

Assignment 4



A–10–2.

**Consider a completely state controllable system**

**Define the controllability matrix as M:**

**Show that**

**where ,,…, are the coefficients of the characteristic polynomial**

Solution. **Let us consider the case where n=3. We shall show that**

**(10-140)**

**The left-hand side of Equation (10-140) is**

**The right-hand side of Equation (10-140) is**

**(10-141)**

**The Cayley-Hamilton theorem states that matrix A satisfies its own characteristic equation or, in the case of n=3.**

**(10-142)**

**Using Equation (10-142), the third column of the right-hand side of Equation (10-141) becomes**

**Thus Equation (10-141) becomes**

**Hence, the left-hand side and the right-hand side of Equation (10-140) are the same.**

**We have thus shown that Equation (10-140\_) is true. Consequently,**

**The preceding derivation can be easily extended to the general case of any positive integer n.**

A-10-4.

**Consider the state equation**

**where**

**The rank of the controllability matrix M,**

**is 2. Thus, the system is completely state controllable. Transform the given state equation into the controllable canonical form.**

Solution. **Since**

**we have**

**Define**

**where**

**Then**

**and**

**Define**

**Then the state equation becomes**

**Since**

**we have**

**which is in the controllable canonical form.**

A–10–6.

**A regulator system has a plant**

**Define state variable as**

**By use of the state-feedback control , it is described to place the closed-loop poles at**

**Obtain the necessary state-feedback gain matrix K with MATLAB.**

**Solution.** **The state-space equations for the system become**

**Hence,**

**(Note that, for the pole placement, matrices C and D do not affect the state-feedback gain matrix K.)**

**Two MATLAB programs for obtaining state-feedback gain matrix K are given in MATLAB Programs 10-24 and 10-25**

|  |
| --- |
| MATLAB Program 10–24 |
| A = [0 1 0;0 0 1;-6 -11 -6];  B = [0;0;10];  J = [-2+j\*2\*sqrt(3) -2-j\*2\*sqrt(3) -10];  K = acker(A,B,J)  K =  15.4000 4.5000 0.8000 |

|  |
| --- |
| MATLAB Program 10–25 |
| A = [0 1 0;0 0 1; -6 -11 -6];  B = [0;0;10];  J = [-2+j\*2\*sqrt(3) -2-J\*2\*Sqrt(3) -10];  K = place(A,B,J)  place: ndigits= 15  K =  15.4000 4.5000 0.8000 |

A–10–9.

**Consider the system defined by**

**where**

**The rank of the observability matrix N,**

**is 2. Hence, the system is completely observable. Transform the system equations into the observable canonical form.**

**Solution.**

**Since**

**we have**

**Define**

**where**

**and**

**Define**

**Then the state equation becomes**

**or**

**The output equation becomes**

**or**

A–10–10.

**For the system defined by**

**consider the problem of designing a state observer such that the desired eigenvalues for the observer gain matrix are**

**Show that the observer gain matrix given by Equation (10–61), rewritten as**

**(10-159)**

**can be obtained from Equation (10-13) by considering the dual problem. That is, the matrix can be determined by considering the pole-placement problem for the dual system, obtaining the state-feedback matrix K, and taking its conjugate transpose, or .**

**Solution.**

**The dual of the given system is**

(10-160)

**Using the state-feedback control**

**Equation (10-160) becomes**

**Equation (10-13), which is rewritten here, is**

(10-161)

**where**

**Hence, matrix T can also be written as**

**Since , we have**

**and**

**Taking the conjugate transpose of both sides Equation (10-146), we have**

**Since , this last equation is the same as Equation (10-159). Thus, we obtained Equation (10-159) by considering the dual program.**