EECE-5666 Midterm-1 Exam Solutions 2023-SPRG

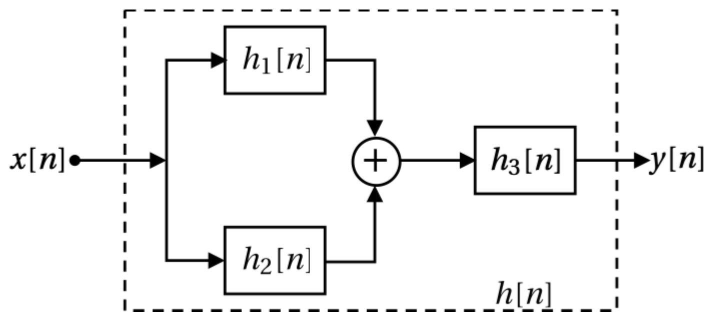
[40-Points]

Default Plot Parameters:

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

Problem-1 (10-points)

Consider the system shown below.



The subsystem impulse responses are

$$h_1(n) = \delta(n) - \frac{2}{3}\delta(n-1); \qquad h_2(n) = \frac{1}{9}\delta(n-2); \qquad h_3(n) = (3)^{1-n}u(n).$$

Let $h[n]$ be the overall system impulse response for which $x[n]$ is the input and $y[n]$ is the output.

(a) [3-Points] Using ***time-domain method only***, determine the impulse response, $h[n]$. Simplify as much as possible to get full credit.

Solution: Using the impulse response properties (impulse responses add in parallel connection of systems while convolve in series connection of systems), we obtain

$$\begin{aligned}
h(n) &= \left[\delta(n) - \frac{2}{3}\delta(n-1) + \frac{1}{9}\delta(n-2)\right] * 3^{1-n}u(n) \\
&= 3\left(\frac{1}{3}\right)^{n}u(n) - 2\left(\frac{1}{3}\right)^{n-1}u(n-1) + \frac{1}{3}\left(\frac{1}{3}\right)^{n-2}u(n-2) \\
&= 3\left(\frac{1}{3}\right)^{n}u(n) - 6\left(\frac{1}{3}\right)^{n}u(n-1) + 3\left(\frac{1}{3}\right)^{n}u(n-2) = 3\left(\frac{1}{3}\right)^{n}\big(u(n) - 2u(n-1) + u(n-2)\big) \\
&= 3\left(\frac{1}{3}\right)^{n}\Big(\underbrace{u(n) - u(n-1)}_{\delta(n)} - \{\underbrace{u(n-1) - u(n-2)}_{\delta(n-1)}\}\Big) = 3\left(\frac{1}{3}\right)^{n}[\delta(n) - \delta(n-1)] \\
&= 3\left(\frac{1}{3}\right)^{0}\delta(n) - 3\left(\frac{1}{3}\right)^{1}\delta(n-1) = 3\delta(n) - \delta(n-1). \qquad (1.1)
\end{aligned}$$

If you are unable to obtain $h(n)$, use $h(n) = \delta(n+1) - \delta(n-2)$ for the rest of the problem, which is not the correct answer. Indicate clearly if you are using this impulse response for the rest of the problem.

(b) [2-Points] Is this overall system ***causal***? BIBO (bounded-Input Bounded-Output) ***stable***? Explain clearly to receive full credit.

Solution: Since $h(n) = 0$ for $n < 0$, the system is **causal**. Since $h(n)$ is of finite duration (only two samples), $h(n)$ is absolutely summable. Hence the system is **BIBO stable**.

For impulse response $h(n) = \delta(n+1) - \delta(n-2)$: The system is **noncausal** but **stable**.

(c) [2-Points] Determine the ***difference equation representation*** of the overall system that relates the output $y[n]$ to the input $x[n]$.

Solution: From $h(n) = 3\delta(n) - \delta(n-1)$, the difference equation is

$$y(n) = h(n) * x(n) = \big(3\delta(n) - \delta(n-1)\big) * x(n) = y(n) = 3x(n) - x(n-1). \tag{1.2}$$

For the impulse response $h(n) = \delta(n+1) - \delta(n-2)$, the difference equation is

$$y(n) = x(n+1) - x(n-2).$$

(d) [3-Points] Analytically determine the ***frequency response*** $H(\omega)$ of the overall system and provide expressions for its magnitude $|H(\omega)|$ and phase $\angle H(\omega)$. Do not provide plots of these responses.

Solution: From (1.1), we have

$$\begin{aligned} H(\omega) &= 3e^{\jmath\omega 0} - e^{\jmath\omega 1} = 3 - e^{-\jmath\omega} \\ &= \underbrace{\left(\sqrt{10 - 6\cos(\omega)}\right)}_{|H(\omega)|} \exp\left[\jmath \underbrace{\tan^{-1}\left(\frac{\sin(\omega)}{3-\cos(\omega)}\right)}_{\angle H(\omega)}\right]. \end{aligned}$$

For the impulse response $h(n) = \delta(n+1) - \delta(n-2)$, the frequency response is

$$\begin{aligned} H(\omega) &= e^{\jmath\omega} - e^{\jmath 2\omega} = e^{-\jmath\omega/2}\left(e^{\jmath 3\omega/2} - e^{-\jmath 3\omega/2}\right) \\ &= e^{-\jmath\omega/2}(2\jmath)\sin(3\omega/2) = 2\sin(3\omega/2)\exp\big(\jmath(\pi/2 - \omega/2)\big). \end{aligned}$$

Hence

$$\begin{aligned} |H(\omega)| &= 2|\sin(3\omega/2)| \\ \angle\big(H(\omega)\big) &= \begin{cases} \pi/2 - \omega/2, & \sin(3\omega/2) \geq 0, \\ -\pi/2 - \omega/2, & \sin(3\omega/2) < 0. \end{cases} \end{aligned}$$

Problem-2 (10-Points)

Consider the z-transform expression: $X(z) = \dfrac{(z-0.91)\left(z^2+0.3z+0.4\right)}{\left(z+1.5\right)\left(z^2-0.6z+0.6\right)}$.

(a) [3-Points] Determine and plot the ***pole-zero pattern*** of $X(z)$.

MATLAB script:

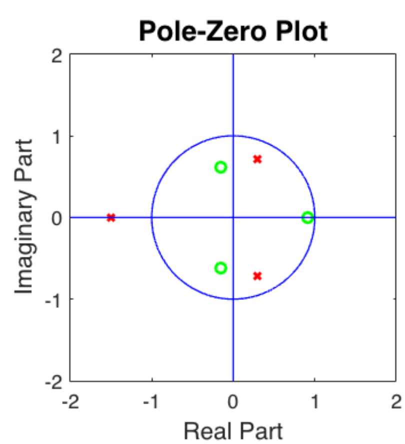
```
clc; close all; clear;
% Given z-Transform numerator and denominator
b = conv([1,-0.91],[1,0.3,0.4]);
a = conv([1,-0.6,0.6],[1,1.5]);
% Zero-pole locations
Z = roots(b).'; P = roots(a).';
Zmag = abs(Z), Zpharad = angle(Z)/pi, Zphadeg = angle(Z)*180/pi
```

```
Zmag = 1×3
    0.9100    0.6325    0.6325
Zpharad = 1×3
         0    0.5762   -0.5762
Zphadeg = 1×3
         0  103.7196 -103.7196
```

```
Pmag = abs(P), Ppharad = angle(P)/pi, Pphadeg = angle(P)*180/pi
```

```
Pmag = 1×3
    1.5000    0.7746    0.7746
Ppharad = 1×3
    1.0000    0.3734   -0.3734
Pphadeg = 1×3
  180.0000   67.2135  -67.2135
```

```
figure('position',[1,1,5,3]*72,'paperposition',[0,0,5,3]*72);
[Hz,Hp,Hl] = zplane(b,a); axis([-2,2,-2,2]);
set(Hz,'markersize',5,'color','g','linewidth',1.5);
set(Hp,'markersize',5,'color','r','linewidth',1.5);
set(Hl,'linestyle','-','linewidth',0.75,'color','b');
title('Pole-Zero Plot');
```



(b) [3-Points] List all possible ***regions of convergence*** (ROCs) for this z-transform. Do not provide plots of these ROCs.

Answer: From the magnitudes of pole locations in part (a), we observe that there are three possible ROCs, given by

$$\text{ROC-1: } |z| < 0.7746, \qquad \text{ROC-2: } 0.7746 < |z| < 1.5, \qquad \text{ROC-3: } |z| > 1.5\,.$$

(c) [4-Points] Determine the inverse z-transform so that the resulting sequence $x[n]$ is bounded. Your sequence $x[n]$ should not contain any complex numbers.

Solution: From part (b) above, the bounded sequence is given by the ROC-2 which contains the unit circle. The residues at the pole locations as well as their magnitudes and angles are computed by the following script:

```
[R,PL,C] = residuez(b,a); C
```

```
C = -0.4044
```

```
magR = abs(R)'
```

```
magR = 1×3
    0.9426    0.2834    0.2834
```

```
phaR = (angle(R)*180/pi)' % Residue phase angles in degrees
```

```
phaR = 1×3
         0   35.4334  -35.4334
```

```
magPL = abs(PL)'
```

```
magPL = 1×3
    1.5000    0.7746    0.7746
```

```
phaPL = (angle(PL)/pi)' % Pole phase angles in units of pi
```

```
phaPL = 1×3
    1.0000    0.3734   -0.3734
```

Therefore, the bounded sequence containing no complex-numbers is given by

$$\begin{aligned} x(n) &= -0.4044\delta(n) - (0.9426)(-1.5)^n u(-n-1) + 2(0.2834)(0.7746)^n \cos(0.3734\pi n + 35.4334^\circ)u(n) \\ &= -0.4044\delta(n) - 0.9426(-1.5)^n u(-n-1) + 0.5668(0.7746)^n \cos\left(0.3734\pi n + 35.4334^\circ\right)u(n)\,. \end{aligned}$$

Note that the sequence contains both causal and anticausal parts.

Problem-3 (11-Points)

We want to design a digital filter that is a cascade of an **IIR notch filter** and a **digital resonator**. The IIR notch filter is a second-order LTI system that is designed to eliminate the digital frequency of $\pm 0.4\pi$ rad/sam. The digital resonator is also a second-order system that is designed to amplify the digital frequency of $\pm 0.2\pi$ rad/sam. Hence the system function of this filter can be given as

$$H(z) = \left[\frac{\left(1-e^{j0.4\pi}z^{-1}\right)\left(1-e^{-j0.4\pi}z^{-1}\right)}{\left(1-re^{j0.4\pi}z^{-1}\right)\left(1-re^{-j0.4\pi}z^{-1}\right)}\right]\left[\frac{b_0}{\left(1-qe^{j0.2\pi}z^{-1}\right)\left(1-qe^{-j0.2\pi}z^{-1}\right)}\right]$$

where $b_0 > 0$ is the overall gain and $0 < r, q < 1$ are the respective pole magnitudes. The first bracketed term is the IIR notch filter while the second bracketed term is the digital resonator. The filter design involves choosing values of r, q, and b_0 for a particular magnitude response. Choose $r = 0.95$ for the rest of the problem.

(a) (4-points) Using MATLAB determine values of

1. q so that the DC response (that is, the response at $\omega = 0$) is no more than 10% of the maximum filter magnitude response, that is, $H(0) \le (0.1)|H(\omega)|_{\max}$ and
2. b_0 so that the maximum filter magnitude response is unity, that is, $|H(\omega)|_{\max} = 1$.

Use these values for the rest of the problem.

Solution: Since b_0 affects only the overall gain, we will set it to unity, $b_0 = 1$, and determine its final value in the end to satisfy Condition-2 above. Now q should be close to the unit circle so that we have a proper digital resonator. First, let us choose $q = 0.95$ as a starting value and use MATLAB to determine magnitude response of $H(z)$ filter. The maximum response should be around $\omega = 0.2\pi$. Then we will compute the ratio of $H(0)/H_{\max}$ and check if it satisfies Condition-1. If not then we will increase (or decrease) the value of q until it does.

MATLAB script:

```
clc; close all; clear;
om = linspace(0,pi,10001); % Digital frequency samples
% IIR Notch Filter (NF)
NFz1 = exp(1j*0.4*pi); NFz2 = conj(NFz1);
bNF = real(conv([1,-NFz1],[1,-NFz2])); % Numerator of Notch Filter
r = 0.95; NFp1 = r*exp(1j*0.4*pi); NFp2 = conj(NFp1);
aNF = real(conv([1,-NFp1],[1,-NFp2])); % Denominator of Notch Filter
% Digital Resonator (DR)
b0 = 1; q = 0.95; % Initial value
DRp1 = q*exp(1j*0.2*pi); DRp2 = conj(DRp1);
aDR = real(conv([1,-DRp1],[1,-DRp2])); % Denominator of Digital Resonator
% Overall Filter
B = bNF*b0; A = conv(aNF,aDR); % Overall filter parameters
H = freqz(B,A,om); % Unnormalized freq resp
Hmag = abs(H); Hmax = max(Hmag);
ratio = (H(1)/Hmax)*100, % Condition-1 ratio
```

```
ratio = 15.7169
```

Clearly the DC response is more than 10%. So, we will increase q to $q = 0.96$ and recompute the ratio.

```
q = 0.96; % New value
```

```
DRp1 = q*exp(1j*0.2*pi); DRp2 = conj(DRp1);
aDR = real(conv([1,-DRp1],[1,-DRp2])); % Denominator of Digital Resonator
% Overall Filter
B = bNF*b0; A = conv(aNF,aDR); % Overall filter parameters
H = freqz(B,A,om); % Unnormalized freq resp
Hmag = abs(H); Hmax = max(Hmag);
ratio = (H(1)/Hmax)*100, % Condition-1 ratio
```

```
ratio = 12.5380
```

So we are on correct track. Now increase q to $q = 0.97$ and recompute the ratio.

```
q = 0.97; % New value
DRp1 = q*exp(1j*0.2*pi); DRp2 = conj(DRp1);
aDR = real(conv([1,-DRp1],[1,-DRp2])); % Denominator of Digital Resonator
% Overall Filter
B = bNF*b0; A = conv(aNF,aDR); % Overall filter parameters
H = freqz(B,A,om); % Unnormalized freq resp
Hmag = abs(H); Hmax = max(Hmag);
ratio = (H(1)/Hmax)*100, % Condition-1 ratio
```

```
ratio = 9.3721
```

So $q = 0.97$ is acceptable (any value greater than 0.97 is acceptable). Now b_0 can be determined to make the maximum magnitude equal to 1.

```
% Overall Filter
B = bNF*b0; A = conv(aNF,aDR); H = freqz(B,A,om); % Unnormalized freq resp
Hmag = abs(H); [Hmax,I] = max(Hmag);
b0 = 1/Hmax; b0, % Condition-2; Display b0 value
```

```
b0 = 0.0331
```

```
B = B*b0; om_max = om(I)/pi, % Display freq at maximum mag resp
```

```
om_max = 0.1998
```

The needed value of b_0 is $b_0 = 0.0331$ and the maximum response occurs at $\omega = 0.1998\pi \approx 0.2\pi$. Note also that $b_0 \approx 1 - q$ as expected. The overall filter system function is

```
B,A
```

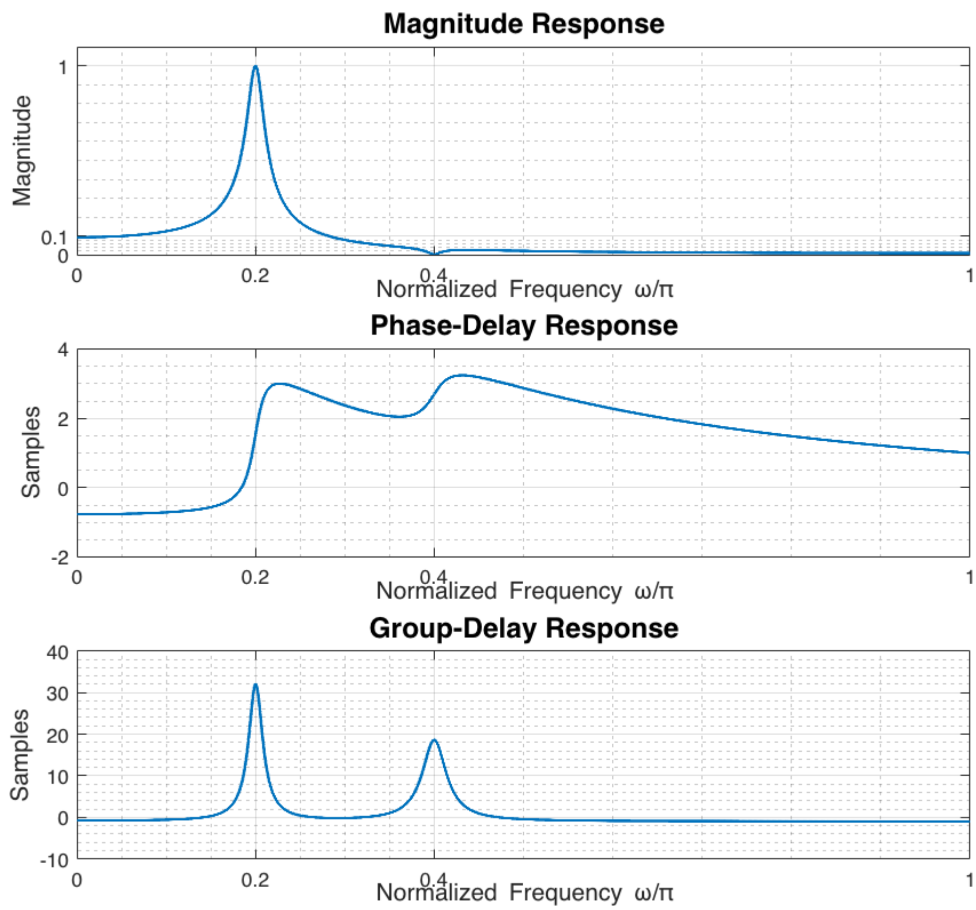
```
B = 1×3
    0.0331   -0.0205    0.0331
A = 1×5
    1.0000   -2.1566    2.7649   -1.9689    0.8492
```

Hence $H(z) = \dfrac{0.0331 - 0.0205z^{-1} + 0.0331z^{-2}}{1 - 2.1566z^{-1} + 2.7649z^{-2} - 1.9689z^{-3} + 0.8492z{-4}}$.

(b) (3-points) Plot the magnitude, phase-delay, and group-delay responses of the above designed filter in one figure using 3×1 subplots.

MATLAB script and plots:

```
H = freqz(B,A,om); Hmag = abs(H); % Normalized freq resp
Hphadelay = phasedelay(B,A,om); % Phase-delay Response
Hgrpdelay = grpdelay(B,A,om); % Group-delay Response
figure('position',[0,0,8,7]*72,'paperposition',[0,0,8,7]*72);
subplot(3,1,1); % Plot for magnitude response
plot(om/pi,Hmag,'linewidth',1.5); axis([0,1,0,1.1]);
ylabel('Magnitude'); title('Magnitude Response'); grid('minor'); grid;
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
set(gca,'xtick',[0,0.2,0.4,1],'ytick',[0,0.1,1]);
subplot(3,1,2); % Plot for phase-delay response
plot(om/pi,Hphadelay,'linewidth',1.5); axis([0,1,-2,4]);
ylabel('Samples'); title('Phase-Delay Response'); grid('minor'); grid;
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
set(gca,'xtick',[0,0.2,0.4,1],'ytick',(-2:2:4));
subplot(3,1,3); % Plot for group-delay response
plot(om/pi,Hgrpdelay,'linewidth',1.5); axis([0,1,-10,40]);
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
ylabel('Samples'); title('Group-Delay Response'); grid('minor'); grid;
set(gca,'xtick',[0,0.2,0.4,1],'ytick',(-10:10:40));
```



(c) (2-points) How much phase- and group-delay, in samples, is there for the dominant frequency in the steady-state response of $y[n]$ in the above plot?

Answer: Using MATLAB

```
PD = Hphadelay(2001), % Display phase-delay at 0.2*pi
```

```
PD = 1.5623
```

```
GD = Hgrpdelay(2001), % Display group-delay at 0.2*pi
```

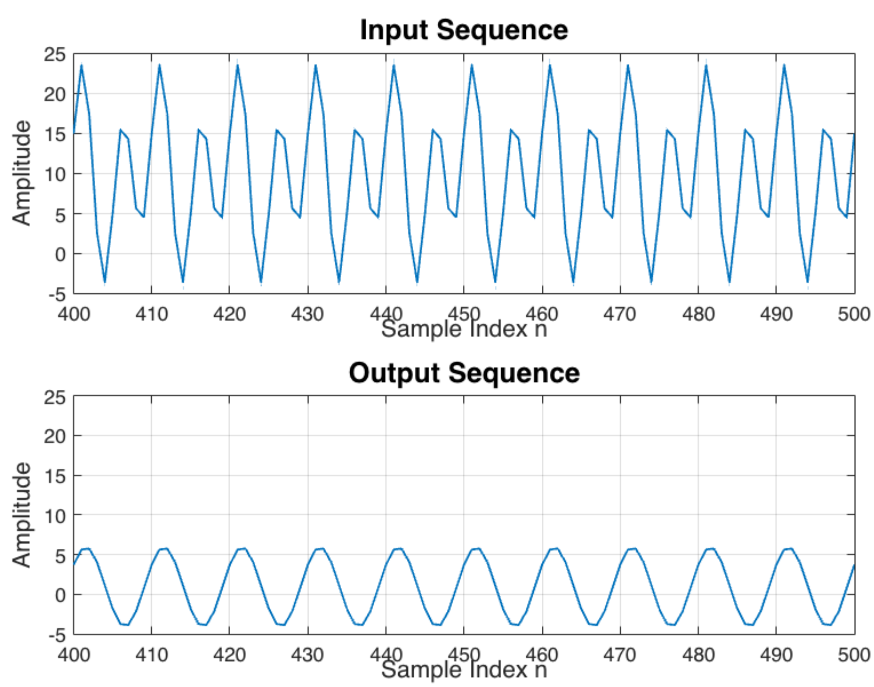
```
GD = 32.0084
```

the phase-delay is 1.56 samples while the group delay is 32 samples.

(d) (2-points) Let $x[n] = 10 + 5\cos(0.2\pi n) + 10\sin(0.4\pi n)$, $0 \le n \le 500$ be the input to the filter designed in part (a). Using the **`filter`** function, determine the output $y[n]$. Using the **`plot`** function (and not the **`stem`** function), provide a graph of $x[n]$ and $y[n]$ for $400 \le n \le 500$ in one figure with 2×1 subplots.

MATLAB script and plots:

```
N = 501; n = 0:N-1; xn = 10+5*cos(0.2*pi*n)+10*sin(0.4*pi*n); yn = filter(B,A,xn);
figure('position',[0,0,7,5]*72,'paperposition',[0,0,7,5]*72);
subplot(2,1,1); % Plot for x[n] 400 <= n <= 500
plot(n(401:501),xn(401:501),'linewidth',1); axis([400,500,-5,25]);
ylabel('Amplitude'); title('Input Sequence'); grid;
xlabel('Sample Index n','VerticalAlignment','middle');
subplot(2,1,2); % Plot for y[n] 400 <= n <= 500
plot(n(401:501),yn(401:501),'linewidth',1); axis([400,500,-5,25]);
ylabel('Amplitude'); title('Output Sequence'); grid;
xlabel('Sample Index n','VerticalAlignment','middle');
```



Alternate Problem Solution

If you used $b_0 = 0.05$ and $q = 0.95$ in this problem then the solution is given below.

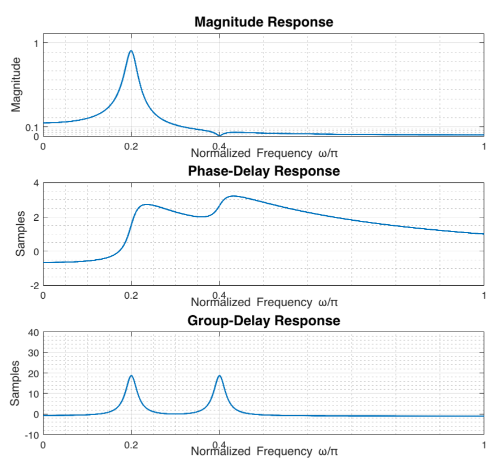
```
b0 = 0.05; q = 0.95; % Alternate values
DRp1 = q*exp(1j*0.2*pi); DRp2 = conj(DRp1);
aDR = real(conv([1,-DRp1],[1,-DRp2])); % Denominator of Digital Resonator
% Overall Filter
B = bNF*b0, A = conv(aNF,aDR), % Overall filter parameters
```

```
B = 1×3
    0.0500   -0.0309    0.0500
A = 1×5
    1.0000   -2.1243    2.7075   -1.9171    0.8145
```

Hence $H(z) = \dfrac{0.05 - 0.0309z^{-1} + 0.05z^{-2}}{1 - 2.1243z^{-1} + 2.7075z^{-2} - 1.9171z^{-3} + 0.8145z{-4}}$.

(b) Frequency Response Plots:

```
H = freqz(B,A,om); Hmag = abs(H); % Normalized freq resp
Hphadelay = phasedelay(B,A,om); % Phase-delay Response
Hgrpdelay = grpdelay(B,A,om); % Group-delay Response
figure('position',[0,0,8,7]*72,'paperposition',[0,0,8,7]*72);
subplot(3,1,1); % Plot for magnitude response
plot(om/pi,Hmag,'linewidth',1.5); axis([0,1,0,1.1]);
ylabel('Magnitude'); title('Magnitude Response'); grid('minor'); grid;
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
set(gca,'xtick',[0,0.2,0.4,1],'ytick',[0,0.1,1]);
subplot(3,1,2); % Plot for phase-delay response
plot(om/pi,Hphadelay,'linewidth',1.5); axis([0,1,-2,4]);
ylabel('Samples'); title('Phase-Delay Response'); grid('minor'); grid;
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
set(gca,'xtick',[0,0.2,0.4,1],'ytick',(-2:2:4));
subplot(3,1,3); % Plot for group-delay response
plot(om/pi,Hgrpdelay,'linewidth',1.5); axis([0,1,-10,40]);
xlabel('Normalized Frequency \omega/\pi','VerticalAlignment','middle');
ylabel('Samples'); title('Group-Delay Response'); grid('minor'); grid;
set(gca,'xtick',[0,0.2,0.4,1],'ytick',(-10:10:40));
```



(c) Phase- and group-delay:

```
PD = Hphadelay(2001), % Display phase-delay at 0.2*pi
```

```
PD = 1.5395
```

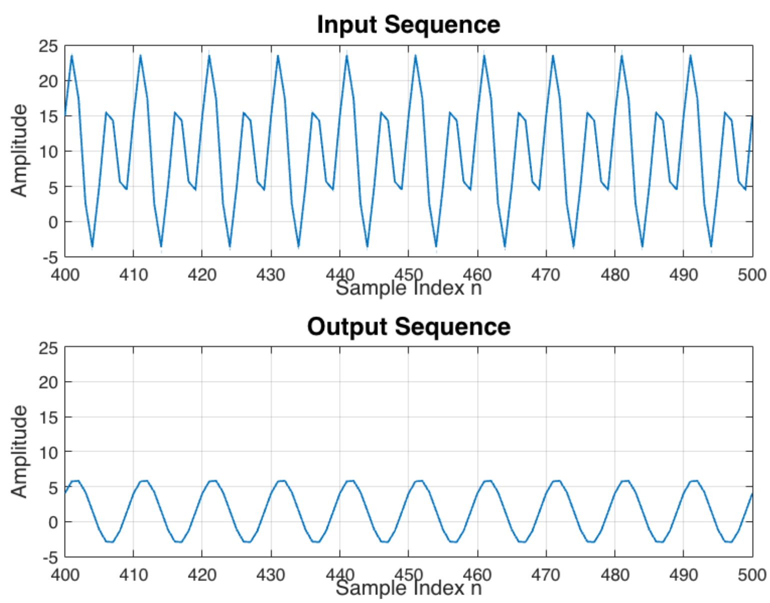
```
GD = Hgrpdelay(2001), % Display group-delay at 0.2*pi
```

```
GD = 18.6901
```

The phase-delay is 1.54 samples while the group delay is 18.69 samples.

(d) MATLAB script and plots:

```
N = 501; n = 0:N-1; xn = 10+5*cos(0.2*pi*n)+10*sin(0.4*pi*n); yn = filter(B,A,xn);
figure('position',[0,0,7,5]*72,'paperposition',[0,0,7,5]*72);
subplot(2,1,1); % Plot for x[n] 400 <= n <= 500
plot(n(401:501),xn(401:501),'linewidth',1); axis([400,500,-5,25]);
ylabel('Amplitude'); title('Input Sequence'); grid;
xlabel('Sample Index n','VerticalAlignment','middle');
subplot(2,1,2); % Plot for y[n] 400 <= n <= 500
plot(n(401:501),yn(401:501),'linewidth',1); axis([400,500,-5,25]);
ylabel('Amplitude'); title('Output Sequence'); grid;
xlabel('Sample Index n','VerticalAlignment','middle');
```



Problem-4 (9-points)

In the following signal processing structure, the Analog-to-Digital (A/D) and Digital-to-Analog (D/A) converters are ideal devices.

$$x_a(t) \rightarrow \boxed{\text{A/D}} \rightarrow x(n) \rightarrow \boxed{H(z)} \rightarrow y(n) \rightarrow \boxed{\text{D/A}} \rightarrow y_a(t)$$

The signals $x_a(t)$ and $y_a(t)$ are analog signals while $x(n)$ and $y(n)$ are discrete-time (DT) signals. Let system function be

$$H(z) = 1 + z^{-2}$$

and let the input signal be $x_a(t) = 5\cos(600\pi t)u(t)$ where $u(t)$ is a unit step function.

(a) (2-points) Determine $x(n)$ if the sampling interval in the A/D and D/A is 5 ms/sample. The digital frequencies (in rad/sam) of the DT signal $x[n]$ must be within the fundamental $(-\pi, \pi]$ range.

Solution: Since $T = 5$ ms/sample, the sampling frequency is $F_s = 200$ Hz. The DT signal is then given by

$$\begin{aligned} x(n) &= x_a(n/200) = 5\cos(600\pi n/200)u(n) \\ &= 5\cos(3\pi n)u(n) = 5\cos(3\pi n - 2\pi n)u(n). \\ &= 5\cos(\pi n)u(n) = 5(-1)^n u(n) \end{aligned}$$

(b) (3-points) Determine the zero-state output response $y(n)$ due to the input $x(n)$. The most compact answer will receive the maximum credit.

Solution: The input z-transform is $X(z) = \mathscr{Z}\{5(-1)^n u(n)\} = \dfrac{5}{1+z^{-1}}$, ROC: $|z| > 1$. Hence, the output transform is

$$Y(z) = H(z)X(z) = (1+z^{-2})\frac{5}{1+z^{-1}} = 5\frac{1+z^{-2}}{1+z^{-1}}, \text{ ROC: } |z| > 1.$$

```
clc; clear; [R,p,C] = residuez(5*[1,0,1],[1,1])
```

```
R = 10
p = -1
C = 1×2
    -5      5
```

Using the PFE given above, we can express $Y(z)$ as

$$Y(z) = -5 + 5z^{-1} + \frac{10}{1+z^{-1}}$$

or the output response is given by

$$y(n) = -5\delta(n) + 5\delta(n-1) + 10(-1)^n u(n).$$

(c) (2-points) Determine the transient $y_{tr}(n)$ and the steady-state components $y_{ss}(n)$ in $y(n)$.

Solution: From part (b), the transient response is $y_{tr}(n) = -5\delta(n) + 5\delta(n-1) = \{\underset{\uparrow}{-5}, 5\}$ while the steady-state response is $y_{ss}(n) = 10(-1)^n u(n)$.

(d) (2-points) Determine the *steady-state* analog response $y_{\mathrm{ss,a}}(t)$ due to the input $x_{\mathrm{a}}(t)$. The most compact answer will receive the maximum credit.

Solution: Since the digital frequency is within the primary range of $-\pi < \omega \le \pi$, the steady-state analog response $y_{\mathrm{ss,a}}(t)$ is obtained by replacing n in $y_{\mathrm{ss}}(n)$ by $tF_{\mathrm{s}} = 200t$ due to ideal interpolation. Hence,

$$y_{\mathrm{ss,a}}(t) = y_{\mathrm{ss}}(n)\big|_{n=200t} = 10\cos(200\pi t)u(t) = 10\cos(2\pi[100]t)u(t).$$

Clearly, there is an aliasing since the input analog signal of 300 Hz is aliased into 100 Hz.