

Assignment 5



A–10–3.

**Consider a completely state controllable system**

**Define**

**and**

**where ’s are the coefficients of the characteristic polynomial**

**Define also**

**Show that**

Solution. **Let us consider the case where n=3. We shall show that**

**(10-143)**

**Referring to Problem A-10-2, we have**

**Hence Equation (10-143) can be rewritten as**

**Therefore, we need to show that**

**(10-144)**

**The left-hand side of Equation (10-144) is**

**The right-hand side of Equation (10-144) is**

**Clearly, Equation (10-144) holds true. Thus, we have shown that,**

**Next, we shall show that**

**Note that Equation (10-145) can be written as**

**Nothing that**

**we have**

**The derivation shown here can be easily extended to the general case of any positive integer n.**

A-10-12.

**Consider a completely state controllable system defined by**

**(10-167)**

**where**

**Suppose that the rank of the following matrix**

**is . Show that the system defined by**

(10-168)

**where**

**is completely state controllable.**

Solution. **Define**

**Because the system given by Equation (10-167) is completely state controllable, the rank of matrix M is n. Then the rank of**

**is n+1. Consider the following equation:**

**Since matrix**

**Is of rank n+1, the left-hand side of Equation (10-169) is of rank n+1. Therefore, the right-hand side of Equation (10-169) is also of rank n+1. Since**

**We find that the rank of**

**is n+1. Thus, the system defined by Equation (10-168) is completely state controllable.**

B–10–5.

**Consider the system defined by**

**Show that this system cannot be stabilized by state-feedback control , whatever matrix is chosen.**

**Solution.** **Substituting**

**into the state equation, we obtain**

**The characteristic equation becomes**

**Because of the presence of one eigenvalue (s=2) in the right-half s plane, the system is unstable whatever values and may assume.**

B–10–17.

**Consider the system defined by**

**Where**

**Determine the value of the parameter a so as to minimize the following performance index:**

**Assume that the initial state is given by**

**Solution.**

**To determine the parameter a in matrix A, we first determine matrix**

**or**

**The result is**

**Then we can obtain the optimal value of the parameter a that minimizes the performance index J for any given initial condition x(0). Since x(0) is given by**

**the performance index J can be simplified to**

**Therefore, we obtain**

**To minimize J, we determine a from =0, or**

**from which we get**

**Since is specified to be positive, we discard the negative value of .**

**Thus we choose Nothing that satisfies the condition for the minimum (>0), the optimal value of is 1.823.**

B–10–19.

**Determine the optimal control signal u for the system defined by**

**where**

**such that the following performance index is minimized:**

**Solution.**

**The optimal control signal u will have the form . Therefore, the performance index J becomes**

**Since R=I in this problem, Equation (10-15) becomes**

**and Equation (10-117) becomes**

**where P is determined from the reduced matrix Riccati equation:**

**Solving for P, requiring that it be positive definite, we obtain**

**The optimal feedback gain matrix K becomes**

**Thus, the optimal control signal u is given by**