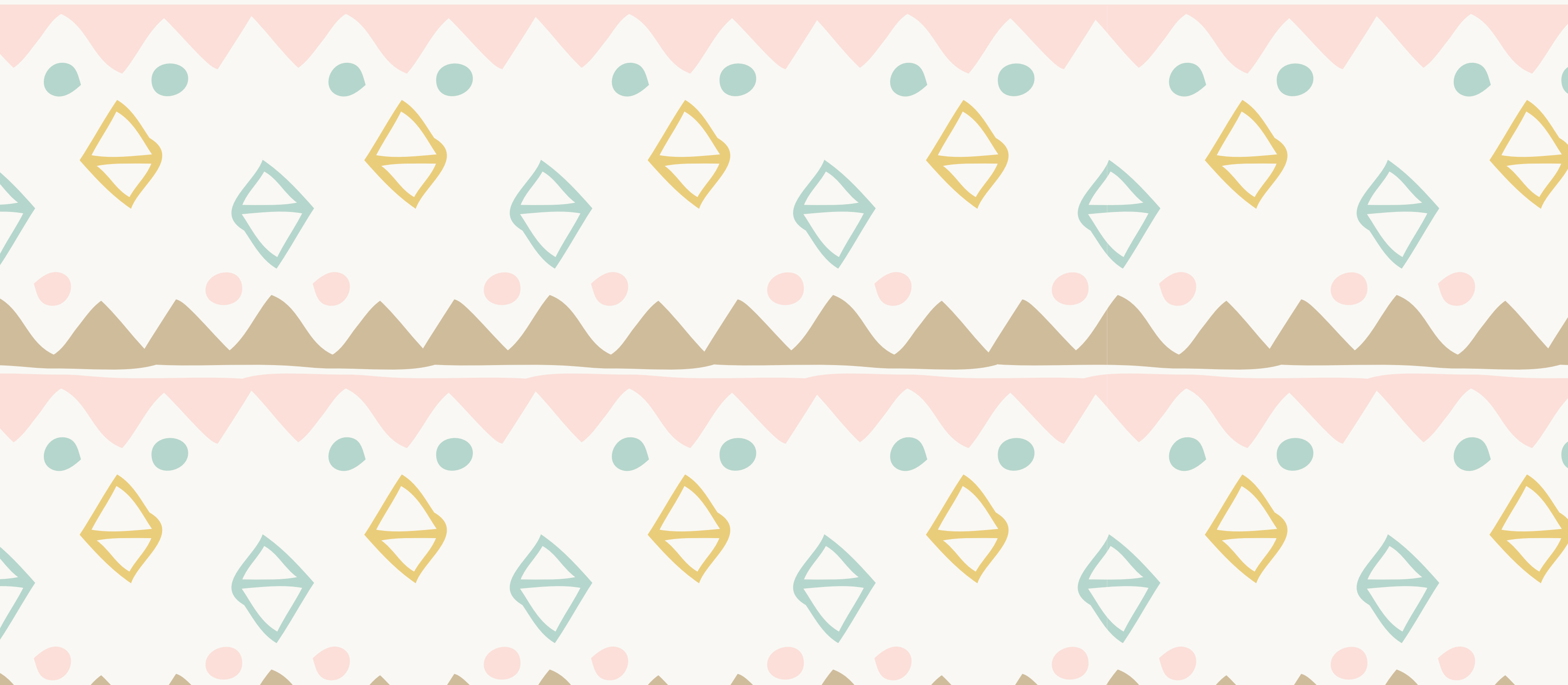


impulso di Kronecker

$$\delta(K)=\begin{cases}1 & K=0\\ 0 & K\neq 0\end{cases}$$

gradino unitario discreto

$$1(K)=\begin{cases}1 & K\geq 0\\ 0 & K<0\end{cases}\qquad \mathcal{Z}\{1(K)\}=\frac{z}{z-1}$$

$$r(t)=\begin{cases}t & t\geq 0\\ 0 & t<0\end{cases}\qquad \mathcal{Z}\{r(t)\}=T\frac{z}{(z-1)^2}$$

$$x(K)=\begin{cases}a^K & K\geq 0\\ 0 & K<0\end{cases}\qquad \mathcal{Z}(a^K)=\frac{z}{z-a}\qquad \mathcal{Z}\{e^{aKT}\}=\frac{z}{z-e^{aT}}$$

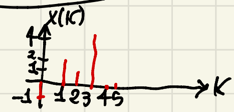
$$\mathcal{Z}\{c_1x_1(K)+c_2x_2(K)\}=c_1X_1(z)+c_2X_2(z)$$

$$\mathcal{Z}\{\sin(K\omega T)\}=\frac{z\sin(\omega T)}{z^2-2\cos(\omega T)z+1}$$

$$\mathcal{Z}\{\cos(K\omega T)\}=\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$$

$$X(K)=a^Kx(K)\qquad \mathcal{Z}\{a^Kx(K)\}=X\left(\frac{z}{a}\right)$$

$$\mathcal{Z}\{e^{at}\sin(\omega t)\}=\frac{\frac{z}{e^{aT}}\sin(\omega T)}{\frac{z^2}{e^{2aT}}-\frac{z\cdot 2}{e^{aT}}\cos(\omega T)+1}$$



$$x(K)=-\delta(K)+2\delta(K-1)+\delta(K-2)+4\delta(K-3)$$

$$X(z)=-1+2z^{-1}+z^{-2}+4z^{-3}$$

$$\mathcal{Z}\{Kx(K)\}=-z\frac{dX(z)}{dz}$$

$$\mathcal{Z}\{KT1(K)\}=-z\frac{d\left(KT\frac{z}{z-1}\right)}{dz}=\frac{+zKT}{(z-1)^2}$$

$$f(K)=\left|\cos\left(\frac{2}{3}\pi K\right)\right|$$

T=0.5

K	f(K)
0	1
1	1/2
2	1/2
3	1

$$P=3\quad =\left[1+\frac{z^{-1}}{2}+\frac{z^{-2}}{2}\right]\frac{1}{1-z^{-P}}$$

$$\mathcal{Z}\{Ka^K\}=\frac{z}{(z-a)^2}a$$

$$\mathcal{Z}\left\{\frac{(K+n-1)!}{K!(n-1)!}a^K\right\}=\left\{\binom{K+n-1}{K}a^K\right\}=\frac{z^n}{(z-a)^n}$$

$$\mathcal{Z}(Ka^{K-1})=\mathcal{Z}\left(\frac{Ka^K}{a}\right)=\frac{z}{(z-a)^2}$$

$$\mathcal{Z}\left\{\frac{10}{s^2(s+1)}\right\}=10\mathcal{L}^{-1}\left[\frac{1}{s^2}+\frac{-1}{s}+\frac{1}{s+1}\right]=\mathcal{Z}\{10KT-10\,1(K)+10e^{-KT}\}$$

$$=10T\frac{z}{(z-1)^2}-10\frac{z}{z-1}+10\frac{z}{z-e^{-T}}$$

$$X(z)=\frac{z^2}{z^2-z-1}$$

$z^2 \quad | \quad z^2-z-1$

$z^2-z-1 \quad | \quad 1+z^{-1}+2z^{-2}$

// $z+1$

$z-1-z^{-1}$

// $2+z^{-1}$

$2-2z^{-1}-2z^{-2}$

// $3z^{-1}+2z^{-2}$

$$E(z)\rightarrow\boxed{G(z)}\rightarrow U(z)\qquad E(z)=\delta(K)$$

$$G(z)=\frac{z}{(z+1)(z+2)}=\frac{z^{-1}}{1+3z^{-1}+2z^{-2}}=\frac{U(z)}{E(z)}$$

$$u(K)+3u(K-1)+2u(K-2)=e(K-1)$$

K

0 $u(0)=0$

1 $u(1)=1-3\cdot 0+2\cdot 0=1$

2 $u(2)=0-3\cdot 1-2\cdot 0=-3$

$$X(z)=\frac{z}{(z+1)(z+2)}=\frac{A}{z+1}+\frac{B}{z+2}=\frac{-z^{-1}z}{z+1}+2z^{-1}\frac{z}{z+2}=-(-1)^{K-1}+2(-2)^{K-1}=-(-1)^{K-1}\underbrace{1(K-1)}_{\text{gradino modulante}}+2(-2)^{K-1}1(K-1)$$

A = -1

B = 2

$$\cos x=\frac{e^{ix}+e^{-ix}}{2}\qquad \sin x=\frac{e^{ix}-e^{-ix}}{2i}$$

$$X(z)=\frac{z^{-1}}{(1+z^{-1})(1+2z^{-1})}=\frac{1}{z+1}+\frac{-1}{z+2}=(-1)^K-(-2)^K$$

$$j=e^{+j\pi/2}\qquad -j=e^{-j\pi/2}$$

$$X(z)=\frac{10(z+2)}{(z^2+1)z}=\frac{A}{z}+\frac{B}{z-j}+\frac{C}{z+j}=\frac{20}{z}+(-10-5j)\frac{z^{-1}z}{z-j}+(10+5j)\frac{z^{-1}z}{z+j}=$$

A = 20

B = -10-5j

C = -10+5j

$$=(-10-5j)j^K+(-10+5j)(-j)^K+20\delta(K)=20\delta(K-1)+\left[-20\cos(K-1)\frac{\pi}{2}+10\sin(K-1)\frac{\pi}{2}\right]1(K-1)$$

$$X(z)=\frac{z}{(z+1)(z+2)}\qquad x(K)=(-1)^K-(-2)^K$$

$$\left.\frac{z}{z+1}z^{K-1}\right|_{z=-1}=(-1)^K$$

$$\left.\frac{z^K}{z+1}\right|_{z=-2}=-(-2)^K$$

$$X(z)=\left(\frac{10(z+2)}{z(z+1)(z-1)}\right)z^{K-1}=(R_1+R_2+R_3)1(K-2)$$

K<2 0, 1, 1

K≥2 -1, 1

$$R_1=\sum res\left(\frac{10(z+2)}{z(z+1)(z-1)}\right)\quad K=1$$

$$R_2=\left.\frac{10(z+2)z^{K-2}}{(z-1)}\right|_{z=-1}=-5(-1)^{K-2}$$

$$R_3=\left.\frac{10(z+2)z^{K-2}}{z+1}\right|_{z=1}=15$$

$$X(z) = \frac{50z^3}{(z+1)(z^2+4)}$$

$$R_1 = \left.\frac{50\, z^3 z^{K-1}}{z^2+4}\right|_{z=-1} = \frac{50}{5}(-1)^{K+2}$$

$$R_2 + R_3 = \frac{50(j2)^{K+2}}{(1+2j)j4} + \frac{50(-j2)^{K+2}}{(1-2j)(-j4)}$$

$$2j = 2e^{j\pi/2} \quad -2j = 2e^{-j\pi/2}$$

$$5 \cdot 2^{K+2} \sin[(K+2)\tfrac{\pi}{2}] - 10\, 2^{K+2} \cos[(K+2)\pi/2]$$

PROVA DI ESAME DEL 18/07/2023

QUESITO A

- Si determini l'espressione del segnale $f(k) = Z^{-1}\left\{\frac{z^3}{(z+1)(z^2+4)}\right\}$ utilizzando esclusivamente il metodo dell'integrale di inversione; sfruttando l'espressione così ottenuta, si calcolino i primi due campioni del segnale.
- Si verifichi che i campioni calcolati al punto precedente corrispondono a quelli ottenuti con il metodo della lunga divisione.

$$\frac{z^{K+2}}{(z+1)(z^2+4)} = R_1 + R_2 + R_3 \qquad R_2 + R_3 = \left.\frac{z^{K+2}}{(z+1)(z+2j)}\right|_{z=2j} + \left.\frac{z^{K+2}}{(z+1)(z-2j)}\right|_{z=-2j}$$

$$R_1 = \left.\frac{z^{K+2}}{(z+1)(z^2+4)}\right|_{z=-1} = \frac{(-1)^{K+2}}{5}$$

$$\frac{3}{10} 2^{K+2} \sin((K+2)\pi/2) - \frac{2^{K+2}}{5}\cos((K+2)\pi/2)$$

$$z^3 \;\Big|\; z^3+z^2+4z+4 \rightarrow 1 - z^{-1}$$

$$z^3+z^2+4z+4$$

$$-z^2-4z-4$$

$$-z^2-z-4-4z^{-1}$$

$$-3z \qquad -4z^{-1}$$

K	
0	$f(0) = 1$
1	$f(1) = -1$

campione 200esimo

$K = 199$

$$f(K)\frac{z^{-3}}{z^{-3}} = \frac{1}{1+z^{-1}+4z^{-2}+4z^{-3}} = \frac{U}{E} \Rightarrow$$

$u(K) = f(1)$

$u(K) = e(K) - u(K-1)$

$-4u(K-2)$

$-4u(K-3)$

indipendente dal segnale

$u(0) = 1$

$u(1) = 0 - 1 = -1$

$u(2) = 0 + 1 - 4 = -3$

PROVA DI ESAME DEL 25/02/2021

QUESITO A

Si calcolino la z-trasformata e l'antitrasformata (in forma esplicita) riportate di seguito, fissando un passo di campionamento $T = 0.5$ secondi:

- $X(z) = Z\left\{\frac{4}{s^2(s+4)}\right\}$
- $x(k) = Z^{-1}\left\{\frac{z}{(z-1)^2(z+1)}\right\}$

$$\frac{1}{s^2} + \frac{-1/4}{s} + \frac{1/4}{s+4} = \frac{zT}{(z-1)^2} - \frac{z}{(z-1)4} + \frac{z}{4(z-e^{-4T})}$$

$$\frac{A}{(z-1)^2} + \frac{B}{z-1} + \frac{C}{z+1} = \frac{1}{2T}r(K) - \frac{1}{4}1(K) + \frac{(-1)^K}{4}$$

TURNO 1 - PROVA DI ESAME DEL 15/06/2021

QUESITO A

Sia assegnata il sistema tempo-discreto descritto dalla seguente equazione alle differenze:

$$y(k) = -4y(k-2) + u(k-3) - u(k-4)$$

con $y(k) = 0$ per $k \leq 0$.

Supponendo di applicare in ingresso al sistema il seguente segnale $u(k)$:

$$u(k) = 0 \quad \text{per } k < 0 \text{ e } k > 2$$
$$u(0) = 0$$
$$u(1) = 1$$
$$u(2) = 1.5$$

- Si determini la Z-trasformata di $y(k)$;
- Si determini in forma esplicita il segnale di uscita $y(k)$ mediante antitrasformata-Z;
- Se ha senso, si determini il valore finale dell'uscita $y(k)$ del sistema.

$$G(z) = \frac{Y(z)}{U(z)} = \frac{z^{-3}(1-z^{-1})}{1+4z^{-2}}$$

$$U(z) = z^{-1} + 1{,}5z^{-2}$$

$$Y = G \cdot U \Rightarrow 0{,}25 \cdot 2^{K-5}\cos((K-5)\tfrac{\pi}{2}) - 2{,}75\, 2^{K-5}\sin((K-5)\pi/2)$$

PROVA DI ESAME DEL 19/09/2023

QUESITO A

Un sistema digitale è descritto dalla seguente funzione di trasferimento:

$$G(z) = \frac{4}{(z+0.5)(z^2+1)}$$

Si calcoli, in forma esplicita, l'espressione temporale della risposta $y(k)$ del sistema al seguente ingresso (avendo indicato con $\mathbf{1}(k)$ il gradino unitario discreto e con $\mathbf{r}(k)$ la rampa unitaria discreta):

$$u(k) = \mathbf{r}(k-1) - 2 \cdot \mathbf{1}(k-2)$$

$$U = \frac{-(z-2)}{z(z-1)^2}$$

$$Y = Z^{-1}\{G \cdot U\} = \left(\frac{32}{9}(-0{,}5)^{K-2} + 4\sin((K-2)\pi/2) + \frac{4}{3}r(K-2) - \frac{3}{2}1(K-2)\right) \cdot 1(K-2)$$

ricostruttore di ordine 0 (zero order hold)

$$H_S = \frac{1-e^{-TS}}{S}$$

$$x_R(t) = x(KT) + \left(\frac{x(KT) - x((K-1)T)}{T}\right)(t-KT)$$

ordine 1° $\quad H_1(S) = \frac{1+TS}{T}\left(\frac{1-e^{-TS}}{S}\right)^2$

ordine frazionario $0° < x < 1°$

$$H_F(S) = \frac{K+TS}{T}\left(\frac{1-e^{-TS}}{S}\right) + (1-K)e^{-TS}\left(\frac{1-e^{-TS}}{S}\right)$$

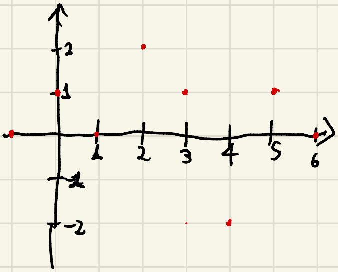
$H_0, H_1, H_F \rightarrow$ discontinuità

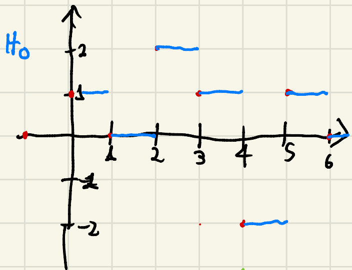
$$x_R = x((K-1)T) + (t-KT)\left(\frac{x(KT) - x((K-1)T)}{T}\right)$$

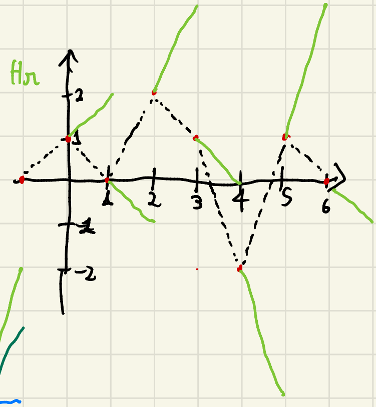
$$KT \leq t < (K+1)T$$

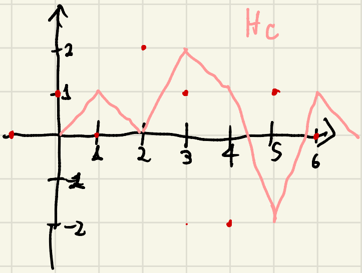
ricostruttore continuo

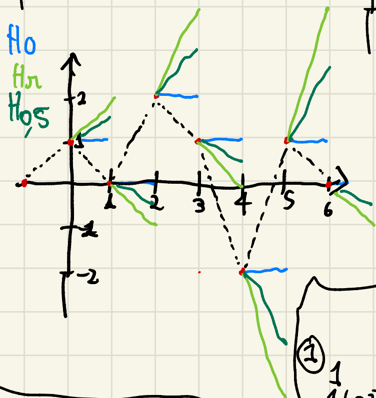
$$H_C = \frac{1}{T}\left(\frac{1-e^{-TS}}{S}\right)^2$$





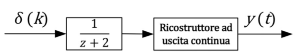


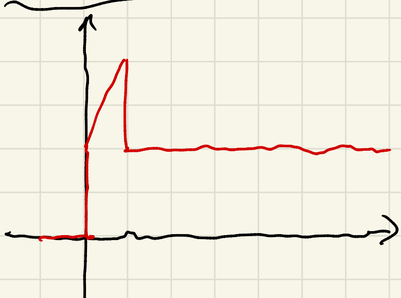




QUESITO B

Si rappresenti graficamente (con precisione) l'andamento nei primi tre intervalli di campionamento del segnale tempo-continuo $y(t)$ ottenuto secondo lo schema in figura.





$$X(S) = \frac{2}{S} + \frac{2}{TS^2} - \frac{2}{S}e^{-TS} - \frac{2}{TS^2}e^{-TS} = 2\,\frac{(1+TS)}{TS^2}\left(1-e^{-TS}\right)$$

$$X^*(S) = \mathcal{Z}\{1(k)\} = \left(\frac{z}{z-1}\Big|_{z=e^{TS}}\right)\frac{e^{-TS}}{e^{-TS}} = \frac{1}{1-e^{-TS}}$$

$$\frac{X(S)}{X^*(S)} \Rightarrow 1^\circ \text{ ordine}$$

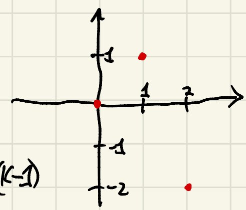
(1)

$$\frac{1}{1+2z^{-1}} \quad \Big| \quad \frac{z+2}{z^{-1}-2z^{-2}}$$

$// \; -2z^{-1}$

(2) $\mu(k) = (-2)^{k-1}\,1(k-1)$

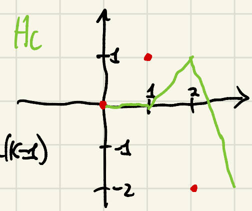
k	$\mu(k)$
0	0
1	1
2	-2



(3) $G(z) = \frac{z^{-1}}{1+2z^{-1}}$

$\mu(k) = \delta(k-1) - 2\mu(k-1)$

k	$\mu(k)$
0	0
1	1
2	-2

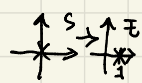


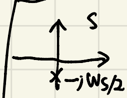
$H_0 \rightarrow$ ritardo $T/2$

$H_1 \rightarrow$ ritardo T

$H_c \rightarrow$ ritardo T

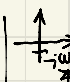
$$\begin{cases} \rho = e^{\sigma T} \\ \theta = \omega T \end{cases}$$





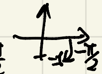
$\rho = e^{\sigma T} = 1$

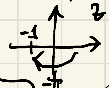
$\theta = \omega T = -\frac{\omega_s}{2}T = -\frac{2\pi}{2T}T = -\pi$

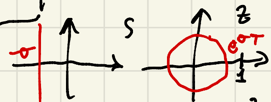


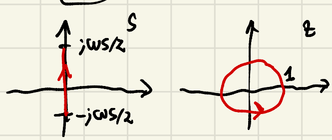
$\rho = 1$

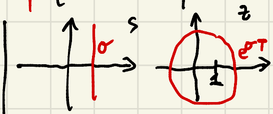
$\theta = -\frac{\pi}{2}$

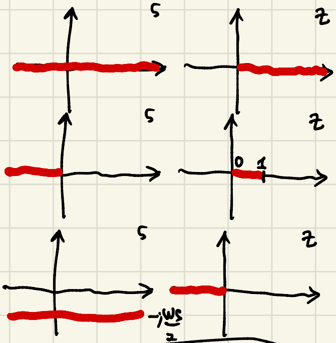












$$-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$$

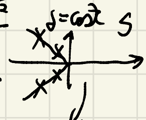
ω_d

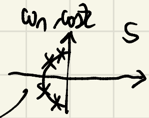
$$z = \begin{cases} \rho = e^{-\delta\omega_n T} \\ \theta = \omega_n\sqrt{1-\delta^2}\,T \end{cases}$$

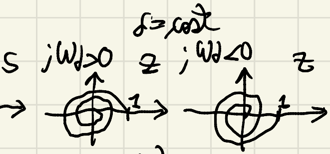
$\Downarrow$

$$\log \rho = -\frac{\delta}{\sqrt{1-\delta^2}}\theta$$

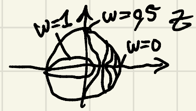
spirale logaritmica

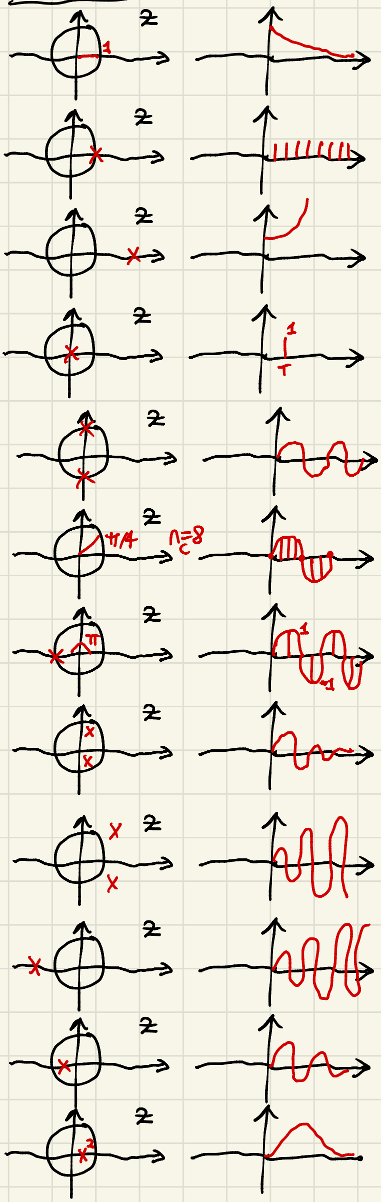


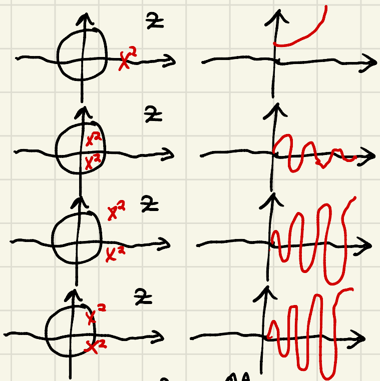




$\omega_n = \text{cost}$

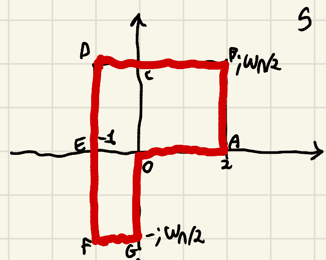


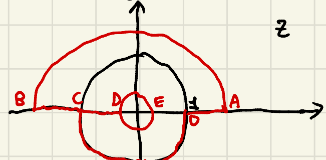




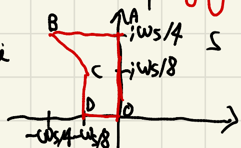
n. campioni

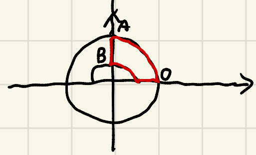
$$n_c = \frac{2\pi}{\theta}$$





$$B = \begin{cases} \rho = e^{2T} \\ \theta = \frac{\omega}{2}T = \frac{2\pi}{2T}T = \pi \end{cases}$$





$$C = \rho = e^{-\pi/4}$$

$$\theta = \frac{2\pi}{8T}T = \frac{\pi}{4}$$

$$C = e^{-\pi/4} \cdot e^{j\pi/4}$$

$X(S)$ / $X^*(S)$ $G(S)$ $Y(S)$ / $Y^*(S)$

$$Y^*(S) = G^*(S)X^*(S)$$

* = campionamento

$X(S)$ $G(S)$ $Y(S)$ / $Y^*(S)$

$$Y(S) = G(S).Y(S)$$

$$Y^*(S) = [G(S)\cdot Y(S)]^* \neq G^*(S)X^*(S)$$

$X(S)$ / $X^*(S)$ $G(S)$ $X_1(S)$ / $X_1^*(S)$ $G_2(S)$ $Y(S)$ / $Y^*(S)$

$$\begin{cases} Y(S) = X_1^*(S)G_2(S) \\ X_1(S) = G(S)X^*(S) \end{cases}$$

$$Y^*(S) = G_1^*(S)G_2^*(S)X^*(S)$$

$X(z)$ $G_1(z)G_2(z)$ $Y(z)$

$X(S) \,/\, X^*(S)\; G_1(S) - G_2(S) \xrightarrow{Y(S)} \,/\, Y^*(S)$

$$Y(S) = G_2(S)\, G_1(S)\, X^*(S)$$

convoluzione

$$Y^*(S) = \left[G_1(S) G_2(S)\right]^* X^*(S) = \overline{G_1 G_2 (z)}\, X(z)$$

$$G_1 = \frac{1}{S+1} \qquad G_2 = \frac{1}{S+2}$$

$$G_1 = \frac{z}{z-e^{-T}} \qquad G_2 = \frac{z}{z-e^{-2T}}$$

$$G_1 G_2 = \mathcal{Z}\left\{\frac{1}{S+1} - \frac{1}{S+2}\right\} = \frac{z}{z-e^{-T}} + \frac{-z}{z-e^{-2T}}$$

$\mu(S) \,/\, \mu^*(S)\; H_0(S) - G_p(S) \xrightarrow{Y(S)} \,/\, Y^*(S)$

$$H_0 G_p = \mathcal{Z}\left\{\frac{G_p(S)}{S}\right\} - z^{-1} \mathcal{Z}\left\{\frac{G_p(S)}{S}\right\}$$

$X(S) \,/\, X^*(S)\; G(S) \xrightarrow{X_1(S)} \,/\, X_1^*(S)\; G_2(S) \xrightarrow{Y(S)} \,/\, Y^*(S)$

$$\begin{cases} Y(S) = G_2(S) X_1^*(S) \\ X_1(S) = G(S) X^*(S) \end{cases}$$

$$Y^*(S) = G_2^*(S) G_1^*(S) X^*(S)$$

$$Y(z) = G_2(z) G_1(z) X(z)$$

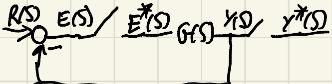
$X(z) \; G_1(z) G_2(z) \; Y(z)$

$X(S) - G_1(S) \xrightarrow{X_1(S)} \,/\, X_1^*(S)\; G_2(S) \xrightarrow{Y(S)} \,/\, Y^*(S)$, $X(S) \,/\, X^*(S)$

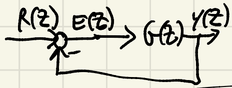
$$Y(S) = G_2(S) X_1^*(S)$$

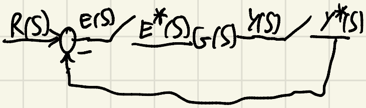
$$X_1(S) = G_1(S) X(S)$$

$$Y(z) = G_2(z)\, X G_1(z)$$



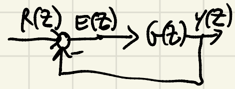
$$\begin{cases} E(S) = R(S) - Y(S) \\ Y(S) = G(S) E^*(S) \end{cases} \qquad \begin{cases} E(z) = R(z) - G(z) E(z) \\ Y(z) = G(z) E(z) \end{cases}$$

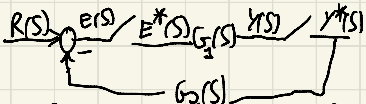




$$\begin{cases} E(S) = R(S) - Y^*(S) \\ Y(S) = G(S) E^*(S) \end{cases}$$

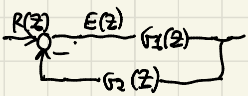
$$E(z) = R(z) - G(z) E(z)$$

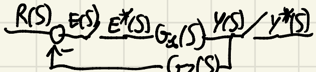




$$\begin{cases} E(S) = R(S) - G_2(S) Y^*(S) \\ Y(S) = G_1(S) \cdot E^*(S) \end{cases} \Rightarrow \begin{cases} E^*(S) = R^*(S) - G_2^*(S) Y^*(S) \\ Y^*(S) = G_1^*(S) E^*(S) \end{cases}$$

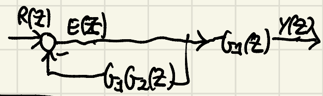
$$E(z) = R(z) - G_1(z) G_2(z)\, E(z) \qquad Y(z) = G_1(z) E(z)$$

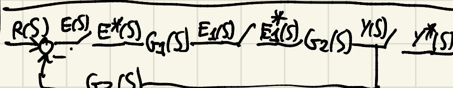




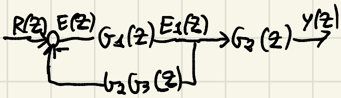
$$\begin{cases} E(z) = R(z) - G_1 G_2(z) E(z) \\ Y(z) = G_1(z) E(z) \end{cases}$$

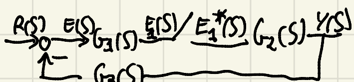
$$\begin{cases} E(S) = R(S) - G_1(S) G_2(S) E^*(S) \\ Y = G_1(S) E^*(S) \end{cases}$$





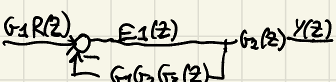
$$\begin{cases} E(S) = R(S) - G_2(S) G_3(S) E_1^*(S) \\ Y(S) = G_2(S) E_1^*(S) \\ E_1(S) = G_1(S) E^*(S) \end{cases} \qquad \begin{cases} E(z) = R(z) - G_2 G_3(z) E_1(z) \\ Y(z) = G_2(z) E_1(z) \\ E_1(z) = G_1(z) E(z) \end{cases}$$

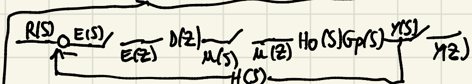




$$\begin{cases} E_1(S) = G_1(S)\left[R(S) - G_2(S) G_3(S) E_1^*(S)\right] \\ Y(S) = G_2(S) E_1^*(S) \end{cases}$$

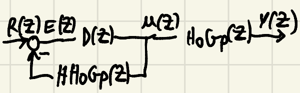
$$\begin{cases} E_1(z) = G_1 R(z) - G_1 G_2 G_3(z) E_1(z) \\ Y(z) = G_2(z) E_1(z) \end{cases}$$

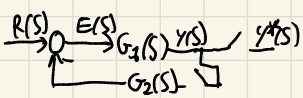




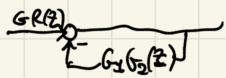
$$\begin{cases} E(S) = R(S) - H(S) H_0(S) G_p(S) \mu^*(S) \\ Y(S) = H_0(S) G_p(S) \mu^*(S) \end{cases}$$

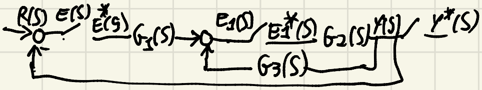
$$\begin{cases} E(z) = R(z) - H H_0 G_p(z) \mu(z) \\ Y(z) = H_0 G_p(z) \mu(z) \\ \mu(z) = D(z) E(z) \end{cases}$$





$$\begin{cases} Y(S) = E(S) G_1(S) \\ E(S) = R(S) - G_2(S) Y^*(S) \end{cases} \qquad Y(z) = GR(z) - G_1 G_2(z) Y(z)$$



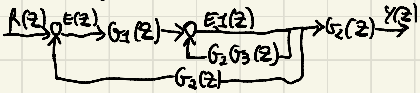


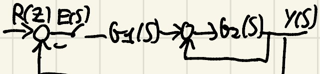
$$\begin{cases} E(S) = R(S) - Y(S) \\ E_1(S) = G_1 E^*(S) + G_3(S) Y(S) \\ E_1^*(S) G_2(S) = Y(S) \end{cases}$$

$E(z) = R(z) - Y(z)$

$E_1(z) G_2(z) = Y(z)$

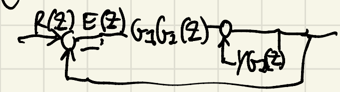
$R(z) - G_2 E_1(z) \left(G_1(z) E(z) + G_2 G_3(z) E_1(z) \right)$

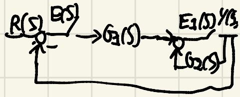




$$\begin{cases} Y(S) = \left(E^*(S) G_1(S) + Y(S)\right) G_2(S) \\ E(S) = R(S) - Y(S) \end{cases}$$

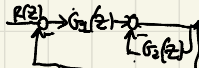
$$\begin{cases} Y(z) = E(z) G_1G_2(z) + YG_2(z) \\ E(z) = R(z) - \left(E(z) G_1G_2(z) + YG_2(z)\right) \end{cases}$$





$$\begin{cases} E(S) = R(z) - Y(S) \\ Y(S) = E_1^*(S) \\ E_1(S) = G_1(S) E^*(S) - G_2(S) \end{cases}$$

$$\begin{cases} E(z) = R(z) - \left(G_2(z) E(z)\right. \\ Y(z) = E_1(z) G_2(z) \left.\right) \\ E_1(z) = G_1(z) E(z) - G_2(z) \end{cases}$$



$$Z_m\left\{\frac{1}{S+a}\right\} = \frac{z}{z - e^{-aT}} z^{-1} e^{-amT}$$

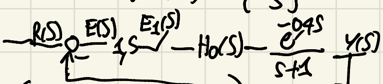
$K_T = KT + mT$

$$Z_m\left\{\frac{1}{S}\right\} = \frac{z^{-1} \cdot z}{z-1}$$

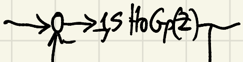
$$Z_m(t\,1(t)) = \frac{z^{-1} T z}{(z-1)^2} + \frac{z^{-1} m T z}{z-1}$$

$\mu^*(S)$ $H_0(S)$ $G(S)$ $Y(S)$ $e^{-\tau S}$ $Y'(S)$

$$Y'(z) = (1 - z^{-1}) Z_m\left\{\frac{G(S)}{S}\right\}$$



$$\begin{cases} E_1(S) = 1{,}5\, R(S) - 1{,}5\, Y^*(S) \\ Y(S) = H_0(S) G_p(S) E_1^*(S) \end{cases}$$

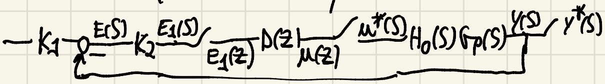


$$G_0 = \frac{1{,}5\, H_0 G_p(z)}{1 + 1{,}5\, H_0 G_p(z)}$$

$$m = 1 - \frac{\tau}{T} = 1 - \frac{0{,}4}{1} = 0{,}6$$

$$H_0G_p = 1 - z^{-1} Z_m\left\{\frac{1}{S(S+1)}\right\}$$

$$(1 - z^{-1})\left(\frac{1}{z-1} - \frac{e^{-0{,}6}}{z - e^{-1}}\right)$$



$K_1 = 5$

$K_2 = 2$

$D(z) = \frac{1}{z}$

$G(S) = \frac{2}{S+2} e^{-0{,}5S}$

$$m = 1 - \frac{0{,}5}{1} = 0{,}5$$

$\tau < T$

$$\boxed{\begin{aligned} &\text{se } \tau = 1{,}3 = 2T + \tau \\ &m = 1 - \frac{0{,}3}{1} \\ &H_0G_p(z) = z^{-T}(1 - z^{-1}) Z_m\left\{\frac{G(S)}{S}\right\} \end{aligned}}$$

$Y(S) = H_0(S) G_p(S) \mu^*(S)$

$E_1(S) = K_2 \left[K_1 R(S) - H_0(S) G_p(S) \mu^*(S) \right]$

$$H_0G_p(S) = (1 - z^{-1}) Z\left\{\frac{2}{S(S+2)} e^{-0{,}5S}\right\}$$

$$H_0G_p(z) = \frac{0{,}64 z + 0{,}225}{z(z - 0{,}135)}$$

$$G(z) = \frac{z}{(z-0{,}2)(z-0{,}5)}$$

$x(k) = 2 \sin(kwT)$

$w = \pi/2$ rad/s

$T = 1$ s

$$|G(e^{jwT})| = \frac{|e^{jwT}|}{|e^{jwT} - 0{,}2|\,|e^{jwT} - 0{,}5|}$$

$$\frac{1}{\sqrt{\left[(\cos wT - 0{,}2)^2 + \sin(wT)^2\right]\left[(\cos(wT) - 0{,}5)^2 + (\sin(wT))^2\right]}}\Bigg|_{\substack{w = \pi/2 \\ T = 1}} = 0{,}88$$

$$\measuredangle G(e^{jwT}) = \measuredangle(e^{jwT}) - \measuredangle(e^{jwT} - 0{,}2) - \measuredangle(e^{jwT} - 0{,}5) =$$

$$= wT - \left[\operatorname{tg}^{-1}\left(\frac{\sin(wT)}{\cos(wT) - 0{,}2}\right) + \pi\right] - \left[\operatorname{tg}^{-1}\left(\frac{\sin(wT)}{\cos(wT) - 0{,}5}\right) + \pi\right]\Bigg|_{\substack{w = \pi/2 \\ T = 1}} -2{,}23 \text{ rad/s}$$

$$Y_n(k) = 2 \cdot 0{,}88 \sin(k\pi/2 - 2{,}23)$$

ingresso durata limitata

asintotica stabilità:

- uscita converge 0 per $K \to +\infty$

semplicemente stabile:

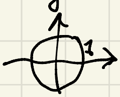
- uscita limitata in y per $K \to +\infty$

instabile

- uscita diverge per $K \to +\infty$

stabilità BIBO $\iff$ ingressi in y limitati uscita

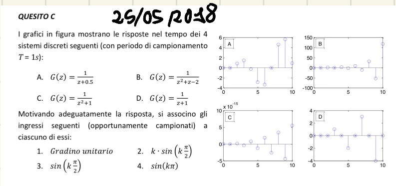
$$G(z) = \frac{K(z - z_i)\ldots}{(z - p_j)\ldots}$$



$|P| < 1$ uscita convergere

$|P| > 1$ uscita diverge

$|P|=1$ uscita limitata in y



QUESITO C — 25/05/2018

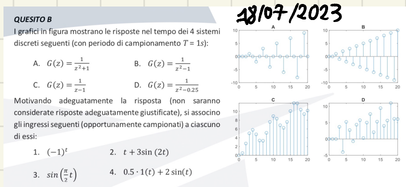
I grafici in figura mostrano le risposte nel tempo dei 4 sistemi discreti seguenti (con periodo di campionamento $T = 1s$):

A. $G(z)=\frac{1}{z+0.5}$ B. $G(z)=\frac{1}{z^2+z-2}$

C. $G(z)=\frac{1}{z^2+1}$ D. $G(z)=\frac{1}{z+1}$

Motivando adeguatamente la risposta, si associno gli ingressi seguenti (opportunamente campionati) a ciascuno di essi:

1. Gradino unitario 2. $k\cdot \sin\left(k\frac{\pi}{2}\right)$
3. $\sin\left(k\frac{\pi}{2}\right)$ 4. $\sin(k\pi)$



QUESITO B — 18/07/2023

I grafici in figura mostrano le risposte nel tempo dei 4 sistemi discreti seguenti (con periodo di campionamento $T = 1s$):

A. $G(z)=\frac{1}{z^2+1}$ B. $G(z)=\frac{1}{z^2-1}$

C. $G(z)=\frac{1}{z-1}$ D. $G(z)=\frac{1}{z^2-0.25}$

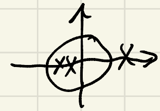
Motivando adeguatamente la risposta (non saranno considerate risposte adeguatamente giustificate), si associno gli ingressi seguenti (opportunamente campionati) a ciascuno di essi:

1. $(-1)^t$ 2. $t+3\sin(2t)$
3. $\sin\left(\frac{\pi}{2}t\right)$ 4. $0.5\cdot 1(t)+2\sin(t)$

$P(z)=z^3+z^2+1$ NO $\rightarrow G(z)=K\dfrac{\dots}{\dots}$

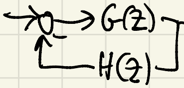
$P\left(z=\frac{w+1}{w-1}\right)=4w^3+2w^2+4w-2=0$

3	4	4	0
2	2	-2	0
1	4	0	
0	-2		



$A\rightarrow 2$ $B\rightarrow 4$
$C\rightarrow 3$ $D\rightarrow 1$

$D\rightarrow 2$
$B\rightarrow 1$
$A\rightarrow 3$
$C\rightarrow 4$



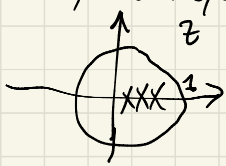
$G(z)=\dfrac{z+1}{z^2+1{,}4z+0{,}7}$ $H(z)=\dfrac{1}{z-1{,}3}$

$G_0(z)=\dfrac{G}{1+GH}$ prendo questo

$P(z)=z^3+0{,}1z^2-0{,}12z+0{,}09=0$

$P\left(\frac{w+1}{w-1}\right)=1{,}07w^3+2{,}95w^2+3{,}29w+0{,}69=0$

3	1,07	3,29
2	2,95	0,69
1	3,04	0
0	0,69	0



criterio di Jury → asintoticamente stabile / non asin. stabile / utile per soluzioni elevati

$P(z)=a_0z^n+\dots+a_n$ no $G(z)=K\dfrac{\dots}{\dots}$

$a_0>0$

1. $P(1)>0$
2. $(-1)^nP(-1)>0$
3. $a_0>|a_n|$

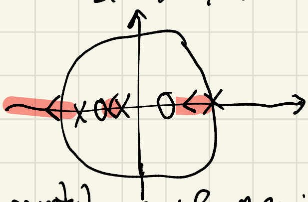
$b_{n-1}=\begin{vmatrix}a_n & a_0\\ a_0 & a_n\end{vmatrix}$ $b_{n-2}=\begin{vmatrix}a_n & a_1\\ a_0 & a_{n-1}\end{vmatrix}$

	z^0	z^1	z^2	…	z^{n-2}	z^{n-1}	z^n
1	a_n	a_{n-1}	a_{n-2}	…	a_2	a_1	a_0
2	a_0	a_1	a_2	…	a_{n-2}	a_{n-1}	a_n
3	b_{n-1}	b_{n-2}	b_{n-3}	…	b_1	b_0	0
4	b_0	b_1	b_2	…	b_{n-1}	b_{n-2}	0
5							
6							

$c_{n-2}=\begin{vmatrix}b_{n-1} & b_0\\ b_0 & b_{n-1}\end{vmatrix}$ $c_{n-3}=\begin{vmatrix}b_{n-1} & b_1\\ b_0 & b_{n-2}\end{vmatrix}$

4. $|b_{n-1}|>|b_0|$
5. $|c_{n-2}|>|c_0|$

$G(z)=\dfrac{K(z^2-0{,}25)}{z(z-1)(z+0{,}75)}$



asintoti = n. poli − n. zeri

$|K(z)|=\dfrac{1}{|G(z)|}=\dfrac{1}{|G(-1)|}=2/3$

$K>2/3$ instabile
$K=2/3$ sempl. stabile
$K<2/3$ asint. stabile

$G(z)=\dfrac{K(z-0{,}5)}{(z-1)(z-0{,}8)}$

$K(1)=\left|\dfrac{1}{G(1)}\right|=2{,}4$

centroide $=\dfrac{\sum p_i-\sum z_i}{n-m}=$

$=\dfrac{-2+0{,}5-0{,}2+j0{,}5-0{,}2-j0{,}5}{3-1}=0{,}95$

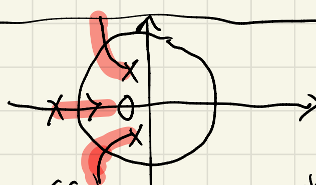
asintoti del centroide $=\dfrac{(2\nu+1)\pi}{n-m}=\pi/2,\ -\pi/2$

centroide $=\dfrac{2\nu\pi}{n-m}$

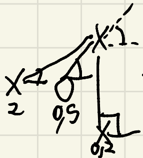
$K(z)=\dfrac{1}{|G(z)|}$ $\dfrac{\partial K}{\partial z}=0$

$z=\begin{cases}0{,}014\\ -0{,}98\pm j0{,}51\end{cases}$ $\notin$ asse reale di LdR^+

$G(z)=\dfrac{K(z+0{,}5)}{(z^2+0{,}4z+0{,}29)(z+2)}$



$\varphi_{p_i}=(2\nu+1)\pi+\sum(p_i-z_i)-\sum(p_i-p_j)$



$180+\sin^{-1}\left(\dfrac{0{,}5}{\sqrt{0{,}5^2+0{,}3^2}}\right)-\sin^{-1}\left(\dfrac{0{,}5}{\sqrt{0{,}5^2+1{,}8^2}}\right)-90=133{,}5°$

$$K=\left|\frac{1}{G(-1)}\right| = 1{,}78$$

$$1{,}78 = \frac{1}{|G(z)|}$$

$$\begin{cases} P(z) = (z^2+0{,}4z+0{,}29)(z+2)+K(z+0{,}5) \\ K = 1{,}78 \end{cases}$$

$$P(z) = z^3+2{,}4z^2+2{,}87z+1{,}47$$

$$\begin{array}{l|l} z^3+2{,}4z^2+2{,}87z+1{,}47 & z+1 \\ z^3+z^2 & z^2+1{,}4z+1{,}47 \\ // \quad 1{,}4z^2+2{,}87z+1{,}47 & \\ \quad 1{,}4z^2+1{,}4z & \\ \quad // \quad 1{,}47z+1{,}47 & \\ \quad\quad 1{,}47z+1{,}47 & \\ \quad\quad // \quad // & \end{array}$$

$z^2+1{,}4z+1{,}47 \rightarrow -0{,}7 \pm 0{,}99i$ instabili

$$G(z) = \frac{2}{z^2(z^2-\sqrt{3}z+1)}$$

$x^2=0$

$x_{1,2} = \pm\frac{\sqrt{3}}{2} \pm j0{,}5$

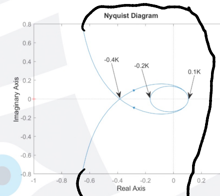
$|x_{1,2}| = 1$

criterio di Nyquist

$N_{giri} = P - Z$ instabili $\quad N<0$

6. *Sia assegnato il diagramma polare in figura, corrispondente ad un sistema tempo-discreto privo di zeri e con 2 poli di modulo inferiore a uno, un polo di modulo unitario, un polo di modulo superiore a uno. Supponendo di chiudere il sistema in retroazione unitaria ed assumendo un guadagno K variabile reale, si indichino le due affermazioni vere.*

A	*Per K=0.4 il sistema è semplicemente stabile*
B	*Per K<0 il sistema è asintoticamente stabile*
C	*Il sistema è instabile per qualsiasi valore di K*
D	*Il sistema è asintoticamente stabile per qualsiasi valore di K*
E	*Per 0<K<2.5 il sistema è asintoticamente stabile*
F	*Per 2.5<K<5 il sistema presenta 3 poli fuori dal cerchio unitario*
G	*Per K>5 il sistema presenta 3 poli fuori dal cerchio unitario*
H	*Per -10<K<0 il sistema è asintoticamente stabile*
I	*Solo una delle precedenti affermazioni è vera*



instabile 1 polo

$0 < K < 1/0{,}4$

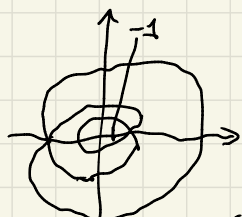
$N = P - Z \quad Z = 1$ poli 3 stabili, 1 instabile
$\;0 \quad\;\; 1$

$-0{,}4K < -1 < -0{,}2K$

$N = P - Z \quad Z = 3$ poli instabili: 3, stabili: 1
$-2 \quad +1$

$-0{,}2K < 1$

$N = P - Z \Rightarrow Z = 4$ poli tutti instabili
$-3 \quad\;\; 1$



$0 < -1 < +0{,}1K$

$N = P - Z \quad Z = 4$
$-3 \quad\;\; 1$

$-1 > 0{,}1K$

$N = P - Z$
$-1 \quad\;\; 1$

$Z = 2$

$$M_F = \pi + \measuredangle G(e^{j\omega_c T})$$

$$M_A = \frac{1}{|G(e^{j\omega_c T})|}$$

$$S\% = \frac{y(t_P)-y(\infty)}{y(\infty)} \cdot 100 = 100\, e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}$$

$\delta = \cos\alpha$

$-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$

$r(k)=1 \qquad R(z) = \frac{z}{z-1}$ errore di posizione e_p

$r(k)=kT \qquad R(z) = \frac{Tz}{(z-1)^2}$ // di velocità e_v

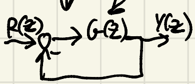
$r(k) = \frac{k^2T^2}{2} \qquad R(z) = \frac{T^2z(z+1)}{2(z-1)^3}$ // di accelerazione e_a

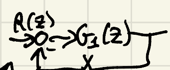
tipo	e_p	e_v	e_a
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$1/K_v$	∞
2	0	0	$1/K_a$

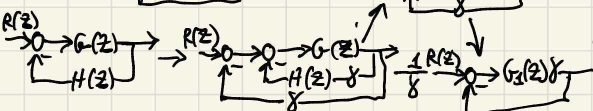
$K_p = \lim_{z\to1} G(z)$

$K_v = \lim_{z\to1} \frac{(z-1)}{T} G(z)$

$K_a = \lim_{z\to1} \frac{(z-1)^2}{T^2} G(z)$







$G_E(z) = \gamma G_1(z)$

$e_p = \frac{1}{1+K_p} \qquad K_p = \lim_{z\to1} G_E(z)$

$e_v = \frac{1}{K_v} \qquad K_v = \lim_{z\to1} \frac{z-1}{T} G_E(z)$

$e_a = \frac{1}{K_a} \qquad K_a = \lim_{z\to1} \frac{(z-1)^2}{T^2} G_E(z)$

risposta $\rightarrow$ 1° ordine
$t_{a\,2\%} = 4\tau,\ t_{a\,5\%} = 3\tau,\ t_{a\,1\%} = 4{,}6\tau$

$\rightarrow$ 2° ordine

$t_{a\,1\%} = \frac{4{,}6}{\delta\omega_n} \qquad t_{a(0-100\%)} = \frac{\pi-\alpha}{\omega_d}$

$t_{a\,2\%} = \frac{4}{\delta\omega_n} \qquad t_p = \frac{\pi}{\omega_d}$

$t_{a\,5\%} = \frac{3}{\delta\omega_n}$

2° ordine

$t_r = 0{,}4\,\omega_B$

$r \to \bigcirc \to D(z) \to \bigcirc \xleftarrow{d_1(z)} \to HG(z) \to \bigcirc \xleftarrow{d_2(z)} \to y(z)$, retroazione con $d_3(z)$

$Y_{d1}(z) = \dfrac{HG(z)}{1+D(z)HG(z)}\, d_1(z) \Leftarrow d_3, d_1, r=0$

$Y(z) \neq 0$

$Y_{d3}(z) = \dfrac{-D(z)HG(z)}{1+D(z)HG(z)}\, d_3(z) \Leftarrow d_2, d_1, r=0$

$\to \bigcirc \to D(z)\,G_p(z) \to$ (retroazione unitaria)

$G_p(z) = \dfrac{z+0{,}1}{z(z-1)}$

$D(z) = K$

$e_p = 0$

$e_v = 1/K_v = \dfrac{1}{\lim_{z\to 1} \frac{z-1}{T} K \frac{z+0{,}1}{z-1} = \frac{1{,}1K}{T}}$

$e_a = \infty$

$\to \bigcirc \to G \to$, retroazione H

$G_0 = \dfrac{G}{1+GH}$

$a \in G$ $\quad a \in H$

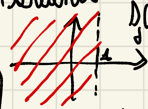
$S_a^{G_0} = \dfrac{a}{G(1+GH)}\dfrac{\partial G}{\partial a} \qquad = -a G_0 \dfrac{\partial H}{\partial a}$

$H_0 = \dfrac{1}{1+\frac{TS}{2}}$ con padé

1 metodo delle differenze all'indietro $D_d\left(s = \dfrac{z-1}{zT}\right)$

stabile

2 metodo delle differenze in avanti $D_d\left(s = \dfrac{z-1}{T}\right)$



non stabile

3 metodo dei trapezi (trasformazione di Tustin)

$D_d\left(s = \dfrac{2}{T}\dfrac{z-1}{z+1}\right)$

$\to \bigcirc \to G_1(z) \to \bigcirc \xleftarrow{d_1(z)} \to G_2(z) \to \bigcirc \xleftarrow{d_2(z)} \to$ (retroazione unitaria)

$G_1(z) = \dfrac{3}{z-1} \quad G_2(z) = \dfrac{z-0{,}25}{z-1} \quad T = 0{,}5$

$P(z) = (z+0{,}5)^2$ asintoticamente stabile

G_1G_2 = tipo 2

$e_p = e_v = 0$

$e_a = \dfrac{1}{\lim_{z\to1} \frac{(z-1)^2}{T^2}\frac{3(z-0{,}25)}{(z-1)^2}} = \dfrac{1}{9}$

d_2 = gradino $\quad Y_{d2} = 0$

d_2 = rampa $\quad Y_{d2} = 0$

d_1 = gradino $\quad Y_{d1} = 0$

d_1 = rampa $\quad Y_{d1} =$

$\lim_{z\to1} \dfrac{(z-1)}{1}\dfrac{G_2(z)}{1+G_1(z)G_2(z)}\dfrac{z}{(z-1)^2} = 1/6$

$\to \bigcirc \to D_c(s) \to \dfrac{1}{1+\frac{TS}{2}} \to \dfrac{1}{S(S+2)} \to$ (retroazione unitaria)

$e_p = 0 \quad \to \bigcirc \to D_c(z) H_0(s) G_p(s) \to$

$S\% = 17\%$

$t_a(1\%) \leq 2s$

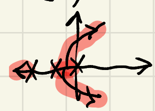
$G_p(S) = \dfrac{K}{S(S+2)}$

$17 = 100\, e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}}$

$\delta = 0{,}5$

$\dfrac{4}{\delta\omega_n} = 2 \quad \delta\omega_n = 2$

$-2 \pm j2\sqrt{3}$



---

invarianza della risposta all'impulso

$Z(D_c(s)) = D_d(z)$

invarianza della risposta al gradino

$D_d(z) = (1-z^{-1})\, Z\left\{\dfrac{D_c(s)}{s}\right\}$

trasformate poli/zeri

$D_c(s) = \dfrac{K(s-z_i)(s^2+2\delta\omega_n s+\omega_n^2)}{s^h(s-p_i)(s^2+2\delta\omega_n s+\omega_n^2)}$

$D_d(z) = \dfrac{K_d(z-e^{z_iT})(z^2-2e^{-\delta\omega_n T}\cdot 2\cdot \cos(\omega_d T)+e^{-2\delta\omega_n T})}{(z-1)^h(z-e^{p_iT})(\quad // \quad)}$

$\lim_{s\to0} D_c(s) = \lim_{z\to1} (z-1)^h D_d(z)$

---

1 ω_b banda passante

$\omega_s = 4 \div 10 \div 20\,\omega_b$

$\omega_b \cong \dfrac{2{,}5}{t_r} \cong 14{,}14$ rad/s

$t_r(0-100\%) = \dfrac{\pi-\alpha}{\omega_d} = 0{,}605$

$\delta = \cos\alpha$

da a da a

$T = \dfrac{2\pi}{\omega_s} = (0{,}076 : 0{,}152 : 0{,}38)\,s$

2 $\tau = 0{,}5$

$T = \dfrac{0{,}5}{2\div10} = (0{,}05 \div 0{,}25)\,s$

3 $T = \dfrac{\frac{2\pi}{2\sqrt3}}{8\div10} = (0{,}181 \div 0{,}227)\,s$

1 2 3 non sempre applicabili

$T = 0{,}2$

$G(s) = \dfrac{10K}{S(S+2)(S+10)}$

$D_c(S) = 1$

$D_c = \dfrac{(S+2)}{S+a} \qquad G''(S) = \dfrac{K'}{S(S+a)(S+10)} \qquad K' = 10K$

$K = \dfrac{1}{G''(S)} = S^3 + (10+a)S^2 + 10aS + K'$

$S^3 + 4S^2 + 16S$

$// \quad (6+a)S^2 + (10a-16)S + K'$

$(6+a)S^2 + (24+4a)S + 16(6+a)$

$// \quad (6a-40)S - 96 - 16a + K'$

$S^2+4S+16$ | $S+6+a$

$\begin{cases} 6a - 40 = 0 \\ -96 - 16a + K' = 0 \end{cases}$

$K' = 202$

$a = 6{,}67$

$D_c(S)=\frac{S+2}{S+6{,}67}$ $\quad T=0{,}2$

$\rightarrow \bigcirc \rightarrow D_c(S) \rightarrow \frac{1}{1+\frac{TS}{2}} \rightarrow G_p(S) \rightarrow$

① differenze in avanti

$D_d(z)=D_c(S)\big|_{S=5(z-1)} = \frac{1-0{,}6z^{-1}}{1+0{,}334z^{-1}} = \frac{u(z)}{e(z)}$

$u(k)=-0{,}334\,u(k-1)+e(k)-0{,}66\,e(k-1)$

② differenze all'indietro

$D_d(z)=D_c(S)\big|_{S=\frac{5(z-1)}{z}} = \frac{0{,}599z-0{,}428}{z-0{,}429}$

$u(k)=0{,}428\,u(k-1)+0{,}599\,e(k)-0{,}428\,e(k-1)$

③ metodo di Tustin

$D_d(z)=D_c(S)\big|_{S=\frac{2}{T}\frac{z-1}{z+1}} = \frac{0{,}7199z-0{,}4799}{z-0{,}1998}$

$u(k)=0{,}1998\,u(k-1)+0{,}7199\,e(k)-0{,}4799\,e(k-1)$

④ Tustin precompensato

$\frac{S+2}{S+6{,}67}=K\frac{(1+\tau S)}{1+\alpha\tau S}$

$\omega=\frac{1}{\tau\sqrt{\alpha}}=3{,}65\,rad/s$

$\tau=0{,}5,\ \alpha=0{,}3$

$D_d(z)=D_c(S)\big|_{S=\frac{\omega}{tg\frac{\omega T}{2}}\frac{z-1}{z+1}} =$

$= \frac{0{,}712z-0{,}465}{z-0{,}177}$

$u(k)=0{,}177\,u(k-1)+0{,}712\,e(k)-0{,}465\,e(k-1)$

⑤ invarianza risposta all'impulso

$D_d(z)=\mathcal{Z}\{D_c(S)\}=\mathcal{Z}\{\delta(t)-4{,}67e^{-6{,}67t}\}$

⑥ invarianza risposta a gradino

$D_d(z)=(1-z^{-1})\mathcal{Z}\left\{\frac{D_c(S)}{S}\right\}=\frac{z-0{,}779}{z-0{,}163}$

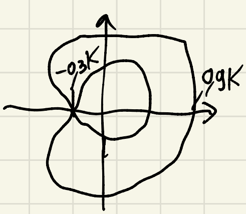
⑦ polo/zero

$D_d(z)=K_d\frac{(z-e^{-2T})}{(z-e^{-6{,}67T})}$

$K_d=\lim_{S\to 0}\underbrace{\frac{S+2}{S+6{,}67}}_{0{,}29}=\lim_{z\to 1}\underbrace{\left(\frac{z-e^{-2T}}{z-e^{-6{,}67T}}\right)}_{0{,}44}\cdot 9{,}66$

$D_d(z)=0{,}669\,\frac{z-0{,}67}{z-0{,}265}$

$u(k)=0{,}263\,u(k-1)+0{,}669\,e(k)-0{,}449\,e(k-1)$



$-1<0{,}3K$ asin. stabile

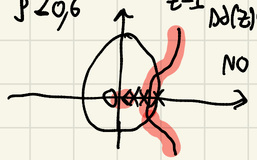
$-1>-0{,}3K$ instabile

$-1=-0{,}3K$ semplic. stabile

$G_p(z)=\frac{z+0{,}5}{(z-0{,}7)(z-0{,}2)}$ $\quad \rightarrow \bigcirc \rightarrow D_d(z)G_p(z) \rightarrow$

$e_p=0$

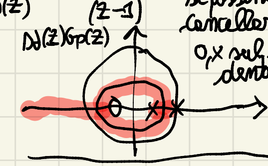
$\rho<0{,}6$ $\quad D_d(z)=\frac{K}{z-1}$ $\quad D_d(z)G_p(z)$



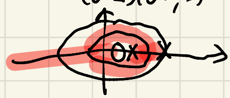
NO

$D_d(z)=\frac{K(z-0{,}2)}{(z-1)}$ non si possono cancellare 0,x sul/dentro il cerchio

$D_d(z)G_p(z)$



$D_d(z)=\frac{K(z-0{,}2)(z-a)}{(z-1)(z+0{,}5)}$



$K(z)=\frac{1}{|G_p(z)D_d(z)|}=0{,}9$

$\frac{dK(z)}{dz}=0$

$z^2-az+1{,}7a-0{,}7=0$

$z_{1/2}=a\pm\sqrt{a^2-1{,}7a+0{,}7}$

$a<0{,}7 \vee a>1$

$a=0{,}6$

$D_d=\frac{0{,}9(z-0{,}2)(z-0{,}6)}{(z+0{,}5)(z-1)}$

$\rightarrow \bigcirc \rightarrow D(z) \rightarrow G_p(z) \rightarrow$ $\quad$ dead beat #1

$G_p(z)=\frac{z+0{,}8}{z^2+0{,}7z+0{,}1}$ dentro il cerchio

$G_m=z^{-1}\Rightarrow n=1$

$D(z)=\frac{1}{G_p(z)}\frac{1}{z^n-1}=\frac{z^2+0{,}7z+0{,}1}{(z+0{,}8)(z-1)}$

$H_0G_p(z)$ $\quad$ dead beat #2

$G_p(z)=\frac{z^{-2}(0{,}143+0{,}109z^{-1})}{1-0{,}741z^{-1}}$

$\sum p_i=0{,}143+0{,}109=0{,}25$

$G_p(z)=\frac{[z^{-2}(0{,}143+0{,}109z^{-1})]/0{,}258}{(1-0{,}741z^{-1})/0{,}258}=\frac{z^{-2}(0{,}577+0{,}423z^{-1})}{3{,}86-2{,}86z^{-1}}$

$D(z)=\frac{3{,}86-2{,}86z^{-1}}{1-z^{-2}(0{,}577+0{,}423z^{-1})}$

$G(S)=K_P+\frac{K_I}{S}+K_DS=K_P\left(1+\frac{1}{T_iS}+T_DS\right)$

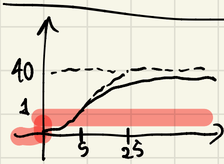
differenze in avanti

$G(S)\big|_{S=\frac{z-1}{T}}\Rightarrow D(z)=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{1-z^{-1}}=\frac{U}{E}$

differenze all'indietro

$$G(S)\Big|_{S=\frac{T(z-1)}{z}} = K_P\left(1+\frac{\alpha}{z-1}+\beta\frac{(z-1)}{z-\gamma}\right) = \frac{\mu}{E} = \frac{q_0+q_1z^{-1}+q_2z^{-2}}{(1-z^{-1})(1-\gamma z^{-1})}$$

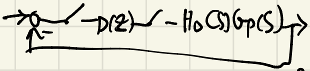
$$\alpha = \frac{T}{T_I} \quad \beta = \frac{NT_D}{T_D+NT} \quad \gamma = \frac{T_D}{T_D+NT}$$



$\theta = 5-0=5$
$\tau = 25-5=20$

$$G_p = 40\frac{e^{-5S}}{1+20S}$$

$K=40$



$$T=\frac{\tau}{10}=2,1$$

$$\theta' = \theta+\frac{T}{2} = 6,1$$

$$P \to K_p = \frac{\tau}{K\theta'} = 0,083$$

PID → $K_p = 0,1$
$T_I = 12,1$
$T_D = 3,1$

PI → $K_p = 0,075$
$T_I = 19,81$

$$D(z) = \frac{q_0+q_1z^{-1}+q_2z^{-2}}{1-z^{-1}}$$

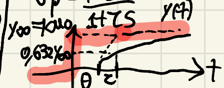
$$q_0 = \left(1+\frac{T_D}{T}\right)K_p = 0,075$$

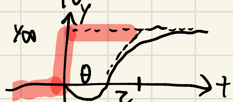
$$q_1 = \left(-1+\frac{T}{T_I}-2\frac{T_D}{T}\right)K_p = -0,067$$

$$q_2 = K_D\frac{T_D}{T} = 0$$

1° ordine con ritardo

$$G_p = K\frac{e^{-\theta S}}{1+\tau S}$$





Ziegler-Nichols in anello aperto

	K_p	T_I	T_D
P	$\frac{\tau}{K\theta}$	X	X
PI	$0,9\frac{\tau}{K\theta}$	$3,3\theta$	X
PID	$1,2\frac{\tau}{K\theta}$	2θ	$0,5\theta$

Z-N in anello chiuso

K_p	T_I	T_D
$0,5K_C$	X	X
$0,45K_C$	$0,85P_C$	X
$0,6K_C$	$0,5K_C$	$0,125K_C$

$$ISE = \int_0^\infty e^2(t)\,dt$$

$$ITSE = \int_0^\infty e(t)^2 t\,dt$$

$$IAE = \int_0^\infty |e(t)|\,dt$$

$$ITAE = \int_0^\infty |e(t)|\,t\,dt$$

$$Y = A+B\left(\frac{\theta}{\tau}\right)$$

$$Y = A+B\left(\frac{\theta}{\tau}\right)$$

$$Y = KK_P$$

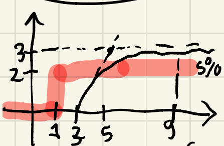
$$Y = \frac{\tau}{T_I} \quad Y = \frac{T_D}{\tau}$$

$$G_p = 40\frac{e^{-5S}}{1+20S}$$

$\tau = 20$
$\theta = 6$

disturbo sull'uscita ⇒ Δ di carico

ottimizzazione nella risposta ⇒ Δ di setpoint



$\tau = 5-3=2$
$\theta = 3-1=2$
$K = \frac{3}{2} = 1,5$

$$G_p = 1,5\frac{e^{-2S}}{1+2S}$$

$$\frac{9-3}{3} = 2,1$$

IAE

$$Y_p = A\left(\frac{\theta}{\tau}\right)^B = 4,35$$

$$K_p = \frac{Y_p}{K} = 0,109$$

$$Y_I = 0,878\left(\frac{6}{20}\right)^{-0,749}$$

$$T_I = \frac{\tau}{Y_I} = 9,45$$

$$Y_D = 0,482\,(6/20)^{1,137}$$

$$T_D = \tau Y_D$$

$$D(z) = K_p\left(1+\frac{\alpha}{z-1}+\beta\frac{z-1}{z-\gamma}\right) = \frac{\mu}{E}$$

$$\alpha = \frac{T}{T_I} = \frac{2}{9,45} = 0,216$$

$$\beta = \frac{NT_D}{NT+T_D} = \frac{10 \cdot 2,452}{20+2,452} = 1,1$$

$$\gamma = \frac{T_D}{NT+T_D} = 0,11$$

$$\mu(k) = 0,228\,e(k) - 0,335\,e(k-1) + 0,128\,e(k-2) + 1,109\,\mu(k-1) - 0,109\,\mu(k-2)$$

regione lineare

errore (controllo) > 0
integratore accumula
↓
azione integrale ↑↑↑
↓
wind up

soluzione
↓
bloccare integratore
(se ∃ un saturazione)