EECE-5666 : Final Exam : 2023-SPRG

[80-Points]

Firstname Lastname

Instructions:

1. Replace **Firstname** by your first name and **Lastname** by your last (or familiy) name. **FAILURE TO ENTER YOUR NAME WILL RESULT IN A LOSS OF 5% OF THE TOTAL GRADE.**
2. You are required to read the NU Academic Integrity policy (given below) and sign below that the submitted work is your own work. (This is a COE requirement.)
3. You are required to complete this exam using Live Editor.
4. Use of the equation editor to typeset mathematical material such as variables, equations, etc., is strongly recommended. However, in the interest of time, a properly scanned and oriented (avoid sideways or upsidedown) image of a neatly hand-written fragment can be inserted as your work in the required space.
5. After completing this exam, export this Live script to PDF. If you encounter problems in the direct export to PDF, export to Microsoft Word and then print to PDF.
6. Submit the PDF file only. **DO NOT SUBMIT YOUR LIVE EDITOR (.MLX) FILE, IT WILL NOT BE GRADED**.
7. You should complete this exam in three hours and twenty minutes with additional 10 minutes for submission activities.
8. You have only one attempt to take and submit this exam.

Academic Integrity Policy

A commitment to the principles of academic integrity is essential to the mission of Northeastern University. The promotion of independent and original scholarship ensures that students derive the most from their educational experience and their pursuit of knowledge. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University.

As members of the academic community, students must become familiar with their rights and responsibilities. In each course, they are responsible for knowing the requirements and restrictions regarding research and writing, examinations of whatever kind, collaborative work, the use of study aids, the appropriateness of assistance, and other issues. Students are responsible for learning the conventions of documentation and acknowledgment of sources in their fields. Northeastern University expects students to complete all examinations, tests, papers, creative projects, and assignments of any kind according to the highest ethical standards, as set forth either explicitly or implicitly in this Code or by the direction of instructors. The full academic integrity policy is available at

http://www.northeastern.edu/osccr/academic-integrity-policy/

Declaration

By signing (i.e., entering my name above) and submitting this exam through the submission portal, I declare that I have read the Academic Integrity Policy and that the submitted work is my own work.

Default Plot Parameters:

The following code fragment sets up default values for some plot parameters. Do not issue `'fontsize'` values in `title`, `xlabel`, and `ylabel` commands in your plots.

```
set(0,'defaultaxesfontsize',10); % default 10-point axis tickmark fontsize
set(0,'defaultaxestitlefontsize',1.2); % default 12-point title font size
set(0,'defaultaxeslabelfontsize',1.1); % default 11-point axis label font
size
set(0,'defaultaxeslinewidth',1.5);
```

Problem-1 [20-points] The DFT, FFT, and Digital Filter Structures

The following three parts, (a), (b), and (c), are not related to each other.

(a) [6-points] DFT Properties

Use only direct calculations and/or DFT properties to solve each subpart. Use of Symbolic Math toolbox or Mathematica or Wolfram Alpha, etc. will not be graded or credited.

Let $x(n)$ be an N-point sequence, that is, $x(n)$ is zero outside of $0 \le n \le (N-1)$ interval. Let $X(z)$ be its z-transform. Listed below are two sequences of length $2N$ derived from $x(n)$.

`[i]` **[3-points]** $y_1(n) = x(n) + x(n-N)$: Determine a compact expression for the DFT $Y_1(k)$ in terms of samples of $X(z)$.

Note: Detailed steps must be shown to get full credit.

Solution:

`[ii]` **[3-points]** $y_2(n) = \begin{cases} x\left(\frac{n}{2}\right), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$: Determine an expression for the DFT $Y_2(k)$ in terms of samples of $X(z)$. Detailed steps must be shown to get full credit.

Solution:

(b) [7-points] Computation of DTFT using the FFT

Let $X(\omega)$ be the DTFT of a finite-length sequence

$$x(n) = \begin{cases} 5(0.9)^{|n|}, & |n| \leq 10 \\ 0, & \text{otherwise.} \end{cases}$$

Using a 512-point **fft** as a computation tool, determine and plot $X(\omega)$ over $-1 \leq \omega/\pi \leq 1$.

Notes:

1. Use the **plot** function and not the **stem** function.
2. Do not analytically determine the DTFT $X(\omega)$ for plotting (it will not only be tedious but also time consuming). This approach will not be graded.
3. Use the following code fragment for plotting:

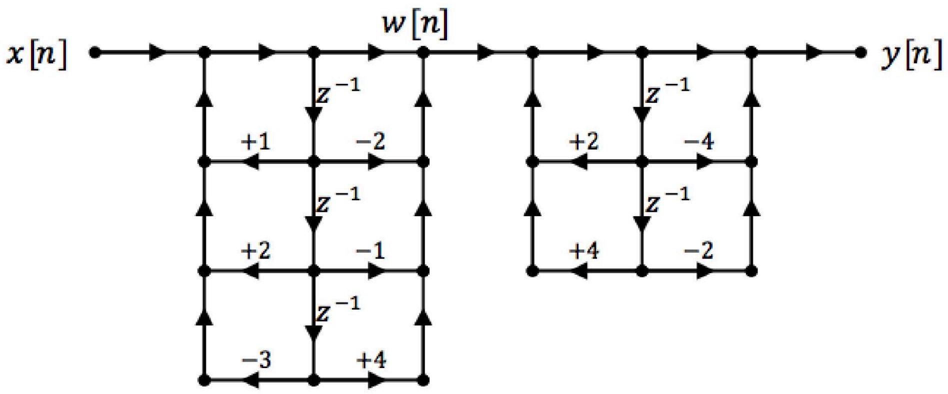
```
figure('units','inches','Position',[0,0,8,3]);
% your plot commands here

xlabel('Normalized Frequency in \pi Units');
ylabel('Amplitude'); title('DTFT of x[n]');
axis([-1,1,-5,70]); grid;
```

MATLAB script:

```
clear; clc;
% Enter your code here
```

(c) [7-Points] IIR Filter Structure

An IIR system with the signal flow graph is shown below. The gains not shown are equal to one.



Note: In the following parts all steps must be shown to get full credit.

(i) [2-Point] Determine analytically the difference equation relating $w(n)$ to $x(n)$.

Solution:

`(ii)` **[2-Point]** Determine analytically the difference equation relating $y(n)$ to $w(n)$.

Solution:

`(iii)` **[3-Points]** Determine analytically the difference equation relating $y(n)$ to $x(n)$.

Solution:

Problem-2 [20-Points] FIR Filter Design

We want to design a linear-phase FIR filter that satisfies the following requirements:

$$\begin{array}{llll} \text{Band-1:} & 0 & \le |\omega| \le 0.4\pi & A(e^{j\omega}) = 0.5 \pm 0.05 \\ \text{Band-2:} & 0.45\pi & \le |\omega| \le 0.63\pi & A(e^{j\omega}) = 0.0 \pm 0.01 \\ \text{Band-3:} & 0.7\pi & \le |\omega| \le \pi & A(e^{j\omega}) = 1.0 \pm 0.025 \end{array}$$

The above design parameters are given in the following MATLAB fragment. Use it where appropriate.

```
f1 = 0.4; f2 = 0.45; f3 = 0.63; f4 = 0.7; % Cutoff freq in pi rad
fedge = [f1,f2,f3,f4]; % Bandpedge frequencies
A1 = 0.5; A2 = 0; A3 = 1; % Nominal Values
Aband = [A1,A2,A3]; % Ideal band amplitude response
d1 = 0.05; d2 = 0.01; d3 = 0.025; % Ripples
dband = [d1,d2,d3]; % Band ripples
XT = [0,fedge,1]; % x-axis tickmarks
YT = sort([Aband-dband,Aband,Aband+dband]); % y-axis tickmarks
```

```
clc; close all; clear;
```

Do not use GUI-based apps such as `FilterDesigner` to solve any part of this problem. It will not be graded.

`(a)` [6-Points] FIR Filter Design using Parks-McClellan Algorithm

Design a minimum-order FIR filter using the ***Parks-McClellan algorithm***. Plot its amplitude response and provide the value of filter order M_{PM}. Use the following MATLAB fragment for your plot.

```
figure('units','inches','position',[0,0,8,4]);
```

```
% your plot commands below

set(gca,'xtick',XT,'ytick',YT); grid;
xlabel('Digital frequency (in \pi radians)'); ylabel('Amplitude');
title('Amplitude Response: Parks-McCellan Algorithm');
```

MATLAB script:

```
% Your code below
```

`(b)` **[6-Points]** **FIR Filter Design using Kaiser Window**

Design a minimum-order FIR filter using the ***Kaiser window***. Plot its amplitude response and provide the value of filter order M_{KW}. Use the following MATLAB fragment for your plot.

```
figure('units','inches','position',[0,0,8,4]);
% your plot commands below

set(gca,'xtick',XT,'ytick',YT); grid;
xlabel('Digital frequency (in \pi radians)'); ylabel('Amplitude');
title('Amplitude Response: Kaiser Window Design');
```

MATLAB script:

```
% Your code below
```

`(c)` **[6-Points]** **FIR Filter Design using Frequency Sampling Technique**

Design a minimum-order filter via the ***Frequency sampling approach***. Use the ***raised cosine profile*** in the transition band. Provide the amplitude response as well as frequency samples in the same plot and indicate the value of filter order M_{FS}. Use the following MATLAB fragment for your plot.

```
figure('units','inches','position',[0,0,8,4]);
% your plot commands below

set(gca,'xtick',XT,'ytick',YT); grid;
xlabel('Digital frequency (in \pi radians)'); ylabel('Amplitude');
title('Amplitude Response: Frequency sampling Design');
```

MATLAB script:

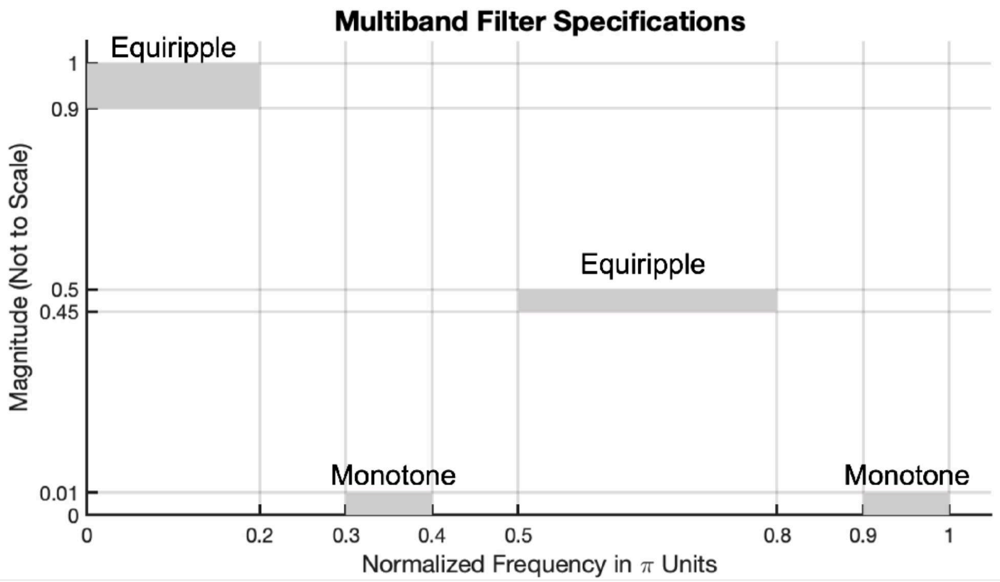
```
% Your code below
```

`(d)` **[2-Points]** Compare the obtained filter orders from each design.

Comparison:

Problem-3 [20-Points]   IIR Filter Design

The multiband filter specifications shown below can be considered as a combination of a lowpass filter and a scaled bandpass filter.



Note: The shaded areas indicate the band tolerance regions. Thus, in the passband of the lowpass section the magnitude response should be equiripple between 0.95 and 1. Similarly, in the passband of the bandpass section, the magnitude response should be equiripple between 0.45 and 0.5. In the two stopbands, the magnitude responses should be monotone between 0 and 0.01.

Hint: Design the lowpass and bandpass sections separately. Then consider if these sections should be connected in cascade or in parallel.

`(a)` **[10-Points]** Design a minimum-order IIR digital filter that will satisfy the above requirements. Determine the system function $H(z)$ in the cascade form. You are not required to draw the cascade structure but to list them in the sos-matrix format.

MATLAB code for the design:

```
clc; close all; clear;
```

`(b)` **[5-Points]** Provide a magnitude response plot of your designed filter over $0 \leq \omega/\pi \leq 1$. Use the following code fragment for your plot.

MATLAB code fragment for the magnitude response plot:

```
figure('units','inches','position',[0,0,8,4]);
% Your plot commands here

xlabel('Normalized Digital Frequency in \pi Units'); ylabel('Magnitude');
title('Magnitude Response of a Multiband Filter');
set(gca,'ytick',[0,0.01,0.45,0.5,0.9,1]);
set(gca,'xtick',[0,0.2,0.3,0.4,0.5,0.8,0.9,1]);
```

MATLAB code for the plot:

```
% Your code here
```

`(c)` **[1-Point]** From your design what is the order N of your multiband filter?

Answer:

`(d)` **[4-Points]** From your design, determine the exact attenuations at the band-edge frequencies and verify that the specifications are satisfied.

Answer:

Verification:

Problem-4 [20-Points] Finite Word Length Effects

A linear, time-invariant system is characterized by the following parameters:

$$\begin{aligned} \text{Gain:}\ & 1.0 \\ \text{Zero locations:}\ & +\jmath, -\jmath \\ \text{Pole locations:}\ & 0.9\angle(+40^{\text{deg}}), 0.9\angle(-40^{\text{deg}}), 0.9\angle(+50^{\text{deg}}), 0.9\angle(-50^{\text{deg}}). \end{aligned}$$

Coefficients of the above system $H(z)$ are quantized using 2's-complement arithmetic with rounding to 2 integer bit and 4 fractional bits for a total of $L = 1 + 2 + 4 = 7$ bits. Let the system with quantized coefficents be denoted by $\widehat{H}(z)$. The following code fragment is given for convenience. Do not edit it.

```
clc; close all; clear;
z1 = 1j; z2 = -1j;
r = 0.9; theta1 = 40*pi/180; theta2 = 50*pi/180;
p1 = r*exp(1j*theta1); p2 = conj(p1);
p3 = r*exp(1j*theta2); p4 = conj(p3);
```

(a) [4-Points] Effect on Frequency response

Compute the frequency responses of the system $H(z)$ and $\widehat{H}(z)$ over $0 \le \omega \le \pi$. Plot the magnitudes of these frequency responses in one figure using the **hold** function. Comment on their shapes.

MATLAB script: Use the following code fragment for plotting.

```
figure('units','inches','position',[0,0,8,3.5]);
% Your plot commands

xlabel('Digital Frequency (in \pi radians)'); ylabel('Magnitude');
title('Magnitude Responses'); set(gca,'xtick',[0,0.25,1],'ytick',0:20:60); grid;
```

```
% your code below
```

Comment:

(b) [4-Points] Effect on Pole-Zero Plot

In one figure, using one row and two columns, plot pole-zero diagrams of $H(z)$ and $\widehat{H}(z)$. Comment on your diagrams.

MATLAB script: Use the following code fragment for figure size. Provide appropriate titles.

```
figure('units','inches','position',[0,0,8,3.5]);
```

```
% Your code here
```

Comment:

(c) [4-Points] Effect on Variance Gain

Determine variance gains of $H(z)$ and $\widehat{H}(z)$. Is there an increase or decrease in the variance gain of the $\widehat{H}(z)$ system? Explain.

Solution:

Explanation:

(d) [8-Points] Input/Output Signal-to-Noise Ratios

Signal $x(n) = 0.99\cos(0.8n)$ is quantized to 4 fractional bits plus one sign bit using 2's-complement arithmetic with rounding. The quantized signal $x_q(n) = x(n) + e_q(n)$ is applied as an input to the system $H(z)$ given above where $e_q(n)$ is the input quantization error. The output signal is $y_q(n) = y(n) + v(n)$ where $y(n)$ is the response due to $x(n)$ and $v(n)$ is the response due to $e_q(n)$.

(i) [4-Points] Determine analytically the input signal power P_X, quantization noise power $\sigma^2_{e_q}$, and the input signal-to-quantization-noise ratio $\mathrm{SQNR_I} = 10\log_{10}\left(P_x/\sigma^2_{e_q}\right)$. Verify using MATLAB by generating 100000 samples of $x(n)$, quantizing it to $x_q(n)$, and then computing the signal and noise powers.

Solution:

MATLAB verification:

```
% Your code here
```

(ii) [4-Points] Determine analytically the output signal power P_Y, output noise power σ^2_V, and the output signal-to-noise ratio $\mathrm{SNR_O} = 10\log_{10}(P_Y/\sigma^2_V)$ where P_Y is the average power in $y(n)$ and σ^2_V is the noise power in $v(n)$. Verify using MATLAB by filtering the above $x(n)$ and $x_q(n)$ sequences through $H(z)$ and the computing the resulting signal and noise powers.

Solution:

MATLAB verification:

```
% Your code here
```
