25.05.2018

SEMINAR 7

$P(x_1^0, \dots\dots, x_n^0)$

$v = (v_1, \dots, v_n)$

$$d: \frac{X_1 - x_1^0}{v_1} = \frac{X_2 - x_2^0}{v_2} = \dots\dots = \frac{X_n - x_n^0}{v_n} = t$$

Ec. param.
$$\begin{cases} X_1 = t \cdot v_1 + x_1^0 \\ \vdots \\ X_n = t \cdot v_n + x_n^0 \end{cases}$$

$P_1(x_1^1, \dots\dots, x_n^1)$

$P_2(x_1^2, \dots\dots, x_n^2)$

$$P_1P_2: \frac{X_1 - x_1^1}{x_1^2 - x_1^1} = \dots\dots = \frac{X_n - x_n^1}{x_n^2 - x_n^1} = t$$

Ex: Ec. param. pt. dreapta ce trece prin punctul $P = (2, 0, 4)$ și are vectorul director $v = (1, 0, -1)$

$$d: \frac{X_1 - 2}{1} = \frac{X_2 - 0}{0} = \frac{X_3 - 4}{-1} = t$$

$$d: \begin{cases} X_1 = t + 2 \\ X_2 = 0 \\ X_3 = -t + 4 \end{cases}$$

$A=(1,-1,2)$, $B=(3,1,-4)$

$$AB: \frac{x_1-1}{3-1}=\frac{x_2+1}{1+1}=\frac{x_3-2}{-4-2}$$

$$AB: \frac{x_1-1}{2}=\frac{x_2+1}{2}=\frac{x_3-2}{-6}$$

Ex 1) Ec. planului ce trece prin $P=(2,0,4)$ și are ca subspațiu director subspațiul generat de vectorii $(1,0,-1)$, $(1,1,-1)$

$$\begin{vmatrix} x_1-2 & 1 & 1 \\ x_2-0 & 0 & 1 \\ x_3-4 & -1 & -1 \end{vmatrix}=0$$

$\cancel{(x_1-2)\cdot 1+(x_2-0)\cdot}$

$$(x_1-2)\cdot(1)+(-1)\cdot(x_2-0)\cdot 0+(x_3-4)\cdot 1=0$$

$$x_1-2+x_3-4=0$$

$$\boxed{\pi: x_1+x_3-6=0}$$

$\mathcal{A}_1=(A_1,V_1,\varphi_1)$

$\mathcal{A}_1 \parallel \mathcal{A}_2 \overset{def}{\iff} V_1\subseteq V_2$ sau $V_2\subseteq V_1$

$d \parallel \pi$ (în $\mathbb{R}^3$)

$$d: \frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}=t$$

$$\pi: ax+by+cz+d=0$$

$$\begin{cases} x = mt + x_0 \\ y = nt + y_0 \\ z = pt + z_0 \\ ax + by + cz + d = 0 \end{cases}$$

$$a(mt + x_0) + b(nt + y_0) + c(pt + z_0) + d = 0$$

$$t(am + bn + cp) + \underbrace{ax_0 + by_0 + cz_0 + d}_{\neq 0} = 0 \quad (\text{pt. că } P \notin \pi)$$

$$P = (x_0, y_0, z_0) \in d$$

$$d \parallel \pi \iff am + bn + cp = 0$$

$$\sum_{i,j=1}^{n} a_{ij} x_i x_j + 2 \sum_{i=1}^{n} b_i x_i + c = 0$$

$$A = (a_{ij})_{i,j=\overline{1,n}}$$

~~$b = (b_1, \ldots, b_n)$~~

$$B = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$\overline{A} = \left(\begin{array}{c|c} A & B \\ \hline {}^tB & c \end{array}\right)$$

$$\delta = \det A$$

$$\Delta = \det \overline{A}$$

$$d(A,B) = \|\overrightarrow{AB}\|$$

$$\overrightarrow{AB}$$

Ex: $\Gamma: 5x^2+8xy+5y^2-18x-18y+9=0$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0 \quad \text{elipsă}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0 \quad \text{hiperbolă}$$

$$x^2=2py \quad , p\geq 0$$

Consider forma pătratică q_h ~~$(x,y)=5x^2+8xy$~~

$q_h(x,y)=5x^2+8xy+5y^2$ (Folosesc un algoritm ~~...~~ la aducere la forma canonică ... de ex Gauss)

$$A=\begin{pmatrix} 5 & 4 \\ 4 & \boxed{5} \end{pmatrix}$$

· Aflu valorile proprii

$$\begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix}=0 \Leftrightarrow (5-\lambda)^2-16=0$$

$$(5-4-\lambda)(5-\lambda+4)=0$$

$$(1-\lambda)(9-\lambda)=0$$

$$\lambda_1=1 \quad , \lambda_2=9$$

$$V_{\lambda}=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, A\cdot\begin{pmatrix}x\\y\end{pmatrix}=\lambda\begin{pmatrix}x\\y\end{pmatrix}\right\}$$

$$V_{\lambda_1=1}=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, \begin{cases}5x+4y=x\\4x+5y=y\end{cases}\right\}$$

$$\Leftrightarrow 4x+4y=0$$

$$x=-y$$

$V_{\lambda_1} = \{ (x, -x) \mid x \in \mathbb{R} \}$

Alegem o bază ~~ortogonală~~ ortonormată în V_{λ_1}

$x \cdot (1, -1)$

$$\|(1,-1)\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$\Rightarrow$ baza $B_{\lambda_1} = \{ (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \} = e_1$

$$V_{\lambda_2} = \{ (x,y) \in \mathbb{R}^2 \mid A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 9 \begin{pmatrix} x \\ y \end{pmatrix} \}$$

$$= \{ (-x, x) \mid x \in \mathbb{R} \}.$$

aleg o bază ortonormată în V_{λ_2}: $\{ (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \} = e_2$

$$R = \begin{pmatrix} e_1 & e_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\det R = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow R$ e o rotație.

Efectuăm rotația

$$\begin{pmatrix} x \\ y \end{pmatrix} = R \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \Leftrightarrow \begin{cases} x = \frac{1}{\sqrt{2}} x' + \frac{1}{\sqrt{2}} y' \\ y = -\frac{1}{2} x' + \frac{1}{\sqrt{2}} y' \end{cases}$$

$$x \cdot y = \left(\frac{1}{\sqrt{2}} y'\right)^2 - \left(\frac{1}{\sqrt{2}} x'\right)^2$$

Γ va avea ~~ecuația~~ urm. formă: $\Gamma: 9(x')^2 + (y')^2 -$
$- 18\sqrt{2}\, x' + 9 = 0$

$$(3x')^2 - 2\cdot 3x'\cdot 3\sqrt{2} + (3\sqrt{2})^2 - (3\sqrt{2})^2$$

$$= (3x' - 3\sqrt{2})^2 - 18$$

$$\Gamma: (3x' - 3\sqrt{2})^2 - 18 + (y')^2 + 9 = 0$$

$$\Gamma: 9(x' - \sqrt{2})^2 + (y')^2 - 9 = 0$$

Facem translația:

$$\begin{cases} x'' = x' - \sqrt{2} \\ y'' = y' \end{cases} \Rightarrow \Gamma: \cancel{9x} \; (9x'')^2 + (y'')^2 - 9 = 0$$

$$\Gamma \Big| \cdot \frac{1}{9}$$

$$\Rightarrow \Gamma: \frac{(x'')^2}{1} + \frac{(y'')^2}{9} - 1 = 0$$

$$x = x''$$

$$y = y''$$

$$\Gamma: \frac{x^2}{1} + \frac{y^2}{9} - 1 = 0 \quad \text{(elipsă)}$$

$$P(0, 3) \in \Gamma$$

$T_P\Gamma$ (dreapta tg în P la Γ)

$T_P\Gamma$

$\cancel{\mathcal{E}: \frac{x^2}{a^2} \; x^2}$

$$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

$$P(x_0, y_0) \in \mathcal{E}$$

$$T_{P,\mathcal{E}}: \frac{x \cdot x_0}{a^2} + \frac{y \cdot y_0}{b^2} - 1 = 0$$

$$T_{P,\Gamma}: \frac{x \cdot 0}{1} + \frac{y \cdot 3}{9} - 1 = 0$$

$$\frac{y}{3} - 1 = 0$$

$$\boxed{y = 3}$$

$$\Gamma: \underbrace{x^2 + y^2 + 5z^2 - 6xy + 2xz - 2yz} - 4x + 8y - 12z + 14 = 0$$

Consider forma patratica $q(x,y,z) = x^2 + y^2 + 5z^2 - 6xy + 2xz - 2yz$

și o aduc la forma canonică.

$$A = \begin{pmatrix} 1 & -3 & 1 \\ -3 & 1 & -1 \\ 1 & -1 & 5 \end{pmatrix}$$

· Val. proprii $\begin{vmatrix} 1-\lambda & -3 & 1 \\ -3 & 1-\lambda & -1 \\ 1 & -1 & 5-\lambda \end{vmatrix} = 0$

$$(\lambda+2)(\lambda^2 - 9\lambda + 18) = 0$$

$\lambda_1 = -2$

$\lambda_2 = 3$

$\lambda_3 = 6$

$$V_{\lambda_1} = \left\{(x,y,z) \in \mathbb{R}^3 \mid A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -2\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right\}$$

$$= \{(x, x, 0) \mid x \in \mathbb{R}\}$$

O bază ortonormată în V_{λ_1}: $\{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)\}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad e_1$

$V_{\lambda_2}: \{(x,y,z) \in \mathbb{R}^3 \mid A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix}\}$

$V_{\lambda_2}: \begin{cases} x - 3y + z = 3x \\ -3x + y - z = 3y \\ x - y + 5z = 3z \end{cases}$

$\begin{cases} -2x - 3y + z = 0 \\ -3x - 2y - z = 0 \\ x - y + 2z = 0 \end{cases} (+)$

$-5x - 5y = 0 \Rightarrow x = -y$

$2x + 2z = 0$

$2x = -2z$

$x = -z$

$V_{\lambda_2} = \{(x, -x, -x) \mid x \in \mathbb{R}\}$

bază ortonormată: $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad e_2$

$V_{\lambda_3} = \{(x, -x, 2x) \mid x \in \mathbb{R}\}$

bază ortonormată: $\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad e_3$

$$R = (e_1 \; e_2 \; e_3) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \end{pmatrix}$$

$\det R = 1 \Rightarrow R$ e rotație

* <u>OBSV:</u> dacă îmi dă $R = -1$ schimb

2 coloane între a.î. să dea 1 !

Efectuăm rotația $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

$$X = \frac{1}{\sqrt{2}} x' + \frac{1}{\sqrt{3}} y' + \frac{1}{\sqrt{6}} z'$$

$$Y = \frac{1}{\sqrt{2}} x' - \frac{1}{\sqrt{3}} y - \frac{1}{\sqrt{6}} z'$$

$$Z = -\frac{1}{\sqrt{3}} y' + \frac{2}{\sqrt{6}} z'$$

$$\Gamma: -2(x')^2 + 3(y')^2 + 6(z')^2$$
$$-4\left(\frac{1}{\sqrt{2}} x' + \frac{1}{\sqrt{3}} y' + \frac{1}{\sqrt{6}} z'\right) + 8\left(\frac{1}{\sqrt{2}} x' - \frac{1}{\sqrt{3}} y' - \frac{1}{\sqrt{6}} z'\right)$$
$$+ (-12)\left(-\frac{1}{\sqrt{3}} y' + \frac{2}{\sqrt{6}} z'\right) + 14 = 0$$

$$\Gamma: -2(x')^2 + 3(y')^2 + 6(z')^2 + \frac{4}{\sqrt{2}} x' - \frac{24}{\sqrt{6}} z' + 14 = 0$$

$$\frac{\cancel{2}4\sqrt{2}}{\cancel{2}} x' \qquad \frac{4\cancel{24}\sqrt{6}}{\cancel{6}} z'$$

$$-2\left((x')^2 + \sqrt{2} x' + \left(\frac{\sqrt{2}}{2}\right)^2\right) + 1$$

$$6(z')^2 - 4\sqrt{6} z'$$

$$(\sqrt{6} z')^2 - 2\sqrt{6} z' \cdot 2 + 4 - 4$$

$$\Gamma: -2 \cdot \left(x' + \frac{\sqrt{2}}{2}\right)^2 + 3(y')^2 + (\sqrt{6} z' - 2)^2 + 1 - 4 + 14 = 0$$

$$6\left(z' - \frac{2}{\sqrt{6}}\right)^2$$

$$x'' = x' + \frac{\sqrt{2}}{2}$$

$$y'' = y'$$

$$z'' = z' - \frac{2}{\sqrt{6}}$$

$$\Gamma: -2(x'')^2 + 3(y')^2 + 6(z'')^2 + 11 = 0 \;\Big|\cdot\left(-\frac{1}{11}\right)$$

$$\Gamma : \frac{(x'')^2}{\frac{11}{2}} + \frac{(y'')^2}{\frac{11}{3}} + \frac{(z'')^2}{\frac{11}{6}} - 1 = 0$$

hiperboloid cu două pânze.