Assignment 8

Chapter 10: PID Control

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CHAPTER 10 (ASSIGNMENT 8) PID Control

10.1(Ideal PID controllers) Consider the systems represented by the block diagrams in Figure10.1.Assume that the process has the transfer function P(s)=b/(s+a) and show that the transfer functions from r to y are

$$(a)\quad G_{yr}(s) = \frac{bk_d s^2 + bk_p s + bk_i}{(1 + bk_d)s^2 + (a + bk_d)s + bk_i},$$

$$(b)\quad G_{yr}(s) = \frac{bk_i}{(1 + bk_d)s^2 + (a + bk_d)s + bk_i}.$$

Pick some parameters and compare the step responses of the systems.

Answer

$$P(s) = \frac{b}{s+a}$$

In PID system

$$C(s) = k_p + k_d s + k_i/s$$

$$T.F = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

$$P(s)C(s) = \frac{bk_p s + bk_d s^2 + bk_i}{s^2 + as}$$

$$T.F = \frac{bk_p s + bk_d s^2 + bk_i}{s^2 + as + bk_p s + bk_d s^2 + bk_i}$$

$$T.F = \frac{bk_p s + bk_d s^2 + bk_i}{(1 + bk_d)s^2 + (a + bk_p)s + bk_i} = G_{yr}(s)$$

Due to the difficulties in implementing and studying the effects on the system, so we can use PI instead of PID with small gain to reach reference signal

$$G_{yr}(s) = \frac{bk_i}{(1 + bk_d)s^2 + (a + bk_p)s + bk_i}$$

10.2 Consider a second-order process with the transfer function

$$P(s) = \frac{b}{s^2 + a_1 s + a_2}.$$

The closed loop system with a PI controller is a third-order system. Show that it is possibletopositiontheclosedlooppolesaslongasthesumofthepolesis−a1.Give equations for the parameters that give the closed loop characteristic polynomial

$$(s + \alpha_0)(s^2 + 2\zeta_0\omega_0 s + \omega_0^2).$$

Answer:

$$P(s) = \frac{b}{s^2 + a_1 s + a_2}$$

In PI system

$$C(s) = k_p + k_i/s$$

$$T.F = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

$$T.F = \frac{bk_p s + bk_i}{s^3 + a_1 s^2 + (a_2 + bk_p)s + bk_i}$$

the parameters that give the closed loop characteristic polynomial

$$(s + \alpha_0)(s^2 + 2\zeta_0\,\omega_0 s + \omega_0{}^2)$$
$$= s^3 + 2\zeta_0\,\omega_0 s^2 + \omega_0{}^2 s + \alpha_0 s^2 + 2\zeta_0\,\omega_0\alpha_0 s + \alpha_0\omega_0{}^2$$

$$a_1 = 2\zeta_0\,\omega_0 + \alpha_0$$

$$k_i = \frac{\alpha_0\omega_0{}^2}{b}$$

$$(a_2 + bk_p) = \omega_0{}^2 + 2\zeta_0\,\omega_0\alpha_0$$

$$a_2 = \omega_0{}^2 + 2\zeta_0\,\omega_0\alpha_0 - bk_p$$

10.3 Consider a system with the transfer function P(s) = (s + 1)−2. Find an integral controller that gives a closed loop pole at s =– a and determine the value of a that maximizes the integral gain. Determine the other poles of the system and judge if the pole can be considered dominant. Compare with the value of the integral gain given by equation (10.6).

Answer

$$P(s) = (s+1)^{-2}$$

$$C(s) = \frac{k_i}{s}$$

$$T.f = \frac{P(s)C(s)}{1+P(s)C(s)}$$

$$T.f = \frac{k_i}{s(s+1)^2 + k_i}$$

$$T.f = \frac{k_i}{s^3 + 2s^2 + s + k_i}$$

$$T.f = \frac{k_i}{(s-1)(as^2 + bs + c)}$$

$$a = 1, b - a = 2\,, c - b = 1$$

$$a = 1, b = 3, c = 4$$

$$T.f = \frac{k_i}{(s-1)(s^2 + 3s + 4)}$$

$$s = 1\,, -1.5 + 1.32j\,, -1.5 - 1.35j$$

$$k_i = \frac{1}{2\grave{P}(0)}$$

$$\grave{P}(0) = -2(s+1)^{-3} = -2$$

$$k_i = 0.25$$

$$T.f = \frac{0.25}{s^3 + 2s^2 + s + 0.25}$$

$$s = -1.42\,, -0.29 + 0.3j\,, -0.29 - 0.3j$$

Dominant poles because it locate near the origin and have a huge effect on stability

10.4 (Ziegler–Nichols tuning) Consider a system with transfer function P(s) = e−s/s. Determine the parameters of P, PI and PID controllers using Ziegler–Nichols step and frequency response methods. Compare the parameter values obtained by the different rules and discuss the results.

Answer

$$P(s) = e^{-s}/s$$

	Ziegler- Nichols method			Frequency response method		
	k_p	T_i	T_d	k_p	T_i	T_d
P	1/a	-	-	$0.5k_c$		
PI	0.9/a	3Ʈ	-	$0.4k_c$	$0.8T_c$	
PID	1.2/a	2Ʈ	0.5Ʈ	$0.6k_c$	$0.5T_c$	$0.125T_c$

Drawbacks:

1- tiny process in formation is used
2- lack robustness for closed loop system response

$$P(t) = L^{-1}\left(\frac{e^{-s}}{s}\right)$$
$$F(t-1)u(t-1) = u(t-1)$$

Data form frequency response method more robustness than Ziegler- Nichlos method

10.5 (Vehicle steering) Design a proportional-integral controller for the vehicle steering system that gives the closed loop characteristic polynomial

$$s^3 + 2\omega_\circ s^2 + 2\omega_\circ s + \omega_\circ{}^3$$

Answer

Vehicle steering

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} \gamma \\ 1 \end{bmatrix}, C = [1 \quad 0], \; D = [0]$$

$$P(s) = s^2 + 2\eta_c\omega_c s + \omega_c{}^2$$

$$T.f = s^2 + (\gamma k_1 + k_2)s + k_1$$

$$k_1 = \omega_c{}^2 \quad , \; k_2 = 2\eta_c\omega_c - \gamma k_1$$

$$C(s) = k_p + \frac{k_i}{s}$$

$$T.f = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

$$T.f = \frac{(k_p s + k_i)(s^2 + 2\eta_c\omega_c s + \omega_c{}^2)}{s + (k_p s + k_i)(s^2 + 2\eta_c\omega_c s + \omega_c{}^2)}$$

$$2\omega_0 = 2\eta_c\omega_c k_p + k_i$$

$$\omega_0 = \eta_c\omega_c k_p + 0.5k_i$$

$$\omega_0{}^3 = k_i\omega_c{}^2$$

$$\omega_0 = 0.5(\omega_c{}^2 k_p + 2\eta_c\omega_c k_i + 1)$$