Applied Kalman Filtering
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Part 2: One-Dimensional Tracking Simulation Construction – Solution

Objective: Begin the development of the simulation of the continuous-time dynamics of the falling body with discrete-time measurements of range

Background: Consider the tracking problem from Part 1.

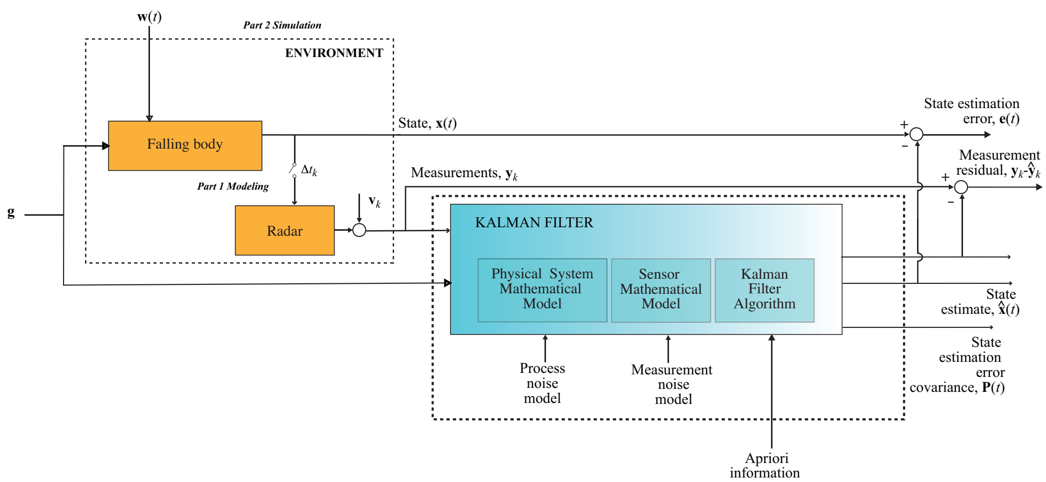

Figure 1: Simulation functional diagram

Falling Body

The motion of the falling body was simulated from the initial time $t_0 = 0$ to the final time $t_f = 14$ s with constant

$$\Delta t = 0.1 \text{ s.}$$

The state vector is

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \end{bmatrix}$$

where $\mathbf{r}(t)$ is the position of the target in the fixed reference frame and $\mathbf{v}(t)$ is the velocity. The initial position and velocity are

$$r_0 = \begin{pmatrix} 0 \\ 1000 \end{pmatrix} \text{ m} \quad \text{and} \quad v_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ m/s}^2$$

respectively. The motion utilized the discrete-time model given by

$$\boxed{\mathbf{x}_k = \Phi_{k-1}\mathbf{x}_{k-1} + \mathbf{u}_{k-1} + \mathbf{w}_{k-1}}$$

where $\mathbf{x}_k := \mathbf{x}(t_k)$ and

$$\Phi_{k-1} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{u}_{k-1} = -g \begin{bmatrix} 0 \\ \Delta t^2/2 \\ 0 \\ \Delta t \end{bmatrix},$$

and $E\{\mathbf{w}_{k-1}\} = \mathbf{0}$ with

$$\mathbf{Q}_{k-1} = E\{\mathbf{w}_{k-1}\mathbf{w}_{k-1}^{\mathrm{T}}\} = q_s \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta t^3/3 & 0 & \Delta t^2/2 \\ 0 & 0 & 0 & 0 \\ 0 & \Delta t^2/2 & 0 & \Delta t \end{bmatrix}$$

and $q_s = 0.01\ \mathrm{m}^2/\mathrm{s}^5$ and $g = 9.806\ \mathrm{m/s}^2$.

Radar

The measurement is the range to the target given by the discrete-time measurement model

$$\boxed{\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + b + v_k}$$

where

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \quad b = -h$$

where

$$h = 2\ \mathrm{m}$$

is the height of the radar antenna above the ground and v_k is a zero-mean white noise sequence with $E\{v_k^2\} = R$ where

$$R = 4\ \mathrm{m}^2.$$

The measurement update rate is 2 Hz.

Simulation results

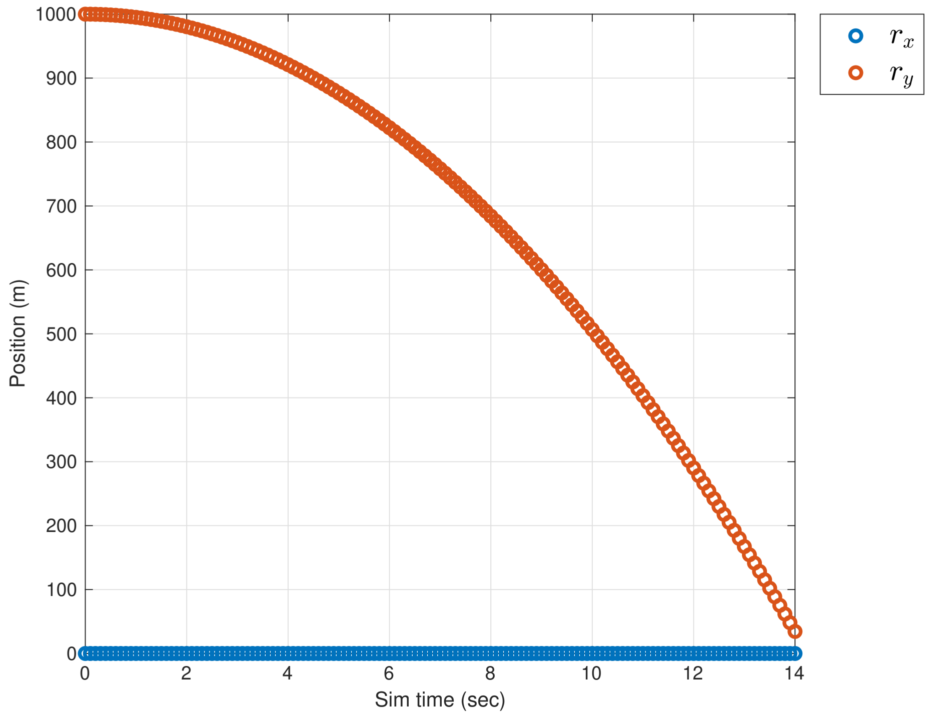

Figure 2: Trajectory Position

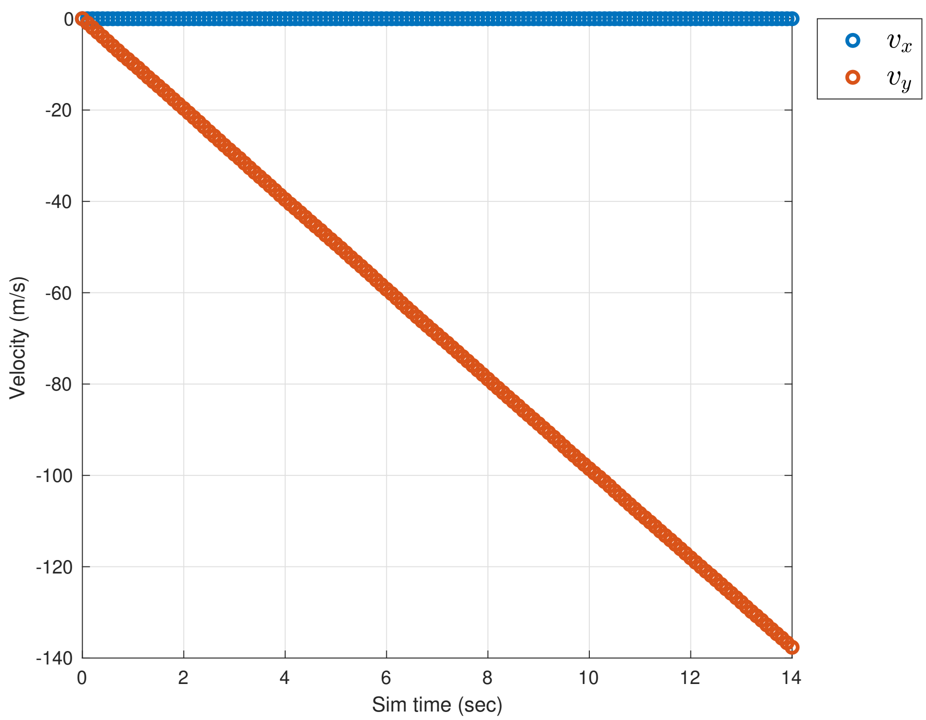

Figure 3: Trajectory Velocity

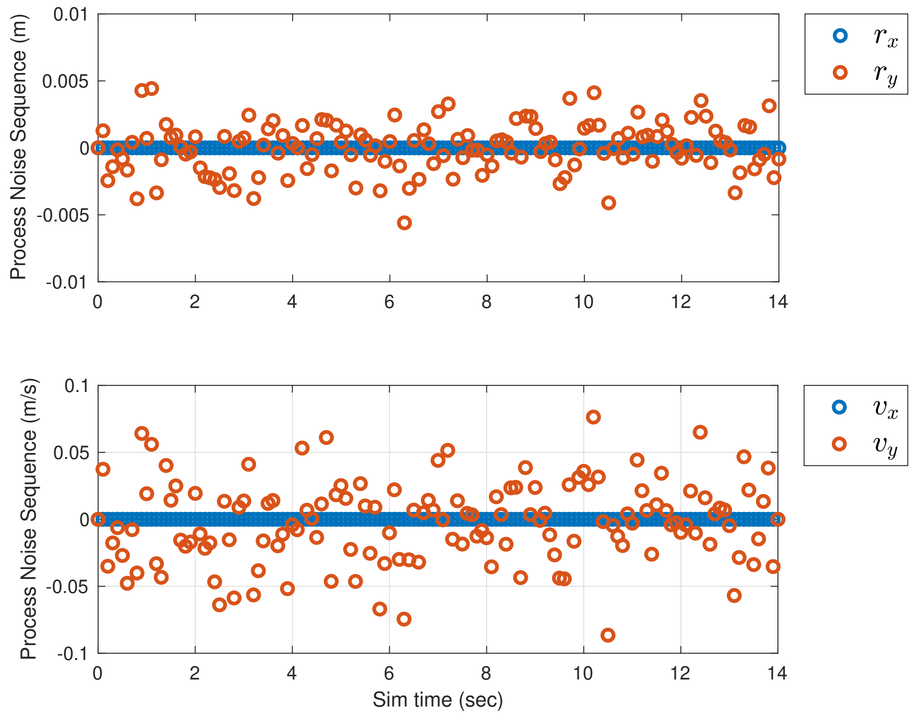

Figure 4: Process Noise, $\mathbf{w}_k$

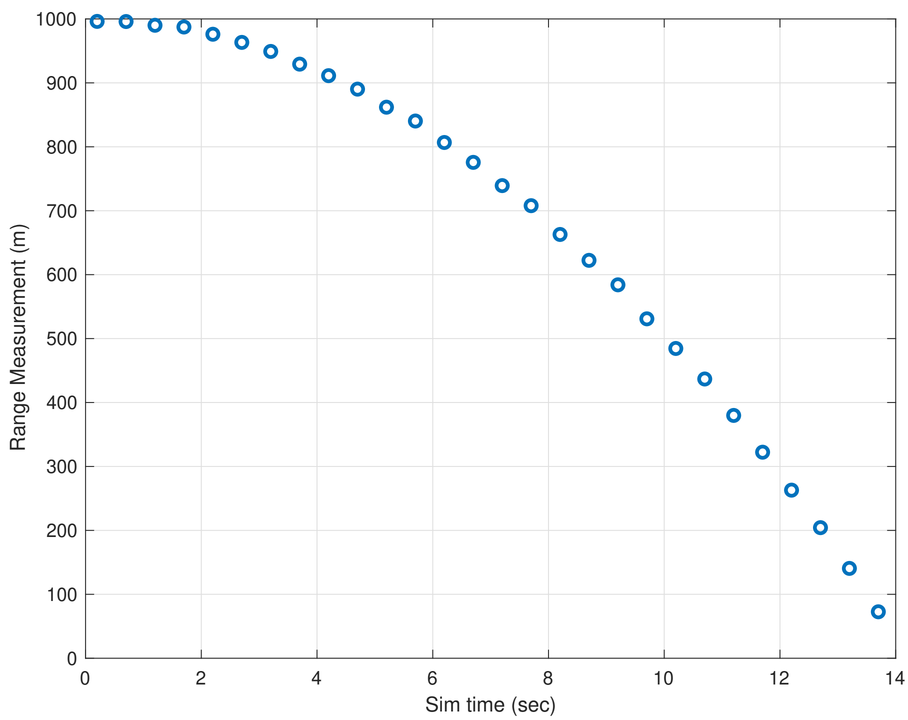

Figure 5: Range measurements, $\mathbf{y}_k$

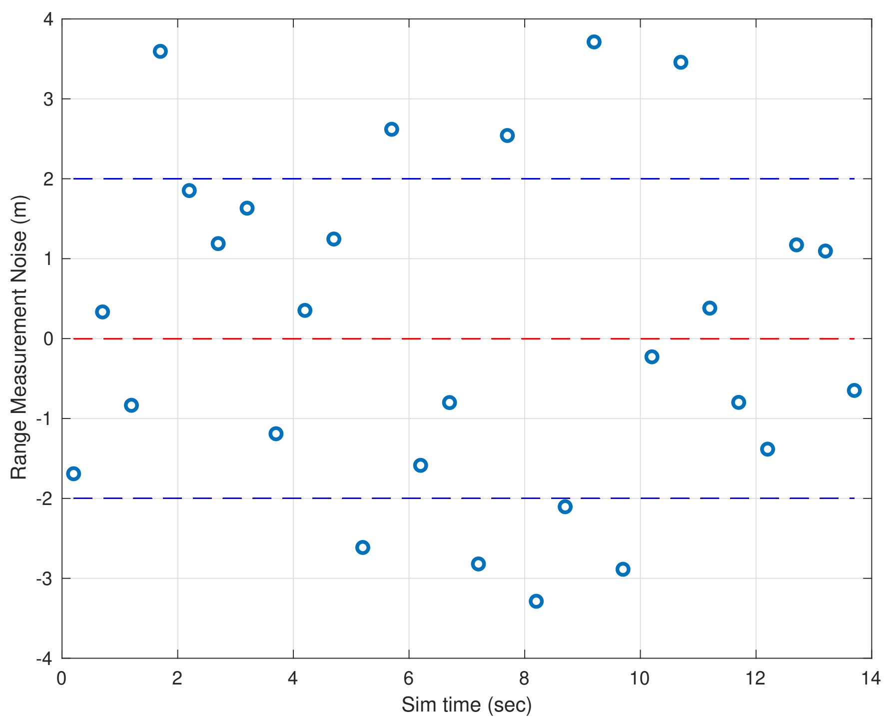

Figure 6: Range measurement noise, $\mathbf{v}_k$