

Muchii critice

Muchii critice

- ▶ O muchie este critică \Leftrightarrow nu este conținută într-un ciclu
- ▶ Găsirea unui ciclu – parcurgere DF
 - **muchii de avansare** – ale arborelui DF (memorat cu vector tata), prin care se descoperă vârfuri noi
 - **muchii de întoarcere** – închid ciclu, nu pot fi critice

Muchii critice



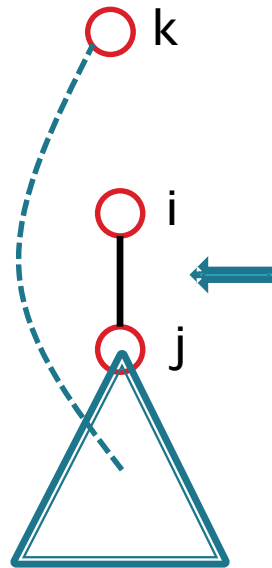
Cum testăm dacă o muchie de avansare (i,j) este critică?

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Cum testăm dacă o muchie de avansare (i,j) este critică?



- nu este conținută într-un ciclu închis de o muchie de întoarcere



Muchii critice

O muchie de avansare (i,j) este critică

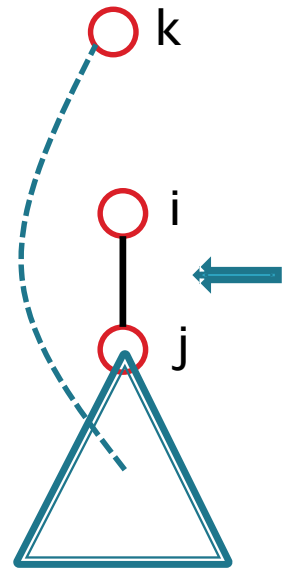
\Leftrightarrow

nu este conținută într-un ciclu închis de o muchie de întoarcere

\Leftrightarrow

nu există nicio muchie de întoarcere cu

- o extremitate în j sau într-un descendent al lui j și
- cealaltă extremitate în i sau într-un ascendent al lui i (într-un vârf de pe un nivel mai mic sau egal cu nivelul lui i)

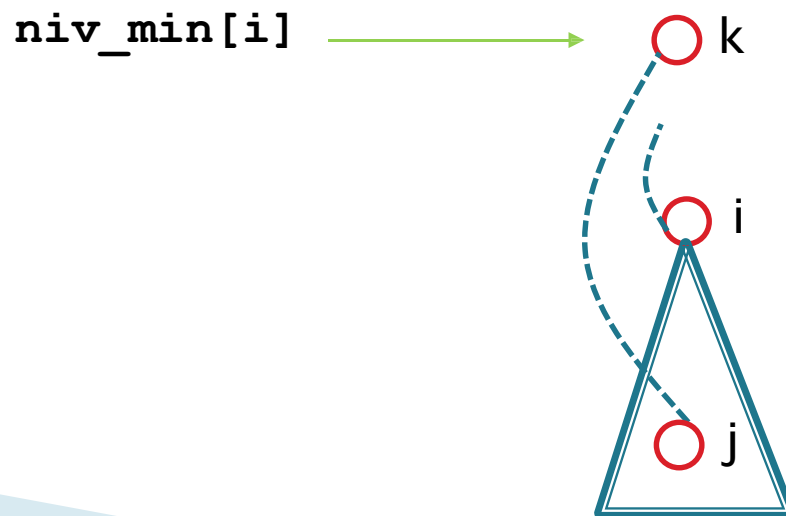


Muchii critice

Memorăm pentru fiecare vârf i :

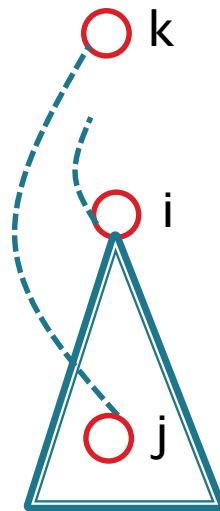
$niv_min[i]$ = nivelul minim al unui vârf care este extremitate a unei muchii de întoarcere din i sau dintr-un descendent al lui i

= nivelul minim la care se închide un ciclu elementar care conține vârful i (printr-o muchie de întoarcere)



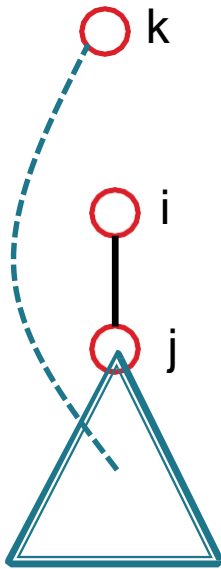
Muchii critice

- ▶ $\text{nivel}[i] = \text{nivelul lui } i \text{ în arborele DF}$
- ▶ $\text{niv_min}[i] = \min \{ \text{nivel}[i], A, B \}$
 - $A = \min \{ \text{nivel}[k] \mid i \text{ k muchie de întoarcere} \}$
 - $B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, j \text{ k muchie de întoarcere} \}$



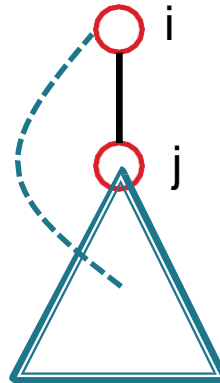
Muchii critice

O muchie de avansare ij este critică \Leftrightarrow



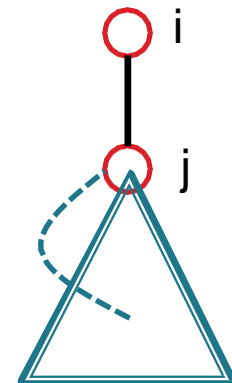
NU este critică

$\text{niv_min}[j] < \text{nivel}[i]$



NU este critică

$\text{niv_min}[j] = \text{nivel}[i]$

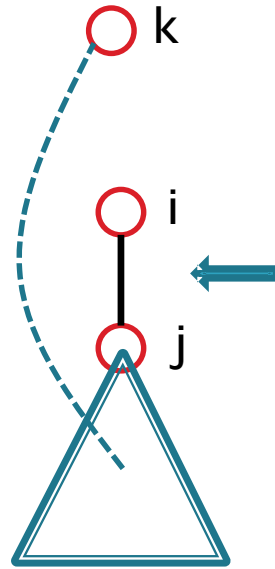


ESTE critică

$\text{niv_min}[j] > \text{nivel}[i]$

Muchii critice

O muchie de avansare ij este critică $\Leftrightarrow \text{niv_min}[j] > \text{nivel}[i]$



Muchii critice



Cum calculăm eficient `niv_min[i]` ?

$$\text{niv_min}[i] = \min \{ \text{nivel}[i], A, B \}$$
$$A = \min \{ \text{nivel}[k] \mid i \text{ k muchie de întoarcere} \}$$
$$B = \min \{ \text{nivel}[k] \mid j \text{ descendent al lui } i, \\ j \text{ k muchie de întoarcere} \}$$

Muchii critice

Cum calculăm eficient `niv_min[i]` ?

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$$A = \min \{ \text{nivel}[k] \mid i \text{ k muchie de întoarcere} \}$$

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B se poate calcula recursiv

Muchii critice

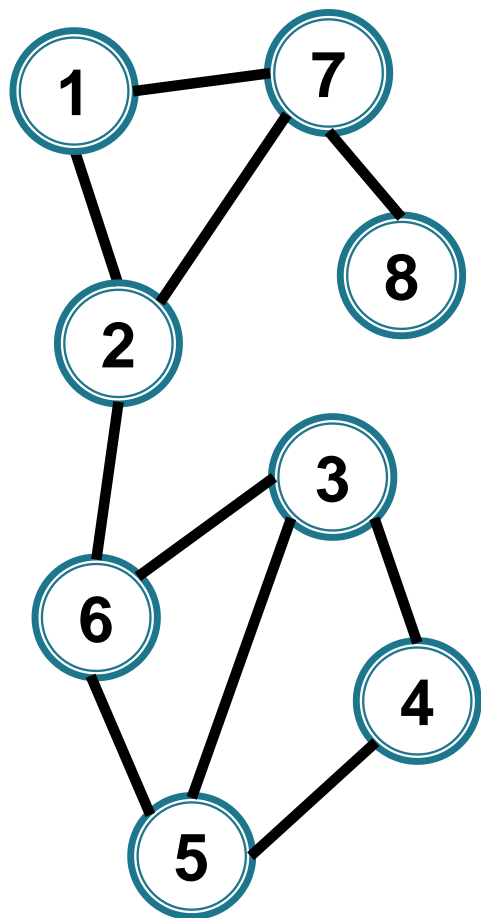
Cum calculăm eficient $niv_min[i]$?

$$niv_min[i] = \min \{ nivel[i], A, B \}$$

$$A = \min \{ nivel[k] \mid i \rightarrow k \text{ muchie de întoarcere} \}$$

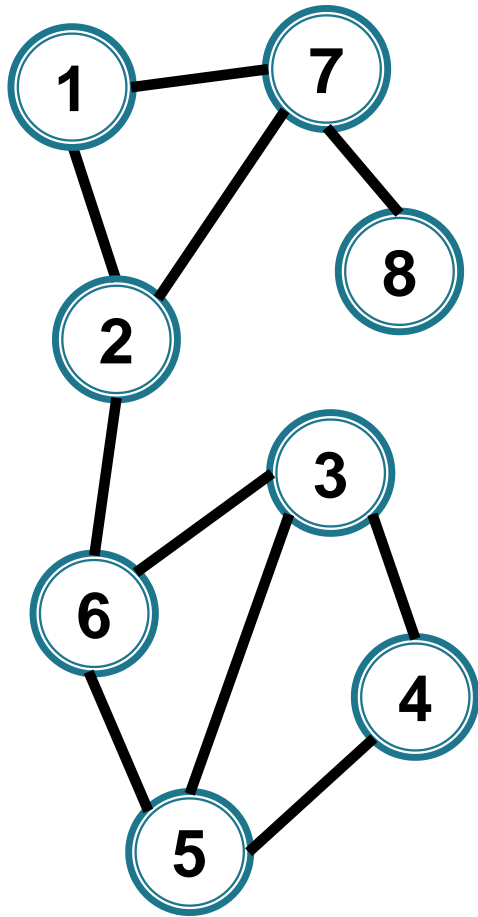
$$B = \min \{ nivel[k] \mid j \text{ descendent al lui } i, \\ j \rightarrow k \text{ muchie de întoarcere} \}$$

$$B = \min \{ niv_min[j] \mid j \text{ fiu al lui } i \}$$

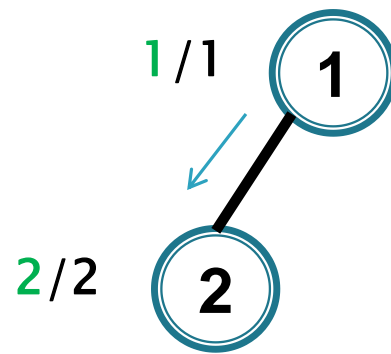
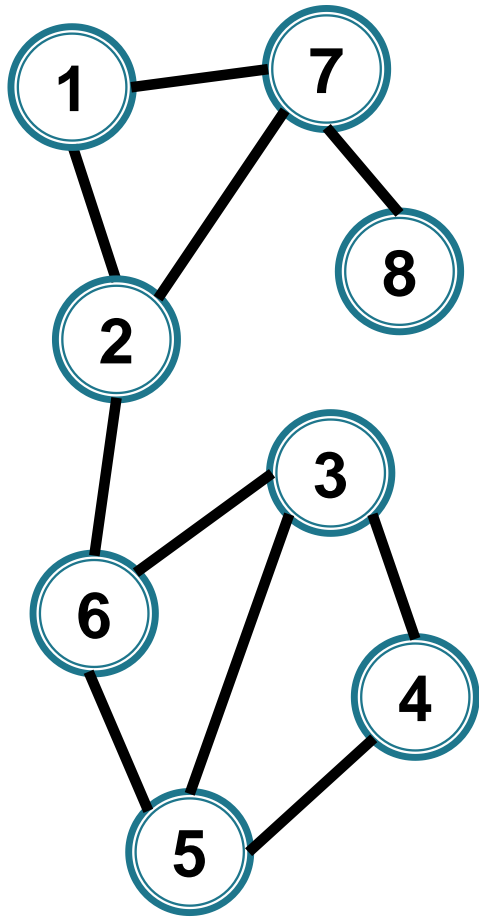


nivel/niv_min

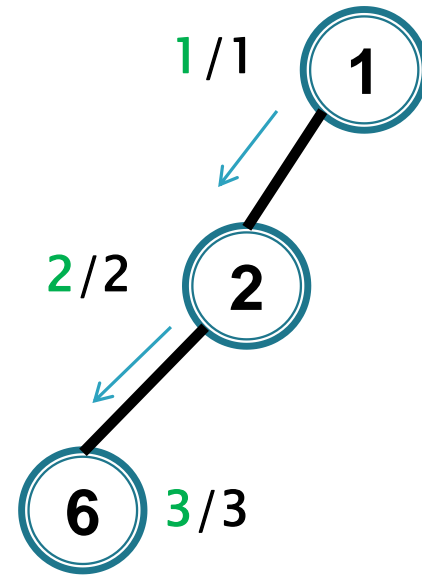
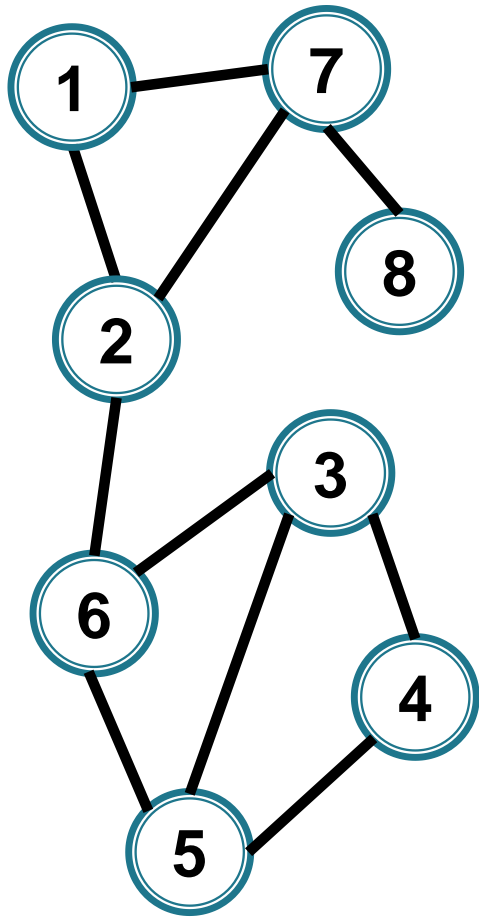
1/1



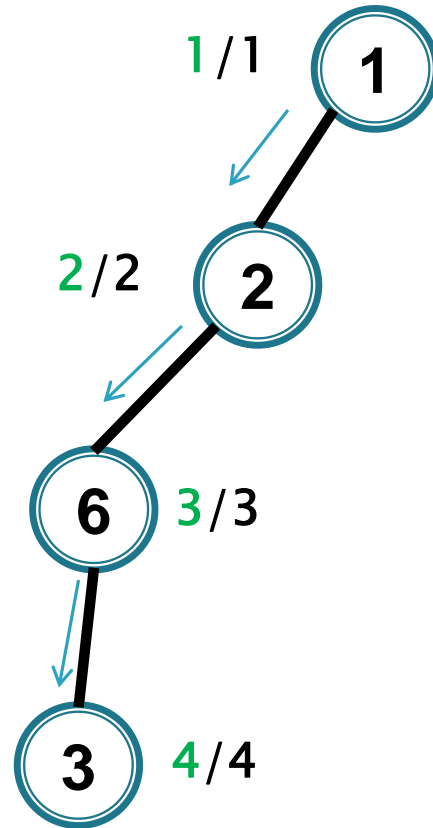
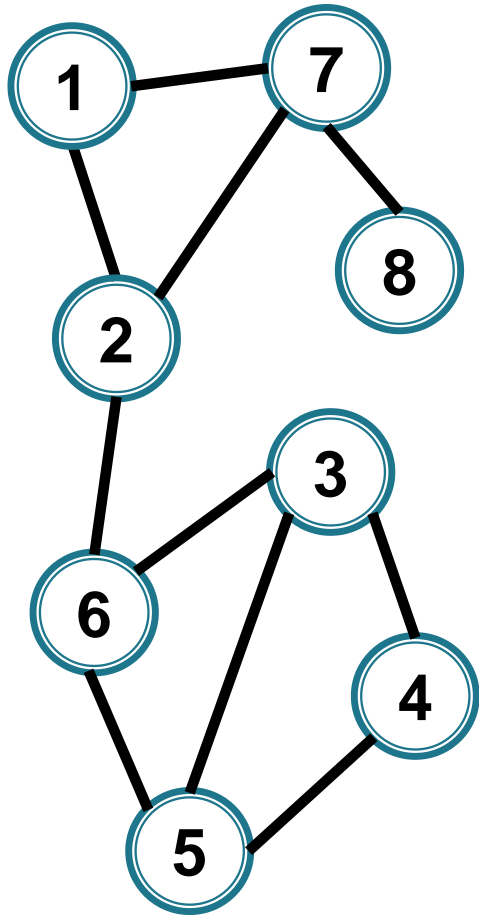
nivel/niv_min



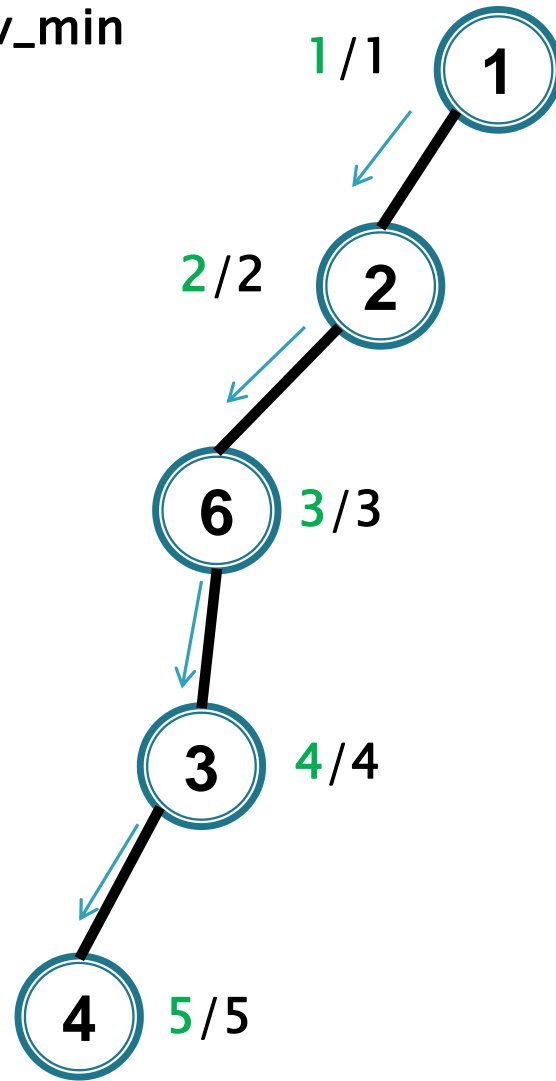
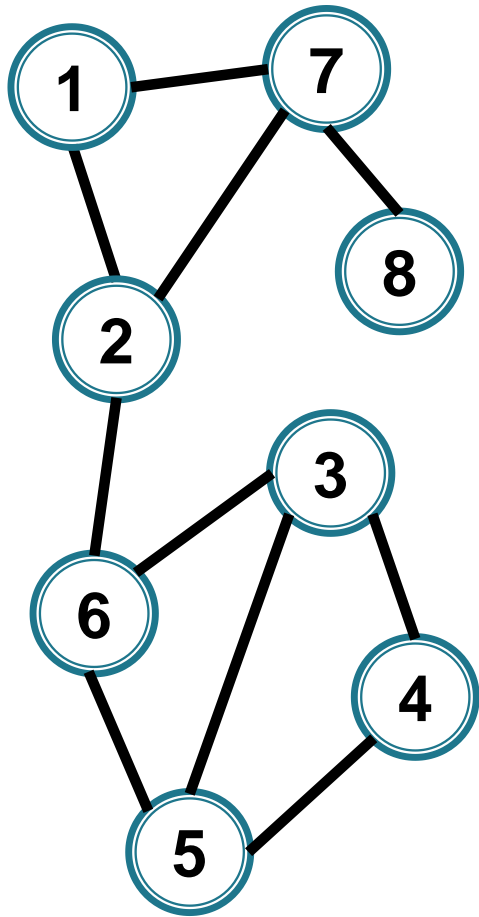
nivel/niv_min

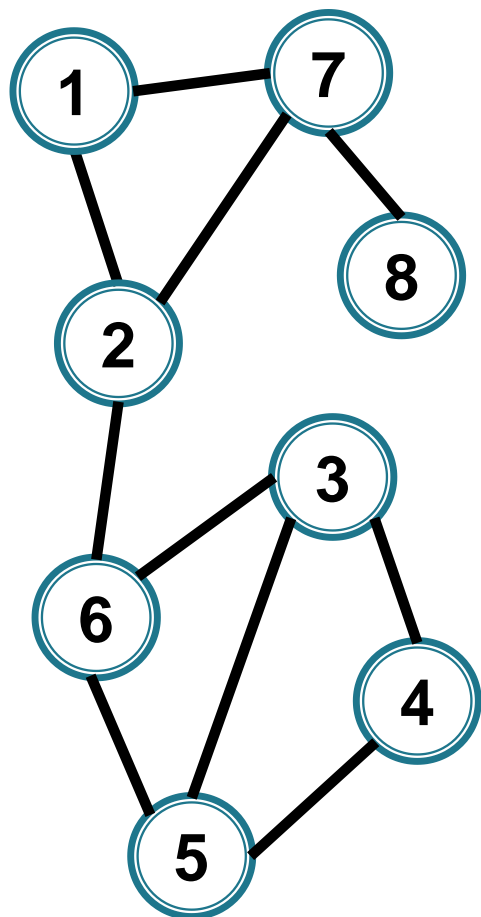


nivel/niv_min

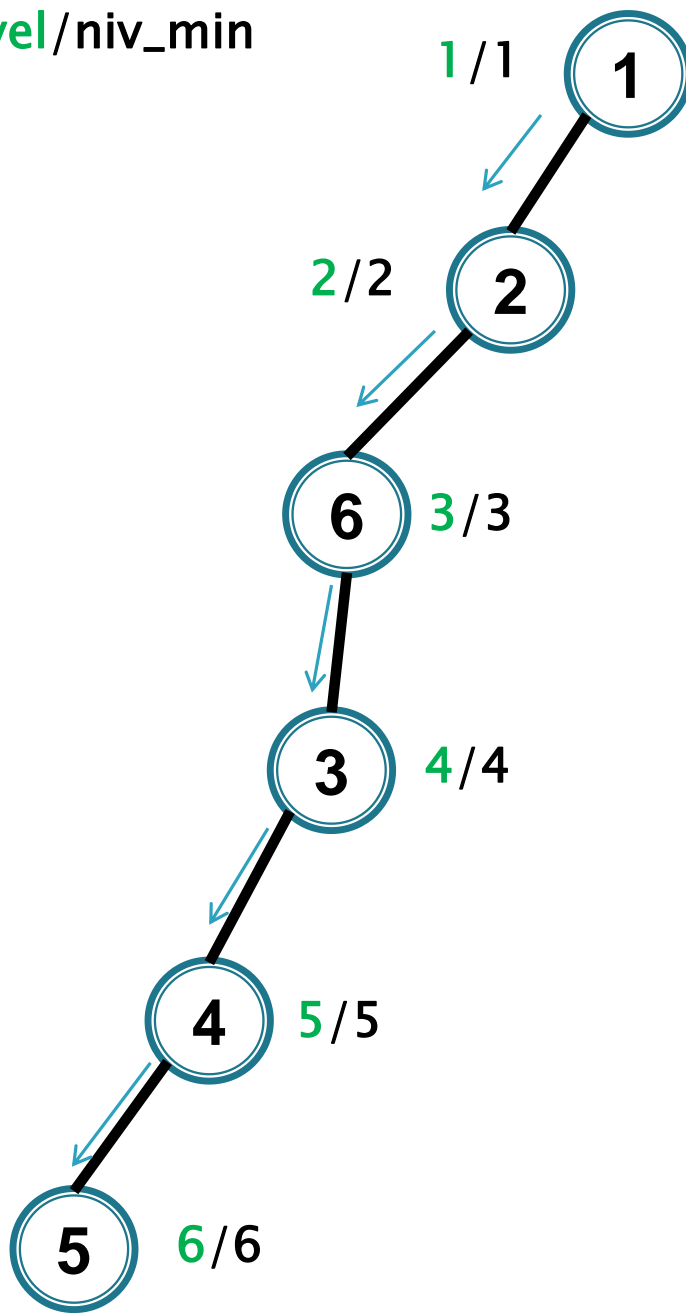


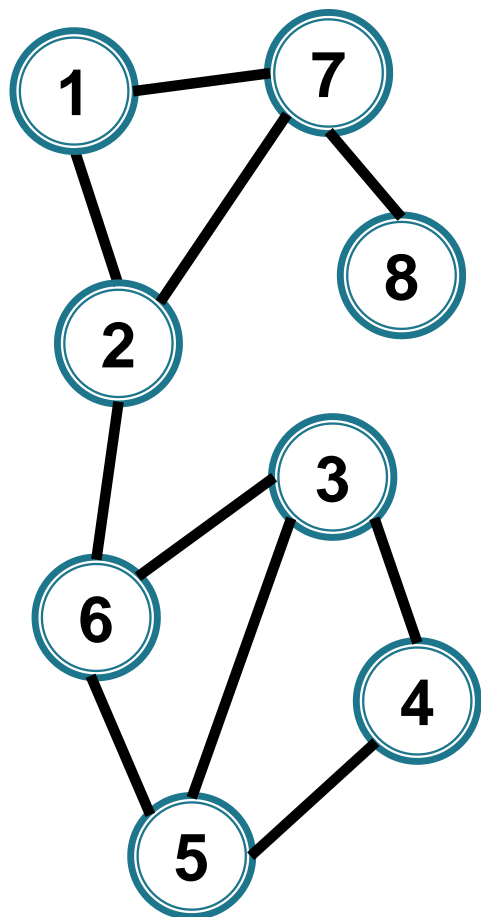
nivel/niv_min



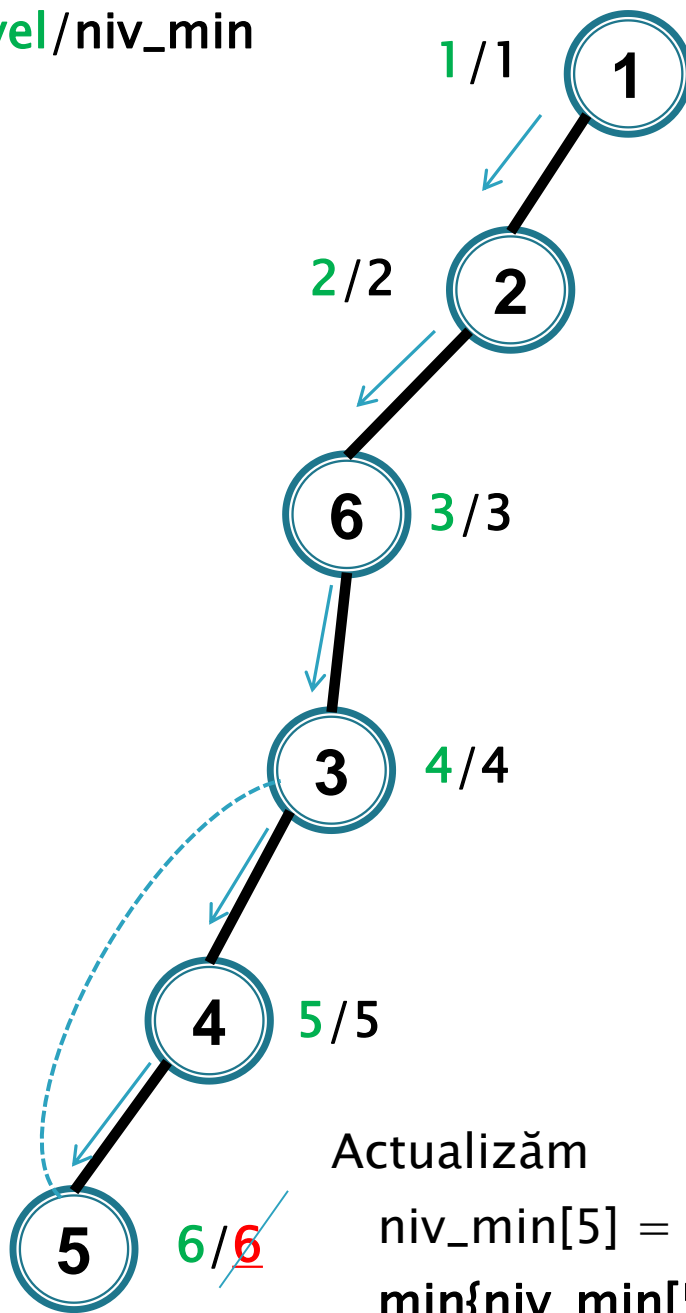


nivel/niv_min





nivel/niv_min

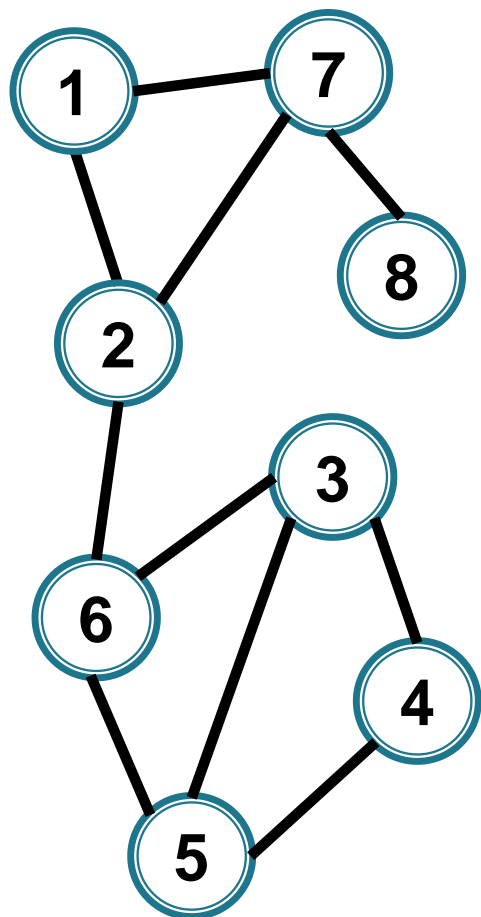


Actualizăm

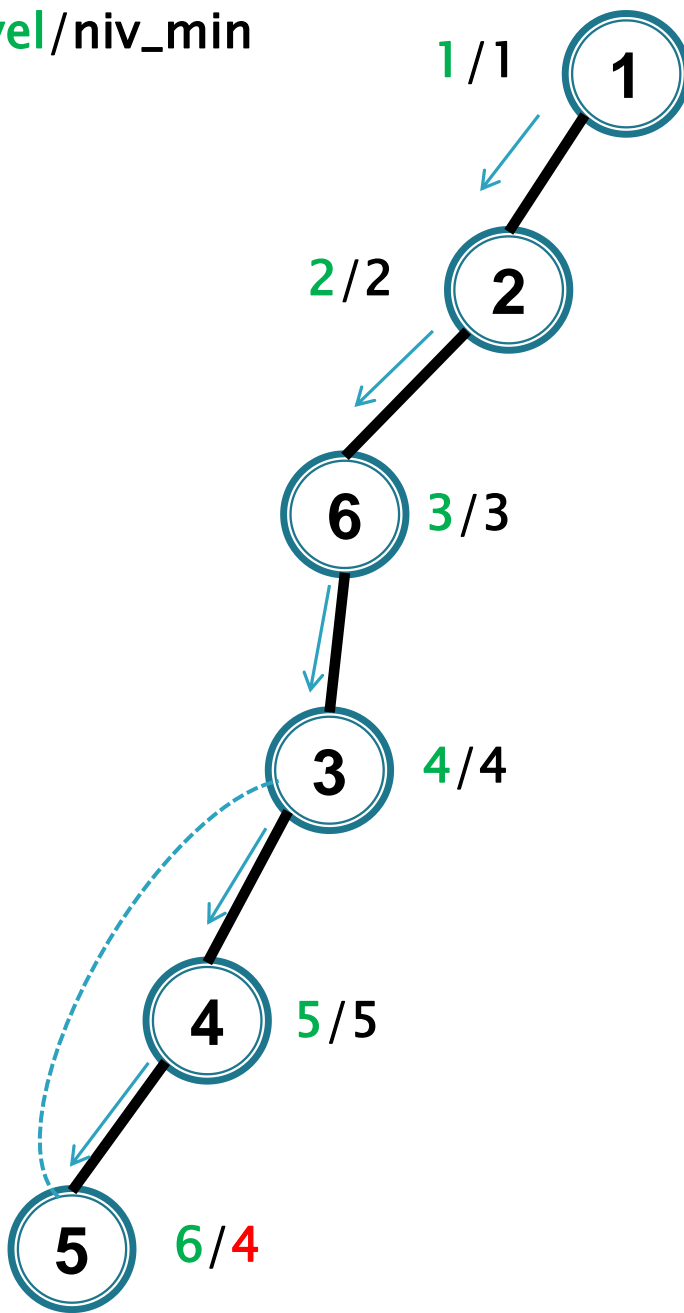
$\text{niv_min}[5] =$

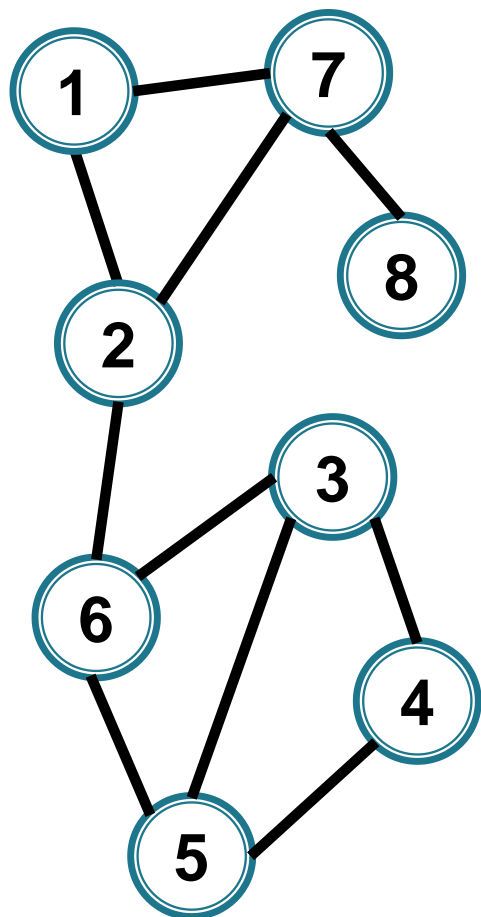
$\min\{\text{niv_min}[5], \text{nivel}[3]\}$

(cazul A)

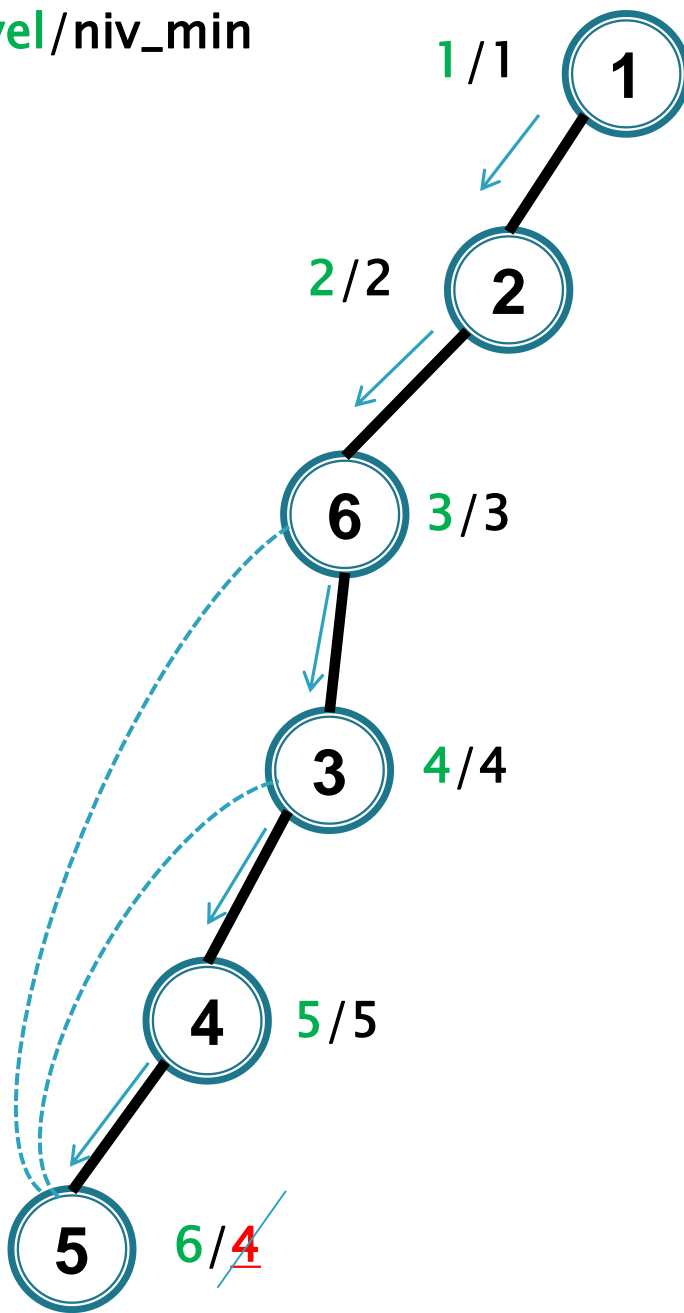


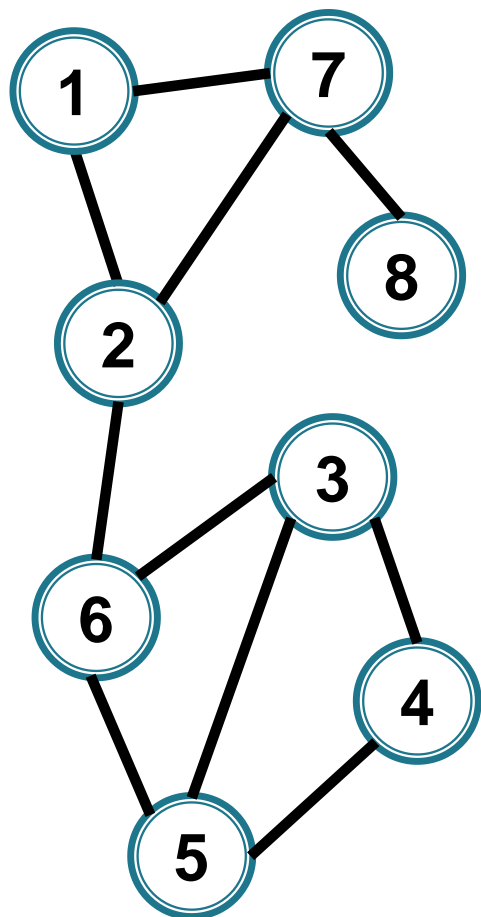
nivel/niv_min



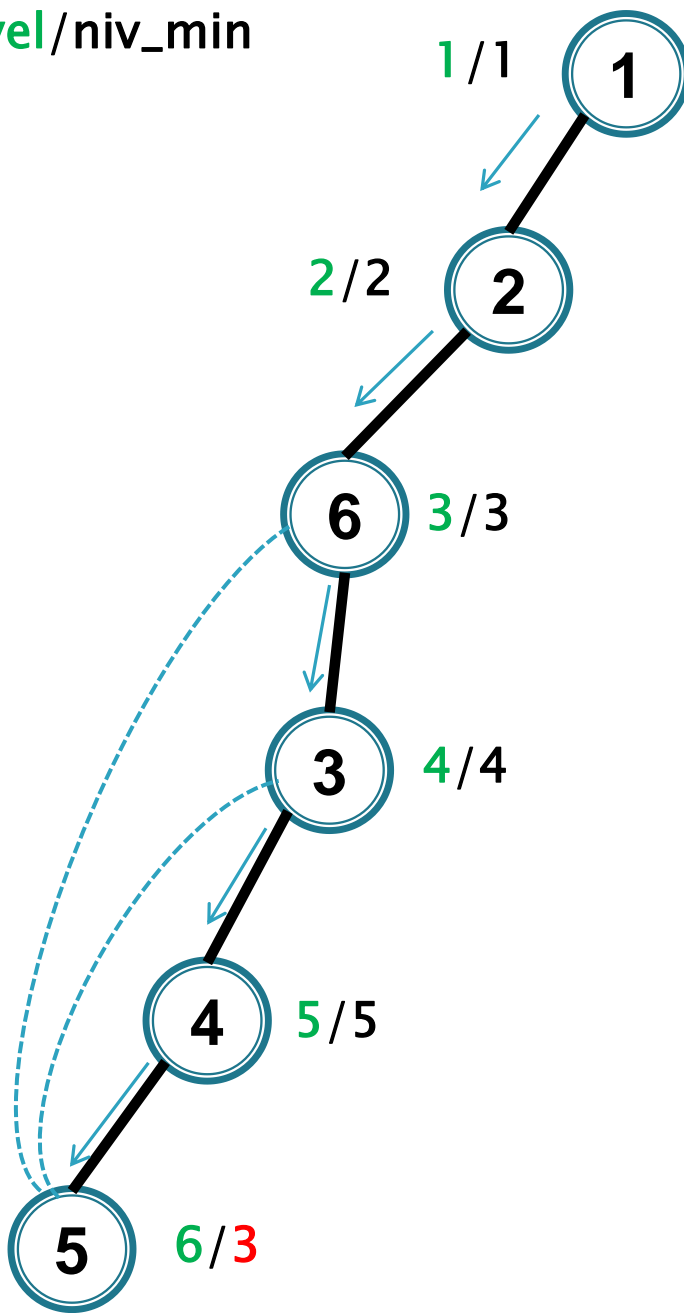


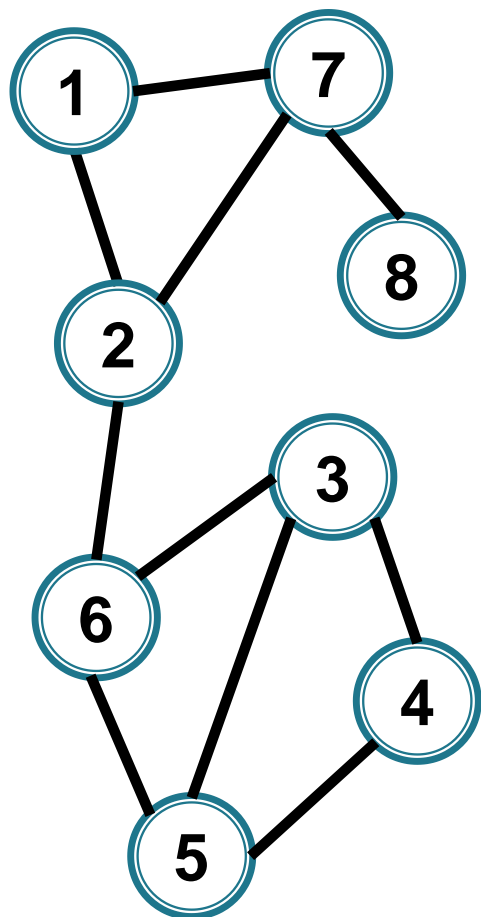
nivel/niv_min



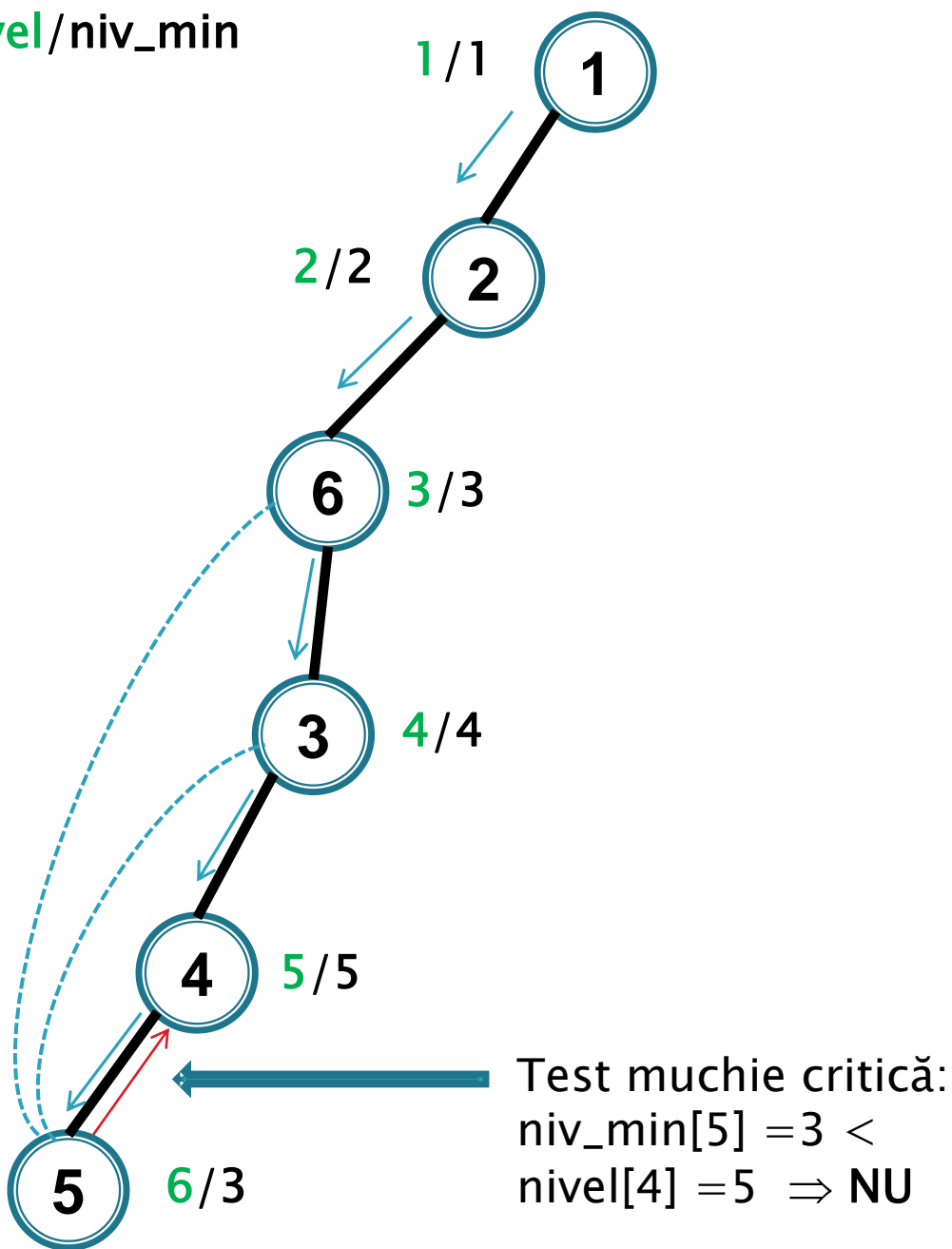


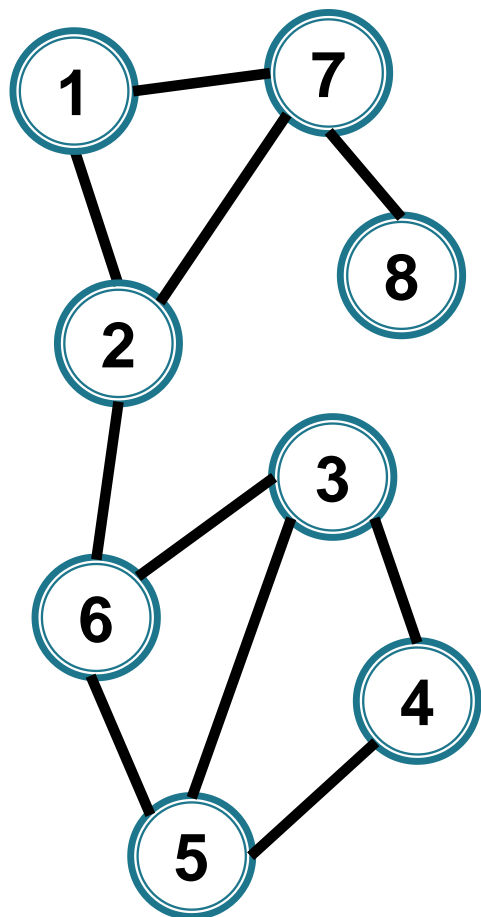
nivel/niv_min



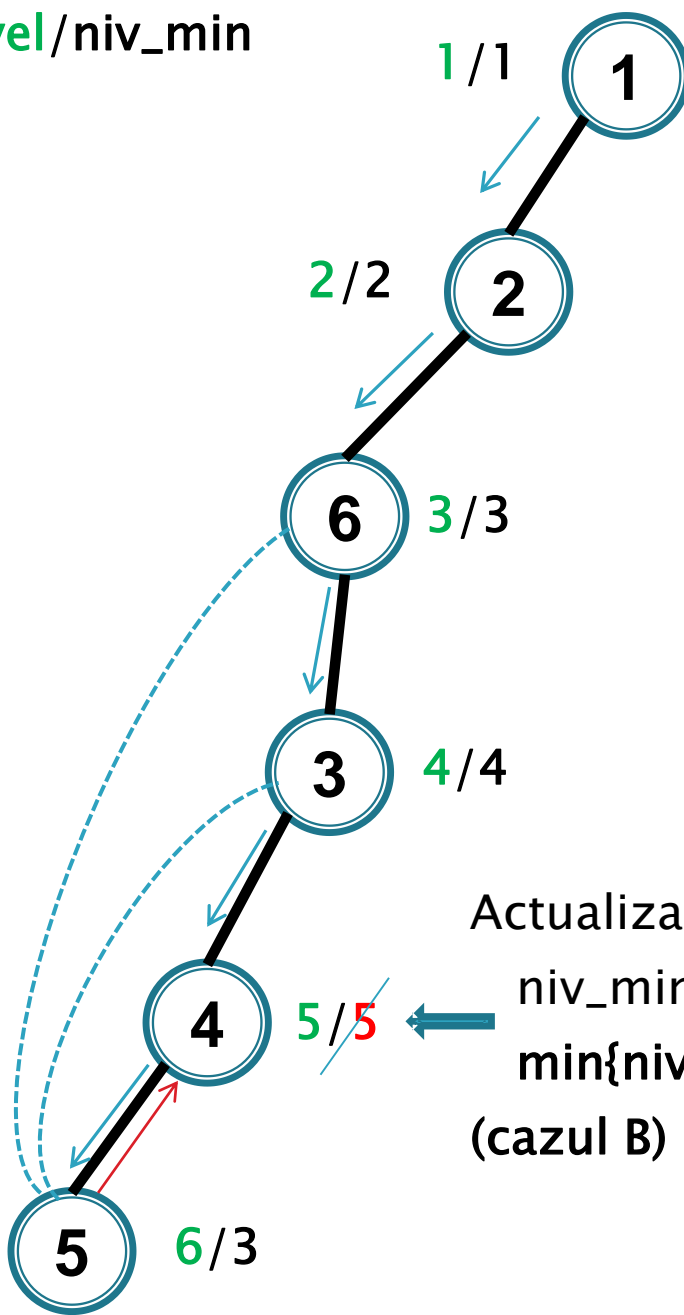


nivel/niv_min





nivel/niv_min

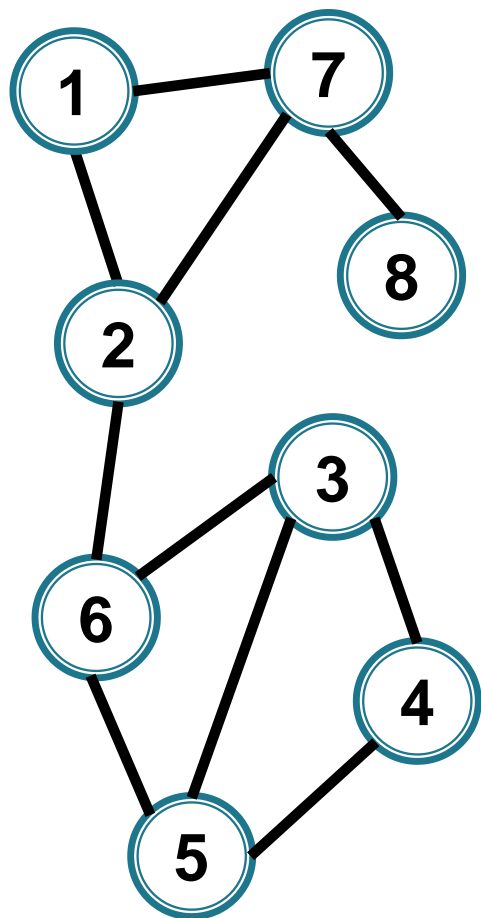


Actualizam

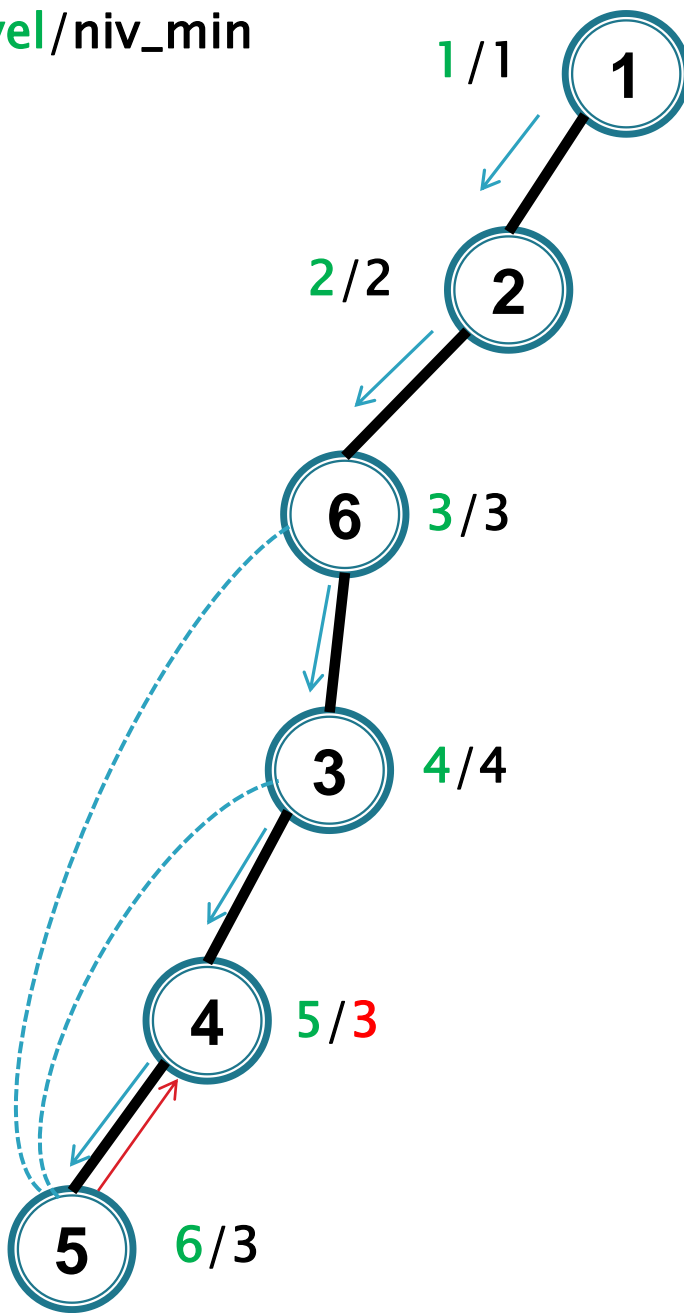
$niv_min[4] =$

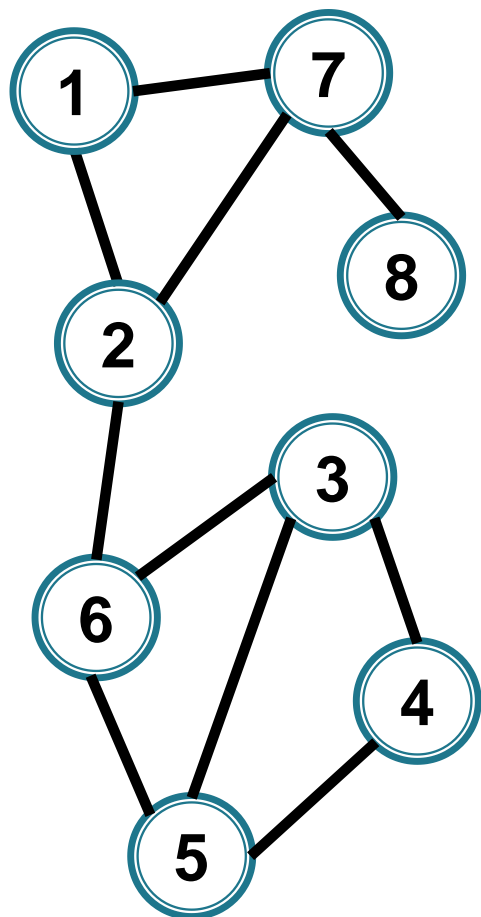
$\min\{niv_min[4], niv_min[5]\}$

(cazul B)

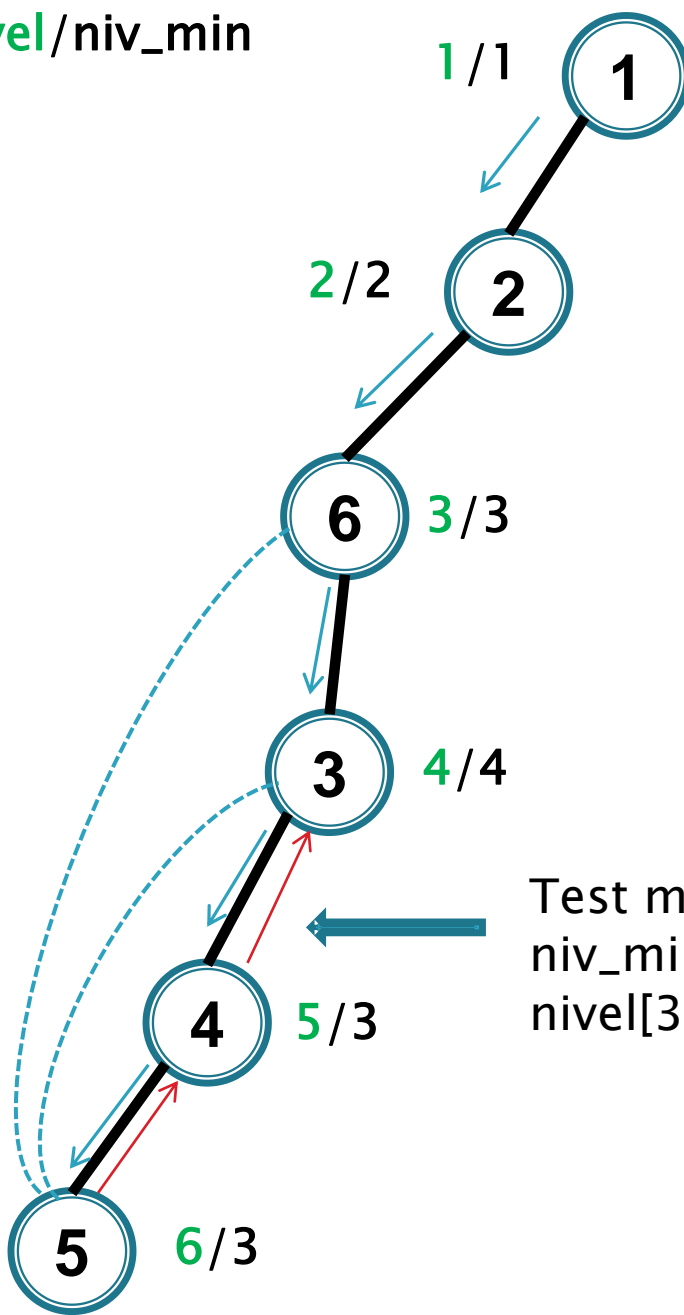


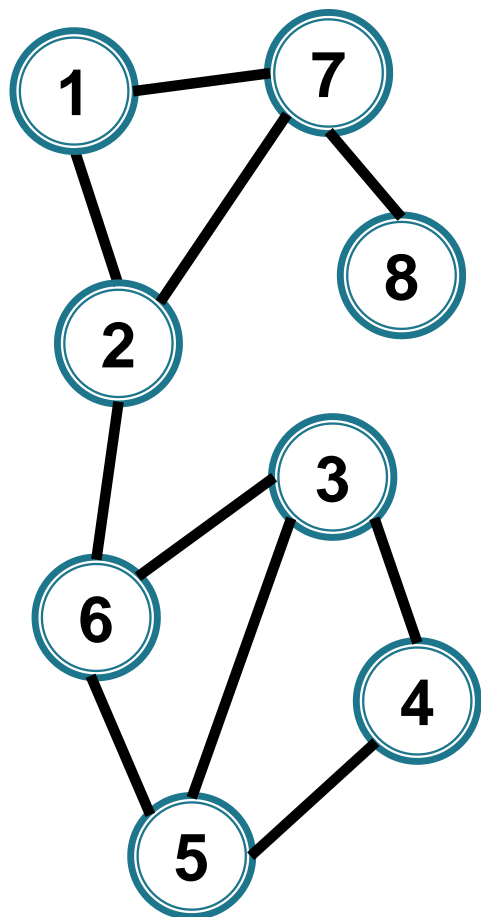
nivel/niv_min



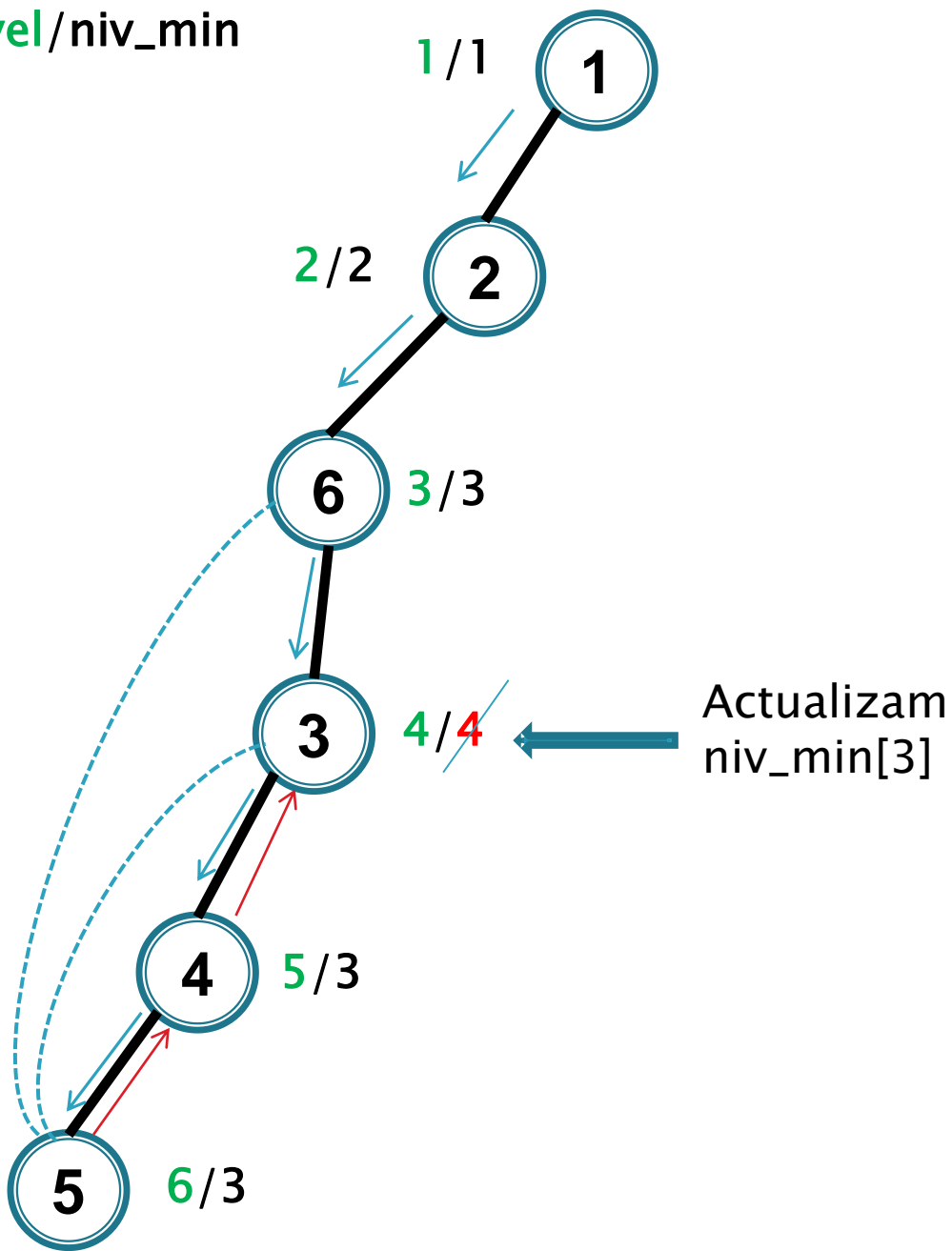


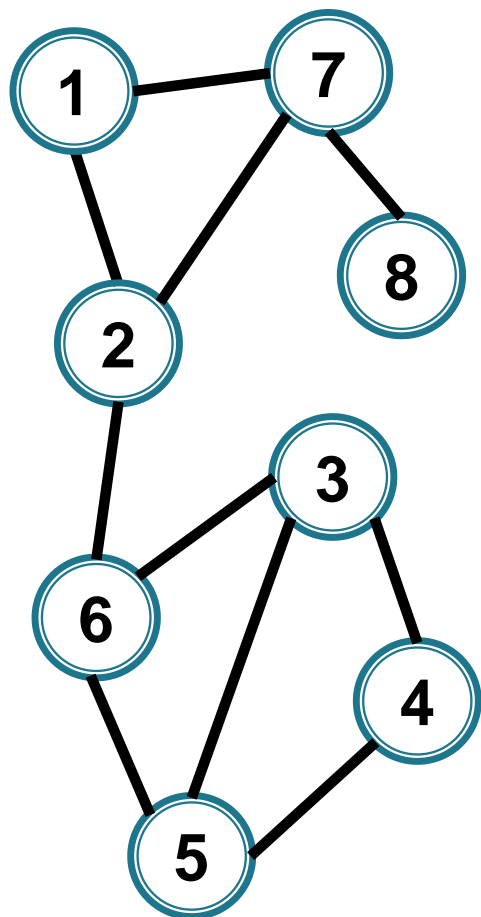
nivel/niv_min



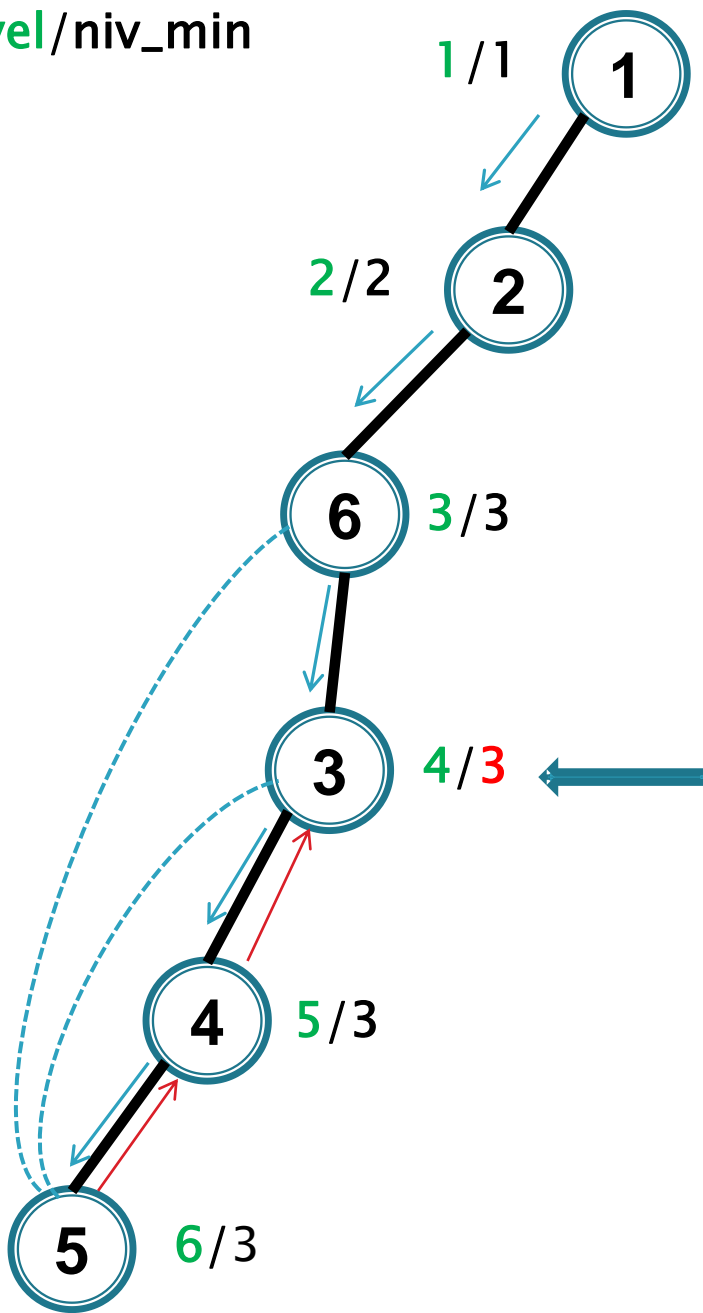


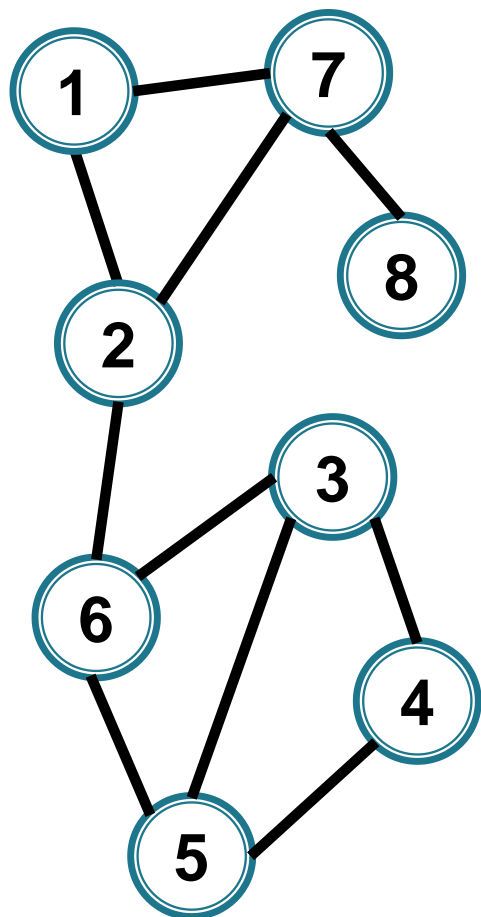
nivel/niv_min



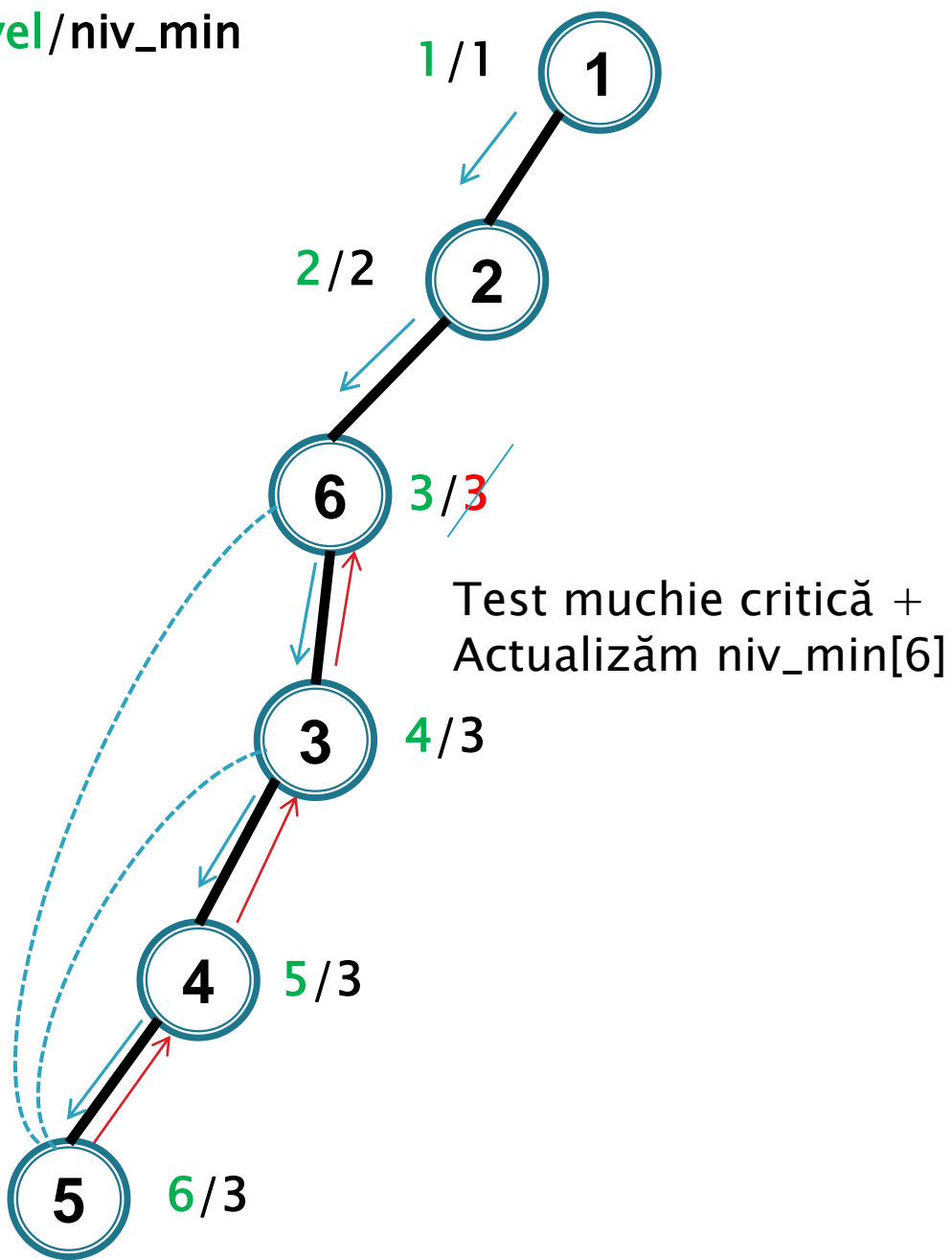


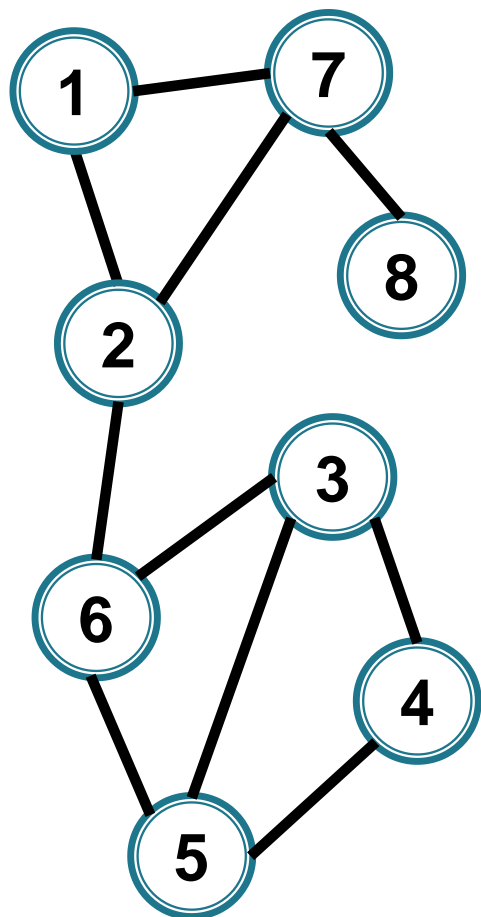
nivel/niv_min



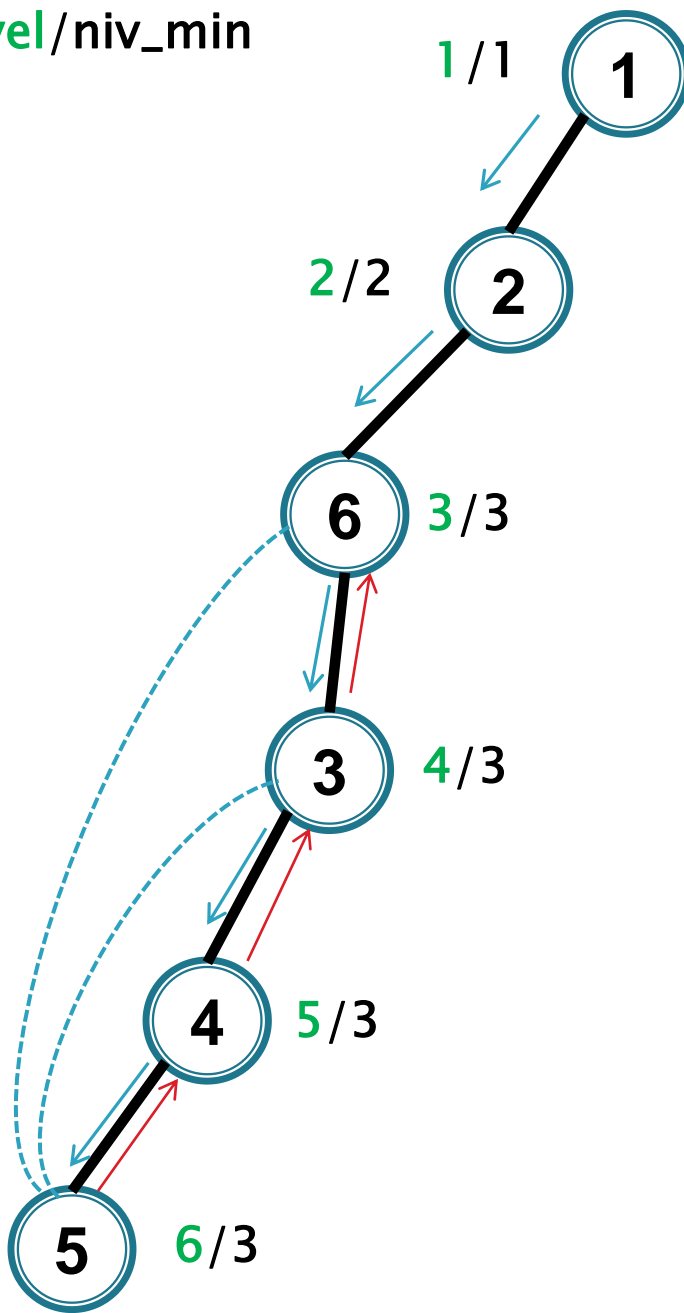


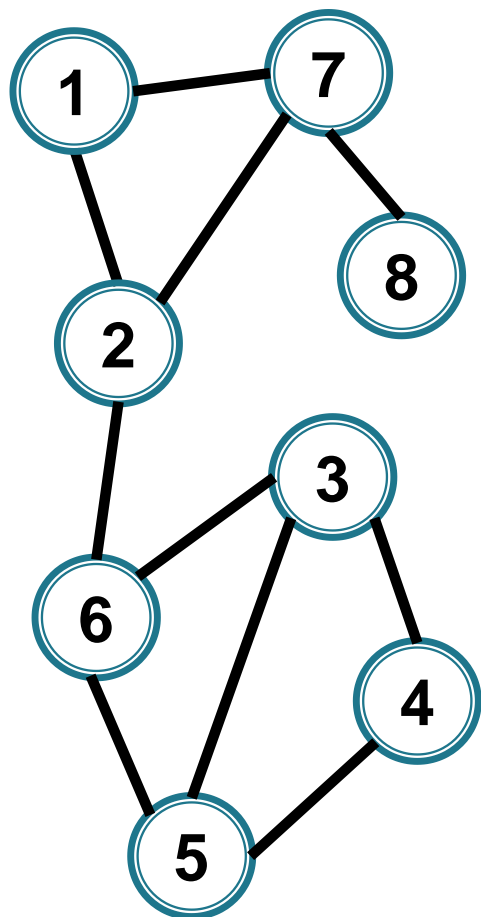
nivel/niv_min



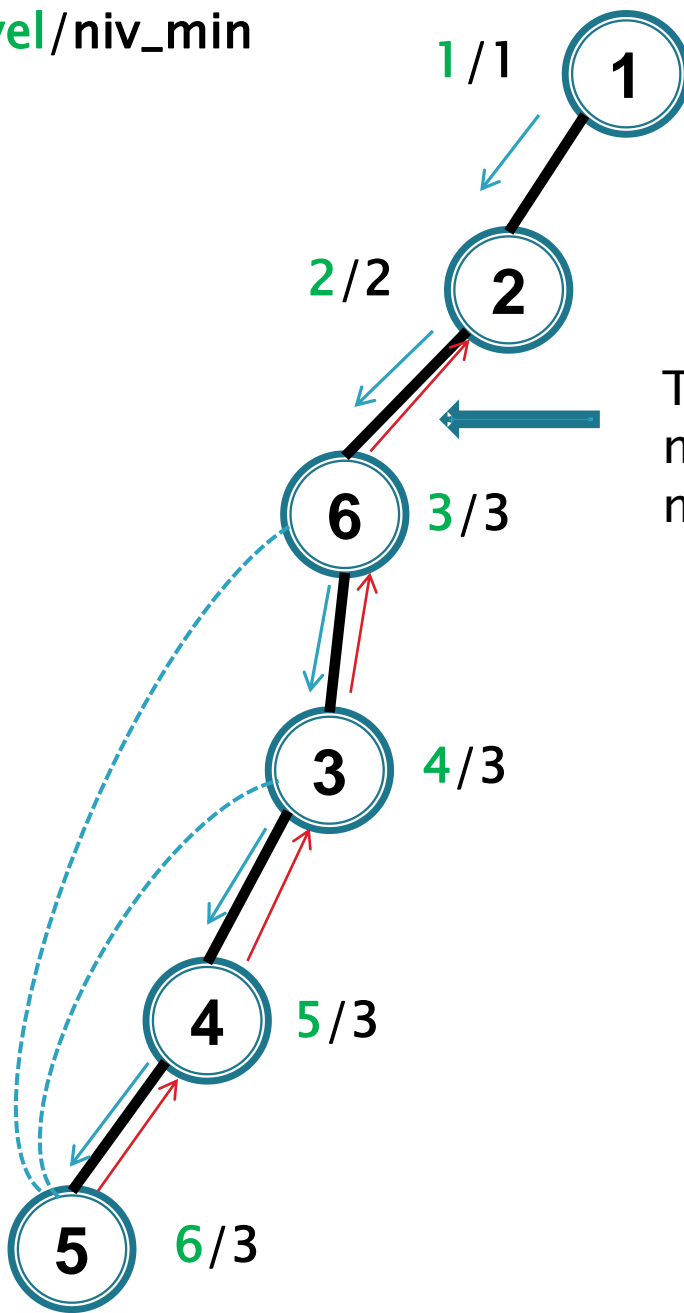


nivel/niv_min

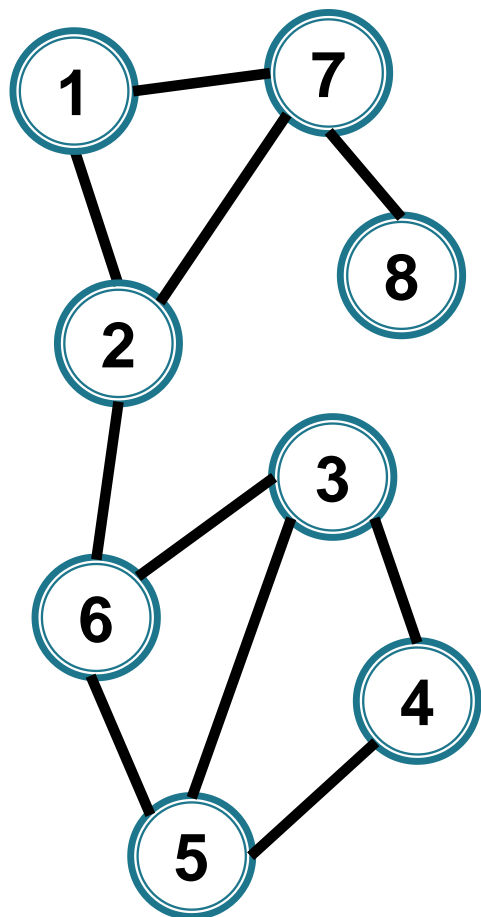




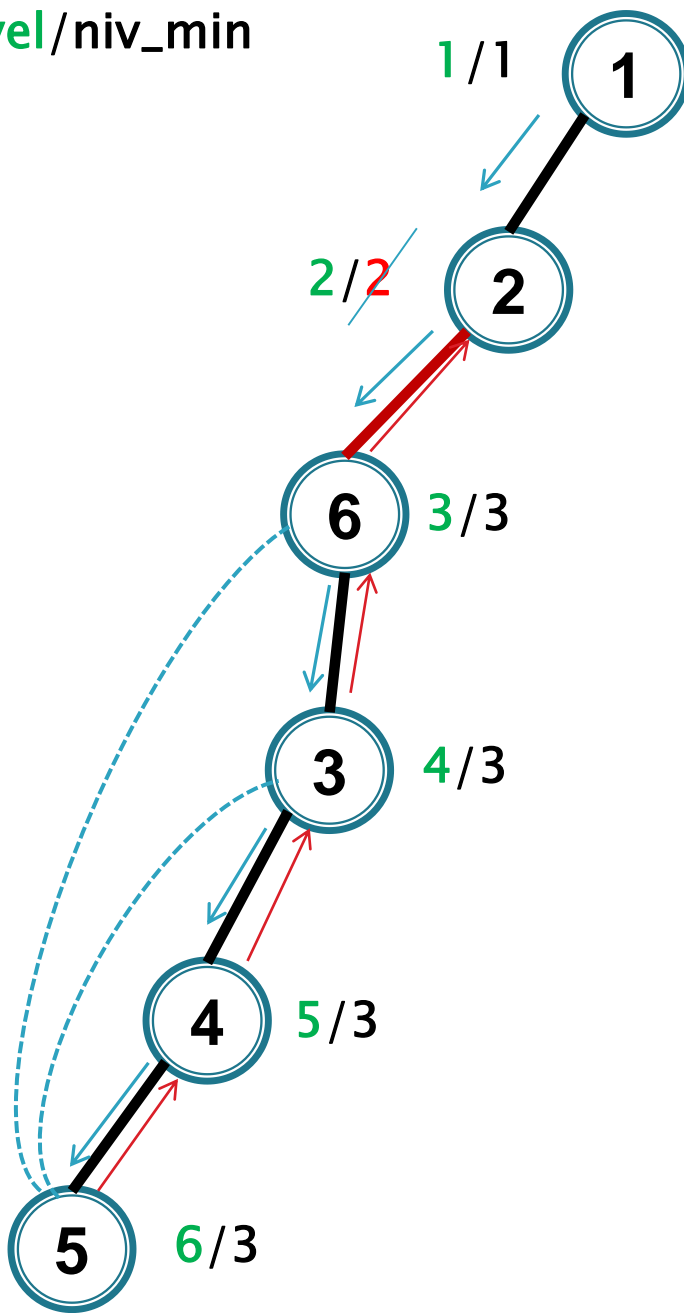
nivel/niv_min

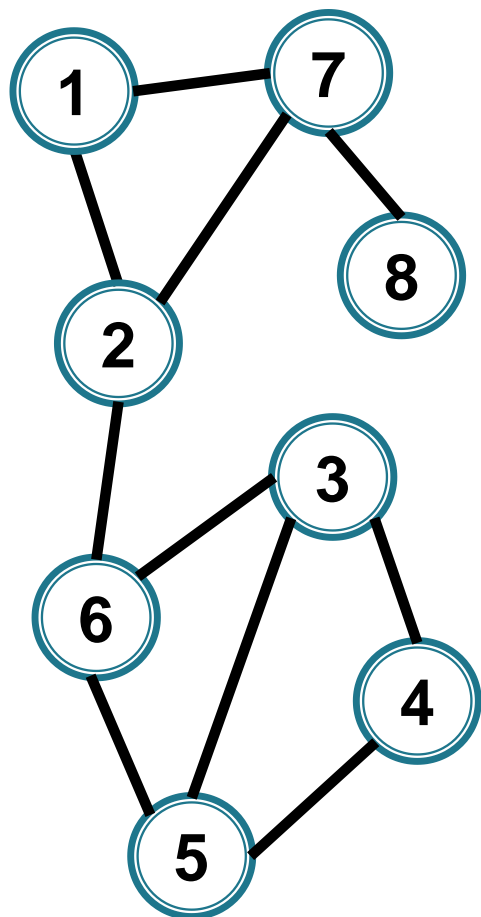


Test muchie critică:
 $\text{niv_min}[6] = 3 >$
 $\text{nivel}[2] = 2 \Rightarrow \text{DA}$

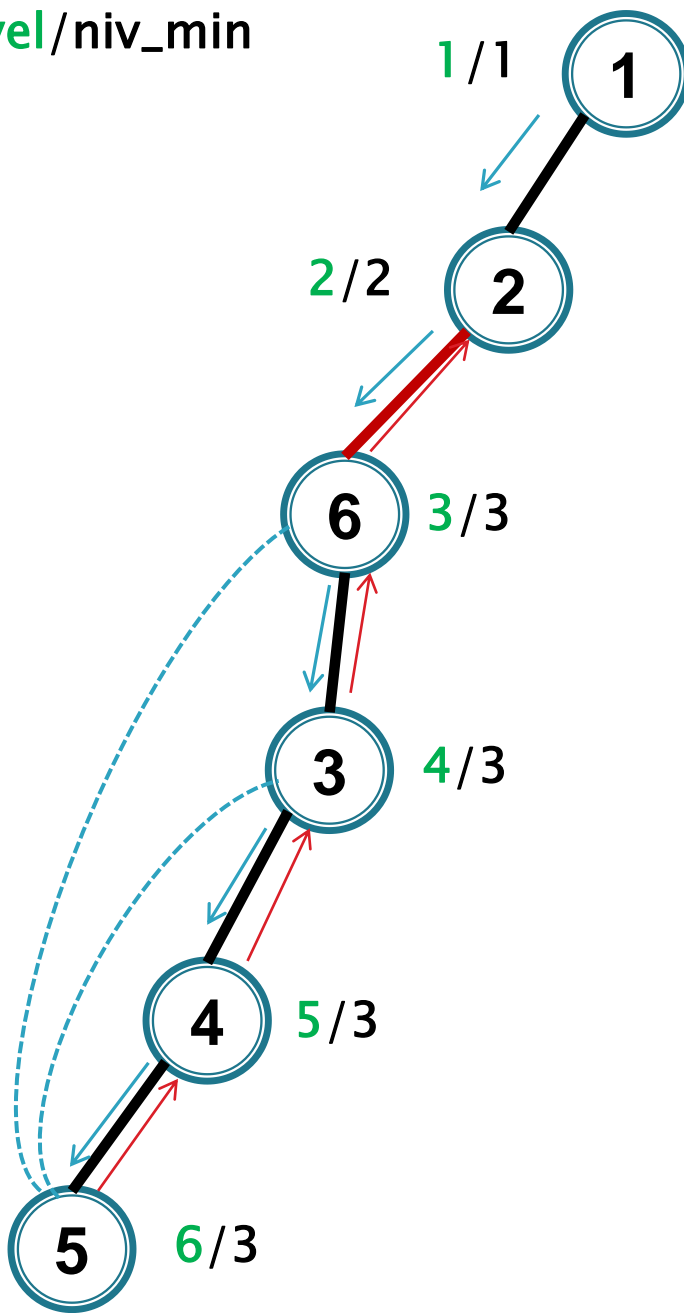


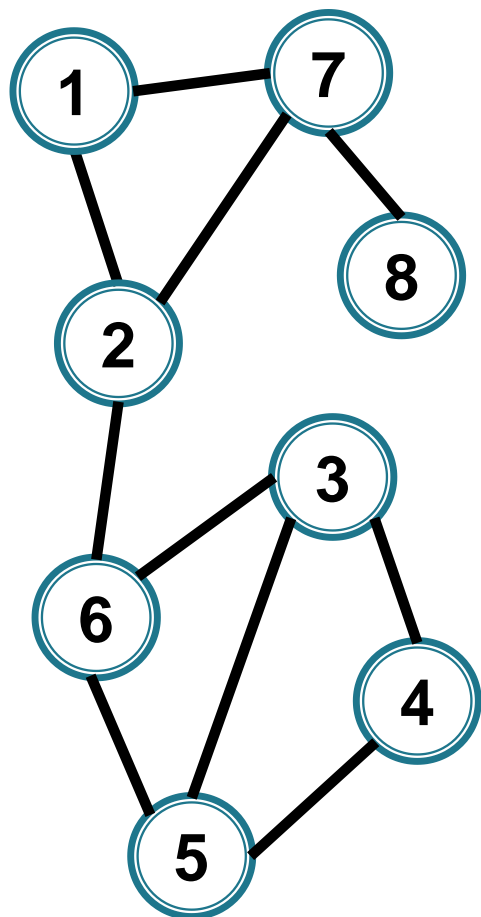
nivel/niv_min



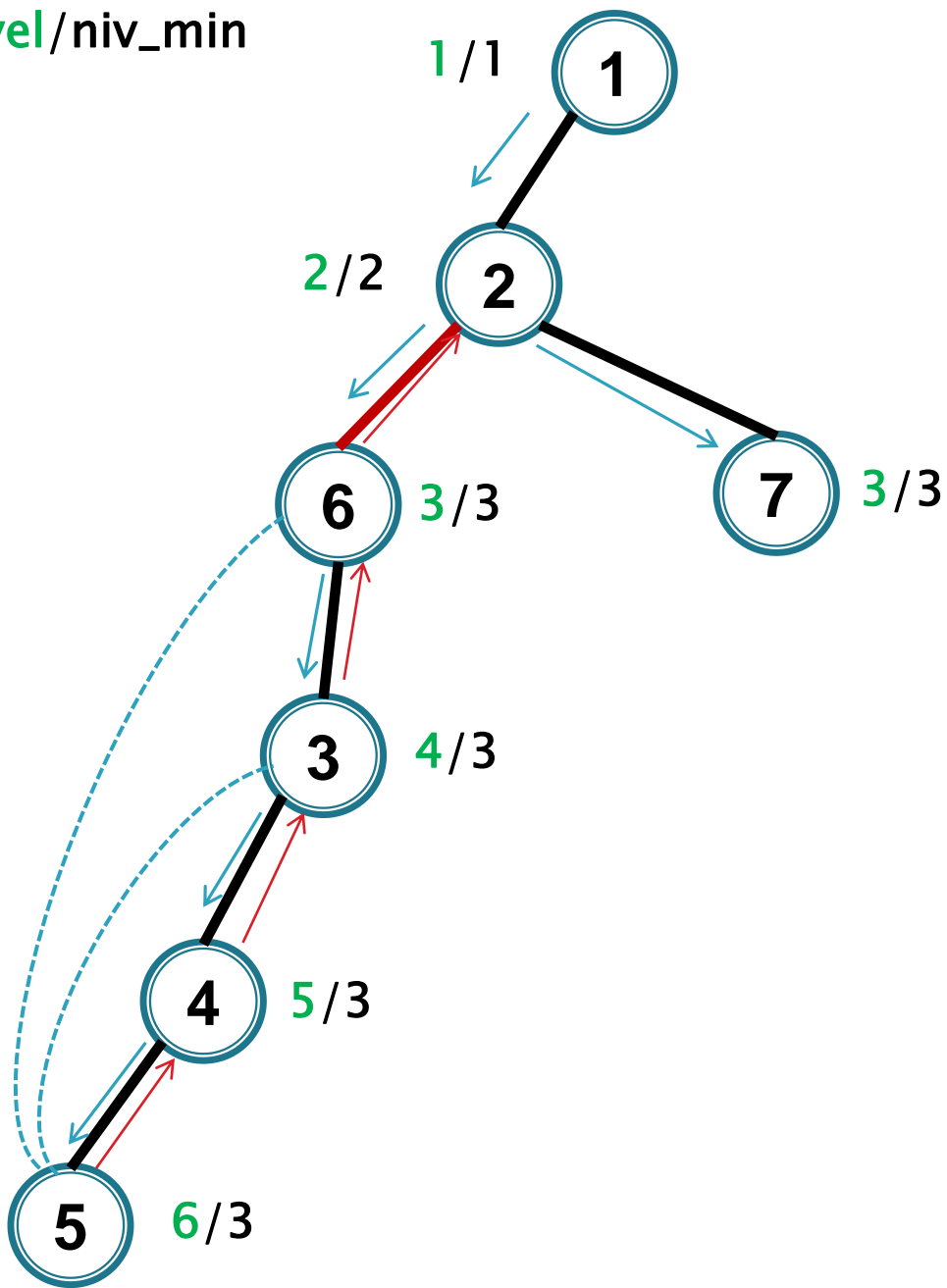


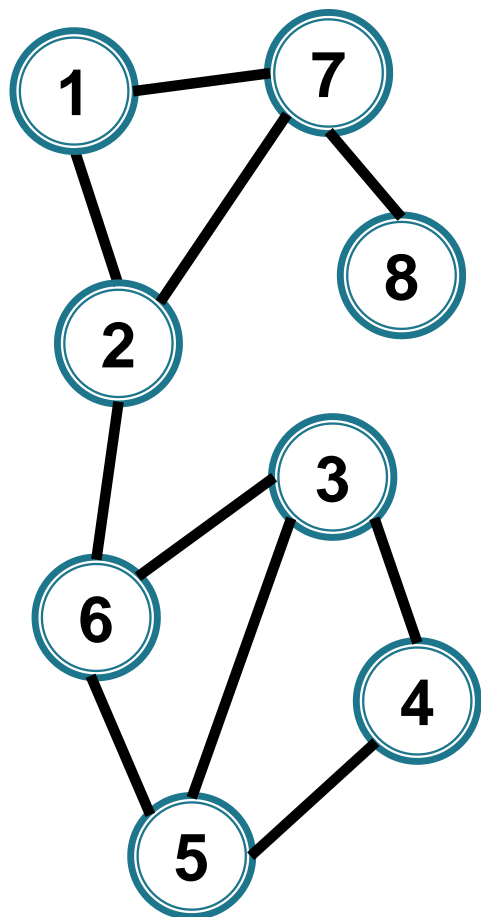
nivel/niv_min



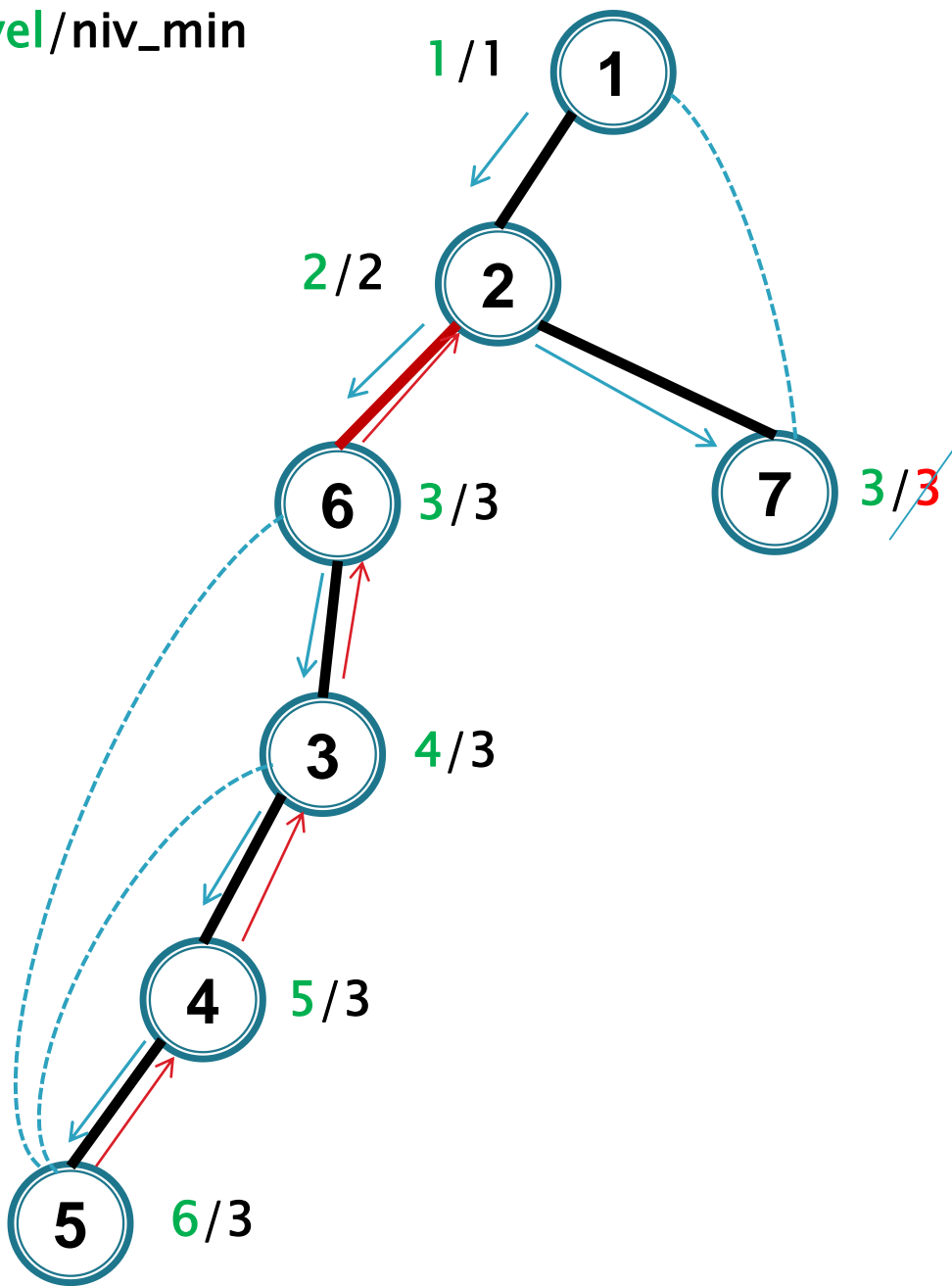


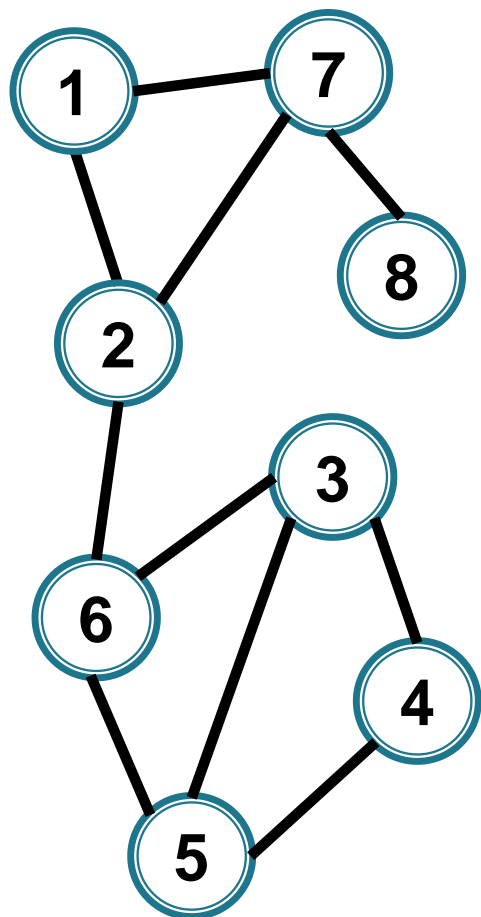
nivel/niv_min



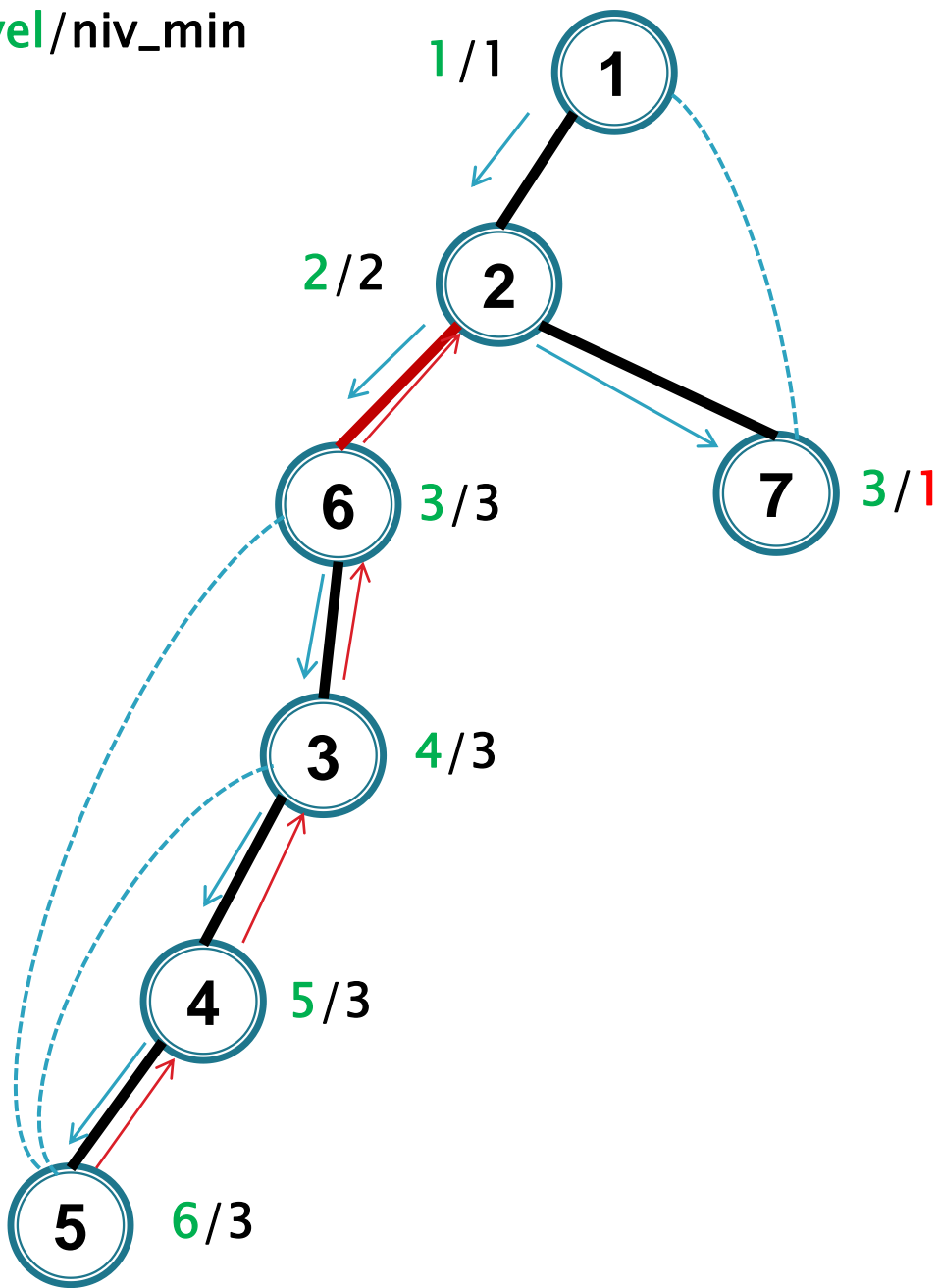


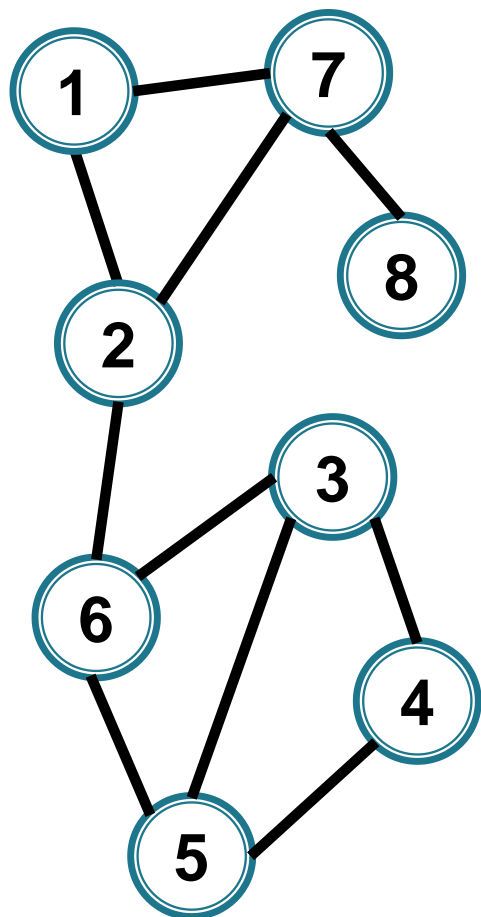
nivel/niv_min



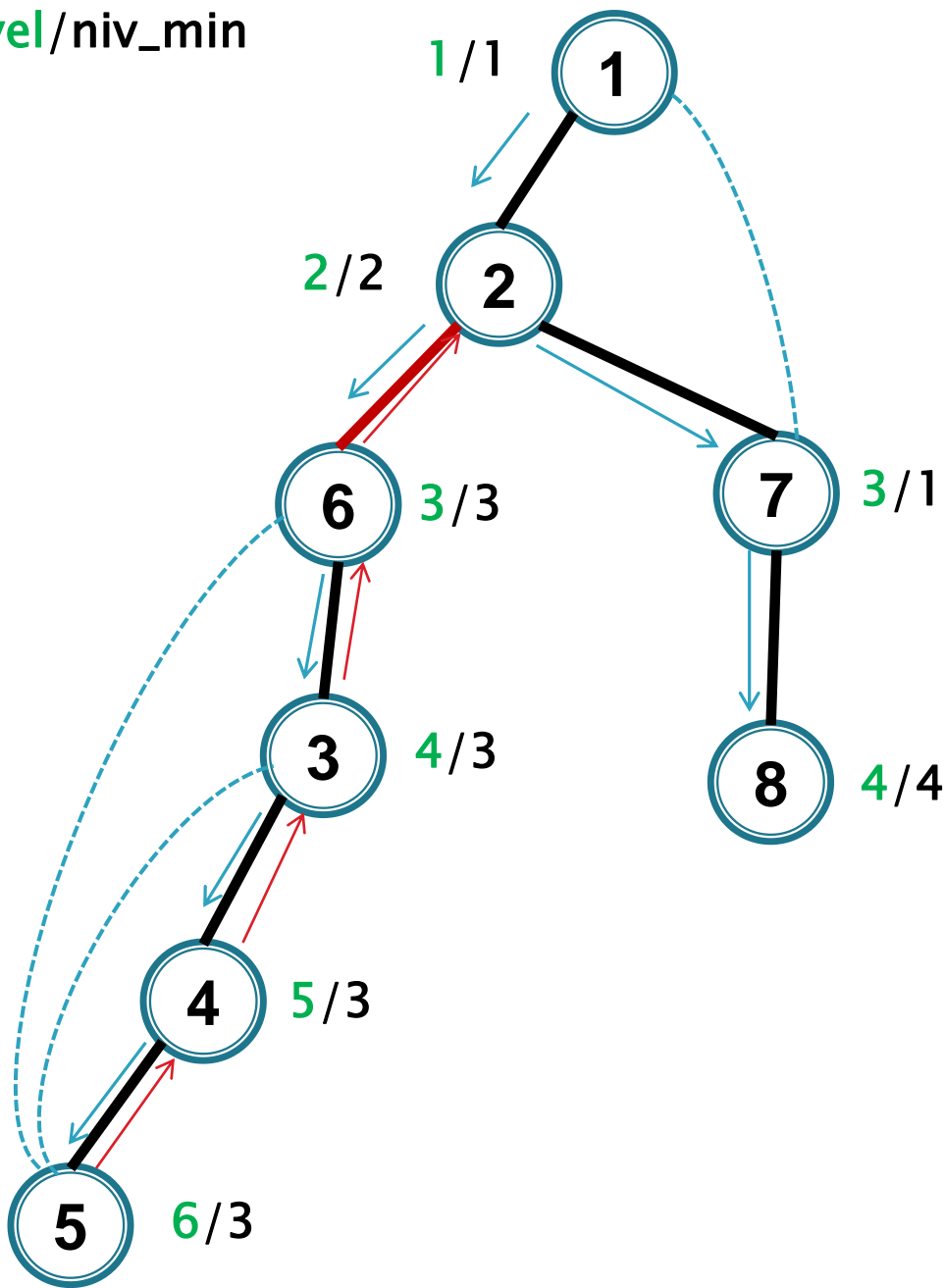


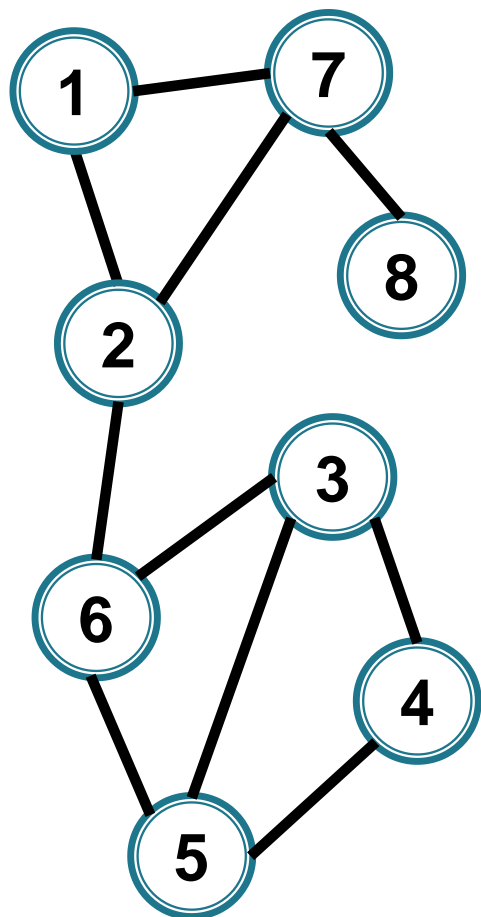
nivel/niv_min



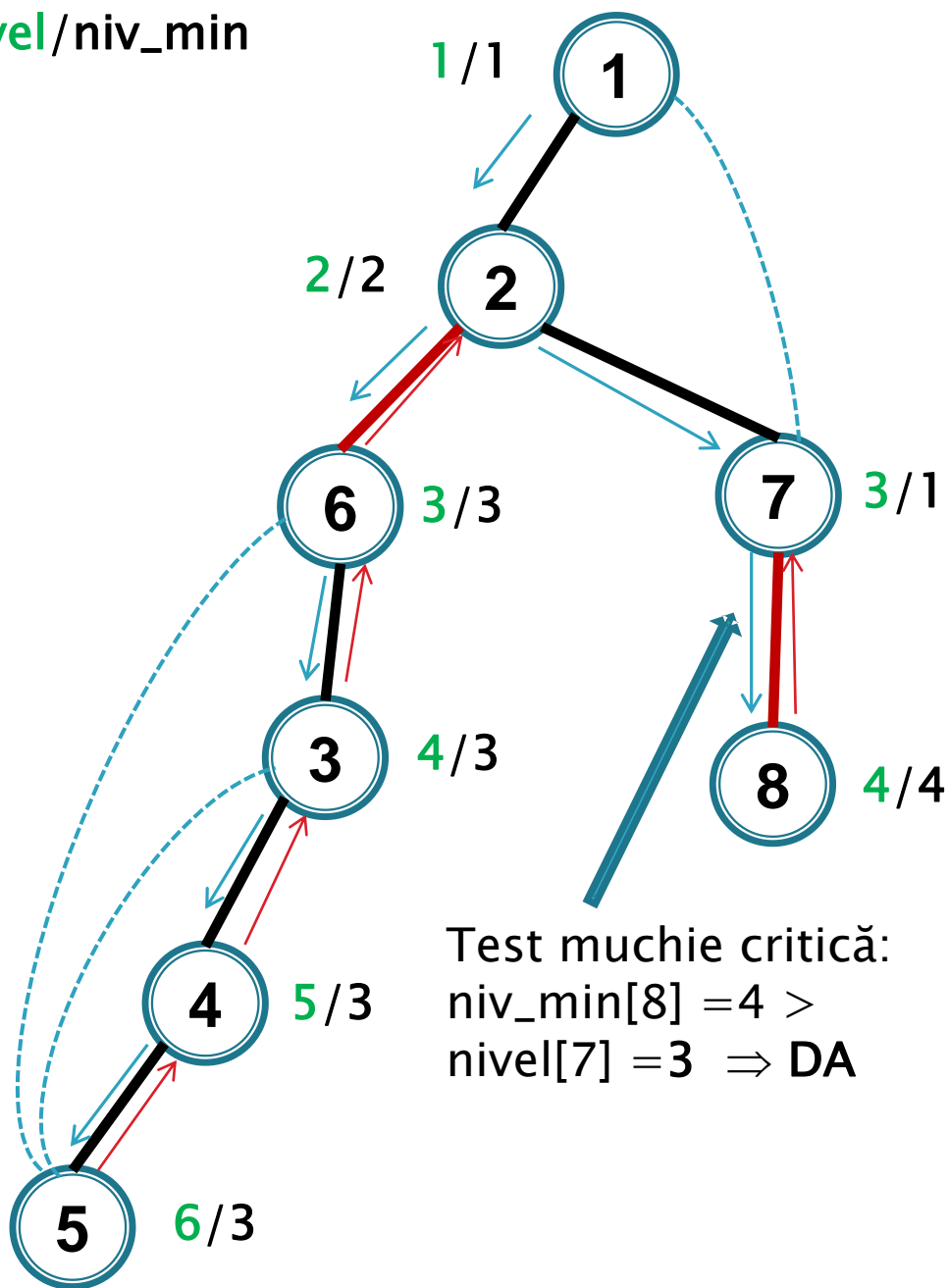


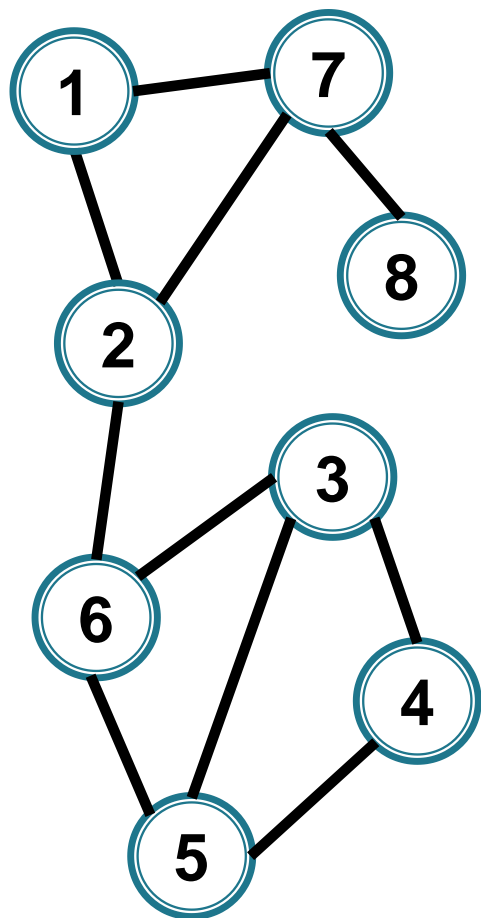
nivel/niv_min



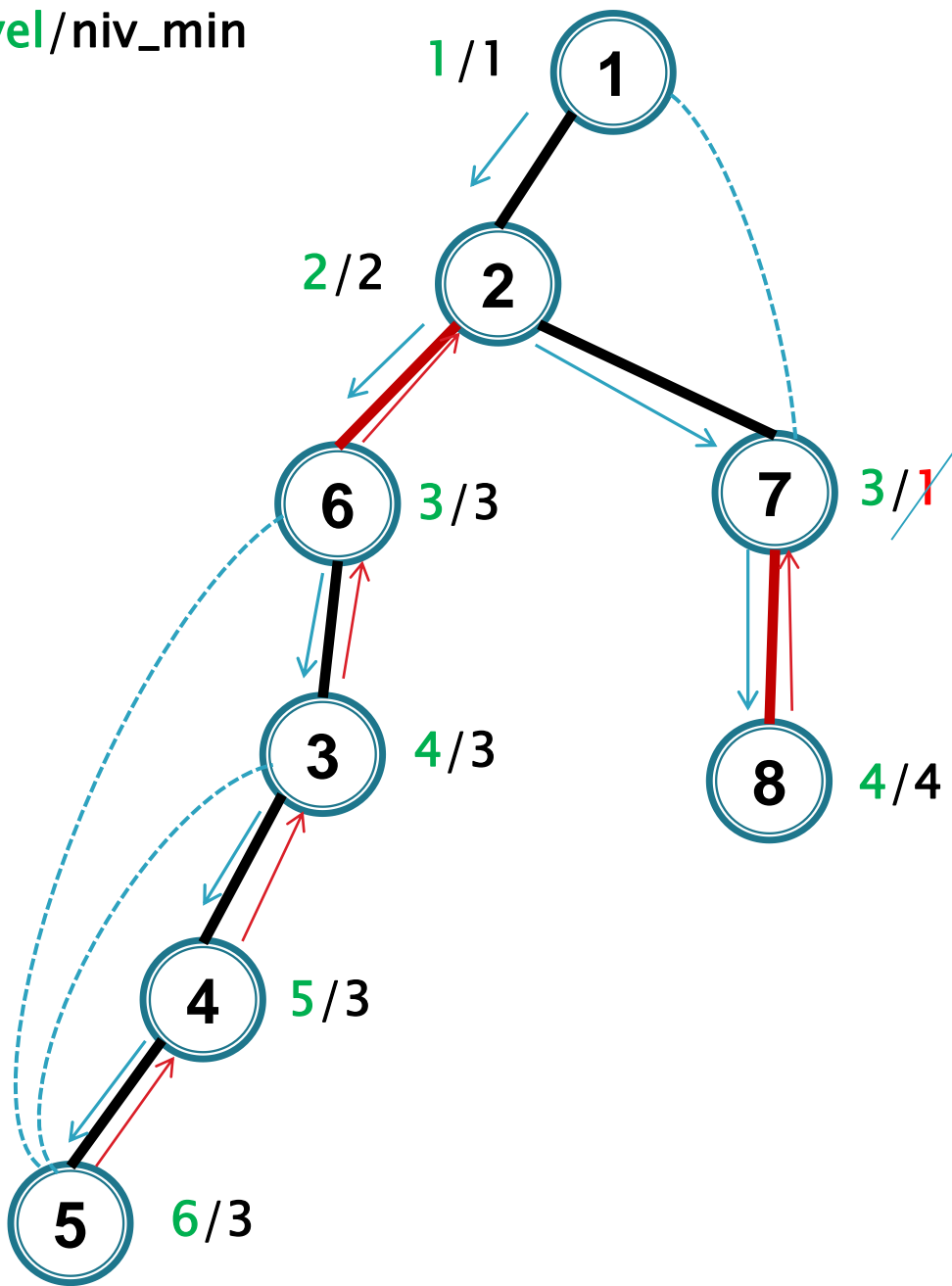


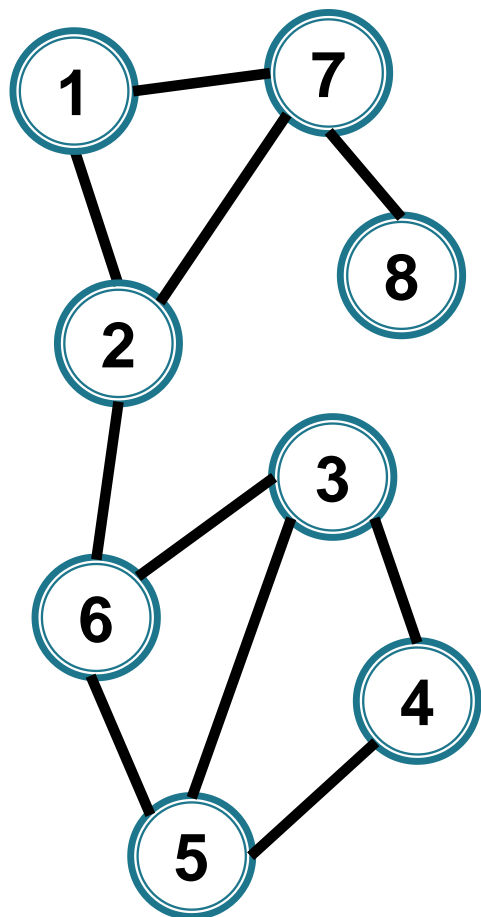
nivel/niv_min



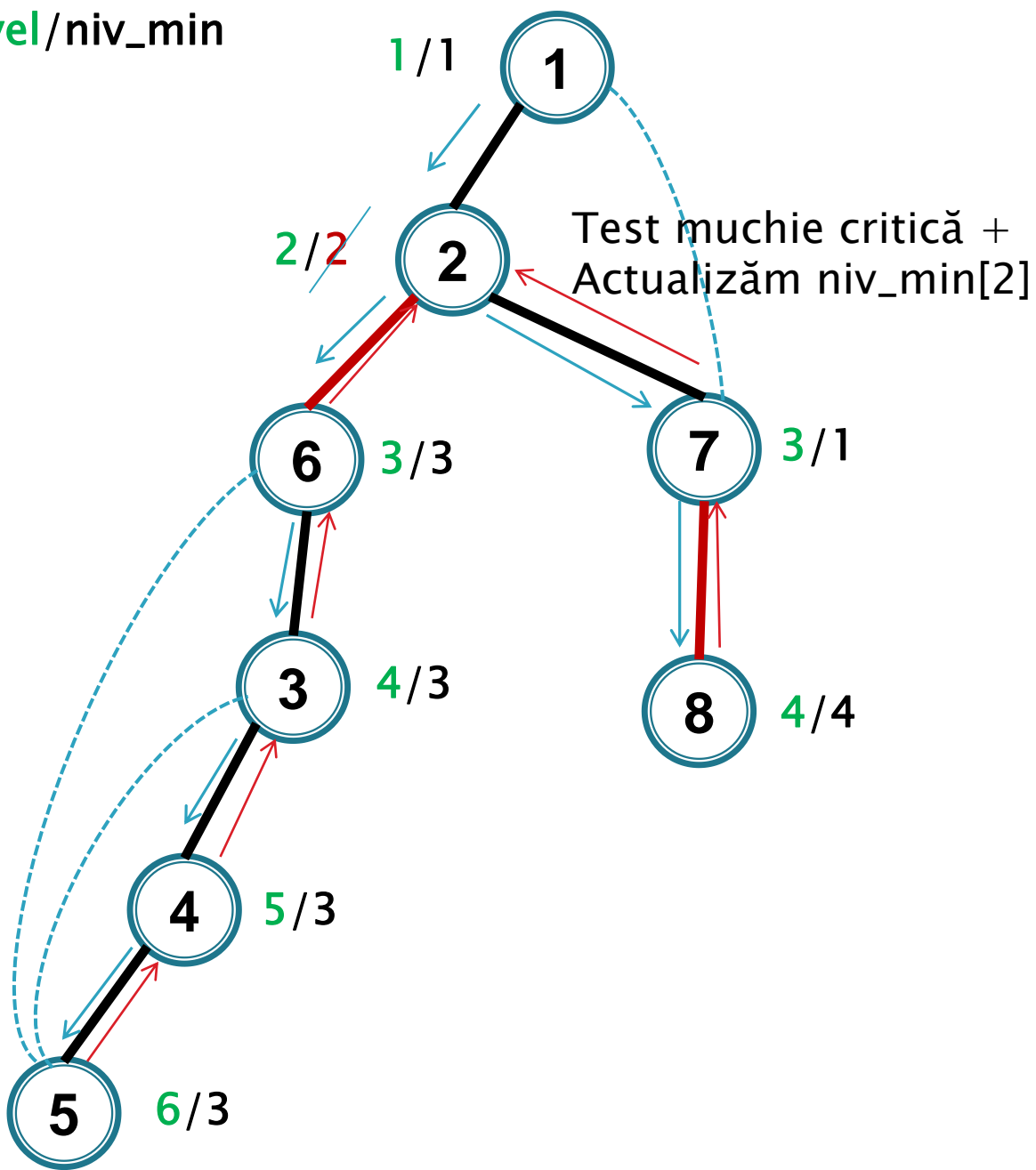


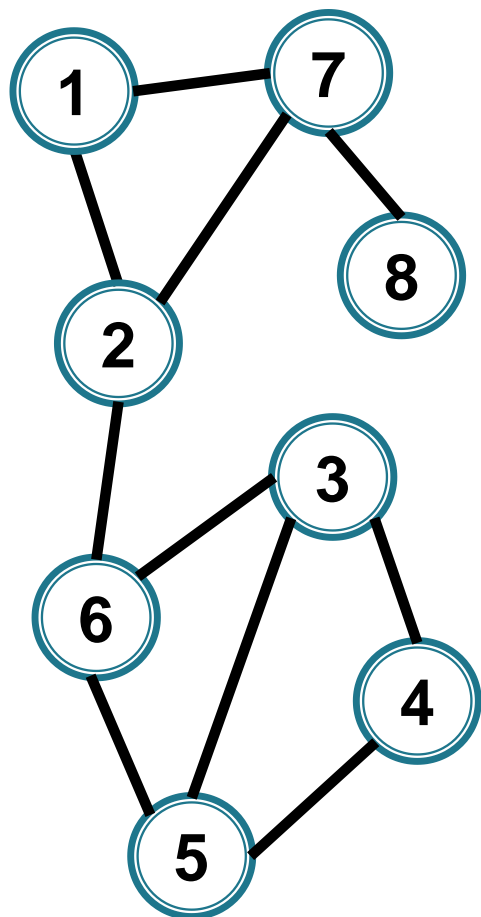
nivel/niv_min



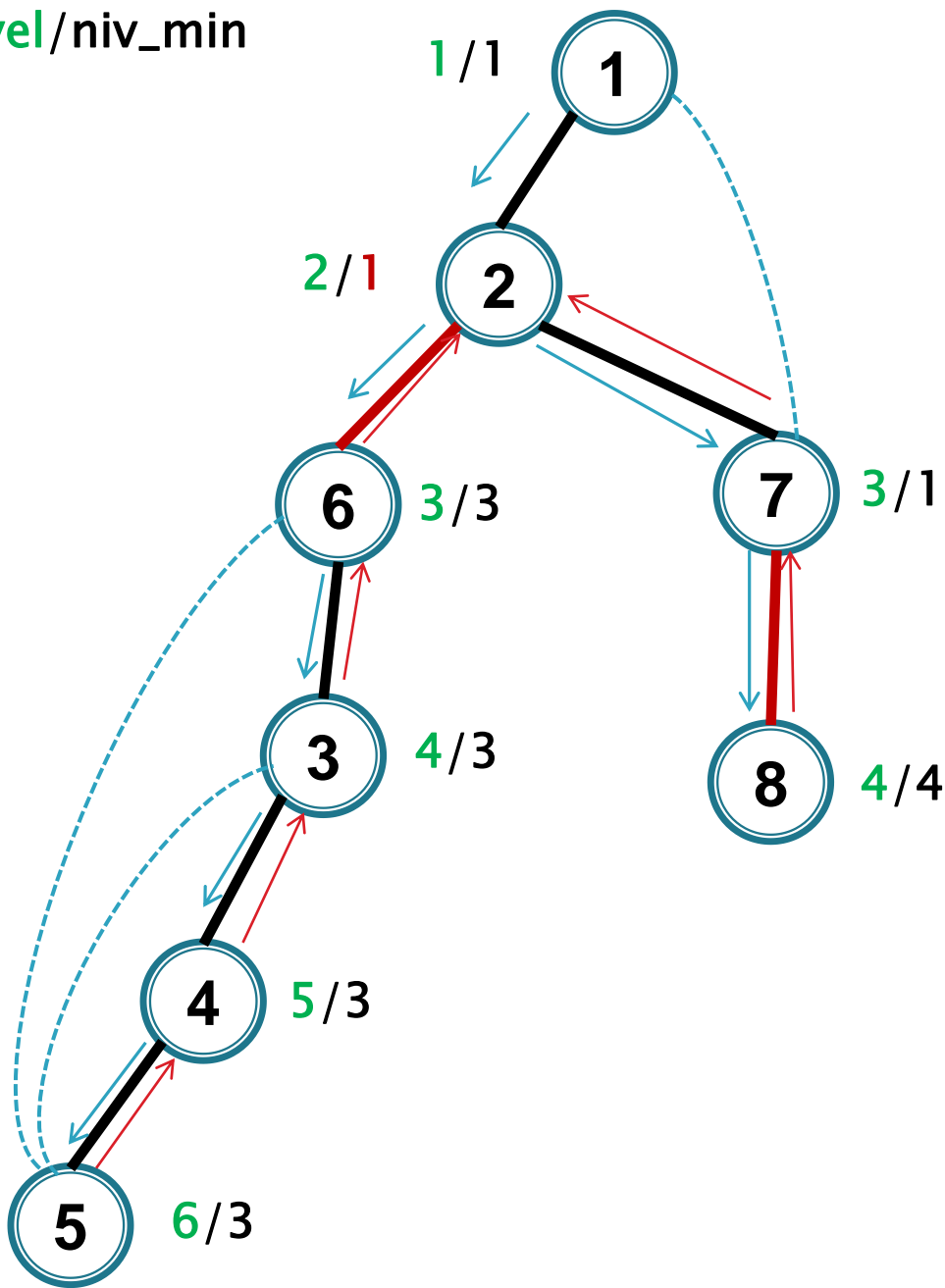


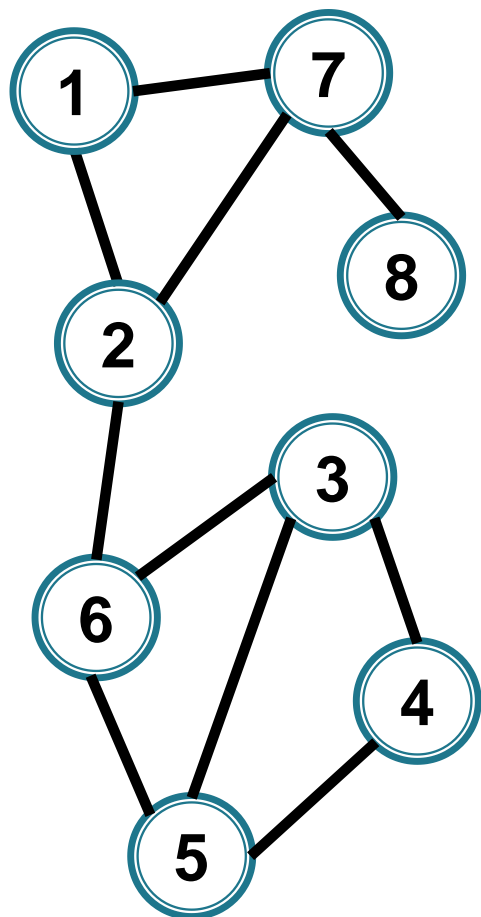
nivel/niv_min





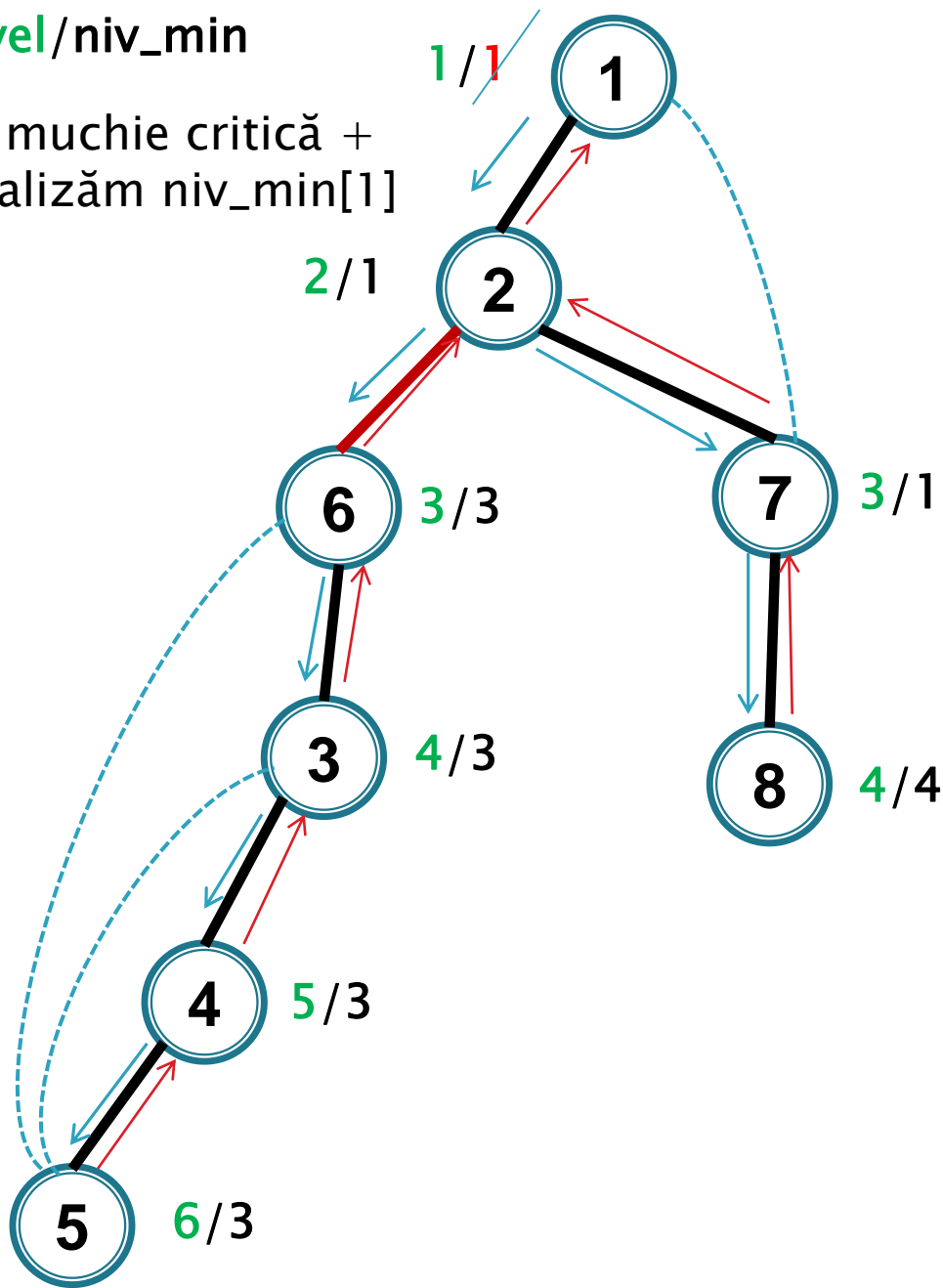
nivel/niv_min

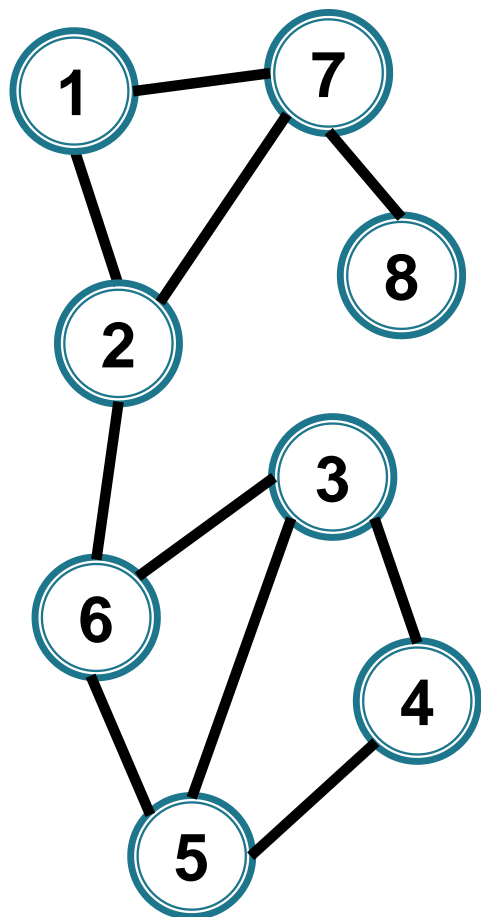




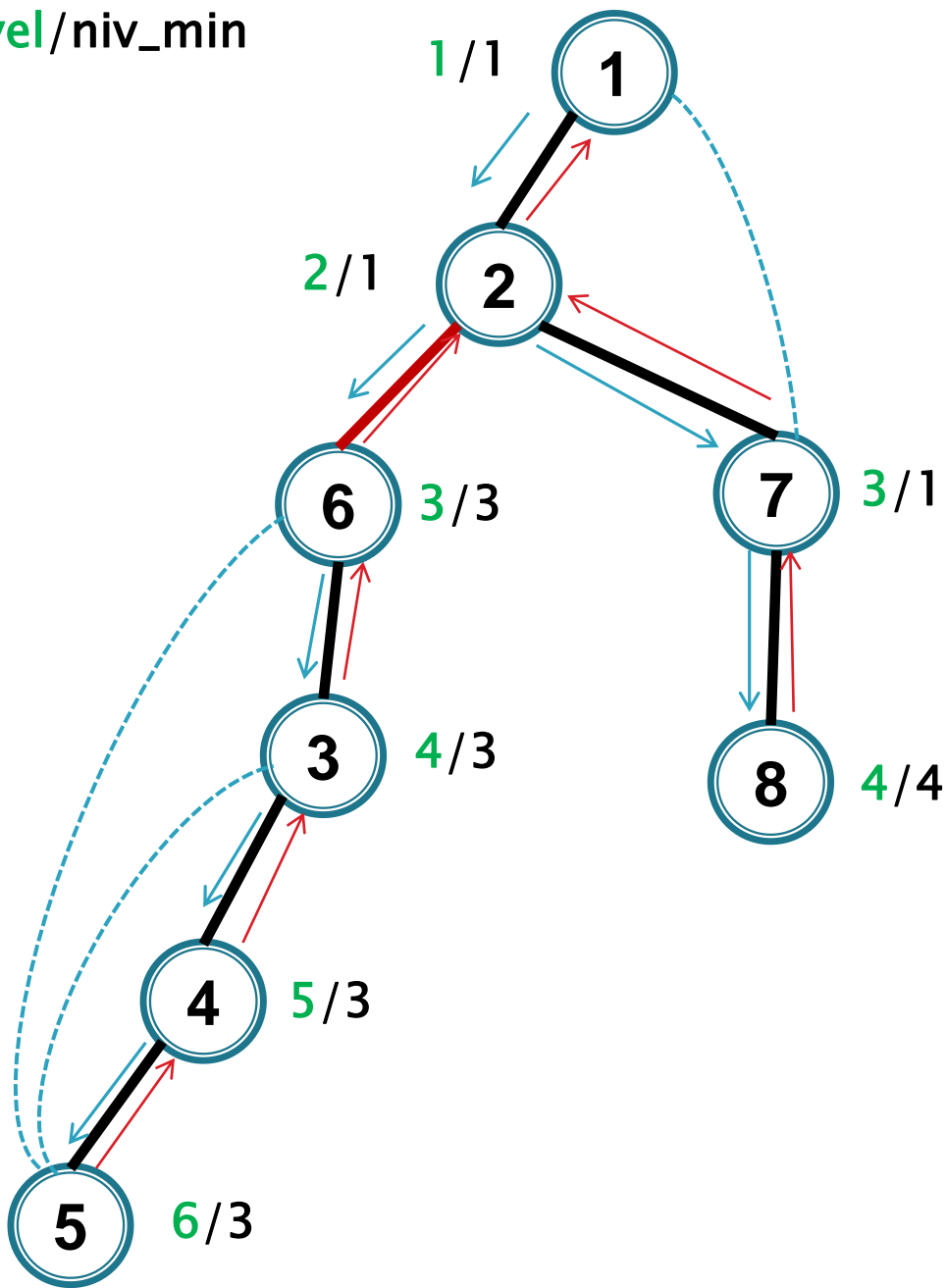
nivel/niv_min

Test muchie critică +
Actualizăm niv_min[1]





nivel/niv_min



Indicații implementare

```
void df(int i){
    viz[i] = 1;
    niv_min[i] = nivel[i];
    for(j vecin al lui i)
        if(viz[j]==0){ //ij muchie de avansare
            nivel[j] = nivel[i]+1;
            df(j);

            //actualizare niv_min[i]- formula B
            niv_min[i] = min{niv_min[i], niv_min[j] }

            //test ij este muchie critica
            ...
        }
    else
        if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
            //actualizare niv_min[i]- formula A
            ...
}
```

Indicații implementare

```
void df(int i){
    viz[i] = 1;
    niv_min[i] = nivel[i];
    for(j vecin al lui i)
        if(viz[j]==0){ //ij muchie de avansare
            nivel[j] = nivel[i]+1;
            df(j);

            //actualizare niv_min[i]- formula B
            niv_min[i] = min{niv_min[i], niv_min[j] }

            //test ij este muchie critica
            if (niv_min[j]>nivel[i]) scrie muchia ij
        }
    else
        if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
            //actualizare niv_min[i]- formula A
            niv_min[i] = min{niv_min[i], nivel[j] }
}
```

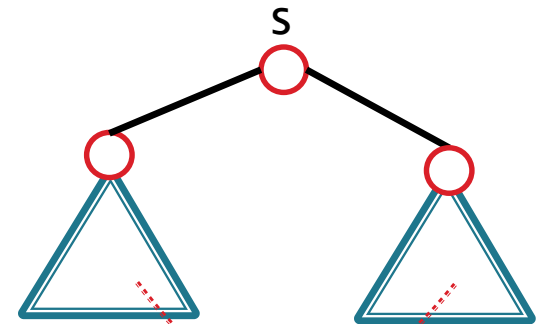
Puncte critice

Puncte critice

- ▶ Un vârf v este punct critic \Leftrightarrow există două vârfuri $x, y \neq v$ astfel încât v aparține oricărui x, y -lanț

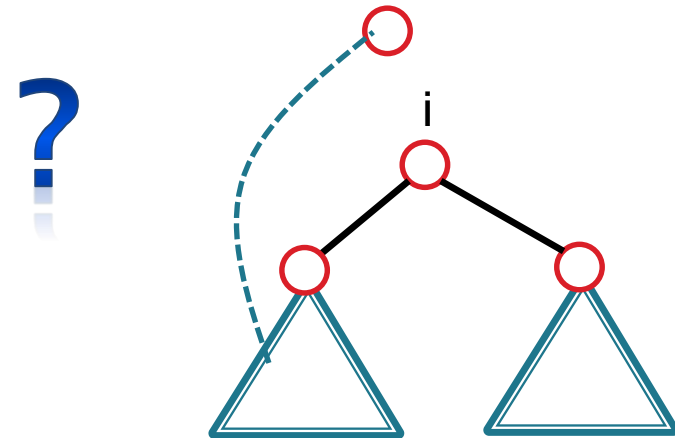
- ▶ Arborele DF

- rădăcina s este punct critic \Leftrightarrow ?



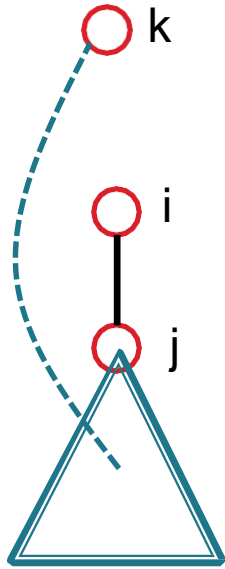
nu există muchii între subarbori
(de traversare)

- un alt vârf i din arbore este critic \Leftrightarrow ?



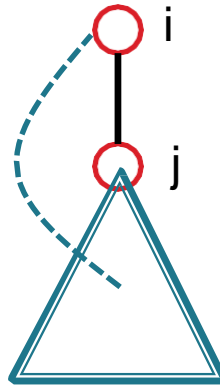
Puncte critice

Pentru $i \neq s$



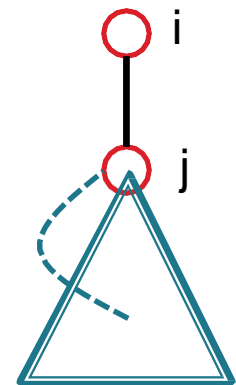
i NU este critic

$niv_min[j] < nivel[i]$



i ESTE critic

$niv_min[j] = nivel[i]$



i ESTE critic

$niv_min[j] > nivel[i]$

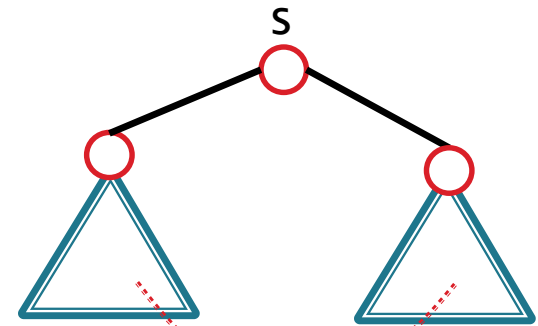
Puncte critice

- ▶ Un vârf v este punct critic \Leftrightarrow există două vârfuri $x, y \neq v$ astfel încât v aparține oricărui x, y -lanț

- ▶ Arborele DF

- rădăcina s este punct critic \Leftrightarrow

are cel puțin 2 fii în arborele DF



nu există muchii între subarbori
(de traversare)

- un alt vârf i din arbore este critic \Leftrightarrow
are cel puțin un fiu j cu
 $niv_min[j] \geq nivel[i]$

