# Passivity based IMC control for multivariable nonlinear systems

**Conference Paper** · August 2004

DOI: 10.1109/ASCC.2004.184757 · Source: IEEE Xplore





# Passivity Based IMC Control for Multivariable Nonlinear Systems

Steven W. Su†, Jie Bao†[1] and Peter L. Lee⋆
†School of Chemical Engineering & Industrial Chemistry
The University of New South Wales,
UNSW, Sydney, NSW 2052, Australia
⋆Division of Engineering & Science
Curtin University of Technology
GPO Box U1987, Perth, WA 6845, Australia

## Abstract

This paper proposes a new control method for multivariable nonlinear processes based on the Internal Model Control (IMC) framework and the Passivity Theorem. The conventional IMC method involves inversion of the process, which is often difficult or even impossible. In the proposed method, the process is approximated using a passive system. The controller is designed to effectively invert the passive approximation. The stability of the closed-loop system is guaranteed by the passivity condition. The effectiveness of the proposed method is illustrated by using a mixing tank control problem.

## 1 Introduction

Nonlinear control is particularly important in the process industries because chemical processes are generally nonlinear. Most current nonlinear controller design methods are based on state feedback. However, they cannot be applied to many process control problems where the complete state information is not available. [1].

Based on the Passivity Theorem [12], this paper develops an output feedback control method for general nonlinear processes under the framework of Internal Model Control (IMC). Internal Model Control (IMC) [10, 5, 4, 15, 14, 13] has been extensively used in the process industries. The main barrier to implement the nonlinear IMC approach is the inversion of the process, which is either difficult or impossible for general nonlinear processes. One approach is to factorize the process into minimum phase and non-minimum phase subsystems [11, 3], and invert the minimum phase part of the process. However, the factorization for a general nonlinear process [11] is often hard to implement. In order to get around this obstacle, the proposed method decomposes the process into a passive approximation subsystem and the reminder. The passive approximation is inverted by using an integral controller at steady state. This guarantees offset free control and has the potential to improve the dynamic performance of the system.

Passivity-based nonlinear control has been widely studied. Most existing approaches are built on feedback passivation [2], which are only applicable to processes that have relative degree one and are weakly minimum phase [2]. The proposed new IMC approach is equivalent to a feedforward passivation design, and thus can be applied to the stable multivariable nonlinear systems, without the above limitations.

The paper is organized as follows. Some notations and concepts are introduced in Section 2. Section 3 gives a description of the passivity based IMC design. The proposed approach is illustrated in Section 4 by using an example of mixing control.

## 2 Preliminaries

In order to clarify our discussion, some notations will be introduced from [4].

**Definition 1** *(Nonlinear operator) A nonlinear operator is the operator $N$, which maps $u$ to $y$ $(= Nu)$ through the relations*

$$\begin{cases} \dot{x} &= f(x,u), \quad x(0) = x_0 \in \mathcal{R}^n \\ y &= g(x,u), \quad u, y \in \mathcal{R}^m \end{cases} \tag{1}$$

*where the $f$ and $g$ are real analytic vector valued functions.*

The nonlinear operator $N$ defined in Definition 1 is "square" in the sense that it has the same number of inputs and outputs.

[1]Author to whom correspondence should be addressed. Email: j.bao@unsw.edu.au. Tel: +61 (2) 9385-6755. Fax: +61 (2) 9385-5966.

For an operator $M$, it would be possible to find an operator $M^{-1}$ with the property

$$MM^{-1}y = y.$$

Where $MM^{-1}$ denotes two cascaded operators. Operator $M^{-1}$ is called the right inverse operator of $M$.

**Definition 2** *(Steady-state operator) Let $M$ be an input-output stable operator in $D_M$ [16] and $u \in D_M$ with $\lim_{t\to\infty} u(t) = u_\infty < \infty$. Letting $y_\infty = \lim_{t\to\infty} M(u(t))$ ($y_\infty < \infty$ because of the stability assumption), the steady-state operator $M_\infty$ is defined by*

$$y_\infty = M_\infty(u_\infty) \tag{2}$$

Although $M$ generally maps function spaces into function spaces, $M_\infty(\cdot)$ is mapping vectors from $\mathcal{R}^m$ into $\mathcal{R}^m$.

As an example of a steady-state operator, consider the operator in (1) where $N$ is assumed input-output stable. For this operator, $N_\infty$ is given by the system of algebraic equations

$$\begin{cases} 0 & = f(x, u_\infty), \\ y_\infty & = g(x, u_\infty), \end{cases} \tag{3}$$

The proposed control method is based on the concept of passive systems [3, 6, 12].

**Definition 3** *[12] Consider a nonlinear system $H$:*

$$\begin{cases} \dot{x} & = & f(x, u) & x \in \mathcal{R}^n \\ y & = & h(x, u), & u, y \in \mathcal{R}^m \end{cases} \tag{4}$$

*and assume that the state $x(t)$, as a function of time, is uniquely determined by its initial value $x(0)$ and the input function $u(t)$. Suppose that the above system has an equilibrium at the origin, that is, $f(0, 0) = 0$, and $h(0, 0) = 0$.*

*Assume that associated with the system $H$ is a function $w : \mathcal{R}^m \times \mathcal{R}^m \mapsto \mathcal{R}$, called the supply rate, which is locally integrable for every $u \in \mathcal{U}$. Let $\mathcal{X}$ be a connected subset of $\mathcal{R}^n$ containing the origin. If there exists a function $S : \mathcal{X} \mapsto \mathcal{R}^+$ (denote $\mathcal{R}^+ = [0, \infty)$), $S(0) = 0$, such that for all $x \in \mathcal{X}$:*

$$S(x(T)) - S(x(0)) \leq \int_0^T s(u(t), y(t))dt \tag{5}$$

*for all $u \in \mathcal{U}$ and all $T \geq 0$ such that $x(t) \in \mathcal{X}$ for all $t \in [0, T]$, then we say that the system $H$ is dissipative in $\mathcal{X}$ with the supply rate $w(u, y)$. The function $S(x)$ is then called a storage function.*

*System $H$ is said to be passive if it is dissipative with supply rate $w(u, y) = u^T y$.*

**Definition 4** *[12] System $H$ is said to have:*

- *Output Feedback Passivity ($OFP(\rho)$) if it is dissipative with respect to $w(u, y) = u^T y - \rho y^T y$ for some $\rho \in \mathcal{R}$.*
- *Input Feedforward Passivity ($IFP(\nu)$) if it is dissipative with respect to $w(u, y) = u^T y - \nu u^T u$ for some $\nu \in \mathcal{R}$.*

**Theorem 1** *[12] If $H_1$ and $H_2$ are dissipative with radially unbounded storage functions $S_1$ and $S_2$ then the equilibrium $(0, 0)$ of their feedback interconnection is Globally Asymptotically Stable (GAS), if $H_1$ is GAS and $IFP(\nu)$, and the system $H_2$ is zero state detectable and $OFP(\rho)$ with $\nu + \rho > 0$.*

**Definition 5** *Assume a nonlinear system $P$ is an input-output operator. If its steady state input and output relationship $P_\infty(\cdot)$ satisfies*

$$u_\infty^T P_\infty(u_\infty) \geq 0, \tag{6}$$

*then, we say the system $P$ is passive at steady state.*

The nonlinear IMC structure proposed in [4] is shown in Figure 1. The nonlinear operators $P$, $M$, and $C$ denote the process, model, and controller. The functions (or signals) $y$, $u$, $e$ and $d$ stand for the output of the process, the input to the process, the reference tracking error and the disturbance.

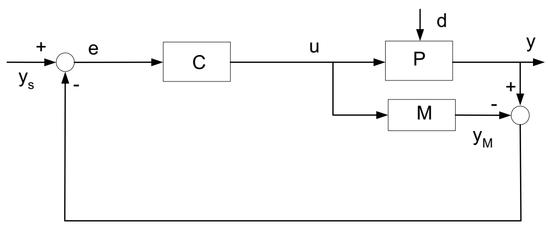


**Figure 1:** Nonlinear IMC control

The main characteristics for nonlinear IMC control structure [4] are:

1) **Stability** Suppose the model is perfect. The closed-loop system in Figure 1 is input-output stable if the controller $C$ and the plant $P$ are input-output stable.

2) **Perfect Control** Assume that the controller is equal to the (right) inverse of the model and that the closed-loop system in Figure 1 is input-output stable with this controller. Then the control will be perfect, i.e., $y = y_s$.

3) **Zero Offset** Assume that the (right) inverse of the steady state model operator $M_\infty^{-1}$ exists, the controller satisfies $C_\infty = M_\infty^{-1}$ and the closed loop system is input-output stable with the controller. Then offset free control can be achieved for asymptotically constant inputs.

## 3 Passivity Based Design

### 3.1 The framework

In this paper, a passivity based IMC control method for stable nonlinear processes is presented. The main idea is to approximate the nonlinear process $P$ using a passive system $P_p$ such that $\|P - P_p\|$ is as small as possible. The controller $C$ is constructed to be the inverse of the passive system $P_p$, i.e. $C = P_p^{-1}$. If the model is perfect, the closed-loop system is $PP_p^{-1}$. Because $P_p$ is passive, it must be invertible and its inverse is stable. Therefore, the input output closed loop **stability** is guaranteed. Moreover, if the process $P$ is passive, then **perfect control** is achievable. If the process is not passive, the achievable performance is determined by $\|P - P_p\|$ . If the steady-state model of the process around the operating point is passive, i.e., $P_\infty(\cdot) = P_{p\infty}(\cdot)$ (after certain input/output transformation), then the controller can achieve **zero offset**.

The key issues of the proposed framework are: (1) to find the approximate passive system $P_p$ for the process to be controlled; (2) to implement the inverse of system $P_p$. Different control design methods can be developed in this framework. Here an easy to implement approach is given. It involves the following steps:

### 3.2 Passivation at steady-state

Offset free control is possible when the process is passive at steady-state. As the steady state input and output relationship $P_\infty(\cdot)$ is continuous, its passivity can be verified numerically using discrete points in the region of interest (as shown in Section 4).

Processes which are not passive at steady state can be passified by using input and output transformations to process $P$ :

$$\tilde{u} = \phi(u) \tag{7}$$

$$\tilde{y} = \psi(y). \tag{8}$$

The functions $\phi(\cdot)$ and $\psi(\cdot)$ are both local diffeomorphism in $\mathcal{U} \subset \mathcal{R}^m$ and $\mathcal{Y} \subset \mathcal{R}^m$[7, 9]. If the process $P$ is described by

$$P: \left\{ \begin{array}{lcl} \dot{x} & = & f(x) + g(x)u \\ y & = & h(x). \end{array} \right. , \tag{9}$$

functions $\phi(\cdot)$ and $\psi(\cdot)$ need to satisfy

$$\left\{ \begin{array}{lcl} 0 & = & f(x) + g(x)\phi^{-1}(\tilde{u}) \\ \tilde{y} & = & \psi(h(x)). \end{array} \right. \tag{10}$$

and

$$\tilde{y}^T \tilde{u} \geq 0.$$

If the steady state input and output functions of system $P$: $y_\infty = P_\infty(u_\infty)$ (see Definition 5) are local diffeomorphism in $\mathcal{U} \subset \mathcal{R}^m$ and $\mathcal{Y} \subset \mathcal{R}^m$, only one transformation is needed, either input or output (e.g., an input transformation $\phi(\cdot) = P_\infty(\cdot)$). However, the use of both the input and output transformations is often preferred as the result may be more effective and/or physically meaningful.

### 3.3 Passive approximation

To find a nonlinear passive system to approximate the dynamics of any given nonlinear process could be difficult. Considering the fact that the stability of the closed loop system can be guaranteed by using any passive approximation, the states of the process will always be forced into the operating region. Thus, it is reasonable to apply a linear passive system to approximate the process dynamics near the operating region, which is easy to implement.

In this paper, the passive approximation of process dynamics is performed around the equilibrium. A passive linear system $P_{pl}(s)$ which has a blocking zero at the origin is designed to approximate $G_0(s)$, which is a linearized model of $P - P_\infty$. This can be implemented by solving Problem 1 (an LMI problem).

Suppose that $\{A_0, B_0, C_0, D_0\}$ and $\{A_{pl}, B_{pl}, C_{pl}, D_{pl}\}$ are minimal realizations of the linear systems $G_0(s)$ and $P_{pl}(s)$ (to be designed) respectively. Further assume $A_{pl} = A_0$ and $B_{pl} = B_0$. As passivity is a phase related property, this assumption does not lead to a conservative design. In order to ensure $P_{pl}(s)$ having a blocking zero at origin, let $D_{pl} = C_{pl}A_0^{-1}B_0$.

The following minimization problem is established to construct the passive linear system $P_{pl}(s)$ based on Positive Real Lemma.

**Problem 1** $\min_{P_1, C_{pl}} \alpha$
*Subject to:*

$$\left[ \begin{array}{cc} \alpha I & C_0 - C_{pl} \\ (C_0 - C_{pl})^T & \alpha I \end{array} \right] > 0, \tag{11}$$

$$\left[ \begin{array}{cc} A_0^T P_1 + P_1 A_0 & P_1 B_0 - C_{pl}^T \\ B_0^T P_1 - C_{pl} & -(D_{pl} + D_{pl}^T) \end{array} \right] < 0, \tag{12}$$

$$P_1 > 0, \tag{13}$$

$$\alpha > 0. \tag{14}$$

The LMIs (11) is equivalent to:

$$\|C_0 - C_{pl}\| < \alpha \tag{15}$$

By minimizing $\alpha$ (11), a good passive approximation of the system of $G_0(s)$ can be achieved.

### 3.4 Construction of controller

The proposed approach decomposes the process model into a passive approximation, $P_p = P_\infty + P_{pl}(s)$, and the remainder, $P - P_p$ as shown in Figure 2. With the integration matrix $K_c/s$ (where gain matrix $K_c$ is diagonal and positive define), the IMC type controller attempts to invert the passive approximation of the process model $P_p$ (when $\|K_c\| \to +\infty$), at least its steady state part $P_\infty$. Assume a perfect model, i.e., $M = P$, the closed-loop system from $y_s$ to $y$ is the open-loop system which comprises the IMC controller and the process. As the process is stable, the closed-loop **stability** is determined by the stability of the IMC controller. As $K_c/s$ is passive ($K_c$ is positive definite), and $P_p$ is constructed to be $IFP(\nu)$, the IMC controller is stable for any $K_c > 0$ according to Theorem 1. When $\|K_c\| \to \infty$, the IMC controller is the inverse of $P_p$. This leads to the best achievable performance. The control performance depends on the difference between the full process model and its passive approximation. If the process itself is passive, then **perfect control** can be achieved. As the steady state input-output relationship $P_\infty$ is rendered passive by input and output transformation, **zero offset** is achievable. If the linear subsystem $P_{pl}$ can approximate the passivity of the nonlinear process in its operating region, the proposed control will also produce reasonable dynamic performance.

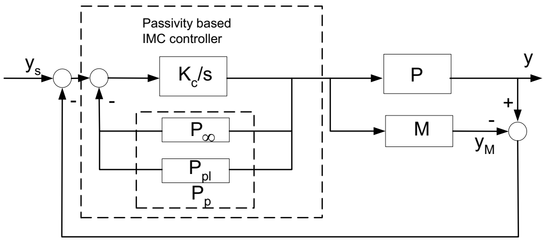


**Figure 2:** The passivity based IMC control

The use of the linear passive approximation $P_{pl}(s)$ can also help improve the robustness of the control system. If the $P_{pl}(s)$ is chosen such that $P - M + P_{pl}(s) + P_\infty$ is strictly passive, the closed-loop stability is ensured in the presence of model uncertainty.

## 4 Illustrative example

Consider the mixing tank process ([8]) as shown in Figure 3. The tank is fed with two inlet flows with flow rates $F_1(t)$ and $F_2(t)$. Both inlet flows contain one dissolved material with concentrations $c_1$ and $c_2$ respectively. The flow rate of the outlet flow is $F(t)$. Assume that the tank is well stirred so that the concentration of the outlet flow is the same as the concentration in the tank. The inlet flow rates $F_1(t)$ and $F_2(t)$ are manipulated to control both the flow rate $F(t)$ and the outlet concentration $c(t)$ to the desired values under the constant input disturbance. The nonlinear system is:

$$\begin{cases} \dot{x}_1 &= -k\sqrt{\frac{x_1}{S}} + u_1 + u_2 \\ \dot{x}_2 &= -\frac{(u_1+u_2)x_2}{x_1} + \frac{(c_1u_1+c_2u_2)}{x_1}. \end{cases} \tag{16}$$

$$\begin{cases} y_1 &= k\sqrt{\frac{x_1}{S}} \\ y_2 &= x_2. \end{cases} \tag{17}$$

where $x_1(t) = V(t)$, $x_2(t) = c(t)$; $u_1(t) = F_1(t)$, $u_2(t) = F_2(t)$; $y_1(t) = F(t)$ and $y_2(t) = c(t)$. Constant $S$ is the cross-sectional area of the tank and $k$ is the discharge coefficient of the exit flow rate. The steady state input-output function can be calculated from equation (16) and (17):

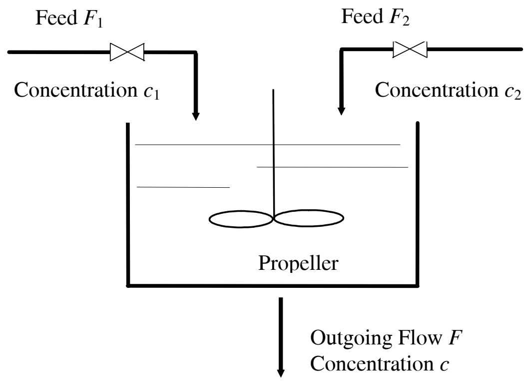


**Figure 3:** The mixing system.

$$\begin{cases} y_1 &= u_1 + u_2 \\ y_2 &= \frac{c_1u_1+c_2u_2}{u_1+u_2}. \end{cases}, \tag{18}$$

The reference input in the simulation study is $r = [1, 1.6]^T$. The corresponding steady state controller output ($u$) is $[0.4, 0.6]^T$. The dissipativity of the steady state nonlinearity $P_\infty(u_\infty)$ needs to be analyzed around this operating point. Because function $P_\infty(u_\infty)$ is continuous, its dissipativity can be verified numerically using discrete points in the region of interest. A three-dimensional plot is given (around $u_0 = [u_{10}, u_{20}]^T = [0.4, 0.6]^T$) in Figure 4. It can be observed that the steady state nonlinearity is $IFP(0.1)$ [12] around $u_0$ in the range of $u_1 \in (u_{10} - 0.3, u_{10} + 0.3)$ and $u_2 \in (u_{20} - 0.3, u_{20} + 0.3)$.

The linearized model of $P - P_\infty$ around the equilibrium point is

$$G_0(s) = \begin{bmatrix} \frac{-s}{s+1.75} & \frac{-s}{s+1.75} \\ \frac{0.6s^2+1.05s}{s^2+5.25s+6.125} & \frac{-0.4s^2-0.7s}{s^2+5.25s+6.125} \end{bmatrix}$$

The linear passive approximation system $P_{pl}(s)$ can be constructed as

$$P_{pl}(s) = \begin{bmatrix} \frac{0.6761s^3+3.261s^2+3.635s}{s^3+7s^2+15.31s+10.72} & \frac{0.4009s^3+2.297s^2+2.793s}{s^3+7s^2+15.31s+10.72} \\ \frac{0.4478s^3+2.379s^2+2.793s}{s^3+7s^2+15.31s+10.72} & \frac{0.4749s^3+2.474s^2+2.875s}{s^3+7s^2+15.31s+10.72} \end{bmatrix}$$

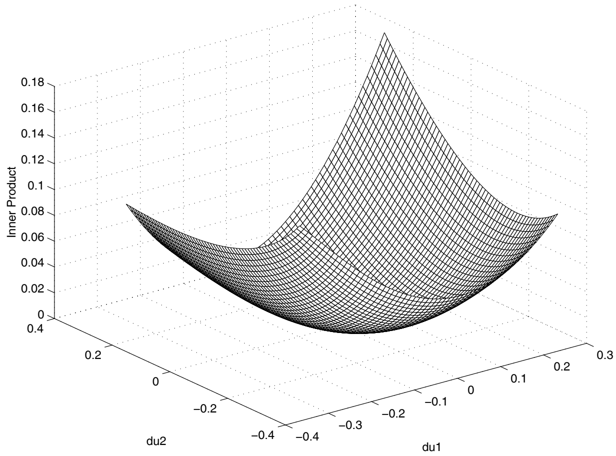


**Figure 4:** Values of $P_\infty^T(u_\infty)u_\infty$.

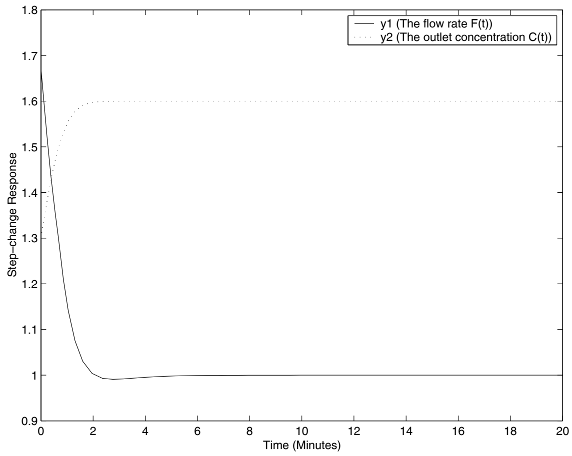


**Figure 5:** Simulation results

by solving Problem 1. The passive approximation is constructed as $P_p = P_\infty + P_{pl}$. The simulation study shows the proposed IMC control approach can achieve zero offset and satisfactory performance (see Figure 5).

# 5 Conclusion

This paper presents an extension of nonlinear IMC control based on the passive decomposition of the controlled process. As the inversion of general nonlinear process is either difficult or even impossible, the new IMC control approach only invert the passive approximation of the process. The proposed control can achieve both zero offset and desired dynamic performance around an operating point.

A linear passive system is applied to approximate the dynamics around the operating point in this paper. Techniques of approximating nonlinear processes using nonlinear passive systems with special model structure are current under investigation.

# 6 Acknowledgments

The authors gratefully acknowledge the financial support of the Australian Research Council (Grant A00104473).

# References

[1] F. Allgower and F.J. Doyle III. Nonlinear process control-which way to the promised land? In J.C. Kantor, C.E. Garcia, and B. Carnahan, editors, *Chemical Process Control-V*, pages 24–45. AIChE Symposium Series No.316, 1997.

[2] C.A. Byrnes, A. Isidori, and J. Willems. Passivity feedback equivalence, and the global stabilization of minimum phase systems. *IEEE Trans. on Automatic Control*, 36:1228–1240, 1991.

[3] A.J. Van der Schaft. *$\mathcal{L}_2$ Gain and Passivity Techniques in Nonlinear Control.* Springer Verlag, New York, 1996.

[4] C.G. Economou and M. Morari. Internal model control. 5. extension to nonlinear systems. *Ind. Eng. Process Des. Dev.*, 25:403–411, 1986.

[5] C.E. Garcia and M. Morari. Internal model control. 3. multivariable control law computation and tuning guidelines. *Ind. Eng. Chem. Res.*, 24:484–494, 1985.

[6] H.Sira-Ramirez and M.I.Angulo-Nunez. Passivity-based control of nonlinear chemical processes. *INT. J. Control*, 68(5):591–996, 1997.

[7] A. Isidori. *Nonlinear Control Systems.* Springer-Verlag, New York, 1989.

[8] H. Kwakanaak and R. Sivan. *Linear Optimal Control Systems.* Wiley Interscience, 1972.

[9] R. Marino and P. Tomei. *Nonlinear Control Design: Geometric, Adaptive and Robust.* Prentice Hall, London, 1995.

[10] M. Morari and E. Zafiriou. *Robust Process Control.* Prentice Hall, Englewood Cliffs, NJ, 1989.

[11] J.M.A. Scherpen and A.J.Van Der Schaft. Normalised coprime factorizations and balancing for unstable nonlinear systems. *INT. J. Control*, 60:1193–1222, 1994.

[12] R. Sepulchre, M. Jankovic, and P. Kokotovic. *Constructive Nonlinear Control.* Springer Verlag, New York, 1996.

[13] G. Tan and M. Chiu. A multiple-model approach to decentralized internal model control design. *Chemical Engineering Science*, 56:6651–6660, 2001.

[14] H.T. Toivonen, K.V. Sandstrom, and R.H. Nystrom. Internal model control of nonlinear systems described by velocity-based linearizations. *Journal of Process Control*, 13:215–224, 2003.

[15] Q. Wang, Y. Zhang, and M. Chiub. Decoupling internal model control for multivariable systems with multiple time delays. *Chemical Engineering Science*, 57:115–124, 2002.

[16] G. Zames. On the input-output stability of time-varying nonlinear feedback systems part one: Conditions derived using concepts of loop gain, conicity, and positivity. *IEEE Trans. on Automatic Control*, 11:228–238, 1966.