Applied Kalman Filtering
Dr. Robert H. Bishop

Part 4: Two-dimensional Tracking with an Extended Kalman Filter

Objective: Develop a state variable mathematical model for the two-dimensional tracking problem and implement an extended Kalman filter to estimate the state

Background: Consider the tracking problem illustrated in Figure 1

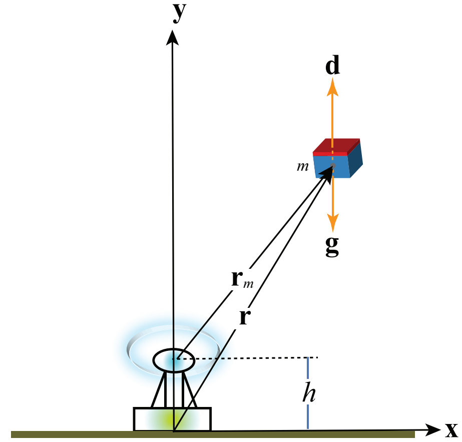

Figure 1: Two-dimensional tracking problem scenario

Assumptions:

1. The magnitude of the gravity is constant with

$$g = ||\mathbf{g}||_2 = \left\| \begin{pmatrix} 0 \\ 9.8066 \end{pmatrix} \right\|_2 = 9.8066$$

2. Range measurements, given by $r_m = ||\mathbf{r}_m||_2$, are available at discrete times and are corrupted by a random noise, denoted by v_k, with $E\{v_k\} = 0$ and $E\{v_k^2\} = R$ (constant)

3. The radar dish is at a height, denoted by h, above the ground which is not precisely known

Tasks:

1. Develop a mathematical model for the two-dimensional tracking problem of the underlying dynamics and the range measurement, denoted by r_m

2. The aero drag magnitude can be modeled via

$$d = ||\mathbf{d}||_2 = \left\| \begin{pmatrix} 0 \\ \rho v_y^2/(2\beta) \end{pmatrix} \right\|_2 = \frac{\rho v_y^2}{2\beta}$$

where

$$\rho = \rho_o e^{-r_y/k_p}$$

and β is the ballistic coefficient of the falling body, ρ is the atmospheric density with ρ_o the atmospheric density at sea level and k_p is a given decay constant.

3. Suppose that nominal values are given, $\bar{h}$, $\bar{\beta}$, $\bar{\rho}_o$, and $\bar{k}_p$ such that

$$\begin{aligned} h &= \bar{h} + \Delta h \\ \beta &= \bar{\beta} + \Delta\beta \\ \rho_o &= \bar{\rho}_o + \Delta\rho_o \\ k_p &= \bar{k}_p + \Delta k_p \end{aligned} \tag{1}$$

where $\bar{h} = 2$ m, $\bar{\beta} = 150$ kg/m^2, $\bar{\rho}_o = 1.225$ kg/m^3, and $\bar{k}_p = 9200$ m.

4. Consider the state vector

$$\mathbf{x} = \begin{pmatrix} \mathbf{r} \\ \mathbf{v} \\ \Delta h \\ \Delta\beta \\ \Delta\rho_o \\ \Delta k_p \end{pmatrix} \in \Re^8 \tag{2}$$

where

$$\mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

5. Using the state variables in Eq. 2, develop a nonlinear state-space model of the falling body and the nonlinear range measurement model. Include the known inputs, process noise, and measurement noise

6. State all your assumptions paying special attention to your modeling of Δh, $\Delta\beta$, $\Delta\rho_o$, and Δk_p. Note that in your simulation the initial EKF state estimates are $\Delta\hat{h}_0 = 0$, $\Delta\hat{\beta}_0 = 0$, $\Delta\hat{\rho}_{o_0} = 0$, and $\Delta\hat{k}_{p_0} = 0$. The truth values in the environment must be consistent with the initial state estimation error covariance matrix for each respective state variable

7. Using the environment and sensor models, integrate the extended Kalman filter into the simulation to generate and process the discrete-time range measurements

8. The inputs to the extended Kalman filter are the initial state estimate and the initial state estimate error covariance, in addition to the process noise spectral density matrix, the measurement noise covariance, the measurement sampling time, and the nominal values $\bar{h}$, $\bar{\beta}$, $\bar{\rho}_o$, and $\bar{k}_p$

9. Remember to save the state estimates (both *apriori* and *aposteriori*), the true states, and the state estimation error covariances (again, both *apriori* and *aposteriori*)

10. After the run completes, generate the time history of the state estimation error making sure to compute both *apriori* and *aposteriori* state estimation errors

11. Start the process of creating a plotting routine to plot the state estimation error for each state and co-plot the $\pm$ standard deviation (the $\pm$ square root of the corresponding diagonal element of the state estimation error covariance)

12. Be sure to adequately comment your code

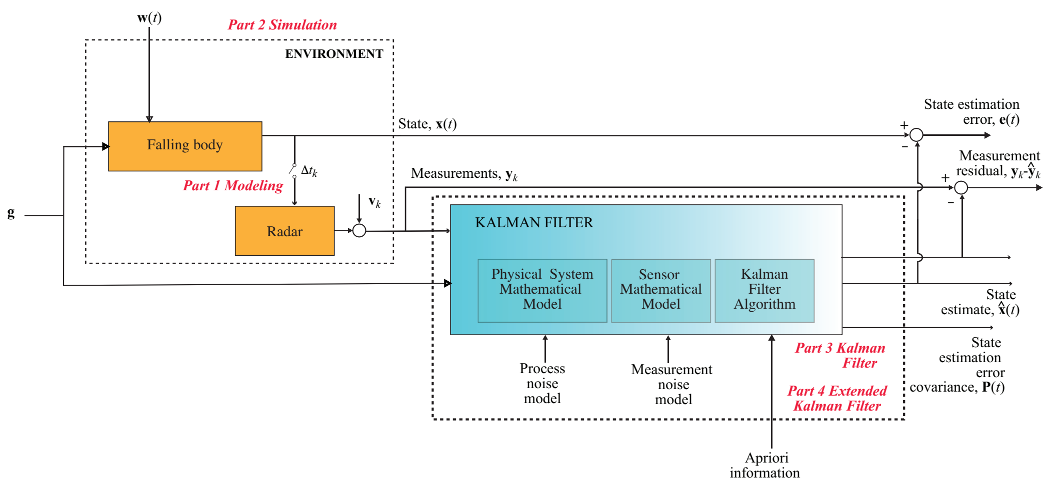

Figure 2: Simulation functional diagram