Missing Tax Instruments:
Attaining Production Efficiency in Disaggregated, Production Network Economies with Nominal Rigidities*

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Abstract

We study optimal fiscal and monetary policy in multi-sector, input-output economies with nominal rigidities and a rich set of tax instruments. Irrespective of the network structure, with $N>1$ sectors, production efficiency can be achieved with $2N$ state-contingent, sector-specific tax instruments. The optimal tax scheme involves jointly stabilizing all prices set by sellers while generating efficient movements in after-tax prices faced by buyers. Optimal fiscal and monetary policy are independent of the vector of price rigidities; optimal sector-specific tax rates are moreover independent of the availability of lump-sum taxes. We provide several tax equivalences and non-equivalences.

Keywords: fiscal policy, monetary policy, production efficiency, production networks, misallocation.

JEL codes: E32, E52, E62, E63, D61

1 Introduction

In multi-sector, input-output economies with nominal rigidities, the first best allocation is generically unattainable with monetary policy alone (Aoki, 2001; Woodford, 2003b, 2010; La'O and Tahbaz-Salehi, 2022). Even when shocks are exclusively technological, as long as tax instruments are non-state-contingent, monetary policy can attain at most a "third-best" allocation: one that features production inefficiency and an endogenous, state-contingent labor wedge (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). Optimal monetary policy in multi-sector economies thereby contrasts sharply to its one-sector counterpart: in the standard New Keynesian (NK) model with technology shocks, the first best is achievable with price level stabilization and a constant sales subsidy.

From this observation one might conclude that there are "missing" tax instruments in the multi-sector, input-output NK framework. Indeed, it is clear that a state-contingent labor income or consumption tax could eliminate the state-contingent labor wedge. It is less clear, however, what tax instruments would restore production efficiency. The heart of the problem is the following. In this class of models, monetary policy can target at most one price index. But with sector-specific technology shocks and nominal rigidities, no price index can eliminate misallocation both within and across sectors. To this end, finer-tuned fiscal instruments appear necessary.

But what instruments? And in what manner should they to be used? We address these questions in this paper. Within a broad class of multi-sector NK models with input-output linkages, and allowing for a rich set of fiscal instruments, we jointly characterize optimal monetary and fiscal policy. Our main result is to demonstrate how distortionary taxation can be used to restore production efficiency and thereby circumvent the problem faced by monetary policy. The tax scheme we engineer involves jointly stabilizing all prices set by sellers while generating efficient movements in after-tax prices faced by buyers.

In any environment with nominal rigidities, fiscal and monetary policy interact: the efficacy and optimality of monetary policy depend on the details of fiscal policy. The contribution of this paper is to thereby establish the theoretical benchmark of *sufficiently rich* fiscal instruments within the class of multi-sector, input-output NK economies. Our benchmark differs markedly from—and, in fact, constitutes the polar opposite of—the standard benchmark of "inactive" fiscal policy in which subsidies are used only to eliminate steady-state markups. As argued above, the distance between the two benchmarks is small to non-existent in the one-sector NK framework but widens with multiple sectors and intermediate good trade. Recent papers consider environments in the interior of this gap.

Prior to studying monetary policy in environments featuring restricted sets of "ac-

tive" fiscal instruments, it would be useful to first understand the sufficiently rich fiscal instruments benchmark. What are the missing fiscal tools that restricted sets may or may not compensate for? Answering this question, characterizing these instruments, and establishing this benchmark is what we do in this paper.

The Model and Results. Firms are linked to one another via an input-output network. They produce goods using labor, capital, and intermediate inputs and are subject to industry-level productivity shocks, demand shocks, and shocks to elasticities of substitution (or markups). Following the NK framework, firms set prices and face Calvo price rigidities; we allow the Calvo parameter to vary by sector. A representative household consumes the final good, supplies labor, saves, and invests in capital. A government finances exogenous public expenditure and sets fiscal and monetary policy jointly.

We take the standard Ramsey approach and assume the government has access to a rich set of distortionary, linear tax instruments—including multiple types of sector-specific, state-contingent tax rates—but cannot raise money with lump-sum taxation. The latter restriction on fiscal instruments implies that full efficiency, i.e. the "first best," cannot be achieved; we later relax this restriction in an extension.

Our main analysis centers on implementation of the Ramsey optimum, which in our economy coincides with the canonical Lucas and Stokey (1983) benchmark. We focus on the Ramsey optimum for the following reason: this benchmark exhibits production efficiency in the sense of Diamond and Mirrlees (1971); see Chari and Kehoe (1999). Implementation of the Ramsey optimum therefore demonstrates how a sufficiently rich set of fiscal instruments can restore production efficiency in this class of economies.

Our main theorem characterizes monetary and fiscal policy that jointly implement the Ramsey optimum. Without loss of generality, we focus on implementations in which monetary policy stabilizes the final good price level. Our characterization of optimal fiscal policy yields the following six lessons:

1. The optimal tax structure achieves production efficiency. In order to do so, sector-specific tax rates play two roles: they stabilize seller prices and generate efficient fluctuations in after-tax prices.

2. Optimal sector-specific tax rates are state-contingent. They depend on the realized nominal wage, sectoral technologies, and sectoral elasticities of substitution, but they do not depend on sectoral demand shocks and government spending. Optimal monetary and fiscal policy are invariant to the vector of price rigidities.

3. Regardless of the network structure, with N sectors production efficiency can be attained with $2N$ sector-specific tax instruments: N sales taxes levied on sellers and N sales taxes levied on buyers. Unlike in standard models with price flexibil-

ity, taxes levied on sellers and on buyers when sellers face price rigidities are not equivalent.

4. Sector-specific input taxes on labor, capital, and intermediate goods can substitute for the seller-imposed sales taxes. In this implementation, intermediate good input taxes are destination-specific but not origin-specific.

5. Sector-specific intermediate good and final good taxes can substitute for the buyer-imposed sales taxes. In this implementation, intermediate good input taxes are origin-specific but not destination-specific.

6. Optimal sector-specific tax rates are independent of the availability of lump-sum taxes.

What roles do the sector-specific tax instruments play in restoring production efficiency? Lesson 1 answers this question. Production efficiency requires no misallocation within and across sectors—a goal unattainable with monetary policy alone. The tax scheme we engineer involves jointly stabilizing seller prices while simultaneously generating efficient movements in after-tax prices faced by buyers. This tax scheme eliminates misallocation within and across sectors, ensuring efficient production of the final good.

These dual roles require tax state-contingency. Lesson 2 tells us what dimensions of the state space the optimal sector-specific taxes are contingent upon. It also answers the more practical question of what information is needed by the fiscal authority in order to administer such taxes. We find that the optimal tax rates are contingent on realized marginal costs and elasticities of substitution or, more generally, desired markups. While this is arguably a heavy information burden for the fiscal authority, what is perhaps more surprising are the dimensions of the state space the optimal tax rates are *not* contingent upon. Conditional on realized marginal costs, sector-specific tax rates are independent of sector-specific demand and government spending.

We find that for any implementation of the Ramsey optimum, tax rates are independent of the vector of sectoral price rigidities (Calvo parameters). The irrelevance of sectoral price rigidities for optimal fiscal and monetary policy, even when the optimal allocation does not coincide with the first best, generalizes previously-established results by Correia, Nicolini and Teles (2008) to multi-sector NK economies with input-output linkages.

Lessons 1 and 2 are general in that they apply to all implementations of the Ramsey optimum. Moving on, we explore specific tax implementations. The first is the "least instruments" implementation: we show that, irrespective of the input-output structure,

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with $N>1$ sectors and two roles for taxes, $2N$ sector-specific taxes can implement the Ramsey optimum (Lesson 3). This implementation requires N sales taxes levied on sellers and N sales taxes levied on buyers. We thus exploit a non-equivalence between sales taxes collected on sellers and on buyers when sellers face nominal rigidities, as highlighted by Poterba, Rotemberg and Summers (1986).

Despite this particular tax non-equivalence, we present two more implementations that showcase two different tax equivalences. In the first, we assume that the seller-imposed sales tax is unavailable. The Ramsey optimum can still be implemented: sector-specific taxes on labor, capital, and intermediate good inputs can substitute for the missing seller-imposed sales tax (Lesson 4). In the second, we assume that the buyer-imposed sales tax instrument is unavailable. The Ramsey optimum can again be implemented: sector-specific intermediate good and final good taxes can substitute for the missing buyer-imposed sales tax (Lesson 5).

Whether or not the first-best is attainable depends on the availability of lump-sum tax instruments. A question arises as to whether the availability of lump-sum taxation alters the optimal sector-specific tax instruments. We show that the answer is no: whether or not the Ramsey optimum coincides with the first-best changes only the optimal aggregate tax rates—it has no effect on the optimal sector-specific tax rates (Lesson 6). The irrelevance of lump-sum taxation for the optimal sector-specific tax rates drives home the point that their main function is to restore within-period production efficiency.

Related literature. This paper demonstrates how a rich set of distortionary tax instruments can be used to implement the Ramsey optimum and completely circumvent the problem faced by monetary policy. In this sense, our paper is closest in spirit to Correia, Nicolini and Teles (2008) and Correia, Farhi, Nicolini and Teles (2013). Our contribution vis-à-vis their work is to extend this insight to multi-sector, input-output NK economies and, specifically, to characterize within this class of models the sector-level tax rates that attain production efficiency.

Our results apply to a broad class of multi-sector NK models with or without input-output linkages. Our framework is fairly general in that it can flexibly nest the economic environments found in Erceg, Henderson and Levin (2000); Aoki (2001); Mankiw and Reis (2003); Benigno (2004); Huang and Liu (2005); Woodford (2010); Eusepi, Hobijn and Tambalotti (2011); Ozdagli and Weber (2017); Pastén, Schoenle and Weber (2020, 2024); Ghassibe (2021); La'O and Tahbaz-Salehi (2022); Rubbo (2023); Bouakez, Rachedi and Santoro (2023); Afrouzi and Bhattarai (2023); Xu and Yu (2024), among others.[1] Relative

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[1]Related, too, is the work of Faria-e-Castro (2021), Guerrieri et al. (2022), Baqaee and Farhi (2022), and Rubbo (2024). These papers focus on disaggregated, multi-sector NK models with and without input-output linkages but feature some form of household heterogeneity. For this reason, our model does not

to most multi-sector NK models, we add physical capital as a factor of production.

As we have noted, the contribution of our paper vis-à-vis the multi-sector, input-output NK literature is to establish a benchmark in which the set of tax instruments is sufficiently rich to achieve production efficiency. A standard benchmark in this literature is to assume non-state-contingent sales subsidies that eliminate steady-state markups (as in Woodford, 2003b; Galí, 2008). Recent papers, however, move past this benchmark and consider the implications of sector-specific and state-contingent government purchases: see Proebsting (2022); Cox et al. (2024); Flynn et al. (2024); Bouakez et al. (2024) as well as Ramey and Shapiro (1998). In a taxonomy of models, one can think of these environments as featuring a set of fiscal instruments that is strictly larger than in the standard benchmark, yet restricted relative to our sufficiently rich fiscal instrument benchmark.

We complement this growing body of work but focus instead on linear tax instruments. Furthermore, an understanding of both benchmarks—or, both extremes—is useful for interpreting the interior.

Layout. This paper is organized as follows. In Section 2 we describe the economic environment; in Section 3 we characterize equilibria. In Section 4 we state and solve the Ramsey problem, and in Section 5 we study its implementation. In Section 6 we explore several specific implementations that illustrate the main results of the paper. In Section 7 we conclude. All proofs are found in the Appendix.

2 The Environment

Time is discrete, indexed by $t = 0, 1, \ldots, \infty$. We denote the aggregate state at time t by $s_t \in S$ where S is a finite set. We let $s^t = \{s_0, ..., s_t\} \in S^t$ denote a history of states up to and including s_t. We let $\mu(s^t|s^{t-1})$ denote the probability of history s^t conditional on s^{t-1}, and with slight abuse of notation we let $\mu(s^t)$ denote the unconditional probability of history s^t.

There is a representative household with time-separable utility:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(C(s^t), L(s^t)), \tag{1}$$

explicitly capture these environments. In another related vein, Ghassibe and Ferrari (2024) develop and analyze a multi-sector, production network model with endogenous price rigidities due to menu costs. Our model instead features exogenous nominal rigidities: in our baseline we assume Calvo pricing, but our model can also accommodate informational frictions. Finally, Qiu, Wang, Xu and Zanetti (2024) study monetary policy in a small open economy with domestic and cross-border input-output linkages. Our model is closely related but features no international trade.

with $\beta \in (0,1)$, where $C(s^t)$ and $L(s^t)$ denote the household's consumption basket and labor supply at date t, history s^t. The function U satisfies the typical regularity conditions: it is continuous, twice-differentiable, strictly increasing and strictly concave in $(C_t, -L_t)$, and satisfies Inada conditions. Throughout, we let $U_C(s^t) \equiv \partial U(\cdot)/\partial C(s^t)$ and $U_L(s^t) \equiv \partial U(\cdot)/\partial L(s^t)$.

The household supplies labor, consumes, saves, and invests in capital. The household's nominal budget constraint at time t, history s^t is given by:

$$
(1 + \tau^C(s^t))\mathcal{P}(s^t)C(s^t) + \mathcal{P}(s^t)H(s^t) + B(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)Z(s^{t+1}|s^t)
$$
$$
\leq (1 - \tau^L(s^t))W(s^t)L(s^t) + (1 - \tau^K(s^t))R(s^t)K(s^{t-1}) + (1 + \iota(s^{t-1}))B(s^{t-1}) + Z(s^t|s^{t-1}) \tag{2}
$$

where $\mathcal{P}(s^t)$ is the price of the final good, $W(s^t)$ is the nominal wage, and $R(s^t)$ is the nominal rental rate on capital. The household faces taxes on consumption, labor income, and capital income, denoted by $\tau^C(s^t)$, $\tau^L(s^t)$, and $\tau^K(s^t)$, respectively.

The household can borrow and save via three separate instruments. The first is a one-period, risk-free nominal bond, $B(s^t)$, which the household can buy or sell at time t, history s^t, and which pays $(1 + \iota(s^t))B(s^t)$ units of money one period later. The household's debt holdings are bounded by $B(s^t) \geq -D$ for some large constant D. We let B_{-1} denote initial holdings of government debt. The second instrument is a complete set of state-contingent Arrow securities, indexed by $s^{t+1}|s^t$. We let $Q(s^{t+1}|s^t)$ denote the price at time t, history s^t, of an Arrow security that pays 1 unit of money in $t+1$ if s^{t+1} is realized and 0 otherwise. The corresponding quantity purchased of this Arrow security is denoted by $Z(s^{t+1}|s^t)$.

Third, the household can invest in capital; we let $H(s^t)$ denote the household's investment in capital at date t, history s^t. The law of motion of capital is given by:

$$
K(s^t) = (1 - \delta)K(s^{t-1}) + H(s^t), \tag{3}
$$

where $\delta \in (0,1]$ is the rate of depreciation. We assume the household is endowed with $K_{-1} > 0$ initial units of capital at time 0. The household solves the following problem.

Household's Problem. Given initial capital and bond holdings, K_{-1} and B_{-1}, the household chooses a complete contingent plan, $\{C(s^t), L(s^t), K(s^t), H(s^t), B(s^t), (Z(s^{t+1}|s^t))_{s^{t+1}}\}_{t \geq 0, s^t \in S^t}$, in order to maximize its lifetime expected utility (1) subject to (2), (3), and $B(s^t) \geq -D$, for all $t \geq 0, s^t \in S^t$.

Intermediate good production. Production takes place in a finite set of $N>1$ industries indexed by $i \in \mathcal{I} \equiv \{1, \ldots, N\}$. Each industry consists of two types of firms: (i)

a measure one continuum of monopolistically-competitive, differentiated good firms, indexed by $m \in [0, 1]$, and (ii) a perfectly-competitive producer that aggregates the industry's differentiated goods and produces a uniform sectoral good. We begin by describing the latter.

Sectoral good production. Within each industry $i \in \mathcal{I}$, a perfectly-competitive firm produces a uniform, sectoral good using a constant returns to scale, constant elasticity of substitution (CES) production technology over the sector's differentiated varieties, $m \in [0, 1]$, given by:

$$Y_i(s^t) = \left[\int_0^1 y_i^m(s^t)^{\frac{\theta_i(s_t)-1}{\theta_i(s_t)}} \, \mathrm{d}m \right]^{\frac{\theta_i(s_t)}{\theta_i(s_t)-1}}, \tag{4}$$

where $\theta_i(s_t) > 1$ denotes the elasticity of substitution across varieties in state s_t, and $y_i^m(s^t)$ denotes the quantity purchased of variety m. The sectoral good producer takes prices as given and maximizes profits, given by:

$$P_i(s^t)Y_i(s^t) - (1 + \hat{\tau}_i^y(s^t)) \int_0^1 p_i^m(s^t)y_i^m(s^t)\mathrm{d}m$$

where $P_i(s^t)$ is the price of sectoral good i, $\hat{\tau}_i^y(s^t)$ is the sales tax the firm pays on its purchases of the differentiated varieties, and $p_i^m(s^t)$ is the price of differentiated good m. Profit maximization yields the following CES demand curves:

$$y_i^m(s^t) = \left[\frac{(1 + \hat{\tau}_i^y(s^t))p_i^m(s^t)}{P_i(s^t)} \right]^{-\theta_i(s^t)} Y_i(s^t). \tag{5}$$

By constant returns to scale, this producer makes zero profits.[2] The good produced by sector i can be used for final good purposes or as an intermediate input to production.

Differentiated goods production. Within each industry $i \in \mathcal{I}$, a measure one continuum of monopolistically-competitive firms use a common constant-returns-to-scale technology to transform labor, capital, and intermediate inputs into differentiated varieties. The production function of firm m in industry i is given by

$$y_i^m(s^t) = A_i(s_t)F_i(\ell_i^m(s^t), k_i^m(s^t), x_{i1}^m(s^t), \ldots, x_{iN}^m(s^t)), \tag{6}$$

where $y_i^m(s^t)$ is firm output, $\ell_i^m(s^t)$ and $k_i^m(s^t)$ are the firm's labor and capital inputs, and for every $j \in \mathcal{I}$, $x_{ij}^m(s^t)$ is the firm's usage of intermediate good j. We let $A_i(s_t)$ denote an industry-specific productivity shock. The production function F_i is homogeneous of degree one. Throughout, we assume that labor is an essential input for the production

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[2]We include this producer to ensure that a homogeneous sectoral good is produced while also allowing for monopolistic competition among the differentiated good firms.

technology of all goods, in the sense that $F_i(0, k_i, x_{i1}, \ldots, x_{iN}) = 0$ and that $\partial F_i / \partial \ell_i > 0$ whenever all other inputs are used in positive amounts. Unless otherwise noted—and without much loss of generality—we also assume that $\partial F_i / \partial k_i > 0$ and that $\partial F_i / \partial x_{ij} > 0$ for all pairs i and j. The nominal profits of firm m in industry i are given by:

$$
\begin{aligned}
\pi_i^m(s^t) =& (1 - \tau_i^y(s^t))p_i^m(s^t)y_i^m(s^t) - (1 + \tau_i^\ell(s^t))W(s^t)\ell_i^m(s^t) - (1 + \tau_i^k(s^t))R(s^t)k_i^m(s^t) \\
& - \sum_{j \in \mathcal{I}} (1 + \tau_{ij}^x(s^t))P_j(s^t)x_{ij}^m(s^t),
\end{aligned}
$$

where $p_i^m(s^t)$ is the price charged by firm m and $P_j(s^t)$ is price of intermediate good j. The firm faces a sector-specific sales tax, $\tau_i^y(s^t)$, sector-specific labor and capital input taxes, $\tau_i^\ell(s^t)$ and $\tau_i^k(s^t)$, and sector-input-specific intermediate good taxes, $\tau_{ij}^x(s^t)$, for every input $j \in \mathcal{I}$.

The differentiated good firms are price-setters and are subject to nominal pricing frictions. We model the nominal rigidity à la Calvo, as in Woodford (2003b). In any given period, a firm m in sector i can adjust its price with probability $1 - \alpha_i$. If at time t the firm can adjust its price, the firm solves the following maximization problem:

$$
p_i^*(s^t) \in \arg\max_p \sum_{v=0}^{\infty} \sum_{s^{t+v}} (\alpha_i \beta)^v \frac{U_C(s^{t+v})}{(1 + \tau^C(s^t))\mathcal{P}(s^{t+v})} \pi_i(s^{t+v}; p)\mu(s^{t+v}|s^t) \tag{7}
$$

subject to demand (5) and technology (6). In this problem, $\pi_i(s^{t+v}; p)$ are the firm's per-period profits in history s^{t+v} given price p and $\beta^v \frac{U_C(s^{t+v})}{(1 + \tau^C(s^t))\mathcal{P}(s^{t+v})}$ is the household's stochastic discount factor. We call $p_i^*(s^t)$ the reset price.

A firm m in sector i that is allowed to adjust its price in history s^t sets its price equal to the reset price: $p_i^m(s^t) = p_i^*(s^t)$. All other firms in sector i charge a price equal to their previous period's price: $p_i^m(s^t) = p_i^m(s^{t-1})$. We assume that at time 0, the firms in sector i that do not change their price, begin with a price of $p_{i,0}^m = \kappa_i$, where $\kappa_i > 0$ is an arbitrary, strictly-positive scalar. We let $\kappa \equiv (\kappa_i) \in \mathbb{R}_+^N$ denote the vector of initial prices.

Remark. The tax rates $\tau_i^y(s^t)$ and $\hat{\tau}_i^y(s^t)$ are sales taxes on the purchase of the differentiated varieties in sector i. These taxes differ only in which party is legally obligated to pay: $\tau_i^y(s^t)$ is the sales tax paid by sellers, while $\hat{\tau}_i^y(s^t)$ is the sales tax paid by buyers.

Final good production. A perfectly-competitive firm produces the final good, $\mathcal{Y}(s^t)$. This firm operates a constant returns to scale technology, given by:

$$
\mathcal{Y}(s^t) = \mathcal{F}(V_1(s_t)X_1^f(s^t), \ldots, V_N(s_t)X_N^f(s^t)), \tag{8}
$$

where $X_i^f(s^t)$ is the quantity of sectoral good i used in final good production and $V_i(s_t)$ is a sector-specific demand shock. We assume that $\mathcal{F} : \mathbb{R}_+^N \to \mathbb{R}_+$ is homogeneous of

degree one, twice-differentiable in all arguments, has positive and diminishing marginal products, satisfies $\mathcal{F}(\mathbf{0}) = 0$, and satisfies Inada conditions. The final good firm maximizes profits, given by: $\mathcal{P}(s^t)\mathcal{Y}(s^t) - \sum_{i \in \mathcal{I}}(1 + \tau_i^f(s^t))P_i(s^t)X_i^f(s^t)$, where $\tau_i^f(s^t)$ is a final good tax on the purchase of sectoral good i. Under constant returns to scale, the price of the final good is equal to the firm's marginal cost of its cost-minimizing bundle:

$$\mathcal{P}(s^t) = \min_{(X_1^f(s^t),\ldots,X_I^f(s^t))} \sum_{i \in \mathcal{I}}(1 + \tau_i^f(s^t))P_i(s^t)X_i^f(s^t) \tag{9}$$

subject to $\mathcal{F}(V_1(s_t)X_1^f(s^t),\ldots,V_N(s_t)X_N^f(s^t)) = 1$. The final good can be used for private consumption, government consumption, or capital investment.

The government and market clearing. The government consists of a consolidated monetary and fiscal authority with a commitment technology. The government finances government spending with state-contingent debt, non-state-contingent debt, and with distortionary taxation. Its nominal period-t budget constraint is given by:

$$(1 + \iota(s^{t-1}))B(s^{t-1}) + Z(s^t|s^{t-1}) + \mathcal{P}(s^t)G(s_t) = B(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t)Z(s^{t+1}|s^t) + \mathcal{T}(s^t) \tag{10}$$

where $G(s_t)$ is the exogenous level of government consumption in state s_t. We let $\mathcal{T}(s^t)$ denote the nominal level of total tax revenue:

$$\mathcal{T}(s^t) \equiv \tau^C(s^t)\mathcal{P}(s^t)C(s^t) + \tau^L(s^t)W(s^t)L(s^t) + \tau^K(s^t)R(s^t)K(s^{t-1}) + \sum_{i \in \mathcal{I}}\tau_i^f(s^t)P_i(s^t)X_i^f(s^t)$$

$$+ \sum_{i \in \mathcal{I}}\left(\tau_i^y(s^t) + \hat{\tau}_i^y(s^t)\right)\int_0^1 p_i^m(s^t)y_i^m(s^t)\mathrm{d}m + \sum_{i \in \mathcal{I}}\tau_i^\ell(s^t)W(s^t)L_i(s^t)$$

$$+ \sum_{i \in \mathcal{I}}\tau_i^k(s^t)R(s^t)K_i(s^t) + \sum_{i \in \mathcal{I}}\sum_{j \in \mathcal{I}}\tau_{ij}^x(s^t)P_j(s^t)X_{ij}(s^t) + \sum_{i \in \mathcal{I}}\Pi_i(s^t)$$

where, for every $i \in \mathcal{I}$, $L_i(s^t) = \int \ell_i^m(s^t)\mathrm{d}m$ and $K_i(s^t) = \int k_i^m(s^t)\mathrm{d}m$ denote total labor and capital usage in sector i, and $X_{ij}(s^t) = \int x_{ij}^m(s^t)\mathrm{d}m$ denotes total intermediate input usage of good j in sector i. Similarly, for every $i \in \mathcal{I}$, we let $\Pi_i(s^t) = \int \pi_i^m(s^t)\mathrm{d}m$ denote total sector-level profits. As in Correia, Nicolini and Teles (2008) and Angeletos and La'O (2020), we assume a 100% dividend (profit) tax on the differentiated good firms. In order to fully rule out replication of lump-sum taxes, we require further restrictions on period zero policies; we discuss these restrictions in Section 4 when we introduce the Ramsey problem. Finally, we abstract from the zero lower bound on the nominal interest rate.[3]

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[3]Ignoring the ZLB is done purely out of convenience and is not central to our results: our model features state-contingent consumption taxes and hence we could implement "unconventional fiscal policy" and circumvent the ZLB as in Correia, Farhi, Nicolini and Teles (2013).

We assume that the monetary authority directly controls nominal aggregate demand according to the following ad hoc, cash-in-advance constraint: $\mathcal{M}(s^t) = \mathcal{P}(s^t)C(s^t)$. The monetary authority can freely choose $\mathcal{M}(s^t) > 0$ in every history. By modeling the monetary instrument in this way, as opposed to the nominal interest rate, we avoid well known issues of indeterminacy.

For every $t \geq 0, s^t \in S^t$, market clearing in the sectoral goods markets, the aggregate labor market, and the aggregate capital market, imply:

$$Y_i(s^t) = X_i^f(s^t) + \sum_{j \in \mathcal{I}} X_{ji}(s^t), \ \forall i \in \mathcal{I}; \ L(s^t) = \sum_{i \in \mathcal{I}} L_i(s^t); \ \text{and} \ K(s^{t-1}) = \sum_{i \in \mathcal{I}} K_i(s^t). \quad (11)$$

Market clearing in the final good for every $t \geq 0, s^t \in S^t$ implies:

$$\mathcal{Y}(s^t) = C(s^t) + H(s^t) + G(s^t). \quad (12)$$

2.1 Timing and Equilibrium Definition

At each date t, Nature draws the state $s_t \in S$ according to probability distribution μ. The aggregate state determines sectoral productivities, sectoral elasticities of substitution, final good demand, and aggregate government spending. Formally, we define functions $A_i : S \to \mathbb{R}_+, \theta_i : S \to \mathbb{R}_+,$ and $V_i : S \to \mathbb{R}_+,$ for all $i \in I$, and $G : S \to \mathbb{R}_+$, as exogenous mappings from the state space to these outcomes.

We assume that the state $s_t \in S$ is common knowledge in period t. In addition, Nature randomly selects which intermediate-good firms are able to set prices that period. Once prices are set, the sectoral aggregator firm for each sector purchases differentiated goods and produces the uniform sectoral good, the final good firm purchases sectoral goods and produces the final good, and the representative household makes its consumption, investment, and labor supply decisions. Intermediate-good firms purchase inputs and produce output in order to meet realized demand. All allocations adjust so that supply equals demand and markets clear.

Throughout, we denote an allocation in this economy by χ:

$$\chi \equiv \{ \mathcal{Y}(s^t), C(s^t), L(s^t), K(s^t), H(s^t), ((Y_i(s^t), L_i(s^t), K_i(s^t), X_i^f(s^t), (X_{ij}(s^t))_{j \in \mathcal{I}})_{i \in \mathcal{I}},$$
$$((y_i^m(s^t), \ell_i^m(s^t), k_i^m(s^t), (x_{ij}^m(s^t))_{j \in \mathcal{I}})_{m \in [0,1]})_{i \in \mathcal{I}} \}_{t \geq 0, s^t \in S^t}.$$

An equilibrium is defined as follows:

Definition 1. An equilibrium is an allocation χ, a price system

$$\{ \mathcal{P}(s^t), W(s^t), R(s^t), (Q(s^{t+1}|s^t))_{s^{t+1}|s^t}, ((p_i^m(s^t))_{m \in [0,1]}, P_i(s^t))_{i \in \mathcal{I}}, \iota(s^t) \}_{t, s^t},$$

a policy

$$\{\tau^C(s^t), \tau^L(s^t), \tau^K(s^t), (\tau_i^y(s^t), \tau_i^\ell(s^t), \tau_i^k(s^t), \hat{\tau}_i^y(s^t), \tau_i^f(s^t), (\tau_{ij}^x(s^t))_{j\in\mathcal{I}})_{i\in\mathcal{I}}, \mathcal{M}(s^t)\}_{t,s^t},$$

and financial market positions $\{B(s^t), Z(s^{t+1}|s^t)\}_{t,s^t}$, such that given K_{-1} and B_{-1}, $\{C(s^t), L(s^t), K(s^t), H(s^t), B(s^t), (Z(s^{t+1}|s^t))_{s^{t+1}|s^t}\}_{t,s^t}$ solves the household's problem and, for all t, s^t: (i) prices and allocations satisfy (5) and the price of the sectoral good i is given by $P_i(s^t) = \left[\int p_i^m(s^t)^{1-\theta_i(s_t)}\mathrm{d}i\right]^{\frac{1}{1-\theta_i(s_t)}}$, for all $i \in \mathcal{I}$; (ii) in sector i, given initial price κ_i, the price of an adjusting firm m satisfies $p_i^m(s^t) = p_i^*(s^t)$, where $p_i^*(s^t)$ solves (7), and the price of a non-adjusting firm m satisfies $p_i^m(s^t) = p_i^m(s^{t-1})$, for all $i \in \mathcal{I}$; (iii) given prices, $\mathcal{Y}(s^t)$ and $\{X_1^f(s^t), \ldots, X_I^f(s^t)\}$ solve the final good firm's problem and the aggregate price level satisfies (9); (iv) the government budget constraint is satisfied; (v) $M(s^t) = \mathcal{P}(s^t)C(s^t)$; and (vi) markets clear.

3 Equilibrium Characterization

We begin by characterizing an equilibrium in this economy. The household's problem is standard, we thus focus on the problem of the intermediate good firms. Consider a typical firm in industry i. Once prices are set, this firm chooses its cost-minimizing bundle of inputs. Given constant returns to scale and linear tax rates, marginal cost is the same for all firms within a given industry. Industry-level marginal cost, $\mathrm{mc}_i(s^t)$, is given by:

$$\mathrm{mc}_i(s^t) \equiv \min_{\{\ell_i,k_i,x_{i1},\ldots,x_{iN}\}} (1 + \tau_i^\ell(s^t))W(s^t)\ell_i + (1 + \tau_i^k(s^t))R(s^t)k_i \tag{13}$$
$$+ \sum_{j\in\mathcal{I}}(1 + \tau_{ij}^x(s^t))P_j(s^t)x_{ij}$$

subject to $A_i(s_t)F_i(\ell_i, k_i, x_{i1}, \ldots, x_{iN}) = 1$. Optimality requires:

$$\mathrm{mc}_i(s^t) = \frac{(1 + \tau_i^\ell(s^t))W(s^t)}{A_i(s_t)\frac{\partial F_i(s^t)}{\partial \ell_i(s^t)}} = \frac{(1 + \tau_i^k(s^t))R(s^t)}{A_i(s_t)\frac{\partial F_i(s^t)}{\partial k_i(s^t)}} = \frac{(1 + \tau_{ij}^x(s^t))P_j(s^t)}{A_i(s_t)\frac{\partial F_i(s^t)}{\partial x_{ij}(s^t)}}, \quad \forall j \in \mathcal{I}. \tag{14}$$

If the firm can reset its price at date t, history s^t, it maximizes the following objective:

$$\sum_{v=0}^{\infty}\sum_{s^{t+v}}(\alpha_i\beta)^v\frac{U_C(s^{t+v})}{(1 + \tau^C(s^t))\mathcal{P}(s^{t+v})}\left\{(1 - \tau_i^y(s^{t+v}))py_i^m(s^{t+v}) - \mathrm{mc}_i(s^{t+v})y_i^m(s^{t+v})\right\}\mu(s^{t+v}|s^t)$$

subject to (5). The firm's optimal reset price satisfies:

$$\sum_{v=0}^{\infty}(\alpha_i\beta)^v\sum_{s^{t+v}}\left[p_i^*(s^t) - \frac{1}{(1 - \tau_i^y(s^{t+v}))}\left(\frac{\theta_i(s^{t+v}) - 1}{\theta_i(s^{t+v})}\right)^{-1}\mathrm{mc}_i(s^{t+v})\right]\hat{\mu}_i(s^{t+v}|s^t) = 0 \tag{15}$$

where $\hat{\mu}_i(s^{t+v}|s^t)$ are sector i's risk-adjusted probabilities; see Appendix A.1 for their derivation. Therefore, the firm's optimal reset price is equal to its expected present discounted value of its desired price in each state. In any state, its desired price is a markup over marginal cost. Combining this with the optimality conditions of the household and the final good firm, we reach the following equilibrium characterization.

Proposition 1. *An allocation χ, a price system, a policy, and financial positions constitute an equilibrium if and only if, for all $t \geq 0$, $s^t \in S^t$:*

$$\frac{-U_L(s^t)}{U_C(s^t)} = \left[\frac{1 - \tau^L(s^t)}{1 + \tau^C(s^t)}\right] \frac{W(s^t)}{\mathcal{P}(s^t)}, \tag{16}$$

$$\frac{U_C(s^t)}{1 + \tau^C(s^t)} = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_C(s^{t+1})}{1 + \tau^C(s^{t+1})} \left[1 - \delta + (1 - \tau^K(s^{t+1}))\frac{R(s^{t+1})}{\mathcal{P}(s^{t+1})}\right], \tag{17}$$

$$\frac{U_C(s^t)}{(1 + \tau^C(s^t))\mathcal{P}(s^t)} = \beta(1 + \iota(s^t)) \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_C(s^{t+1})}{(1 + \tau^C(s^{t+1}))\mathcal{P}(s^{t+1})}, \tag{18}$$

$$Q(s^{t+1}|s^t) = \frac{\beta U_C(s^{t+1})}{U_C(s^t)} \frac{(1 + \tau^C(s^t))\mathcal{P}(s^t)}{(1 + \tau^C(s^{t+1}))\mathcal{P}(s^{t+1})} \mu(s^{t+1}|s^t), \quad \forall s^{t+1}|s^t, \tag{19}$$

$$\lim_{t\to\infty} \sum_{s^t} \beta^t \mu(s^t) \frac{U_C(s^t)}{1 + \tau^C(s^t)} \frac{B(s^t)}{\mathcal{P}(s^t)} = 0, \quad \lim_{t\to\infty} \sum_{s^t} \beta^t \mu(s^t) \frac{U_C(s^t)}{1 + \tau^C(s^t)} K(s^t) = 0, \tag{20}$$

the household budget constraint (2), *law of motion of capital* (3), *and initial conditions K_{-1} and B_{-1}, are all satisfied; prices and allocations satisfy* (5); *given initial price $P_{i,-1} = \kappa_i$, the price of sectoral good i satisfies*

$$P_i(s^t) = \left[(1 - \alpha_i)p_i^*(s^t)^{1-\theta_i(s_t)} + \alpha_i P_i(s^{t-1})^{1-\theta_i(s_t)}\right]^{\frac{1}{1-\theta_i(s_t)}},$$

where $p_i^*(s^t)$ satisfies (15) *and marginal cost satisfies* (14), *for all $i \in \mathcal{I}$;*

$$V_i(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{X}_i(s^t)} = (1 + \tau_i^f(s^t))\frac{P_i(s^t)}{\mathcal{P}(s^t)}, \qquad \forall i \in \mathcal{I} \tag{21}$$

where $\bar{X}_i(s^t) \equiv V_i(s_t)X_i^f(s^t)$; $\mathcal{P}(s^t)$ is given by (9); *the government budget constraint,* (10)*, is satisfied; $M(s^t) = \mathcal{P}(s^t)C(s^t)$; and markets clear.*

Proof. See Appendix A.1. □

Proposition 1 characterizes an equilibrium in this economy. This characterization includes the standard household intratemporal and intertemporal optimality conditions in (16)-(18), the Arrow price in (19), transversality conditions in (20), and the final good demand functions in (21). These demand functions are derived from the final good firm's profit maximization problem; they indicate that the final demand for good i depends on the good's demand shock, $V_i(s_t)$, and after-tax relative price.

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4 The Ramsey Problem

In this section we state and solve the Ramsey problem. We begin by defining feasibility.

Definition 2. An allocation χ is *feasible* if it satisfies technology and resource constraints: (6) for all $i \in \mathcal{I}$ and $m \in [0, 1]$, (4) for all $i \in \mathcal{I}$, (3), (8), (11), and (12).

Let $\mathcal{X}$ denote the set of feasible allocations. Following Correia, Nicolini and Teles (2008), we take a "relaxed" Ramsey approach. The relaxed planning problem we solve is one of choosing an allocation that maximizes welfare, (1), subject to feasibility, $\chi \in \mathcal{X}$, and the following constraint:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left[U_C(s^t) C(s^t) + U_L(s^t) L(s^t) \right] = \mathcal{V}_0. \tag{22}$$

Condition (22) is similar to the standard implementability condition that arises in Ramsey planning problems when lump-sum taxes are unavailable (Lucas and Stokey, 1983; Chari, Christiano and Kehoe, 1991; Chari and Kehoe, 1999). Any equilibrium must satisfy this constraint; it is derived from the household's budget constraint, with all equilibrium prices and taxes substituted out using the first-order conditions of the household.[4]

In terms of restrictions on period 0 policies, we follow Chari, Nicolini and Teles (2020) and Armenter (2008) and assume that the Ramsey planner faces a wealth constraint. Specifically, the household must be allowed to keep an exogenous value of initial wealth of $\mathcal{V}_0 > 0$, measured in units of utility. If we denote initial nominal wealth at time 0 by $\mathcal{A}(s_0)$, then[5]

$$\frac{U_C(s_0)}{1 + \tau^C(s_0)} \frac{\mathcal{A}(s_0)}{\mathcal{P}(s_0)} \geq \mathcal{V}_0.$$

With this restriction, as long as the household receives an initial value of wealth (in utility terms) of $\mathcal{V}_0$, policies, including initial policies, can be chosen arbitrarily. We formally state the Ramsey planning problem as follows.

Ramsey Planning Problem. The Ramsey planner chooses an allocation, χ, that maximizes (1) subject to $\chi \in \mathcal{X}$ and (22).

Following Chari and Kehoe (1999), we write the Ramsey problem in Lagrangian form:

$$\max_{\chi \in \mathcal{X}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \mathcal{W}(C(s^t), L(s^t), \Gamma) - \Gamma \mathcal{V}_0,$$

where the function $\mathcal{W}$ incorporates the implementability condition into the planner's maximand as follows:

$$\mathcal{W}(C(s^t), L(s^t), \Gamma) \equiv U(C(s^t), L(s^t)) + \Gamma(U_C(s^t) C(s^t) + U_L(s^t) L(s^t)). \tag{23}$$

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[4]We derive condition (22) in Appendix A.2. The government's budget constraint holds by Walras's law.
[5]See Appendix A.2 for an expression for initial wealth, $\mathcal{A}(s_0)$.

We let Γ denote the Lagrange multiplier on the implementability condition in (22); aside from feasibility, this is the only constraint on the planner. We now characterize the first-order necessary conditions for an interior solution to the Ramsey problem; for short-hand we let $\mathcal{W}_C(s^t) \equiv \partial \mathcal{W}(\cdot)/\partial C(s^t)$ and $\mathcal{W}_L(s^t) \equiv \partial \mathcal{W}(\cdot)/\partial L(s^t)$.

Proposition 2. *The following three sets of first-order conditions are necessary for an interior solution to the Ramsey problem. For all $s^t \in S^t$,*
(i) for all $i \in \mathcal{I}$ and $m \in [0,1]$:

$$y_i^m(s^t) = Y_i(s^t), \ \ \ell_i^m(s^t) = L_i(s^t), \ \ k_i^m(s^t) = K_i(s^t), \ \ \textbf{and} \ \ x_{ij}^m(s^t) = X_{ij}(s^t), \ \forall j \in \mathcal{I};$$

and for all pairs $(i,j) \in \mathcal{I} \times \mathcal{I}$:

$$\frac{V_j(s_t)}{V_i(s_t)} \frac{\partial \mathcal{F}(s^t)/\partial \bar{X}_j(s^t)}{\partial \mathcal{F}(s^t)/\partial \bar{X}_i(s^t)} = A_i(s_t) \frac{\partial F_i(s^t)}{\partial X_{ij}(s^t)}; \tag{24}$$

(ii) for all $i \in \mathcal{I}$:

$$\frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)} = A_i(s_t) V_i(s_t) \frac{\partial \mathcal{F}(s^t)}{\partial \bar{X}_i(s^t)} \frac{\partial F_i(s^t)}{\partial L_i(s^t)}, \tag{25}$$

$$\mathcal{W}_C(s^t) = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \mathcal{W}_C(s^{t+1}) \left[1 + A_i(s_{t+1}) V_i(s_{t+1}) \frac{\partial \mathcal{F}(s^{t+1})}{\partial \bar{X}_i(s^{t+1})} \frac{\partial F_i(s^{t+1})}{\partial K_i(s^{t+1})} - \delta \right]. \tag{26}$$

Proof. See Appendix A.3. □

Part (i) of Proposition 2 indicates that at a Ramsey optimum, for every sector $i \in \mathcal{I}$, there is no input dispersion and, hence, no output dispersion. Furthermore, for every pair of goods $(i,j) \in \mathcal{I} \times \mathcal{I}$, the marginal rate of substitution is equal to the marginal rate of transformation. Therefore, at a Ramsey optimum, there is no misallocation within or across sectors.

Part (ii) of Proposition 2 provides the planner's intratemporal and intertemporal marginal conditions. Equation (25) indicates that for every technology $i \in \mathcal{I}$, the social marginal rate of substitution between aggregate labor and aggregate consumption, $-\mathcal{W}_L(s^t)/\mathcal{W}_C(s^t)$, is equal to its marginal rate of transformation under technology i. The latter is given by the marginal product of labor in the production of good i times the marginal product of good i in the production of the final good. Similarly, equation (26) indicates that for every technology $i \in \mathcal{I}$, the expected social marginal rate of substitution between consumption today and consumption next period is equal to its expected marginal rate of transformation under technology i.

A Ramsey optimum in this economy is familiar: the marginal conditions in Proposition 2 extend the marginal conditions in Chari and Kehoe (1999) and Correia, Nicolini

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and Teles (2008) to multi-sector, input-output economies. The binding implementability condition in (22) results in a divergence between the planner's marginal rates of substitution and those of the representative household: $\mathcal{W}_C(s^t)$ and $\mathcal{W}_L(s^t)$ differ from $U_C(s^t)$ and $U_L(s^t)$. Therefore the marginal conditions in (25) and (26) pin down optimal equilibrium wedges.

These optimal wedges imply that a Ramsey optimum does not coincide with the first best. Despite this, a Ramsey optimum preserves production efficiency in the sense of Diamond and Mirrlees (1971); see Chari and Kehoe (1999) and Chari, Nicolini and Teles (2020) for discussions on this general point. For our purposes, it is sufficient to note that zero misallocation within or across sectors in every period and every history is a necessary condition for Ramsey optimality.

Although a Ramsey optimum does not coincide with the first best, our characterization in Proposition 2 in fact nests the first best, defined as follows.

First Best Planning Problem. The first best planner chooses an allocation, χ, that maximizes (1) subject to $\chi \in \mathcal{X}$.

The first best is the welfare-maximizing allocation in the version of the economy with all possible tax instruments, including lump-sum taxation. This problem is nested in our formulation of the Ramsey problem with Γ, the multiplier on the implementability constraint, set equal to zero: $\Gamma = 0$. It follows that the first-order necessary conditions for the first best are the same conditions stated in Proposition 2, but with $\mathcal{W}_C(s^t)$ and $\mathcal{W}_L(s^t)$ replaced by $U_C(s^t)$ and $U_L(s^t)$, respectively.

5 Implementation

Recall that $\kappa \equiv (\kappa_i) \in \mathbb{R}_+^N$ is the vector of initial prices. A Ramsey optimum can be implemented as follows.

Theorem 1. *A Ramsey optimum, χ^*, can be implemented as an equilibrium allocation with: (i) a monetary policy that targets $\mathcal{P}(s^t) = 1$ and aggregate tax rates that satisfy, for all $t \geq 0$ and $s^t \in S^t$,*

$$\frac{1 - \tau^L(s^t)}{1 + \tau^C(s^t)} = \frac{U_L(s^t)/\mathcal{W}_L(s^t)}{U_C(s^t)/\mathcal{W}_C(s^t)}, \tag{27}$$

and, for all $t \geq 1$ and $s^t \in S^t$,

$$\tau^K(s^t) = 0 \quad and \quad 1 + \tau^C(s^t) = \varrho \frac{U_C(s^t)}{\mathcal{W}_C(s^t)}, \tag{28}$$

where $\varrho > 0$ is an arbitrary positive scalar; and (ii) sector-specific tax rates that jointly satisfy, for all $t \geq 0$ and $s^t \in S^t$,

$$\frac{1 + \tau_i^\ell(s^t)}{1 - \tau_i^y(s^t)} = \kappa_i \left[\frac{\theta_i(s^t) - 1}{\theta_i(s^t)}\right] \left[\frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)}\right]^{-1} A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)}, \tag{29}$$

$$(1 + \hat{\tau}_i^y(s^t))(1 + \tau_i^f(s^t)) = \kappa_i^{-1} \left[\frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)}\right] \left[A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)}\right]^{-1}, \tag{30}$$

and $\tau_i^\ell(s^t) = \tau_i^k(s^t)$ for every $i \in \mathcal{I}$, and

$$1 + \tau_{ij}^x(s^t) = (1 + \tau_i^\ell(s^t))(1 + \tau_j^f(s^t)). \tag{31}$$

for every $(i, j) \in \mathcal{I} \times \mathcal{I}$.

Nominal wages, rental rates, interest rates, and prices in this equilibrium satisfy, for all $t \geq 0$ and $s^t \in S^t$,

$$W(s^t) = \frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)}, \qquad R(s^t) = \left[\frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)}\right] \frac{\partial F_i(s^t)/\partial k_i(s^t)}{\partial F_i(s^t)/\partial \ell_i(s^t)}, \tag{32}$$

$$1 = \beta(1 + \iota(s^t)) \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{\mathcal{W}_C(s^{t+1})}{\mathcal{W}_C(s^t)}, \tag{33}$$

and, for all $i \in I$, $P_i(s^t) = (1 + \hat{\tau}_i^y(s^t))\kappa_i$ and $p_i^m(s^t) = \kappa_i$ for all $m \in [0, 1]$.

Proof. See Appendix A.4. □

A Ramsey optimum can be implemented as an equilibrium allocation in this economy. Theorem 1 provides implementation in two parts: (i) monetary policy and aggregate tax rates, and (ii) sector-specific tax rates. The final part of the theorem provides the nominal wages, rental rates, interest rates, and prices associated with this equilibrium.

Part (i) extends the optimal monetary policy results in Correia, Nicolini and Teles (2008) and the optimal taxation results in Lucas and Stokey (1983) and Chari and Kehoe (1999) to the class of economies under consideration. We focus on implementations in which monetary policy targets price stability for the final good: $\mathcal{P}(s^t) = 1$, in all periods and histories.[6] This is an arbitrary choice: as we are implementing a Ramsey optimum, the particular price index we target and its level are largely irrelevant. One can thereby think of $\mathcal{P}(s^t) = 1$ as a normalization. Given this normalization, the money supply and the equilibrium nominal interest rate consistent with this policy satisfy $M(s^t) = C(s^t)$ and (33), respectively.

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[6]The final good price, or consumer price index (CPI), is a standard price index targeted by central banks.

We furthermore focus on implementations in which the aggregate tax rates satisfy (27) and (28). Namely, we set the tax rate on capital equal to zero at all horizons (except for the initial period), and we use the labor income and consumption taxes to match the planner's marginal rates of substitution. This is a familiar class of implementations when labor income, capital income, and consumption taxes are all available (Chari and Kehoe, 1999; Chari, Nicolini and Teles, 2020; Angeletos and La'O, 2020).

The contribution of our paper lies in the second part of Theorem 1. Part (ii) characterizes the sector-specific tax rates that, together with the monetary policy and aggregate tax rates described in part (i), implement the Ramsey marginal conditions. Herein, we refer to these as the optimal sector-specific taxes.

We find that the optimal sector-specific tax rates play two roles: they stabilize seller prices and they simultaneously generate efficient movements in after-tax prices faced by buyers. Production efficiency requires uniform prices across all intermediate good sellers within a given sector. If firms within a sector were to set different prices, they would face different realized demands and, as such, would produce different levels of output. Price dispersion thereby results in within-sector misallocation—a feature incompatible with Ramsey optimality.

Stabilization of within-sector prices cannot be achieved in this class of economies with monetary policy and aggregate taxes alone.[7] To this end, sector-specific tax rates are used: in particular, condition (29) ensures that the optimal reset price of a firm in sector i satisfies: $p_i^*(s^t) = \kappa_i$, where κ_i is also equal to the price of the non-resetting firms. Zero price dispersion within sector i results in zero within-sector misallocation.

Stabilization of seller prices eliminates misallocation within sectors, but without further intervention this would result in misallocation across sectors. This is because constant prices cannot, by construction, reflect state-contingent fluctuations in marginal costs; as a result, constant prices lead to an inefficient allocation of goods across their final and intermediate good uses. In order to correct this problem and facilitate an efficient allocation of sectoral goods, after-tax prices faced by buyers must reflect their efficient shadow values. The optimal sector-specific tax rates accomplish this second objective: in particular, condition (30) guarantees that after-tax prices faced by buyers accurately reflect their state-contingent marginal costs of production.

We state these dual roles of the taxes as the first general lesson of our analysis.

Lesson 1. The optimal tax structure achieves production efficiency. In order to do so, sector-specific tax rates play two roles: they stabilize seller prices and generate efficient fluctuations in after-tax prices. This eliminates misallocation within and across sectors, ensuring efficient production of the final good.

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[7]See La'O and Tahbaz-Salehi (2022) for a formal proof of this statement.

We illustrate these dual roles in a number of examples below. Prior to these examples, note that our implementation of the Ramsey optimum requires at least some sector-specific tax rates to be state-contingent. In particular, the optimal tax wedges in (29) and (30) depend on the realization of the nominal wage, given by $W(s^t) = -\mathcal{W}_L(s^t)/\mathcal{W}_C(s^t)$, and the sector's marginal product of labor, $A_i(s^t)\partial F_i(s^t)/\partial \ell_i(s^t)$; more generally, the optimal tax wedges are contingent on the sector's marginal cost. The optimal tax wedges in (29) furthermore vary with the sector's elasticity of substitution $\theta_i(s^t)$ or, more generally, its intended mark-up: $\theta_i(s^t)/(\theta_i(s^t) - 1)$ under CES.

Theorem 1 thereby informs us on the state-contingency of the optimal tax structure. It answers the practical question of what information the fiscal authority must have in order to administer the optimal tax rates: namely, marginal costs and markups. By the same token, it also answers the question of what information the fiscal authority need *not* have. While the optimal tax wedges in (29) and (30) vary with marginal costs and desired mark-ups, conditional on these objects, they are independent of the vector of demand shocks, $(V_i(s_t))_{i \in \mathcal{I}}$, and shocks to government expenditure, $G(s_t)$.[8]

Finally, note that the monetary policy, aggregate tax rates, and sector-specific tax rates characterized in Theorem 1 are all independent of the vector of Calvo parameters, $(\alpha_i)_{i \in \mathcal{I}}$. The irrelevance of the strength of sectoral price rigidities for optimal fiscal and monetary policy, even away from the first best, generalizes previous results in Correia, Nicolini and Teles (2008) to multi-sector, input-output NK economies. We state these observations as the second general lesson of our analysis.

Lesson 2. Optimal sector-specific tax rates are state-contingent. They depend on the realized nominal wage, sectoral technologies, and sectoral elasticities of substitution, but they do not depend on sectoral demand shocks and government spending. Optimal monetary and fiscal policy are invariant to the vector of price rigidities.

6   Implementation Examples

Theorem 1 provides a general characterization of optimal fiscal and monetary policy. In this section, we move away from this general characterization and explore a number of specific implementations, focusing on different configurations of the sector-specific tax instruments. We use these examples to better illustrate the dual roles of the optimal sector-specific tax structure and to highlight a few tax equivalences and non-equivalences. The results in this section are all corollaries of Theorem 1.

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[8]The optimal tax rates are, moreover, sector-specific: they do not depend on any feature that is specific to an individual firm, i.e. its date of last price adjustment.

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The first implementation we consider is what we call the "least number of instruments" implementation. We show that regardless of the network structure, production efficiency can be achieved with only $2N$ sector-specific tax rates.

Corollary 1. Least Number of Instruments Implementation. *A Ramsey optimum can be implemented with the monetary policy and aggregate tax rates described in Theorem 1 and, for every $i \in \mathcal{I}$,*

$$1 - \tau_i^y(s^t) = \kappa_i^{-1} W(s^t) \left[A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \right]^{-1} \left[\frac{\theta_i(s^t) - 1}{\theta_i(s^t)} \right]^{-1}, \quad \forall s^t \in S^t \tag{34}$$

$$1 + \hat{\tau}_i^y(s^t) = \kappa_i^{-1} W(s^t) \left[A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \right]^{-1}, \quad \forall s^t \in S^t \tag{35}$$

and

$$\tau_i^\ell(s^t) = 0, \tau_i^k(s^t) = 0, \tau_i^f(s^t) = 0, \quad \textbf{and} \quad \tau_{ij}^x(s^t) = 0, \forall j \in \mathcal{I}, \quad \forall s^t \in S^t.$$

In standard models with flexible prices, the side of the market on which a tax is levied is irrelevant: it makes no difference whether a sales tax is collected from buyers or from sellers. When prices are rigid, however, this equivalence breaks (Poterba, Rotemberg and Summers, 1986). We exploit this particular non-equivalence in Corollary 1.

In our economy, the sales tax levied on differentiated good sellers in sector i, $\tau_i^y(s^t)$, is not equivalent to the sales tax levied on the buyers, $\hat{\tau}_i^y(s^t)$. Moreover, if these instruments are both available, Ramsey optimality can be attained without the use of the other sector-specific tax instruments. Specifically, in Corollary 1 the sector-specific taxes on labor, capital, intermediate inputs, and final goods are all set equal to zero.

In this implementation, the sales tax on sellers is set according to (34) and the sales tax on buyers is set according to (35). The former satisfies condition (29) in Theorem 1 with $\tau_i^\ell(s^t) = 0$, and the latter satisfies condition (30) with $\tau_i^f(s^t) = 0$. Therefore, $\tau_i^y(s^t)$ ensures that all firms in sector i set uniform prices, while $\hat{\tau}_i^y(s^t)$ ensures that buyers face an after-tax price of:

$$P_i(s^t) = W(s^t) \left[A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \right]^{-1} = \mathrm{mc}_i(s^t). \tag{36}$$

This after-tax price accurately reflects the good's marginal cost of production.[9]

The two types of sales taxes in this implementation are specifically engineered to fulfill the two roles highlighted previously: stabilizing seller prices and generating efficient fluctuations in after-tax prices. This is why, irrespective of the network structure, only two tax instruments per sector are sufficient to attain production efficiency.

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[9]In fact, it is evident from equations (34) and (35) that $\hat{\tau}_i^y(s^t)$ reverses the stabilization effects of $\tau_i^y(s^t)$ on prices, but excludes the markup.

Lesson 3. Regardless of the network structure, with N sectors production efficiency can be attained with $2N$ sectoral tax instruments: N sales taxes levied on sellers and N sales taxes levied on buyers. Unlike in standard models with price flexibility, taxes levied on sellers and on buyers in this economy are not equivalent.

While the two types of sales taxes in this economy are not equivalent, there exists a number of tax equivalences. In what follows we explore two alternative implementations. In one implementation, we assume the seller-imposed sales tax is unavailable; in the other, the buyer-imposed sales tax is unavailable. In either case, greater than $2N$ sector-specific tax instruments are operative.

Corollary 2. *Suppose that the tax instrument $\tau_i^y(s^t)$ is unavailable:*

$$\tau_i^y(s^t) = 0, \qquad \forall i \in \mathcal{I}, t \geq 0, s^t \in S^t.$$

Then a Ramsey optimum can be implemented with the monetary policy and aggregate tax rates described in Theorem 1 and, for every $i \in \mathcal{I}$,

$$\tau_i^\ell(s^t) = \tau_i^k(s^t) = \tau_{ij}^x(s^t), \quad \forall j \in \mathcal{I}, s^t \in S^t,$$

$$1 + \tau_i^\ell(s^t) = \kappa_i W(s^t)^{-1} A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \left[\frac{\theta_i(s^t) - 1}{\theta_i(s^t)} \right], \quad \forall s^t \in S^t, \tag{37}$$

$\hat{\tau}_i^y(s^t)$ set according to (35), and $\tau_i^f(s^t) = 0$, for all $s^t \in S^t$.

When the sales tax levied on sellers is unavailable, Ramsey optimality can still be implemented: in this case, the sector-specific labor, capital, and intermediate input taxes substitute for the missing tax instrument. This tax equivalence first requires that, for every sector $i \in \mathcal{I}$, input taxes are uniform across all inputs—labor, capital, and intermediate goods—so as not to distort the producers' relative use of inputs. Second, these input taxes satisfy (37); this guarantees that all firms within a given sector set uniform prices. The sales tax collected on buyers continues to ensure that after-tax prices reflect marginal costs of production.

Note that in this implementation, the intermediate good taxes, $\tau_{ij}^x(s^t)$, depend only on the destination sector i but not the origin sector j. This is because this implementation requires the input taxes to replicate the missing seller-imposed sales tax on sector i; it therefore need not contain any information about good j. We summarize this in our fourth lesson.

Lesson 4. Sector-specific input taxes on labor, capital, and intermediate goods can substitute for seller-imposed sales taxes. In this case, the optimal intermediate good input taxes are destination-specific but not origin-specific.

Our next example is one in which we restrict the tax structure so that the sales tax imposed on buyers is unavailable.

Corollary 3. *Suppose that the tax instrument $\hat{\tau}_i^y(s^t)$ is unavailable:*

$$\hat{\tau}_i^y(s^t) = 0, \qquad \forall i \in \mathcal{I}, s^t \in S^t.$$

Then a Ramsey optimum can be implemented with the monetary policy and aggregate tax rates described in Theorem 1 and, for every $j \in \mathcal{I}$,

$$1 + \tau_{ij}^x(s^t) = 1 + \tau_j^f(s^t) = \kappa_j^{-1} W(s^t) \left[A_j(s^t) \frac{\partial F_j(s^t)}{\partial \ell_j(s^t)} \right]^{-1}, \quad s^t \in S^t, \tag{38}$$

$\tau_j^y(s^t)$ *set according to* (34) *and* $\tau_j^\ell(s^t) = \tau_j^k(s^t) = 0$.

When the sales taxes levied on buyers is unavailable, Ramsey optimality can still be implemented: in this case, the intermediate good and final good taxes substitute for the missing tax instrument. This tax equivalence requires that, for every good $j \in \mathcal{I}$, intermediate input and final good taxes are equalized so as not to distort the relative use of good j across various producers and consumers. The intermediate good and final good taxes moreover satisfy (38); this guarantees that after-tax prices faced by buyers reflect marginal costs of production. The sales taxes collected on sellers continues to ensure that seller prices are uniformly set.

Note that in this implementation and unlike in our previous example, the intermediate good taxes, $\tau_{ij}^x(s^t)$, depend only on the origin sector j but not the destination sector i. This is because this implementation requires the input taxes to replicate the missing buyer-imposed sales tax on good j; it therefore need not contain any information about the user of the good, sector i. We summarize this in our fifth lesson.

Lesson 5. Sector-specific intermediate good and final good taxes can substitute for buyer-imposed sales taxes. In this case, the optimal intermediate good input taxes are origin-specific but not destination-specific.

Our final example distinguishes the role of the aggregate tax rates vis-à-vis the sector-specific tax rates. Recall from our implementation of Ramsey optimality in Theorem 1, aggregate tax rates were set to replicate the Ramsey planner's marginal rates of substitution, while the sector-specific tax rates were engineered to restore within-period production efficiency.

We now consider the case in which lump-sum taxes are available. The following corollary of Theorem 1 illustrates that the availability of lump-sum taxes affects only the nature of the aggregate tax instruments—it has no bearing on the optimal sector-specific tax structure.

Corollary 4. *Suppose lump-sum taxes are available. Then $\Gamma = 0$ and Ramsey optimality coincides with first best optimality: $\mathcal{W}_C(s^t) = U_C(s^t)$ and $\mathcal{W}_L(s^t) = U_L(s^t)$. The first best can be implemented with aggregate tax rates that satisfy $\tau^L(s^t) = 0$, $\tau^C(s^t) = 0$, and $\tau^K(s^t) = 0$, for all $t \geq 0$, $s^t \in S^t$, and monetary policy and sector-specific tax rates as described in Theorem 1.*

When lump sum taxes are available, the first best is attainable. It is nested in our more general framework with Γ, the multiplier on the implementability constraint, set equal to zero. It follows from Theorem 1 that the first best can be implemented with a monetary policy that stabilizes the price of the final good, sector-specific tax rates as described in part (ii) of the theorem, and all aggregate tax rates set equal to zero.

In the first best, there are no longer any optimal wedges in the planner's marginal conditions. It follows that all aggregate tax rates can be set to zero. However, this does not imply that all sector-specific tax rates can also be set to zero. On the contrary, whether or not lump-sum taxes are available does not alter the nature of the optimal sector-specific tax instruments: in either case they are responsible for ensuring within-period production efficiency. We state this as the sixth and final lesson of our paper.

Lesson 6. Optimal sector-specific tax rates are independent of the availability of lump-sum taxes.

Remark on Robustness. In our baseline model, the nominal rigidity is assumed to be a Calvo pricing friction, in line with the standard assumption in the New Keynesian literature. However, if the nominal rigidity were instead driven by an information friction in price setting as in, e.g., Woodford (2003a) and Mankiw and Reis (2002), our results on optimal policy would still hold.

To see why, suppose that firms were to set prices every period under dispersed and incomplete information. In this case, each seller's price would be constrained to be measurable in the firm's individual information set. Given the tax structure specified in Theorem 1, all firms in sector i would set its price equal to κ_i, regardless of its information set. This pricing strategy would not only satisfy the firm's measurability constraint but would also be optimal from the firm's perspective.

The tax structure and monetary policy in Theorem 1 can thereby implement a Ramsey optimum as described in Proposition 2 regardless of whether the nominal rigidity is driven by informational frictions, Calvo pricing frictions, or a combination of the two.

7 Conclusion

In this paper we study optimal monetary and fiscal policy in a multi-sector, NK economy with input-output linkages and a rich set of tax instruments. With $N>1$ sectors, production efficiency can be achieved with $2N$ state-contingent, sector-specific taxes; the optimal tax scheme we engineer involves jointly stabilizing all prices set by sellers while generating efficient movements in after-tax prices faced by buyers.

The optimal sector-specific tax rates we find are contingent on realized marginal costs and desired mark-ups. Keeping track of these sector-level statistics at business cycle frequency is arguably a heavy informational burden on the fiscal authority. The best that fiscal and monetary policy can do given limited information of the government is an interesting question that we leave for future research.

The contribution of this paper, as we have noted, is to establish a benchmark in which the set of tax instruments is rich enough to attain production efficiency and to characterize these instruments. Our benchmark differs prominently from the standard benchmark in multi-sector NK models of "inactive" fiscal policy in which sales subsidies eliminate steady-state markups. As the literature on multi-sector, input-output NK models moves away from the standard benchmark and explores richer environments with larger, more active sets of fiscal instruments, we believe that an understanding and appreciation of both benchmarks—i.e. both extremes—will prove useful.

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A Appendix

A.1 Proof of Proposition 1

We first derive the optimality conditions of the representative household. We let $\beta^t \mu(s^t) \Lambda(s^t)$ denote the Lagrange multiplier on the household budget set at time t, history s^t. The household's first-order necessary conditions with respect to consumption and labor are given by, respectively:

$$U_C(s^t) - \Lambda(s^t)(1 + \tau^C(s^t))\mathcal{P}(s^t) = 0, \tag{39}$$

$$U_L(s^t) + \Lambda(s^t)(1 - \tau^L(s^t))W(s^t) = 0. \tag{40}$$

The first-order condition with respect to capital $K(s^t)$ is given by:

$$-\mu(s^t)\Lambda(s^t)\mathcal{P}(s^t) + \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1})\Lambda(s^{t+1})[(1 - \tau^K(s^{t+1}))R(s^{t+1}) + \mathcal{P}(s^{t+1})(1 - \delta)] = 0, \tag{41}$$

the first-order condition with respect to the bond $B(s^t)$ is given by:

$$-\mu(s^t)\Lambda(s^t) + \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1})\Lambda(s^{t+1})(1 + \iota(s^t)) = 0, \tag{42}$$

and the first-order condition with respect to the Arrow security $Z(s^{t+1}|s^t)$ is given by:

$$-\mu(s^t)\Lambda(s^t)Q(s^{t+1}|s^t) + \beta\mu(s^{t+1})\Lambda(s^{t+1}) = 0. \tag{43}$$

The household transversality conditions are given by:

$$\lim_{t \to \infty} \sum_{s^t} \beta^t \mu(s^t)\Lambda(s^t)B(s^t) = 0 \quad \text{and} \quad \lim_{t \to \infty} \sum_{s^t} \beta^t \mu(s^t)\Lambda(s^t)\mathcal{P}(s^t)K(s^t) = 0. \tag{44}$$

Combining (39) and (40), we obtain the household's intratemporal condition (16). Next, using $\Lambda(s^t) = U_C(s^t)/[(1 + \tau^C(s^t))\mathcal{P}(s^t)]$, conditions (41) and (42) imply the Euler equations for capital and bonds in (17) and (18). The first order condition in (43) pins down the Arrow security price in (19), and (44) imply the transversality conditions in (20).

The final good firm solves the following profit-maximization problem:

$$\max_{(X_1^f(s^t),\dots,X_I^f(s^t))} \mathcal{P}(s^t)\mathcal{F}(V_1(s_t)X_1^f(s^t),\dots,V_I(s_t)X_I^f(s^t)) - \sum_{i \in \mathcal{I}} P_i(s^t)X_i^f(s^t).$$

The first-order conditions with respect to $X_i^f(s^t)$ yield the demand functions in (21).

The firm's cost-minimization problem, stated in (13), yields the optimality conditions in (14). The firm's price-setting problem is stated in Section 3. Substituting in the

demand function, we can rewrite this problem as follows:

$$\max_p \sum_{v=0}^{\infty}\sum_{s^{t+v}}(\alpha_i\beta)^v \frac{U_C(s^{t+v})Y_i(s^{t+v})P_i(s^{t+v})^{\theta_i(s^{t+v})}}{(1+\tau^C(s^t))\mathcal{P}(s^{t+v})(1+\hat{\tau}_i^y(s^{t+v}))^{\theta_i(s^{t+v})}}\left[(1-\tau_i^y(s^{t+v}))p^{1-\theta_i(s^{t+v})}\right]\mu(s^{t+v}|s^t)$$
$$-\sum_{v=0}^{\infty}\sum_{s^{t+v}}(\alpha_i\beta)^v \frac{U_C(s^{t+v})Y_i(s^{t+v})P_i(s^{t+v})^{\theta_i(s^{t+v})}}{(1+\tau^C(s^t))\mathcal{P}(s^{t+v})(1+\hat{\tau}_i^y(s^{t+v}))^{\theta_i(s^{t+v})}}\left[\mathbf{mc}_i(s^{t+v})p^{-\theta_i(s^{t+v})}\right]\mu(s^{t+v}|s^t)$$

The firm's first order condition with respect to its price is given by (15), where we define

$$\hat{\mu}_i(s^{t+v}|s^t)\equiv \frac{U_C(s^{t+v})}{(1+\tau^C(s^t))\mathcal{P}(s^{t+v})}\left[\frac{(1-\tau_i^y(s^{t+v}))Y_i(s^{t+v})P_i(s^{t+v})^{\theta_i(s^{t+v})}(\theta_i(s^{t+v})-1)}{(1+\hat{\tau}_i^y(s^{t+v}))^{\theta_i(s^{t+v})}}\right]\mu(s^{t+v}|s^t),$$

for all s^{t+v}, $v=0,\ldots,\infty$.

A.2   Derivation of Implementability Condition

We take the household's budget constraint, multiply both sides by $\Lambda(s^t)$, and use the household's FOCs in (39) and (40) to substitute out consumption and labor prices. Doing so, we obtain:

$$U_C(s^t)C(s^t)+\Lambda(s^t)\mathcal{P}(s^t)K(s^t)+\Lambda(s^t)B(s^t)+\Lambda(s^t)\sum_{s^{t+1}|s^t}Q(s^{t+1}|s^t)Z(s^{t+1}|s^t)$$
$$=-U_L(s^t)L(s^t)+\Lambda(s^t)(1-\tau^K(s^t))R(s^t)K(s^{t-1})+\Lambda(s^t)\mathcal{P}(s^t)(1-\delta)K(s^{t-1})$$
$$+\Lambda(s^t)(1+\iota(s^{t-1}))B(s^{t-1})+\Lambda(s^t)Z(s^t|s^{t-1})$$

Multiplying both sides by $\beta^t\mu(s^t)$, summing over t and s^t, and using the household's FOCs for financial assets, (41)-(43), to cancel terms, we obtain:

$$\sum_t\sum_{s^t}\beta^t\mu(s^t)\left[U_C(s^t)C(s^t)+U_L(s^t)L(s^t)\right]=\Lambda(s_0)\left[(1-\tau^K(s_0))R(s_0)K_{-1}+\mathcal{P}(s_0)(1-\delta)K_{-1}\right]$$
$$+\Lambda(s_0)(1+\iota_{-1})B_{-1}$$

Using $\Lambda(s_0)=U_C(s_0)/(1+\tau^C(s_0))\mathcal{P}(s_0)$, we obtain:

$$\sum_t\sum_{s^t}\beta^t\mu(s^t)\left[U_C(s^t)C(s^t)+U_L(s^t)L(s^t)\right]=\frac{U_C(s_0)}{1+\tau^C(s_0)}\frac{\mathcal{A}(s_0)}{\mathcal{P}(s_0)}$$

where initial real wealth is given by:

$$\frac{\mathcal{A}(s_0)}{\mathcal{P}(s_0)}\equiv\left[1+(1-\tau^K(s_0))\frac{R(s_0)}{\mathcal{P}(s_0)}-\delta\right]K_{-1}+(1+\iota_{-1})\frac{B_{-1}}{\mathcal{P}(s_0)}.$$

Although the real rental rate $R(s_0)/\mathcal{P}(s_0)$ is endogenous to both monetary and fiscal policy, for simplicity we have assumed that time zero policies must satisfy the constraint that $\frac{U_C(s_0)}{1+\tau^C(s_0)}\frac{\mathcal{A}(s_0)}{\mathcal{P}(s_0)}=\mathcal{V}_0$.

A.3 Proof of Proposition 2

We write the Ramsey planner's Lagrangian as follows:

$$
\begin{aligned}
\mathcal{L} =& \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \mu(s^t) \mathcal{W}(C(s^t), L(s^t)) - \Gamma \mathcal{V}_0 \\
&- \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \mu(s^t) \lambda^C(s^t) \left[C(s^t) + K(s^t) - (1-\delta)K(s^{t-1}) + G(s^t) - \mathcal{F}(\bar{X}_1^f(s^t), \ldots, \bar{X}_I^f(s^t)) \right] \\
&- \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \mu(s^t) \sum_{i\in\mathcal{I}} \lambda_i^Y(s^t) \left[X_i^f(s^t) + \sum_{j\in\mathcal{I}} X_{ji}(s^t) - A_i(s_t) F_i(L_i(s^t), K_i(s^t), X_{i1}(s^t), \ldots, X_{iI}(s^t)) \right] \\
&+ \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \mu(s^t) \lambda^L(s^t) \left[L(s^t) - \sum_{i\in\mathcal{I}} L_i(s^t) \right] + \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \mu(s^t) \lambda^K(s^t) \left[K(s^{t-1}) - \sum_{i\in\mathcal{I}} K_i(s^t) \right]
\end{aligned}
$$

The planner's FOCs w.r.t. $C(s^t)$, $L(s^t)$, and $K(s^t)$ are given by:

$$
\begin{aligned}
\mathcal{W}_C(s^t) - \lambda^C(s^t) &= 0, \\
\mathcal{W}_L(s^t) + \lambda^L(s^t) &= 0, \\
-\beta^t \mu(s^t) \lambda^C(s^t) + \sum_{s^{t+1}|s^t} \beta^{t+1}\mu(s^{t+1})\lambda^C(s^{t+1}) \left[1 + \lambda^K(s^{t+1}) - \delta \right] &= 0,
\end{aligned}
$$

respectively, for all s^t. The planner's FOCs w.r.t. $X_i^f(s^t)$, $L_i(s^t)$, and $K_i(s^t)$ are given by:

$$
\begin{aligned}
\lambda^C(s^t) V_i(s_t) \frac{\partial \mathcal{F}(s^t)}{\partial \bar{X}_i(s^t)} - \lambda_i^Y(s^t) &= 0, \quad \forall i \in \mathcal{I} \\
-\lambda^L(s^t) + \lambda_i^Y(s^t) A_i(s_t) \frac{\partial F_i(s^t)}{\partial L_i(s^t)} &= 0, \quad \forall i \in \mathcal{I}, \\
-\lambda^K(s^t) + \lambda_i^Y(s^t) A_i(s_t) \frac{\partial F_i(s^t)}{\partial K_i(s^t)} &= 0, \quad \forall i \in \mathcal{I}.
\end{aligned}
$$

The FOCs w.r.t. $X_{ij}(s^t)$ are given by

$$
-\lambda_j^Y(s^t) + \lambda_i^Y(s^t) A_i(s_t) \frac{\partial F_i(s^t)}{\partial X_{ij}(s^t)} = 0, \quad \forall i, j \in \mathcal{I} \times \mathcal{I}
$$

Combining these conditions so as to eliminate the multipliers $\lambda^C(s^t), \lambda^L(s^t), \lambda^K(s^t)$ and $\lambda_i^Y(s^t)$ for all $i \in \mathcal{I}$ results in the optimality conditions stated in Proposition 2.

A.4 Proof of Theorem 1

First, for each $i \in \mathcal{I}$, we set the tax rates $(\tau_i^y(s^t), \tau_i^\ell(s^t))$ such that they jointly satisfy:

$$
\frac{1 + \tau_i^\ell(s^t)}{1 - \tau_i^y(s^t)} = \kappa_i \left[\frac{\theta_i(s^t) - 1}{\theta_i(s^t)} \right] \frac{1}{W(s^t)} A_i(s^t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \tag{45}
$$

29

We will later verify that (45) is consistent with (29). With tax rates set according to (45), the optimal price for each firm $m \in [0, 1]$ satisfies:

$$p_i^m(\omega_i^t) = \sum_{s_t \in S} \kappa_i q_i(s^t | \omega_i^t) = \kappa_i.$$

At the Ramsey optimum $y_i^m(s^t) = Y_i(s^t)$; it follows from (5) that $P_i(s^t) = (1 + \hat{\tau}_i^y(s^t))\kappa_i$.

Next, we set $\tau_i^k(s^t) = \tau_i^\ell(s^t)$. Equilibrium condition (14) implies that the equilibrium rental rate satisfies:

$$R(s^t) = W(s^t) \frac{\partial F_i(s^t)/\partial k_i(s^t)}{\partial F_i(s^t)/\partial \ell_i(s^t)} \qquad \forall i. \tag{46}$$

Household optimality conditions (16) and (17), when combined with condition (21), result in:

$$\frac{-U_L(s^t)}{U_C(s^t)} = \frac{(1 - \tau^L(s^t))}{(1 + \tau^C(s^t))(1 + \tau_i^f(s^t))} \nu_i(s_t) \frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_i(s^t)} \frac{W(s^t)}{P_i(s^t)},$$

$$\frac{U_C(s^t)}{(1 + \tau^C(s^t))} = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \frac{U_C(s^{t+1})}{(1 + \tau^C(s^{t+1}))} \left[1 - \delta + \frac{(1 - \tau^K(s^{t+1}))}{(1 + \tau_i^f(s^{t+1}))} V_i(s_{t+1}) \frac{\partial \mathcal{F}(s^{t+1})}{\partial \bar{x}_i(s^{t+1})} \frac{R(s^{t+1})}{P_i(s^{t+1})} \right].$$

where $P_i(s^t) = (1 + \hat{\tau}_i^y(s^t))\kappa_i$. Combining these conditions with the aggregate tax rates in (27), yield:

$$\frac{-\mathcal{W}_L(s^t)}{\mathcal{W}_C(s^t)} = \frac{\kappa_i^{-1} W(s^t) V_i(s_t) \frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_i(s^t)}}{(1 + \hat{\tau}_i^y(s^t))(1 + \tau_i^f(s^t))}, \tag{47}$$

$$\mathcal{W}_C(s^t) = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \mathcal{W}_C(s^{t+1}) \left[1 - \delta + \frac{\kappa_i^{-1} R(s^{t+1}) V_i(s_{t+1}) \frac{\partial \mathcal{F}(s^{t+1})}{\partial \bar{x}_i(s^{t+1})}}{(1 + \hat{\tau}_i^y(s^{t+1}))(1 + \tau_i^f(s^{t+1}))} \right]. \tag{48}$$

Next, we set the tax rates $(\hat{\tau}_i^y(s^t), \tau_i^f(s^t))$ such that they jointly satisfy:

$$\frac{\kappa_i^{-1}}{(1 + \hat{\tau}_i^y(s^t))(1 + \tau_i^f(s^t))} W(s^t) = A_i(s_t) \frac{\partial F_i(s^t)}{\partial \ell_i(s^t)} \tag{49}$$

We will later verify that (49) is consistent with (30). Substituting this expression into (47), we obtain the Ramsey intratemporal optimality condition in (25). Similarly substituting this expression into (48) yields:

$$\mathcal{W}_C(s^t) = \beta \sum_{s^{t+1}|s^t} \mu(s^{t+1}|s^t) \mathcal{W}_C(s^{t+1}) \left[1 - \delta + A_i(s_{t+1}) V_i(s_{t+1}) \frac{\partial \mathcal{F}(s^{t+1})}{\partial \bar{x}_i(s^{t+1})} \frac{R(s^{t+1})}{W(s^{t+1})} \frac{\partial F_i(s^{t+1})}{\partial \ell_i(s^{t+1})} \right].$$

Combining this with the equilibrium rental rate in (46), we obtain the Ramsey intertemporal optimality condition in (26).

Finally, taking the ratio of equilibrium condition (21) for sectors j and i, yields:

$$\frac{V_j(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_j(s^t)}}{V_i(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_i(s^t)}} = \frac{(1+\tau_j^f(s^t))P_j(s^t)}{(1+\tau_i^f(s^t))P_i(s^t)}. \tag{50}$$

Equilibrium condition (14) implies that $P_j(s^t)$ satisfies:

$$P_j(s^t) = W(s^t)\left[\frac{1+\tau_i^\ell(s^t)}{1+\tau_{ij}^x(s^t)}\right]\frac{\partial F_i(s^t)}{\partial x_{ij}(s^t)}\left(\frac{\partial F_i(s^t)}{\partial \ell_i(s^t)}\right)^{-1}$$

Substituting this expression for $P_j(s^t)$ into (50) we get:

$$\frac{V_j(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_j(s^t)}}{V_i(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_i(s^t)}} = \frac{1+\tau_j^f(s^t)}{1+\tau_i^f(s^t)}\left[\frac{1+\tau_i^\ell(s^t)}{1+\tau_{ij}^x(s^t)}\right]\frac{W(s^t)}{P_i(s^t)}\frac{\partial F_i(s^t)}{\partial x_{ij}(s^t)}\left(\frac{\partial F_i(s^t)}{\partial \ell_i(s^t)}\right)^{-1}$$

where $P_i(s^t) = (1+\hat{\tau}_i^y(s^t))\kappa_i$. Combining this expression with the taxes set in (49), yields:

$$\frac{V_j(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_j(s^t)}}{V_i(s_t)\frac{\partial \mathcal{F}(s^t)}{\partial \bar{x}_i(s^t)}} = \left[\frac{(1+\tau_j^f(s^t))(1+\tau_i^\ell(s^t))}{1+\tau_{ij}^x(s^t)}\right]A_i(s_t)\frac{\partial F_i(s^t)}{\partial x_{ij}(s^t)}$$

We thus set intermediate input taxes according to (31) and satisfy the Ramsey optimality condition in (24).

Next, we show that the tax rates set in (45) and (49) coincide with those stated in the Theorem. Recall that the household's intratemporal condition is given by (16). Using this condition we find that under this implementation, the nominal wage satisfies: $W(s^t) = -\mathcal{W}_L(s^t)/\mathcal{W}_C(s^t)$. Substituting this expression for the nominal wage into (45) and (49) results in equations (29) and (30), respectively.

What remains to be shown is that we can construct financial asset holdings such that the household's budget constraint is satisfied at this allocation in every date t and history. First, we take the household's budget constraint in (2) for all periods and states following and including period r, history s^r; we multiply these budget constraints by $\beta^{t-r}\mu(s^t|s^r)\Lambda(s^t)$ and sum over all periods and states following and including period r, history s^r. Doing so, we get:

$$\sum_{t=r}^{\infty}\sum_{s^t|s^r}\beta^{t-r}\mu(s^t|s^r)\Lambda(s^t)\left[(1+\tau^C(s^t))\mathcal{P}(s^t)C(s^t) + \mathcal{P}(s^t)K(s^t) + B(s^t) + \sum_{s^{t+1}|s^t}Q(s^{t+1}|s^t)Z(s^{t+1}|s^t)\right]$$

$$= \sum_{t=r}^{\infty}\sum_{s^t|s^r}\beta^{t-r}\mu(s^t|s^r)\Lambda(s^t)\left[(1-\tau^L(s^t))W(s^t)L(s^t) + (1+\iota(s^{t-1}))B(s^{t-1}) + Z(s^t|s^{t-1})\right]$$

$$+ \sum_{t=r}^{\infty}\sum_{s^t|s^r}\beta^{t-r}\mu(s^t|s^r)\Lambda(s^t)\left[(1-\tau^K(s^t))R(s^t)K(s^{t-1}) + (1-\delta)\mathcal{P}(s^t)K(s^{t-1})\right]$$

Using the household's FOCs for financial assets, (41)-(43), the above equation reduces to:

$$\sum_{t=r}^{\infty} \sum_{s^t | s^r} \beta^{t-r} \mu(s^t | s^r) \Lambda(s^t) \left[(1 + \tau^C(s^t)) \mathcal{P}(s^t) C(s^t) - (1 - \tau^L(s^t)) W(s^t) L(s^t) \right] \qquad (51)$$
$$= \Lambda(s^r) \left[(1 + \iota(s^{r-1})) B(s^{r-1}) + Z(s^r | s^{r-1}) \right]$$
$$+ \Lambda(s^r) \left[(1 - \tau^K(s^r)) R(s^r) K(s^{r-1}) + (1 - \delta) \mathcal{P}(s^r) K(s^{r-1}) \right]$$

Next, we define $\mathcal{A}(s^r)$ as follows:

$$\mathcal{A}(s^r) \equiv (1 + \iota(s^{r-1})) B(s^{r-1}) + Z(s^r | s^{r-1}) + (1 - \tau^K(s^r)) R(s^r) K(s^{r-1}) + (1 - \delta) \mathcal{P}(s^r) K(s^{r-1});$$

$\mathcal{A}(s^r)$ represents the total nominal assets that the household carries into period r, history s^r. Rearranging (51) gives us:

$$\Lambda(s^r) \mathcal{A}(s^r) = \sum_{t=r}^{\infty} \sum_{s^t | s^r} \beta^{t-r} \mu(s^t | s^r) \Lambda(s^t) \left[(1 + \tau^C(s^t)) \mathcal{P}(s^t) C(s^t) - (1 - \tau^L(s^t)) W(s^t) L(s^t) \right]$$

Next, using conditions (39) and (40), we obtain the following expression for the total *real* financial assets that household i carries into period r, history s^r:

$$\frac{\mathcal{A}(s^r)}{\mathcal{P}(s^t)} = \left(\frac{U_C(s^r)}{1 + \tau^C(s^r)} \right)^{-1} \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu(s^t | s^r) \left[U_C(s^t) C(s^t) + U_L(s^t) L(s^t) \right].$$

Finally, the government's budget constraint holds by Walras's law.