

Assignment 6



A–7–1.

**(Coordinate transformations) Consider a system under a coordinate transformation , where****is an invertible matrix. Show that the observability matrix for the transformed system is given by and hence observability is independent of the choice of coordinates.**

**Solution.**

**where is an invertible matrix**

**, ,**

A–7–2.

**Show that the system depicted in Figure 7.2 is not observable.**

Solution.

**Where the observability matrix is:**

**So, system is observable only if**

**If**

**Rank of matrix is 1 not fully ranked, so the system is not observable because the system is composed of two identical.**

A–7–3.

**7.3 (Observable canonical form) Show that if a system is observable, then there exists a change of coordinates that puts the transformed system into observable canonical form.**

**Solution.**

**System observable canonical form**

.

**Where \* represents an entry whose exact value is not important.**

**The rows of this matrix are linearly independent (since it is lower triangular), and hence is full rank**

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**As in the case of reachability, it turns out that if a system is observable then there always exists a transformation T that converts the system into observable canonical form. This is useful for proofs since it lets us assume that a system is in reachable canonical form without any loss of generality.**

A–7–4.

**(Bicycle dynamics) The linearized model for a bicycle is given in equation (3.5), which has the form**

**where is the tilt of the bicycle and *δ* is the steering angle. Give conditions under which the system is observable and explain any special situations where it loses observability.**

**Solution.**

**Where the elements of the2 ×2matrices and depend on the geometry and the mass distribution of the bicycle.**

**Since the matrix is full ranked, so the system is observable for the more accurate model called “whipple model” causing of missing observability when neglect the degree of freedom and treat with model as two plane with fixed coordinates with respect to bike or wheel and neglect fraction with earth.**

A–7–7.

**(Uniqueness of observers) Show that the design of an observer by eigenvalue assignment is unique for single-output systems. Construct examples that show that the problem is not necessarily unique for systems with many outputs.**

**Solution.**

**With 1, 0 input, 1, 0 output**

**Characteristic polynomial of A if system is observable dynamics is:**

**is an observable of system, L is chosen as**

**Resulting observer error is given by**

**Dynamic system is called observer of the system from its input and output. It is much more useful from the one given by pure differential unique because of depending on system input and the change from system to system.**

**The example of vehicle steering system**

**Steering angle is**

**Observability matrix**

**This matrix is an identic matrix, so the system is observable**

**Assume observer polynomial**

**Observer is**