# A cascaded supervised learning approach to inverse reinforcement learning

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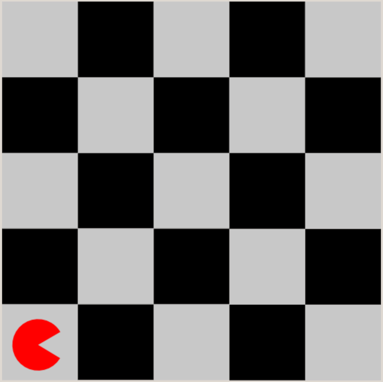
# Imitation: Expert



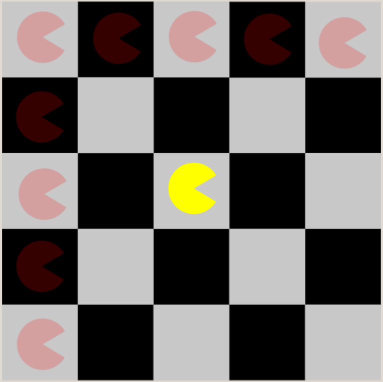
## Expert

- The expert is an optimal agent in an MDP
- Its behavior is observed

## Apprenticeship learning

Reward inference

# Imitation: Expert



### Expert

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### Apprenticeship learning

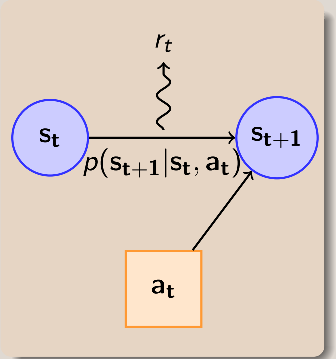
Reward inference

# Contribution

## CSI

- CSI Algorithm
  - Classification step. . .
  - . . . followed by a regression step that introduces the temporal structure of the MDP
  - Only needs data from the expert (if we use the heuristics)
  - Can use other data if available
  - Able to use off-the-shelf components
- Theoretical results
- Experimental results

# Quick definitions




## Notions

- State $s_t \in \mathcal{S}$
- Action $a_t \in \mathcal{A}$
- Reward $r_t = R(s_t) \in \mathbb{R}$
- Transition $(s_t, a_t, s_{t+1}, r_t) \in \mathcal{S} \times \mathcal{A} \times \mathcal{S} \times \mathbb{R}$
- $\pi : \mathcal{S} \to \mathcal{A}$

## Markovian criterion

Past states are irrelevant

# RL problem and solution

## Value function

$$V_R^\pi(s) = E\left[\sum_{t\geq 0}\gamma^t R(s_t)\middle| s_0 = s, \pi\right] \quad (1)$$

## Goal

Optimal policy $\pi_R^* = \arg\max_\pi V_R^\pi$ $\pi_R^*(s) = \arg\max_a Q_R^{\pi^*}(s, a)$

# IRL problem

## Goal

Finding the reward $R$ so that the observed behavior is optimal

## Ill-posed

The null reward $\forall s, R(s) = 0$ is a solution

## Existing solutions

- Most algorithms follow (Abbeel and Ng, 2004), they need to repeatedly solve the MDP
- Most others need to know the transition probabilities $p$
- The two least data greedy algorithms are :
  - RelEnt from (Boularias et al, 2011)
  - SCIRL

# A certain class of classifiers

## Score function based classifiers

- Classifier: map inputs $s \in \mathcal{S}$ to labels $a \in \mathcal{A}$
- Data: $D_{sa}^{\pi_E} = \{(s_i, a_i)_{1 \leq i \leq N}\}$
- Decision rule : $\pi^C \in \mathcal{A}^{\mathcal{S}}$
- Score function : $\pi^C(s) \in \arg\max_{a \in \mathcal{A}} q(s, a)$
- Very few exceptions (*e.g.* decision trees)

# The idea behind CSI

## Score function based classifiers

$$\pi^C(s) \in \arg\max_{a \in \mathcal{A}} q(s, a)$$

## Expert policy

$$\pi_E(s) = \arg\max_a Q^{\pi_E}(s, a)$$

## Bellman Equation for the expert

$$Q^{\pi_E}_{R^E}(s, a) = R^E(s, a) + \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s, a\right) Q^{\pi_E}_{R^E}(s', \pi_E(s')) \tag{3}$$

# The idea behind CSI

### Score function based classifiers

$$\pi^C(s) \in \arg\max_{a \in \mathcal{A}} q(s,a)$$

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$$\pi_E(s) = \arg\max_a Q^{\pi_E}(s,a)$$

### Bellman Equation for the expert

$$R^E(s,a) = Q^{\pi_E}_{R^E}(s,a) - \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s,a\right) Q^{\pi_E}_{R^E}(s', \pi_E(s')) \tag{3}$$

# The idea behind CSI

## We view $q$ as a quality function

$$R^C(s,a) = q(s,a) - \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s,a\right) q(s', \pi^C(s')) \tag{2}$$

$\pi^C$ is optimal for $R^C$ and $\pi^C \approx \pi_E$, ergo we would be happy to find $R^C$.

## Score function based classifiers

$$\pi^C(s) \in \arg\max_{a \in \mathcal{A}} q(s,a)$$

## Expert policy

$$\pi_E(s) = \arg\max_a Q^{\pi_E}(s,a)$$

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$$R^E(s,a) = Q^{\pi_E}_{R^E}(s,a) - \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s,a\right) Q^{\pi_E}_{R^E}(s', \pi_E(s')) \tag{3}$$

# The idea behind CSI

**After a classifier has learned a score function $q$**

$$R^C(s,a) = q(s,a) - \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s,a\right) q(s', \pi^C(s')) \tag{4}$$

**Non expert data**

$$D_{sas}^{\sim} = \{s_i, a_i, s_i'\}_{0 \leq i \leq N}. \tag{5}$$

**Sampled version of Eq. 7**

$$\hat{r}_i = q(s_i, a_i) - \gamma q(s_i', \pi^C(s_i')). \tag{6}$$

# CSI Pseudo-code

**Algorithm 1:** CSI algorithm

***Given*** a training set $D_{sa}^{\pi_E} = \{(s_i, a_i = \pi_E(s_i))\}_{1 \leq i \leq N}$
and another training set $D_{sas}^{\sim} = \{(s_j, a_j, s'_j)\}_{1 \leq j \leq N'}$;
***Train*** a score function-based classifier on $D_{sa}^{\pi_E}$, obtaining decision rule $\pi^C$
and score function $q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$;
***Learn*** a reward function $\hat{R}^C$ from the dataset $\{((s_j, a_j), \hat{r}_j)\}_{1 \leq j \leq N'}$,
$\forall (s_j, a_j, s'_j) \in D_{sas}^{\sim}, \hat{r}_j = q(s_j, a_j) - \gamma q(s'_j, \pi_C(s'_j))$;
***Output*** the reward function $\hat{R}^C$ ;

# Heuristics

## After a classifier has learned a score function $q$

$$R^C(s,a) = q(s,a) - \gamma \sum_{s' \in \mathcal{S}} p\left(s' \middle| s,a\right) q(s', \pi^C(s')) \tag{7}$$

## Non expert data

$$\cancel{D_{sas}^{\sim} = \{s_i, a_i, s_i'\}_{0 \le i \le N}} \tag{8}$$

## Expert data

$$D_{sas}^{\pi_E} = \{(s_i, a_i, s_i')_{1 \le i \le N}\}$$

## Sampled version of Eq. 7

$$(s_i, \pi_E(s_i)), \hat{r}_i = q(s_i, \pi_E(s_i)) - \gamma q(s_i', \pi^C(s_i')). \tag{9}$$

## Heuristics

$$(s_i, \forall a \neq \pi_E(s_i)), \hat{r}_{min} = \min_{i \in [\![1;N]\!]} \hat{r}_i - 1. \tag{10}$$

# Error bound

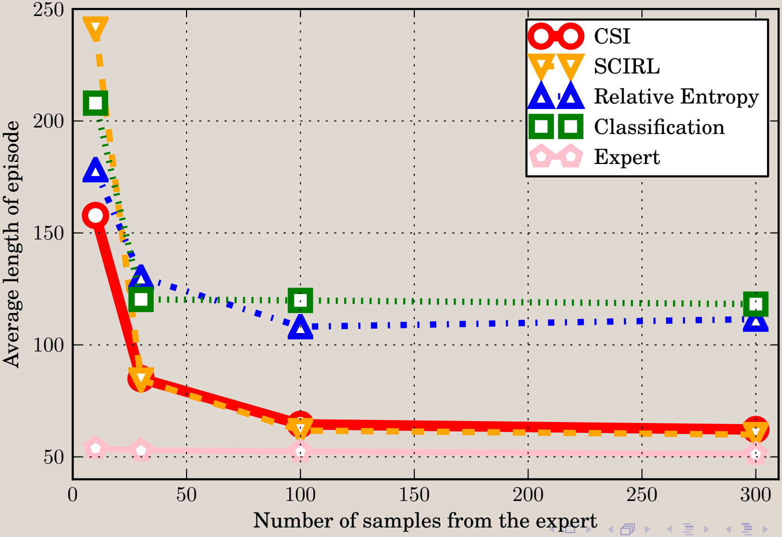
## Theorem

$$0 \leq \mathbf{E}\left[ V^{\pi^*_{\hat{R}^C}}_{\hat{R}^C}(s) - V^{\pi_E}_{\hat{R}^C}(s) \middle| s \sim \rho_E \right] \leq \frac{1}{1-\gamma}\left( \epsilon_C \Delta q + \epsilon_R (1 + C_{\pi^*_{\hat{R}^C}}) \right). \quad (11)$$

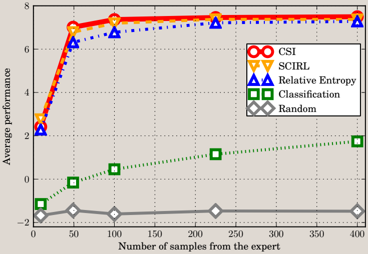
## Notation

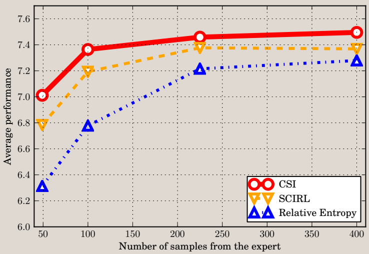
- $\hat{R}^C$: Reward learned by CSI
- $\rho_E$: Expert distribution
- $\epsilon_C$ : classification error
- $\epsilon_R$ : regression error
- $C_{\pi^*_{\hat{R}^C}}$: concentration coefficient

# Results on the mountain car



# Results on the driving problem




## Description

- Widespread benchmark
- Goal of the expert : avoid other cars, do not go off-road, go fast
- Using only data from the expert and natural features

# Possible future work

## CSI

- A theoretically sound, empirically promising new IRL algorithm.
- Can use most off-the-shelf classifiers and any off-the-shelf regressor
- Favorably compares to the most efficient existing approaches

## Real world problems

The difficult part is solving the MDP once the reward has been found by CSI

# Thank you. . .

. . . for your attention