

# **Government Spending in Multi-Sector Open Economies with Production Networks**

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# Motivation and Plan

**Question.** What is the welfare effect of redistributing sector-specific government spending under a fixed aggregate budget, relative to fully endogenous fiscal policy?

- ▶ Production networks amplify and reshape the transmission of shocks across sectors (Acemoglu et al., 2012; Baqaee & Farhi, 2020; Rubbo, 2023).
- ▶ Optimal fiscal allocation across sectors depends on sectoral heterogeneity in price stickiness, public-good shares, and network position (Cox et al., 2024).
- ▶ No existing framework combines these two dimensions in an open-economy setting.

**Plan — building up step by step:**

- (i) A **toy model** with non-distortionary lump-sum transfers and government procurements → a *relative allocation rule*.
- (ii) The **global production network model** of Aguilar et al. (2025) with distortionary taxation and subsidisation.
- (iii) **Planned extensions:** combine lump-sum procurements with proportional taxation/subsidies to production and labour in the richer model.

## A Simple Multi-Sector Model: Setup

Closed NK economy,  $K$  sectors, Calvo pricing ( $\alpha_k$ ), following Cox et al. (2024).

Household utility over private and public composites:

$$U = \sum_t \beta^t \left[ (1 - \chi) \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \sum_k \nu_k \frac{N_{k,t}^{1+\varphi}}{1+\varphi} \right]$$

**Key constraint:** the aggregate public-good bundle  $\bar{G}_t$  is *exogenous*. The planner chooses only the sectoral composition  $\{g_{k,t}\}$ .

*What does the planner's loss function look like under this constraint?*

# Welfare Loss under a Fixed Budget

Setting  $\sigma = 1$  and isolating the budget channel, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[ \underbrace{(1 + \varphi) y_{k,t}^2}_{\text{output gaps}} + \underbrace{\frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2}_{\text{inflation}} + \underbrace{\chi_k^* (g_{k,t} - y_{k,t})^2}_{\text{public-good gaps}} \right]$$

Three tensions the planner must balance:

- 1 Output-gap stabilisation:** penalises  $y_{k,t}^2$ .
- 2 Inflation stabilisation:** penalises  $\pi_{k,t}^2$ , weighted inversely by the Phillips curve slope  $\lambda_k$ .
- 3 Public-good allocation:** penalises the gap  $g_{k,t} - y_{k,t}$ ; under a fixed budget the planner can only reshuffle spending across sectors.

# The Relative Allocation Rule

Under exogenous  $\bar{G}_t$ , optimal spending in sector  $k$  relative to a residual sector  $i$ :

$$g_{k,t} = \underbrace{\Phi_{ki}}_{\text{structural}} g_{i,t}$$

$$- \underbrace{\frac{\varphi}{1+\lambda_k+\varphi\lambda_k} y_{k,t} - \frac{\theta\varphi(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k} \pi_{k,t}}_{\text{sector } k: \text{ spending falls when own gaps are large}}$$

$$+ \underbrace{\Phi_{ki} \left( \frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\varphi(1-\chi_i)\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right)}_{\text{sector } i: \text{ spending rises when residual-sector gaps are large}}$$

## Key property

The rule is inherently *relative*: spending is **reallocated** toward sectors with lower output gaps and lower inflation *relative* to the residual sector. What matters is the cross-sectional dispersion of gaps, not their level.

# The Global Production Network Model (Aguilar et al., 2025)

- ▶  $K=4$  countries,  $I=44$  sectors
- ▶ Nested CES: energy/non-energy, domestic/foreign
- ▶ Sector- and country-specific Calvo pricing
- ▶ Country-specific Taylor rules
- ▶ Balanced budget, lump-sum taxes, static production subsidies, tariff revenue

Marginal cost in sector  $i$ , country  $k$ :

$$\widehat{mc}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki} \widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\sum_{I,j} \mathcal{M}_{ki}\omega_{klj} \widehat{p}_{klj,t}}_{\text{IO inputs}}$$

Sectoral Phillips curve:

$$\pi_{ki,t} = \kappa_{ki}(\widehat{mc}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^P$$

Tariffs enter as price wedges:

$$P_{k,I,i,t} = (1 + \tau_{k,I,i,t}) \widetilde{P}_{I,k,i,t}$$

## Original Extensions: Fiscal Instruments in the Phillips Curves

**Why extend?** The Aguilar et al. model features lump-sum taxes and static production subsidies—no active fiscal stabilisation. To study the welfare cost of constrained fiscal policy in this environment, we introduce two time-varying distortionary instruments that enter the Phillips curves directly.

Define  $\hat{\tau}_{k,t}^w \equiv (\tau_{k,t}^w - \bar{\tau}_k^w)/(1 - \bar{\tau}_k^w)$  and  $\hat{\tau}_{ki,t}^s \equiv (\tau_{ki,t}^s - \bar{\tau}_{ki}^s)/(1 - \bar{\tau}_{ki}^s)$ .

**Wage Phillips curve** — adding a labour income tax:

$$\pi_{wk,t} = \kappa_{wk} \left( \sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

**Price Phillips curve** — making the production subsidy time-varying:

$$\pi_{ki,t} = \kappa_{ki} \left( \widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Tax hike  $\rightarrow$  inflationary cost-push. Subsidy hike  $\rightarrow$  disinflationary cost-push.

# Research Agenda: Fixed vs. Endogenous Fiscal Envelopes

**Goal:** introduce sector-specific  $G_{ki,t}$  in the goods market clearing condition of the global production network model:

$$Y_{ki,t} = \sum_l C_{lki,t} + \sum_l \sum_j X_{lkji,t} + G_{ki,t}$$

and compare two policy regimes:

## Fully endogenous spending

Planner chooses level *and* composition.  
Benchmark for first-best fiscal stabilisation.

## Fixed aggregate budget

$\bar{G}_{k,t}$  exogenous; only the sectoral composition adjusts. Relevant when total spending is politically constrained.

**Central question:** how large is the welfare gap between the two regimes? If it is small, compositional reallocation alone—the relative allocation rule—may approximate the first-best, even without aggregate fiscal flexibility.

- 1 Under a fixed fiscal envelope, optimal sectoral spending follows a **relative allocation rule**: reallocate toward sectors with larger gaps.
- 2 The global production network model of **Aguilar et al. (2025)** provides a quantitative environment with IO linkages and sectoral tariffs, but lacks non-trivial fiscal dynamics.
- 3 **Original extensions**—labour income tax in the wage PC, time-varying subsidy in the price PC—introduce fiscal instruments without altering the core model.
- 4 The **research agenda**: derive spending rules in the networked model; quantify the welfare cost of fixing the aggregate budget relative to fully endogenous spending.

**Thank you**

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# Appendix: Related Literature

## Production Networks & NK

- ▶ Acemoglu et al. (2012)
- ▶ Baqaee & Farhi (2020, 2024)
- ▶ Pasten, Schoenle & Weber (2020)
- ▶ Rubbo (2023)

## Fiscal Policy in Disaggregated Economies

- ▶ Aoki (2001)
- ▶ Cox, Müller, Pasten, Schoenle & Weber (2024)

## Tariffs & Open-Economy NK

- ▶ Galí & Monacelli (2005)
- ▶ Comin & Johnson (2023)
- ▶ Aguilar et al. (2025)

## Fiscal–Price Effects (Empirical)

- ▶ Nekarda & Ramey (2013)
- ▶ Ben Zeev & Pappa (2017)
- ▶ Barattieri et al. (2023)

## Appendix: Welfare Objective

Under  $\sigma = 1$ , abstracting from government-demand pass-through, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[ (1 + \varphi) y_{k,t}^2 + \frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

Three tensions:

- 1 Output-gap stabilisation:** penalises  $y_{k,t}^2$ .
- 2 Inflation stabilisation:** penalises  $\pi_{k,t}^2$ , weighted inversely by the Phillips curve slope  $\lambda_k$ .
- 3 Public-good allocation:** penalises the gap  $g_{k,t} - y_{k,t}$ .

When  $\bar{G}_t$  is unconstrained,  $g_{k,t}$  can be set independently in each sector. When only the composition is a choice variable, the problem changes qualitatively: the planner can only *reshuffle* spending.

## Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$\begin{aligned} & -\frac{1}{2} \sum_k \mu_k \left( (1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ & \quad \left. + (\sigma-1) \left[ (1-\chi_k) \omega_{c,k} \left( \frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right) \end{aligned}$$

- The  $\sigma-1$  term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- $\kappa > 0$  steepens Phillips curves ( $\lambda'_k > \lambda_k$ ), making fiscal policy a supply-side instrument.

## Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous  $\bar{G}_t$  (with  $\sigma = 1$ ,  $\kappa = 0$ ):

$$\begin{aligned} g_{k,t} &= \frac{1-\chi_k}{1-\chi_i} \left( \frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \left( \frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left( \frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} g_{i,t} \\ &\quad - \frac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta \varphi (1-\chi_k) \pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ &\quad + \frac{1-\chi_k}{1-\chi_i} \left( \frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left( \frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} \\ &\quad \times \left( \frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta \varphi (1-\chi_i) \pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \end{aligned}$$

$\omega_{g,k}$ : weight of sector  $k$  in the government Cobb–Douglas aggregator.  $\rho$ : CES elasticity of public-good bundle.

## Appendix: Optimal Monetary Rule under Aggregate Constraint

Under exogenous  $\bar{G}_t$ , optimal monetary policy sets:

$$\sum_k \mu_k \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\lambda_k + \varphi\lambda_k(1-\chi_k)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \pi_{k,t} =$$
$$\sum_k \mu_k \left( \frac{\chi_k g_{k,t}}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} - \frac{y_{k,t}(1-\chi_k)(1+\varphi+\chi_k^*)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \right)$$

- ▶ Inflation weights depend on private-consumption share and  $\lambda_k$ .
- ▶ Government spending enters the target because the constraint links  $g_{k,t}$  and  $y_{k,t}$  across sectors.

## Appendix: Aguilar et al. — Household Problem

Per-period utility:  $U_t = \left( C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 N_{gk,t}^{1+\varphi}/(1+\varphi) dg \right) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[ \tilde{\beta}_k^{1/\gamma} C_{kE,t}^{(\gamma-1)/\gamma} + (1-\tilde{\beta}_k)^{1/\gamma} C_{kM,t}^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$C_{k,t}^{-\sigma} = \beta \mathbb{E}_t C_{k,t+1}^{-\sigma} \frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}}$$

$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* nfa_{k,t} + \varepsilon_{kK,t}^e \quad (\text{UIP})$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk} \left( \sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

## Appendix: Aguilar et al. — Firm Problem

CES production:  $Y_{ki,f,t} = A_{ki,t} \left[ \tilde{\alpha}_{ki}^{1/\psi} N_{fki,t}^{(\psi-1)/\psi} + \tilde{\vartheta}_{ki}^{1/\psi} X_{fki,t}^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{mc}_{ki,t} = -a_{ki,t} + M_{ki}\alpha_{ki} \widehat{w}_{k,t} + \sum_{l=1}^K \sum_{j=1}^I M_{ki}\omega_{klj} \widehat{p}_{klj,t}$$

where  $\alpha_{ki}$ : labour share,  $\omega_{klj}$ : IO expenditure share,  $M_{ki}$ : steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki} (\widehat{mc}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p \quad \kappa_{ki} = \frac{(1 - \theta_{ki}^p)(1 - \beta\theta_{ki}^p)}{\theta_{ki}^p}$$

## Appendix: Aguilar et al. — Tariff Propagation

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \tilde{P}_{l,k,i,t}$$

### Propagation:

- 1 Tariff on country  $l$  raises input prices  $\hat{p}_{klj_i,t}$  for domestic sectors sourcing from sector  $j$  in  $l$ .
- 2 Higher input costs raise  $\widehat{\text{mc}}_{ki,t}$ , feeding into  $\pi_{ki,t}$ .
- 3 Cost increases cascade downstream through the IO network.
- 4 Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_i \tau_{kli,t} P_{kli,t}^l \left( C_{kli,t} + \sum_j X_{klji,t} \right) = B_{k,t-1} + \sum_i \tau_{ki}^s \text{MC}_{ki,t} Y_{ki,t}$$

# Appendix: Aguilar et al. — Calibration

## Households

- ▶  $\beta = 0.99$ ,  $\sigma = 1$ ,  $\varphi = 1$
- ▶ Energy/non-energy elast.  $\gamma = 0.4$
- ▶ Trade elasticity  $\delta = 1$
- ▶ Calvo wage  $\theta_k^w = 0.75$
- ▶ Consumption shares from OECD ICIO (2019)

## Monetary policy

- ▶  $\rho_r = 0.7$ ,  $\phi_\pi = 1.5$ ,  $\phi_y = 0.125$
- ▶ Target: headline inflation

## Firms

- ▶ Labour/input elast.  $\psi = 0.5$
- ▶ Energy/non-energy elast.  $\phi = 0.4$
- ▶ Trade elasticity  $\mu = 1$
- ▶ IO shares from OECD ICIO (2019)
- ▶ Markups from Eurostat Figaro
- ▶ Calvo prices from ECB PRISMA

## Tariff shocks

- ▶  $\rho^\tau = 0.96$ ,  $\sigma^\tau = 1$

## Appendix: Aguilar et al. — Goods Market Clearing and GDP

Market clearing:

$$Y_{ki,t} = \sum_{l=1}^K C_{lki,t} + \sum_{l=1}^K \sum_{j=1}^I X_{lkji,t}$$

Log-linearised:

$$\lambda_{ki} \hat{y}_{ki,t} = \sum_{l=1}^K \mathcal{Y}_{lk} \left( \beta_{lki} \hat{c}_{lki,t} + \sum_{j=1}^I \lambda_{lj} \omega_{lkji} \hat{x}_{lkji,t} \right)$$

where  $\lambda_{ki} = P_{ki} Y_{ki} / \mathcal{Y}_k$  is the Domar weight.

Real GDP:

$$\hat{y}_{k,t} = \hat{c}_{k,t} + \Upsilon_k (\hat{\exp}_{k,t} - \hat{\text{imp}}_{k,t})$$

where  $\Upsilon_k$  is the trade-to-GDP ratio.

## Appendix: Derivation — Wage Phillips Curve with Tax

Household FOC with labour income tax  $\tau_{k,t}^w$ :

$$\sum_{l=0}^{\infty} (\beta \theta_k^W)^l \mathbb{E}_t \left[ N_{k,t+l|t} C_{t+l|t}^{-\sigma} \left( \frac{(1-\tau_{k,t+l}^w) W_{k,t}^*}{P_{t+l}} - M_{wk,t} \text{MRS}_{k,t+l|t} \right) \right] = 0$$

Log-linearised reset wage:

$$w_{k,t}^* = (1-\beta \theta_k^W) \sum_{l=0}^{\infty} (\beta \theta_k^W)^l \mathbb{E}_t [ \text{mrs}_{t+l|t} + \mu_{wk,t+l}^n + p_{kC,t+l} + \hat{\tau}_{k,t+l}^w ]$$

Calvo aggregation ( $\pi_{wk,t} = w_{k,t} - w_{k,t-1}$ ):

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

## Appendix: Derivation — Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy  $\tau_{ki,t}^s$ :

$$\sum_{l=0}^{\infty} (\beta \theta_{ki}^P)^l \mathbb{E}_t [\Lambda_{t,t+l} Y_{ki,t+l|t} (P_{ki,t}^* - M_{pk,t+l} (1 - \tau_{ki,t+l}^s) MC_{ki,t+l|t}^n)] = 0$$

Log-linearised reset price:

$$p_{ki,t}^* = (1 - \beta \theta_{ki}^P) \sum_{l=0}^{\infty} (\beta \theta_{ki}^P)^l \mathbb{E}_t [mc_{ki,t+l|t}^n + \mu_{pk,t+l}^n - \hat{\tau}_{ki,t+l}^s]$$

Calvo aggregation ( $\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$ ):

$$\pi_{ki,t} = \kappa_{ki} (\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^P$$

Subsidy increase lowers effective marginal cost  $\rightarrow$  disinflationary cost-push.

## Appendix: Government Budget Dynamics

Starting from the nominal budget constraint:

$$B_{t+1} = B_t(1 + i_{t+1}) + G_{t+1} - T_{t+1}$$

In ratios to GDP ( $b_t \equiv B_t/Y_t$ ):

$$b_{t+1} = b_t \frac{1 + i_{t+1}}{1 + g_{Y,t+1}} + s_{t+1} - \mathcal{T}_{t+1}$$

where  $s_t = G_t/Y_t$  and  $\mathcal{T}_t = T_t/Y_t$ .

Linearised:

$$\hat{b}_{t+1} = \underbrace{\frac{1 + \bar{i}}{1 + \bar{g}_Y}}_{\rho_b} \hat{b}_t + \frac{\bar{i}}{1 + \bar{g}_Y} \hat{i}_t - \frac{\bar{g}_Y}{1 + \bar{g}_Y} \hat{g}_{Y,t+1} + \frac{\bar{s}}{\bar{b}} \hat{s}_{t+1} - \frac{\bar{\mathcal{T}}}{\bar{b}} \hat{\mathcal{T}}_{t+1}$$

Extension: replace lump-sum rebate of tariff revenue with  $G_{ki,t}$  financing, creating a feedback loop between trade and fiscal policy.

## Appendix: Optimal Fiscal Rule (Unconstrained Budget, $\sigma > 1$ , $\kappa > 0$ )

$$g_{k,t} = \frac{H_k}{X_k} y_{k,t} + \frac{J_k}{X_k} a_{k,t} - \frac{\theta}{\lambda_k X_k} \pi_{k,t} + \frac{\sigma-1}{(1-\chi)^{1/\sigma} X_k} \sum_k \mu_k \lambda_k \phi_{k,t}^\pi$$

$$H_k = \chi_k^* + 1 + \varphi + (\sigma-1) \frac{\omega_{c,k}}{1-\chi_k} - \varphi \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

$$J_k = -(1+\varphi) \left( 1 - \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$X_k = \chi_k^* + (\sigma-1) \chi_k^* \omega_{c,k} \chi_k + \left( 1 + (\sigma-1) \omega_{g,k} \right) \left( \lambda_k + \frac{1+\chi_k^*+\varphi}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$\phi_{k,t}^\pi = \left( -\varphi y_{k,t} - g_{k,t} (1 + (\sigma-1) \omega_{g,k}) + (1+\varphi) a_{k,t} \right) \left( \lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k} \right)^{-1}$$

Comparing welfare under this vs. the constrained rule quantifies the cost of fiscal inflexibility.

## Appendix: Optimal Monetary Rule (Unconstrained Budget)

Optimal monetary policy sets a weighted inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t} = - \sum_k \mu_k \frac{\varphi y_{k,t} + g_{k,t}(1+(\sigma-1)\omega_{g,k}) + (1+\varphi) a_{k,t}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

- ▶ Inflation weights increase with private-consumption share, decrease with  $\lambda_k$ .
- ▶ Government spending enters the target when  $\kappa > 0$  (government demand affects marginal costs).

## Appendix: Fixed vs. Endogenous Budgets — Detail

### Fully endogenous spending

- ▶ Planner chooses both  $\bar{G}_{k,t}$  and  $\{G_{ki,t}\}$ .
- ▶ Level and composition respond to shocks.
- ▶ Benchmark: first-best fiscal stabilisation.
- ▶ Fiscal rule is *absolute*: each  $g_{k,t}$  set independently.

### Fixed aggregate budget

- ▶  $\bar{G}_{k,t}$  exogenous; only composition adjusts.
- ▶ Fiscal rule is *relative*: gaps wrt residual sector.
- ▶ Welfare loss from constraint is concentrated in aggregate stabilisation.
- ▶ Cross-sectional allocation may remain close to first-best.

**Implications.** If the welfare gap is small, a budget-constrained government can approximate first-best outcomes through compositional reallocation alone. This is the central hypothesis to be tested in the networked model.

## Appendix: Open Questions

- ▶ **Network centrality and fiscal allocation.** How does a sector's position in the IO network affect its optimal public-good allocation? The IO weights  $\omega_{klj}$  introduce cross-sector spillovers absent in the simple model.
- ▶ **Fiscal–trade policy interaction.** With tariff revenue financing government purchases, trade policy changes alter the fiscal envelope. The welfare implications of this feedback are to be characterised.
- ▶ **Dimensionality.** The Ramsey problem involves  $K \times I$  spending instruments. Practical approaches may require restricting the class of admissible rules.
- ▶ **Political economy.** The exogenous-budget assumption abstracts from the determination of  $\bar{G}_{k,t}$ . Endogenising this would require a political-economy layer.

## Q&A: Why Not Derive the Ramsey Problem Directly?

### The question

Why modify Phillips curves with fiscal instruments rather than solving a full Ramsey taxation problem?

- ▶ The Phillips-curve approach is a **reduced-form shortcut**: it isolates the cost-push channel of taxation while preserving the existing IO structure of Aguilar et al. (2025).
- ▶ A full Ramsey problem in a  $4 \times 44$  networked model is computationally demanding—it requires solving for  $K \times I$  optimal tax instruments simultaneously.
- ▶ The **research agenda goal** is precisely to move toward the full Ramsey characterisation. The current extensions are a tractable first step.
- ▶ La'O & Tahbaz-Salehi (2024) show that even in simpler networks, the Ramsey problem has a rich structure. Our approach builds intuition before scaling up.

## The question

How does the relative allocation rule relate to optimal policy results in production networks?

### Rubbo (2023):

- ▶ Optimal *monetary* policy in production networks.
- ▶ Divine coincidence fails; price-stability target depends on network topology.
- ▶ Our work: fiscal policy adds a second instrument that can target sectoral gaps directly.

### La’O & Tahbaz-Salehi (2024):

- ▶ Optimal monetary policy in production networks (*Econometrica*).
- ▶ Network structure shapes the optimal inflation target and creates trade-offs absent in one-sector models.
- ▶ Our contribution: government *spending* as a complementary fiscal instrument, with an aggregate budget constraint.

**Key distinction:** our relative allocation rule operates under a *fixed fiscal envelope*—a constraint absent in both papers.

# Q&A: When Does the Budget Constraint Bind?

## The question

What happens when  $\bar{G}_t$  is close to optimal vs. far from it?

- ▶ When  $\bar{G}_t$  is **close to the first-best level**, compositional reallocation suffices: the relative allocation rule can approximate optimal welfare by reshuffling spending across sectors.
- ▶ When  $\bar{G}_t$  is **far from optimal** (e.g., a deep austerity constraint), the welfare gap grows because the level channel is shut off. The planner cannot compensate for an insufficient aggregate envelope by reallocating alone.
- ▶ The **welfare gap** between fixed and endogenous budgets is therefore increasing in  $|\bar{G}_t - G_t^*|$ , where  $G_t^*$  is the first-best aggregate level.
- ▶ Quantifying this gap in the  $4 \times 44$  networked model is the central objective of the research agenda.

## Q&A: Why 4 Countries and 44 Sectors?

### The question

How sensitive are results to this level of granularity?

- ▶ The  $4 \times 44$  structure follows Aguilar et al. (2025), calibrated to the **OECD ICIO 2019 tables** (Inter-Country Input-Output).
- ▶ 4 countries: a practical choice balancing model tractability with open-economy realism (e.g., US, EA, CN, RoW).
- ▶ 44 sectors: the full ISIC Rev. 4 classification available in ICIO—no aggregation required.
- ▶ **Robustness considerations:**
  - Coarser aggregations (e.g., 10–15 sectors) can be tested by collapsing IO tables.
  - The key qualitative predictions—relative reallocation, countercyclical—should survive aggregation.
  - Quantitative magnitudes (welfare gaps) are likely sensitive to granularity, as network effects depend on the density of the IO matrix.

## Q&A: The Pass-Through Parameter $\kappa$

**Motivation.** Cox et al. (2024) assume government demand does not affect firms' pricing decisions ( $\kappa = 0$ ). Empirical evidence (Ben Zeev & Pappa, 2017; Barattieri et al., 2023) suggests fiscal spending does affect prices.

### Definition

$\kappa \in [0, 1]$  controls how much inelastic government demand passes through to marginal cost. When  $\kappa > 0$ :

- ▶ The Phillips curve steepens:  $\lambda'_k = \lambda_k / (1 - \delta)$ , where  $\delta = \frac{\kappa}{\theta-1} \frac{\bar{G}}{C}$ .
- ▶ Government spending becomes a **supply-side instrument**:  $g_{k,t}$  enters the Phillips curve directly.

**Nesting:**  $\kappa = 0$  recovers Cox et al. (2024). At  $\kappa = 1$  (full pass-through), for  $\theta = 6$  and  $\chi_k^* = 1$ :  
 $\lambda'_k = 1.25 \lambda_k$ .

## Q&A: CRRA Preferences and the Insurance Motive

With  $\sigma \neq 1$ , the welfare objective acquires an additional term (in red):

$$\begin{aligned} & -\frac{1}{2} \sum_k \mu_k \left( (1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ & \quad \left. + (\sigma-1) \left[ (1-\chi_k) \omega_{c,k} \left( \frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right) \end{aligned}$$

Fiscal policy now balances **three motives**:

- 1 Allocation:** match public-good provision to sectoral needs.
- 2 Stabilisation:** dampen output and inflation gaps.
- 3 Insurance:** hedge against aggregate consumption risk ( $\sigma-1$  term).

## Q&A: Negativity Condition for Countercyclicity

Setting  $\kappa = 1$ , the overall coefficient on  $y_{k,t}$  in the fiscal rule is:

$$\underbrace{\frac{1}{(\theta-1)(1-\chi_k)\varphi + 1}}_{\text{always } > 0} \times \underbrace{\left[ \frac{1}{1-\chi_k} + \varphi + (\sigma-1) \frac{\omega_{c,k}}{1-\chi_k} - \frac{(\theta-1)(1-\chi_k)\varphi}{(1-\chi_k)\lambda_k} \right]}_{\text{sign determines cyclicity}}$$

Government spending is **countercyclical** ( $g_{k,t}$  falls when  $y_{k,t}$  rises) if and only if:

$$\sigma > \left( \frac{(\theta-1)(1-\chi_k)\varphi}{\lambda_k} - 1 - \varphi(1-\chi_k) \right) \frac{1}{\omega_{c,k}} + 1$$

- ▶ Under **flexible prices** ( $\lambda_k \rightarrow \infty$ ): always satisfied.
- ▶ Under **moderate rigidity** ( $\alpha_k < 0.3$ ): satisfied for standard calibrations.
- ▶  $\kappa < 1$  implies a **less countercyclical** stance (the pass-through channel weakens).

## Q&A: Welfare Approximation — Full Derivation Steps

**Step 1.** Second-order Taylor expansion of  $U$  around the efficient steady state:

$$U - \bar{U} \approx \sum_k \mu_k \left[ \bar{C}(1-\chi) \hat{c}_{k,t} + \bar{G}\chi \hat{g}_{k,t} - \bar{N}_k^{1+\varphi} \hat{n}_{k,t} \right] + \text{second-order terms}$$

**Step 2.** Use the production function  $Y_{k,t} = A_{k,t} N_{k,t}$  and goods market clearing  $Y_{k,t} = C_{k,t} + G_{k,t}$  to eliminate  $\hat{n}_{k,t}$  and  $\hat{c}_{k,t}$ :

$$\hat{n}_{k,t} = y_{k,t} - a_{k,t}, \quad (1-\chi_k) \hat{c}_{k,t} = y_{k,t} - \chi_k g_{k,t}$$

**Step 3.** Calvo price dispersion contributes  $\sum_k \frac{\theta}{\lambda_k} \pi_{k,t}^2$  (Woodford, 2003, Ch. 6).

**Step 4.** Collecting terms (setting  $\sigma = 1$ ,  $\kappa = 0$ ):

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[ (1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

where  $\chi_k^* \equiv \chi_k / (1-\chi_k)$ . The public-good gap arises from the resource constraint binding at the sectoral level.

## Q&A: Relative Allocation Rule — FOC Details

**Lagrangian.** Minimise  $\mathcal{W}$  subject to  $\sum_k \omega_{g,k} g_{k,t} = \bar{g}_t$  (aggregate budget):

$$\mathcal{L} = \mathcal{W} + \eta_t \left( \bar{g}_t - \sum_k \omega_{g,k} g_{k,t} \right)$$

**FOC for sector  $k$ :**

$$\frac{\partial \mathcal{W}}{\partial g_{k,t}} = \mu_k \left[ \chi_k^* (g_{k,t} - y_{k,t}) + \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\partial \pi_{k,t}}{\partial g_{k,t}} + (1+\varphi) \frac{\partial y_{k,t}}{\partial g_{k,t}} \right] = \eta_t \omega_{g,k}$$

**Eliminating  $\eta_t$ :** Equate the FOC for sector  $k$  with sector  $i$  (the residual):

$$\frac{1}{\omega_{g,k}} \frac{\partial \mathcal{W}}{\partial g_{k,t}} = \frac{1}{\omega_{g,i}} \frac{\partial \mathcal{W}}{\partial g_{i,t}}$$

Substituting the Phillips curve  $\pi_{k,t} = \lambda_k [(1+\varphi) y_{k,t} + \chi_k g_{k,t}] + \beta \mathbb{E}_t \pi_{k,t+1}$  and solving for  $g_{k,t}$  in terms of  $g_{i,t}$ ,  $y_{k,t}$ ,  $\pi_{k,t}$ ,  $y_{i,t}$ ,  $\pi_{i,t}$  yields the relative allocation rule.

See Appendix A4 for the full structural coefficients.