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2016 J. Phys.: Conf. Ser. 735 012053

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# The analysis of optical wave beams propagation in lens systems

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**Abstract.** In this paper some aspects of the formation and propagation of optical wave beams in lens systems were considered. As an example, the two-lens optical information processing system was considered. Analysis of the two-lens optical circuit has been made with a systems approach perspective. As part of the radio-optical analogies had been applied certain provisions of the theory of dynamical systems to the spatial optical system. The lens system is represented as a simple series-connected optical elements with known spatial impulse response. General impulse response of such a system has been received, as well as consider some special cases of the impulse response. The question of the relationship between the parameters and the size of the input aperture lenses for undistorted transmission of the optical signal has been considered. Analysis of the energy loss resulting from the finite aperture of the lens. It's based on an assessment of the fraction of radiation that propagates beyond the lens. Analysis showed that the energy losses depend explicitly on the following parameters: radiation wavelength, distance between input aperture and lens, and ratio of the input aperture and lens aperture. With the computer help simulation the dependence of losses was shown on the above parameters

## 1. Introduction

In system of optical information processing (optical signals), a central issue is the formation of wave beams with the desired properties. Device forming such beams represent a plurality of layers are free space and lenses. Analysis of the propagation of optical signals in a lens system based mainly on the principles of geometric optics. This approach greatly simplifies the task of calculating the losses and distortions introduced by the system, but however, has several drawbacks. Firstly, the principles of geometrical optics ignore diffraction divergence of the optical beams, since the path of rays in the free space is rectilinear. Secondly, according to the principles of geometrical optics, an optical beam focused by a lens into a point, which corresponds to an infinitely large energy density and inappropriate physical picture of the phenomenon. In some cases this description is not acceptable. The above-mentioned disadvantages of the principles of geometrical optics do not allow to describe transformation spatial optical signal by optical system. This gives rise to the need to apply the methods of wave propagation analysis of optical beams in optical systems. However, the analysis of wave beams in lens system insufficient attention is given. This is due primarily to the complexity of the calculations required to obtain the fields at the system output.

Any optical information system is characterized by the size of the input aperture, lengths of layers of space and lens systems apertures<sup>1,2</sup>. In terms of structure, an important issue is the relationship



between the size of the input aperture of the optical information processing system, and lens aperture systems.

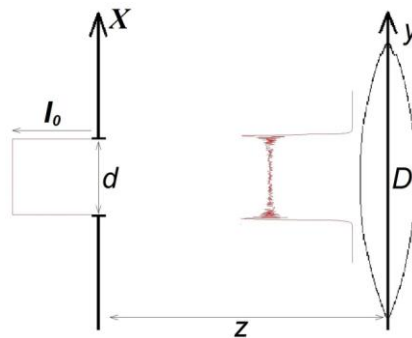
For the analysis of energy losses in the two-lens optical system is sufficient to consider a simple optical system comprising:

- input aperture of finite size, on which supply the optical signal;
- a layer of free space, the impact of which is determined by Fresnel diffraction;
- finite lens aperture, the amount of which exceeds the size of the input aperture.

This simple optical system is a part of complex optical information systems. Known mathematical analysis of such optical systems supposes the lens system of infinite size. However, the real structure of any optical system involves the installation of a lens system with an aperture of finite size. This leads to losses of radiation energy affecting further processing of the diffracted optical field. Loss estimate is based on the part of optical radiation, which partly extends outside of the lens system.

## 2. Evaluation of energy losses two-lens system

Simple optical system is a link in a complex optical information systems (figure 1).



**Figure 1.** Simple optical system consisting of the input aperture size  $d$ , space length layer  $z$  and the lens diameter  $D$

Consider the process of converting an input field with intensity  $I_0$  in this system. It is obvious that the lens diameter  $D$  should be larger than the input aperture  $d$ , otherwise it will lead to loss of the diffracted field propagating outside the lens aperture. However, the question of the relationship between the parameters  $d$ ,  $D$  and  $z$ , in which the loss of fields are permissible is small, so far was not raised. Finite apertures lenses cause error of the optical system signal. In this case, the criterion of error for evaluating is the relative proportion of the energy which is not involved in the further conversion of the optical signal. It refers to that portion of the optical radiation, which partially extends outside the lens system.

The losses of optical radiation, arising due to the limited aperture of the lens, can be written as:

$$\Delta = 1 - \frac{\left( \int_{-\frac{D}{2}}^{\frac{D}{2}} I(y) dy \right)}{\left( \int_{-\frac{d}{2}}^{\frac{d}{2}} I_0 dx \right)}, \quad (1)$$

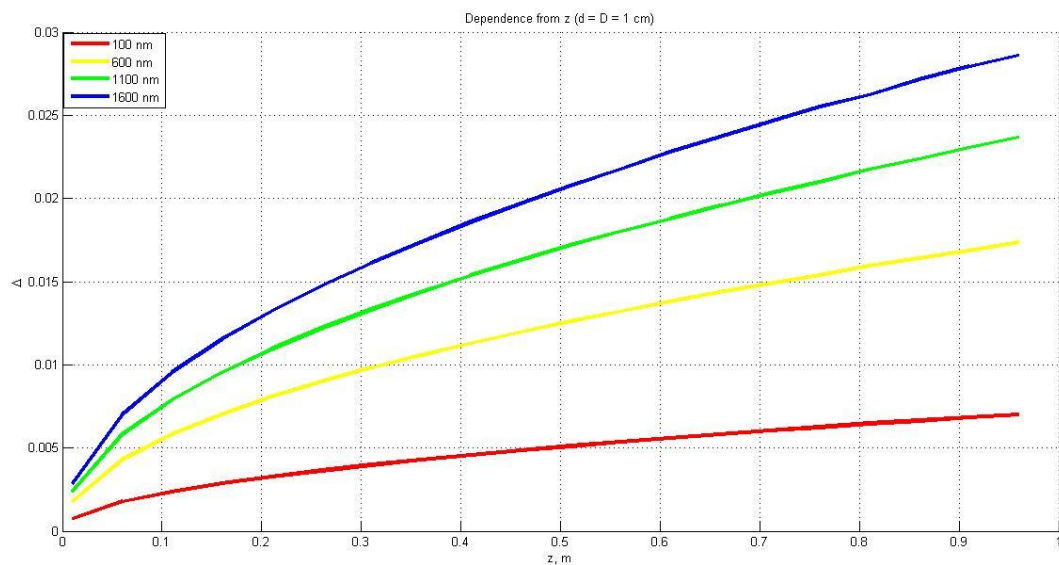
where  $I$  – optical field intensity.

The distribution of the field intensity  $I(y)$  can be written as<sup>3</sup>:

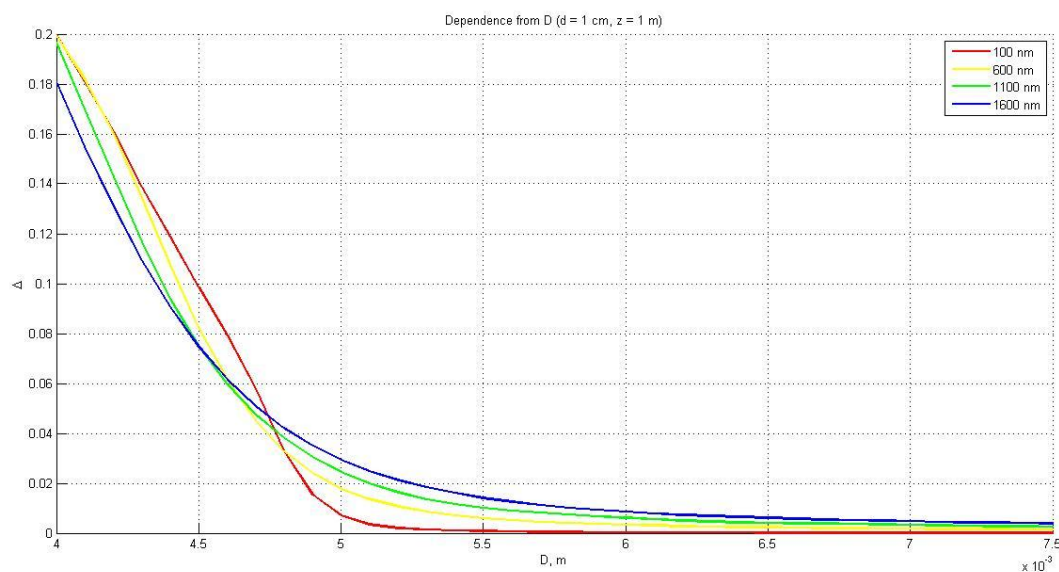
$$I(y) = \frac{I_0}{2} \left[ (C(\xi_2) - C(\xi_1))^2 + (S(\xi_2) - S(\xi_1))^2 \right], \quad (2)$$

where  $C(\xi) = \int_0^\xi \cos(\frac{\pi t^2}{2}) dt$ ,  $S(\xi) = \int_0^\xi \sin(\frac{\pi t^2}{2}) dt$  – Fresnel integrals,  $\xi_1(x) = -\sqrt{\frac{k}{\pi z}}(\frac{d}{2} + x)$ ,  $\xi_2(x) = \sqrt{\frac{k}{\pi z}}(\frac{d}{2} - x)$  – integration variables,  $k = \frac{2\pi}{\lambda}$  – wavenumber.

From the equation (2) implies that the losses of the diffracted optical field explicitly dependent on the following parameters: wavelength, distance between the input aperture and lens, lens aperture, which allows to evaluate the distortion of the transmitted signal depending on these parameters. With the help of Matlab the program was created that allows calculating the losses of optical radiation in the considered system. The calculation results are shown in figure 2 and figure 3.



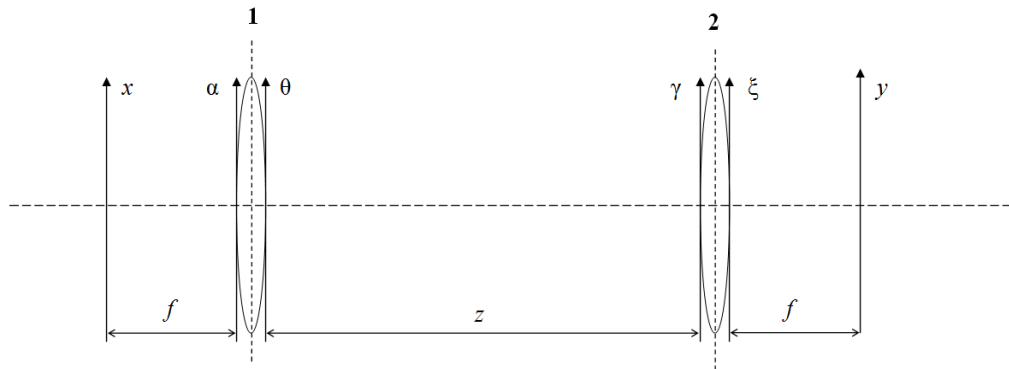
**Figure 2.** The calculation results of dependence of losses from  $z$



**Figure 3.** The calculation results of dependence of losses from  $D$

Method of errors estimation can be applied to more complex optical systems that consist of several layers and lenses. A graphical representation of the results enables the selection of parameters for the selected lens in advance margin of error that is acceptable for further calculations.

As a more complicated optical system can be considered two-lens optical circuit (figure 4). Two-lens optical scheme (figure 1) is sequentially located on the one optical axis of the collimating (1) and focusing (2) lenses with the focal length  $f$ . A feature of such schemes, in contrast to dual scheme spatial Fourier transform is that the distance between the lenses is chosen arbitrarily.



**Figure 4.** The two-lens optical scheme ( $f$  - focal length lens,  $z$  - the distance between the lenses)

In the paper we used some of the theory of a systematic approach to the analysis of the spatial optical lens systems. Known ratio, allowing establishing the input-output linear system in the form of:

$$U_{out}(y) = \int_{-\infty}^{\infty} U_{in}(x) \cdot h(x, y) dx, \quad (3)$$

where  $h(x, y)$  – impulse response of the system.

The lens system is represented as a simple series-connected optical elements with known spatial impulse response. In the case of such a two-lens system are: a layer of free space in front of the lens 1, the lens 1, a layer of free space between the lenses 1 and 2, the lens 2, a layer of free space after the lens 2. For further analysis, the basis of the known relationship was taken, establishing a total impulse response of two cells in series A and B<sup>4</sup>:

$$h_{BA}(t, \xi) = \int_{-\infty}^{\infty} h_B(t, \eta) \cdot h_A(\eta, \xi) d\eta. \quad (4)$$

where  $h_A(t, \eta)$  and  $h_B(\eta, \xi)$  – impulse response elements B and A, respectively.

It should be noted that this ratio is known for dynamic systems. For spaces systems, such a transition is based on the use of radio-optical analogy<sup>5</sup>, for a two-lens optical system, such an analysis is made for the first time.

The main result of this research is to establish a ratio between input and output of two-lens system which allows for a given input field  $u(x)$  by convolution with a total impulse response of the system to obtain the distribution function of the field at the output of  $u(y)$ . For the two-lens optical system impulse response can be written as:

$$h(x, y) = \exp\left(i \left[ y^2 \left( \beta - \frac{\beta^2}{\mu} \right) \right]\right) \cdot \exp(i\beta x^2) \cdot \delta(x + y). \quad (5)$$

where  $\beta = \frac{k}{2f}$ ,  $\mu = \frac{k}{2z}$ ,  $k = \frac{2\pi}{\lambda}$ ,  $f$  – lens focal length,  $z$  – distance between lenses.

Thus, the ratio is obtained, which depends directly on two variables: the focal length lens ( $f$ ) and the distance between the focusing and collimating lenses ( $z$ ), which can be used to calculate the form of the optical field at the output of the system.

To illustrate the use of the expression (5), consider a special case of a two-lens optical system - the double Fourier transform scheme (the distance between lenses is  $2f$ ). The result of a double spatial Fourier transform is a function that is equal to the original. But the sign of the argument is converted<sup>5</sup>.

Apply the equation (5) to calculate the field at the output of the scheme, which perform a double Fourier transform. Taking into account that  $z=2f$ , get  $\mu=\beta/2$ . Thus, obtained:

$$\begin{aligned} U(y) &= \int_{-\infty}^{\infty} u(x) \cdot \exp\left(i \left[ y^2 \left( \beta - \frac{\beta^2}{\mu} \right) \right]\right) \cdot \exp(i\beta x^2) \cdot \delta(x+y) dx = \\ &= \int_{-\infty}^{\infty} u(x) \cdot \exp\left(i \left[ y^2 \left( \beta - \frac{2\beta^2}{\beta} \right) \right]\right) \cdot \exp(i\beta x^2) \cdot \delta(x+y) dx = \\ &= \int_{-\infty}^{\infty} u(x) \cdot \exp(-i\beta y^2) \cdot \exp(i\beta x^2) \cdot \delta(x+y) dx = \\ &= u(-y) \cdot \exp(i\beta y^2 - i\beta y^2) = u(-y). \end{aligned} \quad (6)$$

Thus, the expression (6) shows that the field at the output is a mirror image of the field at the input scheme of the double spatial Fourier transform.

## Conclusion

In this work it was discussed in general terms two-lens optical circuit of formation of optical beams. As result of analysis based on the use of radio-optical analogies, was obtained spatial impulse response of the system at its various representations. These representations of impulse responses allowed to establish a connection between input and output of system at any distance between the lenses. This representation of impulse response allows the simple, but at the same time clearly and fully describe the process of transformation of the field in lens system. Detailed analysis of expression is shows that the field at the output of the system is equivalent to a mirror image of the field at the entrance, but with additional phase distortions. Since in optics traditionally accepted to work with the intensity of the optical field, i.e., with the square of the modulus of the optical field, phase distortions can be neglected.

This work was funded by the RFBR - The Russian Foundation for Basic Research: project No. 15-37-20446\15

## References

- [1] Tarasov K I 1968, Spectral devices (Leningrad.: Mechanical engineering).
- [2] Belyakov Y M and Pavlycheva N K 2007, Spectral devices (Kazan: publishing state tech. Un-ty)
- [3] Gudmen G 1970, Introduction to Fourier optics (Moscow: MIR)
- [4] Zade L and Desoer C 1970, The theory of linear systems (Method of state space) (Moscow: Science, Home edition of Physical and Mathematical Literature)
- [5] Papulis A 1971, Systems theory and transformation in optics (Moscow: MIR)