EECE-5666 (DSP) : Homework-7 Solutions

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Default Plot Parameters:

```
set(0,'defaultfigurepaperunits','inches','defaultfigureunits','inches');
set(0,'defaultaxesfontsize',10);
set(0,'defaultaxestitlefontsize',1.2,'defaultaxeslabelfontsize',1.1);
```

Problem-1 Zero-Phase IIR Filtering

One approach for **zero-phase** IIR filtering is as follows. Let $x(n)$ be an input signal and $h(n)$ denote the causal and stable IIR system. First, $x(n)$ is filtered through $h(n)$ to obtain output $y_1(n)$. Next, the signal $x(n)$ is flipped and the flipped signal $x(-n)$ is filtered through $h(n)$ to obtain $y_2(n)$. Finally, the zero-phase output $y(n)$ is given by $y(n) = y_1(n) + y_2(-n)$.

(a) Let $h_{zp}(n)$ be the impulse response of the filter with input $x(n)$ and output $y(n)$. Determine $h_{zp}(n)$ in terms of $h(n)$.

Solution: The first output $y_1(n)$ is given by $y_1(n) = h(n) * x(n)$. The second output $y_2[n]$ is given by $y_2(n) = h(n) * x(-n)$. Finaly, the zero-phase output $y(n)$ is given by

$$\begin{aligned} y(n) &= y_1(n) + y_2(-n) = h(n) * x(n) + h(-n) * x(n) \\ &= \underbrace{\big(h(n) + h(-n)\big)}_{\triangleq h_{zp}(n)} * x(n) = h_{zp}(n) * x(n). \end{aligned}$$

Hence, $h_{zp}(n) = h(n) + h(-n)$.

(b) Determine the frequency response $H_{zp}(\omega)$ and show that its phase response is zero.

Solution: Assuming $H(\omega) = \mathscr{F}\{h(n)\}$ and using DTFT properties, we have

$$H_{zp}(\omega) = H(\omega) + H(-\omega) = H(\omega) + H^*(\omega) = 2\mathrm{Re}\{H(\omega)\}.$$

Since $H_{zp}(\omega)$ is a real-valued function, it is also an amplitude function. Thus, the phase response is zero.

(c) Let $x(n) = u(n) - u(n-10)$ and $H(z) = 1/(1-0.9z^{-1})$. Determine the zero-phase response $y_{zp}(n)$.

Solution: We will use the z-transform approach. Since the given system is causal, the ROC of the system function is $|z| > 0.9$. Now the system function of the zero-phase IIR system is

$$H_{zp}(z) = H(z) + H(1/z) = \frac{1}{1-0.9z^{-1}} + \frac{1}{1-0.9z}, \quad 0.9 < |z| < 10/9$$
$$= \frac{2-0.9z^{-1}-0.9z}{1.81-0.9z^{-1}-0.9z} = \frac{1-\frac{20}{9}z^{-1}+z^{-2}}{1-\frac{181}{90}z^{-1}+z^{-2}}, \quad 0.9 < |z| < 10/9$$

Hence the zero-phase output transform is

$$Y_{zp}(z) = H_{zp}(z)X(z) = \left(\frac{1-\frac{20}{9}z^{-1}+z^{-2}}{1-\frac{181}{90}z^{-1}+z^{-2}}\right)\left(\frac{1-z^{-10}}{1-z^{-1}}\right), \quad 0.9 < |z| < 10/9$$

```
% clear; close all; clc;
bHzp = [1,-20/9,1]; aHzp = [1,-181/90,1];
bX = [1,zeros(1,9),-1]; aX = [1,-1];
bYzp = conv(bHzp,bX); aYzp = conv(aHzp,aX);
[A,p,C] = residuez(bYzp,aYzp); A = A.', p = p.', C15 = C(1:5), C610 = C(6:10)
```

```
A = 1×3
   -6.5132    0.0000   16.8117
p = 1×3
    1.1111    1.0000    0.9000
C15 = 1×5
   -9.2985   -7.1048   -5.2122   -3.5997   -2.2495
C610 = 1×5
   -1.1465   -0.2784    0.3643    0.7889    1.0000
```

Hence the zero-phase response is (note that the residue at pole $z = 1$ is zero)

$$\begin{aligned} y_{zp}(n) = &-9.2985\delta(n) - 7.1048\delta(n-1) - 5.2122\delta(n-2) - 3.5997\delta(n-3) - 2.2495\delta([n-4) - 1.1465\delta(n-5) \\ &- 0.2784\delta(n-6) + 0.3643\delta(n-7) + 0.7889\delta(n-8) + \delta([n-9) + 16.8117(0.9)^n u(n) \\ &+ 6.5132(0.9)^{-n}u([-n-1) \qquad \text{(P1.1)} \end{aligned}$$

The above zero-phase response can be expressed as a symmetric sequence using

```
yzp0to9 = C+A(3)*(0.9).^(0:9); yzp0to9(1:5), yzp0to9(6:10)
```

```
ans = 1×5
    7.5132    8.0258    8.4053    8.6560    8.7807
ans = 1×5
    8.7807    8.6560    8.4053    8.0258    7.5132
```

Hence,

$$
\begin{aligned}
y_{zp}(n) = {} & 6.5132(0.9)^{-n}u(-n-1) \\
& + 7.5132\delta(n) + 8.0258\delta(n-1) + 8.4053\delta(n-2) + 8.6560\delta(n-3) + 8.7807\delta(n-4) \\
& + 8.7807\delta(n-5) + 8.6560\delta((n-6) + 8.4053\delta(n-7) + 8.0258\delta(n-8) + 7.5132\delta(n-9) \\
& + 6.5132(0.9)^{n-9}u(n-10) \qquad (\text{P1.2})
\end{aligned}
$$

Equation (P1.2) shows that $y_{zp}[n]$ is symmetric with respect to $n = 4.5$, which is also the center of symmetry of the input $x[n]$ pulse. Thus, $y_{zp}[n]$ is indeed a zero-phase response.

MATLAB verification: (not required)

The **`filter`** function implements only causal input and causal operation while the **`conv0`** function can use noncausal input. So, we will use the **`conv0`** function. At $n = 100$, the impulse response is $h(100) = 0.9^{100} = 2.6561 \times 10^{-5} \approx 0$. Hence, we will use first 101 samples of $h(n)$ in the **`conv0`** function.

Generate $y_{zp}[n]$ samples using the given procedure:

```
nx = 0:9; x = ones(1,10); % input sequence
nh = 0:100; h = (0.9).^nh; % impulse response
[y1,ny1] = conv0(h,nh,x,nx); % output y1[n]
[x2,nx2] = fold(x,nx); % folded input x[-n]
[y2,ny2] = conv0(h,nh,x2,nx2); % output y2[n]
[y2,ny2] = fold(y2,ny2); % fold output y2[n]
[y1,y2,nzp] = timealign(y1,ny1,y2,ny2); % synchronize y1 and y2 samples
yzp = y1 + y2; % zero-phase output y[n]
```

Generate $y_{zp}[n]$ samples from the analytic expression in (5.1):

```
nC = 0:9; % time array for the polynomial part
yC = C; % polynomial samples of (5.1)
ncausal = 0:nzp(end); % timing for the causal part
ycausal = A(3)*(p(3).^ncausal); % causal part from (5.1)
nanticausal = nzp(1):-1; % timing for the anticausal part
yanticausal = -A(1)*(p(1).^nanticausal); % anticausal part from (5.1)
[ycausal,yanticausal,nnoncausal] =
timealign(ycausal,ncausal,yanticausal,nanticausal);
ynoncausal = ycausal+yanticausal; % Add causal and anticausal parts
[~,Czp,~] = timealign(ynoncausal,nnoncausal,yC,nC); % sync polynomial and
noncausal parts
yzpcheck = ynoncausal + Czp; % add noncausal and polynomial parts as the
final yzp
```

Due to the boundary (i.e., left- and right-ends of **`yzp`** array) effects of convolution, we will ignore the first and last 9 samples before computing the absolute maximum difference.

```
Difference = max(abs(yzp(10:end-9)-yzpcheck(10:end-9)))
```

```
Difference = 1.5900e-11
```

Problem-2 Text Problem 10.12 (Page 745)

An ideal analog integrator is described by the system function $H_a(s) = 1/s$. A digital integrator with system function $H(z)$ can be obtained by use of the bilinear transformation. That is,

$$H(z) = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \equiv H_a(s)\Big|_{s=(2/T)(1-z^{-1})/(1+z^{-1})}$$

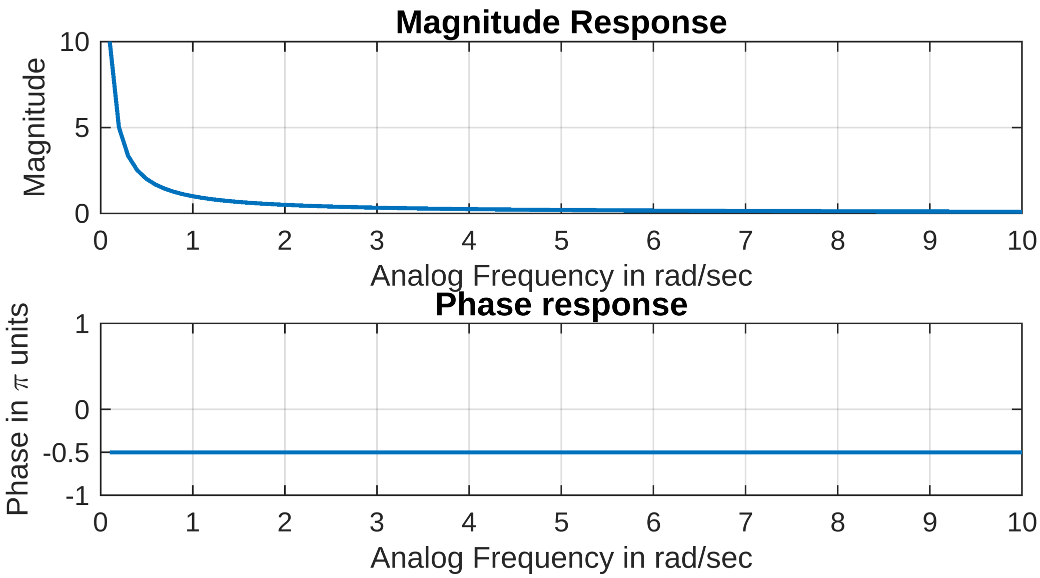
(a) Write the difference equation for the digital integrator relating the input $x(n)$ to the output $y(n)$.

Solution: GIven the transfer function, we have the difference equation as $y(n) = y(n-1) + \frac{T}{2}[x(n) + x(n-1)]$

(b) Plot the magnitude $|H_a(\mathrm{j}\Omega)|$ and phase $\Theta(\Omega)$ of the analog integrator.

Solution:

```
clc; close all; clear; b = 1; a = [1, 0];
Om = linspace(0.1,10,101); %om = linspace(0,1,1001)*pi;
h = freqs(b,a,Om); mag = abs(h); phase = angle(h);
figure('units','inches','Position',[0,0,6,3]);
subplot(2,1,1); plot(Om,mag,'linewidth',1.5); grid on;
ylabel('Magnitude'); title('Magnitude Response');
xlabel('Analog Frequency in rad/sec');
subplot(2,1,2); plot(Om,phase/pi,'linewidth',1.5); grid on;
xlabel('Analog Frequency in rad/sec'); axis([0,10,-1,1]);
ylabel('Phase in \pi units'); title('Phase response'); set(gca,'ytick',
[-1,-0.5,0,1]);
```



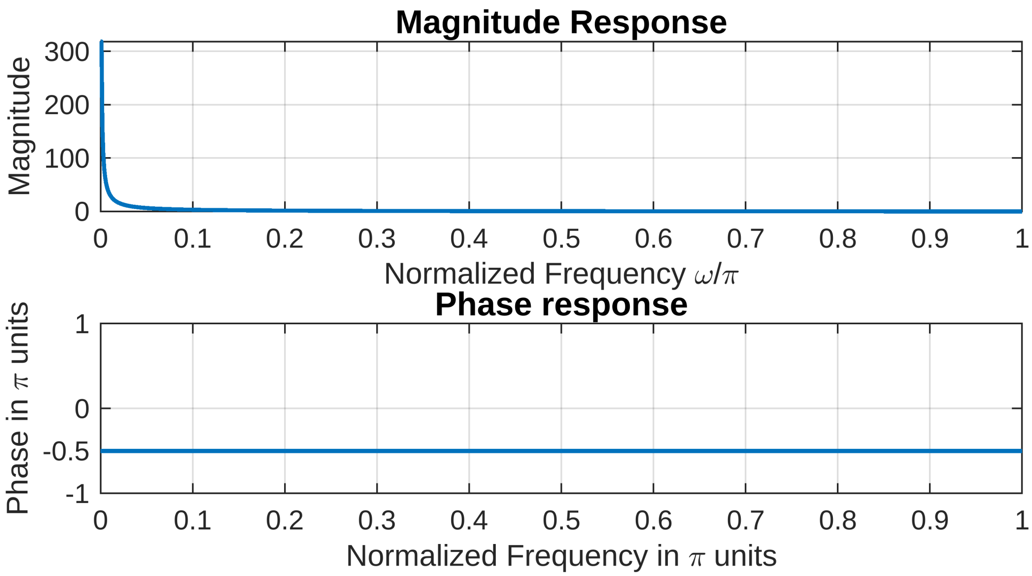
(c) It is easily verified that the frequency response of the digital integrator is

$$H(\omega) = -\jmath \frac{T}{2} \frac{\cos(\omega/2)}{\sin(\omega/2)} = -\jmath \frac{T}{2} \cot\left(\frac{\omega}{2}\right)$$

Plot $|H(\omega)|$ and $\theta(\omega)$ over $0 \le \omega/\pi \le 1$ interval using 2 rows and one columns of subplots.

MATLAB script:

```
% clc; close all; clear;
T = 1; om = linspace(0,1,1001)*pi;
H = -1j*(T/2)*cot(om/2);
H(1) = H(2); % tp avoid the 1/0 = inf value at om = 0% h = freqz(b,a,om);
mag = abs(H); phase = angle(H);
figure('units','inches','Position',[0,0,6,3]);
subplot(2,1,1); plot(om/pi,mag,'linewidth',1.5); grid on;
ylabel('Magnitude'); title('Magnitude Response');
xlabel('Normalized Frequency \omega/\pi');
subplot(2,1,2); plot(om/pi,phase/pi,'linewidth',1.5); grid on;
xlabel('Normalized Frequency in \pi units');
ylabel('Phase in \pi units'); title('Phase response');
set(gca,'ytick',[-1,-0.5,0,1]); axis([0,1,-1,1]);
```



(d) Compare the magnitude and phase characteristics obtained in parts (b) and (c). How well does the digital integrator match the magnitude and phase characteristics of the analog integrator?

Solution: The phase responses match exactly with each other. However, the magnitude response of the digital integrator is a compressed version of the analog integrator response, which is to be expected for the bilinear transformation.

(e) The digital integrator has a pole at $z = 1$. If you implement this filter on a digital computer, what restrictions might you place on the input signal sequence to avoid computational difficulties?

Solution: The digital integrator having a pole at $z = 1$ means that the pole is at $\omega = 0$. Therefore, the input signal should have no dc component.

Problem-3 Text problems 10.20 (Page 748)

In this problem you will be comparing some of the characteristics of analog and digital implementations of the single-pole lowpass analog system

$$H_a(s) = \frac{\alpha}{s+\alpha} \Leftrightarrow h_a(t) = \alpha e^{-\alpha t} u(t)$$

(a) What is the gain at dc? At what radian frequency is the analog frequency response 3 dB down from its dc value? At what frequency is the analog frequency response zero? At what time has the analog impulse response decayed to $1/e$ of its initial value?

Solution:

1. The dc gain is $H_a(0) = 1$.
2. For the 3 dB frequency $\Omega_{3\text{dB}}$, we want $|H(j\Omega_{3\text{dB}})|^2 = \frac{1}{2}|H(0)|^2 = \frac{1}{2}$, or

   $$\frac{1}{2} = |H_\mathrm{a}(\jmath\Omega_{3\text{dB}})|^2 = \frac{\alpha^2}{\alpha^2 + \Omega_{3\text{dB}}^2} \quad \Rightarrow \quad \Omega_{3\text{dB}}^2 + \alpha^2 = 2\alpha^2 \quad \Rightarrow \quad \Omega_{3\text{dB}}^2 = \alpha^2$$, leading to $\Omega_{3\text{dB}} = \alpha$.
3. The frequency response is zero at frequency is $\Omega = \infty$.
4. From the given impulse response, we want $h_a(t_0) = \alpha\frac{1}{\mathrm{e}} = \alpha\mathrm{e}^{-1} = \alpha\mathrm{e}^{-\alpha(1/\alpha)}$, leading to $t_0 = \frac{1}{\alpha}$.

(b) Give the digital system function $H(z)$ for the impulse-invariant design for this filter. What is the gain at dc? Give an expression for the 3-dB radian frequency. At what (real-valued) frequency is the response zero? How many samples are there in the unit sample time-domain response before it has decayed to $1/\mathrm{e}$ of its initial value?

Solution: Analog impulse response is $h_a(t) = \alpha\mathrm{e}^{-\alpha t}u(t)$.

1. Using $h(n) = h_a(nT) = \alpha\mathrm{e}^{-\alpha nT}u(n)$, we have $H(z) = \dfrac{\alpha}{1 - \mathrm{e}^{-\alpha T}z^{-1}}$,
2. The impulse response is $H(\omega) = \dfrac{\alpha}{1 - \mathrm{e}^{-\alpha T}\mathrm{e}^{-\jmath\omega}}$ so the dc gain is $H(0) = \dfrac{\alpha}{1 - \mathrm{e}^{-\alpha T}}$.
3. For the 3dB frequency ω_0, we have $|H(\omega_0)|^2 = \frac{1}{2}|H(0)|^2$, leading to $\omega_0 = 2\sin^{-1}\left[\sinh\left(\frac{\alpha T}{2}\right)\right]$. The proof is given below.
4. The frequency response will never go to zero.
5. From the given impulse response, we want $h(n_0) = \alpha\mathrm{e}^{-\alpha T n_0} = \alpha\mathrm{e}^{-1}$, leading to $n_0 = \dfrac{1}{\alpha T}$. Therefore, the sample index should be the the smallest integer that is larger than $\dfrac{1}{\alpha T}$, that is, $n_0 = \left\lceil\dfrac{1}{\alpha T}\right\rceil$.

Proof: Let ω_0 be the 3dB frequency. Then we want

$$|H(\omega_0)|^2 = \left|\frac{\alpha}{1 - \mathrm{e}^{-\alpha T}\mathrm{e}^{-\jmath\omega_0}}\right|^2 = \frac{1}{2}|H(0)|^2 = \frac{1}{2}\left|\frac{\alpha}{1 - \mathrm{e}^{-\alpha T}}\right|^2.$$

This leads to

$$\alpha^2\left|\frac{1}{1 - \mathrm{e}^{-\alpha T}\mathrm{e}^{-\jmath\omega_0}}\right|^2 = \frac{\alpha^2}{2}\left|\frac{1}{1 - \mathrm{e}^{-\alpha T}}\right|^2 \qquad (3\text{b}.1)$$

Let $a = \mathrm{e}^{-\alpha T}$. Then from $(3\text{b}.1)$, we have

$$2(1-a)^2 = \left|1 - a\mathrm{e}^{-\jmath\omega_0}\right|^2 = 1 + a^2 - 2a\cos(\omega_0)$$

or

$$\begin{aligned}\cos(\omega_0) &= \frac{4a-a^2-1}{2a} = \frac{2a-(a-1)^2}{2a} = 1-\frac{(a-1)^2}{2a} \\ 1-2\sin^2(\omega_0/2) &= 1-\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2 \quad\Rightarrow\quad \sin^2(\omega_0/2) = \frac{1}{4}\left(\frac{1}{\sqrt{a}}-\sqrt{a}\right)^2 \quad (3b.2)\end{aligned}$$

Substituting $a = \mathrm{e}^{-\alpha T}$ in $(3b.2)$ and recognizing that $\left(\frac{1}{\sqrt{a}}-\sqrt{a}\right) = \mathrm{e}^{\alpha T/2}-\mathrm{e}^{-\alpha T/2} = 2\sinh\left(\frac{\alpha T}{2}\right)$, we obtain

$$\sin^2(\omega_0/2) = \sinh^2(\alpha T/2) \quad\Rightarrow\quad \omega_0 = 2\sin^{-1}\big(\sinh(\alpha T/2)\big)$$

which completes the proof.

(c) "Prewarp" the parameter α and perform the bilinear transformation to obtain the digital system function $H(z)$ from the analog design. What is the gain at dc? At what (real-valued) frequency is the response zero? Give an expression for the 3-dB radian frequency. How many samples are there in the unit sample time-domain response before it has decayed to $1/\mathrm{e}$ of its initial value?

Solution: With bilinear transformation, we have $H(z) = \dfrac{\alpha}{\dfrac{2}{T}\dfrac{1-z^{-1}}{1+z^{-1}}+\alpha} = \dfrac{\alpha T(1+z^{-1})}{2+\alpha T+(\alpha T-2)z^{-1}}$.

1. The dc gain of the digital filter is $H(z)|_{z=1} = 1$.
2. The response of the digital filter is zero when $(1+z^{-1})$ in the numerator above is zero, or when $z=-1$ or $\omega = \pi$.
3. The 3-dB frequency of the analog filter is $\Omega_{3\mathrm{dB}} = \alpha$. Using the warping relation, $\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$, the 3-dB frequency of the digital filter is $\omega_0 = 2\tan^{-1}\left(\frac{\alpha T}{2}\right)$.
4. Using $a = \dfrac{2-\alpha T}{2+\alpha T}$, $H(z)$ can be simplified as $H(z) = \dfrac{1-a}{2}\left[1+\dfrac{(1+a)z^{-1}}{1-az^{-1}}\right]$, leading to $h(n) = \dfrac{1-a}{2}[\delta(n)+(1+a)a^{n-1}u(n-1)] \quad\Rightarrow\quad h(0) = \dfrac{1-a}{2}$. Now, we want, for $n_0>0$, $h(n_0) = \dfrac{h(0)}{\mathrm{e}} = \dfrac{1-a}{2\mathrm{e}}$, leading to $n_0 = \dfrac{\ln\frac{a}{1+a}-1}{\ln a}$. Therefore, n_0 should be the smallest integer that is larger than $\dfrac{\ln\left(\frac{a}{1+a}\right)-1}{\ln a}$ or $n_0 = \left\lceil\dfrac{\ln\left(\frac{a}{1+a}\right)-1}{\ln a}\right\rceil$ where $a = \dfrac{2-\alpha T}{2+\alpha T}$.

Problem-4 Text problems CP 10.16 and CP 10.17 (Page 755)

A lowpass digital filter has the following specifications:

$$\begin{array}{rclrcl} \omega_p & = & 0.2\pi & \omega_s & = & 0.3\pi \\ \text{passband ripple} & = & 1\text{ dB} & \text{stopband ripple} & = & 20\text{ dB} \end{array}$$

(a) Design a lowpass digital filter using a Chebyshev-II prototype to satisfy the specifications given above. Provide the system function $H(z)$ in cascade form. Plot the log-magnitude frequency response over $0 \le \omega/\pi \le 1$ interval.

MATLAB script:

```
clc; close all; clear;
fp = 0.2; fs = 0.3; Ap = 1; As = 20;
[N,omegac] = cheb2ord(fp,fs,Ap,As);N, omegac
```

```
N = 4
omegac = 0.3000
```

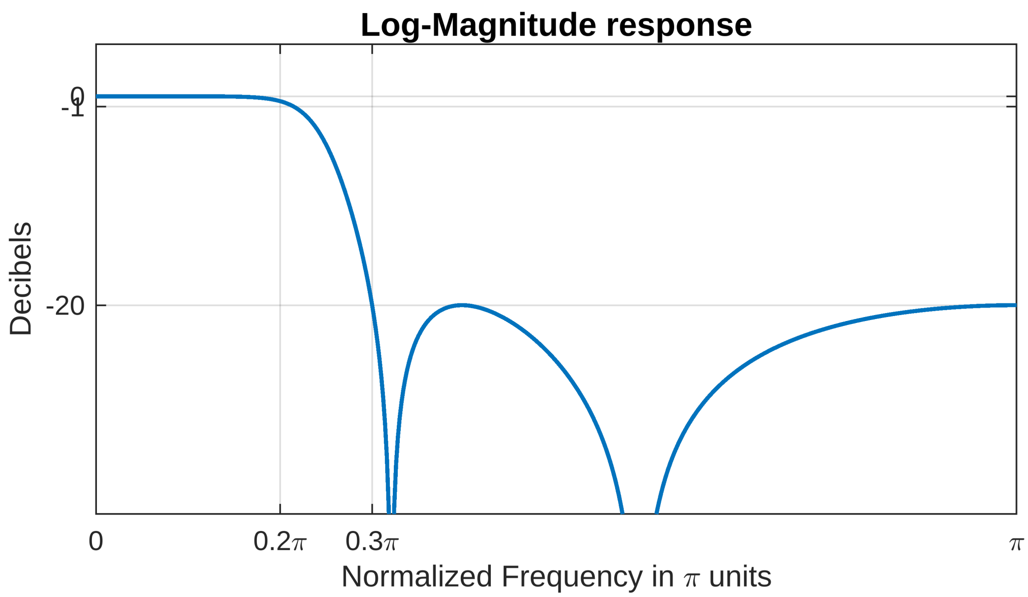
```
[b,a] = cheby2(N,As,omegac); [sos,G] = tf2sos(b,a)
```

```
sos = 2×6
    1.0000    0.5574    1.0000    1.0000   -0.6045    0.1656
    1.0000   -1.0671    1.0000    1.0000   -1.2030    0.6962
G = 0.1160
```

Therefore,

$$H(z) = 0.1160\frac{(1+0.5574z^{-1}+z^{-2})(1-1.0671z^{-1}+z^{-2})}{(1-0.6045z^{-1}+0.1656z^{-2})((1-1.2030z^{-1}+0.6962z^{-2}))}$$

```
om = linspace(0,1,501)*pi; H = freqz(b,a,om);
figure('units','inches','Position',[0,0,6,3]);
Hmag = abs(H); Hdb = 20*log10(Hmag);
plot(om/pi,Hdb,'linewidth',1.5); axis([0,1,-40,5]);
xlabel('Normalized Frequency in \pi units');
ylabel('Decibels'); title('Log-Magnitude response');
set(gca,'xtick',[0,0.2,0.3,1],'ytick',[-20,-1,0]); grid;
set(gca,'xticklabel',{'0','0.2\pi','0.3\pi','\pi'});
```



(b) Design a lowpass digital filter using a elliptic prototype to satisfy the specifications given above. Provide the system function $H(z)$ in the rational function form. Plot the log-magnitude frequency response over $0 \le \omega/\pi \le 1$ interval.

MATLAB script:

```
[N,omegac] = ellipord(fp,fs,Ap,As); N, omegac
```

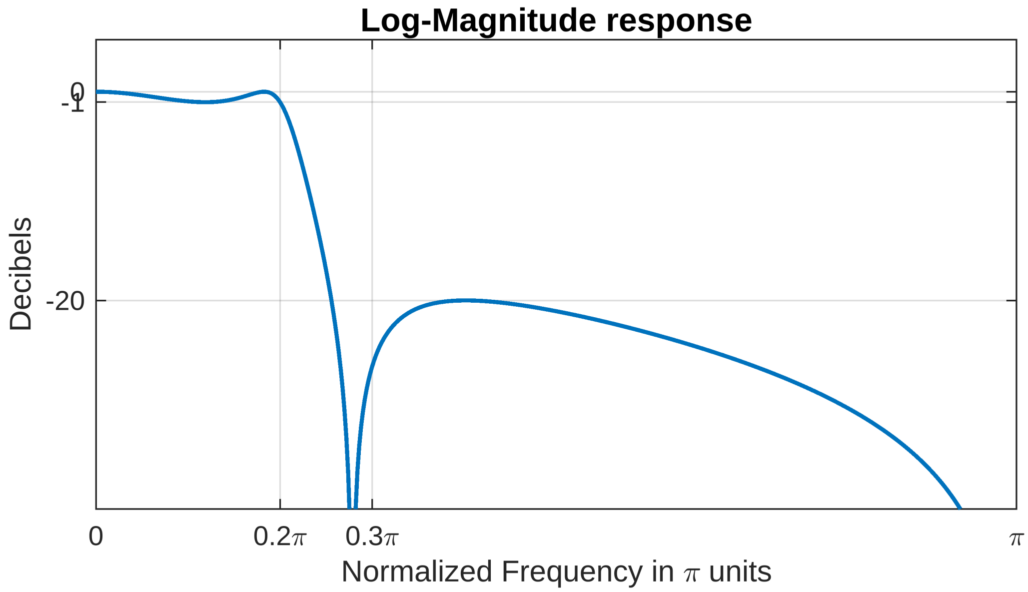
```
N = 3
omegac = 0.2000
```

```
[b,a] = ellip(N,Ap,As,omegac)
```

```
b = 1×4
    0.0865   -0.0244   -0.0244    0.0865
a = 1×4
    1.0000   -2.1220    1.7872   -0.5410
```

Therefore, $H(z) = \dfrac{0.0865 - 0.0244z^{-1} - 0.0244z^{-2} + 0.0865z^{-3}}{1 - 2.122z^{-1} + 1.7872z^{-2} - 0.541z^{-3}}$

```
om = linspace(0,1,501)*pi; H = freqz(b,a,om);
figure('units','inches','Position',[0,0,6,3]);
Hmag = abs(H); Hdb = 20*log10(Hmag);
plot(om/pi,Hdb,'linewidth',1.5); axis([0,1,-40,5]);
xlabel('Normalized Frequency in \pi units');
ylabel('Decibels'); title('Log-Magnitude response');
set(gca,'xtick',[0,0.2,0.3,1],'ytick',[-20,-1,0]); grid;
set(gca,'xticklabel',{'0','0.2\pi','0.3\pi','\pi'});
```



Problem-5: Text Problem CP 10.20 (Page 745) (Page 755)

Use the bilinear transformation to design a 10^{th}-order Butterworth bandstop filter to remove a digital frequency $\omega = 0.4\pi$ with a bandwidth of 0.1π. Choose appropriate value for the stopband attenuation.

(a) Provide the system function $H(z)$ in the cascade form.

MATLAB script for design: Since we want to remove the digital frequency $\omega = 0.4\pi$ with a bandwidth of 0.1π, the stopband edge frequencies are $\omega_{s_1} = 0.35\pi$ and $\omega_{s_2} = 0.45\pi$, assuming equal spread around the center frequency. Now, we do not know passband edges nor attenuations at these edges. These are then the free variables and we have many choices to select them. First, we assume reasonable values of attenuations: $A_p = 1$dB and $A_s = 50$dB. Then the free variable is the transition bandwidth $\Delta\omega$, Assuming an equal transition bandwidth on each side of the stopband, we have

$$\omega_{p_1} = 0.35\pi - \Delta\omega \text{ and } \omega_{p_2} = 0.45\pi + \Delta\omega$$

Now we vary $\Delta\omega$ to obtain the bandstop filter order of 10 or the prototype lowpass filter order of 5 using the the **`buttord`** function. We start with a small $\Delta\omega$ value and increase it until $N = 5$.

```
% clc; close all; clear;
ws = [0.35*pi,0.45*pi]; As = 40; Ap = 1; % fixed parameters
Deltaf = 0.05; % starting value
wp = [0.35-Deltaf,0.45+Deltaf]*pi;
[N, ~] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 8
```

```
Deltaf = 0.06; % new value
wp = [0.35-Deltaf,0.45+Deltaf]*pi;
```

```
[N, ~] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 7
```

```
Deltaf = 0.08; % new value
wp = [0.35-Deltaf,0.45+Deltaf]*pi;
[N, ~] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 6
```

```
Deltaf = 0.09; % new value
wp = [0.35-Deltaf,0.45+Deltaf]*pi;
[N, ~] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 6
```

```
Deltaf = 0.095; % new value
wp = [0.35-Deltaf,0.45+Deltaf]*pi;
[N, ~] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 5
```

```
% We now use these values
[N, wn] = buttord(wp/pi,ws/pi,Ap,As),
```

```
N = 5
wn = 1×2
    0.2856    0.5267
```

```
[b,a] = butter(N,wn,'stop');
fprintf('The system function in the cascade form is\n\n')
```

```
The system function in the cascade form is
```

```
[b0,B,A] = dir2cas(b,a)
```

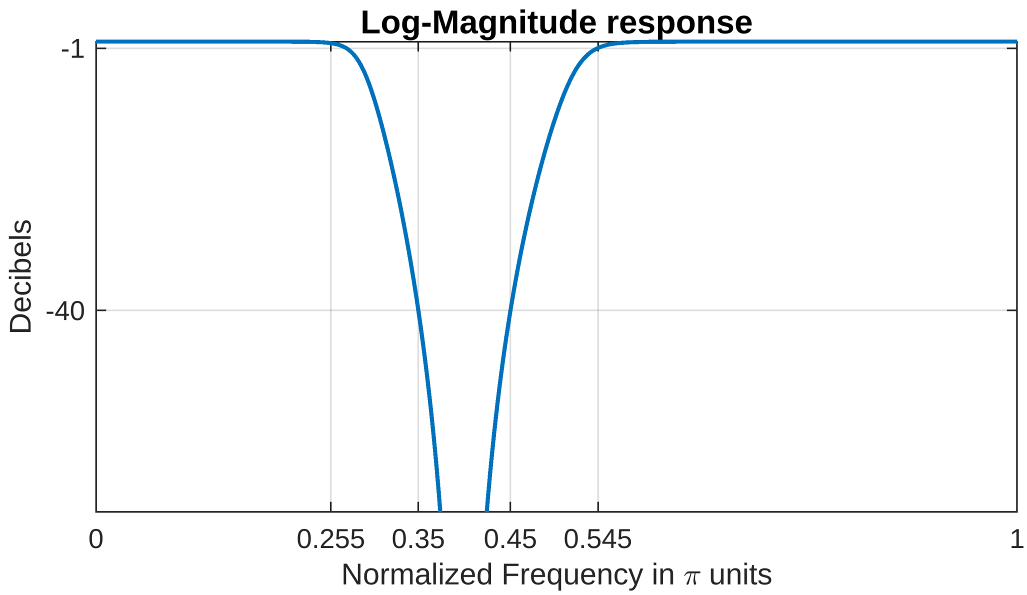
```
b0 = 0.2827
B = 5×3
    1.0000   -0.6242    0.9994
    1.0000   -0.6252    1.0012
    1.0000   -0.6254    0.9984
    1.0000   -0.6270    1.0013
    1.0000   -0.6271    0.9996
A = 5×3
    1.0000    0.1204    0.7859
    1.0000   -0.0992    0.5068
    1.0000   -0.4477    0.4307
    1.0000   -0.8188    0.5632
    1.0000   -1.1213    0.8267
```

(b) Plot the log-magnitude frequency response of the filter over $0 \leq \omega/\pi \leq 1$ interval.

MATLAB script for plot:

```
om = linspace(0,1,501)*pi; H = freqz(b,a,om);
figure('units','inches','Position',[0,0,6,3]);
Hmag = abs(H); Hdb = 20*log10(Hmag);
plot(om/pi,Hdb,'linewidth',1.5); %axis([0,1,-80,5]);
```

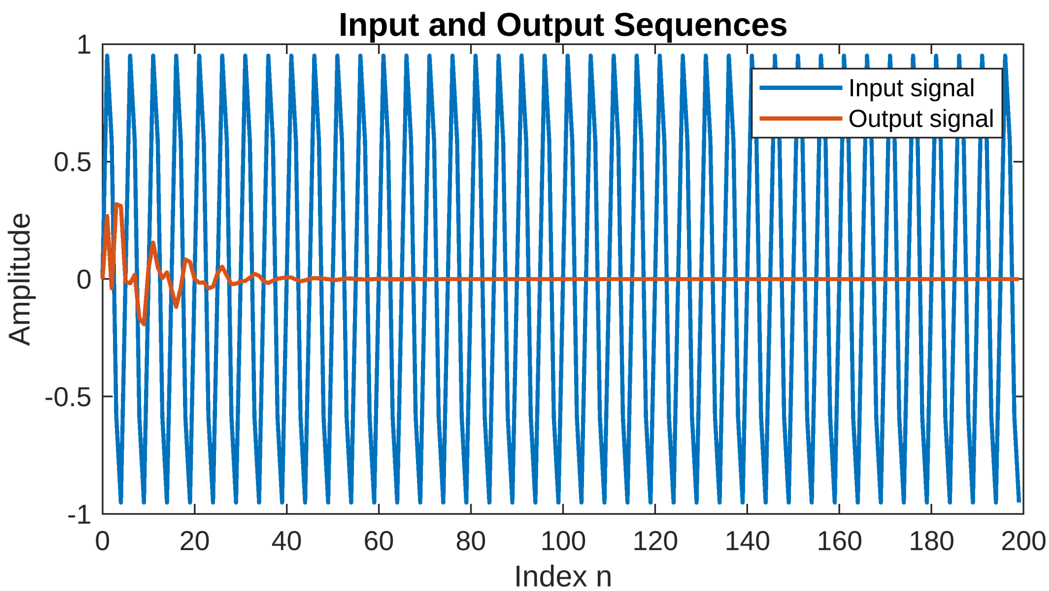
```
xlabel('Normalized Frequency in \pi units');
ylabel('Decibels'); title('Log-Magnitude response');
set(gca,'xtick',sort([0,wp,ws,pi]/pi),'ytick',[-As,-Ap]); grid;
%set(gca,'xticklabel',{'0','0.35','0.4','0.45','1'});
ylim([-70, 0]);
```



(c) Generate 200 samples of the sequence and pass the sequence through the bandstop filter. Plot the filter output sequence (using the **`plot`** function) and comment on its effectiveness in suppressing the sinusoidal signal.

MATLAB script for signal processing:

```
n = 0:200-1; x = sin(0.4*pi.*n); y = filter(b,a,x);
figure('units','inches','Position',[0,0,6,3]);
plot(n,x,'linewidth',1.5); hold on
plot(n,y,'linewidth',1.5); xlabel('Index n'); ylabel('Amplitude');
title('Input and Output Sequences');
legend('Input signal', 'Output signal')
```



According to the figure, the sinusoidal signal samples at 0.4π are totally removed after around 120 samples.

Problem-6: Lowpass Digital Filter Design using Impulse Invariance

A lowpass digital filter's specifications are given by:

$$\omega_\mathrm{p} = 0.15\pi \ \text{radians}, \quad A_\mathrm{p} = 1 \ \text{dB}, \qquad \omega_\mathrm{s} = 0.3\pi \ \text{radians}, \quad A_\mathrm{s} = 45 \ \text{dB}.$$

(a) Using an **impulse invariance** approach obtain a system function $H(z)$ in the **cascade form** that satisfies the above specifications with **equiripple passband** and **monotone stopband**.

MATLAB script for design: Since the design requires equiripple passband but monotone stopband, the analog prototype is Chebyshev-1. Furthermore, since impulse invariance approach is required for filter transformation, we cannot use MATLAB's digital filter design function **cheby1** because it uses bilinear transformation. Therefore, we will first obtain the analog prototype specifications from the given digital filter specifications via inverse impulse invariance mapping. Then we will design the analog prototype filter. Finally, we will transform the analog filter into a digital filter using impulse invariance transformation.

```
clc; close all; clear;
omp = 0.15*pi; Ap = 1; oms = 0.3*pi; As = 45; % Digital filter specs
% Analog filter specifications: Inverse mapping for frequencies
T = 0.01; % This is arbitrary
Omp = omp/T; % Analog passband cutoff
Oms = oms/T; % Analog stopband cutoff
% Analog filter design
[N,Omegac] = cheb1ord(Omp,Oms,Ap,As,'s'); % Analog Chebyshev-1 design
parameters
fprintf('Order of the prototype analog lowpass filter: %I \n',N);
```

```
Order of the prototype analog lowpass filter:
```

```
[Cs,Ds] = cheby1(N,Ap,Omegac,'s'); % Analog Chebyshev-1 filter design
% Digital filter design
[b,a] = impinvar(Cs,Ds,1/T);
% Cascade form representation
[sos,G] = tf2sos(b,a); format long; SoS = sos*G; SoS(:,1:3), SoS(:,4:6)
```

```
ans = 3×3
                  0   0.000107796344187                   0
   0.000107796344187   0.001057854864898   0.000878375912318
   0.000107796344187   0.000010152192715                   0
ans = 3×3
10^-3 ×
   0.107796344187054  -0.094049686448922                   0
   0.107796344187054  -0.185093155545258   0.086445201266747
   0.107796344187054  -0.184600279034811   0.099080358700973
```

$$H(z) = \left(\frac{-0.22\times10^{-16} + 0.1078\times10^{-3}z^{-1}}{-0.2220\times10^{-16} + 0.1937\times10^{-16}z^{-1}}\right)$$
$$\times\left(\frac{-0.22\times10^{-16} - 0.218\times10^{-15}z^{-1} - 0.181\times10^{-15}z^{-2}}{-0.2220\times10^{-16} + 0.3813\times10^{-16}z^{-1} - 0.1781\times10^{-16}z^{-2}}\right)$$
$$\times\left(\frac{-0.22\times10^{-16} - 0.2\times10^{-17}z^{-1}}{-0.2220\times10^{-16} + 0.3802\times10^{-16}z^{-1} - 0.2041\times10^{-16}z^{-2}}\right)$$

(b) What is the order of the digital lowpass filter?

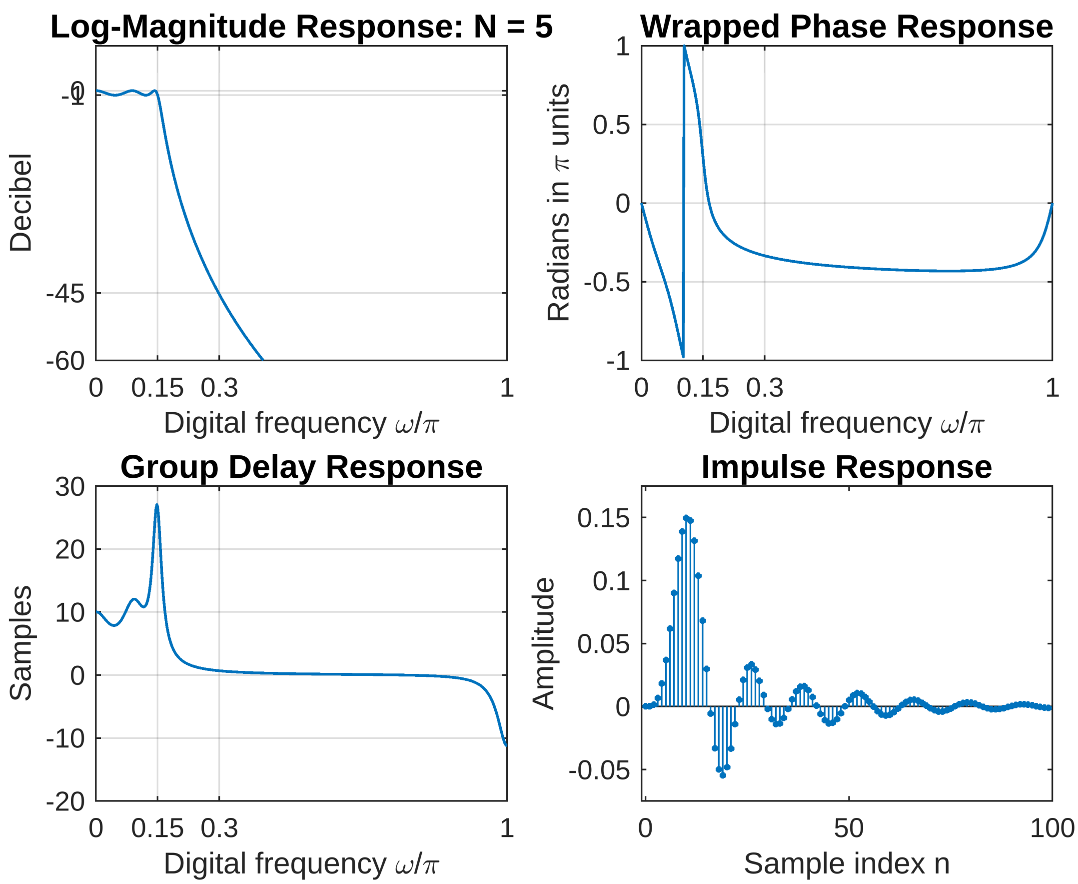
Answer: Order of the digital lowpass filter is $N = 5$.

(c) Provide design plots in the form of log-magnitude, phase, group-delay, and impulse responses.

MATLAB script for plots:

```
f = linspace(0,1,501); om = f*pi; % Digital frequency array
H = freqz(b,a,om); % Frequency response
Hmag = abs(H); HdB = 20*log10(Hmag/max(Hmag)); % Log-magnitude response
Hpha = angle(H)/pi; % Phase response in pi units
Grd = grpdelay(b,a,om); % Group delay
[h,n] = impz(b,a,100); % Impulse response
figure('Units','inches','position',[0,0,8,6]);
subplot(2,2,1); % Log-Magnitude plot
plot(f,HdB,"LineWidth",1); axis([0,1,-60,10]);
set(gca,'xtick',[0,omp/pi,oms/pi,1],'ytick',[-60,-As,-Ap,0]);
xlabel('Digital frequency \omega/\pi'); ylabel('Decibel');
title('Log-Magnitude Response: N = 5'); grid;
subplot(2,2,2); % Phase
plot(f,Hpha,'linewidth',1); axis([0,1,-1,1]);
set(gca,'xtick',[0,omp/pi,oms/pi,1]);
xlabel('Digital frequency \omega/\pi'); ylabel('Radians in \pi units');
title('Wrapped Phase Response'); grid;
subplot(2,2,3); % Group-delay plot
plot(f,Grd,'LineWidth',1); axis([0,1,-20,30]);
set(gca,'xtick',[0,omp/pi,oms/pi,1],'ytick',(-20:10:30));
xlabel('Digital frequency \omega/\pi'); ylabel('Samples');
```

```
title('Group Delay Response'); grid;
subplot(2,2,4); % Impulse response plot
stem(n,h,'filled','LineWidth',0.5,'MarkerSize',2);
axis([-1,100,-0.075,0.175]);
xlabel('Sample index n'); ylabel('Amplitude'); title('Impulse Response');
```



Problem-7: Lowpass Digital Filter Design using Bilinear Transformation

A lowpass digital filter's specifications are given by:

$$\omega_\mathrm{p} = 0.2\pi \text{ radians}, \quad A_\mathrm{p} = 1 \text{ dB}, \qquad \omega_\mathrm{s} = 0.4\pi \text{ radians}, \quad A_\mathrm{s} = 50 \text{ dB}.$$

(a) Using **bilinear transformation** and the **Elliptic approximation** approach obtain a system function $H(z)$ in the **parallel form** that satisfies the above specifications.

MATLAB script for design: Since the design uses bilinear transformation, we can use MATLAB's order calculation and design functions for the elliptic filter.

```
clear; clc; close all; format short
omp = 0.2*pi; Ap = 1; oms = 0.4*pi; As = 50; % Digital filter specs
[N,omegac] = ellipord(omp/pi,oms/pi,Ap,As); % Elliptic filter order
[b,a] = ellip(N,Ap,As,omegac); % Elliptic filter design
```

```
[C,B,A] = dir2par(b,a), % Parallel orm
```

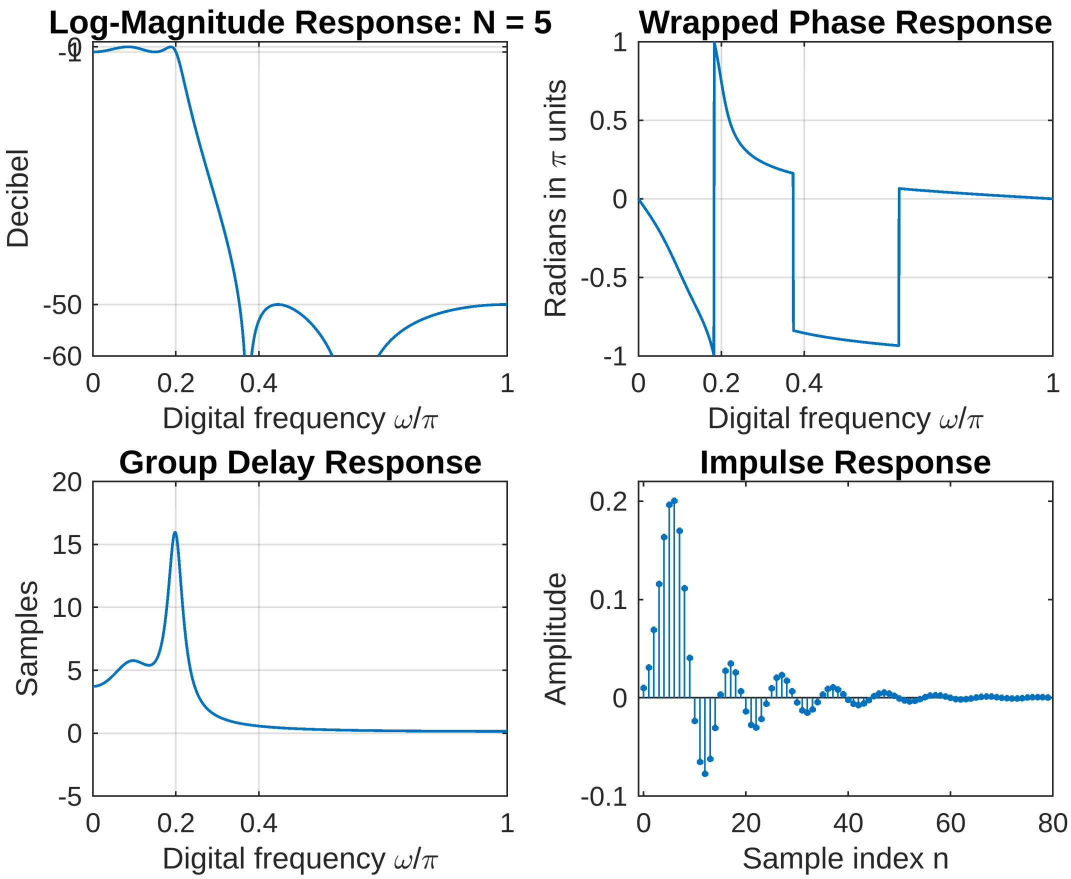
```
C = 0.0182
B = 2×2
   -0.0649   -0.0223
    0.0568    0.0644
A = 2×3
    1.0000   -1.5138    0.8687
    1.0000   -1.5287    0.6370
```

$$H(z) = 0.0182 + \frac{-0.0649 - 0.0223z^{-1}}{1 - 1.5138z^{-1} + 0.8687z^{-2}} + \frac{0.0568 + 0.0644z^{-1}}{1 - 1.5287z^{-1} + 0.637z^{-2}}$$

(b) Provide design plots in the form of log-magnitude, phase, group-delay, and impulse responses.

MATLAB script for plots:

```
% Use the above figure and plot parameters
f = linspace(0,1,1001); om = f*pi; % Digital frequency array
H = freqz(b,a,om); % Frequency response
Hmag = abs(H); HdB = 20*log10(Hmag/max(Hmag)); % Log-magnitude response
Hpha = angle(H)/pi; % Phase response in pi units
Grd = grpdelay(b,a,om); % Group delay
[h,n] = impz(b,a,80); % Impulse response
figure('units','inches','Position',[0,0,8,6]);
subplot(2,2,1); % Log-Magnitude plot
plot(f,HdB,"LineWidth",1); axis([0,1,-60,1]);
set(gca,'xtick',[0,omp/pi,oms/pi,1],'ytick',[-60,-As,-Ap,0]);
xlabel('Digital frequency \omega/\pi'); ylabel('Decibel');
title('Log-Magnitude Response: N = 5'); grid;
subplot(2,2,2); % Phase
plot(f,Hpha,'linewidth',1); axis([0,1,-1,1]);
set(gca,'xtick',[0,omp/pi,oms/pi,1]);
xlabel('Digital frequency \omega/\pi'); ylabel('Radians in \pi units');
title('Wrapped Phase Response'); grid;
subplot(2,2,3); % Group-delay plot
plot(f,Grd,'LineWidth',1); axis([0,1,-5,20]);
set(gca,'xtick',[0,omp/pi,oms/pi,1],'ytick',(-5:5:20));
xlabel('Digital frequency \omega/\pi'); ylabel('Samples');
title('Group Delay Response'); grid;
subplot(2,2,4); % Impulse response plot
stem(n,h,'filled','LineWidth',0.5,'MarkerSize',2); axis([-1,80,-0.1,0.22]);
xlabel('Sample index n'); ylabel('Amplitude'); title('Impulse Response');
```



(c) Determine the exact band-edge frequencies for the given attenuation.

Answer: To determine exact band edges, we need to determine indices into the frequency array at which the attenuation specifications are satisfied in the log-magnitude plot. Alternatively, one could use the "Data Cursor" tool to search for the x-axis value for each of the $-A_P$ and $-A_s$ y-axis attenuations. Also, we may not be able to get the exact real value since we have discrete set of frequencies that we use to plot. We will consider the former approach using the **`find`** function.

```
df = 1/1000; % normalized frequency resolution
fp_exact = (find(HdB >= -Ap,1,'last')-1)*df;
fprintf('Exact passband edge: %g⊠ radians\n',fp_exact);
```

```
Exact passband edge: 0.2⊠ radians
```

```
fs_exact = (find(HdB <= -As,1,'first')-1)*df;
fprintf('Exact stopband edge: %g⊠ radians\n',fs_exact);
```

```
Exact stopband edge: 0.354⊠ radians
```

Problem-8: Highpass Digital Filter Design

Design a highpass digital filter that has an equiripple passband and a monotone stopband and satisfies the following specifications:

$$\omega_s = 0.4586\pi; \quad \omega_p = 0.6\pi; \quad A_s = 15 \text{ dB}; \quad A_p = 1 \text{ dB}.$$

(a) Provide the system function $H(z)$ in the cascade form containing second-order sections with real coefficients.

MATLAB script for design: This is a Chebishev-1 design.

```
% clc; close all; clear;
omp = 0.6*pi; Ap = 1; oms = 0.4586*pi; As = 15; % Digital filter specs
[N,omegac] = cheb1ord(omp/pi,oms/pi,Ap,As); % Chebyshev-1 filter order
fprintf('Order of the prototype lowpass filter: %g \n',N);
```

```
Order of the prototype lowpass filter: 4
```

```
[b,a] = cheby1(N,Ap,omegac,'high'); % Chebyshev-1 filter design
[sos,G] = tf2sos(b,a)
```

```
sos = 2×6
    1.0000   -2.0000    1.0000    1.0000    1.0416    0.4019
    1.0000   -2.0000    1.0000    1.0000    0.5561    0.7647
G = 0.0243
```

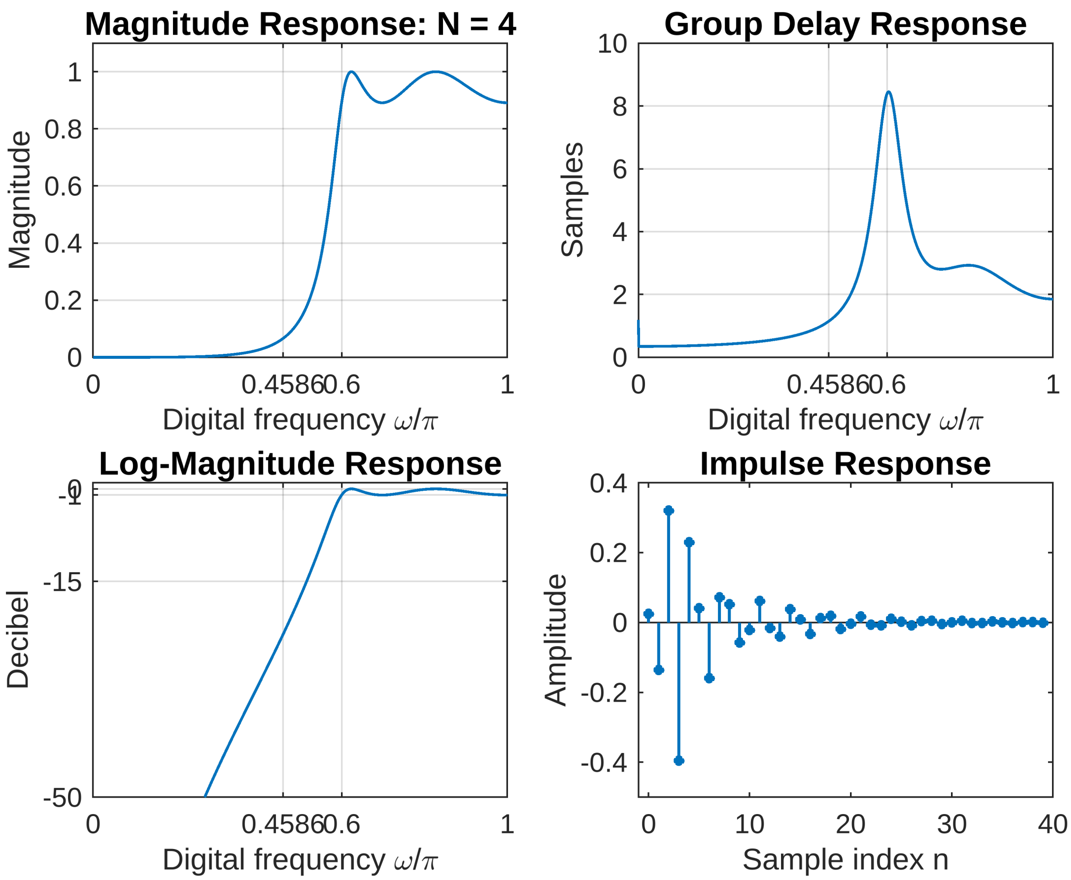
$$H(z) = 0.0243 \times \left(\frac{1 - 2z^{-1} + z^{-2}}{1 + 1.0416z^{-1} + 0.4019z^{-2}}\right)\left(\frac{1 - 2z^{-1} + z^{-2}}{1 + 0.5561z^{-1} + 0.7647z^{-2}}\right)$$

(b) Provide plots of the magnitude, log-magnitude, group-delay, and impulse responses.

MATLAB script for plots:

```
f = linspace(0,1,1001); om = f*pi; % Digital frequency array
H = freqz(b,a,om); % Frequency response
Hmag = abs(H); HdB = 20*log10(Hmag/max(Hmag)); % Log-magnitude response
Grd = grpdelay(b,a,om); % Group delay
[h,n] = impz(b,a,40); % Impulse response
figure('Units','inches','position',[0,0,8,6]);
subplot(2,2,1); % Magnitude plot
plot(f,Hmag,"LineWidth",1); axis([0,1,0,1.1]);
set(gca,'xtick',[0,oms/pi,omp/pi,1],'ytick',(0:0.2:1));
xlabel('Digital frequency \omega/\pi'); ylabel('Magnitude');
title('Magnitude Response: N = 4'); grid;
subplot(2,2,2); % Group delay plot
plot(f,Grd,'LineWidth',1); %axis([0,1,-5,20]);
set(gca,'xtick',[0,oms/pi,omp/pi,1],'ytick',(0:2:10));
xlabel('Digital frequency \omega/\pi'); ylabel('Samples');
title('Group Delay Response'); grid;
subplot(2,2,3); % Log-magnitude (dB) plot
plot(f,HdB,"LineWidth",1); axis([0,1,-50,1]);
set(gca,'xtick',[0,oms/pi,omp/pi,1],'ytick',[-50,-As,-Ap,0]);
xlabel('Digital frequency \omega/\pi'); ylabel('Decibel');
title('Log-Magnitude Response'); grid;
```

```
subplot(2,2,4); % Impulse response plot
stem(n,h,'filled','LineWidth',1,'MarkerSize',3); axis([-1,40,-0.5,0.4]);
xlabel('Sample index n'); ylabel('Amplitude'); title('Impulse Response');
```



(c) What are the exact passband and stopband edges for the given A_p and A_s, respectively?

Answer: We will use the **`find`** function.

```
df = 1/1000; % normalized frequency resolution
fp_exact = (find(HdB >= -Ap,1,'first')-1)*df;
fprintf('Exact passband edge: %g⊠ radians\n',fp_exact);
```

```
Exact passband edge: 0.6⊠ radians
```

```
fs_exact = (find(HdB <= -As,1,'last')-1)*df;
fprintf('Exact stopband edge: %g⊠ radians\n',fs_exact);
```

```
Exact stopband edge: 0.515⊠ radians
```

(d) What is the order N of the highpass filter?

Answer: Order of the highpass filter is $N = 4$.

Problem-9: Bandpass Digital Filter Design using Elliptic Prototype

Design an elliptic bandpass filter that satisfies the specifications given below.

$$\text{Lower stopband} : |\omega| \le 0.3\pi,\ A_{s_1} = 40\ \text{dB}$$

$$\text{Passband} : 0.4\pi \le |\omega| \le 0.6\pi, A_{\text{p}} = 1\ \text{dB}$$

$$\text{Upper stopband} : 0.75\pi \le |\omega| \le \pi,\ A_{s_2} = 40\ \text{dB}$$

(a) Provide the system function $H(z)$ in the parallel form containing second-order sections with real coefficients.

MATLAB script for design:

```
clc; close all; clear;
fp = [0.4,0.6]; % Passband edges
fs = [0.3,0.75]; % stopband edges
Ap = 1; As = 40; % Attenuations in dB
[N,omegac] = ellipord(fp,fs,Ap,As); % Order of the lowpass elliptic filter
fprintf('Order of the prototype lowpass filter: %g \n',N);
```

```
Order of the prototype lowpass filter: 4
```

```
[b,a] = ellip(N,Ap,As,omegac,'bandpass'); % design of bandpass elliptic
filter
[C,B,A] = dir2par(b,a), % Parallel form
```

```
C = 0.0354
B = 4×2
   -0.0292   -0.0614
    0.0213    0.2151
    0.0213   -0.2151
   -0.0292    0.0614
A = 4×3
    1.0000    0.5963    0.9399
    1.0000    0.2774    0.7929
    1.0000   -0.2774    0.7929
    1.0000   -0.5963    0.9399
```

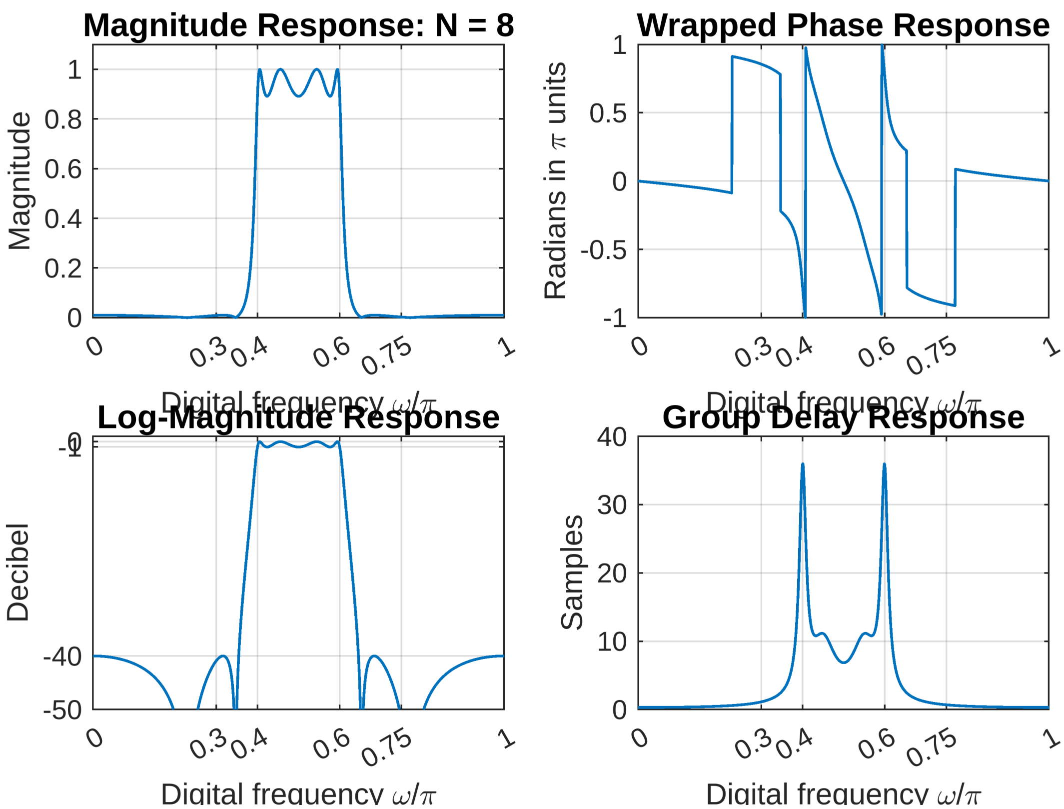
$$\begin{aligned} H(z) = 0.0354 &+ \frac{-0.0292 - 0.0614z^{-1}}{1 + 0.5963z^{-1} + 0.9399z^{-2}} + \frac{0.0213 + 2151z^{-1}}{1 + 0.2774z^{-1} + 0.7929z^{-2}} \\ &+ \frac{0.0213 - 2151z^{-1}}{1 - 0.2774^{-1} + 0.7929^{-2}} + \frac{-0.0292 + 0.0614z^{-1}}{1 - 0.5963z^{-1} + 0.9399z^{-2}} \end{aligned}$$

(b) Provide plots of the magnitude, log-magnitude, group-delay, and phase responses.

MATLAB script for plots:

```
% Use the above figure and plot parameters
f = linspace(0,1,1001); om = f*pi; % Digital frequency array
H = freqz(b,a,om); % Frequency response
Hmag = abs(H); % Magnitude response
HdB = 20*log10(Hmag/max(Hmag)); % Log-magnitude response
Hpha = angle(H)/pi; % Phase response in pi units
Grd = grpdelay(b,a,om); % Group delay
figure('Units','inches','position',[0,0,9,6]);
```

```
subplot(2,2,1); % Magnitude plot
plot(f,Hmag,"LineWidth",1); axis([0,1,0,1.1]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',(0:0.2:1));
xlabel('Digital frequency \omega/\pi'); ylabel('Magnitude');
title('Magnitude Response: N = 8'); grid;
subplot(2,2,2); % phase response plot
plot(f,Hpha,'linewidth',1); axis([0,1,-1,1]);
set(gca,'xtick',sort([0,fp,fs,1]));
xlabel('Digital frequency \omega/\pi'); ylabel('Radians in \pi units');
title('Wrapped Phase Response'); grid;
subplot(2,2,3); % Log-magnitude (dB) plot
plot(f,HdB,"LineWidth",1); axis([0,1,-50,1]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',[-50,-As,-Ap,0]);
xlabel('Digital frequency \omega/\pi'); ylabel('Decibel');
title('Log-Magnitude Response'); grid;
subplot(2,2,4); % Group-delay plot
plot(f,Grd,'LineWidth',1); axis([0,1,0,40]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',(0:10:40));
xlabel('Digital frequency \omega/\pi'); ylabel('Samples');
title('Group Delay Response'); grid;
```



(c) For the given cutoff frequencies, what are the exact passband and stopband attenuations?

Answer: To determine these attenuations, we have to read plot values at cutoff frequencies. This can be done either by using the "Data Cursor" tool or from the **HdB** array. We will use the later approach.

```
df = 1/1000; % normalized frequency resolution
```

```
fsl_exact = (find(round(HdB,2) > -As,1,'first')-1)*df;
fprintf('Exact lower stopband edge: %g⊠ radians\n',fsl_exact);
```

```
Exact lower stopband edge: 0.355⊠ radians
```

```
fpl_exact = (find(round(HdB,2) == -Ap,1,'first')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fpl_exact);
```

```
Exact lower passband edge: 0.4⊠ radians
```

```
fpu_exact = (find(round(HdB,2) == -Ap,1,'last')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fpu_exact);
```

```
Exact lower passband edge: 0.6⊠ radians
```

```
fsu_exact = (find(round(HdB,2) > -As,1,'last')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fsu_exact);
```

```
Exact lower passband edge: 0.645⊠ radians
```

(d) What is the order N of the bandpass filter?

Answer: Since the 4^{th}-order lowpass filter is mapped into a bandpass filter, the order of the bandpass filter is doubled. Hence the order of the bandpass filter is $N = 8$.

Problem-10: Bandstop Digital Filter Design using Chebyshev-II Prototype

Design a Chebyshev-2 bandstop filter that satisfies the specifications given below.

$$\text{Lower passband}: |\omega| \le 0.25\pi,\ A_{\mathrm{p}_1} = 1\ \text{dB}$$

$$\text{Stopband}: 0.4\pi \le |\omega| \le 0.7\pi, A_{\mathrm{s}} = 40\ \text{dB}$$

$$\text{Upper passband}: 0.8\pi \le |\omega| \le \pi,\ A_{\mathrm{p}_2} = 1\ \text{dB}$$

(a) Provide the system function $H(z)$ in the parallel form containing second-order sections with real coefficients.

MATLAB script for design:

```
clc; close all; clear;
fs = [0.4,0.7]; % Passband edges
fp = [0.25,0.8]; % stopband edges
Ap = 1; As = 40; % Attenuations in dB
[N,omegac] = cheb2ord(fp,fs,Ap,As); % Order of the lowpass Cheby-2 filter
fprintf('Order of the prototype lowpass filter: %g \n',N);
```

```
Order of the prototype lowpass filter: 5
```

```
[b,a] = cheby2(N,As,omegac,'stop'); % design of bandpass elliptic filter
[C,B,A] = dir2par(b,a), % Parallel form
```

```
C = 6.5858
B = 5×2
```

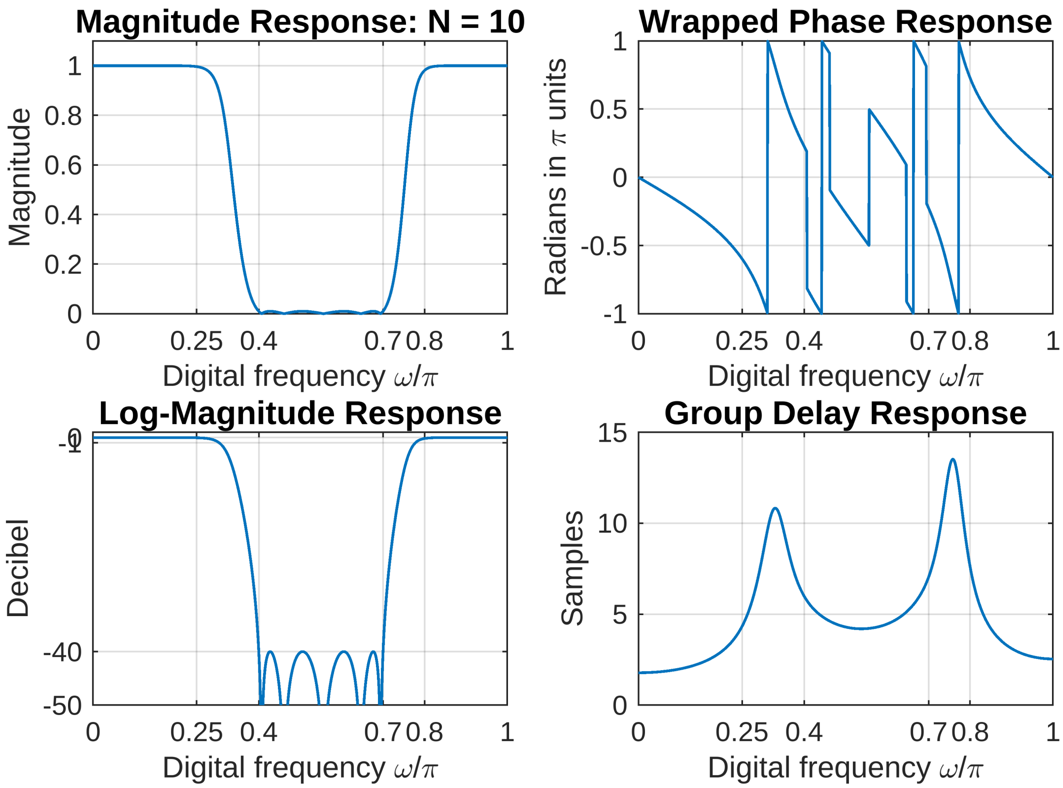
```
   -0.0561    0.1397
   -1.2333   -0.3767
   -3.6386   -0.6388
   -1.4441   -0.0127
   -0.0580   -0.2445
A = 5×3
    1.0000    1.3041    0.8031
    1.0000    0.8901    0.4614
    1.0000    0.2132    0.2145
    1.0000   -0.4713    0.3916
    1.0000   -0.8936    0.7602
```

$$H(z) = 6.5858 + \frac{-0.0561 + 0.1397z^{-1}}{1 + 1.3041z^{-1} + 0.8031z^{-2}} + \frac{-1.2333 - 0.3767z^{-1}}{1 + 0.8901z^{-1} + 0.4614z^{-2}} + \frac{-3.6386 - 0.6388z^{-1}}{1 + 0.2132z^{-1} + 0.2145z^{-2}}$$
$$+ \frac{-1.4441 - 0.0127z^{-1}}{1 - 0.4713z^{-1} + 0.3916z^{-2}} + \frac{-0.058 - 0.2445z^{-1}}{1 - 0.8936z^{-1} + 0.7602z^{-2}}.$$

(b) Provide plots of the magnitude, log-magnitude, group-delay, and phase responses.

MATLAB script for plots:

```
% Use the above figure and plot parameters
f = linspace(0,1,1001); om = f*pi; % Digital frequency array
H = freqz(b,a,om); % Frequency response
Hmag = abs(H); % Magnitude response
HdB = 20*log10(Hmag/max(Hmag)); % Log-magnitude response
Hpha = angle(H)/pi; % Phase response in pi units
Grd = grpdelay(b,a,om); % Group delay
figure('Units','inches','position',[0,0,9,6]);
subplot(2,2,1); % Magnitude plot
plot(f,Hmag,"LineWidth",1); axis([0,1,0,1.1]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',(0:0.2:1));
xlabel('Digital frequency \omega/\pi'); ylabel('Magnitude');
title('Magnitude Response: N = 10'); grid;
subplot(2,2,2); % phase response plot
plot(f,Hpha,'linewidth',1); axis([0,1,-1,1]);
set(gca,'xtick',sort([0,fp,fs,1]));
xlabel('Digital frequency \omega/\pi'); ylabel('Radians in \pi units');
title('Wrapped Phase Response'); grid;
subplot(2,2,3); % Log-magnitude (dB) plot
plot(f,HdB,"LineWidth",1); axis([0,1,-50,1]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',[-50,-As,-Ap,0]);
xlabel('Digital frequency \omega/\pi'); ylabel('Decibel');
title('Log-Magnitude Response'); grid;
subplot(2,2,4); % Group-delay plot
plot(f,Grd,'LineWidth',1); axis([0,1,0,15]);
set(gca,'xtick',sort([0,fp,fs,1]),'ytick',(0:5:15));
xlabel('Digital frequency \omega/\pi'); ylabel('Samples');
title('Group Delay Response'); grid;
```



(c) What are the exact passband and stopband edges for the given A_p and A_s, respectively?

Answer: To determine these attenuations, we have to read plot values at cutoff frequencies. This can be done either by using the "Data Cursor" tool or from the **HdB** array. We will use the later approach.

```
df = 1/1000; % normalized frequency resolution
fpl_exact = (find(round(HdB,2) < -Ap,1,'first')-1)*df;
fprintf('Exact lower stopband edge: %g⊠ radians\n',fpl_exact);
```

```
Exact lower stopband edge: 0.305⊠ radians
```

```
fsl_exact = (find(round(HdB,2) == -As,1,'first')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fsl_exact);
```

```
Exact lower passband edge: 0.4⊠ radians
```

```
fsu_exact = (find(round(HdB,2) == -As,1,'last')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fsu_exact);
```

```
Exact lower passband edge: 0.7⊠ radians
```

```
fpu_exact = (find(round(HdB,2) < -Ap,1,'last')-1)*df;
fprintf('Exact lower passband edge: %g⊠ radians\n',fpu_exact);
```

```
Exact lower passband edge: 0.777⊠ radians
```

(d) What is the order N of the bandstop filter?

Answer: Since the 5^{th}-order lowpass filter is mapped into a bandstop filter, the order of the bandstop filter is doubled. Hence the order of the bandstop filter is $N = 10$.