

The background features a complex network of thin, light gray lines and dots, forming a web-like structure. Scattered throughout are various triangles of different sizes and orientations, some with solid outlines and others with dashed or dotted outlines. The overall aesthetic is technical and minimalist.

Memory-efficient aggregations Tutorial 6

Gabriele Santin

<https://gabrielesantin.github.io/>



Sparse matrices and
SparseTensor

01

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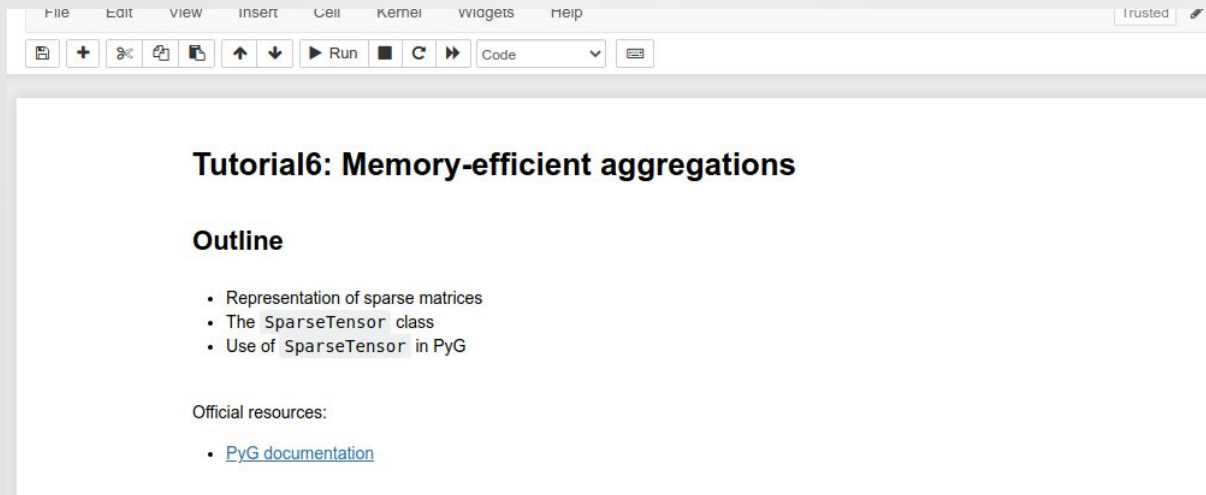
02

Use of SparseTensor in
MessagePassing



01 Sparse matrices and SparseTensor

... notebook ...



02 Use of SparseTensor in MessagePassing

Message passing: an example layer

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02 Use of SparseTensor in MessagePassing

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from torch_geometric.nn import MessagePassing

x = ...           # Node features of shape [num_nodes, num_features]
edge_index = ...  # Edge indices of shape [2, num_edges]

class MyConv(MessagePassing):
    def __init__(self):
        super().__init__(aggr="add")

    def forward(self, x, edge_index):
        return self.propagate(edge_index, x=x)

    def message(self, x_i, x_j):
        return MLP(x_j - x_i)
```

02 Use of SparseTensor in MessagePassing

Message passing: an example layer

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class MyConv(MessagePassing):
    def __init__(self):
        super().__init__()

    def forward(self, x, edge_index):
        x_j = x[edge_index[0]] # Source node features [num_edges, num_features]
        x_i = x[edge_index[1]] # Target node features [num_edges, num_features]

        msg = MLP(x_j - x_i) # Compute message for each edge

        # Aggregate messages based on target node indices
        out = scatter(msg, edge_index[1], dim=0, dim_size=x.size(0), reduce="add")
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02 Use of SparseTensor in MessagePassing

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from torch_geometric.nn import MessagePassing
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x = ... # Node features of shape [num_nodes, num_features]  
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```

```
class MyConv(MessagePassing):  
    def __init__(self):  
        super().__init__()  
        from torch_scatter import scatter
```

```
    def forward(self, x: Tensor, edge_index: Tensor):  
        x = ... # Node features of shape [num_nodes, num_features]  
        edge_index = ... # Edge indices of shape [2, num_edges]
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        x_j = x[edge_index[0]] # Source node features [num_edges, num_features]  
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    def forward(self, x, edge_index):  
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    def message(self, x_j = x[edge_index[0]] # Source node features [num_edges, num_features]  
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gather


scatter

02 Use of SparseTensor in MessagePassing



gather

- Collect all features and apply the message
- It requires the explicit storage of x_i, x_j (lots of possible duplicates, especially on dense graphs)



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02 Use of SparseTensor in MessagePassing



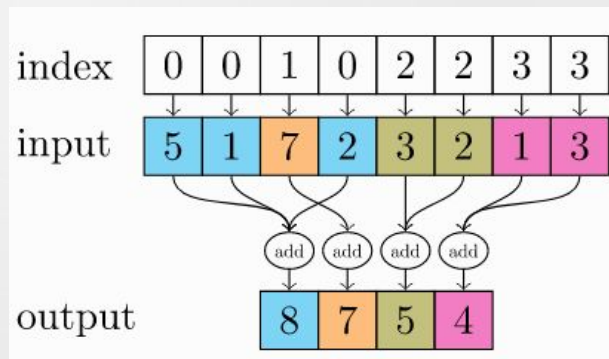
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scatter

- Aggregate the message according to the target node index
- It requires a lot of aggregation operations




02 Use of SparseTensor in MessagePassing



gather

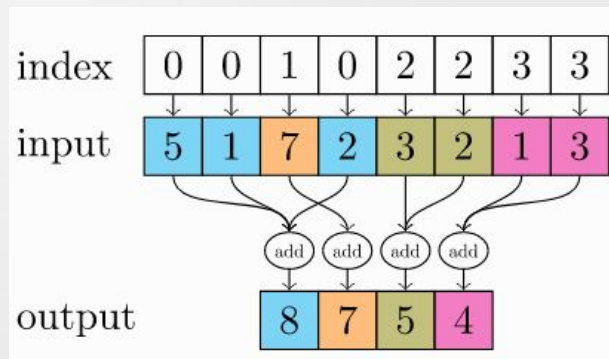
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Sparse matrix multiplication as an efficient replacement of gather-scatter

02 Use of SparseTensor in MessagePassing

GINConv example:

$$\mathbf{x}'_i = \text{MLP} \left((1 + \epsilon) \cdot \mathbf{x}_i + \sum_{j \in \mathcal{N}(i)} \mathbf{x}_j \right)$$

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02 Use of SparseTensor in MessagePassing

GINConv example:

$$\mathbf{x}'_i = \text{MLP} \left((1 + \epsilon) \cdot \mathbf{x}_i + \sum_{j \in \mathcal{N}(i)} \mathbf{x}_j \right) \rightarrow \mathbf{X}' = \text{MLP} ((1 + \epsilon) \cdot \mathbf{X} + \mathbf{A}\mathbf{X})$$

- **Lower memory** footprint
- **Faster execution** time
- Possible for most, but not all layers

02 Use of SparseTensor in MessagePassing

The method `message_and_aggregate()` :

- It combines the `message` and `aggregate` methods in one single call that uses sparse matrix operations.
- It is automatically called whenever implemented!

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from torch_sparse import matmul

class GINConv(MessagePassing):
    def __init__(self):
        super().__init__(aggr="add")

    def forward(self, x, edge_index):
        out = self.propagate(edge_index, x=x)
        return MLP((1 + eps) * x + out)

    def message(self, x_j):
        return x_j

    def message_and_aggregate(self, adj_t, x):
        return matmul(adj_t, x, reduce=self.aggr)
```

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Sparse matrix-vector multiplication

THANKS

Questions?

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