Government Spending in Multi-Sector Open Economies with Production Networks

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Motivation

Question. What is the welfare effect of redistributing sector-specific government spending under a fixed aggregate budget, relative to fully endogenous fiscal policy?

- Production networks amplify and reshape the transmission of shocks across sectors (Acemoglu et al., 2012; Baqaee & Farhi, 2020; Rubbo, 2023).
- Optimal fiscal allocation across sectors depends on sectoral heterogeneity in price stickiness and public-good shares (Cox et al., 2024), amplified by network position (Rubbo, 2023).

Plan

(i) A **simple closed-economy model** with lump-sum transfers and government procurements $\to$ a *relative allocation rule*.

(ii) The **global production network model** of Aguilar et al. (2025): $K{=}4$ countries, $I{=}44$ sectors, with IO linkages and sectoral tariffs.

(iii) **Original extensions**: time-varying labour income tax and production subsidy that enter the Phillips curves directly.

(iv) **Research agenda**: introduce sector-specific government purchases in the networked model; quantify the welfare cost of fixing the aggregate budget.

Status: steps (i)–(iii) are derived; the networked model implementation (iv) is in progress.

A Simple Multi-Sector Model

Closed NK economy, N sectors, Calvo pricing (α_k), linear production $Y_{k,t} = A_{k,t}\, N_{k,t}$, following Cox et al. (2024).

Key constraint: the aggregate public-good bundle $\bar{G}_t$ is *exogenous*. The planner chooses only the sectoral composition $\{g_{k,t}\}$.

Setting $\sigma = 1$, the second-order welfare approximation is (where $\chi_k^* \equiv \chi_k/(1-\chi_k)$):

$$\mathcal{W} \;\approx\; -\frac{1}{2}\sum_k \mu_k \Bigg[\underbrace{(1+\varphi)\, y_{k,t}^2}_{\text{output gaps}} + \underbrace{\frac{\theta(1-\chi_k)}{\lambda_k}\,\pi_{k,t}^2}_{\text{inflation}} + \underbrace{\chi_k^*\,(g_{k,t}-y_{k,t})^2}_{\text{public-good gaps}}\Bigg]$$

Three tensions the planner must balance:

1. **Output-gap stabilisation:** penalises $y_{k,t}^2$.
2. **Inflation stabilisation:** penalises $\pi_{k,t}^2$, weighted inversely by the PC slope λ_k.
3. **Public-good allocation:** penalises $g_{k,t} - y_{k,t}$; under a fixed budget the planner can only *reshuffle* spending across sectors.

The Relative Allocation Rule

Under exogenous $\bar{G}_t$, the spending gap between sector k and a residual sector i satisfies:[1]

$$\underbrace{g_{k,t} - \Phi_{ki}\, g_{i,t}}_{\text{spending gap}} = - \underbrace{\left(a_k\, y_{k,t} - \Phi_{ki}\, a_i\, y_{i,t}\right)}_{\text{output-gap differential}} - \underbrace{\left(b_k\, \pi_{k,t} - \Phi_{ki}\, b_i\, \pi_{i,t}\right)}_{\text{inflation differential}}$$

where $a_k \equiv \frac{\varphi}{1+\lambda_k+\varphi\lambda_k}$, $b_k \equiv \frac{\theta\,\varphi\,(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k}$, and analogously for sector i.

Key property

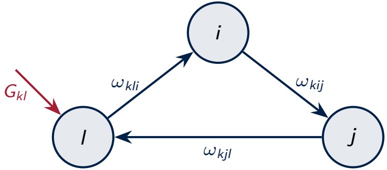
The rule is inherently *relative*: spending is **reallocated** toward sectors with lower inflation and lower output gaps *relative* to the residual sector. What matters is the cross-sectional differences in gaps, not their level.

[1] Φ_{ki} collects structural parameters: public-good shares χ_k, χ_i, aggregator weights $\omega_{g,k}/\omega_{g,i}$, and PC slopes λ_k, λ_i. ▸ Extended form

The Global Production Network Model (Aguilar et al., 2025)

The closed-economy model delivers the intuition; we need a quantitative framework to test it.

- $K=4$ countries, $I=44$ sectors
- Nested CES: energy/non-energy, domestic/foreign
- Sector- and country-specific Calvo pricing
- Country-specific Taylor rules
- Balanced budget, lump-sum taxes, static production subsidies, tariff revenue



IO network with fiscal entry point

Marginal cost in sector i, country k:

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki}\,\widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\textstyle\sum_{l,j}\mathcal{M}_{ki}\omega_{klij}\,\widehat{p}_{klij,t}}_{\text{IO inputs}}$$

Sectoral Phillips curve:

$$\pi_{ki,t} = \kappa_{ki}(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}$$

Tariffs enter as price wedges (Details).

Our Contribution: Fiscal Instruments in the Phillips Curves

Why extend? The Aguilar et al. model features lump-sum taxes and static production subsidies but no active fiscal stabilisation. We introduce two time-varying distortionary instruments that enter the Phillips curves directly. This is a tractable first step; the full Ramsey problem in a $K \times I$ networked model is the research goal.

Define $\hat{\tau}^w_{k,t} \equiv (\tau^w_{k,t} - \bar{\tau}^w_k)/(1 - \bar{\tau}^w_k)$ and $\hat{\tau}^s_{ki,t} \equiv (\tau^s_{ki,t} - \bar{\tau}^s_{ki})/(1 - \bar{\tau}^s_{ki})$.

Wage Phillips curve, adding a labour income tax:

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma\,\hat{c}_{k,t} + \varphi\,\hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}^w_{k,t}\right) + \beta\,\mathbb{E}_t\pi_{wk,t+1} + u^w_{k,t}$$

Price Phillips curve, making the production subsidy time-varying:

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}^s_{ki,t}\right) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}$$

Key implication

Tax hike $\rightarrow$ inflationary cost-push. Subsidy hike $\rightarrow$ disinflationary cost-push.

Research Agenda: Fixed vs. Endogenous Fiscal Envelopes

Goal: introduce sector-specific $G_{ki,t}$ in the goods market clearing condition of the global production network model:

$$Y_{ki,t} = \sum_l C_{lki,t} + \sum_l \sum_j X_{lkji,t} + G_{ki,t}$$

and compare two policy regimes:

Fully endogenous spending

Planner chooses level *and* composition. Unconstrained fiscal benchmark (first-best within the class of spending instruments).

Fixed aggregate budget

$\bar{G}_{k,t}$ exogenous; only the sectoral composition adjusts. Relevant when total spending is politically constrained.

Central question: how large is the welfare gap between the two regimes? If it is small, compositional reallocation alone (the relative allocation rule) may approximate the first-best, even without aggregate fiscal flexibility.

Summary

1. **Key result:** under a fixed fiscal envelope, optimal spending follows a *relative allocation rule*; what matters is the cross-sectional dispersion of output and inflation gaps, not their level.

2. **Central question:** can compositional reallocation alone approximate the first-best, even without aggregate fiscal flexibility?

3. **What's next:** quantify the welfare gap between fixed and endogenous budgets in the $K \times I$ networked model of Aguilar et al. (2025), extended with distortionary fiscal instruments.

Thank you

Appendix: Related Literature

Production Networks & NK

- Acemoglu et al. (2012)
- Baqaee & Farhi (2020, 2024)
- Pasten, Schoenle & Weber (2020)
- Rubbo (2023)

Fiscal Policy in Disaggregated Economies

- Aoki (2001)
- Antonova & Müller (2025)
- Cox, Feng, Müller, Pasten, Schoenle & Weber (2024)

Tariffs & Open-Economy NK

- Galí & Monacelli (2005)
- Comin & Johnson (2023)
- Aguilar et al. (2025)

Fiscal–Price Effects (Empirical)

- Nekarda & Ramey (2020)
- Ben Zeev & Pappa (2017)

Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$-\frac{1}{2}\sum_k \mu_k \Bigg((1+\varphi)\, y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k}\, \pi_{k,t}^2 + \chi_k^*\, (g_{k,t}-y_{k,t})^2 \\ + (\sigma-1)\left[(1-\chi_k)\,\omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t} \right)^2 + \omega_{g,k}\,\chi_k\, g_{k,t}^2 \right] \Bigg)$$

- The $\sigma-1$ term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- $\kappa > 0$ steepens Phillips curves ($\lambda_k' > \lambda_k$), making fiscal policy a supply-side instrument.

Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous $\bar{G}_t$ (with $\sigma = 1$, $\kappa = 0$):

$$\begin{aligned} g_{k,t} \;=\; & \frac{1-\chi_k}{1-\chi_i}\left(\frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)}\right)\left(\frac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)}\right)^{-1} g_{i,t} \\ & - \frac{\varphi\, y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta\,\varphi\,(1-\chi_k)\,\pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ & + \frac{1-\chi_k}{1-\chi_i}\left(\frac{\omega_{g,k}}{\omega_{g,i}}\right)^{\rho}\left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)}\right)^{-1} \\ & \times\left(\frac{\varphi\, y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\,\varphi\,(1-\chi_i)\,\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)}\right) \end{aligned}$$

$\omega_{g,k}$: weight of sector k in the government Cobb–Douglas aggregator. ρ: CES elasticity of public-good bundle.

Note: the asymmetric denominators ($1+\lambda_k+\varphi\lambda_k$ for own-sector terms vs. $1+\lambda_i+\varphi\lambda_i(1-\chi_i)$ for the residual-sector terms) arise from solving the equated FOCs for $g_{k,t}$; the $(1-\chi)$ factor enters through the resource-constraint substitution in the residual sector's Phillips curve.

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Appendix: Optimal Monetary Rule under Aggregate Constraint

Under exogenous $\bar{G}_t$, optimal monetary policy sets:

$$\sum_k \mu_k \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\lambda_k + \varphi\lambda_k(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \pi_{k,t} =$$

$$\sum_k \mu_k \left(\frac{\chi_k \, g_{k,t}}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} - \frac{y_{k,t}(1-\chi_k)(1+\varphi+\chi_k^*)}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)$$

- Inflation weights depend on private-consumption share and λ_k.
- Government spending enters the target because the constraint links $g_{k,t}$ and $y_{k,t}$ across sectors.

Appendix: Aguilar et al. —Household Problem

Per-period utility: $U_t = \Big(C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi)\, dg\Big) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[\widetilde{\beta}_k^{1/\gamma}\, C_{kE,t}^{(\gamma-1)/\gamma} + (1-\widetilde{\beta}_k)^{1/\gamma}\, C_{kM,t}^{(\gamma-1)/\gamma}\right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$C_{k,t}^{-\sigma} = \beta\, \mathbb{E}_t\, C_{k,t+1}^{-\sigma}\, \frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}}\, \frac{Z_{k,t+1}}{Z_{k,t}}$$

$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* \, \mathrm{nfa}_{k,t} + \varepsilon^e_{kK,t} \qquad \text{(UIP)}$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk}\Big(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t}\Big) + \beta\, \mathbb{E}_t \pi_{wk,t+1} + u^w_{k,t}$$

Appendix: Aguilar et al. —Firm Problem

CES production: $Y_{ki,f,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{1/\psi}\, N_{fki,t}^{(\psi-1)/\psi} + \widetilde{\vartheta}_{ki}^{1/\psi}\, X_{fki,t}^{(\psi-1)/\psi}\right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\,\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I}\mathcal{M}_{ki}\omega_{klij}\,\widehat{p}_{klij,t}$$

where α_{ki}: labour share, ω_{klij}: IO expenditure share, $\mathcal{M}_{ki}$: steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}\right) + \beta\,\mathbb{E}_t\pi_{ki,t+1} + u_{ki,t}^{p} \qquad \kappa_{ki} = \frac{(1-\theta_{ki}^{p})(1-\beta\theta_{ki}^{p})}{\theta_{ki}^{p}}$$

Appendix: Aguilar et al. —Tariff Propagation

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t})\, \widetilde{P}_{l,k,i,t}$$

Propagation:

1. Tariff on country l raises input prices $\widehat{p}_{klij,t}$ for domestic sectors sourcing from sector j in l.
2. Higher input costs raise $\widehat{\mathrm{mc}}_{ki,t}$, feeding into $\pi_{ki,t}$.
3. Cost increases cascade downstream through the IO network.
4. Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_{i} \tau_{kli,t}\, P^{l}_{kli,t} \Big(C_{kli,t} + \textstyle\sum_j X_{klji,t} \Big) = B_{k,t-1} + \sum_{i} \tau^{s}_{ki}\, \mathrm{MC}_{ki,t}\, Y_{ki,t}$$

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Appendix: Aguilar et al. —Calibration

Households

- $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$
- Energy/non-energy elast. $\gamma = 0.4$
- Trade elasticity $\delta = 1$
- Calvo wage $\theta_k^w = 0.75$
- Consumption shares from OECD ICIO (2019)

Monetary policy

- $\rho_r = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.125$
- Target: headline inflation

Firms

- Labour/input elast. $\psi = 0.5$
- Energy/non-energy elast. $\phi = 0.4$
- Trade elasticity $\mu = 1$
- IO shares from OECD ICIO (2019)
- Markups from Eurostat Figaro
- Calvo prices from ECB PRISMA

Tariff shocks

- $\rho^\tau = 0.96$, $\sigma^\tau = 1$

Appendix: Aguilar et al. —Goods Market Clearing and GDP

Market clearing:

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K} \sum_{j=1}^{I} X_{lkji,t}$$

Log-linearised:

$$\lambda_{ki}\, \widehat{y}_{ki,t} = \sum_{l=1}^{K} \mathcal{Y}_{lk} \left(\beta_{lki}\, \widehat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj}\, \omega_{lkji}\, \widehat{x}_{lkji,t} \right)$$

where $\lambda_{ki} = P_{ki} Y_{ki} / \mathcal{Y}_k$ is the Domar weight.

Real GDP:

$$\widehat{y}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k \left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t} \right)$$

where Υ_k is the trade-to-GDP ratio.

Appendix: Derivation —Wage Phillips Curve with Tax

Household FOC with labour income tax $\tau^{w}_{k,t}$:

$$\sum_{l=0}^{\infty} (\beta\theta^{W}_{k})^{l}\, \mathbb{E}_t \left[N_{k,t+l|t}\, C^{-\sigma}_{t+l|t} \left(\frac{(1-\tau^{w}_{k,t+l})\, W^{*}_{k,t}}{P_{t+l}} - \mathcal{M}_{wk,t}\, \mathrm{MRS}_{k,t+l|t} \right) \right] = 0$$

Log-linearised reset wage:

$$w^{*}_{k,t} = (1-\beta\theta^{W}_{k}) \sum_{l=0}^{\infty} (\beta\theta^{W}_{k})^{l}\, \mathbb{E}_t \left[\mathrm{mrs}_{t+l|t} + \mu^{n}_{wk,t+l} + p_{kC,t+l} + \hat{\tau}^{w}_{k,t+l} \right]$$

Calvo aggregation ($\pi_{wk,t} = w_{k,t} - w_{k,t-1}$):

$$\pi_{wk,t} = \kappa_{wk} \left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}^{w}_{k,t} \right) + \beta\, \mathbb{E}_t \pi_{wk,t+1} + u^{w}_{k,t}$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

Appendix: Derivation —Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy $\tau^s_{ki,t}$:

$$\sum_{l=0}^{\infty}(\beta\theta^p_{ki})^l\,\mathbb{E}_t\left[\Lambda_{t,t+l}\,Y_{ki,t+l|t}\left(P^*_{ki,t}-\mathcal{M}_{pk,t+l}\,(1-\tau^s_{ki,t+l})\,\mathrm{MC}^n_{ki,t+l|t}\right)\right]=0$$

Log-linearised reset price:

$$p^*_{ki,t}=(1-\beta\theta^p_{ki})\sum_{l=0}^{\infty}(\beta\theta^p_{ki})^l\,\mathbb{E}_t\left[\mathrm{mc}^n_{ki,t+l|t}+\mu^n_{pki,t+l}-\hat{\tau}^s_{ki,t+l}\right]$$

Calvo aggregation ($\pi_{ki,t}=p_{ki,t}-p_{ki,t-1}$):

$$\pi_{ki,t}=\kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t}-\hat{p}_{ki,t}-\hat{\tau}^s_{ki,t}\right)+\beta\,\mathbb{E}_t\pi_{ki,t+1}+u^p_{ki,t}$$

Subsidy increase lowers effective marginal cost $\rightarrow$ disinflationary cost-push.

Appendix: Government Budget Dynamics

Starting from the nominal budget constraint:

$$B_{t+1} = B_t(1 + i_{t+1}) + G_{t+1} - T_{t+1}$$

In ratios to GDP ($b_t \equiv B_t / Y_t$):

$$b_{t+1} = b_t \, \frac{1 + i_{t+1}}{1 + g_{Y,t+1}} + s_{t+1} - \mathcal{T}_{t+1}$$

where $s_t = G_t / Y_t$ and $\mathcal{T}_t = T_t / Y_t$.

Linearised:

$$\hat{b}_{t+1} = \underbrace{\frac{1 + \bar{\imath}}{1 + \bar{g}_Y}}_{\rho_b} \, \hat{b}_t + \frac{\bar{\imath}}{1 + \bar{g}_Y} \, \hat{\imath}_t - \frac{\bar{g}_Y}{1 + \bar{g}_Y} \, \hat{g}_{Y,t+1} + \frac{\bar{s}}{\bar{b}} \, \hat{s}_{t+1} - \frac{\bar{\mathcal{T}}}{\bar{b}} \, \hat{\mathcal{T}}_{t+1}$$

Extension: replace lump-sum rebate of tariff revenue with $G_{ki,t}$ financing, creating a feedback loop between trade and fiscal policy.

Appendix: Optimal Fiscal Rule (Unconstrained Budget, $\sigma > 1$, $\kappa > 0$)

$$g_{k,t} = \frac{H_k}{X_k}\, y_{k,t} + \frac{J_k}{X_k}\, a_{k,t} - \frac{\theta}{\lambda_k X_k}\, \pi_{k,t} + \frac{\sigma-1}{(1-\chi)^{1/\sigma} X_k} \sum_j \mu_j \lambda_j\, \phi^{\pi}_{j,t}$$

$$H_k = \chi_k^* + 1 + \varphi + (\sigma-1)\frac{\omega_{c,k}}{1-\chi_k} - \varphi\lambda_k\, \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}$$

$$J_k = -(1+\varphi)\left(1 - \lambda_k\, \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}\right)$$

$$X_k = \chi_k^* + (\sigma-1)\chi_k^*\omega_{c,k}\chi_k + \Big(1+(\sigma-1)\omega_{g,k}\Big)\left(\lambda_k + \frac{1+\chi_k^*+\varphi}{\lambda_k\varphi+\frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}}\right)$$

$$\phi^{\pi}_{k,t} = \Big(-\varphi\, y_{k,t} - g_{k,t}(1+(\sigma-1)\omega_{g,k}) + (1+\varphi)\, a_{k,t}\Big)\left(\lambda_k\varphi + \frac{\kappa}{\theta-1}\,\frac{\lambda_k}{1-\chi_k}\right)^{-1}$$

Comparing welfare under this vs. the constrained rule quantifies the cost of fiscal inflexibility.

Appendix: Optimal Monetary Rule (Unconstrained Budget)

Optimal monetary policy sets a weighted inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k}\,\pi_{k,t} = -\sum_k \mu_k\,\frac{\varphi\, y_{k,t} + g_{k,t}\big(1+(\sigma-1)\omega_{g,k}\big) + (1+\varphi)\,a_{k,t}}{\lambda_k\varphi + \frac{\kappa}{\theta-1}\frac{\lambda_k}{1-\chi_k}}$$

- Inflation weights increase with private-consumption share, decrease with λ_k.
- Government spending enters the target when $\kappa > 0$ (government demand affects marginal costs).

Appendix: Fixed vs. Endogenous Budgets — Detail

Fully endogenous spending

- Planner chooses both $\bar{G}_{k,t}$ and $\{G_{ki,t}\}$.
- Level and composition respond to shocks.
- Unconstrained fiscal benchmark (first-best within spending instruments).
- Fiscal rule is *absolute*: each $g_{k,t}$ set independently.

Fixed aggregate budget

- $\bar{G}_{k,t}$ exogenous; only composition adjusts.
- Fiscal rule is *relative*: gaps wrt residual sector.
- Welfare loss from constraint is concentrated in aggregate stabilisation.
- Cross-sectional allocation may remain close to first-best.

Implications. If the welfare gap is small, a budget-constrained government can approximate first-best outcomes through compositional reallocation alone. This is the central hypothesis to be tested in the networked model.

Appendix: Open Questions

- **Network centrality and fiscal allocation.** How does a sector's position in the IO network affect its optimal public-good allocation? The IO weights ω_{klij} introduce cross-sector spillovers absent in the simple model.
- **Fiscal–trade policy interaction.** With tariff revenue financing government purchases, trade policy changes alter the fiscal envelope. The welfare implications of this feedback are to be characterised.
- **Dimensionality.** The Ramsey problem involves $K \times I$ spending instruments. Practical approaches may require restricting the class of admissible rules.
- **Political economy.** The exogenous-budget assumption abstracts from the determination of $\bar{G}_{k,t}$. Endogenising this would require a political-economy layer.

Q&A: Why Not Derive the Ramsey Problem Directly?

The question

Why modify Phillips curves with fiscal instruments rather than solving a full Ramsey taxation problem?

- The Phillips-curve approach is a **reduced-form shortcut**: it isolates the cost-push channel of taxation while preserving the existing IO structure of Aguilar et al. (2025).
- A full Ramsey problem in a 4×44 networked model is computationally demanding: it requires solving for $K \times I$ optimal tax instruments simultaneously.
- The **research agenda goal** is precisely to move toward the full Ramsey characterisation. The current extensions are a tractable first step.
- La'O & Tahbaz-Salehi (2024) show that even in simpler networks, the Ramsey problem has a rich structure. Our approach builds intuition before scaling up.

Q&A: Relation to Rubbo (2023) and La'O & Tahbaz-Salehi (2024)

The question

How does the relative allocation rule relate to optimal policy results in production networks?

Rubbo (2023):

- Optimal *monetary* policy in production networks.
- Divine coincidence fails; price-stability target depends on network topology.
- Our work: fiscal policy adds a second instrument that can target sectoral gaps directly.

La'O & Tahbaz-Salehi (2024):

- Optimal monetary policy in production networks (Econometrica).
- Network structure shapes the optimal inflation target and creates trade-offs absent in one-sector models.
- Our contribution: government *spending* as a complementary fiscal instrument, with an aggregate budget constraint.

Key distinction: our relative allocation rule operates under a *fixed fiscal envelope*, a constraint absent in both papers.

Q&A: When Does the Budget Constraint Bind?

The question

What happens when $\bar{G}_t$ is close to optimal vs. far from it?

- When $\bar{G}_t$ is **close to the first-best level**, compositional reallocation suffices: the relative allocation rule can approximate optimal welfare by reshuffling spending across sectors.
- When $\bar{G}_t$ is **far from optimal** (e.g., a deep austerity constraint), the welfare gap grows because the level channel is shut off. The planner cannot compensate for an insufficient aggregate envelope by reallocating alone.
- The **welfare gap** between fixed and endogenous budgets is therefore increasing in $|\bar{G}_t - G_t^*|$, where G_t^* is the first-best aggregate level.
- Quantifying this gap in the 4×44 networked model is the central objective of the research agenda.

Q&A: Why 4 Countries and 44 Sectors?

The question

How sensitive are results to this level of granularity?

- The 4×44 structure follows Aguilar et al. (2025), calibrated to the **OECD ICIO 2019 tables** (Inter-Country Input-Output).
- 4 countries: a practical choice balancing model tractability with open-economy realism (e.g., US, EA, CN, RoW).
- 44 sectors: the full ISIC Rev. 4 classification available in ICIO (no aggregation required).
- **Robustness considerations:**
  - Coarser aggregations (e.g., 10–15 sectors) can be tested by collapsing IO tables.
  - The key qualitative predictions (relative reallocation, countercyclicality) should survive aggregation.
  - Quantitative magnitudes (welfare gaps) are likely sensitive to granularity, as network effects depend on the density of the IO matrix.

Q&A: The Pass-Through Parameter κ

Motivation. Cox et al. (2024) assume government demand does not affect firms' pricing decisions ($\kappa = 0$). Empirical evidence (Ben Zeev & Pappa, 2017) suggests fiscal spending does affect prices.

Definition

$\kappa \in [0, 1]$ controls how much inelastic government demand passes through to marginal cost. When $\kappa > 0$:

- The Phillips curve steepens: $\lambda'_k = \lambda_k/(1-\delta)$, where $\delta = \frac{\kappa}{\theta-1}\frac{\bar{G}}{\bar{C}}$.
- Government spending becomes a **supply-side instrument**: $g_{k,t}$ enters the Phillips curve directly.

Nesting: $\kappa = 0$ recovers Cox et al. (2024). At $\kappa = 1$ (full pass-through), for $\theta = 6$ and $\chi^*_k = 1$: $\lambda'_k = 1.25\,\lambda_k$.

Q&A: CRRA Preferences and the Insurance Motive

With $\sigma \neq 1$, the welfare objective acquires an additional term (in red):

$$-\frac{1}{2}\sum_k \mu_k \Bigg((1+\varphi)\, y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k}\, \pi_{k,t}^2 + \chi_k^*\, (g_{k,t} - y_{k,t})^2 \\ + (\sigma-1) \left[(1-\chi_k)\, \omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^*\, g_{k,t} \right)^2 + \omega_{g,k}\, \chi_k\, g_{k,t}^2 \right] \Bigg)$$

Fiscal policy now balances **three motives**:

1. **Allocation:** match public-good provision to sectoral needs.
2. **Stabilisation:** dampen output and inflation gaps.
3. **Insurance:** hedge against aggregate consumption risk ($\sigma - 1$ term).

Q&A: Negativity Condition for Countercyclicality

Setting $\kappa = 1$, the overall coefficient on $y_{k,t}$ in the fiscal rule is:

$$\underbrace{\frac{1}{(\theta-1)(1-\chi_k)\varphi + 1}}_{\text{always } >0} \times \underbrace{\left[\frac{1}{1-\chi_k} + \varphi + (\sigma-1)\frac{\omega_{c,k}}{1-\chi_k} - \frac{(\theta-1)(1-\chi_k)\varphi}{(1-\chi_k)\lambda_k}\right]}_{\text{sign determines cyclicality}}$$

Government spending is **countercyclical** ($g_{k,t}$ falls when $y_{k,t}$ rises) if and only if:

$$\sigma > \left(\frac{(\theta-1)(1-\chi_k)\varphi}{\lambda_k} - 1 - \varphi(1-\chi_k)\right)\frac{1}{\omega_{c,k}} + 1$$

- Under **flexible prices** ($\lambda_k \to \infty$): always satisfied.
- Under **moderate rigidity** ($\alpha_k < 0.3$): satisfied for standard calibrations.
- $\kappa < 1$ implies a **less countercyclical** stance (the pass-through channel weakens).

Q&A: Welfare Approximation — Full Derivation Steps

Step 1. Second-order Taylor expansion of U around the efficient steady state:

$$U - \bar{U} \approx \sum_k \mu_k \left[\bar{C}\,(1-\chi)\,\hat{c}_{k,t} + \bar{G}\,\chi\,\hat{g}_{k,t} - \bar{N}_k^{1+\varphi}\,\hat{n}_{k,t} \right] + \text{second-order terms}$$

Step 2. Use the production function $Y_{k,t} = A_{k,t}N_{k,t}$ and goods market clearing $Y_{k,t} = C_{k,t} + G_{k,t}$ to eliminate $\hat{n}_{k,t}$ and $\hat{c}_{k,t}$:

$$\hat{n}_{k,t} = y_{k,t} - a_{k,t}, \qquad (1-\chi_k)\hat{c}_{k,t} = y_{k,t} - \chi_k\, g_{k,t}$$

Step 3. Calvo price dispersion contributes $\sum_k \frac{\theta}{\lambda_k}\,\pi_{k,t}^2$ (Woodford, 2003, Ch. 6).

Step 4. Collecting terms (setting $\sigma = 1$, $\kappa = 0$):

$$\mathcal{W} \approx -\frac{1}{2}\sum_k \mu_k \left[(1+\varphi)\,y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k}\,\pi_{k,t}^2 + \chi_k^*\,(g_{k,t} - y_{k,t})^2 \right]$$

where $\chi_k^* \equiv \chi_k/(1-\chi_k)$. The public-good gap arises from the resource constraint binding at the sectoral level.

Q&A: Relative Allocation Rule — FOC Details

Lagrangian. Minimise $\mathcal{W}$ subject to $\sum_k \omega_{g,k}\, g_{k,t} = \bar{g}_t$ (aggregate budget):

$$\mathcal{L} = \mathcal{W} + \eta_t \left(\bar{g}_t - \sum_k \omega_{g,k}\, g_{k,t} \right)$$

FOC for sector k:

$$\frac{\partial \mathcal{W}}{\partial g_{k,t}} = \mu_k \left[\chi_k^* \, (g_{k,t} - y_{k,t}) + \frac{\theta(1-\chi_k)}{\lambda_k} \, \frac{\partial \pi_{k,t}}{\partial g_{k,t}} + (1+\varphi) \, \frac{\partial y_{k,t}}{\partial g_{k,t}} \right] = \eta_t \, \omega_{g,k}$$

Eliminating η_t: Equate the FOC for sector k with sector i (the residual):

$$\frac{1}{\omega_{g,k}} \, \frac{\partial \mathcal{W}}{\partial g_{k,t}} = \frac{1}{\omega_{g,i}} \, \frac{\partial \mathcal{W}}{\partial g_{i,t}}$$

Substituting the Phillips curve $\pi_{k,t} = \lambda_k \left[(1+\varphi+\chi_k^*) \, y_{k,t} - \chi_k^* \, g_{k,t} \right] + \beta \, \mathbb{E}_t \pi_{k,t+1}$ (using the resource constraint $\hat{c}_{k,t} = (\hat{y}_{k,t} - \chi_k \, g_{k,t})/(1-\chi_k)$) and solving for $g_{k,t}$ in terms of $g_{i,t}$, $y_{k,t}$, $\pi_{k,t}$, $y_{i,t}$, $\pi_{i,t}$ yields the relative allocation rule.

See ▸ *Structural coefficients* *for the full extended form.*