PROBLEM #1 (15 Points) PART1- Conceptual Questions

Indicate whether the following statements are **true or false. If true, give a brief explanation. If false, give a simple counter-example or a clear reason.**

1) All periodic continuous-time signals remain periodic after sampling.

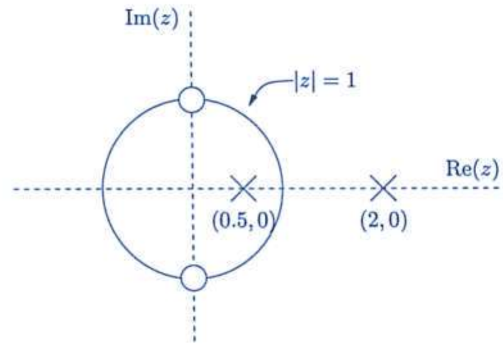
2) An all-pass filter can be FIR.

3) Increasing the window length can lower stopband ripples.

4) Bilinear transformation of analog IIR filter to digital IIR filter can preserve linear phase.

5) A modulator $y(n) = (-1)^n\, x(n)$ is a linear and time-invariant system.

PROBLEM #1 (30 Points) PART2- Filter Analysis



The pole-zero plot for $H(z)$ for a DT-LTI system is shown below.

1) Write the expression for the transfer function that is associated with the shown pole-zero plot. You can use K for a scaling constant.
2) Can the system be stable? If yes, what kind of a signal would $h(n)$ $be\ right\ sided,\ left\ sided,\ double\ sided$?
3) Can this system be both stable and causal? Explain your answer?
4) Write an expression for $H(\omega)$, $and\ magnitude\ and\ phase\ of\ H(\omega)$. $Use\ K = 1$
5) Plot magnitude of $H(\omega)$ (not bode plot), i.e $|H(\omega)|\ versus\ frequency\ \omega$.
6) What kind of a filter can this be and why?

PROBLEM #2 FIR DESIGN USING WINDOWING METHOD

Use the specifications of the following ideal filter to design an FIR Filter, answer the following questions:

$$H_d(\omega) = \begin{cases} 0 & |\omega| < \pi/4 \\ 1 & \pi/4 \le |\omega| \le \pi/3 \\ 0 & \pi/3 \le |\omega| \le \pi \end{cases}$$

And using,

$$W_v(n) = \begin{cases} 0.5 + 0.5\cos\left(\frac{\pi n}{9}\right) & -9 \le n \le 9 \\ 0 & otherwise \end{cases}$$

1) Sketch the frequency response for the desired filter.
2) Determine the impulse response this FIR filter which approximates this frequency response.
3) Compute by hand, show all your work, the first 4 coefficients of the impulse response h(n) coefficients.
4) Use MATLAB to plot both h(n), and the frequency response magnitude and phase of the filter you designed.
5) Comment on the expected performance of the filter you designed.

PROBLEM #3   Filters and frequency response

Consider the following filter $H(z)$

$$H(\omega) = \frac{0.75(1 + z^{-2})}{1 + 0.5\, z^{-2}}$$

1) Give the expression for its frequency response
2) Provide a plot of the magnitude of the frequency response (i.e $|H(\omega)| versus\ frequency\ \omega$.
3) Determine the exact form of the steady state output (do not use z-transform) when the input is

$$h(n) = 2u(n) + 3\sin\left(\frac{\pi n}{2}\right) + 4\cos\left(\frac{\pi n}{2}\right) u(n) + 5\cos(\pi n)\, u(n)$$

4) Based on the filter's frequency response, explain the relation between $y(n)\ and\ x(n)(that\ is\ why\ some\ sinusoids\ from\ x(n) passed\ while\ others\ were\ filtered\ out)$.