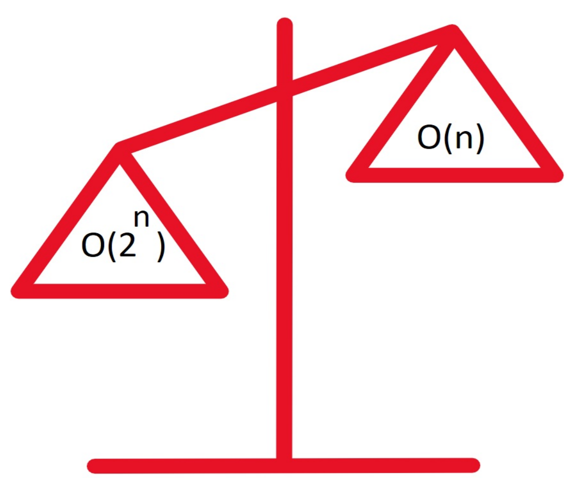
Criptografía post-cuántica basada en retículos

Lección 1: Minicurso Mar del Plata, Noviembre 2025

Syllabus

- **Criptografia asimetrica y asimetrica**
- **Principio de Kerckhoff**
- **Complexidad computacional y Security Level**
- **El objetivo CPA-IND**



Índice

1.3 Criptografia: confidencialidad y otros usos modernos

The art and science of keeping messages secure is **cryptography**, and it is practiced by **cryptographers**. **Cryptanalysts** are practitioners of **cryptanalysis**, the art and science of breaking ciphertext; that is, seeing through the disguise. The branch of mathematics encompassing both cryptography and cryptanalysis is **cryptology** and its practitioners are **cryptologists**. Modern cryptologists are generally trained in theoretical mathematics **they have to be**.

[Schneier15, Chapter 1, page 1]

Authentication, Integrity, and Nonrepudiation

In addition to providing confidentiality, cryptography is often asked to do other jobs:

- **Authentication**. It should be possible for the receiver of a message to ascertain its origin; an intruder should not bc able to masqueradc as someone else.
- **Integrity**. It should be possible for the receiver of a message to verify that it has not been modified in transit; an intruder should not be able to substitute a false message for a legitimate one.
- **Nonrepudiation**. A sender should not be able to falsely deny later that he sent a message.

[Schneier15, page 2]

1.4 Criptografia simetrica y asimetrica : primitivas

A cryptographic primitive is a fundamental building block used in cryptographic protocols and algorithms. These primitives provide basic functionalities and are combined to implement more complex cryptographic operations.

1.4.1 Symmetric Cryptography and primitives

Figure 1.4.1.1: Symmetric Key-Gen

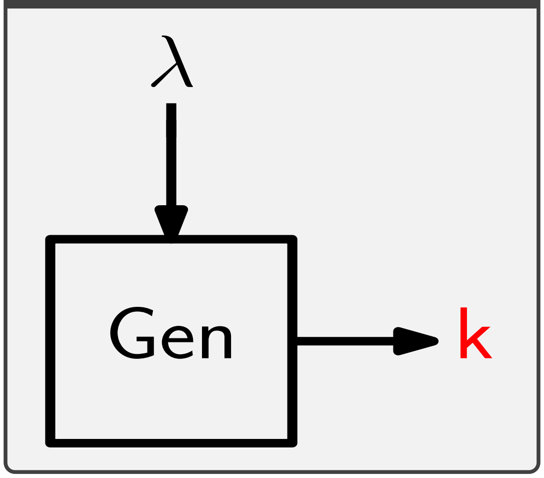
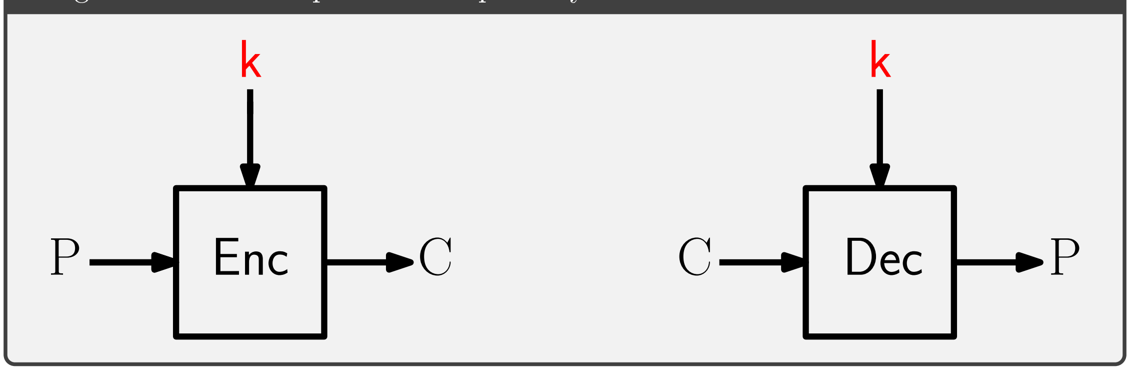


Figure 1.4.1.2: Encipher & Decipher: symmetric



Most important symmetric primitives :

- **Block Ciphers**: Encrypt data in fixed-size blocks..
  - DES (Data Encryption Standard)
  - AES (Advanced Encryption Standard), KeyGen NIST SP 800-133
  - 3DES (Triple DES)
  - Blowfish
  - Twofish
  - Serpent
- **Stream Ciphers**: Encrypt data one bit or byte at a time.
  - RC4
  - Salsa20
  - ChaCha20
- **Message Authentication Codes (MACs)**: Provide authentication and integrity.
  - HMAC (Hash-based Message Authentication Code)
  - CMAC (Cipher-based Message Authentication Code)
  - GMAC (Galois Message Authentication Code)
- **Key Derivation Functions (KDFs)**: Derive one or more secret keys from a secret value.
  - PBKDF2 (Password-Based Key Derivation Function 2)
  - bcrypt
  - scrypt
  - Argon2
- **Random Number Generators**: Generate random numbers for cryptographic use.
  - Cryptographically Secure PRNGs (CSPRNGs)
- **Hash Functions**
  - SHA-256 (Secure Hash Algorithm)
  - SHA-3
  - MD5 (though considered weak)
  - RIPEMD-160

NOTE 1.4.1.3

Besides mining blocks Bitcoin also uses SHA-256 for the 24 words of wallets (BIP-39).

1.4.2 Asymmetric Cryptography also called Public Key Cryptography

Figure 1.4.2.1: KeyGen: Asymmetric

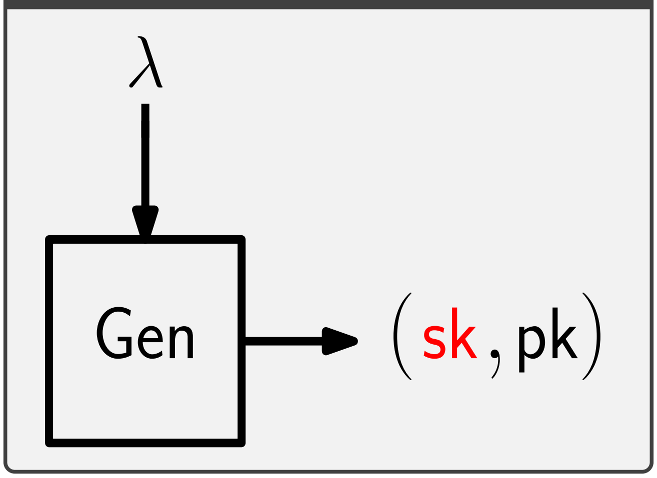
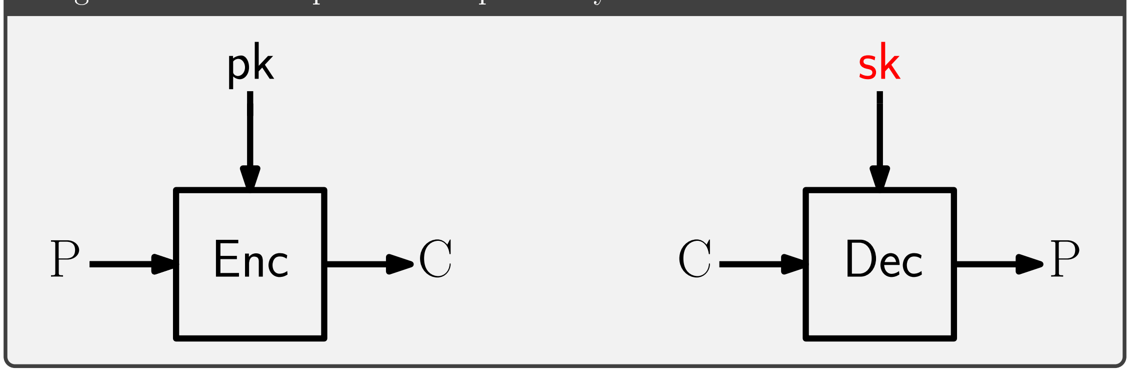


Figure 1.4.2.2: Encipher & Decipher: Asymmetric



Most important asymmetric primitives :

- **Public Key Cryptography**: Algorithms that use a pair of keys.
  - RSA (Rivest-Shamir-Adleman)
  - ECC (Elliptic Curve Cryptography)
  - DSA (Digital Signature Algorithm)
  - ElGamal Encryption
- **Digital Signatures**: Verify the authenticity and integrity of a message.
  - RSA Signatures
  - ECDSA (Elliptic Curve Digital Signature Algorithm)
  - EdDSA (Edwards-Curve Digital Signature Algorithm)
- **Key Exchange Protocols**: Allow secure key agreement between parties.
  - Diffie-Hellman Key Exchange
  - ECDH (Elliptic Curve Diffie-Hellman)
- **Identity-Based Cryptography**: Uses identity as a public key.
  - Identity-Based Encryption (IBE) systems

1.5 Security Level and Computationally infeasible

Security level λ

An encryption algorithm has a security level of λ bits if the best known attack has a computational cost of $\mathcal{O}(2^\lambda)$ e.g. requires $\mathcal{O}(2^\lambda)$ steps. This allows us to compare algorithms and is useful when we combine several primitives in a hybrid cryptosystem to understand any weaknesses. The security level is usually written in unary representation as 1^λ.

The standard security levels are related to the best knowns attacks to AES-128, 192, 256, corresponding to NIST levels 1,3 and 5. Such attacks have complexity 143, 207 and 272 bits, respectively. NIST : Security (Evaluation Criteria)

We will call a task *computationally infeasible* if its cost as measured by either the amount of memory used or the runtime is finite but impossible large.

[DH76, page 646]

To get a clue of the meaning of *computationally infeasible* imagine you are looking for a key k of m bits:

$$\mathsf{k} \in \{0,1\}^m$$

1.5.0.1 Brute Force: 30 bits

```
from timeit import default_timer as timer

#loop with 2**m rounds
m=30
start = timer()
j=0
while j < 2**m:
        # try with key k_j
        print(j)
        j+=1
end = timer()

#print the total time employed to check all keys
print(m, (end - start)/60 ,"minutes") # Time in minutes.
```

Ejercicio 1.5.0.2

How long it takes for m=64 bits ?

In most cryptographic functions, the key length is an important security parameter.

1.5.1 NIST–Standardized Post-Quantum Digital Signatures

As of 2024, the **National Institute of Standards and Technology (NIST)** has standardized the following post-quantum digital signature schemes:

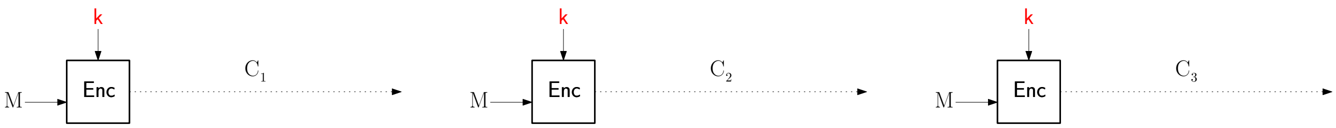
- **CRYSTALS-Dilithium**
  - **Type:** Lattice-based (Module-LWE)
  - **Security Levels:** 1, 3, and 5 (NIST categories)
  - **Status: Primary standard for general-purpose signatures**
  - **Features:** Efficient, well-balanced performance
- **FALCON**
  - **Type:** Lattice-based (NTRU-like, short signatures)
  - **Security Levels:** 1 and 5
  - **Status: Standardized (for use when smaller signatures are needed)**
  - **Features:** Very compact signatures but more complex implementation
- **SPHINCS+**

- **Type:** Hash-based (stateless)
- **Security Levels:** 1, 3, and 5
- **Status: Standardized as a backup (for long-term security)**
- **Features:** Conservative security (based on hash functions), larger signatures

NOTE 1.5.1.1

- These algorithms were selected in **July 2022** as part of **NIST's PQC Standardization Round 3**.
- **CRYSTALS-Dilithium** is the **recommended general-purpose signature scheme**, while **FALCON** is suggested for cases requiring smaller signatures.
- **SPHINCS+** is included as a **hedge against potential future attacks on lattice-based schemes**.

1.6 CPA-IND and Probabilistic Encryption (Non deterministic)



Different ciphertexts for the same cleartext M encrypted with a key k.

The CPA indistinguishability experiment $\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n)$:

1. *A key k is generated by running* $\mathsf{Gen}(1^n)$.
2. *The adversary $\mathcal{A}$ is given input 1^n and oracle access to* $\mathsf{Enc}_k(\cdot)$, *and outputs a pair of messages m_0, m_1 of the same length.*
3. *A random bit $b \leftarrow \{0,1\}$ is chosen, and then a ciphertext $c \leftarrow \mathsf{Enc}_k(m_b)$ is computed and given to $\mathcal{A}$. We call c the* challenge ciphertext.
4. *The adversary $\mathcal{A}$ continues to have oracle access to* $\mathsf{Enc}_k(\cdot)$, *and outputs a bit b'.*
5. *The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise. (In case $\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A},\Pi}(n) = 1$, we say that $\mathcal{A}$* succeeded.)

1.6.0.1 CPA-IND secure

The symmetric crypto system $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is **CPA-IND**-secure (or just CPA-secure) if no adversary $\mathcal{A}$ can succeed with probability better than $1/2$.

1.6.1 Kerckhoffs's principle and models COA, KPA, CPA and CCA

Kerckhoffs's principle

The enemy knows the system (Shannon's Maxim). This means that security should just depend on the secrecy of the key.

Kerckhoffs principle
Attack models

Referencias

[Schneier15] Bruce Schneier, *Applied Cryptography: Protocols, Algorithms and Source Code in C* ,Wiley; 20th Anniversary edition,2015.

[DH76] Diffie, W.; Hellman, M. *New directions in cryptography*, (1976). IEEE Transactions on Information Theory. 22 (6): 644-654.

[QUBIP23] QUBIP: *Transition to Post-Quantum Cryptography*
QUBIP project is co-funded by the European Union under the Horizon Europe framework programme [grant agreement no. 101119746].
https://qubip.eu/
https://github.com/QUBIP
http://www.youtube.com/@qubip_eu