A Kalman filter extension for the analysis of imprecise time series

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A KALMAN FILTER EXTENSION FOR THE ANALYSIS OF IMPRECISE TIME SERIES

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ABSTRACT

The Kalman filter combines given physical information for a linear system and external observations of its state in an optimal way. Conventionally, the uncertainty is assessed in a stochastic framework: measurement and system errors are modelled using random variables and probability distributions. However, the quantification of the uncertainty budget of empirical measurements is often too optimistic due to, e.g., the ignorance of non-stochastic errors in the analysis process. For this reason a more general formulation is required which is closer to the situation in real-world applications. Here, the Kalman filter is extended with respect to non-stochastic data imprecision which is caused by hidden systematic errors. The paper presents both the theoretical formulation and a numerical example.

1. INTRODUCTION

The Kalman filter combines physical information about a linear system and the observation of its behaviour in an optimal way. Because of this fundamental property there have been many significant applications. In Geodesy it is used for, e.g., multi-sensor navigation systems and the analysis of deformation processes.

The basic concept is straightforward. The system state at time t_k is predicted using the system equations and the knowledge of a previous (or initial) system state. The accuracy of this prediction can be improved when external observations of the system state at time t_k are taken into account. The present system state is then optimally estimated from both types of information (filtering) in terms of unbiasedness and minimum filtering error variance.

Conventionally, the stochastic filter component is given in terms of the expectation vectors and the variance-covariance matrices of system and measurement noise. It implies only random variability for the error sources but does not take non-stochastic errors such as imprecision into account. *Random variability* describes the deviations of the results of repeated measurements due to the laws of probability. It corresponds with random errors which are assumed as white noise throughout this paper.

In contrast, *imprecision* is considered as a non-stochastic type of uncertainty. It is due to remaining systematic deviations in the data which could not be eliminated by data preprocessing ([1]). It is possible to assess and to quantify data imprecision by means of a sensitivity analysis of the mostly sophisticated observation models; see [7] for relevant terrestrial geodetic observations, and, e.g., [8] for GPS observations. The approach is directly transferable to other multi-sensor systems in signal processing.

The classic Kalman filter algorithm is briefly reviewed in the following section, based on the notation in [10]. Afterwards it is extended to take imprecision into account.

2. KALMAN FILTER ALGORITHM

The Kalman filter algorithm is recursive. It starts with the definition of the input data vector $\mathbf{L}$ and its associated variance-covariance matrix (VCM) $\boldsymbol{\Sigma}_{LL}$:

$$\mathbf{L}=\begin{bmatrix}\mathbf{x}_k^*\\ \mathbf{u}_k\\ \mathbf{w}_k\\ \mathbf{l}_{k+1}\end{bmatrix},\ \boldsymbol{\Sigma}_{LL}=\begin{bmatrix}\boldsymbol{\Sigma}_{xx,k} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Sigma}_{uu,k} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma}_{ww,k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma}_{ll,k+1}\end{bmatrix},$$

with the initial conditions or the state estimates $\mathbf{x}_k^*$, the vectors of system input variables $\mathbf{u}_k$ and of model disturbances $\mathbf{w}_k$ for time t_k , as well as the vector of measurements $\mathbf{l}_{k+1}$ for time t_{k+1} . The vectors $\mathbf{x}_k^*$, $\mathbf{u}_k$ and $\mathbf{w}_k$ are assigned to the physical model, which represents the theoretical component of the filter (system equation) whereas the vector of measurements $\mathbf{l}_{k+1}$ represents the experimental component (measurement equation).

The evaluation continues with the extrapolation of the system equation (prediction) and the error propagation for time t_{k+1} . The predicted system state $\overline{\mathbf{x}}_{k+1}$, and its VCM $\boldsymbol{\Sigma}_{\overline{xx},k+1}$ are given by:

$$\overline{\mathbf{x}}_{k+1}=\underbrace{\begin{bmatrix}\mathbf{T}_{k+1,k} & \mathbf{B}_{k+1,k} & \mathbf{C}_{k+1,k}\end{bmatrix}}_{=:\mathbf{P}_{k+1,k}}\begin{bmatrix}\mathbf{x}_k^*\\ \mathbf{u}_k\\ \mathbf{w}_k\end{bmatrix}=\mathbf{P}_{k+1,k}\mathbf{x}_k\text{, and}$$

$$\boldsymbol{\Sigma}_{\overline{xx},k+1}=\mathbf{T}_{k+1,k}\boldsymbol{\Sigma}_{xx,k}\mathbf{T}_{k+1,k}^T+\mathbf{B}_{k+1,k}\boldsymbol{\Sigma}_{uu,k}\mathbf{B}_{k+1,k}^T+\mathbf{C}_{k+1,k}\boldsymbol{\Sigma}_{ww,k}\mathbf{C}_{k+1,k}^T,$$

with the transition matrix $\mathbf{T}_{k+1,k}$, the Jacobi matrix $\mathbf{B}_{k+1,k}$, and the coefficient matrix of disturbing variables $\mathbf{C}_{k+1,k}$.

The matrices $\mathbf{T}_{k+1,k}$, $\mathbf{B}_{k+1,k}$, and $\mathbf{C}_{k+1,k}$ depend on the physical model to be observed and must be chosen accordingly. Examples for a dynamic structural model for the calculation of thermal deformations of bar-shaped machine elements are given in [2].

The optimal estimation of the system state $\hat{\mathbf{x}}_{k+1}$ at time t_{k+1} (filtering) in a Gauss-Markov-Model is now based on the predicted system state $\overline{\mathbf{x}}_{k+1}$ at time t_{k+1}, and on the (indirect) external measurements $\mathbf{l}_{k+1}$:

$$\hat{\mathbf{x}}_{k+1} = \overline{\mathbf{x}}_{k+1} + \mathbf{K}_{k+1}\mathbf{d}_{k+1}\,, \quad (1)$$

with $\mathbf{K}_{k+1} = \mathbf{\Sigma}_{\overline{x}\overline{x},k+1}\mathbf{A}_{k+1}^T\left(\mathbf{\Sigma}_{ll,k+1} + \mathbf{A}_{k+1}\mathbf{\Sigma}_{\overline{x}\overline{x},k+1}\mathbf{A}_{k+1}^T\right)^{-1}$ the Kalman gain matrix and the vector of innovation $\mathbf{d}_{k+1} = \mathbf{l}_{k+1} - \boldsymbol{\varphi}(\overline{\mathbf{x}}_{k+1})$ as well as the design matrix $\mathbf{A}_{k+1}$ which is the Jacobi matrix of the function $\boldsymbol{\varphi}$ with respect to **x**. This function models the relation between the predicted system state and the collected measurements at time t_{k+1}. In the linearised case considered here the vector of innovation $\mathbf{d}_{k+1}$ is given by $\mathbf{d}_{k+1} = \mathbf{l}_{k+1} - \mathbf{A}_{k+1}\overline{\mathbf{x}}_{k+1}$.

Due to the lack of space only the VCM $\mathbf{\Sigma}_{dd,k+1}$ of the innovation $\mathbf{d}_{k+1}$, and the VCM $\mathbf{\Sigma}_{\hat{x}\hat{x},k+1}$ of the estimated system state $\hat{\mathbf{x}}_{k+1}$ at time t_{k+1} are introduced. For further studies on stochastic uncertainty measures the reader is referred to, e. g., ([3]):

$$\begin{aligned}\mathbf{\Sigma}_{dd,k+1} &= \mathbf{\Sigma}_{ll,k+1} + \mathbf{A}_{k+1}\mathbf{\Sigma}_{\overline{x}\overline{x},k+1}\mathbf{A}_{k+1}^T\,,\\ \mathbf{\Sigma}_{\hat{x}\hat{x},k+1} &= \mathbf{\Sigma}_{\overline{x}\overline{x},k+1} - \mathbf{K}_{k+1}\mathbf{\Sigma}_{dd,k+1}\mathbf{K}_{k+1}^T\,. \quad (2)\end{aligned}$$

3. MODELLING OF IMPRECISION

In this paper *imprecision* is treated in terms of intervals (e. g., [6]), see Fig. 1. With an interval $[x]$ it is possible to describe uncertainty quantities by upper ($x_{\min}$) and lower ($x_{\max}$) bounds or by midpoint (x_m) and radius (x_r):

$$x_{\min} = x_m - x_r$$
$$x_{\max} = x_m + x_r$$
$$[x] = [x_{\min}, x_{\max}] = [x_m - x_r, x_m + x_r]$$

Intervals can also be defined by their indicator functions $m(x)$:

$$m(x) = \begin{cases} 1, & x_{\min} \le x \le x_{\max} \\ 0, & \text{else} \end{cases}$$

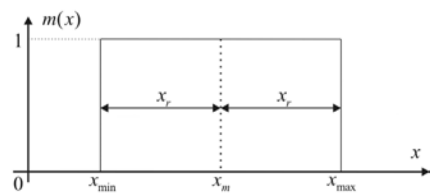

Figure 1 – Interval with midpoint x_m and radius x_r

Random variability is then introduced through the interval midpoint which is modelled as a random variable and hence treated by methods of stochastics. Here random variability is superposed by imprecision which is due to non-stochastic errors of the measurements and the physical model with respect to reality. This extends the approach presented in Section 2 which may thus be considered as the precise case. The standard deviation σ_x is the carrier of the stochastic uncertainty, and the radius x_r is the carrier of imprecision. The propagation of uncertainty is then separated into two parts. Whereas the stochastic part is treated with the law of variance covariance propagation, imprecision is linearly propagated to the reduced measurements and the parameters of interest (see Section 4.2 and 4.3).

For the modelling of imprecision it is important that the original measurement results are typically preprocessed before they are introduced in the measurement equation of the Kalman filter. These preprocessing steps comprise several factors **s** influencing the observations (see also Fig. 2):

- Physical parameters (model constants) for the reduction and correction steps from the original to the reduced measurements
- Sensor parameters (e. g., remaining error sources that cannot be modelled)
- Additional information (e. g., temperature and pressure measurements for the reduction steps of a distance measurement)

Most of these influence factors are uncertain realisations of random variables; their imprecision is meaningful by many reasons:

- The model constants are only partially representative for the given situation (e. g., the model constants for the refraction index for distance measurements).
- The number of additional information (measurements) may be too small to estimate reliable distributions
- Displayed measurement results are affected by rounding errors
- Other non-stochastic errors of the reduced observations occur due to neglected correction and reduction steps and for effects that cannot be modelled.

Figure 2 shows the interaction between the system and measurement equation and their influence factors. While correction and reduction steps are systematic, the imprecision of the influence parameters is directly transferred to the measurements, which are now carrier of random variability and imprecision.

The quantification of the uncertainty budget of empirical measurements is often too optimistic due to, e.g., the ignorance of non-stochastic errors in the analysis process. For this reason, one has to find (exact) enclosures for the non stochastic part of the influence factors. This step is based on expert knowledge and on error models concerning the deterministic behavior of these parameters.

Based on the assumption that imprecision is small in comparison with the measured values, we derive the data imprecision by means of a sensitivity analysis of the mostly sophisticated observation models (see, e. g., [7]). This is fulfilled in most applications. Section 4.2 shows the propagation of imprecision to the reduced observations.

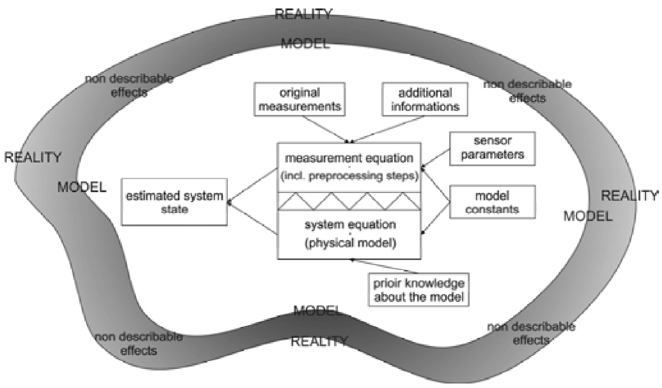

Figure 2 – Interaction between the system and the measurement equation and their influence factors

4. KALMAN FILTER EXTENSION FOR IMPRECISE TIME SERIES

4.1 General introduction

In this section we introduce a Kalman filter extension for the analysis of imprecise time series which takes both random variability and imprecision into account. In Section 4.2 the propagation of imprecision from the influence factors **s** according to Section 3 to the observations **l** is presented. In Section 4.3 this approach is directly applied to the Kalman filter algorithm. Note that this is fully consistent with the precise (stochastic) case as it leads to the same results if imprecision is absent.

The procedure can be generalized to fuzzy sets (see, e.g., [1]) which allow to model imprecision in a more refined way. This is straightforward as the so called α-cut discretisation of a fuzzy set can be used directly. Due to the lack of space, in this paper only the interval mathematical extension is shown. The reader is referred to [7] for the fuzzy evaluation of a geodetic deformation analysis with respect to imprecision. Furthermore, the presented approach can be applied directly to adaptive Kalman filter techniques (e. g., [3]).

4.2 Propagation of Imprecision

Imprecision in the Kalman filter approach is caused by non-stochastic errors of the observations and the physical model which are due to neglected systematic effects. Respective influence factors **s** were introduced in Section 3. In the interval approach the imprecision of **s** is described by lower and upper bounds which refer to worst case conditions. The corresponding interval radii $\mathbf{s}_r$ which serve as direct measures of imprecision are now propagated to the observations which are introduced to the measurement equation. In case of intervals this propagation is linear; it guarantees that imprecision is not be reduced by averaging repeated measurements when computing a mean value.

The observations $\mathbf{l}$ are considered as a function of the influence factors $\mathbf{s}$:

$$\mathbf{l} = \boldsymbol{f}(\mathbf{s}),$$

with the midpoints $\mathbf{l}_m$ of the observations and $\mathbf{s}_m$ of the influence parameters representing the stochastic case

$$\mathbf{l}_m = \boldsymbol{f}(\mathbf{s}_m) .$$

According to the fundamental rules of interval arithmetic ([6]), the imprecision of the observations $\mathbf{l}_r$ is obtained as:

$$\mathbf{l}_r = |\mathbf{F}|\mathbf{s}_r ,$$

with the Jacobi matrix $\mathbf{F} = \dfrac{\partial \mathbf{l}}{\partial \mathbf{s}}$ of $\boldsymbol{f}$, and the imprecision $\mathbf{s}_r$ of the influence factors **s**. The element-by-element absolute value of the matrix $\mathbf{F}$ is indicated by $|\mathbf{F}|$. The interval $[\mathbf{l}]$ of the observations is given by:

$$[\mathbf{l}] = [\mathbf{l}_{\min}, \mathbf{l}_{\max}] = \left[\mathbf{l}_m - |\mathbf{F}|\mathbf{s}_r, \mathbf{l}_m + |\mathbf{F}|\mathbf{s}_r\right].$$

The influence of the imprecision on the parameters of interest is strongly connected with the applied measurement method, as, e. g., observation differences in differential GPS. The matrix $\mathbf{M}$ models the influence of the measurement method on the propagation of imprecision:

$$\mathbf{l}_r = |\mathbf{MF}|\mathbf{s}_r . \tag{3}$$

Examples for the matrix $\mathbf{M}$, e. g., in GPS measurements are given in ([5], p. 348).

4.3 The interval extension of the system state

The interval extension of the system state $\hat{\mathbf{x}}_{k+1}$ at time t_{k+1} is obtained by the interval mathematical evaluation of Eq. (1). In order to reduce the overestimation of imprecision, caused by the sub-distributivity property of the intervals (cf. [1] and [6]), the analysis is referred to the imprecise influence factors $\mathbf{s}$. This requires a non-recursive formulation of the filter algorithm. Note that the interval extension of the system state is only required for the propagation of imprecision to the estimated parameters, the precise midpoint of the estimated system state remains equal to the stochastic case.

The calculation may start with the linearisation (First order Taylor series expansion) of the innovation $\mathbf{d}_{k+1}$ in point $\left(\overline{\mathbf{x}}_{k+1} + \hat{\mathbf{x}}_k\right)/2$:

$$\begin{aligned}\mathbf{d}_{k+1} &= \mathbf{l}_{k+1} - \boldsymbol{\varphi}(\overline{\mathbf{x}}_{k+1}) = \mathbf{l}_{k+1} - \boldsymbol{\varphi}(\overline{\mathbf{x}}_{k+1} + \hat{\mathbf{x}}_k - \hat{\mathbf{x}}_k) \\ &\approx \Delta\mathbf{l}_{k+1} + \mathbf{A}_{m,k}(\mathbf{E} - \mathbf{P}_{k+1,k})\frac{\hat{\mathbf{x}}_k}{2},\end{aligned}$$

with the required design matrix $\mathbf{A}_{m,k} = \boldsymbol{\varphi}'\left(\left(\overline{\mathbf{x}}_{k+1} + \hat{\mathbf{x}}_k\right)/2\right)$ in the Gauß-Markov-Model. The state estimate update $\hat{\mathbf{x}}_{k+1}$ is then given by

$$\hat{\mathbf{x}}_{m,k+1} = \begin{bmatrix}\mathbf{K}_{k+1} & \left(\dfrac{-\mathbf{K}_{k+1}\mathbf{A}_{m,k}\mathbf{P}_{k+1,k} + \mathbf{K}_{k+1}\mathbf{A}_{m,k}}{2} + \mathbf{P}_{k+1,k}\right)\mathbf{H}_k\end{bmatrix}\begin{bmatrix}\Delta\mathbf{l}_{k+1} \\ \vdots \\ \Delta\mathbf{l}_1 \\ \hat{\mathbf{x}}_0\end{bmatrix},$$

with the auxiliary matrix

$$\mathbf{H}_k = \begin{bmatrix}\mathbf{K}_k & \left(\dfrac{-\mathbf{K}_{k+1}\mathbf{A}_{m,k}\mathbf{P}_{k+1,k} + \mathbf{K}_{k+1}\mathbf{A}_{m,k}}{2} + \mathbf{P}_{k,k-1}\right)\mathbf{H}_{k-1}\end{bmatrix}$$

and $\mathbf{H}_0 = \mathbf{E}$ for the initial conditions.

Based on Eq. (1) and Eq. (3) the imprecision $\hat{\mathbf{x}}_{r,k+1}$ of the estimated state $\hat{\mathbf{x}}_{k+1}$ can be obtained as:

$$\hat{\mathbf{x}}_{r,k+1} = \left| \left[\mathbf{K}_{k+1} \quad \left(\frac{-\mathbf{K}_{k+1}\mathbf{A}_{m,k}\mathbf{P}_{k+1,k} + \mathbf{K}_{k+1}\mathbf{A}_{m,k}}{2} + \mathbf{P}_{k+1,k} \right) \mathbf{H}_k \right] \mathbf{MF} \right| \mathbf{s}_r , \tag{4}$$

and the interval vector of the state estimate update $\hat{\mathbf{x}}_{k+1}$ is now computed as:

$$\left[\hat{\mathbf{x}}_{k+1} \right] = \left[\hat{\mathbf{x}}_{m,k+1} - \hat{\mathbf{x}}_{r,k+1}, \hat{\mathbf{x}}_{m,k+1} + \hat{\mathbf{x}}_{r,k+1} \right] . \tag{5}$$

The parameter vector is exact component by component but overestimates the correct range of values which is a convex polyhedron (zonotope). See [9] for a detailed description of zonotopes.

In the present implementation, the imprecision of the estimated state vector is obtained by a projection, see Eq. (4). While this projection is a function of all observations, the computational complexity rises significantly for a large number of observations. For this reason, the propagation of imprecision in case of long time series is based on optimal filters with limited memory ([4]). This means that only a reasonable number of epochs for the optimal state estimate are considered (for the propagation of imprecision). This does not affect the state estimate and stochastic uncertainty of the computations in Kalman filter techniques.

5. EXAMPLE FOR A KALMAN FILTER FOR IMPRECISE TIME SERIES

5.1 The lock Uelzen

This numerical example demonstrates an application of the extended Kalman filter in a geodetic deformation analysis for the monitoring of a lock (see Fig. 3). Due to changing water levels (from 42 m to 65 m) inside the lock, different deformations of the lock chamber occur. We focus on the periodical expansion of the lock chamber during the time when ships are passing through the lock.

Figure 3 – The lock Uelzen

Therefore, the 3d-coordinates of different points at the left and right position of the lock chamber were observed with special geodetic tacheometers (Leica TPS 1101), measuring horizontal directions (a), zenith angles (b), and distances (c). Table 1 shows the theoretical standard deviations (from the manufacturer) and interval radii of the measurements, obtained by the sensitivity analysis from section 3. Due to the definition of the geodetic reference system, the expansion of the lock chamber is visible in the y-coordinate of the point positions. On that account, the numerical examples were mainly reduced to the y-coordinates of the points. The time series consists of about 300 epochs observed in periods of one minute. There are four nearly identical expansions of the lock chamber during the 300 epochs. For this reason only the first 120 epochs with two expansions are shown. The goal is to compute the displacements, velocities and accelerations of the y-coordinates. The uncertainties of the y-coordinate are mainly influenced by the direction measurements and the uncertainties of the x-coordinate are mainly influenced by the distance measurements.

Measurement	*Standard deviations*	*Interval radii*
(a) Horiz. direction	0.7 mgon	0.5 mgon
(b) Zenith angle	0.7 mgon	0.5 mgon
(c) Distance	2.0 mm	1.5 mm

Table 1 – Standard deviations and interval radii of the observations

5.2 Numerical results

Figure 4 shows the estimated system states (y-coordinate) and Figure 5 the stochastic uncertainty (standard deviations, according to Eq. 2) of the system state. The characteristic of the implemented Kalman filter in this example is as follows: if the innovation is determined as significant, the standard deviations of the measurements are increased. This leads to maximum standard deviations of the system state in case of strong innovations. Due to the lack of space, the determination of the significance of the innovation with statistical tests in case of imprecise data cannot be shown; the reader is referred to [7] for the general procedure. The main influence factors for the imprecision of the measurements are given in Table 2. Additionally, the imprecision of the measurement period (time step) must be considered. The measurements of the temperature and the pressure are only partially representative for the given situation, because they are measured at the position of the instrument, only. On this account, their imprecision is stronger than their uncertainty determined in terms of standard deviations.

Influence factor	*Interval radii (imprecision)*	*Affected measurements*
Temperature	1.0 °C	(c)
Pressure	1.0 hPa	(c)
Visual axis error	0.1 mgon	(a)
Collimation error	0.1 mgon	(a)
Vertical axis error	0.2 mgon	(a) and (b)
Time step	3sec	(a), (b) and (c)

Table 2 – An assortment of the most important influence factors

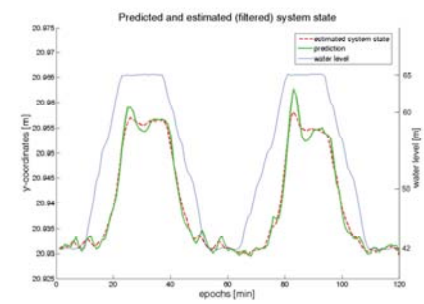

Figure 4 – y-coordinates of the filtered 3d-point positions and the corresponding water levels

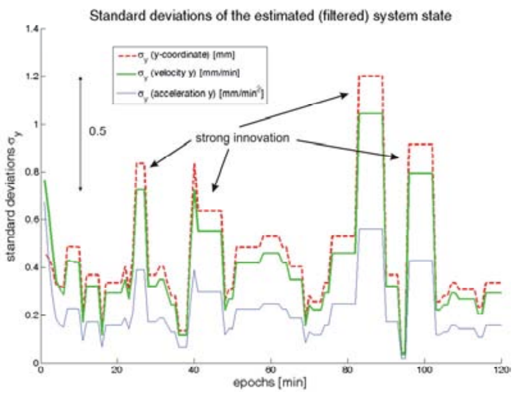

Figure 5 – Standard deviations of the estimated state vector

The imprecision obtained from Eq. (4) and (5) is depicted in Fig. 6. Due to possible time delays of the measurements (time step), the imprecision is strongly correlated with the characteristics of the deformations. These effects become more and more important for higher velocities and accelerations (see Fig. 6). This is due to the proceeding expansion of the lock during a time delay, which is stronger for high velocities and accelerations.

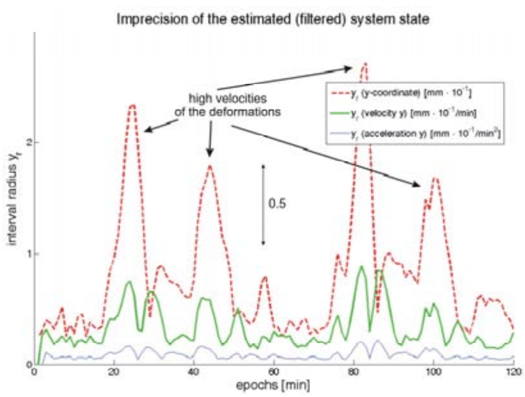

Figure 6 – Imprecision of the estimated state vector (y-coordinate)

Whereas the chosen measurement configuration leads to a stronger imprecision of the point positions, the first (velocity) and second (acceleration) derivatives of the process (in the y-coordinates) are well determined (see Fig. 6). This is in full accordance with the theoretical expectations because the imprecision of the inner geometry (derivatives) of processes is significantly reduced by observing only process differences.

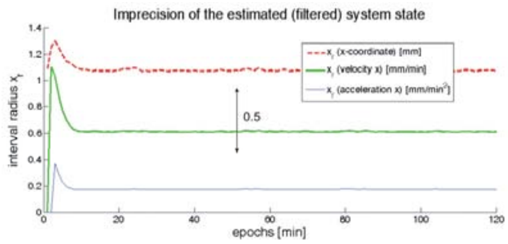

Figure 7 – Imprecision of the estimated state vector (x-coordinate)

Figure 7 shows the significantly stronger imprecision of the x-coordinate of the system state, which is due to the stronger imprecision of distance measurements in comparison to the angle measurements (in this example). While there are no significant velocities and accelerations in the x-coordinate, the influence of imprecision is almost constant during the process.

6. CONCLUSIONS

The presented paper shows a Kalman filter extension for the analysis of imprecise time series. Therefore stochastic and non-stochastic uncertainties reflecting random variability and imprecision were taken into account. Whereas random variability can be modelled in terms of variance-covariance matrices, imprecision is given in terms of intervals which are derived from a sensitivity analysis of the mostly sophisticated observation models. It was shown, that the consideration of imprecision yields an extended uncertainty budget. If imprecision is absent, the presented approach leads to the same results than in the stochastic case.

Both the evaluation of statistical tests in case of imprecision ([7]), e.g., for the determination of the significance of the innovation, and the application of the presented approach to adaptive Kalman filter techniques are possible. Further work has to deal with the reduction of the computational complexity and with extended numerical examples to make more refined statements on an realistic uncertainty budget in Kalman filter techniques.

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