Government Spending in Disaggregated Economies

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Abstract

Effective stabilisation of asymmetric shocks requires understanding the sectoral dynamics through which fiscal and monetary policies transmit in disaggregated economies. Multi-sector New-Keynesian frameworks are well suited to this task, yet they often rely on two restrictive assumptions that this paper relaxes. First, government demand does not enter firms' consumer-price-setting problem; we drop this assumption by setting a pass-through parameter $\kappa \in [0, 1]$, nesting the original framework at $\kappa = 0$. Second, government spending at both the aggregate and sectoral level is entirely under the planner's control; we assume that the aggregate volume of government goods is exogenously set, reflecting political constraints, while the planner can flexibly allocate spending across sectors. Allowing for pass-through of government demand on consumer prices introduces government spending into the Phillips curve and steepens its slope by as much as 25%. The optimal fiscal rule becomes less counter-cyclical and turns pro-cyclical in sectors with high price flexibility. Assuming an exogenously set aggregate government good provision, optimal fiscal policy is characterised by relative (rather than absolute) sectoral interventions. Our results underscore the critical roles of government demand pass-through to prices and politically set aggregate government good provision in shaping optimal fiscal policy.

Contents

1. Introduction

Recent large asymmetric shocks, such as the 2020 pandemic and the 2022 energy-price spike, have demonstrated that sectoral heterogeneity can be relevant for aggregate dynamics. This relevance stems from differences across sectors in their exposure (Baqaee and Farhi, 2022; Guerrieri et al., 2022) and responses to shocks (Pasten et al., 2020), and in the implications that such heterogeneity has for the design of optimal policy (Aoki, 2001; Cox et al., 2024; Rubbo, 2023).

Within this last strand of the literature, Cox et al. (2024) study the interaction between optimal monetary and fiscal policy in disaggregated economies. They set up a multi-sector New-Keynesian model in which the planner jointly chooses a sector-specific pattern of government purchases and an aggregate monetary instrument. They find that fiscal policy should focus on sectors, while monetary policy should prioritise aggregate stabilisation.

Their model assumes that (i) public demand does not enter the firm's consumption good optimal pricing problem (i.e., that consumer and government prices are independent) and (ii) the social planner freely chooses aggregate and sectoral government spending.

Contribution. This paper relaxes two assumptions within the Cox et al. (2024) framework.

1. **Partial pass-through.** We drop the assumption that government and consumer prices are independent. We introduce a parameter $\kappa \in [0, 1]$ to represent the degree of pass-through of inelastic government demand to the firms' optimal private consumption pricing problem. By allowing κ to take on any generic value on the $[0, 1]$ interval, we nest the Cox et al. (2024) model for $\kappa = 0$. The introduction of inelastic government expenditure in the firms' private consumption pricing problem allows us to match both the strong empirical effects of fiscal expansions on prices documented by Ben Zeev and Pappa (2017) and Barattieri et al. (2023), and the empirical lack of support for strong procyclical markups (Nekarda and Ramey, 2013).

2. **Exogenous aggregate government good provision.** We relax the assumption that aggregate real government good provision $\bar{G}$ is completely endogenous. Specifically, we assume that changes to the aggregate government good basket from its optimal steady-state value are exogenously set. At the same time, the planner can flexibly allocate sectoral spending between sectors to reach this exogenous level. This setup allows us to account for the politically determined nature of aggregate government good provision, which Cox et al. (2024) abstract from.

Results. Relaxing the zero government expenditure **pass-through** to consumer prices assumption has two effects on the Phillips curve. First, government spending enters the Phillips curve directly: all else equal, a positive (negative) shift in government spending increases (decreases) markups. Second, as firms internalise inelastic government demand in the optimal price-setting problem, the slope of the Phillips curve increases ($\lambda_k^{'} \geq \lambda_k$). We also introduce CRRA Utilities to allow for generic degrees of risk aversion σ. We find that the optimal policy becomes less countercyclical in the output gap, to the point of changing sign in extreme specifications (price reset probability greater than 0.8). At the same time, monetary policy becomes less responsive to changes in the output gap.

By introducing exogenous aggregate **government good** provision, we are introducing an additional constraint to the Ramsey problem. Fiscal policy now becomes relative, setting sectoral government expenditure as a function of each sector's difference in inflation and output gaps with residual sector i. Monetary policy remains similar to the Cox et al. (2024) baseline, while generalising it by making the inflation objective an explicit function of sectoral government expenditure.

Roadmap Section 2 positions the work within the literature; Section 3 presents the model, derives the equilibrium conditions, and presents the log-linearised model. Section 4 solves the Ramsey problem with $\kappa \neq 0$. Section 5 solves the Ramsey problem with constrained government expenditure. Section 6 concludes.

2. Literature Review

In the canonical one-sector New-Keynesian (NK) model, monetary policy faces no tradeoff between stabilising inflation and closing the output gap; a result often referred to as the divine coincidence (Galí, 2008). In this baseline framework, keeping inflation at target automatically stabilises output at its potential. However, as noted by Galí (2008), extensions of the NK model featuring multiple sectors or additional nominal rigidities inherently generate policy tradeoffs in which pure inflation targeting is suboptimal.

2.1 Optimal policy

In a simple two-sector setup, a central bank cannot simultaneously stabilise inflation in every sector and keep output at potential. Suppose sectors experience asymmetric shocks or have different price stickiness. In that case, the monetary authority faces a choice: stabilising aggregate inflation will leave some sector-specific output gaps open, whereas closing the aggregate output gap will allow relative price disparities to emerge. As a result, "whenever the divine coincidence fails, a tradeoff emerges between stabilising inflation and closing the output gap" Rubbo (2023). For instance, Aoki (2001) shows that in a model with a flexible-price sector and a sticky-price sector, optimal policy largely ignores inflation in the flexible sector (which adjusts on its own) and concentrates on stabilising inflation in the sticky sector. This effectively amounts to targeting "core" inflation.

Index targeting. Later research has formalised the idea of designing optimal inflation indices for policy in disaggregated economies. Eusepi et al. (2011) propose a "cost-of-nominal-distortions index" (CONDI) that weights sectoral inflation rates according to each sector's contribution to welfare losses from price stickiness. Targeting the CONDI, rather than headline consumer price index (CPI) inflation, can closely replicate the welfare-optimal outcome in models with multiple sticky-price sectors. Intuitively, sectors with stickier prices or larger shares in expenditures receive greater weight in the optimal index, since price misalignments in those sectors are more harmful. Similarly, Rubbo (2023) extends these results by constructing a "divine coincidence inflation index" in a production-network model, showing that if the central bank targets this specially weighted index, it can close the aggregate output gap. Still, some misallocations persist because monetary policy has only one instrument and cannot simultaneously eliminate all relative price distortions.

Cox et al. (2024). Targeted fiscal interventions, such as sector-specific government spending or subsidies, can, in theory, address sectoral imbalances and help achieve what pure monetary policy cannot. Recent work by Cox et al. (2024) formalises this idea. In their framework, optimal policy consists of a time-consistent mix of monetary and sectoral fiscal measures that together stabilise both aggregate and sectoral gaps. Pure monetary policy on its own cannot enjoy an "aggregate divine coincidence" in a multi-sector economy, meaning it cannot close output gaps across all sectors by stabilising a single inflation index. However, jointly deploying fiscal policy at the sector level alongside monetary policy can restore a form of efficiency. Fiscal tools, such as government spending or taxes, can be tailored to specific sectors, directly affecting sectoral demand or costs. By using these tools optimally, policymakers can achieve what Cox et al. term a "disaggregated divine coincidence."

2.2 Monetary Policy Transmission in Multi-Sector Economies

Multi-sector New-Keynesian research has shown that aggregate dynamics can depend highly on sector-level characteristics. Carvalho (2006) and Bils and Klenow (2004) document dispersion in price-adjustment frequencies; this heterogeneity lengthens the persistence of monetary shocks, a result confirmed for real exchange rates by Carvalho and Nechio (2011) and for output by Nakamura and Steinsson (2010) when intermediate inputs are included. Production-network calibration by Pasten et al. (2020) indicates that a small set of sticky-price sectors drives most of the aggregate response to policy, and structural estimation by Atalay (2017) attributes about 83 per cent of U.S. GDP variance to industry-specific shocks. Analytical studies by Afrouzi and Bhattarai (2023) and Baqaee and Farhi (2022) show that sectors with negligible expenditure shares but rigid prices explain a large share of monetary non-neutrality, and that nonlinear sectoral interactions can produce aggregate demand shortfalls, as in Guerrieri et al. (2022).

2.3 Extensions

Government-demand pass-through to prices. In models with imperfect competition, an increase in government purchases can squeeze markups: in the classic framework of Rotemberg and Woodford (1992), higher government spending raises output and real wages by inducing a countercyclical decline in price markups. However, if firms instead raise prices in response to public demand (maintaining markups), the inflationary impact is larger, an outcome more likely when supply is constrained.

Traditional aggregate analyses often struggled to find the strong markup compression predicted by theory. In fact, Nekarda and Ramey (2013) find that markups are procyclical or acyclical (not countercyclical) in response to demand shocks, implying that the textbook NK mechanism (government spending lowering the average markup) may not hold quantitatively. At the same time, disaggregated studies uncover significant price effects along supply chains. Barattieri et al. (2023) use U.S. federal procurement data (2001–2019) to identify sector-specific government spending shocks. They find that a demand shock in a given industry raises prices for intermediate inputs economy-wide, as the procuring industry bids up input costs. Ghate et al. (2018) develop a three-sector New-Keynesian model of India in which the government procures food grains at a support price; this "demand shock" in the grain sector leads to higher food price inflation and labour reallocation across sectors.

The introduction of price-insensitive government spending in the firm's price-setting model allows for accommodating both results: on the one hand, prices increase more in government expenditure, as it now enters the inflation equation directly; on the other hand, markups are now less pro-cyclical, as they are increasing in the counter-cyclical G_k[1].

Government Spending with Fixed Total Provision and Flexible Composition. Ramey and Shapiro (1998) demonstrate that changes in government spending often induce large demand shifts across sectors, and a two-sector DSGE with costly factor mobility yields a much richer array of responses than an aggregate one-sector model. A key mechanism is relative price adjustment: as the government reallocates its fixed spending, the relative prices of sectoral goods change, influencing consumption and production decisions economy-wide.

Recent multi-sector models highlight that directing government purchases to certain sectors yields higher aggregate returns: for example, Bouakez et al. (2023) show that the aggregate spending multiplier is larger when government purchases originate in sectors that have a small share in private consumption (thus spare capacity) and are downstream (using inputs from many other industries). Reallocating public demand is especially valuable in response to asymmetric shocks. If one sector suffers a recession (e.g. a collapse in private demand or a supply disruption),

[1] See equation B.1

the government can compensate by purchasing more from that sector, while reducing expenditures in sectors that are performing well. This kind of targeted fiscal policy can stabilise output and employment across sectors without altering aggregate spending.

Beyond allowing us to abstract from aggregate procurement and focus on government spending shares, setting exogenous aggregate good provision allows us to consider part of the underlying political budget determination process. As Cox et al. (2024) state:

> "To be clear, we do not envision fiscal policy decisions to be taken as they are in the model. In reality, the allocation of government demand follows a complicated political process involving Congress, lobbying and stakeholders interacting in many layers, none of which is in the model. Yet, we consider useful to contrast the outcome of such a process with an optimal fiscal policy that balances its direct welfare effects with stabilisation to find, remarkably, that are not so different empirically."

Our expansion of the model goes in the direction set by Cox et al. (2024), and confirms that indeed, even after accounting for an exogenous "budget level" political process, the optimal sectoral allocation follows similar rules.

Drazen and Eslava (2010) also show that this aggregate government spending mechanism is well within political incentives. They find that political incumbents strategically allocate budget shares to sectors before elections to signal their post-election preferences. Crucially, such attempts can be successful only when done within the previously established budgets. Attempts to increase the overall budget are often met with strong resistance, making a pre-election sectoral re-distribution policy out of an exogenous budget the optimal strategy for incumbents.

CRRA Utility (with $\sigma \geq 1$) Many macroeconomic models assume moderate household risk aversion (often $\sigma \approx 1$, logarithmic utility). Here we examine the implications of high risk aversion – for instance, CRRA coefficients $\sigma \geq 2$.

One insight is that high risk aversion can strengthen the motive for stabilisation policy. When σ is large, households experience bigger utility losses from fluctuations (since the marginal utility of consumption rises steeply as consumption falls). Consequently, stabilising **consumption** and government spending becomes relatively more valuable than in the low σ case. Diercks (2015) incorporates an equity-premium-matching level of risk aversion into a NK model and finds that the Ramsey-optimal monetary policy puts greater weight on output stability than in standard models. In his calibrations, the optimal policy tolerates a higher average inflation (above 3.5%, vs $\tilde{0}$% in the baseline) and allows inflation to be more volatile if that helps stabilise the real economy. Intuitively, when recessions are very costly to risk-averse households, optimal policy becomes more "dovish," accepting some inflation in order to avoid deep output contractions.

Empirical calibration of σ in macro models often yields values in the 1–3 range, notably, Smets and Wouters (2007) estimated a medium-scale NK model and obtained a consumption elasticity parameter consistent with $\sigma \approx 1.5$ (along with habit formation). In asset pricing, resolving the equity premium puzzle requires σ on the order of 10–50 under CRRA, or alternative preferences (Epstein–Zin or habit formation) to effectively raise risk aversion without crushing IES.

3. The model

We hereby build on the multi-sector model developed in Cox et al. (2024).

The economy is made up of k sectors, each with a continuum of profit-maximising price-setting (a la Calvo) firms operating under monopolistic competition. A representative household maximises its utility by choosing present and future labour supply and consumption subject to its resource constraint. Meanwhile, the Government maximises aggregate welfare by buying goods from firms and providing them to the household. Government purchases are financed via lump-sum taxes under a balanced budget. Monetary policy maximises aggregate welfare by setting the short-term nominal interest rate.

3.1 Differences with Cox et al. (2024)

We expand the model introduced in Cox et al. (2024) in three ways:

Partial pass-through. The notion of two separate firm price-setting problems (one for government and one for private demand) stems from the focus Cox et al. (2024) put on public procurements, and is quite fitting in such a context. However, we expand our focus to all kinds of government expenditure, including the purchase of goods from the private market. We will, therefore, assume at least some degree $\kappa \geq 0$ of pass-through to consumer prices.

CRRA Utility. The macro-finance literature has highlighted how asset pricing requires extremely high risk-aversion parameters to match empirical facts. High risk aversion implies that governments should act relatively more as consumption insurers, increasing government spending when output is high and decreasing it when it is low.

Exogenous government good basket. We will drop the assumption that the planner has complete control over aggregate and sectoral spending. In our model, while the planner retains the ability to flexibly move government consumption between sectors, steady-state deviations from the aggregate government good basket, defined as $\bar{g}_t = \sum_k \omega_{g,k}^{\rho} g_{k,t}$, are set exogenously. For $\rho = 1$, changes to the aggregate government good basket become equivalent to changes to aggregate consumption, which are, therefore, exogenous.

We will abstract from the third expansion when studying the first and second in Section 4, and, conversely, we will abstract from the first and the second when studying the third in Section 5.

3.2 Setup

We begin by detailing the optimisation problem of a representative household, the decisions faced by firms, policy instruments and objectives, and equilibrium conditions.

The Household. Consider a single representative household that lives forever and obtains utility from both private and public consumption, while experiencing disutility from labour. Its objective is to maximise expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi) \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \chi \frac{G_t^{1-\sigma} - 1}{1-\sigma} - \sum_k \nu_k \frac{\left(N_{k,t}\right)^{1+\varphi}}{1+\varphi} \right], \tag{3.1}$$

where $0 < \beta < 1$ is the discount factor, $\mathbb{E}_0$ denotes the expectations operator, C_t and G_t represent composite consumption indices for private and government goods, respectively, and $N_{k,t}$ is labor supplied in period t in sector k. The parameters $\chi \in (0,1)$ and $\nu_k > 0$ govern the utility weights of public and private consumption, and of labor disutility. σ and φ are the degree of risk aversion and the curvature of labour disutility, respectively.

Private consumption, C_t, aggregates sectoral goods indexed by $k \in [0,1]$. We adopt a Cobb-Douglas structure:

$$C_t = \prod_{k=1}^{K} C_{kt}^{\omega_{ck}}, \tag{3.2}$$

where $\omega_{c,k} < 1$ is the weight associated to goods produced in sector k in the private consumption index.

Government consumption, G_t, is defined analogously:

$$G_t \;=\; \prod_{k=1}^{K} G_{kt}^{\omega_{gk}}, \tag{3.3}$$

with $\omega_{g,k}$ the weight associated to goods produced in sector k in the public consumption index.

Sectoral bundles aggregate the continuum of goods produced in sector k according to a CES structure:

$$C_{kt} = \left[\mu_k^{-\frac{1}{\theta}} \int_0^1 C_{kt}(j)^{\frac{\theta-1}{\theta}}\, dj\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad G_{kt} = \left[\mu_k^{-\frac{1}{\theta}} \int_0^1 G_{kt}(j)^{\frac{\theta-1}{\theta}}\, dj\right]^{\frac{\theta}{\theta-1}}. \tag{3.4}$$

Each sector k encompasses a continuum of firms, indexed by (j), that produce good (j) using labour (j). A sector k is defined as the set of firms producing goods indexed by $j \in J_k$ with mass μ_k such that $\sum_{k=1}^{K} \mu_k = 1$. Each sector has the same degree of elasticity of (public and private) product substitution across varieties $\theta > 1$.

At each date t, the household chooses how much of each variety $C_{k,t}$ to consume and how many hours N_t to supply, subject to its budget constraint

$$\sum_k P_{k,t} C_{k,t} \;+\; \sum_k P^G_{k,t} G_{k,t} \;+\; Q_{t-1} B_{t-1} \;=\; \sum_k W_{k,t} N_{k,t} \;+\; B_t \;+\; \Pi_t. \tag{3.5}$$

Where $P^C_{k,t}$, $P^G_{k,t}$, and $W_{k,t}$ denote, respectively, the prices consumers and the government face, and the nominal wages paid by firms, in sector k. The budget constraint shows total government expenditure $G_{k,t}$, rather than tax revenue, reflecting our assumption of a balanced budget. In this setting, Q_{t-1} represents the period-t price of a one-period discount bond B_t (maturing at $t+1$), and Π_t captures dividend payouts. $P_{k,t}(j)$ is the price of variety j, while $P_{c,t}$ is the aggregate consumer price index. Moreover, we rule out Ponzi schemes.

The resulting First Order Conditions (FOCs) are:

$$C_{k,t} = \omega_{c,k} \left(\frac{P_{k,t}}{P_t}\right)^{-1} C_t, \quad C_{k,t}(j) = \frac{1}{\mu_k} \left(\frac{P_k(j)}{P_{k,t}}\right)^{-\theta} C_{k,t}. \tag{3.6}$$

Where the consumer price index and the sector-specific price index are constructed as:

$$P_t = \prod_k \left(P_{k,t}^{\omega_{c,k}}\right), \quad P_{k,t} = \left(\int_{j\in J_k} P_{k,t}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}. \tag{3.7}$$

Sectoral labour supply satisfies:

$$\nu_k\, C_t^{\sigma}\, N_{k,t}^{\varphi} = (1-\chi) \left(\frac{W_{k,t}}{P_t}\right). \tag{3.8}$$

Finally, the household's intertemporal choice satisfies the Euler equation:

$$1 = \beta\, E_t\left[I_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_{c,t}}{P_{c,t+1}}\right]. \tag{3.9}$$

where $\beta \in (0,1)$ is the discount factor and the gross nominal interest rate is given by $I_t = 1/E_t(Q_t)$.

Firms. Firm j in sector k produces a single variety j with input labour $N_{k,t}(j)$ according to the production function

$$Y_{k,t}(j) \;=\; A_{k,t}\, N_{k,t}(j). \tag{3.10}$$

The government subsidizes labour at a constant rate τ_k.

Firms set prices à la Calvo with sector-specific adjustment incidence probability $1-\alpha_k$ to maximize:

$$\max_{P^*_{k,t}} E_t \left\{ \sum_{t=0}^{\infty} (\alpha\beta)^{\tau} \frac{C_{k,t}\, P_{k,t}}{C_{k,t+\tau}\, P_{k,t+\tau}} \left[P_{k,t}\, Y^d_{k,t+\tau|t}(j) \;-\; (1-\tau_k)\, W_{k,t+\tau}\, N^d_{k,t+\tau|t}(j) \right] \right\}.$$

Where $Y^d_{k,t+\tau|t}(j)$ is the demand at period $t+\tau$ relevant to the firm j pricing problem, and $N^d_{k,t+\tau|t}(j) \;=\; \frac{Y^d_{k,t+\tau|t}(j)}{A_{k,t}}$ is the relevant labour demand. Here, we assume a partial degree κ of pass-through from government demand to maximisation problem relevant demand, so that $Y^d_{k,t+\tau|t}(j) \;=\; C_{k,t+\tau|t}(j) \;+\; \kappa\; G_{k,t+\tau|t}(j)$. If we were to set $\kappa = 0$, we would have that $Y^d_{k,t+\tau|t}(j) \;=\; C_{k,t+\tau|t}(j)$, and fall back into the baseline setup.

Government. The government sets jointly monetary and fiscal policy to maximise total welfare by setting the nominal interest rate and real sectoral government spending, under a balanced budget.

Equilibrium conditions. Goods market clear such that:

$$Y_{k,t}(j) \;=\; C_{k,t}(j) \;+\; G_{k,t}(j), \tag{3.11}$$
$$Y_{k,t} \;=\; C_{k,t} \;+\; G_{k,t}, \tag{3.12}$$
$$N_{k,t} \;=\; \int_{j\in J} N_{k,t}(j)dj. \tag{3.13}$$

3.3 Efficient allocation

In order to find the efficient allocation, we solve the planner's problem by maximising lifetime household utility, subject to sectoral constraints, firm-specific production function, and goods market clearing conditions.

We also normalize parameters ν_k to obtain symmetric firms in steady state ($\nu_k = \mu_k^{-\varphi}$), and set the optimal government subsidy $\tau_k \;=\; \theta^{-1}$. Under such conditions, the sectoral variation in steady-state output depends entirely on μ_k. Normalising so that at the steady state[2] $N_k(j) = 1$, and assuming that $A_k = 1$:

$$N_k = \;\mu_k, \qquad Y_k = \;\mu_k, \qquad C_k = \;(1-\chi_k)\, Y_k, \qquad G_k = \;\chi_k\, Y_k, \tag{3.14}$$
$$Y \;=\; \mu, \qquad C = \;\chi^{1/\sigma}, \qquad G = \;(1-\chi)^{1/\sigma}. \tag{3.15}$$

Where $\mu \equiv (1-\chi)^{1/\sigma} + \chi^{1/\sigma}$, $\mu_k = (1-\chi)^{1/\sigma}\, \omega_{c,k} + \chi^{1/\sigma}\, \omega_{g,k}$, , $\nu_k = \mu_k^{-\varphi}$, $\chi_k \equiv \frac{\chi^{1/\sigma}\, \omega_{g,k}}{\mu_k}$, and $1-\chi_k \equiv \frac{(1-\chi)^{1/\sigma}\, \omega_{c,k}}{\mu_k}$.

The introduction of CRRA Utility changed the relationship between steady state ratios and utility weights, as efficient government spending and consumption are now the result not only of their utility weights, but also of the CRRA parameter σ. For $\sigma = 1$, we return to the log-preferences case.

[2]Where steady state values are denoted by the lack of a time index t.

3.4 Log-linearised Model

We now proceed to log-linearise the conditions obtained in the previous section:

The aggregation condition. We know from the accounting identity that

$$Y_t = C_t + G_t,$$
$$Y_{k,t} = C_{k,t} + G_{k,t},$$

Which can immediately be used to obtain the relations:

$$y_t = c_t \frac{(1-\chi)^{1/\sigma}}{\mu} + g_t \frac{\chi^{1/\sigma}}{\mu}, \tag{3.16}$$
$$y_{k,t} = c_{k,t}\, \chi_k + g_{k,t}\,(1-\chi_k). \tag{3.17}$$

Since we also know from consumer optimising behaviour that

$$c_{k,t} \;=\; c_t + p_t - p_{k,t}.$$

We can infer the first of our constraints:

$$\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t} \;=\; \frac{y_t\, \mu}{(1-\chi)^{1/\sigma}} - \chi^*\, g_t + p_t - p_{k,t}. \tag{3.18}$$

Equation (3.18) shows the fundamental link between sectoral and aggregate variables. This condition implies that sectoral consumption depends only on relative prices and aggregate consumption. In the extreme case of a continuum of infinitely small sectors with infinitely rigid prices, the sectoral fiscal multiplier will be equal to χ_k.

The sectoral Phillips curve. as we show in Appendix B:

$$\pi_{k,t} \;=\; \beta E_t[\pi_{k,t+1}] \;+\; \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k\left(1 - \dfrac{\kappa}{\theta-1}\,\chi_k^*\right)}\left[\psi_{k,t} - a_{k,t+\tau} + \frac{\kappa}{\theta-1}\,\chi_k^* g_{k,t}\right],$$

Where, by setting $\kappa = 0$, the expression simplifies to the standard sectoral Phillips Curve:

$$\pi_{k,t} \;=\; \beta\, E_t[\pi_{k,t+1}] \;+\; \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(\psi_{k,t} - a_{k,t+\tau}).$$

κ enters the Phillips curve in two ways. The first is directly as a determinant of inflation: since inelastic government demand enters the firm's optimal pricing problem, the higher the government demand, the higher the markups and, therefore, the prices firms can charge for the same marginal cost. The second is indirectly as a determinant of the Phillips Curve slope λ_k: the introduction of inelastic government demand makes overall demand less elastic, allowing firms to react more strongly to changes in marginal costs or government spending.
The labour supply condition can be log-linearised to:

$$\psi_{k,t} = (\sigma - 1)\left(\frac{y_t\, \mu}{(1-\chi)^{1/\sigma}} - \chi^*\, g_t\right) + y_{k,t}(1 + \chi_k^* + \varphi) - \chi_k^*\, g_{k,t} - \varphi\, a_{k,t}.$$

CRRA preferences lead to another deviation from the baseline. Since utility is not separable in sectoral consumptions, the Marginal Product of Consumption and inflation now depend on both aggregate and sectoral variables. Substituting for the real wage ψ in the Phillips Curve:

$$\pi_{k,t} = \beta\, \pi_{k,t+1} + \lambda'_k(\sigma-1)\left(\frac{y_t\, \mu}{(1-\chi)^{1/\sigma}} - \chi^*\, g_t\right) + \lambda_k\, y_{k,t}(1+\chi^*_k+\varphi) - \lambda'_k\, \chi^*_k\left(1-\frac{\kappa}{\theta-1}\right) g_{k,t} - \lambda_k\, \varphi a_{k,t}. \tag{3.19}$$

CRRA preferences have introduced a further deviation from the baseline model: sectoral inflation depends on aggregate variables.

Identities. We start with aggregation identities, specifically for output, government spending and prices.

$$G_t = \prod_k G_{k,t}^{\omega_{g,k}}, \quad C_t = \prod_k C_{k,t}^{\omega_{c,k}}, \quad Y_t = C_t + G_t, \quad P_t = \prod P_t^{\omega_{c,k}}.$$

Which yield, once log-linearised:

$$g_t = \sum_k \omega_{g,k}\, g_{k,t}, \tag{3.20}$$

$$c_t = \sum_k \omega_{c,k}\, c_{k,t}, \tag{3.21}$$

$$y_t = \sum_k \frac{(1-\chi)^{1/\sigma}}{\mu}\, \omega_{c,k} c_{k,t} + \frac{\chi^{1/\sigma}}{\mu}\, \omega_{g,k} g_{k,t} = \sum_k \frac{\mu_k}{\mu} y_{k,t}, \tag{3.22}$$

$$p_t = \sum_k \omega_{c,k}\, c_{k,t}. \tag{3.23}$$

According to the definition of inflation, we can also write:

$$p_{k,t} = p_{k,t-1} + \pi_{k,t}. \tag{3.24}$$

The Euler equation. The approximation of the Euler equation yields:

$$\mu \Delta\, y_{t+1} = (1-\chi)^{1/\sigma}\left(i_t - \pi_{t+1} + \log(\beta)\right) + \chi^{1/\sigma} \Delta g_{t+1}. \tag{3.25}$$

In both the maximisation problems ahead, we will assume that the government sets the optimal sectoral inflation. This will determine the optimal aggregate inflation, and through the Euler equation (together with optimal output), will set the interest rate consistent with the optimal allocation.

Welfare objective. The government wants to maximise aggregate welfare. The objective will be equal to the second-order approximation of the household's utility. Notice that we assume shocks to be mean-zero and uncorrelated across sectors, allowing us to focus on insurance motives and obtain a within-sector demand comparable to that in Cox et al. (2024).

Under such assumptions, we obtain in Appendix C the Welfare objective:

$$-\frac{1}{2}\sum_k \mu_k \Bigg(\left[(1+\varphi)\, y_{k,t}^2 + \frac{\theta(1-\chi)}{\lambda_k}\, \pi_{k,t}^2 + \chi^*_k\, (g_{k,t} - y_{k,t})^2\right] + (\sigma-1)\left[(1-\chi_k)\, \omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi^*_k\, g_{k,t}\right)^2 + \omega_{g,k}\, \chi_k\, g_{k,t}^2\right]\Bigg). \tag{3.26}$$

Which, again, simplifies to the standard expression under log-utilities ($\sigma = 1$)

4. Optimal policy with $\kappa \neq 1$

We can now solve for the optimal policy by maximising (3.26) subject to (3.18), (3.19), (3.20), (3.22), (3.23), and (3.24). The full solution is shown in Appendix D.

4.1 Optimal Fiscal Policy

The optimal fiscal policy under non-zero government demand pass-through on consumer prices and CRRA preferences is:

$$g_{k,t} = \frac{H_{k,t}}{X_{k,t}}\, y_{k,t} + \frac{J_{k,t}}{X_{k,t}}\, a_{k,t} - \frac{\theta}{\lambda_k\, X_{k,t}} \pi_{k,t} + \frac{\sigma - 1}{(1-\chi)^{\frac{1}{\sigma}}\, X_{k,t}} \sum_k \mu_k \lambda_k \phi^{\pi}_{k,t}\,,$$

Where:

$$\begin{aligned}
H_{k,t} &= \chi^*_k + 1 + \varphi + (\sigma - 1)\tfrac{\omega_{c,k}}{1-\chi_k} - \varphi\lambda_k \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\,\varphi + \frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}\,, \\
J_{k,t} &= -\,(1+\varphi)\left(1 - \lambda_k \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\,\varphi + \frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}\right), \\
X_{k,t} &= \chi^*_k + (\sigma-1)\,\chi^*_k\,\omega_{c,k}\,\chi_k + \Big(1 + (\sigma-1)\,\omega_{g,k}\Big)\left(\lambda_k + \frac{1+\chi^*_k+\varphi}{\lambda_k\,\varphi + \frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}\right), \\
\phi^{\pi}_{k,t} &= \Big(-\,y_{k,t}\,\varphi \;-\; g_{k,t}\Big(1 + (\sigma-1)\,\omega_{g,k}\Big) \;+\; (1+\varphi)\,a_{k,t}\Big)\left(\lambda_k\,\varphi \;+\; \frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}\right)^{-1},
\end{aligned}$$

$X_{k,t} > 0\ \forall\ \sigma, \kappa$, and under $\kappa = 0\ \wedge\ \sigma = 1$ this expression simplifies to the one in Cox et al. (2024).

Let us now focus on the case of a symmetric two-sector economy, with the only asymmetry being that sector 1 has almost flexible prices $\alpha_1 = 0.2$, while sector 2 has rigid prices $\alpha_2 = 0.9$.

Sectoral information. We first consider a government that only acts on sectoral information, ignoring aggregation indices ν. In this fictional, simplified economy, optimal fiscal policy in sector 1, for any $\sigma > 1.7$, will be **pro-cyclical**, that is, when output grows, optimal fiscal policy will entail increasing government spending.

We can show this by simplifying to the case where $\kappa = 0 \wedge \sigma > 1$. The coefficient $H_{k,t}$ simplifies to:

$$(\sigma - 1)\,\frac{\omega_{c,k}}{1-\chi_k} - \frac{1}{\lambda_k\,(1-\chi_k)}\,,$$

Which is negative in sector one if and only if

$$\sigma < 1 + \frac{1}{\omega_{c,1}\,\lambda_1} < 1.7\,.$$

The reason for the sign inversion is that when introducing risk aversion in the Household objective, we make the household more averse to expected changes in both private and government consumption. Let us look back at the three roles of government:

1. Allocation of government goods
2. Stabilisation

3. **Insurance**

Increasing risk aversion σ means increasing the relative weight of the insurance motive vis-a-vis the optimal allocation and stabilisation motives.

Now, consider an output contraction (expansion) under constant inflation. As income decreases (increases), disposable income follows suit. If the government acts by increasing (decreasing) government spending, it will fulfil its obligations to stabilise output, but fail to ensure consumption smoothing. The crux of the problem is in the elements resulting from the expansion of the consumption term in welfare. During a crisis, a government may increase its spending, but in doing so, it crowds out consumption, disrupting consumption patterns. The higher the risk aversion $(\sigma - 1)$ vis-a-vis the stabilisation effect $\left(\frac{1}{\omega_{c,k}\,\lambda_k}\right)$, the higher the disutility of counter-cyclical policies.

Complete information. Let us now consider a government that acts on complete information. We set $\kappa = 1$, $\theta = 6$, and $\chi_1^* = 1$. Under this specification, λ_1 is equal to 4, as $\kappa > 0$ steepens the Phillips curve.

The overall coefficient on $y_{k,t}$ now becomes (by adding the relevant component of ν):

$$\left(\frac{1}{(\theta-1)\,(1-\chi_k)\,\varphi+1}\right)\left(\frac{1}{1-\chi_k}+\varphi+(\sigma-1)\,\frac{\omega_{c,k}}{1-\chi_k}-\frac{(\theta-1)\,(1-\chi_k)\,\varphi}{(1-\chi_k)\,\lambda_k}\right),$$

Which is negative in sector one if and only if

$$\sigma \;>\; \left(\frac{(\theta-1)\,(1-\chi_k)\,\varphi}{\lambda_k}\;-\;1\;-\;\varphi\,(1-\chi_k)\right)\frac{1}{\omega_{c,k}}+1\,.$$

or $\sigma \;\geq 1.25$ under the current specification.

$\kappa \neq 0$. In the full-information scenario, the departure from $\kappa = 0$ is essential, as otherwise all terms would cancel, leaving exactly the same $H_{k,t}$ as in Cox et al. (2024). That is to say, in a scenario with relatively high price flexibility, the assumption of independent consumer and government price-setting is quite demanding. It can lead to very different results through two channels:

- **Direct:** government expenditure has a direct effect on prices beyond the contribution to output.
- **Indirect:** Inelastic government expenditure leads to a steeper Phillips Curve.

The second point stems directly from the Phillips Curve derivation: if inelastic government demand enters the price-setting problem, the same increase in price will lead to a smaller drop in demand, because a non-negligible part of total demand for a good (government demand) will not react to the price change. This allows firms to respond more strongly to changes in real wage (which in turn depends on consumption) and leads to a larger λ_k.

The change in the Phillips curve slope is not marginal. One can easily show that:

$$\lambda_{|\kappa=1} = \lambda_{|\kappa=0}\,\frac{\theta-1}{\theta-1-\chi_k^*}\,,$$

Which, under $\theta = 6$, and $\chi_k^* = 1$, makes the contribution of inelastic government spending to λ equal to 25%.

4.2 Optimal Monetary Policy

We now turn to the optimal monetary policy. As shown in Appendix D, optimal monetary policy entails setting the aggregate inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k}\pi_{k,t} = \\ -\sum_k \mu_k \left(y_{k,t}\,\varphi + g_{k,t}\Big(1+(\sigma-1)\,\omega_{g,k}\Big) + (1+\varphi)\,a_{k,t}\right)\left(\lambda_k\,\varphi + \tfrac{\kappa}{\theta-1}\,\tfrac{\lambda_k}{1-\chi_k}\right)^{-1}.$$

As in Cox et al. (2024), the optimal monetary policy entails setting a weighted price index where sectoral inflation levels are given larger weight the higher the steady state share of private consumption on sectoral output $(1-\chi_k)$ and the lower the Phillips curve slope (λ_k).

As for the effect of our two extensions, positive government demand pass-through to prices $(\kappa > 0)$ will lead, all else equal, to lower responsiveness of the inflation target to sectoral gaps in output, government spending, and productivity. At the same time, allowing for high risk aversion $(\sigma > 2)$ will lead to a higher responsiveness of optimal sectoral inflation to government spending.

Intuitively, the direct pass-through of government spending to prices decreases the planner's tolerance for inflation deviations, as it becomes less costly to bridge price distortions. At the same time, high risk aversion strengthens the insurance motive and makes government policy less counter-cyclical. Monetary policy should then give relatively higher weight to government spending in setting counter-cyclical monetary policy.

5. Optimal Constrained Policy

Next, we introduce a straightforward modification: the government cannot freely adjust its spending each period. Instead, at the beginning of each period, it exogenously determines the change in the total amount of government-provided goods required by the household.

In this revised framework, the government is bound to supply a fixed quantity, denoted by $\bar{G}$, which differs from the simple G previously discussed. Specifically, $\bar{G}$ represents a composite basket of sectoral goods aggregated using a CES (Constant Elasticity of Substitution) aggregator with elasticity parameter ρ. In log-linearised terms, this relationship is expressed as:

$$\bar{g}_t = \sum_k \omega_{g,k}^{1+\rho}\, g_{k,t},$$

here, $\bar{g}_t$ denotes the exogenous log-deviation in the provision of the government's goods basket. All other aspects of the model are the same as in the baseline, as we now abstract from government pass-through and CRRA utilities.

5.1 Optimal Fiscal Policy

As demonstrated in Appendix E, optimal provision involves selecting one sector as the residual (indexed by i) and setting spending in all other sectors as a function of their deviations in output and inflation relative to sector i.

The resulting optimal spending rule is given by:

$$g_{k,t} = \tfrac{1-\chi_k}{1-\chi_i}\left(\tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\right)\left(\tfrac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\right)^{-1} g_{i,t} \\ - \tfrac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \tfrac{\theta\,\varphi(1-\chi_k)\,\pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ + \tfrac{1-\chi_k}{1-\chi_i}\left(\tfrac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\tfrac{1+\lambda_k+\varphi\,\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\right)^{-1}\left(+\tfrac{\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}y_{i,t} + \tfrac{\theta\,\varphi\,(1-\chi_i)}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\pi_{i,t}\right),$$

Where

$$
\begin{aligned}
g_{i,t} = &+ \bar{g}_t \left(\tfrac{1}{1-\chi_i} \tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)} \tfrac{1}{\omega_{g,i}^{\rho}} \sum_{k\neq i} \left(\tfrac{1}{1-\chi_k} \tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)} \tfrac{1}{\omega_{g,k}^{1+2\rho}} \right)^{-1} + \omega_{g,i}^{1+\rho} \right)^{-1} \\
&- \left(+ \tfrac{\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)} y_{i,t} + \tfrac{\theta(1-\chi_i)\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)} \pi_{i,t} \right) \\
&\times \left(\tfrac{1}{1-\chi_i} \tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)} \tfrac{1}{\omega_{g,i}^{\rho}} \sum_{k\neq i} \left(\tfrac{1}{1-\chi_k} \tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)} \tfrac{1}{\omega_{g,k}^{1+2\rho}} \right)^{-1} + \omega_{g,i}^{1+\rho} \right)^{-1} \\
&\times \left(\tfrac{1}{1-\chi_i} \tfrac{1}{\omega_{g,i}^{\rho}} \sum_{k\neq i} \left(\tfrac{1}{1-\chi_k} \tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)} \tfrac{1}{\omega_{g,k}^{1+2\rho}} \right)^{-1} \right) \\
&- \left(\sum_{k\neq i} \tfrac{\varphi}{1+\lambda_k+\varphi\,\lambda_k} \omega_{g,k}^{1+\rho} y_{k,t} - \tfrac{\theta(1-\chi_k)\varphi}{1+\lambda_k+\varphi\,\lambda_k} \omega_{g,k}^{1+\rho} \pi_{k,t} \right) \\
&\times \left(\tfrac{1}{1-\chi_i} \tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)} \tfrac{1}{\omega_{g,i}^{\rho}} \sum_{k\neq i} \left(\tfrac{1}{1-\chi_k} \tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)} \tfrac{1}{\omega_{g,k}^{1+2\rho}} \right)^{-1} + \omega_{g,i}^{1+\rho} \right)^{-1}.
\end{aligned}
$$

The optimal policy now becomes a balancing act, setting relative government expenditure while maintaining a set government basket provision. Still, overall rules remain similar to the ones shown in Cox et al. (2024), the main difference is that budget considerations force the government to make a relative decision, and base optimal sectoral policy not only on sectoral variables, but on relative performance compared to other sectors. If $\rho = 0$, the model simplifies to a world where government good provision is exogenous, while sectoral good provision remains under government control.

This setup also allows for the study of the optimal allocation of exogenous changes in government provision of goods. Imagine a new government taking office after elections. This model shows that even after a choice is made to provide more (or less) of the same basket of government goods, there is an optimal policy for distributing these additional resources.

Such optimal fiscal policy, in this case as in Cox et al. (2024), involves allocating relatively higher government spending to sectors with lower inflation and output log deviations.

5.2 Optimal Monetary Policy

As the constraint we introduce binds exclusively fiscal policy, it should be no surprise that the optimal monetary policy would be very similar to the one portrayed in Cox et al. (2024).

$$
\begin{aligned}
&\sum_k \mu_k \frac{\theta\,(1-\chi_k)}{\lambda_k} \frac{\lambda_k + \varphi\,\lambda_k\,(1-\chi_k)}{1+\lambda_k+\varphi\,\lambda_k\,(1-\chi_k)} \pi_{k,t} = \\
&\quad \sum_k \mu_k \left(\frac{\chi_k\, g_{k,t}}{1+\lambda_k+\varphi\,\lambda_k\,(1-\chi_k)} - \frac{y_{k,t}\,(1-\chi_k)\,(1+\varphi+\chi_k^*)}{1+\lambda_k+\varphi\,\lambda_k\,(1-\chi_k)} \right).
\end{aligned}
$$

However, a difference emerges: given the more complicated optimal fiscal policy, we cannot as easily substitute in for $g_{k,t}$ to reach a pure output target, leading to a slightly more complex version.

As in Cox et al. (2024), the optimal monetary policy entails setting a weighted price index where sectoral inflation levels are given larger weight the higher the steady state share of private consumption on sectoral output $(1-\chi_k)$ and the lower the Phillips curve slope (λ_k).

6. Conclusion

Summary of contributions. This paper revisits Ramsey-optimal policy in a multi-sector New-Keynesian model by (i) allowing partial pass-through of government demand to consumer prices and (ii) imposing an exogenous aggregate government-goods constraint.

Relaxing the zero government pass-through assumption yields three main results:

(i) **Phillips-curve amplification.** Introducing pass-through steepens the sectoral Phillips curve by the factor $\frac{\theta-1}{\theta-1-\chi_k^*}$; under a calibration of $\theta = 6$ and $\chi_k^* = 1$ the slope rises by 25 %.

(ii) **Shift in fiscal cyclicality.** Introducing pass-through also weakens stabilisation motives, making optimal fiscal policy less counter-cyclical. In a two-sector symmetric economy with complete pass-through $\kappa = 1$, low price rigidity $\alpha_k = 0.2$, and $\chi_k^* = 1$, the output-gap coefficient in the fiscal rule changes sign for any $\sigma \geq 1.25$, producing pro-cyclical spending in the more price-flexible sector.

(iii) **Re-weighting of monetary targets.** Optimal monetary policy remains a weighted-inflation rule. The weight on sectoral government spending in determining the aggregate inflation target is increasing in risk aversion σ, and the sensitivity of the optimal inflation target to changes in sectoral variables is decreasing in κ,

Introducing an exogenous level of government good provision, while allowing for flexible sectoral reallocation, leads to **relative fiscal rules.** When aggregate $\bar{g}_t$ is constrained, optimal fiscal policy becomes *relative*: for any residual sector i,

$$g_{k,t} - \delta_i\, g_{i,t} = \eta_\pi(\pi_{k,t} - \delta_i^\pi\, \pi_{i,t}) + \eta_y(y_{k,t} - \delta_i^y\, y_{i,t}), \qquad k \neq i,$$

Where the coefficients $\eta_\pi, \eta_y, \delta_i, \delta_i^\pi, \delta_i^y$ depend only on $(\lambda_k, \rho, \varphi)$ and steady state ratios, and are independent of the level of g_t. Monetary policy is virtually unchanged, if not for the need to internalise a different optimal fiscal policy.

Limitations. The model abstracts from input–output production linkages, capital accumulation, and distortions in labour or sales taxation; it also treats the aggregate budget as exogenous rather than endogenising the underlying decision process.

Future research. Extending the framework to networks of intermediate inputs would reveal whether the steepening of the Phillips curve under $\kappa > 0$ is amplified or dampened by upstream propagation. Endogenising the budget process could explain how electoral incentives interact with optimal relative spending rules. Finally, calibrating the model to micro evidence on government procurement elasticities would help quantitatively pin down welfare effects.

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A. Optimal efficient allocation

$$U_0 \;=\; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi)\, \frac{C_t^{1-\sigma}}{1-\sigma} \;+\; \chi\, \frac{G_t^{1-\sigma}}{1-\sigma} \;-\; \sum_{k=1}^{K} \nu_k\, \frac{(C_{k,t}+G_{k,t})^{\,1+\varphi}}{A_{k,t}^{1+\varphi}\,(1+\varphi)} \right],$$

with $0<\beta<1$, $\sigma>0$ (CRRA), $\varphi>0$, $\nu_k>0$.

$$C_t \;=\; \prod_{k=1}^{K} (\omega_{c,k}^{-1} C_{k,t})^{\omega_{c,k}}, \quad G_t \;=\; \prod_{k=1}^{K} (\omega_{g,k}^{-1} G_{k,t})^{\omega_{g,k}}, \quad \sum_k \omega_{c,k} = \sum_k \omega_{g,k} = 1.$$

$$\text{FOC for } C_{k,t}: \qquad \omega_{c,k}\,(1-\chi)\, C_t^{-\sigma}\, \frac{C_t}{C_{k,t}} = \frac{\nu_k}{A_{k,t}}\, N_{k,t}^{\varphi},$$

$$\text{FOC for } G_{k,t}: \qquad \omega_{g,k}\,\chi\, G_t^{-\sigma}\, \frac{G_t}{G_{k,t}} = \frac{\nu_k}{A_{k,t}}\, N_{k,t}^{\varphi}.$$

We set at the efficient steady state, that $\nu_k = \mu_k^{-\varphi}$, such that $N_k(j) = 1$, and all firms are symmetric. For simplicity, we also assume that at the efficient steady state $A_k = 1$.

We can then find that:

$$N_k \;=\; Y_k \;=\; \mu_k \tag{A.1}$$

$$C_k \;=\; \omega_{c,k}\, C \tag{A.2}$$

$$C \;=\; (1-\chi)^{\frac{1}{\sigma}}, \quad G \;=\; \chi^{\frac{1}{\sigma}} \tag{A.3}$$

$$C_k \;=\; \omega_{c,k}(1-\chi)^{\frac{1}{\sigma}} \;=\; \chi_k\, \mu_k \tag{A.4}$$

$$G_k \;=\; \omega_{g,k}\chi^{\frac{1}{\sigma}} \;=\; (1-\chi_k)\, \mu_k \tag{A.5}$$

$$\mu_k \;=\; \omega_{c,k}(1-\chi)^{\frac{1}{\sigma}} \;+\; \omega_{g,k}\chi^{\frac{1}{\sigma}} \tag{A.6}$$

$$Y \;=\; \mu \;=\; (1-\chi)^{\frac{1}{\sigma}} \;+\; \chi^{\frac{1}{\sigma}} \tag{A.7}$$

Equation (A.1) follows from the normalisation of labour disutility parameters ν_k and the assumption of a linear production function at the firm level, which implies that labour input, output, and sector size all equal μ_k in steady state.
Equation (A.2) is derived from equation (3.6), evaluated at the efficient steady state. It reflects the sectoral consumption share, given by the consumption weights $\omega_{c,k}$.
Equation (A.3) substitutes (A.2) into the first-order conditions for consumption, and derives the closed-form expressions for aggregate private and public consumption levels.
Equations (A.4) and (A.5) use the definitions of sector-specific shares χ_k and $(1-\chi_k)$, together with the goods market clearing conditions, to map sectoral consumption and government demand to sector size μ_k.
Equation (A.6) sums the two components of demand (private and public) using their respective weights and provides a decomposition of sector size in terms of preference parameters and consumption weights.
Finally, equation (A.7) defines aggregate output as the sum of the weighted private and public consumption contributions, consistent with the resource constraint.

B. Phillips Curve derivation

Profit Maximization Problem The firm maximises the future discounted sum of profits:

$$\max_{P_{k,t}(j)} \quad E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \Big[P_{k,t}(j)\, Y^d_{k,t+\tau}(j) \; - \; W_{k,t+\tau}\, N^d_{k,t+\tau}(j) \Big].$$

Here, $\beta^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}}$ is the stochastic discount factor for nominal payoffs, $\alpha_k \in (0,1)$ is the Calvo-style probability of not resetting prices (so $(\alpha_k)^{\tau}$ is the probability the price set at t is still active at $t+\tau$), $Y^d_{k,t+\tau}(j)$ is demand for the j-indexed good in sector k at time $t+\tau$, $N^d_{k,t+\tau}$ is labour used to produce output $Y^d_{k,t+\tau}(j)$, and $W_{k,t+\tau}$ is the nominal wage in sector k.

Demand Functions The firm faces demand composed of consumption and government components within its sector k. We assume that the total demand the firm faces is:

$$Y^d_{k,t+\tau}(j) \; = \; C_{k,t+\tau}(j) \; + \; \kappa\, G_{k,t+\tau}(j).$$

Where $C_{k,t+\tau}(j)$ represents the consumption demand component (and $C_{k,t+\tau}(j) = C_{k,t+\tau}(P_{k,t}(j)/P_{k,t+\tau})^{-\theta}$), $G_{k,t+\tau}(j)$ is the government demand component, and κ captures the fraction of inelastic government expenditure the firm "internalises" in price-setting. Note that κ can alternatively be interpreted as the degree to which government goods prices affect consumption goods pricing.

First-Order Condition Denote the chosen price by $P_{k,t}(j)^*$. The first-order condition (FOC) with respect to $P_{k,t}(j)^*$ is:

$$E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \Bigg[P_{k,t}(j)^* \frac{\partial Y^d_{k,t+\tau}(j)}{\partial P_{k,t}(j)^*} \; + \; Y^d_{k,t+\tau}(j) \\ - \; (1-\tau_k)\, \frac{W_{k,t+\tau}}{A_{k,t+\tau}} \frac{\partial Y^d_{k,t+\tau}(j)}{\partial P_{k,t}(j)^*} \Bigg] = 0.$$

Substituting $\dfrac{\partial Y^d_{k,t+\tau}(j)}{\partial P_{k,t}(j)^*} = C_{k,t+\tau}(j)\big(-\frac{\theta}{P_{k,t}(j)^*}\big)$ (assuming only consumption demand is price-elastic with elasticity θ), we obtain:

$$E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \big[(1-\theta)\, C_{k,t+\tau}(j) + \kappa G_{k,t+\tau}(j) \big] \; = \\ E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \left[-\,\theta\,(1-\tau_k)\, \frac{W_{k,t+\tau}}{P_{k,t}(j)^*} \frac{C_{k,t+\tau}(j)}{A_{k,t+\tau}} \right].$$

Defining the desired markup as $M = \frac{\theta}{\theta-1}$, knowing that $(1-\tau_k) = \frac{\theta-1}{\theta}$ and multiplying both sides by $\frac{P_{k,t}(j)^*}{P_{k,t}}$, we get:

$$E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \left[C_{k,t+\tau}(j)\, \frac{P_{k,t}(j)^*}{P_{k,t}} \right] = \\ E_t \sum_{\tau=0}^{\infty} (\beta\alpha_k)^{\tau} \frac{P_{k,t}\, C_{k,t}}{P_{k,t+\tau}\, C_{k,t+\tau}} \left[\frac{W_{k,t+\tau}}{P_{k,t+\tau}} \frac{P_{k,t+\tau}}{P_{k,t}} \frac{C_{k,t+\tau}(j)}{A_{k,t+\tau}} \; + \; \frac{\kappa}{\theta-1}\, G_{k,t+\tau}(j)\, \frac{P_{k,t}(j)^*}{P_{k,t}} \right].$$

Loglinearising around a symmetric steady state, where $\frac{P_k(j)^*}{P_k} = 1$ and $\frac{W_k}{P_k} = \frac{1}{M}$, and defining the real wage $\Psi_{k,t+\tau} = \frac{W_{k,t+\tau}}{P_{k,t+\tau}}$ yields:

$$E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau} \Big[p_{k,t}(j)^* - p_{k,t}\Big] =$$
$$E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau} \Big[\psi_{k,t+\tau} - a_{k,t+\tau} + p_{k,t+\tau} - p_{k,t} + B_k\big(g_{k,t+\tau}(j) + p_{k,t}(j)^* - p_{k,t}\big)\Big],$$

Where hats denote log-deviations from steady state and

$$B_k \;=\; \frac{\kappa}{\theta-1}\chi_k^*.$$

From Infinite Sum to Difference Equation Using the fact that for a variable $x_{t+\tau}$,

$$E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau} \sum_{i=0}^{\tau-1} x_{t+1+i} \;=\; \frac{\alpha_k\,\beta}{1-\alpha_k\,\beta} E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau}\, x_{t+1+\tau},$$

And applying it to the inflation term $\pi_{k,t+1+\tau} = p_{k,t+1+\tau} - p_{k,t+\tau}$ arising from $p_{k,t+\tau} - p_{k,t} = \sum_{i=0}^{\tau-1} \pi_{k,t+1+i}$, we can rewrite the infinite-sum expression as

$$\big(p_{k,t}(j)^* - p_{k,t}\big) = \frac{1-\alpha_k\beta}{1-B_k}\, E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau} \left[\psi_{k,t+\tau} - a_{k,t+\tau} + B_k\, g_{k,t+\tau}(j)\right] +$$
$$+\; \frac{\alpha_k\beta}{1-B_k}\, E_t \sum_{\tau=0}^{\infty} (\beta\,\alpha_k)^{\tau} \pi_{k,t+1+\tau}.$$

Finally, converting this to difference form (by quasi-differencing, i.e., $X_t = (1-\alpha_k\beta)Y_t + \alpha_k\beta E_t[X_{t+1}]$ where $X_t = E_t \sum (\beta\alpha_k)^{\tau} Y_{t+\tau}$):

$$\big(p_{k,t}(j)^* - p_{k,t}\big) \;= \alpha_k\beta\, E_t\big[p_{k,t+1}(j)^* - p_{k,t+1}\big] +$$
$$+\; \frac{1-\alpha_k\beta}{1-B_k}\left[\psi_{k,t} - a_{k,t+\tau} + B_k\, g_{k,t}(j)\right] + \frac{\alpha_k\beta}{1-B_k}\, E_t\big[\pi_{k,t+1}\big].$$

Final Reduced-Form Inflation Equation Using the standard Calvo relation between the optimal reset price gap and aggregate inflation $\big(p_{k,t}(j)^* - p_{k,t}\big) = \dfrac{\alpha_k}{1-\alpha_k}\,\pi_{k,t}$ and substituting back for B_k gives an inflation equation of the form:

$$\hat{\pi}_{k,t} \;=\; \beta E_t[\hat{\pi}_{k,t+1}] \;+\; \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k\Big(1 - \dfrac{\kappa}{\theta-1}\,\chi_k^*\Big)} \Big[\hat{\psi}_{k,t} - \hat{a}_{k,t+\tau} + \frac{\kappa}{\theta-1}\,\chi_k^* g_{k,t}\Big]. \tag{B.1}$$

C. A Second–Order Approximation to Household Welfare

This appendix derives a *second–order* Taylor expansion of the representative household's period utility around the efficient, stationary steady state. The calculation generalises Woodford (2003) along two dimensions: (i) K production sectors, and (ii) sector-specific price dispersion. Throughout the derivation, we flag by *t.i.p.* any constants that drop out of welfare comparisons.

Notation. For any stationary variable X_t with steady state X we write

$$\hat{x}_t \equiv \log\frac{X_t}{X} \quad\Longrightarrow\quad \frac{X_t - X}{X} = \hat{x}_t + \tfrac{1}{2}\hat{x}_t^2 + O(\hat{x}_t^3). \tag{C.1}$$

Preferences. Period utility is

$$U_t = (1-\chi)u(C_t) + \chi u(G_t) - \sum_{k=1}^{K} \nu_k v(N_{k,t}), \tag{C.2}$$

with $u(X) = \frac{X^{1-\sigma}}{1-\sigma}$ and $v(N) = \frac{N^{1+\varphi}}{1+\varphi}$, where $\sigma, \varphi > 0$.

Derivatives at the steady state Let $f_X \equiv \partial f / \partial X$. Evaluated at $X_t = X$ we have

$$u_X(X) = X^{-\sigma}, \qquad u_{XX}(X) = -\sigma\, X^{-\sigma-1}, \tag{C.3}$$
$$v_N(N) = N^{\varphi}, \qquad v_{NN}(N) = \varphi\, N^{\varphi-1}. \tag{C.4}$$

Hence

$$U_C = (1-\chi)C^{-\sigma}, \qquad U_{CC} = -(1-\chi)\sigma\, C^{-\sigma-1}, \tag{C.5}$$
$$U_G = \chi\, G^{-\sigma}, \qquad U_{GG} = -\chi\sigma\, G^{-\sigma-1}, \tag{C.6}$$
$$U_{N_k} = -\nu_k\, N_k^{\varphi}, \qquad U_{N_k N_k} = -\nu_k \varphi\, N_k^{\varphi-1}. \tag{C.7}$$

Separable utility ensures $U_{CG} = U_{CN_k} = U_{GN_k} = 0$; thus no cross-second derivatives appear in what follows.

Raw second–order expansion A multivariate Taylor expansion of (C.2) around the steady state, combined with (C.1) and (C.7), yields

$$\begin{aligned} U_t - U &= U_C C\big[\hat{c}_t + \tfrac{1}{2}(1-\sigma)\hat{c}_t^2\big] + U_G G\big[\hat{g}_t + \tfrac{1}{2}(1-\sigma)\hat{g}_t^2\big] \\ &\quad + \sum_{k=1}^{K} U_{N_k} N_k \big[\hat{n}_{k,t} + \tfrac{1}{2}(1+\varphi)\hat{n}_{k,t}^2\big] + \text{t.i.p.} \end{aligned} \tag{C.8}$$

Labour and price dispersion Sector k comprises a continuum (mass μ_k) of firms charging $P_{k,t}(j)$. Labour demand aggregates as

$$N_{k,t} = \frac{Y_{k,t}}{A_{k,t}}\, Z_{k,t}, \qquad Z_{k,t} \equiv \int_{J_k} \left(\tfrac{P_{k,t}(j)}{P_{k,t}}\right)^{-\theta} dj. \tag{C.9}$$

Linearising gives $\hat{n}_{k,t} = \hat{y}_{k,t} - \hat{a}_{k,t} + \hat{z}_{k,t}$, with the *price-dispersion lemma* (Woodford, 2003)

$$\hat{z}_{k,t} = \tfrac{\theta}{2}\, \mathrm{var}_j\{\hat{p}_{k,t}(j)\}, \qquad \hat{p}_{k,t}(j) \equiv \log \tfrac{P_{k,t}(j)}{P_{k,t}}. \tag{C.10}$$

Normalising by marginal utility of consumption Set

$$\Lambda \equiv U_C C = (1-\chi)C^{1-\sigma}. \tag{C.11}$$

After dividing (C.8) by Λ and substituting (C.4)–(C.10), we obtain

$$\frac{U_t - U}{\Lambda} = \hat{c}_t + \tfrac{1}{2}(1-\sigma)\hat{c}_t^2 + \gamma_G\big[\hat{g}_t + \tfrac{1}{2}(1-\sigma)\hat{g}_t^2\big] - \sum_k \gamma_{N_k}\big[\hat{n}_{k,t} + \tfrac{1}{2}(1+\varphi)\hat{n}_{k,t}^2\big], \tag{C.12}$$

with dimensionless multipliers

$$\gamma_G \equiv \frac{U_G G}{\Lambda} = \frac{\chi}{1-\chi}\left(\frac{G}{C}\right)^{1-\sigma}, \qquad \gamma_{N_k} \equiv \frac{-U_{N_k} N_k}{\Lambda} = \frac{\nu_k\, N_k^{1+\varphi}}{(1-\chi)C^{1-\sigma}}. \tag{C.13}$$

Knowing that $n_{k,t} = y_{k,t} + (1-\chi_k) z_{k,t}$, and that $z_{k,t}^2 \approx 0$

$$\begin{aligned}\frac{U_t - U}{\Lambda} =& \hat{c}_t + \tfrac{1}{2}(1-\sigma)\hat{c}_t^2 + \gamma_G\big[\hat{g}_t + \tfrac{1}{2}(1-\sigma)\hat{g}_t^2\big] \\ & - \sum_k \gamma_{N_k}\big[\tilde{y}_{k,t} \,+\, (1-\chi_k)\, z_{k,t} + \tfrac{1}{2}\,(1+\varphi)\,\tilde{y}_{k,t}^2\big],\end{aligned}$$

Steady-state values The planner chooses (ν_k) so that, in the efficient steady state, (i) all firms are symmetric, (ii) $A_k = 1$, and (iii) $N_k = Y_k = \mu_k$. These choices imply

$$C = (1-\chi)^{1/\sigma}, \quad G = \chi^{1/\sigma}, \qquad C_k = \omega_{c,k} C = \chi_k \mu_k. \tag{C.14}$$

Plugging (C.14) into (C.13) gives

$$\gamma_G = \frac{\chi}{1-\chi}\,\frac{\chi}{1-\chi}^{\frac{1-\sigma}{\sigma}} \;=\; \frac{\chi}{1-\chi}^{\frac{1}{\sigma}} \equiv \chi^*, \qquad \gamma_{N_k} = \frac{\mu_k}{(1-\chi)\,(1-\chi)^{\frac{1-\sigma}{\sigma}}} = \mu_k(1-\chi)^{-\frac{1}{\sigma}}.$$

Consumption expansion. Now let us consider the expansion in consumption and output:

$$\begin{aligned}\frac{C_{k,t}}{C_k} &= \frac{Y_{k,t} - G_{k,t}}{C_k} = \frac{Y_{k,t}}{Y_k}\cdot\frac{Y_k}{C_k} - \frac{G_{k,t}}{G_k}\cdot\frac{G_k}{C_k} \\ &= \frac{1}{1-\chi_k}\cdot\left(1 + y_{k,t} + \frac{1}{2}\,y_{k,t}^2\right) - \chi_k^*\left(1 + g_{k,t} + \frac{1}{2}\,g_{k,t}^2\right) \\ &= 1 + c_{k,t} + c_{k,t}^2\cdot\frac{1}{2}\,,\end{aligned}$$

$$c_{k,t} = \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right) + \frac{1}{2}\left(\frac{1}{1-\chi_k}\right)^2\left(y_{k,t}^2 - \chi_k\,g_{k,t}^2\right) - \frac{1}{2}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right)^2,$$

$$c_{k,t}^2 = \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right)^2.$$

Combining this result with the definition of c_t we get:

$$c_t + \frac{1}{2}(1-\sigma)\,c_t^2 = \sum_k \omega_{c,k}\,c_{k,t} + \frac{1}{2}(1-\sigma)\sum_k\sum_n \omega_{c,n}\,\omega_{c,k}\,c_{n,t}\,c_{k,t}$$

$$\begin{aligned}c_t + \frac{1}{2}(1-\sigma)\,c_t^2 &= \sum \omega_{c,k}\,c_{k,t} + \frac{1}{2}(1-\sigma)\,\sum \omega_{c,k}^2\,c_{k,t}^2 \\ &= \sum \omega_{c,k}\Bigg[\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right) + \frac{1}{2}\left(\frac{1}{1-\chi_k}\,y_{k,t}^2 - \chi_k^*\,g_{k,t}^2\right) \\ &\quad - \frac{1}{2}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right)^2 + \frac{1}{2}(1-\sigma)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right)^2\Bigg].\end{aligned}$$

Where we have dropped cross-terms, as we assume that sectoral shocks are uncorrelated, and we now drop linear term, as they have zero mean at the steady state.

$$\begin{aligned}\sum \omega_{c,k}\frac{1}{2}\Bigg[&\frac{y_{k,t}^2}{1-\chi_k} - \chi_k^*\,g_{k,t}^2 - \left(\frac{y_{k,t}}{1-\chi_k}\right)^2 - (\chi_k^*\,g_{k,t})^2 + \\ & - 2\frac{\chi_k^*\,y_{k,t}\,g_{k,t}}{(1-\chi_k)} + (1-\sigma)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\,g_{k,t}\right)^2\Bigg]\end{aligned}$$

$$\sum \omega_{c,k} \frac{1}{2}\left[\frac{y_{k,t}^2}{1-\chi_k}\left(1-\frac{1}{1-\chi_k}\right) - \chi_k^*(1+\chi_k^*)\, g_{k,t}^2 + 2\,\frac{\chi_k^*}{1-\chi_k}\, y_{k,t}\, g_{k,t} + (1-\sigma)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t}\right)^2\right]$$

$$\sum \omega_{c,k} \frac{1}{2}\left[-\frac{\chi_k^*}{1-\chi_k}\, y_{k,t}^2 - \frac{\chi_k^*}{1-\chi_k}\, g_{k,t}^2 + 2\,\frac{\chi_k^*}{1-\chi_k}\, g_{k,t}\, y_{k,t} + {} + (1-\sigma)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t}\right)^2\right]$$

$$\sum \omega_{c,k} \frac{1}{2}\left[-\frac{\chi_k^*}{1-\chi_k}\,(y_{k,t}-g_{k,t})^2 + (1-\sigma)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t}\right)^2\right].$$

Final expression. Plugging back in the overall welfare objective, rearranging some steady state ratios, and normalising the objective by $(1-\chi)^{1/\sigma}$ yields:

$$-\frac{1}{2}\sum_k \mu_k\Bigg(\left[(1+\varphi)\,(y_{k,t}-a_{k,t})^2 + \frac{\theta(1-\chi)}{\lambda_k}\,\pi_{k,t}^2 + \chi_k^*\,(g_{k,t}-y_{k,t})^2\right] + (\sigma-1)\left[(1-\chi_k)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t}\right)^2 + \omega_{g,k}\,\chi_k\, g_{k,t}^2\right]. \tag{C.15}$$

D. Optimal Fiscal policy with $\kappa \neq 1$

We begin by defining the Lagrangian of the social planner's problem: the social planner maximises the present-value welfare subject to the disaggregation constraint, the New-Keynesian Phillips Curve (NKPC), the price-indexing identity, and the three aggregation identities.

$$\begin{aligned}
\mathcal{L} = -\sum_{t=0}^{\infty}\beta^t\Big\{\sum_{k=1}^{K}\mu_k\Big[\frac{1}{2}\Big(&(g_{k,t}-y_{k,t})^2\,\chi_k^* + (1+\varphi)\,y_{k,t}^2 + \frac{\theta\,(1-\chi_k)}{\lambda_k}\,\pi_{k,t}^2 + {}\\
&+(\sigma-1)\left[(1-\chi_k)\,\omega_{c,k}\left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t}\right)^2 + \omega_{g,k}\,\chi_k\, g_{k,t}^2\right]\\
&+\phi_{k,t}^{y}\Big(-\tfrac{y_{k,t}}{1-\chi_k} + \chi_k^* g_{k,t} + \tfrac{y_t\,\mu}{(1-\chi)^{1/\sigma}} - \chi^* g_t - p_{k,t} + p_t\Big)\\
&+\phi_{k,t}^{\pi}\Big(-\pi_{k,t} + \beta\,\pi_{k,t+1} + \lambda_k(\sigma-1)\Big(\tfrac{y_t\,\mu}{(1-\chi)^{1/\sigma}} - \chi^* g_t\Big)\\
&\qquad + \lambda_k y_{k,t}(1+\chi_k^*+\varphi) - \lambda_k\chi_k^*\Big(1-\tfrac{\kappa}{\theta-1}\Big)g_{k,t}\Big)\\
&+\phi_{k,t}^{p}\big(-p_{k,t} + p_{k,t-1} + \pi_{k,t}\big)\Big]\Big]\\
&+\nu_{y,t}\Big(-y_t\,\mu + \sum_{k=1}^{K}\mu_k y_{k,t}\Big) + \nu_{p,t}\Big(-p_t + \sum_{k=1}^{K}\omega_{c,k} p_{k,t}\Big)\\
&+\nu_{g,t}\Big(-g_t + \sum_{k=1}^{K}\omega_{g,k} g_{k,t}\Big)\Big\}.
\end{aligned} \tag{D.1}$$

First-Order Conditions. The necessary optimality conditions are obtained by differentiating (D.1) with respect to each choice variable:

$$\begin{aligned}
(\mathrm{FOC})_{y_{k,t}}: \quad & -\chi_k^*(g_{k,t}-y_{k,t})+(1+\varphi)(y_{k,t}-a_{k,t})-(1-\sigma)\omega_{c,k}\Big(\tfrac{y_{k,t}}{1-\chi_k}-\chi_k^*\,g_{k,t}\Big)\\
& -\phi^y_{k,t}\,\tfrac{1}{1-\chi_{k,t}}+\phi^\pi_{k,t}\,\lambda'_k\,(1+\chi_k^*+\varphi)+\nu_{y,t}=0,\\
(\mathrm{FOC})_{g_{k,t}}: \quad & \chi_k^*(g_{k,t}-y_{k,t})-(1-\sigma)\Big(-\chi_k\,\omega_{c,k}\big(\tfrac{y_{k,t}}{1-\chi_k}-\chi_k^*\,g_{k,t}\big)+\omega_{g,k}\,\chi_k g_{k,t}\Big)\\
& +\phi^y_{k,t}\,\chi_k^*-\phi^\pi_{k,t}\,\lambda'_k\,\chi_k^*\Big(1-\tfrac{\kappa}{\theta-1}\Big)+\nu_{g,t}\,\tfrac{\omega_{g,k}}{\mu_k}=0,\\
(\mathrm{FOC})_{\pi_{k,t}}: \quad & \tfrac{\theta\,(1-\chi_k)}{\lambda'_k}\,\pi_{k,t}-\phi^\pi_{k,t}+\phi^p_{k,t}=0,\\
(\mathrm{FOC})_{p_{k,t}}: \quad & -\phi^y_{k,t}-\phi^p_{k,t}+\nu_{p,t}\,\tfrac{\omega_{c,k}}{\mu_k}=0,\\
(\mathrm{FOC})_{y_t}: \quad & \sum_{k=1}^K\mu_k\,\phi^y_{k,t}\,\tfrac{1}{(1-\chi)^{1/\sigma}}+\sum_{k=1}^K\mu_k\,\phi^\pi_{k,t}\,\lambda'_k\,(\sigma-1)\,\tfrac{1}{(1-\chi)^{1/\sigma}}-\nu_{y,t}=0,\\
(\mathrm{FOC})_{g_t}: \quad & -\sum_{k=1}^K\mu_k\,\phi^y_{k,t}\,\chi^*+\sum_{k=1}^K\mu_k\,\phi^\pi_{k,t}\,\lambda'_k\,(\sigma-1)\,\chi^*-\nu_{g,t}=0,\\
(\mathrm{FOC})_{p_t}: \quad & \sum_{k=1}^K\mu_k\,\phi^y_{k,t}-\nu_{p,t}=0.
\end{aligned}$$

Condensed System. Combining and rearranging the above FOCS yields the following reduced–form equations:

$$\begin{aligned}
& y_{k,t}\Big(\chi_k^*+1+\varphi-(1-\sigma)\tfrac{\omega_{c,k}}{1-\chi_k}\Big)-g_{k,t}\Big(\chi_k^*+(\sigma-1)\,\chi_k^*\omega_{c,k}\Big)-(1+\varphi)\,a_{k,t}\\
& \quad -\tfrac{\phi^y_{k,t}}{1-\chi_k}+\phi^\pi_{k,t}\,\lambda'_k\,(1+\chi_k^*+\varphi)+\nu_{y,t}=0,
\end{aligned}\tag{D.2a}$$

$$\begin{aligned}
& -y_{k,t}\Big(\chi_k^*-(1-\sigma)\,\chi_k^*\,\omega_{c,k}\Big)+g_{k,t}\Big(\chi_k^*+(\sigma-1)\,\chi_k^*\,\omega_{c,k}\,\chi_k+(\sigma-1)\,\omega_{g,k}\,\chi_k\Big)\\
& \quad +\phi^y_{k,t}\,\chi_k^*-\phi^\pi_{k,t}\,\lambda_k\,\chi_k^*\Big(1-\frac{\kappa}{\theta-1}\Big)+\nu_{g,t}\,\frac{\omega_{g,k}}{\mu_k}=0,
\end{aligned}\tag{D.2b}$$

$$\frac{\theta\,(1-\chi_k)}{\lambda'_k}\,\pi_{k,t}-\phi^\pi_{k,t}-\phi^y_{k,t}+\nu_{p,t}\,\frac{\omega_{c,k}}{\mu_k}=0,\tag{D.2c}$$

$$\nu_{p,t}=\sum_k\mu_k\,\phi^y_{k,t},\qquad \nu_{g,t}=-\chi^{-1/\sigma}\,\nu_{y,t}.\tag{D.2d}$$

Optimal Fiscal Policy. Combining (D.2a) and (D.2b), we can solve for $\phi^\pi_{k,t}$:

$$\begin{aligned}
& y_{k,t}\Big(\chi_k^*+1+\varphi-(1-\sigma)\,\tfrac{\omega_{c,k}}{1-\chi_k}-\tfrac{1}{1-\chi_k}+(1-\sigma)\tfrac{\omega_{c,k}}{1-\chi_k}\Big)\\
& \quad -g_{k,t}\Big(\chi_k^*+(\sigma-1)\,\chi_k^*\,\omega_{c,k}-\tfrac{1}{1-\chi_k}-(\sigma-1)\,\chi_k^*\omega_{c,k}-(\sigma-1)\,\omega_{g,k}\Big)\\
& \quad -(1+\varphi)\,a_{k,t}+\phi^\pi_{k,t}\,\lambda_k\Big(1+\chi_k^*+\varphi-\tfrac{1}{1-\chi_k}+\tfrac{\kappa}{\theta-1}\,\tfrac{1}{1-\chi_k}\Big)\\
& \quad +\nu_{y,t}\Big(1-\frac{G_k}{G}\cdot\frac{Y}{Y_k}\cdot\frac{G}{Y}\cdot\frac{Y_k}{G_k}\Big)=0,
\end{aligned}$$

$$y_{k,t}\Big(\tfrac{\chi_k}{1-\chi_k}+\tfrac{1-\chi_k}{1-\chi_k}-\tfrac{1}{1-\chi_k}+\varphi\Big)-g_{k,t}\Big(\tfrac{\chi_k^*}{1-\chi_k}-\tfrac{1}{1-\chi_k}-(\sigma-1)\,\omega_{g,k}\Big)$$

$$- \;(1+\varphi)\, a_{k,t} \;+\; \phi^{\pi}_{k,t}\, \lambda_k \Big(\tfrac{1-\chi_k+\chi_k-1}{1-\chi_k} \;+\; \varphi \;+\; \tfrac{\kappa}{\theta-1}\, \tfrac{1}{1-\chi_k} \Big) \;=\; 0,$$

$$\phi^{\pi}_{k,t} \;=\; \Big(- y_{k,t}\, \varphi \;-\; g_{k,t} \Big(1 + (\sigma-1)\, \omega_{g,k} \Big) \;+\; (1+\varphi)\, a_{k,t} \Big) \Big(\lambda_k\, \varphi \;+\; \tfrac{\kappa}{\theta-1}\, \tfrac{\lambda_k}{1-\chi_k} \Big)^{-1}. \quad \text{(D.3)}$$

Combining (D.2a) and (D.2c) then yields:

$$\begin{aligned} & y_{k,t} \Big(\chi^*_k + 1 + \varphi - (1-\sigma) \tfrac{\omega_{c,k}}{1-\chi_k} \Big) - g_{k,t} \Big(\chi^*_k + (\sigma-1)\, \chi^*_k\, \omega_{c,k}\, \chi_k \Big) - \;(1+\varphi)\, a_{k,t} \\ & + \; \frac{\theta}{\lambda_k}\, \pi_{k,t} + \; \phi^{\pi}_{k,t}\, \lambda_k \; \left(1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)} \right) \;+\; \nu_{y,t} - \; \frac{\nu_{p,t}}{(1-\chi)^{1/\sigma}} \;=\; 0. \end{aligned}$$

Where by plugging in (D.3)

$$\begin{aligned} & y_{k,t} \Big(\chi^*_k + 1 + \varphi - (1-\sigma) \tfrac{\omega_{c,k}}{1-\chi_k} \Big) - g_{k,t} \Big(\chi^*_k + (\sigma-1)\, \chi^*_k\, \omega_{c,k}\, \chi_k \Big) - (1+\varphi)\, a_{k,t} \\ & + \lambda_k \left(- y_{k,t}\, \varphi - \; g_{k,t} \Big(1 + (\sigma-1)\, \omega_{g,k} \Big) \;+\; (1+\varphi)\, a_{k,t} \right) \cdot \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \\ & - \; \frac{\theta}{\lambda_k}\, \pi_{k,t} + \nu_{y,t} - \; \frac{\nu_{p,t}}{(1-\chi)^{1/\sigma}} = 0, \end{aligned} \quad \text{(D.4)}$$

$$\begin{aligned} & y_{k,t} \left(\chi^*_k + 1 + \varphi - (1-\sigma) \tfrac{\omega_{c,k}}{1-\chi_k} - \varphi \lambda_k\, \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \right) - \; \frac{\theta}{\lambda_k}\, \pi_{k,t} \\ & - g_{k,t} \left(\chi^*_k + \Big(1 + (\sigma-1)\, \omega_{g,k} \Big) \left(\lambda_k + \frac{1 + \chi^*_k + \varphi}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \right) \right), \\ & - (1+\varphi)\, a_{k,t} \left(1 - \lambda_k\, \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \right) \;+\; \nu_{y,t} - \; \frac{\nu_{p,t}}{(1-\chi)^{1/\sigma}} = 0. \end{aligned} \quad \text{(D.5)}$$

Finally:

$$g_{k,t} \;=\; \frac{H_{k,t}}{X_{k,t}}\, y_{k,t} + \; \frac{J_{k,t}}{X_{k,t}}\, a_{k,t} - \; \frac{\theta}{\lambda_k\, X_{k,t}} \pi_{k,t} + \; \frac{\sigma-1}{(1-\chi)^{\frac{1}{\sigma}}\; X_{k,t}} \sum_k \mu_k \lambda_k \phi^{\pi}_{k,t},$$

Where:

$$\begin{aligned} H_{k,t} \;&= \chi^*_k + 1 + \varphi + (\sigma-1)\, \tfrac{\omega_{c,k}}{1-\chi_k} - \varphi \lambda_k\, \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}}, \\ J_{k,t} \;&= \; - \,(1+\varphi) \; \left(1 - \lambda_k\, \frac{1 + \chi^*_k + \varphi + \frac{1}{\lambda_k\,(1-\chi_k)}}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \right), \\ X_{k,t} \;&= \chi^*_k + \Big(1 + (\sigma-1)\, \omega_{g,k} \Big) \left(\lambda_k + \frac{1 + \chi^*_k + \varphi}{\lambda_k\, \varphi + \frac{\kappa}{\theta-1}\, \frac{\lambda_k}{1-\chi_k}} \right), \\ \phi^{\pi}_{k,t} \;&= \Big(- y_{k,t}\, \varphi \;-\; g_{k,t} \Big(1 + (\sigma-1)\, \omega_{g,k} \Big) \;+\; (1+\varphi)\, a_{k,t} \Big) \Big(\lambda_k\, \varphi \;+\; \tfrac{\kappa}{\theta-1}\, \tfrac{\lambda_k}{1-\chi_k} \Big)^{-1}. \end{aligned}$$

Optimal Monetary Policy. Since we know that $\sum_k (1-\chi_k)\, \mu_k\, \nu_{y,t} = (1-\chi)^{-\sigma} \sum_k \omega_{c,k}\, \nu_{y,t} = (1-\chi)^{-1}\, \nu_{y,t}$, we can sum (D.2a) over all k and substitute in for $\phi_{k,t}$ from (D.3) to obtain the optimal monetary policy:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k}\,\pi_{k,t} \;=\; -\sum_k \mu_k \Big(y_{k,t}\,\varphi \;+\; g_{k,t}\Big(1+(\sigma-1)\,\omega_{g,k}\Big) \;+\; (1+\varphi)\,a_{k,t}\Big)\Big(\lambda_k\,\varphi \;+\; \tfrac{\kappa}{\theta-1}\,\tfrac{\lambda_k}{1-\chi_k}\Big)^{-1}. \tag{D.6}$$

E. Optimal Constrained fiscal policy

The Lagrangian under the proposed setup can be written schematically as:

$$\begin{aligned}
L = \; -\sum_{t=0}^{\infty}\beta^t \Bigg(& \sum_k \mu_k \Bigg\{ \tfrac{1}{2}\Bigg[\tfrac{\theta\,(1-\chi_k)}{\lambda_k}\pi_{k,t}^2 \;+\; (1+\phi)\,y_{k,t}^2 \;+\; \chi_k^*\left(g_{k,t}-y_{k,t}\right)^2\Bigg] \\
& + \;\phi^y_{k,t}\Bigg[-y_{k,t}+y_t+\chi_k^*\left(g_{k,t}-y_{k,t}\right)-\chi^*\left(g_t-y_t\right)-\left(p_{k,t}-p_{c,t}\right)\Bigg] \\
& + \;\phi^\pi_{k,t}\Bigg[-\pi_{k,t}+\beta\,E_t\,\pi_{k,t+1}+\lambda_k\,(1+\phi)\,y_{k,t}-\lambda_k\,\chi_k^*\left(g_{k,t}-y_{k,t}\right)\Bigg] \\
& + \;\phi^p_{k,t}\Bigg[-p_{k,t}+\pi_{k,t}+p_{k,t+1}\Bigg]\Bigg\} \\
& + \;v_{y,t}\Bigg[-y_t+\sum_k \mu_k\,y_{k,t}\Bigg] \\
& + v_{p,t}\Bigg[-p_{c,t}+\sum_k \omega_{c,k}\,p_{k,t}\Bigg] \\
& + v_{g,t}\Bigg[-g_t+\sum_{k\neq i}\omega_{g,k}\,g_{k,t}+\omega_{g,i}\,\frac{\overline{g}_t-\sum_{k\neq i}\omega_{g,k}^{1+\rho}\,g_{k,t}}{\omega_{g,i}^{1+\rho}}\Bigg] \\
& + \tfrac{1}{2}\,\mu_i\,x_i^*\Bigg[-\left(g_{i,t}-y_{i,t}\right)^2+\Bigg(\frac{\overline{g}_t-\sum_{k\neq i}\omega_{g,k}^{1+\rho}\,g_{k,t}}{\omega_{g,i}^{1+\rho}}\;-\;y_{i,t}\Bigg)^2\Bigg] \\
& + \;\mu_i\,\phi^y_{i,t}\,\chi_i^*\Bigg[-g_{i,t}\;+\;\frac{\overline{g}_t-\sum_{k\neq i}\omega_{g,k}^{1+\rho}\,g_{k,t}}{\omega_{g,i}^{1+\rho}}\Bigg] \\
& + \;\mu_i\,\phi^\pi_{i,t}\,\lambda_i\,\chi_i^*\Bigg[g_{i,t}-\frac{\overline{g}_t-\sum_{k\neq i}\omega_{g,k}^{1+\rho}\,g_{k,t}}{\omega_{g,i}^{1+\rho}}\Bigg]\Bigg).
\end{aligned}$$

First Order Conditions. Taking FOCS:

Case: $K \neq i$

$$\begin{aligned}
\mathrm{FOC}_{y_k}: \quad & (1+\phi)\,y_{k,t} \;-\; \chi_k^*\left(g_{k,t}-y_{k,t}\right) \;-\; \phi^y_{k,t} \;-\; \phi^y_{k,t}\,\chi_k^* \;+\; \phi^\pi_{k,t}\,\lambda_k\,(1+\phi)\;+ \\
& \;+\; \phi^\pi_{k,t}\,\lambda_k\,\chi_k^* \;+\; v_{y,t} \;=\; 0, \\
\mathrm{FOC}_{g_k}: \quad & \chi_k^*\left(g_{k,t}-y_{k,t}\right) \;+\; \phi^y_{k,t}\,\chi_k^* \;-\; \phi^\pi_{k,t}\,\lambda_k\,\chi_k^* \;+\; \frac{v_{g,t}}{\mu_k}\left(\omega_{g,k}-\tfrac{\omega_{g,k}^{1+\rho}}{\omega_{g,i}^{\rho}}\right)
\end{aligned}$$

$$- \tfrac{\mu_i}{\mu_k} \chi_i^* \tfrac{\omega_{g,k}^{1+\rho}}{\omega_{g,i}^{1+\rho}} \left(g_{i,t} - y_{i,t}\right) - \phi_{i,t}^y \tfrac{\mu_i}{\mu_k} \chi_i^* \tfrac{\omega_{g,k}^{1+\rho}}{\omega_{g,i}^{1+\rho}} + \phi_{i,t}^\pi \tfrac{\mu_i}{\mu_k} \chi_i^* \tfrac{\omega_{g,k}^{1+\rho}}{\omega_{g,i}^{1+\rho}} \lambda_i = 0,$$

$$\text{FOC}_{\pi_k}: \quad \tfrac{\theta\,(1-\chi_k)}{\lambda_k}\, \pi_{k,t} + \phi_{k,t}^p - \phi_{k,t}^\pi = 0,$$

$$\text{FOC}_{p_k}: \quad - \phi_{k,t}^p - \phi_{k,t}^y + \frac{\omega_{c,k}}{\mu_k}\, \upsilon_{p,t} = 0,$$

$$\text{FOC}_{y}: \quad \sum_k \phi_{k,t}^y\, \mu_k\, (1+\chi^*) - \upsilon_{y,t} = 0,$$

$$\text{FOC}_{p_t}: \quad \sum_k \mu_k\, \phi_{k,t}^y - \upsilon_{p,t} = 0,$$

$$\text{FOC}_{g}: \quad - \sum_k \mu_k\, \phi_{k,t}^y\, \chi^* - \upsilon_{g,t} = 0,$$

Case: $K = i$

$$\text{FOC}_{y_i}: \quad (1+\phi)\, y_{i,t} - \phi_{i,t}^y - \phi_{i,t}^y\, \chi_i^* + \phi_{i,t}^\pi\, \lambda_i\, (1+\varphi) + \phi_{i,t}^\pi\, \lambda_i\, \chi_i^* + \\ + \upsilon_{y,t} - \chi_i^*(g_{i,t} - y_{i,t}) = 0,$$

$$\text{FOC}_{\pi_i}: \quad \tfrac{\theta\,(1-\chi_i)}{\lambda_i}\, \pi_{i,t} - \phi_{i,t}^\pi + \phi_{i,t}^p = 0,$$

$$\text{FOC}_{p_i}: \quad - \phi_{i,t}^p - \phi_{i,t}^y + \frac{\omega_{c,i}}{\mu_i}\, \upsilon_{p,t} = 0.$$

Condensed system. We will be mainly working with the following set of equations obtained from manipulations of the previous set:

$$- \frac{\theta}{\lambda_k}\, \pi_{k,t} + \frac{\phi_{k,t}^\pi}{1-\chi_k} + \frac{\phi_{k,t}^y}{1-\chi_k} - \upsilon_{y,t} = 0, \tag{E.1a}$$

$$(1+\varphi+\chi_k^*)\,(y_{k,t} + \phi_{k,t}^\pi\, \lambda_k) - \chi_k^*\, g_{k,t} - \phi_{k,t}^y\, \tfrac{1}{1-\chi_k} + \upsilon_{y,t} = 0, \tag{E.1b}$$

$$\chi_k^*(g_{k,t} - y_{k,t}) - \tfrac{\chi_k}{1-\chi_i}(g_{i,t} - y_{i,t}) \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} + \phi_{k,t}^y \chi_k^* - \phi_{i,t}^y \tfrac{\chi_k}{1-\chi_i} \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} \\ - \phi_{k,t}^\pi \lambda_k \chi_k^* + \phi_{i,t}^\pi \lambda_i \tfrac{\chi_k}{1-\chi_i} \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} - \chi_k \upsilon_{y,t} \left(1 - \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho}\right) = 0, \tag{E.1c}$$

$$\upsilon_{p,t} = -\upsilon_{g,t}\, \tfrac{1-\chi}{\chi} = \upsilon_{y,t}\, (1-\chi). \tag{E.1d}$$

Where we used:

$$\tfrac{\omega_{c,i}}{\mu_i} = \tfrac{C_k}{C} \tfrac{Y}{Y_k} = \tfrac{1-\chi_k}{1-\chi},$$

$$\tfrac{\omega_{g,i}}{\mu_i} = \tfrac{G_k}{G} \tfrac{Y}{Y_k} = \tfrac{\chi_k}{\chi},$$

$$\tfrac{\mu_i}{\mu_k} \chi_i^* \tfrac{\omega_{g,k}}{\omega_{g,i}} = \tfrac{Y_i}{Y_k} \tfrac{G_i}{C_i} \tfrac{G_k}{G_i} = \tfrac{G_k}{Y_k} \tfrac{Y_i}{C_i} = \tfrac{\chi_k}{1-\chi_i}.$$

Optimal Fiscal policy. We can combine (E.1a) and (E.1b) to obtain:

$$\phi_{k,t}^\pi = \frac{1-\chi_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)} \left(\chi_k^* g_{k,t} - y_{k,t}(1+\phi+\chi_k^*) + \frac{\theta}{\lambda_k}\pi_{k,t}\right). \tag{E.2}$$

Plugging (E.1a) into (E.1c) we get:

$$\tfrac{1}{1-\chi_k}\left(g_{k,t} - y_{k,t} + \tfrac{\theta}{\lambda_k}\pi_k\right) - \tfrac{1}{1-\chi_i}\left(g_{i,t} - y_{i,t} + \tfrac{\theta}{\lambda_k}\pi_i\right) \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} - \phi_{k,t}^\pi \tfrac{1+\lambda_k}{1-\chi_k} + \phi_{i,t}^\pi \tfrac{1+\lambda_i}{1-\chi_i} \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} = 0.$$

Substituting in for $\phi_{k,t}^\pi$ from (E.2):

$$\tfrac{1}{1-\chi_k}\left(g_{k,t} - y_{k,t} + \tfrac{\theta}{\lambda_k}\pi_k\right) \tfrac{1}{1-\chi_i}\left(g_{i,t} - y_{i,t} + \tfrac{\theta}{\lambda_i}\pi_i\right) \tfrac{\omega_{g,k}^\rho}{\omega_{g,i}^\rho} +$$

$$
\begin{aligned}
&+ \frac{1+\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\left(\chi_i^* g_{i,t} - y_{i,t}(1+\phi+\chi_i^*) + \frac{\theta}{\lambda_i}\pi_{i,t}\right)\frac{\omega_{g,k}^{\rho}}{\omega_{g,i}^{\rho}} + \\
&- \frac{1+\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\left(\chi_k^* g_{k,t} - y_{k,t}(1+\phi+\chi_k^*) + \frac{\theta}{\lambda_k}\pi_{k,t}\right) \;=\; 0.
\end{aligned}
$$

Which we solve for $g_{k,t}$:

$$
\begin{aligned}
g_{k,t} \;=\;& \tfrac{1-\chi_k}{1-\chi_i}\left(\tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\right)\left(\tfrac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\right)^{-1} g_{i,t} \\
&- \tfrac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \tfrac{\theta\,\varphi(1-\chi_k)\,\pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\
&+ \tfrac{1-\chi_k}{1-\chi_i}\left(\tfrac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\tfrac{1+\lambda_k+\varphi\,\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\right)^{-1}\left(+\tfrac{\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}y_{i,t} + \tfrac{\theta\,\varphi\,(1-\chi_i)}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\pi_{i,t}\right).
\end{aligned}
\tag{E.3}
$$

We now sum over $g_{k,t}\,\omega_{g,k}^{1+\rho}$ and solve for $g_{i,t}$:

$$
\begin{aligned}
g_{i,t} =& +\overline{g}_t\left(\tfrac{1}{1-\chi_i}\,\tfrac{1+\lambda_i+\varphi\,\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\,\tfrac{1}{\omega_{g,i}^{\rho}}\sum_{k\neq i}\left(\tfrac{1}{1-\chi_k}\,\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\,\tfrac{1}{\omega_{g,k}^{1+2\rho}}\right)^{-1} + \omega_{g,i}^{1+\rho}\right)^{-1} \\
&- \left(+\tfrac{\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}y_{i,t} + \tfrac{\theta(1-\chi_i)\varphi}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\pi_{i,t}\right) \\
&\times\left(\tfrac{1}{1-\chi_i}\,\tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\,\tfrac{1}{\omega_{g,i}^{\rho}}\sum_{k\neq i}\left(\tfrac{1}{1-\chi_k}\,\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\,\tfrac{1}{\omega_{g,k}^{1+2\rho}}\right)^{-1} + \omega_{g,i}^{1+\rho}\right)^{-1} \\
&\times\left(\tfrac{1}{1-\chi_i}\,\tfrac{1}{\omega_{g,i}^{\rho}}\sum_{k\neq i}\left(\tfrac{1}{1-\chi_k}\,\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\,\tfrac{1}{\omega_{g,k}^{1+2\rho}}\right)^{-1}\right) \\
&- \left(\sum_{k\neq i}\tfrac{\varphi}{1+\lambda_k+\varphi\,\lambda_k}\omega_{g,k}^{1+\rho}y_{k,t} - \tfrac{\theta(1-\chi_k)\varphi}{1+\lambda_k+\varphi\,\lambda_k}\omega_{g,k}^{1+\rho}\pi_{k,t}\right) \\
&\times\left(\tfrac{1}{1-\chi_i}\,\tfrac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\phi\lambda_i(1-\chi_i)}\,\tfrac{1}{\omega_{g,i}^{\rho}}\sum_{k\neq i}\left(\tfrac{1}{1-\chi_k}\,\tfrac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\phi\lambda_k(1-\chi_k)}\,\tfrac{1}{\omega_{g,k}^{1+2\rho}}\right)^{-1} + \omega_{g,i}^{1+\rho}\right)^{-1}.
\end{aligned}
\tag{E.4}
$$

Or, in simpler terms:

$$
g_{k,t} \;=\; g_{i,t}\,A_{i,t}^{g} \;-\; y_{k,t}\,A_{y,t} \;-\; \pi_{k,t}\,A_{\pi,t} \;+\; y_{i,t}\,A_{i,t}^{y} \;+\; \pi_{i,t}\,A_{i,t}^{\pi},
\tag{E.5}
$$

Equivalently:

$$
\begin{aligned}
g_{k,t} \;&=\; g_{i,t}\,A_{i,t}^{g} \;-\; A_{y,t}\left(y_{k,t} \;-\; \frac{A_{i,t}^{g}}{A_{y,t}}\,y_{i,t}\right) \;-\; A_{\pi,t}\left(\pi_{k,t} \;-\; \frac{A_{i,t}^{\pi}}{A_{\pi,t}}\,\pi_{i,t}\right) \;-\; (A_{k,t} \;-\; A_{i,t}), \\
g_{i,t} \;&=\; -\,\overline{g}_t\,B_{i,t}^{g} \;-\; y_{i,t}\,B_{i,t}^{y} \;-\; \pi_{i,t}\,B_{i,t}^{\pi} \;+\; \sum_k B_{k,t}^{0}\left(B_{k,t}^{y}\,y_{k,t} \;+\; B_{k,t}^{\pi}\,\pi_{k,t}\right).
\end{aligned}
$$

Optimal Monetary Policy. Since we know that $\sum_k(1-\chi_k)\,\mu_k\,\nu_{y,t} = (1-\chi)^{-1}\sum_k \omega_{c,k}\,\nu_{y,t} = (1-\chi)^{-1}\,\nu_{y,t}$, we can sum (E.1a) over all k and substitute in for $\phi_{k,t}$ from (E.2) to obtain the optimality condition:

$$
\sum_k \mu_k\,\frac{\theta\,(1-\chi_k)}{\lambda_k}\,\frac{\lambda_k+\varphi\,\lambda_k\,(1-\chi_k)}{1+\lambda_k+\varphi\,\lambda_k\,(1-\chi_k)}\,\pi_{k,t} \;=
$$

$$\sum_k \mu_k \left(\frac{\chi_k \, g_{k,t}}{1 + \lambda_k + \varphi \, \lambda_k \, (1 - \chi_k)} - \frac{y_{k,t \, (1-\chi_k) \, (1+\varphi+\chi_k^*)}}{1 + \lambda_k + \varphi \, \lambda_k \, (1 - \chi_k)} \right). \tag{E.6}$$