## Document task "brake squeak"

**Title:**

Numerical calculation of a simplified Karnopp model for simulating brake groan and implementation in MATLAB.

**Task:**

Brake squeal is a low-frequency brake noise phenomenon that results from vibration between the brake disc and caliper at low speeds. The oscillation is excited by the stick-slip effect between the brake lining and the brake disc, which is to be modeled and calculated below.

Given is a simplified wheel brake in disc brake design according to Figure 2 consisting of brake disc, brake pads and brake caliper. The wheel brake system is driven via the hub with an input speed *n in* . The brake pads are pressed against the brake disk by the brake caliper with a clamping force *F clamping* , which, together with the coefficient of friction µ, leads to a friction force *F friction* . The braking torque or the friction torque *M friction is thus generated* via the friction diameter *d friction* .

Figure 3 shows the associated equivalent model of the wheel brake for modeling the stick-slip effect using a simplified Karnopp model. The actual Karnopp model for linear motion needs 5 parameters to calculate:

* Mass / inertia *m* of the moving body
* Coulomb friction *F c* (sliding friction - slip)
* Static friction *F s* (stick)
* Viscose friction *F v*
* Speed range to differentiate stick and slip *dv*

The frictional force is calculated as follows:

(Eq. 1)

*F e (t)* is an external force (eg spring force in the model, see Figure 3 ).

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| --- | --- |
| **Figure 1** : Karnopp 's law of friction | Figure 1 shows the Karnopp friction model again graphically. The hatched speed range 2∙ *dv* corresponds to the static friction range (stick). Outside the range there is sliding friction (slip). |

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| **Figure 2** : Wheel brake (disc brake) | **Figure 3** : Wheel brake replacement model |

**Assumptions, boundary conditions and notes:**

* Moving masses of the wheel brake are combined into the equivalent moment of inertia *J replacement* (see Figure 3 ).
* Stiffnesses of the system, which lead to elastic deformations, are summarized in the equivalent torsional stiffness *c replacement* (see Figure 3 ).
* The frictional force, which acts at two contact points between the brake pad and brake disc, is combined via the effective friction diameter friction *to form* a resulting braking torque/friction torque
* The viscous friction of the Karnopp model (see Eq. 1) is neglected (see Figure 3 )

**The following subtasks are to be processed:**

1. Write down the ODE system in symbolic notation for this model.
2. Develop the numerical calculation rule for this ODE system based on the 4th order Runge- Kutta method.
3. Implement this rule using MATLAB, with the following functionality:

* Parameter entry in a Graphical User Interface (GUI). The parameters should be entered with the units from Table 1 . In addition, the values from Table 1 should be available as default parameters. (Note: It is recommended to convert the angle-related parameters to radian-based quantities for further calculations after reading them in).
* Start the calculation by clicking the "Start calculation" button
* Graphical representation of the entrance angle *ϕin* [°] , of the disc *ϕangle disc* [°] and the disk speed *n disk* [min -1 ] each as a function of time by clicking the "Display" button.

1. Calculate the time course of the input angle, the disk angle and the disk speed for the following parameters:

**Table 1** : Default parameters for parameter input via GUI

|  |  |  |  |
| --- | --- | --- | --- |
| **parameter** | **symbol** | **value** | **Unit** |
| Substitute torsional stiffness | *c replacement* | 5236 | Nm /° |
| equivalent moment of inertia | *J replacement* | 0.122 | kg∙m² |
| coefficient of static friction | *µ Stick* | 0.60 |  |
| coefficient of sliding friction | *µ briefs* | 0.49 |  |
| reaming diameter | *d friction* | 0.3125 | m |
| resilience | *F instep* | 200 | N |
| input speed | *n in* | 0.05 | min -1 |
| initial conditions | *ϕslice (t=0)* | 0 | ° |
|  | *n slice (t = 0)* | 0 | min -1 |
| Speed range static friction | *dn* | 5∙10 -5 | min -1 |
| calculation increment | *Δt* | 0.000001 | s |
| calculation time | *t end* | 0.1 | s |

1. Answer the following questions:
2. What influence do the coefficient of static friction *µ Stick* , the coefficient of sliding friction *µ Slip* and their difference have on the stick-slip effect and thus on brake groaning?
3. What does the phrase "one-step" procedure mean?
4. For which mathematical problems are the Runge- Kutta methods numerical solution methods?
5. Numerical solution methods such as Euler 's line method or the Runge- Kutta method only provide approximate solutions of differential equations. In which cases are particularly big mistakes made here? What options are there to minimize these errors?
6. Outline (principle representation) one possibility of a mechanical solution for a first-order delay element (PT 1 element) and state its differential equation.

**Result for orientation:**

The graphic solution of a calculation **with other parameters is used for orientation** Figure 4 indicated:



**Figure 4** : Sample solution for the "squealing brakes" task

Submission: - Task 1, 2, 5: written documentation

- Task 3, 4: MATLAB program

- please note the MATLAB version used for programming