

Government Spending in Multi-Sector Open Economies with Production Networks

Antonio Eugenio

University of Oxford
Department of Economics

Macro Workshop — February 2026

Motivation and Plan

Question. What is the welfare effect of redistributing sector-specific government spending under a fixed aggregate budget, relative to fully endogenous fiscal policy?

- ▶ Production networks amplify and reshape the transmission of shocks across sectors (Acemoglu et al., 2012; Baqaee & Farhi, 2020; Rubbo, 2023).
- ▶ Optimal fiscal allocation across sectors depends on sectoral heterogeneity in price stickiness, public-good shares, and network position (Cox et al., 2024).
- ▶ No existing framework combines these two dimensions in an open-economy setting.

Plan — building up step by step:

- (i) A **toy model** with non-distortionary lump-sum transfers and government procurements → a *relative allocation rule*.
- (ii) The **global production network model** of Aguilar et al. (2025) with distortionary taxation and subsidisation.
- (iii) **Planned extensions:** combine lump-sum procurements with proportional taxation/subsidies to production and labour in the richer model.

A Simple Multi-Sector Model: Setup

Closed NK economy, K sectors, Calvo pricing (α_k), following Cox et al. (2024).

Household utility over private and public composites:

$$U = \sum_t \beta^t \left[(1 - \chi) \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \sum_k \nu_k \frac{N_{k,t}^{1+\varphi}}{1+\varphi} \right]$$

Key constraint: the aggregate public-good bundle \bar{G}_t is *exogenous*. The planner chooses only the sectoral composition $\{g_{k,t}\}$.

The welfare loss (setting $\sigma = 1$, isolating the budget channel):

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[\underbrace{(1 + \varphi) y_{k,t}^2}_{\text{output gaps}} + \underbrace{\frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2}_{\text{inflation}} + \underbrace{\chi_k^* (g_{k,t} - y_{k,t})^2}_{\text{public-good gaps}} \right]$$

The Relative Allocation Rule

Under exogenous \bar{G}_t , optimal spending in sector k relative to a residual sector i :

$$g_{k,t} = \underbrace{\Phi_{ki}}_{\text{structural}} g_{i,t} - \underbrace{\frac{\varphi}{1+\lambda_k+\varphi\lambda_k} y_{k,t} - \frac{\theta\varphi(1-\chi_k)}{1+\lambda_k+\varphi\lambda_k} \pi_{k,t}}_{\text{sector } k: \text{ spending falls when own gaps are large}} + \underbrace{\Phi_{ki} \left(\frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta\varphi(1-\chi_i)\pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right)}_{\text{sector } i: \text{ spending rises when residual-sector gaps are large}}$$

Key property

The rule is inherently *relative*: spending is **reallocated** toward sectors with lower output gaps and lower inflation *relative* to the residual sector. What matters is the cross-sectional dispersion of gaps, not their level.

The Global Production Network Model (Aguilar et al., 2025)

- ▶ $K=4$ countries, $I=44$ sectors
- ▶ Nested CES: energy/non-energy, domestic/foreign
- ▶ Sector- and country-specific Calvo pricing
- ▶ Country-specific Taylor rules
- ▶ Balanced budget, lump-sum taxes, static production subsidies, tariff revenue

Marginal cost in sector i , country k :

$$\widehat{mc}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\sum_{l,j} \mathcal{M}_{ki}\omega_{klij}\widehat{p}_{klij,t}}_{\text{IO inputs}}$$

Sectoral Phillips curve:

$$\pi_{ki,t} = \kappa_{ki}(\widehat{mc}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Tariffs enter as price wedges:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \widetilde{P}_{l,k,i,t}$$

Original Extensions: Fiscal Instruments in the Phillips Curves

Not part of Aguilar et al. (2025). Low-cost additions that do not alter the IO structure.

Define $\hat{\tau}_{k,t}^w \equiv (\tau_{k,t}^w - \bar{\tau}_k^w)/(1 - \bar{\tau}_k^w)$ and $\hat{\tau}_{ki,t}^s \equiv (\tau_{ki,t}^s - \bar{\tau}_{ki}^s)/(1 - \bar{\tau}_{ki}^s)$.

Wage Phillips curve — adding a labour income tax:

$$\pi_{wk,t} = \kappa_{wk} \left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Price Phillips curve — making the production subsidy time-varying:

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Tax hike \rightarrow inflationary cost-push. Subsidy hike \rightarrow disinflationary cost-push.

Research Agenda: Fixed vs. Endogenous Fiscal Envelopes

Goal: introduce sector-specific $G_{ki,t}$ in the goods market clearing condition of the global production network model:

$$Y_{ki,t} = \sum_l C_{lki,t} + \sum_l \sum_j X_{lkji,t} + G_{ki,t}$$

and compare two policy regimes:

Fully endogenous spending

Planner chooses level *and* composition.
Benchmark for first-best fiscal stabilisation.

Fixed aggregate budget

$\bar{G}_{k,t}$ exogenous; only the sectoral composition adjusts. Relevant when total spending is politically constrained.

Central question: how large is the welfare gap between the two regimes? If it is small, compositional reallocation alone—the relative allocation rule—may approximate the first-best, even without aggregate fiscal flexibility.

- 1 Under a fixed fiscal envelope, optimal sectoral spending follows a **relative allocation rule**: reallocate toward sectors with larger gaps.
- 2 The global production network model of **Aguilar et al. (2025)** provides a quantitative environment with IO linkages and sectoral tariffs, but lacks non-trivial fiscal dynamics.
- 3 **Original extensions**—labour income tax in the wage PC, time-varying subsidy in the price PC—introduce fiscal instruments without altering the core model.
- 4 The **research agenda**: derive spending rules in the networked model; quantify the welfare cost of fixing the aggregate budget relative to fully endogenous spending.

Thank you

antonio.eugenelo@economics.ox.ac.uk

Production Networks & NK

- ▶ Acemoglu et al. (2012)
- ▶ Baqaee & Farhi (2020, 2024)
- ▶ Pasten, Schoenle & Weber (2020)
- ▶ Rubbo (2023)

Tariffs & Open-Economy NK

- ▶ Galí & Monacelli (2005)
- ▶ Comin & Johnson (2023)
- ▶ Aguilar et al. (2025)

Fiscal Policy in Disaggregated Economies

- ▶ Aoki (2001)
- ▶ Cox, Müller, Pasten, Schoenle & Weber (2024)

Fiscal–Price Effects (Empirical)

- ▶ Nekarda & Ramey (2013)
- ▶ Ben Zeev & Pappa (2017)
- ▶ Barattieri et al. (2023)

Appendix: Welfare Objective

Under $\sigma = 1$, abstracting from government-demand pass-through, the second-order welfare approximation is:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[(1 + \varphi) y_{k,t}^2 + \frac{\theta(1 - \chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

Three tensions:

- 1 Output-gap stabilisation:** penalises $y_{k,t}^2$.
- 2 Inflation stabilisation:** penalises $\pi_{k,t}^2$, weighted inversely by the Phillips curve slope λ_k .
- 3 Public-good allocation:** penalises the gap $g_{k,t} - y_{k,t}$.

When \bar{G}_t is unconstrained, $g_{k,t}$ can be set independently in each sector. When only the composition is a choice variable, the problem changes qualitatively: the planner can only *reshuffle* spending.

Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$-\frac{1}{2} \sum_k \mu_k \left((1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ \left. + (\sigma-1) \left[(1-\chi_k) \omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right)$$

- ▶ The $\sigma-1$ term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- ▶ $\kappa > 0$ steepens Phillips curves ($\lambda'_k > \lambda_k$), making fiscal policy a supply-side instrument.

Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous \bar{G}_t (with $\sigma = 1$, $\kappa = 0$):

$$\begin{aligned} g_{k,t} = & \frac{1-\chi_k}{1-\chi_i} \left(\frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} g_{i,t} \\ & - \frac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta \varphi (1-\chi_k) \pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ & + \frac{1-\chi_k}{1-\chi_i} \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} \\ & \times \left(\frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta \varphi (1-\chi_i) \pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \end{aligned}$$

$\omega_{g,k}$: weight of sector k in the government Cobb–Douglas aggregator. ρ : CES elasticity of public-good bundle.

Appendix: Optimal Monetary Rule under Aggregate Constraint

Under exogenous \bar{G}_t , optimal monetary policy sets:

$$\sum_k \mu_k \frac{\theta(1-\chi_k)}{\lambda_k} \frac{\lambda_k + \varphi\lambda_k(1-\chi_k)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \pi_{k,t} =$$
$$\sum_k \mu_k \left(\frac{\chi_k g_{k,t}}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} - \frac{y_{k,t}(1-\chi_k)(1+\varphi+\chi_k^*)}{1 + \lambda_k + \varphi\lambda_k(1-\chi_k)} \right)$$

- ▶ Inflation weights depend on private-consumption share and λ_k .
- ▶ Government spending enters the target because the constraint links $g_{k,t}$ and $y_{k,t}$ across sectors.

Appendix: Aguilar et al. — Household Problem

Per-period utility: $U_t = (C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi) dg) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[\tilde{\beta}_k^{1/\gamma} C_{kE,t}^{(\gamma-1)/\gamma} + (1-\tilde{\beta}_k)^{1/\gamma} C_{kM,t}^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$C_{k,t}^{-\sigma} = \beta \mathbb{E}_t C_{k,t+1}^{-\sigma} \frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}}$$
$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* \text{nfa}_{k,t} + \varepsilon_{kK,t}^e \quad (\text{UIP})$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t}) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Appendix: Aguilar et al. — Firm Problem

CES production: $Y_{ki,f,t} = A_{ki,t} \left[\tilde{\alpha}_{ki}^{1/\psi} N_{fki,t}^{(\psi-1)/\psi} + \tilde{\vartheta}_{ki}^{1/\psi} X_{fki,t}^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{\text{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki} \alpha_{ki} \widehat{w}_{k,t} + \sum_{l=1}^K \sum_{j=1}^I \mathcal{M}_{ki} \omega_{klj} \widehat{p}_{klj,t}$$

where α_{ki} : labour share, ω_{klj} : IO expenditure share, \mathcal{M}_{ki} : steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki} (\widehat{\text{mc}}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p \quad \kappa_{ki} = \frac{(1 - \theta_{ki}^p)(1 - \beta \theta_{ki}^p)}{\theta_{ki}^p}$$

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \tilde{P}_{l,k,i,t}$$

Propagation:

- 1 Tariff on country l raises input prices $\hat{p}_{klij,t}$ for domestic sectors sourcing from sector j in l .
- 2 Higher input costs raise $\widehat{mc}_{ki,t}$, feeding into $\pi_{ki,t}$.
- 3 Cost increases cascade downstream through the IO network.
- 4 Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_i \tau_{kli,t} P_{kli,t}^l \left(C_{kli,t} + \sum_j X_{klji,t} \right) = B_{k,t-1} + \sum_i \tau_{ki}^s MC_{ki,t} Y_{ki,t}$$

Households

- ▶ $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$
- ▶ Energy/non-energy elast. $\gamma = 0.4$
- ▶ Trade elasticity $\delta = 1$
- ▶ Calvo wage $\theta_k^w = 0.75$
- ▶ Consumption shares from OECD ICIO (2019)

Monetary policy

- ▶ $\rho_r = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.125$
- ▶ Target: headline inflation

Firms

- ▶ Labour/input elast. $\psi = 0.5$
- ▶ Energy/non-energy elast. $\phi = 0.4$
- ▶ Trade elasticity $\mu = 1$
- ▶ IO shares from OECD ICIO (2019)
- ▶ Markups from Eurostat Figaro
- ▶ Calvo prices from ECB PRISMA

Tariff shocks

- ▶ $\rho^\tau = 0.96$, $\sigma^\tau = 1$

Market clearing:

$$Y_{ki,t} = \sum_{l=1}^K C_{lki,t} + \sum_{l=1}^K \sum_{j=1}^I X_{lkji,t}$$

Log-linearised:

$$\lambda_{ki} \hat{y}_{ki,t} = \sum_{l=1}^K \mathcal{Y}_{lk} \left(\beta_{lki} \hat{c}_{lki,t} + \sum_{j=1}^I \lambda_{lj} \omega_{lkji} \hat{x}_{lkji,t} \right)$$

where $\lambda_{ki} = P_{ki} Y_{ki} / \mathcal{Y}_k$ is the Domar weight.

Real GDP:

$$\hat{y}_{k,t} = \hat{c}_{k,t} + \Upsilon_k (\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t})$$

where Υ_k is the trade-to-GDP ratio.

Appendix: Derivation — Wage Phillips Curve with Tax

Household FOC with labour income tax $\tau_{k,t}^w$:

$$\sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t \left[N_{k,t+l|t} C_{t+l|t}^{-\sigma} \left(\frac{(1-\tau_{k,t+l}^w) W_{k,t}^*}{P_{t+l}} - \mathcal{M}_{wk,t} \text{MRS}_{k,t+l|t} \right) \right] = 0$$

Log-linearised reset wage:

$$w_{k,t}^* = (1-\beta \theta_k^w) \sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t [\text{mrs}_{t+l|t} + \mu_{wk,t+l}^n + p_{kC,t+l} + \hat{\tau}_{k,t+l}^w]$$

Calvo aggregation ($\pi_{wk,t} = w_{k,t} - w_{k,t-1}$):

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

Appendix: Derivation — Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy $\tau_{ki,t}^s$:

$$\sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[\Lambda_{t,t+l} Y_{ki,t+l|t} \left(P_{ki,t}^* - \mathcal{M}_{pk,t+l} (1 - \tau_{ki,t+l}^s) MC_{ki,t+l|t}^n \right) \right] = 0$$

Log-linearised reset price:

$$p_{ki,t}^* = (1 - \beta \theta_{ki}^p) \sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[mc_{ki,t+l|t}^n + \mu_{pki,t+l}^n - \hat{\tau}_{ki,t+l}^s \right]$$

Calvo aggregation ($\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$):

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Subsidy increase lowers effective marginal cost \rightarrow disinflationary cost-push.

Appendix: Government Budget Dynamics

Starting from the nominal budget constraint:

$$B_{t+1} = B_t(1 + i_{t+1}) + G_{t+1} - T_{t+1}$$

In ratios to GDP ($b_t \equiv B_t/Y_t$):

$$b_{t+1} = b_t \frac{1 + i_{t+1}}{1 + g_{Y,t+1}} + s_{t+1} - \mathcal{T}_{t+1}$$

where $s_t = G_t/Y_t$ and $\mathcal{T}_t = T_t/Y_t$.

Linearised:

$$\hat{b}_{t+1} = \underbrace{\frac{1 + \bar{i}}{1 + \bar{g}_Y}}_{\rho_b} \hat{b}_t + \frac{\bar{i}}{1 + \bar{g}_Y} \hat{i}_t - \frac{\bar{g}_Y}{1 + \bar{g}_Y} \hat{g}_{Y,t+1} + \frac{\bar{s}}{\bar{b}} \hat{s}_{t+1} - \frac{\bar{\mathcal{T}}}{\bar{b}} \hat{\mathcal{T}}_{t+1}$$

Extension: replace lump-sum rebate of tariff revenue with $G_{ki,t}$ financing, creating a feedback loop between trade and fiscal policy.

Appendix: Optimal Fiscal Rule (Unconstrained Budget, $\sigma > 1$, $\kappa > 0$)

$$g_{k,t} = \frac{H_k}{X_k} y_{k,t} + \frac{J_k}{X_k} a_{k,t} - \frac{\theta}{\lambda_k X_k} \pi_{k,t} + \frac{\sigma-1}{(1-\chi)^{1/\sigma} X_k} \sum_k \mu_k \lambda_k \phi_{k,t}^{\pi}$$

$$H_k = \chi_k^* + 1 + \varphi + (\sigma-1) \frac{\omega_{c,k}}{1-\chi_k} - \varphi \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

$$J_k = -(1+\varphi) \left(1 - \lambda_k \frac{1+\chi_k^*+\varphi+\frac{1}{\lambda_k(1-\chi_k)}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$X_k = \chi_k^* + (\sigma-1) \chi_k^* \omega_{c,k} \chi_k + \left(1 + (\sigma-1) \omega_{g,k} \right) \left(\lambda_k + \frac{1+\chi_k^*+\varphi}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}} \right)$$

$$\phi_{k,t}^{\pi} = \left(-\varphi y_{k,t} - g_{k,t} (1 + (\sigma-1) \omega_{g,k}) + (1+\varphi) a_{k,t} \right) \left(\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k} \right)^{-1}$$

Comparing welfare under this vs. the constrained rule quantifies the cost of fiscal inflexibility.

Appendix: Optimal Monetary Rule (Unconstrained Budget)

Optimal monetary policy sets a weighted inflation target:

$$\sum_k \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t} = - \sum_k \mu_k \frac{\varphi y_{k,t} + g_{k,t}(1+(\sigma-1)\omega_{g,k}) + (1+\varphi) a_{k,t}}{\lambda_k \varphi + \frac{\kappa}{\theta-1} \frac{\lambda_k}{1-\chi_k}}$$

- ▶ Inflation weights increase with private-consumption share, decrease with λ_k .
- ▶ Government spending enters the target when $\kappa > 0$ (government demand affects marginal costs).

Appendix: Fixed vs. Endogenous Budgets — Detail

Fully endogenous spending

- ▶ Planner chooses both $\bar{G}_{k,t}$ and $\{G_{ki,t}\}$.
- ▶ Level and composition respond to shocks.
- ▶ Benchmark: first-best fiscal stabilisation.
- ▶ Fiscal rule is *absolute*: each $g_{k,t}$ set independently.

Fixed aggregate budget

- ▶ $\bar{G}_{k,t}$ exogenous; only composition adjusts.
- ▶ Fiscal rule is *relative*: gaps wrt residual sector.
- ▶ Welfare loss from constraint is concentrated in aggregate stabilisation.
- ▶ Cross-sectional allocation may remain close to first-best.

Implications. If the welfare gap is small, a budget-constrained government can approximate first-best outcomes through compositional reallocation alone. This is the central hypothesis to be tested in the networked model.

Appendix: Open Questions

- ▶ **Network centrality and fiscal allocation.** How does a sector's position in the IO network affect its optimal public-good allocation? The IO weights ω_{klij} introduce cross-sector spillovers absent in the simple model.
- ▶ **Fiscal–trade policy interaction.** With tariff revenue financing government purchases, trade policy changes alter the fiscal envelope. The welfare implications of this feedback are to be characterised.
- ▶ **Dimensionality.** The Ramsey problem involves $K \times I$ spending instruments. Practical approaches may require restricting the class of admissible rules.
- ▶ **Political economy.** The exogenous-budget assumption abstracts from the determination of $\bar{G}_{k,t}$. Endogenising this would require a political-economy layer.