Government Spending in Multi-Sector Open Economies with Production Networks

Antonio Eugenelo

University of Oxford
Department of Economics

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Motivation and Plan

Question. What is the welfare effect of redistributing sector-specific government spending under a fixed aggregate budget, relative to fully endogenous fiscal policy?

- Production networks amplify and reshape the transmission of shocks across sectors (Acemoglu et al., 2012; Baqaee & Farhi, 2020; Rubbo, 2023).
- Optimal fiscal allocation across sectors depends on sectoral heterogeneity in price stickiness, public-good shares, and network position (Cox et al., 2024).
- No existing framework combines these two dimensions in an open-economy setting.

Plan — building up step by step:

(i) A **toy model** with non-distortionary lump-sum transfers and government procurements $\to$ a *relative allocation rule*.

(ii) The **global production network model** of Aguilar et al. (2025) with distortionary taxation and subsidisation.

(iii) **Planned extensions**: combine lump-sum procurements with proportional taxation/subsidies to production and labour in the richer model.

A Simple Multi-Sector Model: Setup

Closed NK economy, K sectors, Calvo pricing (α_k), following Cox et al. (2024).

Household utility over private and public composites:

$$U = \sum_t \beta^t \left[(1-\chi) \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \sum_k \nu_k \frac{N_{k,t}^{1+\varphi}}{1+\varphi} \right]$$

Key constraint: the aggregate public-good bundle $\bar{G}_t$ is *exogenous*. The planner chooses only the sectoral composition $\{g_{k,t}\}$.

The welfare loss (setting $\sigma = 1$, isolating the budget channel):

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[\underbrace{(1+\varphi)\, y_{k,t}^2}_{\text{output gaps}} + \underbrace{\frac{\theta(1-\chi_k)}{\lambda_k}\, \pi_{k,t}^2}_{\text{inflation}} + \underbrace{\chi_k^* \left(g_{k,t} - y_{k,t}\right)^2}_{\text{public-good gaps}} \right]$$

The Relative Allocation Rule

Under exogenous $\bar{G}_t$, optimal spending in sector k relative to a residual sector i:

$$g_{k,t} = \underbrace{\Phi_{ki}}_{\text{structural}} \; g_{i,t}$$

$$\underbrace{-\frac{\varphi}{1+\lambda_k+\varphi\lambda_k}\, y_{k,t} - \frac{\theta\,\varphi\,(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k}\,\pi_{k,t}}_{\text{sector } k\text{: spending falls when own gaps are large}}$$

$$\underbrace{+\,\Phi_{ki}\left(\frac{\varphi\, y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\,\varphi\,(1-\chi_i)\,\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)}\right)}_{\text{sector } i\text{: spending rises when residual-sector gaps are large}}$$

Key property

The rule is inherently *relative*: spending is **reallocated** toward sectors with lower output gaps and lower inflation *relative* to the residual sector. What matters is the cross-sectional dispersion of gaps, not their level.

The Global Production Network Model (Aguilar et al., 2025)

- K=4 countries, I=44 sectors
- Nested CES: energy/non-energy, domestic/foreign
- Sector- and country-specific Calvo pricing
- Country-specific Taylor rules
- Balanced budget, lump-sum taxes, static production subsidies, tariff revenue

Marginal cost in sector i, country k:

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki}\,\widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\textstyle\sum_{l,j}\mathcal{M}_{ki}\omega_{klij}\,\widehat{p}_{klij,t}}_{\text{IO inputs}}$$

Sectoral Phillips curve:

$$\pi_{ki,t} = \kappa_{ki}(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t}$$

Tariffs enter as price wedges:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t})\,\widetilde{P}_{l,k,i,t}$$

Original Extensions: Fiscal Instruments in the Phillips Curves

Not part of Aguilar et al. (2025). Low-cost additions that do not alter the IO structure.

Define $\hat{\tau}^w_{k,t} \equiv (\tau^w_{k,t} - \bar{\tau}^w_k)/(1 - \bar{\tau}^w_k)$ and $\hat{\tau}^s_{ki,t} \equiv (\tau^s_{ki,t} - \bar{\tau}^s_{ki})/(1 - \bar{\tau}^s_{ki})$.

Wage Phillips curve — adding a labour income tax:

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma\,\hat{c}_{k,t} + \varphi\,\hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}^w_{k,t}\right) + \beta\,\mathbb{E}_t\pi_{wk,t+1} + u^w_{k,t}$$

Price Phillips curve — making the production subsidy time-varying:

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}^s_{ki,t}\right) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}$$

Tax hike $\rightarrow$ inflationary cost-push. Subsidy hike $\rightarrow$ disinflationary cost-push.

Research Agenda: Fixed vs. Endogenous Fiscal Envelopes

Goal: introduce sector-specific $G_{ki,t}$ in the goods market clearing condition of the global production network model:

$$Y_{ki,t} = \sum_{l} C_{lki,t} + \sum_{l} \sum_{j} X_{lkji,t} + G_{ki,t}$$

and compare two policy regimes:

Fully endogenous spending

Planner chooses level *and* composition. Benchmark for first-best fiscal stabilisation.

Fixed aggregate budget

$\bar{G}_{k,t}$ exogenous; only the sectoral composition adjusts. Relevant when total spending is politically constrained.

Central question: how large is the welfare gap between the two regimes? If it is small, compositional reallocation alone—the relative allocation rule—may approximate the first-best, even without aggregate fiscal flexibility.

Summary

1. Under a fixed fiscal envelope, optimal sectoral spending follows a **relative allocation rule**: reallocate toward sectors with larger gaps.
2. The global production network model of **Aguilar et al. (2025)** provides a quantitative environment with IO linkages and sectoral tariffs, but lacks non-trivial fiscal dynamics.
3. **Original extensions**—labour income tax in the wage PC, time-varying subsidy in the price PC—introduce fiscal instruments without altering the core model.
4. The **research agenda**: derive spending rules in the networked model; quantify the welfare cost of fixing the aggregate budget relative to fully endogenous spending.

Thank you

antonio.eugenelo@economics.ox.ac.uk

Appendix: Related Literature

Production Networks & NK

- Acemoglu et al. (2012)
- Baqaee & Farhi (2020, 2024)
- Pasten, Schoenle & Weber (2020)
- Rubbo (2023)

Tariffs & Open-Economy NK

- Galí & Monacelli (2005)
- Comin & Johnson (2023)
- Aguilar et al. (2025)

Fiscal Policy in Disaggregated Economies

- Aoki (2001)
- Cox, Müller, Pasten, Schoenle & Weber (2024)

Fiscal–Price Effects (Empirical)

- Nekarda & Ramey (2013)
- Ben Zeev & Pappa (2017)
- Barattieri et al. (2023)

Appendix: Welfare Objective

Under $\sigma = 1$, abstracting from government-demand pass-through, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[(1+\varphi)\, y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* \left(g_{k,t} - y_{k,t} \right)^2 \right]$$

Three tensions:

1. **Output-gap stabilisation:** penalises $y_{k,t}^2$.
2. **Inflation stabilisation:** penalises $\pi_{k,t}^2$, weighted inversely by the Phillips curve slope λ_k.
3. **Public-good allocation:** penalises the gap $g_{k,t} - y_{k,t}$.

When $\bar{G}_t$ is unconstrained, $g_{k,t}$ can be set independently in each sector. When only the composition is a choice variable, the problem changes qualitatively: the planner can only *reshuffle* spending.

Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$-\frac{1}{2}\sum_k \mu_k \Bigg((1+\varphi)\, y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k}\, \pi_{k,t}^2 + \chi_k^*\,(g_{k,t}-y_{k,t})^2$$
$$+ {\color{red}(\sigma-1)} \left[(1-\chi_k)\,\omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t} \right)^2 + \omega_{g,k}\,\chi_k\, g_{k,t}^2 \right] \Bigg)$$

- The $\sigma-1$ term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- $\kappa > 0$ steepens Phillips curves ($\lambda_k' > \lambda_k$), making fiscal policy a supply-side instrument.

Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous $\bar{G}_t$ (with $\sigma = 1$, $\kappa = 0$):

$$
\begin{aligned}
g_{k,t} \;=\;& \frac{1-\chi_k}{1-\chi_i}\left(\frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)}\right)\left(\frac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)}\right)^{-1} g_{i,t} \\
& - \frac{\varphi\, y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta\,\varphi\,(1-\chi_k)\,\pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\
& + \frac{1-\chi_k}{1-\chi_i}\left(\frac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)}\right)^{-1} \\
& \quad \times \left(\frac{\varphi\, y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\,\varphi\,(1-\chi_i)\,\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)}\right)
\end{aligned}
$$

$\omega_{g,k}$: weight of sector k in the government Cobb–Douglas aggregator. ρ: CES elasticity of public-good bundle.

Appendix: Optimal Monetary Rule under Aggregate Constraint

Under exogenous $\bar{G}_t$, optimal monetary policy sets:

$$\sum_k \mu_k \, \frac{\theta(1-\chi_k)}{\lambda_k} \, \frac{\lambda_k + \varphi\lambda_k(1-\chi_k)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \, \pi_{k,t} \;=$$

$$\sum_k \mu_k \left(\frac{\chi_k \, g_{k,t}}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} - \frac{y_{k,t}(1-\chi_k)(1+\varphi+\chi_k^*)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \right)$$

- Inflation weights depend on private-consumption share and λ_k.
- Government spending enters the target because the constraint links $g_{k,t}$ and $y_{k,t}$ across sectors.

Appendix: Aguilar et al. — Household Problem

Per-period utility: $U_t = \left(C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi)\, dg\right) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[\widetilde{\beta}_k^{1/\gamma}\, C_{kE,t}^{(\gamma-1)/\gamma} + (1-\widetilde{\beta}_k)^{1/\gamma}\, C_{kM,t}^{(\gamma-1)/\gamma}\right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$C_{k,t}^{-\sigma} = \beta\, \mathbb{E}_t\, C_{k,t+1}^{-\sigma} \frac{1+i_{k,t}}{1+\pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}}$$

$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* \, \mathrm{nfa}_{k,t} + \varepsilon_{kK,t}^{e} \qquad \text{(UIP)}$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t}\right) + \beta\, \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^{w}$$

Appendix: Aguilar et al. — Firm Problem

CES production: $Y_{ki,f,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{1/\psi}\, N_{fki,t}^{(\psi-1)/\psi} + \widetilde{\vartheta}_{ki}^{1/\psi}\, X_{fki,t}^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\,\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I}\mathcal{M}_{ki}\omega_{klij}\,\widehat{p}_{klij,t}$$

where α_{ki}: labour share, ω_{klij}: IO expenditure share, $\mathcal{M}_{ki}$: steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}\right) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t} \qquad \kappa_{ki} = \frac{(1-\theta^{p}_{ki})(1-\beta\theta^{p}_{ki})}{\theta^{p}_{ki}}$$

Appendix: Aguilar et al. — Tariff Propagation

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t})\,\widetilde{P}_{l,k,i,t}$$

Propagation:

1. Tariff on country l raises input prices $\widehat{p}_{klij,t}$ for domestic sectors sourcing from sector j in l.
2. Higher input costs raise $\widehat{\mathrm{mc}}_{ki,t}$, feeding into $\pi_{ki,t}$.
3. Cost increases cascade downstream through the IO network.
4. Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\frac{B_{k,t}}{1+i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_{i} \tau_{kli,t}\, P^{l}_{kli,t} \Big(C_{kli,t} + \textstyle\sum_j X_{klji,t} \Big) = B_{k,t-1} + \sum_{i} \tau^{s}_{ki}\, \mathrm{MC}_{ki,t}\, Y_{ki,t}$$

Appendix: Aguilar et al. — Calibration

Households

- $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$
- Energy/non-energy elast. $\gamma = 0.4$
- Trade elasticity $\delta = 1$
- Calvo wage $\theta_k^w = 0.75$
- Consumption shares from OECD ICIO (2019)

Monetary policy

- $\rho_r = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.125$
- Target: headline inflation

Firms

- Labour/input elast. $\psi = 0.5$
- Energy/non-energy elast. $\phi = 0.4$
- Trade elasticity $\mu = 1$
- IO shares from OECD ICIO (2019)
- Markups from Eurostat Figaro
- Calvo prices from ECB PRISMA

Tariff shocks

- $\rho^\tau = 0.96$, $\sigma^\tau = 1$

Appendix: Aguilar et al. — Goods Market Clearing and GDP

Market clearing:

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K} \sum_{j=1}^{I} X_{lkji,t}$$

Log-linearised:

$$\lambda_{ki}\, \widehat{y}_{ki,t} = \sum_{l=1}^{K} \mathcal{Y}_{lk} \left(\beta_{lki}\, \widehat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj}\, \omega_{lkji}\, \widehat{x}_{lkji,t} \right)$$

where $\lambda_{ki} = P_{ki} Y_{ki} / \mathcal{Y}_k$ is the Domar weight.

Real GDP:

$$\widehat{y}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k \left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t} \right)$$

where Υ_k is the trade-to-GDP ratio.

Appendix: Derivation — Wage Phillips Curve with Tax

Household FOC with labour income tax $\tau^{w}_{k,t}$:

$$\sum_{l=0}^{\infty}(\beta\theta^{W}_{k})^{l}\,\mathbb{E}_t\left[N_{k,t+l|t}\,C^{-\sigma}_{t+l|t}\left(\frac{(1-\tau^{w}_{k,t+l})\,W^{*}_{k,t}}{P_{t+l}}-\mathcal{M}_{wk,t}\,\mathrm{MRS}_{k,t+l|t}\right)\right]=0$$

Log-linearised reset wage:

$$w^{*}_{k,t}=(1-\beta\theta^{W}_{k})\sum_{l=0}^{\infty}(\beta\theta^{W}_{k})^{l}\,\mathbb{E}_t\left[\mathrm{mrs}_{t+l|t}+\mu^{n}_{wk,t+l}+p_{kC,t+l}+\hat{\tau}^{w}_{k,t+l}\right]$$

Calvo aggregation ($\pi_{wk,t}=w_{k,t}-w_{k,t-1}$):

$$\pi_{wk,t}=\kappa_{wk}\left(\sigma\hat{c}_{k,t}+\varphi\hat{n}_{k,t}-\hat{w}_{k,t}+\hat{\tau}^{w}_{k,t}\right)+\beta\,\mathbb{E}_t\pi_{wk,t+1}+u^{w}_{k,t}$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

Appendix: Derivation — Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy $\tau^s_{ki,t}$:

$$\sum_{l=0}^{\infty}(\beta\theta^p_{ki})^l\,\mathbb{E}_t\left[\Lambda_{t,t+l}\,Y_{ki,t+l|t}\left(P^*_{ki,t}-\mathcal{M}_{pk,t+l}\,(1-\tau^s_{ki,t+l})\,\mathrm{MC}^n_{ki,t+l|t}\right)\right]=0$$

Log-linearised reset price:

$$p^*_{ki,t}=(1-\beta\theta^p_{ki})\sum_{l=0}^{\infty}(\beta\theta^p_{ki})^l\,\mathbb{E}_t\left[\mathrm{mc}^n_{ki,t+l|t}+\mu^n_{pki,t+l}-\hat{\tau}^s_{ki,t+l}\right]$$

Calvo aggregation ($\pi_{ki,t}=p_{ki,t}-p_{ki,t-1}$):

$$\pi_{ki,t}=\kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t}-\hat{p}_{ki,t}-\hat{\tau}^s_{ki,t}\right)+\beta\,\mathbb{E}_t\pi_{ki,t+1}+u^p_{ki,t}$$

Subsidy increase lowers effective marginal cost $\rightarrow$ disinflationary cost-push.

Appendix: Government Budget Dynamics

Starting from the nominal budget constraint:

$$B_{t+1} = B_t(1 + i_{t+1}) + G_{t+1} - T_{t+1}$$

In ratios to GDP ($b_t \equiv B_t/Y_t$):

$$b_{t+1} = b_t \, \frac{1 + i_{t+1}}{1 + g_{Y,t+1}} + s_{t+1} - \mathcal{T}_{t+1}$$

where $s_t = G_t/Y_t$ and $\mathcal{T}_t = T_t/Y_t$.

Linearised:

$$\hat{b}_{t+1} = \underbrace{\frac{1 + \bar{\imath}}{1 + \bar{g}_Y}}_{\rho_b} \, \hat{b}_t + \frac{\bar{\imath}}{1 + \bar{g}_Y} \, \hat{\imath}_t - \frac{\bar{g}_Y}{1 + \bar{g}_Y} \, \hat{g}_{Y,t+1} + \frac{\bar{s}}{\bar{b}} \, \hat{s}_{t+1} - \frac{\bar{\mathcal{T}}}{\bar{b}} \, \hat{\mathcal{T}}_{t+1}$$

Extension: replace lump-sum rebate of tariff revenue with $G_{ki,t}$ financing, creating a feedback loop between trade and fiscal policy.

Appendix: Optimal Fiscal Rule (Unconstrained Budget, $\sigma > 1$, $\kappa > 0$)

$$g_{k,t} = \frac{H_k}{X_k}\, y_{k,t} + \frac{J_k}{X_k}\, a_{k,t} - \frac{\theta}{\lambda_k X_k}\, \pi_{k,t} + \frac{\sigma-1}{(1-\chi)^{1/\sigma} X_k} \sum_k \mu_k \lambda_k\, \phi^{\pi}_{k,t}$$

$$H_k = \chi_k^* + 1 + \varphi + (\sigma-1)\frac{\omega_{c,k}}{1-\chi_k} - \varphi\lambda_k\, \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}}$$

$$J_k = -(1+\varphi)\left(1 - \lambda_k\, \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}}\right)$$

$$X_k = \chi_k^* + (\sigma-1)\chi_k^*\omega_{c,k}\chi_k + \Big(1+(\sigma-1)\omega_{g,k}\Big)\left(\lambda_k + \frac{1+\chi_k^*+\varphi}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}}\right)$$

$$\phi^{\pi}_{k,t} = \Big(-\varphi\, y_{k,t} - g_{k,t}(1+(\sigma-1)\omega_{g,k}) + (1+\varphi)\, a_{k,t}\Big)\left(\lambda_k\varphi + \frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}\right)^{-1}$$

Comparing welfare under this vs. the constrained rule quantifies the cost of fiscal inflexibility.

Appendix: Optimal Monetary Rule (Unconstrained Budget)

Optimal monetary policy sets a weighted inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k}\,\pi_{k,t} = -\sum_k \mu_k \,\frac{\varphi\, y_{k,t} + g_{k,t}\big(1+(\sigma-1)\omega_{g,k}\big) + (1+\varphi)\, a_{k,t}}{\lambda_k\varphi + \frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}}$$

- Inflation weights increase with private-consumption share, decrease with λ_k.
- Government spending enters the target when $\kappa > 0$ (government demand affects marginal costs).

Appendix: Fixed vs. Endogenous Budgets — Detail

Fully endogenous spending

- Planner chooses both $\bar{G}_{k,t}$ and $\{G_{ki,t}\}$.
- Level and composition respond to shocks.
- Benchmark: first-best fiscal stabilisation.
- Fiscal rule is *absolute*: each $g_{k,t}$ set independently.

Fixed aggregate budget

- $\bar{G}_{k,t}$ exogenous; only composition adjusts.
- Fiscal rule is *relative*: gaps wrt residual sector.
- Welfare loss from constraint is concentrated in aggregate stabilisation.
- Cross-sectional allocation may remain close to first-best.

Implications. If the welfare gap is small, a budget-constrained government can approximate first-best outcomes through compositional reallocation alone. This is the central hypothesis to be tested in the networked model.

Appendix: Open Questions

- **Network centrality and fiscal allocation.** How does a sector's position in the IO network affect its optimal public-good allocation? The IO weights ω_{klij} introduce cross-sector spillovers absent in the simple model.
- **Fiscal–trade policy interaction.** With tariff revenue financing government purchases, trade policy changes alter the fiscal envelope. The welfare implications of this feedback are to be characterised.
- **Dimensionality.** The Ramsey problem involves $K \times I$ spending instruments. Practical approaches may require restricting the class of admissible rules.
- **Political economy.** The exogenous-budget assumption abstracts from the determination of $\bar{G}_{k,t}$. Endogenising this would require a political-economy layer.