

Government Spending in Multi-Sector Open Economies with Production Networks

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Motivation and Plan

Question. What is the welfare effect of redistributing sector-specific government spending under a fixed aggregate budget, relative to fully endogenous fiscal policy?

- ▶ Production networks amplify and reshape the transmission of shocks across sectors (Acemoglu et al., 2012; Baqaee & Farhi, 2020; Rubbo, 2023).
- ▶ Optimal fiscal allocation across sectors depends on sectoral heterogeneity in price stickiness, public-good shares, and network position (Cox et al., 2024).
- ▶ No existing framework combines these two dimensions in an open-economy setting.

Plan — building up step by step:

- (i) A **toy model** with non-distortionary lump-sum transfers and government procurements → a *relative allocation rule*.
- (ii) The **global production network model** of Aguilar et al. (2025) with distortionary taxation and subsidisation.
- (iii) **Planned extensions:** combine lump-sum procurements with proportional taxation/subsidies to production and labour in the richer model.

A Simple Multi-Sector Model: Setup

Closed NK economy, K sectors, Calvo pricing (α_k), following Cox et al. (2024).

Household utility over private and public composites:

$$U = \sum_t \beta^t \left[(1 - \chi) \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \sum_k \nu_k \frac{N_{k,t}^{1+\varphi}}{1+\varphi} \right]$$

Key constraint: the aggregate public-good bundle \bar{G}_t is *exogenous*. The planner chooses only the sectoral composition $\{g_{k,t}\}$.

What does the planner's loss function look like under this constraint?

Welfare Loss under a Fixed Budget

Setting $\sigma = 1$ and isolating the budget channel, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[\underbrace{(1 + \varphi) y_{k,t}^2}_{\text{output gaps}} + \underbrace{\frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2}_{\text{inflation}} + \underbrace{\chi_k^* (g_{k,t} - y_{k,t})^2}_{\text{public-good gaps}} \right]$$

Three tensions the planner must balance:

- 1 Output-gap stabilisation:** penalises $y_{k,t}^2$.
- 2 Inflation stabilisation:** penalises $\pi_{k,t}^2$, weighted inversely by the Phillips curve slope λ_k .
- 3 Public-good allocation:** penalises the gap $g_{k,t} - y_{k,t}$; under a fixed budget the planner can only *reshuffle* spending across sectors.

The Relative Allocation Rule

Under exogenous \bar{G}_t , optimal spending in sector k relative to a residual sector i :

$$g_{k,t} = \underbrace{\Phi_{ki}}_{\text{structural}} g_{i,t} - \underbrace{\frac{\varphi}{1+\lambda_k+\varphi\lambda_k} y_{k,t} - \frac{\theta\varphi(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k} \pi_{k,t}}_{\text{sector } k: \text{ spending falls when own gaps are large}} + \underbrace{\Phi_{ki} \left(\frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\varphi(1-\chi_i)\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right)}_{\text{sector } i: \text{ spending rises when residual-sector gaps are large}}$$

Key property

The rule is inherently *relative*: spending is **reallocated** toward sectors with lower output gaps and lower inflation *relative* to the residual sector. What matters is the cross-sectional dispersion of gaps, not their level.

The Global Production Network Model (Aguilar et al., 2025)

- ▶ $K=4$ countries, $I=44$ sectors
- ▶ Nested CES: energy/non-energy, domestic/foreign
- ▶ Sector- and country-specific Calvo pricing
- ▶ Country-specific Taylor rules
- ▶ Balanced budget, lump-sum taxes, static production subsidies, tariff revenue

Marginal cost in sector i , country k :

$$\widehat{mc}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\sum_{l,j} \mathcal{M}_{ki}\omega_{klij}\widehat{p}_{klij,t}}_{\text{IO inputs}}$$

Sectoral Phillips curve:

$$\pi_{ki,t} = \kappa_{ki}(\widehat{mc}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Tariffs enter as price wedges:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \widetilde{P}_{l,k,i,t}$$

Original Extensions: Fiscal Instruments in the Phillips Curves

Why extend? The Aguilar et al. model features lump-sum taxes and static production subsidies—no active fiscal stabilisation. To study the welfare cost of constrained fiscal policy in this environment, we introduce two time-varying distortionary instruments that enter the Phillips curves directly.

Define $\hat{\tau}_{k,t}^w \equiv (\tau_{k,t}^w - \bar{\tau}_k^w)/(1 - \bar{\tau}_k^w)$ and $\hat{\tau}_{ki,t}^s \equiv (\tau_{ki,t}^s - \bar{\tau}_{ki}^s)/(1 - \bar{\tau}_{ki}^s)$.

Wage Phillips curve — adding a labour income tax:

$$\pi_{wk,t} = \kappa_{wk} \left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Price Phillips curve — making the production subsidy time-varying:

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Tax hike \rightarrow inflationary cost-push. Subsidy hike \rightarrow disinflationary cost-push.

Research Agenda: Fixed vs. Endogenous Fiscal Envelopes

Goal: introduce sector-specific $G_{ki,t}$ in the goods market clearing condition of the global production network model:

$$Y_{ki,t} = \sum_l C_{lki,t} + \sum_l \sum_j X_{lkji,t} + G_{ki,t}$$

and compare two policy regimes:

Fully endogenous spending

Planner chooses level *and* composition.
Benchmark for first-best fiscal stabilisation.

Fixed aggregate budget

$\bar{G}_{k,t}$ exogenous; only the sectoral composition adjusts. Relevant when total spending is politically constrained.

Central question: how large is the welfare gap between the two regimes? If it is small, compositional reallocation alone—the relative allocation rule—may approximate the first-best, even without aggregate fiscal flexibility.

- 1 Under a fixed fiscal envelope, optimal sectoral spending follows a **relative allocation rule**: reallocate toward sectors with larger gaps.
- 2 The global production network model of **Aguilar et al. (2025)** provides a quantitative environment with IO linkages and sectoral tariffs, but lacks non-trivial fiscal dynamics.
- 3 **Original extensions**—labour income tax in the wage PC, time-varying subsidy in the price PC—introduce fiscal instruments without altering the core model.
- 4 The **research agenda**: derive spending rules in the networked model; quantify the welfare cost of fixing the aggregate budget relative to fully endogenous spending.

Thank you

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Production Networks & NK

- ▶ Acemoglu et al. (2012)
- ▶ Baqaee & Farhi (2020, 2024)
- ▶ Pasten, Schoenle & Weber (2020)
- ▶ Rubbo (2023)

Tariffs & Open-Economy NK

- ▶ Galí & Monacelli (2005)
- ▶ Comin & Johnson (2023)
- ▶ Aguilar et al. (2025)

Fiscal Policy in Disaggregated Economies

- ▶ Aoki (2001)
- ▶ Cox, Müller, Pasten, Schoenle & Weber (2024)

Fiscal–Price Effects (Empirical)

- ▶ Nekarda & Ramey (2013)
- ▶ Ben Zeev & Pappa (2017)
- ▶ Barattieri et al. (2023)

Appendix: Welfare Objective

Under $\sigma = 1$, abstracting from government-demand pass-through, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[(1 + \varphi) y_{k,t}^2 + \frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

Three tensions:

- 1 Output-gap stabilisation:** penalises $y_{k,t}^2$.
- 2 Inflation stabilisation:** penalises $\pi_{k,t}^2$, weighted inversely by the Phillips curve slope λ_k .
- 3 Public-good allocation:** penalises the gap $g_{k,t} - y_{k,t}$.

When \bar{G}_t is unconstrained, $g_{k,t}$ can be set independently in each sector. When only the composition is a choice variable, the problem changes qualitatively: the planner can only *reshuffle* spending.

Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$-\frac{1}{2} \sum_k \mu_k \left((1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ \left. + (\sigma-1) \left[(1-\chi_k) \omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right)$$

- ▶ The $\sigma-1$ term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- ▶ $\kappa > 0$ steepens Phillips curves ($\lambda'_k > \lambda_k$), making fiscal policy a supply-side instrument.

Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous \bar{G}_t (with $\sigma = 1$, $\kappa = 0$):

$$\begin{aligned} g_{k,t} = & \frac{1-\chi_k}{1-\chi_i} \left(\frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} g_{i,t} \\ & - \frac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta \varphi (1-\chi_k) \pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ & + \frac{1-\chi_k}{1-\chi_i} \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} \\ & \times \left(\frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta \varphi (1-\chi_i) \pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \end{aligned}$$

$\omega_{g,k}$: weight of sector k in the government Cobb–Douglas aggregator. ρ : CES elasticity of public-good bundle.

Appendix: Optimal Monetary Rule under Aggregate Constraint

Under exogenous \bar{G}_t , optimal monetary policy sets:

$$\sum_k \mu_k \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\lambda_k + \varphi\lambda_k(1-\chi_k)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \pi_{k,t} =$$
$$\sum_k \mu_k \left(\frac{\chi_k g_{k,t}}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} - \frac{y_{k,t}(1-\chi_k)(1+\varphi+\chi_k^*)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \right)$$

- Inflation weights depend on private-consumption share and λ_k .
- Government spending enters the target because the constraint links $g_{k,t}$ and $y_{k,t}$ across sectors.

Appendix: Aguilar et al. — Household Problem

Per-period utility: $U_t = (C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi) dg) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[\tilde{\beta}_k^{1/\gamma} C_{kE,t}^{(\gamma-1)/\gamma} + (1-\tilde{\beta}_k)^{1/\gamma} C_{kM,t}^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$\begin{aligned} C_{k,t}^{-\sigma} &= \beta \mathbb{E}_t C_{k,t+1}^{-\sigma} \frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}} \\ i_{k,t} - i_{K,t} &= \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* \text{nfa}_{k,t} + \varepsilon_{kK,t}^e \quad (\text{UIP}) \end{aligned}$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t}) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Appendix: Aguilar et al. — Firm Problem

CES production: $Y_{ki,f,t} = A_{ki,t} \left[\tilde{\alpha}_{ki}^{1/\psi} N_{fki,t}^{(\psi-1)/\psi} + \tilde{\vartheta}_{ki}^{1/\psi} X_{fki,t}^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{\text{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki} \alpha_{ki} \widehat{w}_{k,t} + \sum_{l=1}^K \sum_{j=1}^I \mathcal{M}_{ki} \omega_{klj} \widehat{p}_{klj,t}$$

where α_{ki} : labour share, ω_{klj} : IO expenditure share, \mathcal{M}_{ki} : steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki} (\widehat{\text{mc}}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p \quad \kappa_{ki} = \frac{(1 - \theta_{ki}^p)(1 - \beta \theta_{ki}^p)}{\theta_{ki}^p}$$

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \tilde{P}_{l,k,i,t}$$

Propagation:

- 1 Tariff on country l raises input prices $\hat{p}_{klij,t}$ for domestic sectors sourcing from sector j in l .
- 2 Higher input costs raise $\widehat{mc}_{ki,t}$, feeding into $\pi_{ki,t}$.
- 3 Cost increases cascade downstream through the IO network.
- 4 Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_i \tau_{kli,t} P_{kli,t}^l \left(C_{kli,t} + \sum_j X_{klji,t} \right) = B_{k,t-1} + \sum_i \tau_{ki}^s MC_{ki,t} Y_{ki,t}$$

Households

- ▶ $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$
- ▶ Energy/non-energy elast. $\gamma = 0.4$
- ▶ Trade elasticity $\delta = 1$
- ▶ Calvo wage $\theta_k^w = 0.75$
- ▶ Consumption shares from OECD ICIO (2019)

Monetary policy

- ▶ $\rho_r = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.125$
- ▶ Target: headline inflation

Firms

- ▶ Labour/input elast. $\psi = 0.5$
- ▶ Energy/non-energy elast. $\phi = 0.4$
- ▶ Trade elasticity $\mu = 1$
- ▶ IO shares from OECD ICIO (2019)
- ▶ Markups from Eurostat Figaro
- ▶ Calvo prices from ECB PRISMA

Tariff shocks

- ▶ $\rho^\tau = 0.96$, $\sigma^\tau = 1$

Market clearing:

$$Y_{ki,t} = \sum_{l=1}^K C_{lki,t} + \sum_{l=1}^K \sum_{j=1}^I X_{lkji,t}$$

Log-linearised:

$$\lambda_{ki} \hat{y}_{ki,t} = \sum_{l=1}^K \mathcal{Y}_{lk} \left(\beta_{lki} \hat{c}_{lki,t} + \sum_{j=1}^I \lambda_{lj} \omega_{lkji} \hat{x}_{lkji,t} \right)$$

where $\lambda_{ki} = P_{ki} Y_{ki} / \mathcal{Y}_k$ is the Domar weight.

Real GDP:

$$\hat{y}_{k,t} = \hat{c}_{k,t} + \Upsilon_k (\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t})$$

where Υ_k is the trade-to-GDP ratio.

Appendix: Derivation — Wage Phillips Curve with Tax

Household FOC with labour income tax $\tau_{k,t}^w$:

$$\sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t \left[N_{k,t+l|t} C_{t+l|t}^{-\sigma} \left(\frac{(1-\tau_{k,t+l}^w) W_{k,t}^*}{P_{t+l}} - \mathcal{M}_{wk,t} \text{MRS}_{k,t+l|t} \right) \right] = 0$$

Log-linearised reset wage:

$$w_{k,t}^* = (1-\beta \theta_k^w) \sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t [\text{mrs}_{t+l|t} + \mu_{wk,t+l}^n + p_{kC,t+l} + \hat{\tau}_{k,t+l}^w]$$

Calvo aggregation ($\pi_{wk,t} = w_{k,t} - w_{k,t-1}$):

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

Appendix: Derivation — Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy $\tau_{ki,t}^s$:

$$\sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[\Lambda_{t,t+l} Y_{ki,t+l|t} \left(P_{ki,t}^* - \mathcal{M}_{pk,t+l} (1 - \tau_{ki,t+l}^s) MC_{ki,t+l|t}^n \right) \right] = 0$$

Log-linearised reset price:

$$p_{ki,t}^* = (1 - \beta \theta_{ki}^p) \sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[mc_{ki,t+l|t}^n + \mu_{pki,t+l}^n - \hat{\tau}_{ki,t+l}^s \right]$$

Calvo aggregation ($\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$):

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Subsidy increase lowers effective marginal cost \rightarrow disinflationary cost-push.

Appendix: Government Budget Dynamics

Starting from the nominal budget constraint:

$$B_{t+1} = B_t(1 + i_{t+1}) + G_{t+1} - T_{t+1}$$

In ratios to GDP ($b_t \equiv B_t/Y_t$):

$$b_{t+1} = b_t \frac{1 + i_{t+1}}{1 + g_{Y,t+1}} + s_{t+1} - \mathcal{T}_{t+1}$$

where $s_t = G_t/Y_t$ and $\mathcal{T}_t = T_t/Y_t$.

Linearised:

$$\hat{b}_{t+1} = \underbrace{\frac{1 + \bar{i}}{1 + \bar{g}_Y}}_{\rho_b} \hat{b}_t + \frac{\bar{i}}{1 + \bar{g}_Y} \hat{i}_t - \frac{\bar{g}_Y}{1 + \bar{g}_Y} \hat{g}_{Y,t+1} + \frac{\bar{s}}{\bar{b}} \hat{s}_{t+1} - \frac{\bar{\mathcal{T}}}{\bar{b}} \hat{\mathcal{T}}_{t+1}$$

Extension: replace lump-sum rebate of tariff revenue with $G_{ki,t}$ financing, creating a feedback loop between trade and fiscal policy.

Appendix: Optimal Fiscal Rule (Unconstrained Budget, $\sigma > 1$, $\kappa > 0$)

$$g_{k,t} = \frac{H_k}{X_k} y_{k,t} + \frac{J_k}{X_k} a_{k,t} - \frac{\theta}{\lambda_k X_k} \pi_{k,t} + \frac{\sigma-1}{(1-\chi)^{1/\sigma} X_k} \sum_k \mu_k \lambda_k \phi_{k,t}^\pi$$

$$H_k = \chi_k^* + 1 + \varphi + (\sigma-1) \frac{\omega_{c,k}}{1-\chi_k} - \varphi \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

$$J_k = -(1+\varphi) \left(1 - \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$X_k = \chi_k^* + (\sigma-1) \chi_k^* \omega_{c,k} \chi_k + \left(1 + (\sigma-1) \omega_{g,k} \right) \left(\lambda_k + \frac{1+\chi_k^*+\varphi}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$\phi_{k,t}^\pi = \left(-\varphi y_{k,t} - g_{k,t} (1 + (\sigma-1) \omega_{g,k}) + (1+\varphi) a_{k,t} \right) \left(\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k} \right)^{-1}$$

Comparing welfare under this vs. the constrained rule quantifies the cost of fiscal inflexibility.

Appendix: Optimal Monetary Rule (Unconstrained Budget)

Optimal monetary policy sets a weighted inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t} = - \sum_k \mu_k \frac{\varphi y_{k,t} + g_{k,t}(1+(\sigma-1)\omega_{g,k}) + (1+\varphi) a_{k,t}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

- ▶ Inflation weights increase with private-consumption share, decrease with λ_k .
- ▶ Government spending enters the target when $\kappa > 0$ (government demand affects marginal costs).

Appendix: Fixed vs. Endogenous Budgets — Detail

Fully endogenous spending

- ▶ Planner chooses both $\bar{G}_{k,t}$ and $\{G_{ki,t}\}$.
- ▶ Level and composition respond to shocks.
- ▶ Benchmark: first-best fiscal stabilisation.
- ▶ Fiscal rule is *absolute*: each $g_{k,t}$ set independently.

Fixed aggregate budget

- ▶ $\bar{G}_{k,t}$ exogenous; only composition adjusts.
- ▶ Fiscal rule is *relative*: gaps wrt residual sector.
- ▶ Welfare loss from constraint is concentrated in aggregate stabilisation.
- ▶ Cross-sectional allocation may remain close to first-best.

Implications. If the welfare gap is small, a budget-constrained government can approximate first-best outcomes through compositional reallocation alone. This is the central hypothesis to be tested in the networked model.

Appendix: Open Questions

- ▶ **Network centrality and fiscal allocation.** How does a sector's position in the IO network affect its optimal public-good allocation? The IO weights ω_{klij} introduce cross-sector spillovers absent in the simple model.
- ▶ **Fiscal–trade policy interaction.** With tariff revenue financing government purchases, trade policy changes alter the fiscal envelope. The welfare implications of this feedback are to be characterised.
- ▶ **Dimensionality.** The Ramsey problem involves $K \times I$ spending instruments. Practical approaches may require restricting the class of admissible rules.
- ▶ **Political economy.** The exogenous-budget assumption abstracts from the determination of $\bar{G}_{k,t}$. Endogenising this would require a political-economy layer.

Q&A: Why Not Derive the Ramsey Problem Directly?

The question

Why modify Phillips curves with fiscal instruments rather than solving a full Ramsey taxation problem?

- ▶ The Phillips-curve approach is a **reduced-form shortcut**: it isolates the cost-push channel of taxation while preserving the existing IO structure of Aguilar et al. (2025).
- ▶ A full Ramsey problem in a 4×44 networked model is computationally demanding—it requires solving for $K \times I$ optimal tax instruments simultaneously.
- ▶ The **research agenda goal** is precisely to move toward the full Ramsey characterisation. The current extensions are a tractable first step.
- ▶ La'O & Tahbaz-Salehi (2024) show that even in simpler networks, the Ramsey problem has a rich structure. Our approach builds intuition before scaling up.

Q&A: Relation to Rubbo (2023) and La'O & Tahbaz-Salehi (2024)

The question

How does the relative allocation rule relate to optimal policy results in production networks?

Rubbo (2023):

- ▶ Optimal *monetary* policy in production networks.
- ▶ Divine coincidence fails; price-stability target depends on network topology.
- ▶ Our work: fiscal policy adds a second instrument that can target sectoral gaps directly.

La'O & Tahbaz-Salehi (2024):

- ▶ Optimal monetary policy in production networks (Econometrica).
- ▶ Network structure shapes the optimal inflation target and creates trade-offs absent in one-sector models.
- ▶ Our contribution: government *spending* as a complementary fiscal instrument, with an aggregate budget constraint.

Key distinction: our relative allocation rule operates under a *fixed fiscal envelope*—a constraint absent in both papers.

Q&A: When Does the Budget Constraint Bind?

The question

What happens when \bar{G}_t is close to optimal vs. far from it?

- ▶ When \bar{G}_t is **close to the first-best level**, compositional reallocation suffices: the relative allocation rule can approximate optimal welfare by reshuffling spending across sectors.
- ▶ When \bar{G}_t is **far from optimal** (e.g., a deep austerity constraint), the welfare gap grows because the level channel is shut off. The planner cannot compensate for an insufficient aggregate envelope by reallocating alone.
- ▶ The **welfare gap** between fixed and endogenous budgets is therefore increasing in $|\bar{G}_t - G_t^*|$, where G_t^* is the first-best aggregate level.
- ▶ Quantifying this gap in the 4×44 networked model is the central objective of the research agenda.

Q&A: Why 4 Countries and 44 Sectors?

The question

How sensitive are results to this level of granularity?

- ▶ The 4×44 structure follows Aguilar et al. (2025), calibrated to the **OECD ICIO 2019 tables** (Inter-Country Input-Output).
- ▶ 4 countries: a practical choice balancing model tractability with open-economy realism (e.g., US, EA, CN, RoW).
- ▶ 44 sectors: the full ISIC Rev. 4 classification available in ICIO—no aggregation required.
- ▶ **Robustness considerations:**
 - Coarser aggregations (e.g., 10–15 sectors) can be tested by collapsing IO tables.
 - The key qualitative predictions—relative reallocation, countercyclicality—should survive aggregation.
 - Quantitative magnitudes (welfare gaps) are likely sensitive to granularity, as network effects depend on the density of the IO matrix.

Motivation. Cox et al. (2024) assume government demand does not affect firms' pricing decisions ($\kappa = 0$). Empirical evidence (Ben Zeev & Pappa, 2017; Barattieri et al., 2023) suggests fiscal spending does affect prices.

Definition

$\kappa \in [0, 1]$ controls how much inelastic government demand passes through to marginal cost. When $\kappa > 0$:

- ▶ The Phillips curve steepens: $\lambda'_k = \lambda_k / (1 - \delta)$, where $\delta = \frac{\kappa}{\theta - 1} \frac{\bar{G}}{\bar{C}}$.
- ▶ Government spending becomes a **supply-side instrument**: $g_{k,t}$ enters the Phillips curve directly.

Nesting: $\kappa = 0$ recovers Cox et al. (2024). At $\kappa = 1$ (full pass-through), for $\theta = 6$ and $\chi_k^* = 1$: $\lambda'_k = 1.25 \lambda_k$.

Q&A: CRRA Preferences and the Insurance Motive

With $\sigma \neq 1$, the welfare objective acquires an additional term (in red):

$$-\frac{1}{2} \sum_k \mu_k \left((1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ \left. + (\sigma-1) \left[(1-\chi_k) \omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right)$$

Fiscal policy now balances **three motives**:

- 1 Allocation:** match public-good provision to sectoral needs.
- 2 Stabilisation:** dampen output and inflation gaps.
- 3 Insurance:** hedge against aggregate consumption risk ($\sigma-1$ term).

Q&A: Negativity Condition for Countercyclicality

Setting $\kappa = 1$, the overall coefficient on $y_{k,t}$ in the fiscal rule is:

$$\underbrace{\frac{1}{(\theta-1)(1-\chi_k)\varphi + 1}}_{\text{always } > 0} \times \underbrace{\left[\frac{1}{1-\chi_k} + \varphi + (\sigma-1)\frac{\omega_{c,k}}{1-\chi_k} - \frac{(\theta-1)(1-\chi_k)\varphi}{(1-\chi_k)\lambda_k} \right]}_{\text{sign determines cyclicity}}$$

Government spending is **countercyclical** ($g_{k,t}$ falls when $y_{k,t}$ rises) if and only if:

$$\sigma > \left(\frac{(\theta-1)(1-\chi_k)\varphi}{\lambda_k} - 1 - \varphi(1-\chi_k) \right) \frac{1}{\omega_{c,k}} + 1$$

- ▶ Under **flexible prices** ($\lambda_k \rightarrow \infty$): always satisfied.
- ▶ Under **moderate rigidity** ($\alpha_k < 0.3$): satisfied for standard calibrations.
- ▶ $\kappa < 1$ implies a **less countercyclical** stance (the pass-through channel weakens).

Q&A: Welfare Approximation — Full Derivation Steps

Step 1. Second-order Taylor expansion of U around the efficient steady state:

$$U - \bar{U} \approx \sum_k \mu_k [\bar{C} (1-\chi) \hat{c}_{k,t} + \bar{G} \chi \hat{g}_{k,t} - \bar{N}_k^{1+\varphi} \hat{n}_{k,t}] + \text{second-order terms}$$

Step 2. Use the production function $Y_{k,t} = A_{k,t} N_{k,t}$ and goods market clearing $Y_{k,t} = C_{k,t} + G_{k,t}$ to eliminate $\hat{n}_{k,t}$ and $\hat{c}_{k,t}$:

$$\hat{n}_{k,t} = y_{k,t} - a_{k,t}, \quad (1-\chi_k) \hat{c}_{k,t} = y_{k,t} - \chi_k g_{k,t}$$

Step 3. Calvo price dispersion contributes $\sum_k \frac{\theta}{\lambda_k} \pi_{k,t}^2$ (Woodford, 2003, Ch. 6).

Step 4. Collecting terms (setting $\sigma = 1$, $\kappa = 0$):

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[(1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

where $\chi_k^* \equiv \chi_k / (1-\chi_k)$. The public-good gap arises from the resource constraint binding at the sectoral level.

Q&A: Relative Allocation Rule — FOC Details

Lagrangian. Minimise \mathcal{W} subject to $\sum_k \omega_{g,k} g_{k,t} = \bar{g}_t$ (aggregate budget):

$$\mathcal{L} = \mathcal{W} + \eta_t \left(\bar{g}_t - \sum_k \omega_{g,k} g_{k,t} \right)$$

FOC for sector k :

$$\frac{\partial \mathcal{W}}{\partial g_{k,t}} = \mu_k \left[\chi_k^* (g_{k,t} - y_{k,t}) + \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\partial \pi_{k,t}}{\partial g_{k,t}} + (1+\varphi) \frac{\partial y_{k,t}}{\partial g_{k,t}} \right] = \eta_t \omega_{g,k}$$

Eliminating η_t : Equate the FOC for sector k with sector i (the residual):

$$\frac{1}{\omega_{g,k}} \frac{\partial \mathcal{W}}{\partial g_{k,t}} = \frac{1}{\omega_{g,i}} \frac{\partial \mathcal{W}}{\partial g_{i,t}}$$

Substituting the Phillips curve $\pi_{k,t} = \lambda_k [(1+\varphi) y_{k,t} + \chi_k g_{k,t}] + \beta \mathbb{E}_t \pi_{k,t+1}$ and solving for $g_{k,t}$ in terms of $g_{i,t}$, $y_{k,t}$, $\pi_{k,t}$, $y_{i,t}$, $\pi_{i,t}$ yields the relative allocation rule.

See Appendix A4 for the full structural coefficients.