

Fiscal Policy in Multi-Sector Economies with Production Networks

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Motivation: Sectoral Heterogeneity and Stabilisation Policy

Question. How important is sectoral heterogeneity for the design of stabilisation policy, and what role should fiscal instruments play?

Networks matter

In input-output (IO) economies, intermediate-input linkages are a *fundamental* driver of cross-sectoral spillovers. Monetary policy targeting **CPI inflation** is suboptimal (Rubbo, 2023).

Fiscal policy restores divine coincidence

Without IO, **optimal sectoral fiscal policy** makes zero-inflation monetary targeting approximately optimal. Without fiscal, the standard result holds (Cox et al., 2024).

This talk: recent theoretical benchmarks (La'O & Tahbaz-Salehi, 2025; Antonova & Müller, 2025) show that rich fiscal instruments restore efficiency — but under assumptions that break in realistic environments. The gap between benchmark and reality is precisely where fiscal policy design becomes essential.

Rubbo (2023): Networks, Phillips Curves, and Monetary Policy

Setup. Multi-sector New Keynesian economy with input-output linkages and heterogeneous price stickiness.

Key mechanism: intermediate-input propagation. Under Cobb–Douglas production ($\psi=1$):

$$\pi_{i,t} = \kappa_i (\widehat{mc}_{i,t} - \hat{p}_{i,t}) + \beta \mathbb{E}_t \pi_{i,t+1}$$
$$\widehat{mc}_{i,t} = \alpha_i \hat{w}_t + \sum_j \omega_{ij} \hat{p}_{j,t} - a_{i,t}$$

The IO term $\sum_j \omega_{ij} \hat{p}_{j,t}$ links sectoral marginal costs: sticky prices in upstream sectors distort downstream costs even when the downstream sector has flexible prices.

Central result

Divine coincidence fails in IO economies. The optimal monetary rule targets the **divine coincidence index**: a weighted inflation index (not CPI). Monetary policy alone cannot close all sectoral gaps simultaneously.

Setup. Multi-sector New Keynesian economy *without* IO linkages. Sectors differ in Calvo parameter θ_k^p and public-good share χ_k .

Key result: when sectoral fiscal policy is optimally conducted, zero-inflation monetary targeting is close to optimal. The welfare approximation:

$$\mathcal{W} \approx -\frac{1}{2} \sum_k \mu_k \left[(1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right]$$

What they find:

- ▶ Optimal policy: MP stabilises aggregates; fiscal stabilises sectors
- ▶ With optimal fiscal, zero-inflation MP is near-optimal (welfare 3.1 vs. 2.8)
- ▶ With **passive fiscal**, MP must target divine coincidence index (welfare 4.7 vs. 6.3)

What they assume:

- ▶ **No input-output linkages**
- ▶ Linear-in-labour production
- ▶ Sectoral shocks do not propagate through intermediate goods

The Theoretical Benchmark: $2N$ Instruments Restore Efficiency

Two recent papers establish the **polar case**: with sufficiently many fiscal instruments, production efficiency is achievable even in IO economies.

La'O & Tahbaz-Salehi (2025)

$2N$ sector-specific taxes implement the **Ramsey optimum** (production efficiency) in multi-sector IO economies.

- ▶ Extends Correia, Nicolini & Teles (2008) to networks
- ▶ Optimal taxes **independent of Calvo parameters**

Antonova & Müller (2025)

$2N$ targeted taxes replicate the **flexible-price allocation** in a Rubbo-style IO framework.

- ▶ “Prices move as if fully flexible”
- ▶ Closed-economy, N -sector Calvo model

Critical shared assumption: both assume **flexible wages** (competitive labour market). The restoration result relies on price-side instruments being sufficient to close all distortionary wedges.

The Gap: From Benchmark to Reality

Benchmark (L&TS, A&M)

- ▶ $2N$ fiscal instruments
- ▶ **Flexible wages** (competitive labour market)
- ▶ Closed economy (A&M)
- ▶ Production efficiency achievable

Our setting (Aguilar et al.)

- ▶ Restricted instruments: τ_k^W, τ_{ki}^S
- ▶ **Calvo wages** ($\theta_k^W = 0.75$)
- ▶ Open economy ($K=4$ countries)
- ▶ Quantitative IO ($I=44$ sectors)

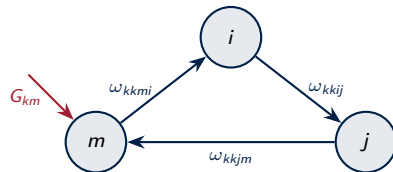
Why staggered wages change everything

L&TS's $2N$ instruments are all price-side taxes; with flexible wages, the wage adjusts freely to clear labour misallocation. With **Calvo wage setting**, the wage is a state variable that price-side instruments cannot directly control. The flexible-price restoration of L&TS and A&M is **off the table**.

Our question: what can *restricted* fiscal instruments achieve in this richer friction environment?

Aguilar et al. (2025): A Rubbo-Style Economy with Staggered Wages

- ▶ $K=4$ countries, $I=44$ sectors
- ▶ Nested CES production with IO linkages (OECD ICIO)
- ▶ Sector-specific Calvo pricing + **Calvo wages** ($\theta_k^w=0.75$)
- ▶ Country-specific Taylor rules



IO network with fiscal entry point

Marginal cost in sector i , country k :

$$\widehat{mc}_{ki,t} = -a_{ki,t} + \underbrace{\mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t}}_{\text{labour}} + \underbrace{\sum_{l,j}\mathcal{M}_{ki}\omega_{klj}\widehat{p}_{klj,t}}_{\text{IO inputs}}$$

This is exactly the IO structure Rubbo identifies as breaking divine coincidence, now at $K \times I$ scale — with the additional complication of staggered wages ([▶ Details](#)).

Our Contribution: Restricted Fiscal Instruments in the Network

We extend Aguilar et al. with two instruments — a **restricted subset** of L&TS's $2N$ instrument set.

Wage Phillips curve — country-level labour income tax τ_k^w :

$$\pi_{wk,t} = \kappa_{wk} \left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

→ Addresses **wage rigidity** that L&TS and A&M do not face (flexible wages).

Price Phillips curve — sector-specific production subsidy τ_{ki}^s :

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

→ Same channel as A&M, but **fewer degrees of freedom** ($K+K \times I$ vs. $2K \times I$).

Key distinction: we operate where the flexible-price restoration of L&TS and A&M is impossible. Both instruments propagate through the IO network that makes monetary policy insufficient.

Goal: what can restricted fiscal instruments achieve when flexible-price restoration is off the table?

1 Sector-specific government purchases in goods market clearing:

$$Y_{ki,t} = \sum_l C_{lki,t} + \sum_l \sum_j X_{lkji,t} + G_{ki,t}$$

2 Optimal fiscal rule in the networked economy with staggered wages: characterise how IO structure and wage rigidity jointly shape optimal spending allocation.

3 The critical test: do the L&TS/A&M efficiency results survive under (a) restricted instruments, (b) Calvo wage setting, (c) open-economy spillovers?

4 Two policy regimes: fully endogenous spending (level + composition) vs. fixed aggregate budget (composition only). Quantify the welfare gap relative to the $2N$ -instrument benchmark.

- 1 The polar case:** La'O & Tahbaz-Salehi (2025) and Antonova & Müller (2025) show that $2N$ fiscal instruments restore production efficiency in multi-sector IO economies — under **flexible wages**.
- 2 The break:** Aguilar et al.'s **Calvo wage setting** fundamentally changes the problem. Price-side instruments alone cannot close wage-rigidity-induced misallocation. The benchmark restoration result is off the table.
- 3 Our agenda:** test what *restricted* fiscal instruments (labour tax + production subsidy) can achieve in this richer environment — a quantitative open-economy IO model with staggered wages.

Conjecture: restricted instruments cannot fully restore production efficiency, but sectoral *composition* of spending will matter more than its aggregate level.

Thank you

Production Networks & NK

- ▶ Acemoglu et al. (2012)
- ▶ Baqaee & Farhi (2020, 2024)
- ▶ Pasten, Schoenle & Weber (2020)
- ▶ **Rubbo (2023)**
- ▶ La'O & Tahbaz-Salehi (2024)

Tariffs & Open-Economy NK

- ▶ Galí & Monacelli (2005)
- ▶ Comin & Johnson (2023)
- ▶ **Aguilar et al. (2025)**

Fiscal Policy in Disaggregated Economies

- ▶ Aoki (2001)
- ▶ **La'O & Tahbaz-Salehi (2025, WP)**
- ▶ **Antonova & Müller (2025)**
- ▶ **Cox et al. (2024)**

Fiscal–Price Effects (Empirical)

- ▶ Nekarda & Ramey (2020)
- ▶ Ben Zeev & Pappa (2017)

Appendix: Rubbo (2023) — Model Detail

Network Phillips curve. Under Cobb–Douglas production ($\psi = 1$) and Calvo pricing:

$$\pi_t = \kappa \Psi \hat{\mathbf{w}}_t + \beta \mathbb{E}_t \pi_{t+1}$$

where $\Psi = (\mathbf{I} - \Omega)^{-1}$ is the Leontief inverse and $\kappa = \text{diag}(\kappa_1, \dots, \kappa_N)$. (The general CES case involves additional relative-price terms.)

Key insight: the Leontief inverse maps wage costs into sectoral inflation. A sector with flexible prices but upstream sticky suppliers still experiences inflation distortions through Ψ .

Optimal monetary policy (divine coincidence index):

$$\sum_i \tilde{\mu}_i \hat{y}_{i,t} = 0 \quad \text{where weights } \tilde{\mu}_i \text{ depend on sales shares } \lambda_i \text{ and price adjustment frequencies}$$

Equivalently, the planner targets a specific weighted inflation index (the “divine coincidence index”), not CPI. The weights over-weight sectors that are: (i) large (high Domar weight λ_i), (ii) sticky (low price adjustment frequency), and (iii) upstream (high influence through the Leontief inverse).

Failure of zero inflation: targeting $\pi_{i,t} = 0 \forall i$ requires all marginal cost gaps to be zero. With IO linkages, this is generically impossible because upstream price distortions propagate to downstream costs.

Appendix: Cox et al. (2024): Key Quantitative Results

Model. N sectors, no IO. Production: $Y_{k,t} = A_{k,t} N_{k,t}$. Calvo pricing (θ_k^p), government share χ_k .

Four policy regimes (Table 3, baseline U.S. calibration, welfare loss):

	Optimal fiscal	Passive fiscal ($\tilde{f}_{kt} = 0$)
Optimal MP	2.8	4.7
Zero-inflation MP	3.1	6.3

Interpretation: with optimal sectoral fiscal, zero-inflation MP is approximately optimal (3.1 vs. 2.8). Without fiscal, the standard result holds: optimal MP must target the divine coincidence index (4.7 vs. 6.3).

Critical assumption: without IO, sectoral marginal costs depend only on own wages and productivity. There is *no channel* for upstream price distortions to affect downstream sectors, precisely the mechanism Rubbo identifies as first-order.

Appendix: Welfare with $\sigma \neq 1$ and $\kappa \neq 0$

When CRRA preferences and government-demand pass-through are active:

$$-\frac{1}{2} \sum_k \mu_k \left((1+\varphi) y_{k,t}^2 + \frac{\theta(1-\chi_k)}{\lambda_k} \pi_{k,t}^2 + \chi_k^* (g_{k,t} - y_{k,t})^2 \right. \\ \left. + (\sigma-1) \left[(1-\chi_k) \omega_{c,k} \left(\frac{y_{k,t}}{1-\chi_k} - \chi_k^* g_{k,t} \right)^2 + \omega_{g,k} \chi_k g_{k,t}^2 \right] \right)$$

- ▶ The $\sigma-1$ term introduces an **insurance motive**: the planner uses sectoral fiscal policy to hedge against aggregate risk.
- ▶ $\kappa > 0$ steepens Phillips curves ($\lambda'_k > \lambda_k$), making fiscal policy a supply-side instrument.

Appendix: Relative Allocation Rule — Structural Coefficients

Under exogenous \bar{G}_t (with $\sigma = 1$, $\kappa = 0$):

$$\begin{aligned} g_{k,t} = & \frac{1-\chi_k}{1-\chi_i} \left(\frac{1+\lambda_i+\varphi\lambda_i}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} g_{i,t} \\ & - \frac{\varphi y_{k,t}}{1+\lambda_k+\varphi\lambda_k} - \frac{\theta \varphi (1-\chi_k) \pi_{k,t}}{1+\lambda_k+\varphi\lambda_k} \\ & + \frac{1-\chi_k}{1-\chi_i} \left(\frac{\omega_{g,k}}{\omega_{g,i}} \right)^\rho \left(\frac{1+\lambda_k+\varphi\lambda_k}{1+\lambda_k+\varphi\lambda_k(1-\chi_k)} \right)^{-1} \\ & \times \left(\frac{\varphi y_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} + \frac{\theta \varphi (1-\chi_i) \pi_{i,t}}{1+\lambda_i+\varphi\lambda_i(1-\chi_i)} \right) \end{aligned}$$

$\omega_{g,k}$: weight of sector k in the government Cobb–Douglas aggregator. ρ : CES elasticity of public-good bundle.

Appendix: Aguilar et al. —Household Problem

Per-period utility: $U_t = \left(C_{k,t}^{1-\sigma} / (1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi} / (1+\varphi) dg \right) Z_{k,t}$

Consumption nested CES (energy/non-energy, domestic/foreign):

$$C_{k,t} = \left[\tilde{\beta}_k^{1/\gamma} C_{kE,t}^{(\gamma-1)/\gamma} + (1-\tilde{\beta}_k)^{1/\gamma} C_{kM,t}^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}$$

Euler equations:

$$C_{k,t}^{-\sigma} = \beta \mathbb{E}_t C_{k,t+1}^{-\sigma} \frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}}$$
$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{kK,t+1} - \gamma_* \text{nfa}_{k,t} + \varepsilon_{kK,t}^e \quad (\text{UIP})$$

Calvo wage setting yields:

$$\pi_{wk,t} = \kappa_{wk} \left(\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} \right) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Appendix: Aguilar et al. —Firm Problem

CES production: $Y_{ki,f,t} = A_{ki,t} \left[\tilde{\alpha}_{ki}^{1/\psi} N_{fki,t}^{(\psi-1)/\psi} + \tilde{\vartheta}_{ki}^{1/\psi} X_{fki,t}^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}$

Intermediate bundle mirrors the household CES nesting (energy/non-energy, domestic/foreign).

Log-linearised marginal cost:

$$\widehat{\text{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki} \alpha_{ki} \widehat{w}_{k,t} + \sum_{l=1}^K \sum_{j=1}^I \mathcal{M}_{ki} \omega_{klj} \widehat{p}_{klj,t}$$

where α_{ki} : labour share, ω_{klj} : IO expenditure share, \mathcal{M}_{ki} : steady-state markup.

Calvo pricing yields:

$$\pi_{ki,t} = \kappa_{ki} (\widehat{\text{mc}}_{ki,t} - \widehat{p}_{ki,t}) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p \quad \kappa_{ki} = \frac{(1 - \theta_{ki}^p)(1 - \beta \theta_{ki}^p)}{\theta_{ki}^p}$$

Appendix: Aguilar et al. —Tariff Propagation

Tariffs enter as price wedges on final and intermediate goods:

$$P_{k,l,i,t} = (1 + \tau_{k,l,i,t}) \tilde{P}_{l,k,i,t}$$

Propagation:

- 1 Tariff on country l raises input prices $\hat{p}_{klj,t}$ for domestic sectors sourcing from sector j in l .
- 2 Higher input costs raise $\widehat{mc}_{ki,t}$, feeding into $\pi_{ki,t}$.
- 3 Cost increases cascade downstream through the IO network.
- 4 Tariff revenue accrues to the government; currently rebated lump-sum.

Government budget constraint:

$$\begin{aligned} \frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_i \tau_{kli,t} P_{kli,t}^l \left(c_{kli,t} + \sum_j x_{klji,t} \right) \\ = B_{k,t-1} + \sum_i \tau_{ki,t}^s MC_{ki,t} Y_{ki,t} \end{aligned}$$

Households

- ▶ $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$
- ▶ Energy/non-energy elast. $\gamma = 0.4$
- ▶ Trade elasticity $\delta = 1$
- ▶ Calvo wage $\theta_k^w = 0.75$
- ▶ Consumption shares from OECD ICIO (2019)

Monetary policy

- ▶ $\rho_r = 0.7$, $\phi_\pi = 1.5$, $\phi_y = 0.125$
- ▶ Target: headline inflation

Firms

- ▶ Labour/input elast. $\psi = 0.5$
- ▶ Energy/non-energy elast. $\phi = 0.4$
- ▶ Trade elasticity $\mu = 1$
- ▶ IO shares from OECD ICIO (2019)
- ▶ Markups from Eurostat Figaro
- ▶ Calvo prices from ECB PRISMA

Tariff shocks

- ▶ $\rho^\tau = 0.96$, $\sigma^\tau = 1$

Appendix: Derivation —Wage Phillips Curve with Tax

Household FOC with labour income tax $\tau_{k,t}^w$:

$$\sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t \left[N_{k,t+l|t} C_{k,t+l|t}^{-\sigma} \left(\frac{(1-\tau_{k,t+l}^w) W_{k,t}^*}{P_{kC,t+l}} - \mathcal{M}_{wk,t} \text{MRS}_{k,t+l|t} \right) \right] = 0$$

Log-linearised reset wage:

$$w_{k,t}^* = (1-\beta \theta_k^w) \sum_{l=0}^{\infty} (\beta \theta_k^w)^l \mathbb{E}_t [\text{mrs}_{t+l|t} + \mu_{wk,t+l}^n + p_{kC,t+l} + \hat{\tau}_{k,t+l}^w]$$

Calvo aggregation ($\pi_{wk,t} = w_{k,t} - w_{k,t-1}$):

$$\pi_{wk,t} = \kappa_{wk} (\sigma \hat{c}_{k,t} + \varphi \hat{n}_{k,t} - \hat{w}_{k,t} + \hat{\tau}_{k,t}^w) + \beta \mathbb{E}_t \pi_{wk,t+1} + u_{k,t}^w$$

Tax deviation enters additively: wage-setters pass the wedge through to the pre-tax wage.

Appendix: Derivation —Price Phillips Curve with Subsidy

Firm FOC with time-varying production subsidy $\tau_{ki,t}^s$:

$$\sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[\Lambda_{t,t+l} Y_{ki,t+l|t} \left(P_{ki,t}^* - \mathcal{M}_{pk,t+l} (1 - \tau_{ki,t+l}^s) MC_{ki,t+l|t}^n \right) \right] = 0$$

Log-linearised reset price:

$$p_{ki,t}^* = (1 - \beta \theta_{ki}^p) \sum_{l=0}^{\infty} (\beta \theta_{ki}^p)^l \mathbb{E}_t \left[mc_{ki,t+l|t}^n + \mu_{pki,t+l}^n - \hat{\tau}_{ki,t+l}^s \right]$$

Calvo aggregation ($\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$):

$$\pi_{ki,t} = \kappa_{ki} \left(\widehat{mc}_{ki,t} - \hat{p}_{ki,t} - \hat{\tau}_{ki,t}^s \right) + \beta \mathbb{E}_t \pi_{ki,t+1} + u_{ki,t}^p$$

Subsidy increase lowers effective marginal cost \rightarrow disinflationary cost-push.

Q&A: Why Not Derive the Ramsey Problem Directly?

The question

Why modify Phillips curves with fiscal instruments rather than solving a full Ramsey taxation problem?

- ▶ The Phillips-curve approach is a **reduced-form shortcut**: it isolates the cost-push channel of taxation while preserving the existing IO structure of Aguilar et al. (2025).
- ▶ A full Ramsey problem in a 4×44 networked model is computationally demanding: it requires solving for $K \times I$ optimal tax instruments simultaneously.
- ▶ The **research agenda goal** is precisely to move toward the full Ramsey characterisation. The current extensions are a tractable first step.
- ▶ La'O & Tahbaz-Salehi (2025) show that even in simpler networks, the Ramsey problem has a rich structure — $2N$ instruments implement production efficiency. Our approach builds intuition before scaling up.

Q&A: How Does This Relate to Antonova & Müller (2025)?

The question

Antonova & Müller already study fiscal policy in Rubbo's IO framework. What is your contribution?

Antonova & Müller (2025):

- ▶ $2N$ targeted taxes replicate flexible-price allocation
- ▶ **Flexible wages** (competitive labour market)
- ▶ Closed-economy framework
- ▶ The **polar case**: sufficient instruments + simple frictions

Our contribution:

- ▶ Restricted instruments ($\tau_k^w + \tau_{ki}^s$)
- ▶ **Calvo wages** ($\theta_k^w = 0.75$)
- ▶ Quantitative open economy ($K \times I$)
- ▶ What can restricted instruments achieve?

Key distinction: A&M establish the polar case (flexible-price restoration with $2N$ instruments and flexible wages). We test what restricted instruments can achieve when staggered wages make that restoration impossible.

The question

La'O & Tahbaz-Salehi have two relevant papers. How do they relate to your work?

Econometrica (2024): optimal *monetary* policy in production networks; network structure shapes the optimal inflation target.

“Missing Tax Instruments” (2025 WP):

- ▶ $2N$ taxes implement Ramsey optimum (production efficiency)
- ▶ Optimal taxes **independent of Calvo parameters**
- ▶ Extends Correia, Nicolini & Teles (2008) to IO

Key distinction: L&TS provide the theoretical ceiling. We ask how far restricted instruments can go when wage rigidity makes that ceiling unattainable.

Our contribution:

- ▶ Restricted instruments ($\tau_k^w + \tau_{ki}^s$, not $2N$)
- ▶ **Calvo wages** break their flexible-wage assumption
- ▶ Quantitative open economy ($K \times I$)
- ▶ Their Ramsey optimum is our benchmark

Q&A: Why Not Just Use $2N$ Instruments?

The question

La'O & Tahbaz-Salehi show $2N$ taxes implement production efficiency. Why not use them?

- 1 Instrument restriction:** $2N$ sector-specific taxes require the government to target each sector individually. In practice, labour income taxes are set at *country level* (τ_k^w). Our instrument set ($K + K \times I$) is a strict subset of $2N$.
- 2 Staggered wages:** L&TS and A&M assume **flexible wages**. With Calvo wage setting ($\theta_k^w = 0.75$), wage-rigidity misallocation creates distortions that *no price-side taxes* can address.
- 3 Open economy:** L&TS study a closed economy. Cross-border IO linkages and exchange rate dynamics introduce additional channels absent from the benchmark.

Bottom line: the $2N$ -instrument result is the polar case. We study the realistic setting where that ceiling is unattainable.