x_3^3 = -2\underbrace{(x_1 + x_2 + x_3)}_{\Delta 1} - 9$

$$\begin{cases} x_1^3+2x_1+3=0 \mid \cdot x_1 \\ x_2^3+2x_2+3=0 \mid x_2 \\ x_3^2+2x_3+3=0 \mid x_3 \end{cases} \Rightarrow x_1^4+x_2^4+x_3^4=-2\underbrace{(x_1^2+x_2^2+x_3^2)}_{\Delta_1^2-2\Delta_2}\underbrace{-3S_1}_{=0}$$

$$=8.$$

la puterea 5:

$$x_1^5+x_2^5+x_3^5-2(\underbrace{x_1^3+x_2^3+x_3^3}_{-9})-3(\underbrace{\Delta_1^2-2\Delta_2}_{-4})=18+12=30$$

la puterea a 6:

$$x_1^6+x_2^6+x_3^6=-2(x_1^4+x_2^4+x_3^4)-3\underbrace{(x_1^3+x_2^3+x_3^3)}_{=9}=-16+27=11\equiv 0(\bmod 11)$$

$2018:6=336$ rest 2

$$x_1^{2018}+x_2^{2018}+x_3^{2018}=-2(x_1^{2016}+x_2^{2016}+x_3^{2016})-3(x_1^{2015}+x_2^{2015}+x_3^{2015})$$

$$x_1^7+x_2^7+x_3^7=-2(x_1^5+x_2^5+x_3^5)-3(x_1^4+x_2^4+x_3^4)$$

$$=-60-24=-84\equiv -4(\bmod 11)$$

$=$