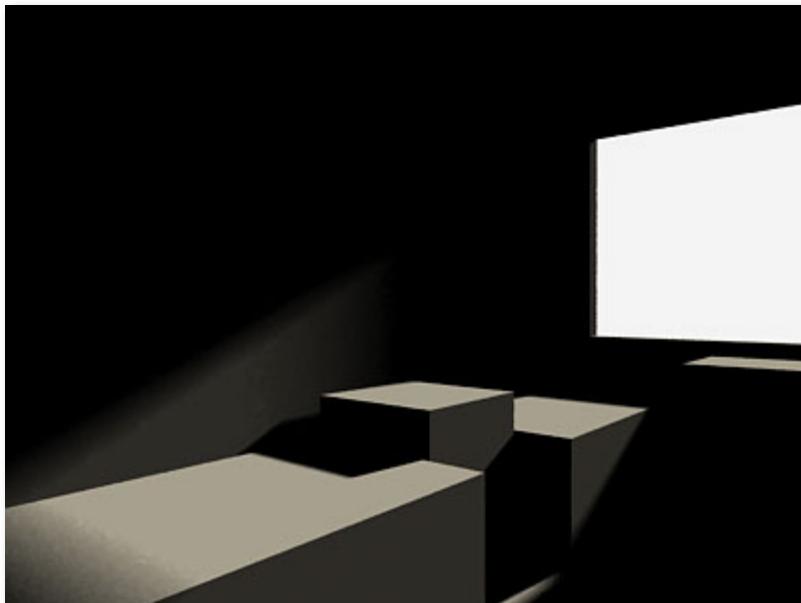


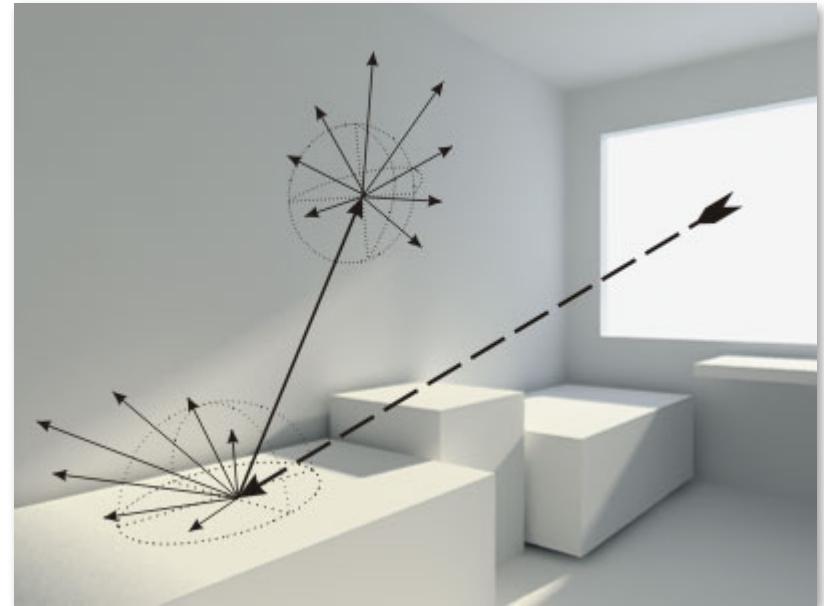
Ray-tracing

(Global Illumination part 1)

Direct and indirect lighting

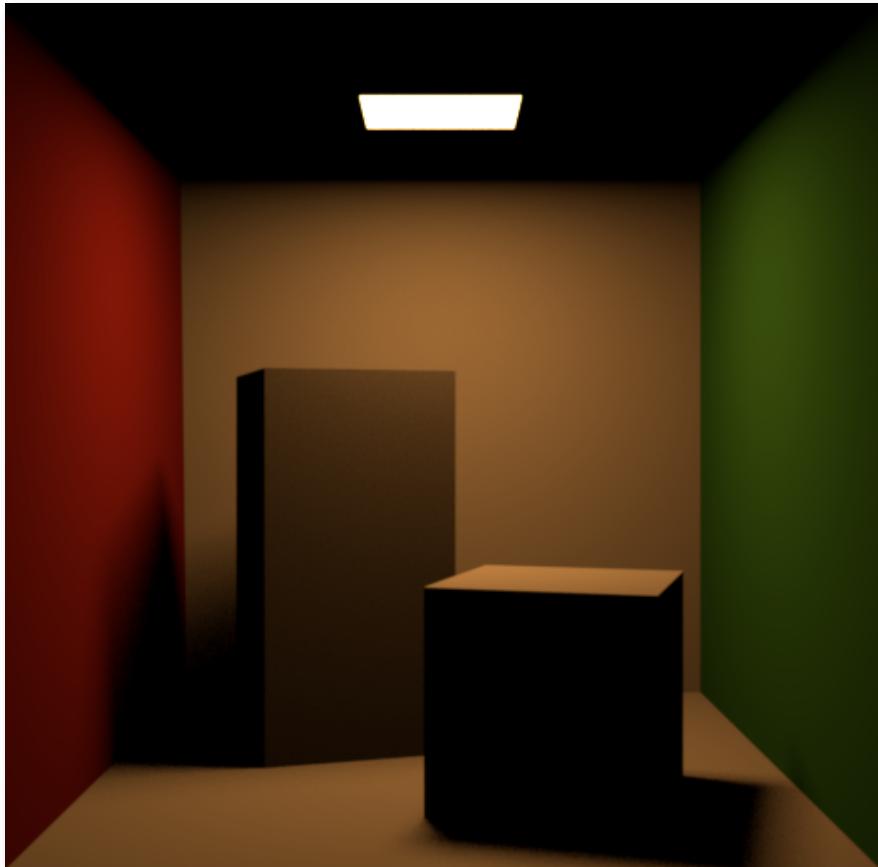


Direct :
local properties

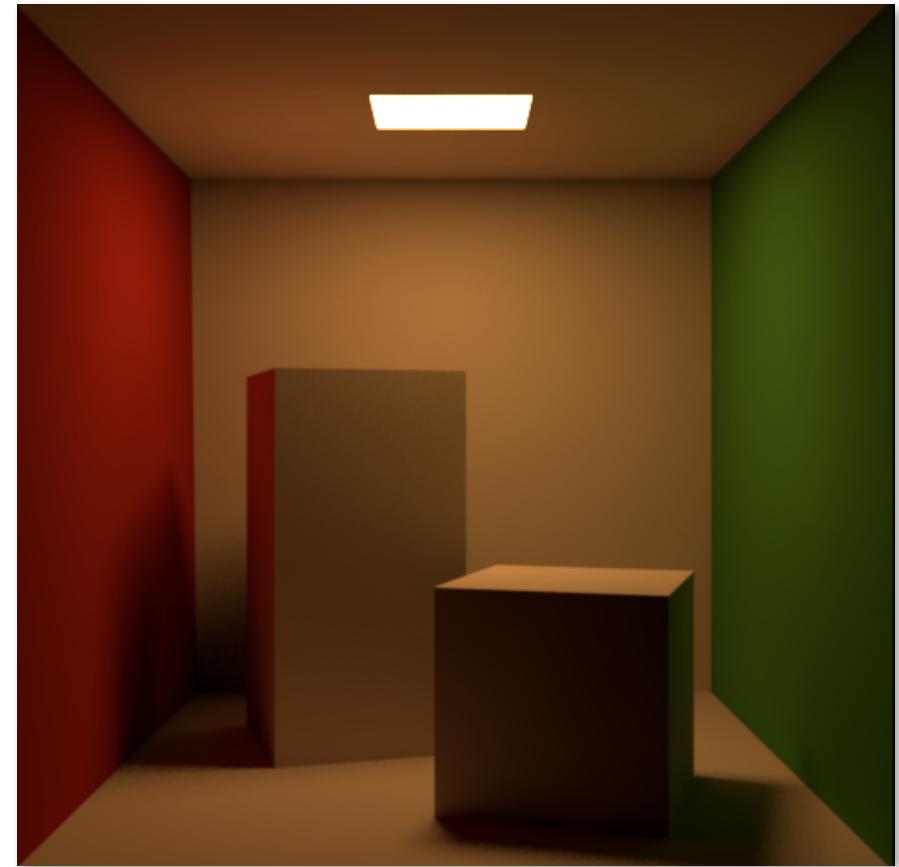


Indirect :
global problem

Direct and indirect lighting

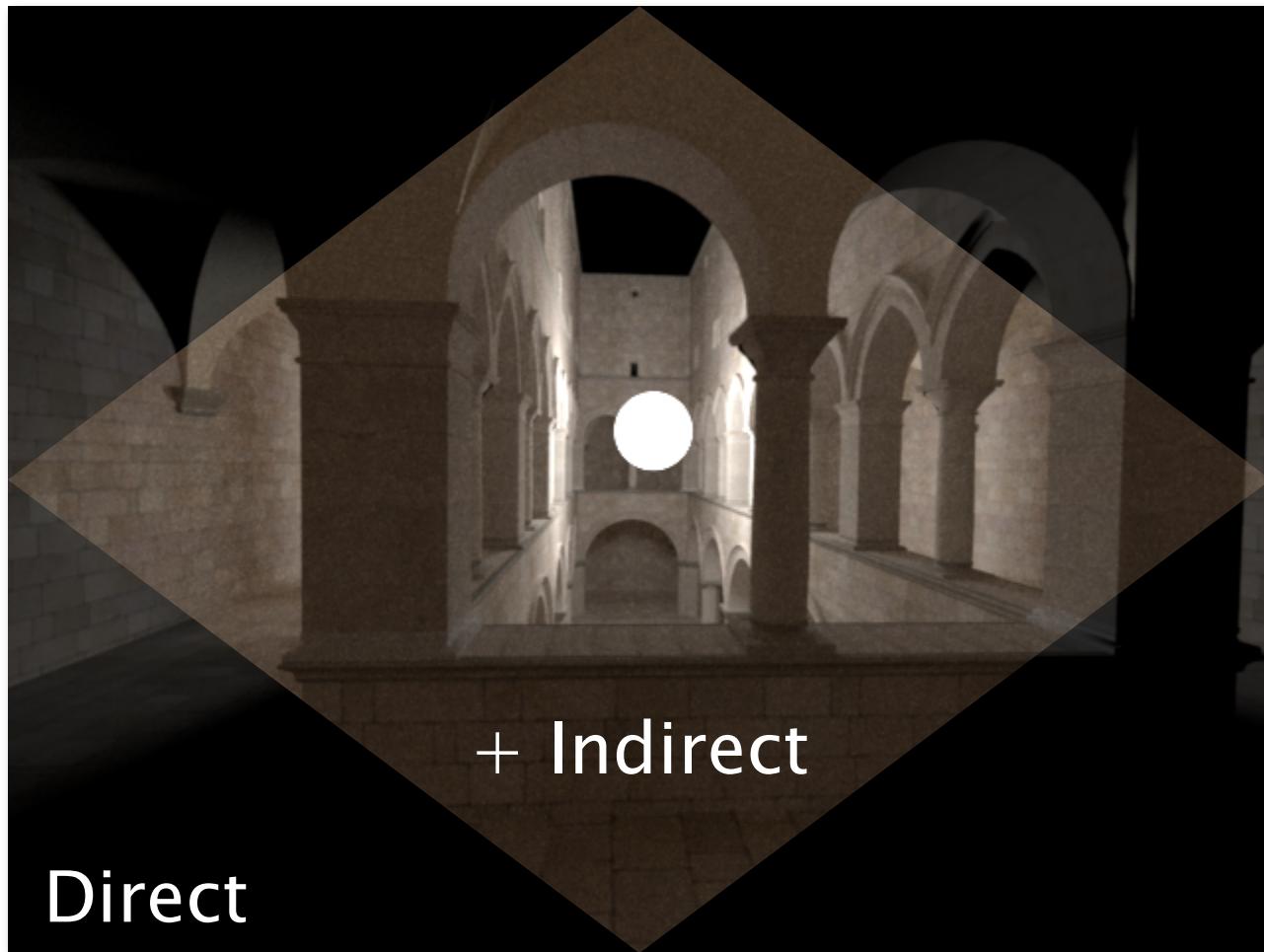


Direct



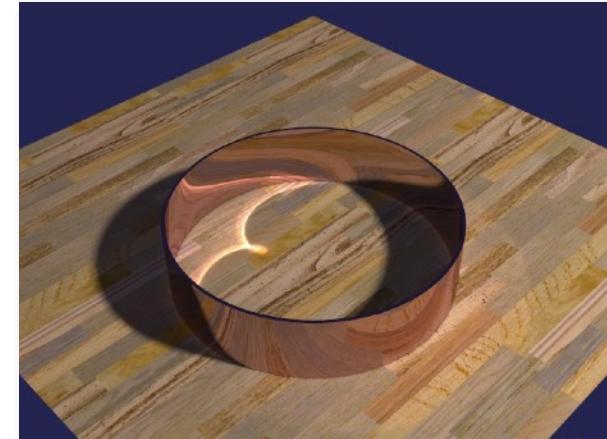
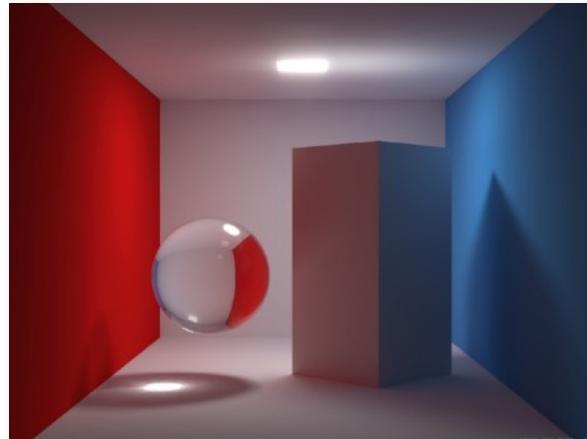
+ Indirect

Direct and indirect lighting

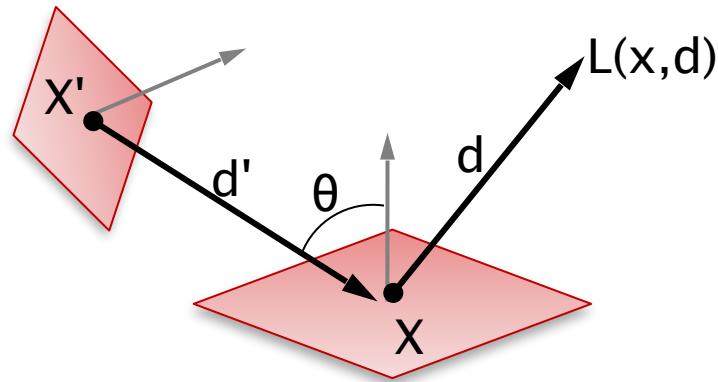


Global illumination

- ▶ Interactions between objects
- ▶ Light transport
- ▶ Reflections, refraction, diffusion
- ▶ Energy Conservation (for light)



The rendering equation



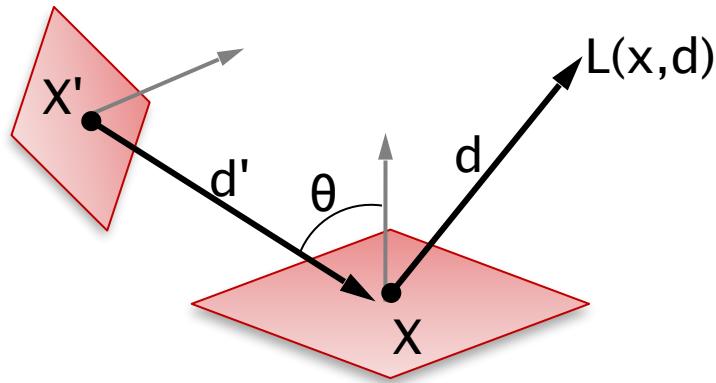
$$L(x,d) = E(x,d) + \underbrace{\int \rho(x,d,d') v(x,x') L(x',d') G(x,x') dA}_{\text{Radiance leaving point } x \text{ in the direction } d}$$

Radiance leaving point x
in the direction d

Radiance?

- ▶ Radiant flux per unit solid angle per unit projected area:
 - ▶ $\text{W}\cdot\text{sr}^{-1}\cdot\text{m}^{-2}$
- ▶ Invariant along a given ray:
 - ▶ $L(x, x \rightarrow y) = L(y, y \leftarrow x)$
 - ▶ No attenuation with distance
 - ▶ Easier for computations
 - ▶ Exception for point light sources

The rendering equation

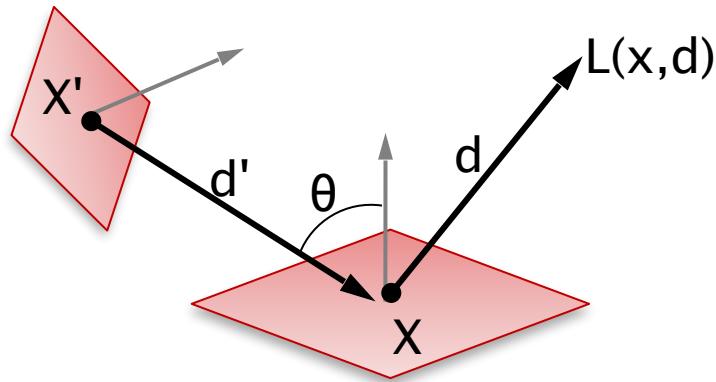


$$L(x,d) = \underbrace{E(x,d)}_{\text{Radiance emitted from } x} + \int \rho(x,d,d') v(x,x') L(x',d') G(x,x') dA$$

Radiance emitted from x:

non-zero only if x belongs to a light source

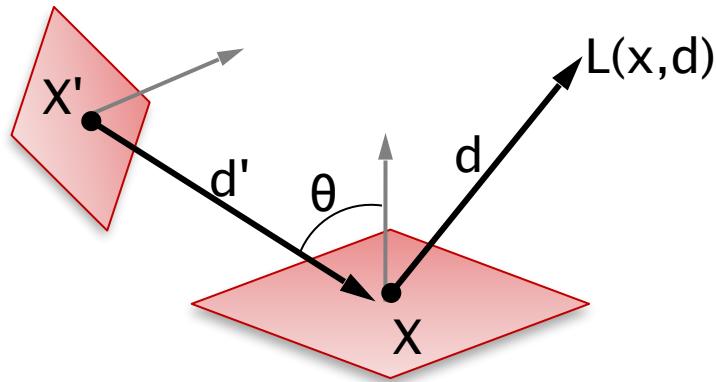
The rendering equation



$$L(x,d) = E(x,d) + \int \rho(x,d,d') v(x,x') L(x',d') G(x,x') dA$$

Integrating the contribution
from all the surfaces

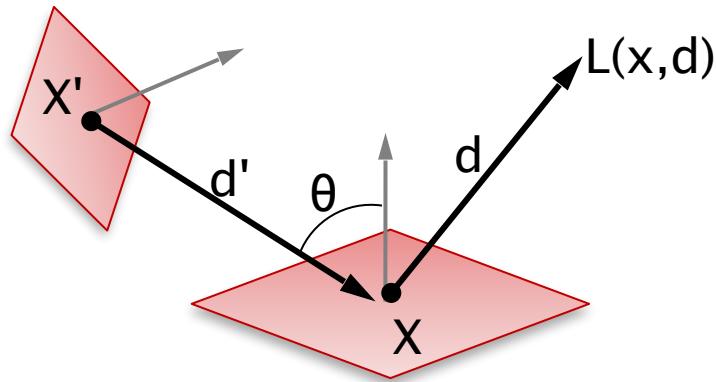
The rendering equation



$$L(x,d) = E(x,d) + \int \rho(x,d,d') v(x,x') \underbrace{L(x',d') G(x,x')}_{\text{Incoming Radiance}} dA$$

Incoming Radiance
from point x' in the direction d'

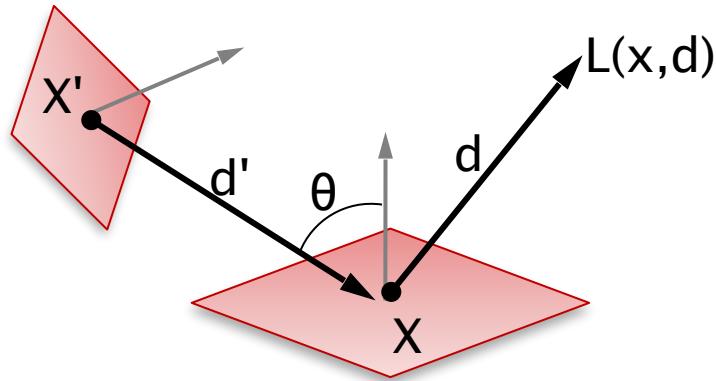
The rendering equation



$$L(x,d) = E(x,d) + \underbrace{\int \rho(x,d,d') v(x,x') L(x',d') G(x,x') dA}_{\text{Multiplication by the reflectance (BRDF) of the surface at point } x}$$

Multiplication by the reflectance (BRDF)
of the surface at point x

The rendering equation

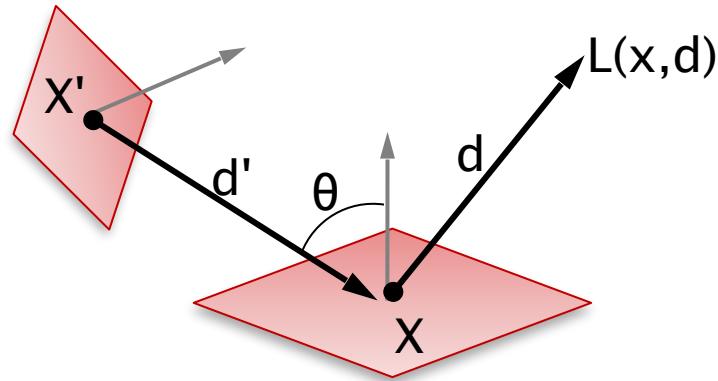


$$L(x,d) = E(x,d) + \int \rho(x,d,d') v(x,x') L(x',d') G(x,x') dA$$

Visibility between x and x'

1 when the two points are visible from each other, 0 otherwise

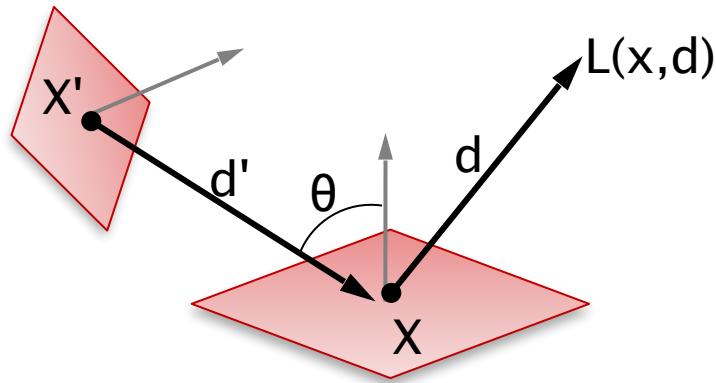
The rendering equation



$$L(x, d) = E(x, d) + \int \rho(x, d, d') v(x, x') L(x', d') \underbrace{G(x, x')}_{\text{Geometric factor}} dA$$

Geometric factor depending on the
surfaces x and x'

The rendering equation

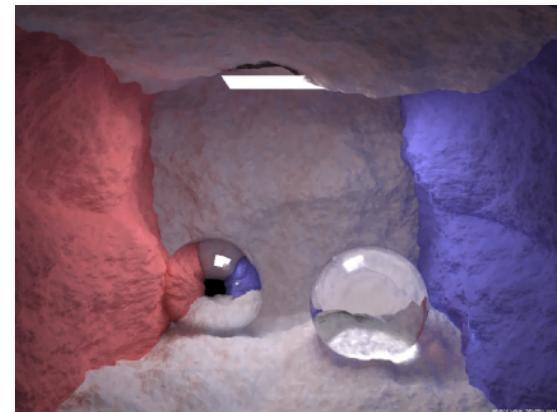


$$L(x, d) = E(x, d) + \int \rho(x, d, d') v(x, x') L(x', d') G(x, x') dA$$

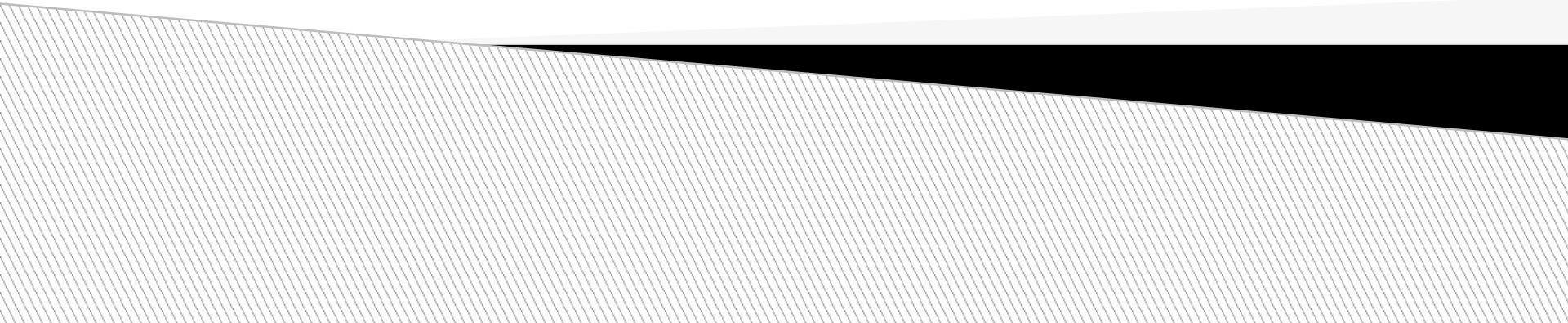
Full general analytical solution impossible

Two discretisation

- ▶ Ray-tracing and its extensions
(Monte-Carlo path tracing, Photon mapping...)
 - Sampling the integral
 - Optical laws
- ▶ Radiosity
 - Discretize the geometry:
exchanges between patches
 - All objects are diffuse

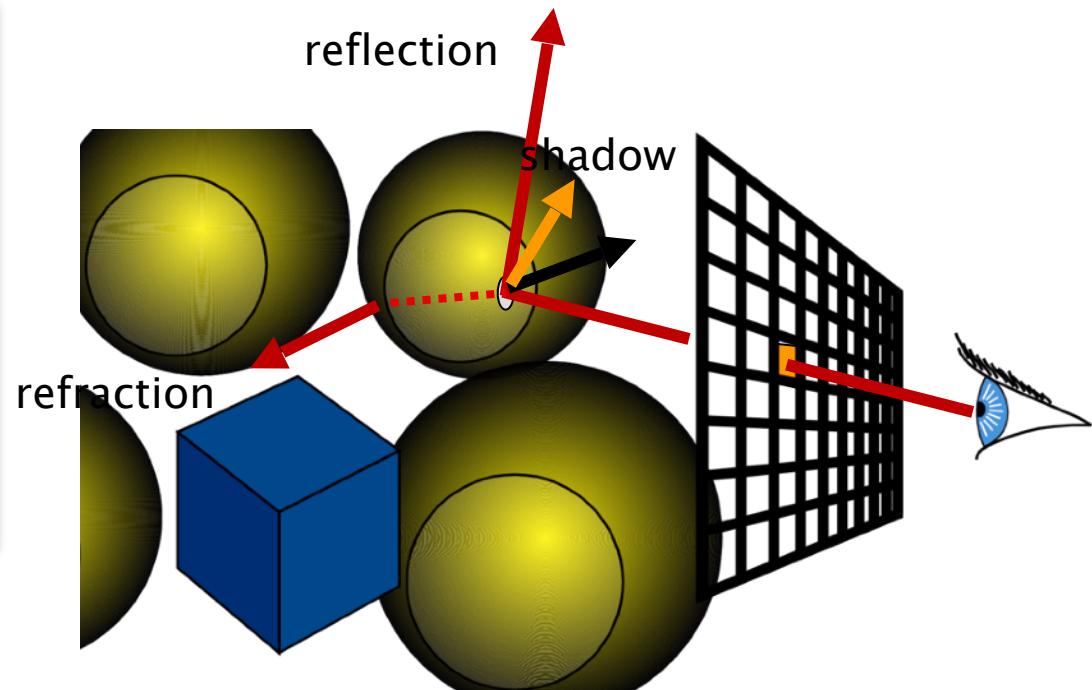
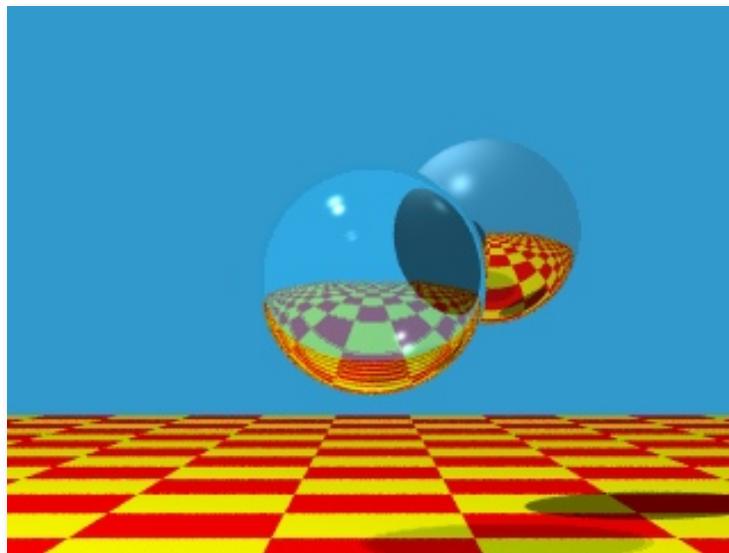


Ray tracing



« Whitted Ray Tracing » (1980)

- ▶ One ray per pixel
- ▶ Three new rays are created

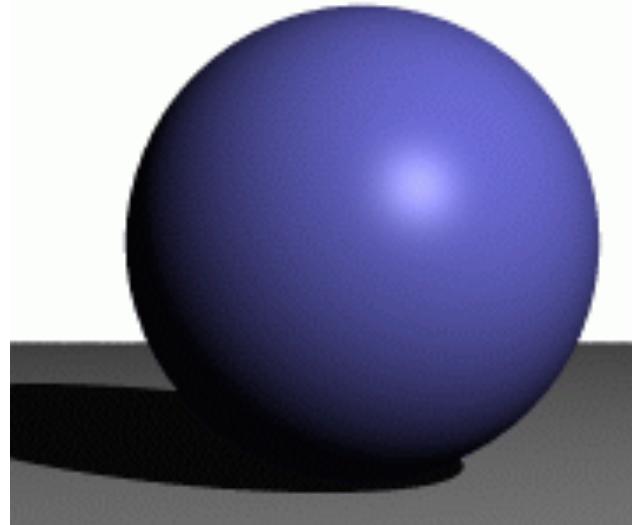


Even more rays

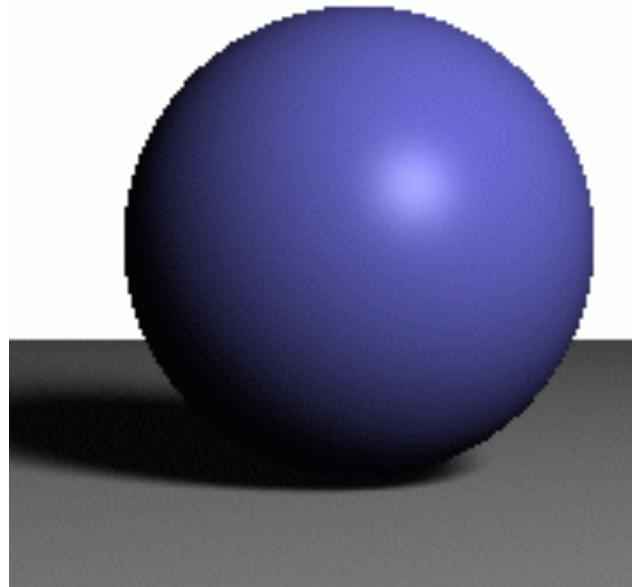
« Distributed Ray Tracing » Cook et al. (1984)

- ▶ Soft shadows
 - Several rays for each extended light source

Point light source



Extended light source



Even more rays

« Distributed Ray Tracing » Cook et al. (1984)

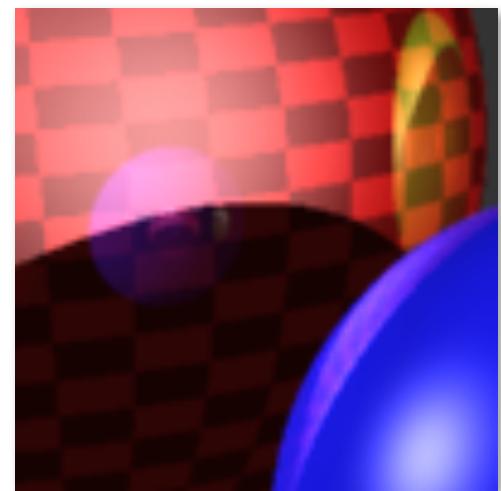
- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel



1 ray



2 rays

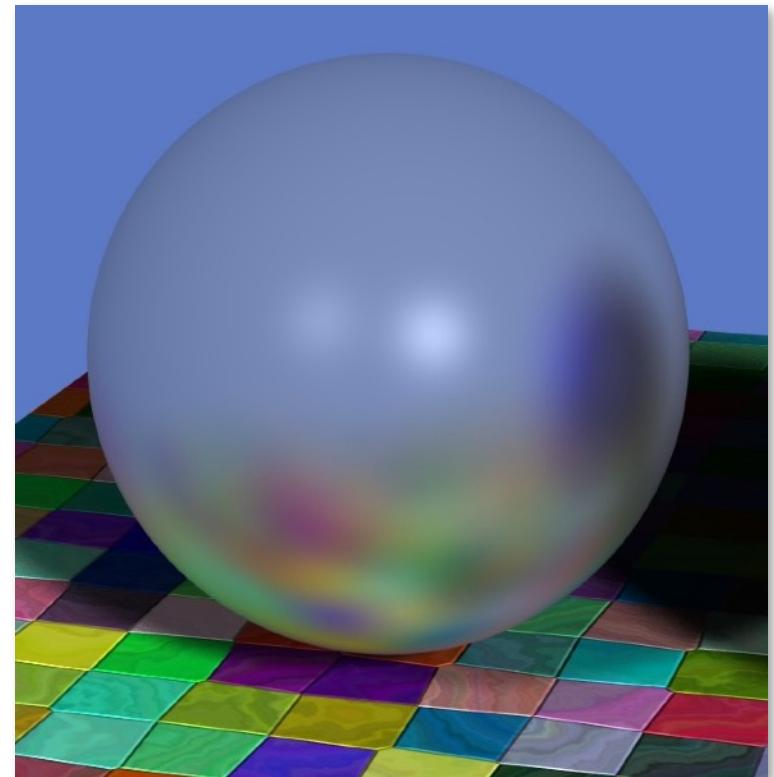


3 rays

Even more rays

« Distributed Ray Tracing » Cook et al. (1984)

- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel
- ▶ Glossy Reflection
 - Several rays are reflected



Even more rays

« Distributed ray tracing » Cook et al. (1984)

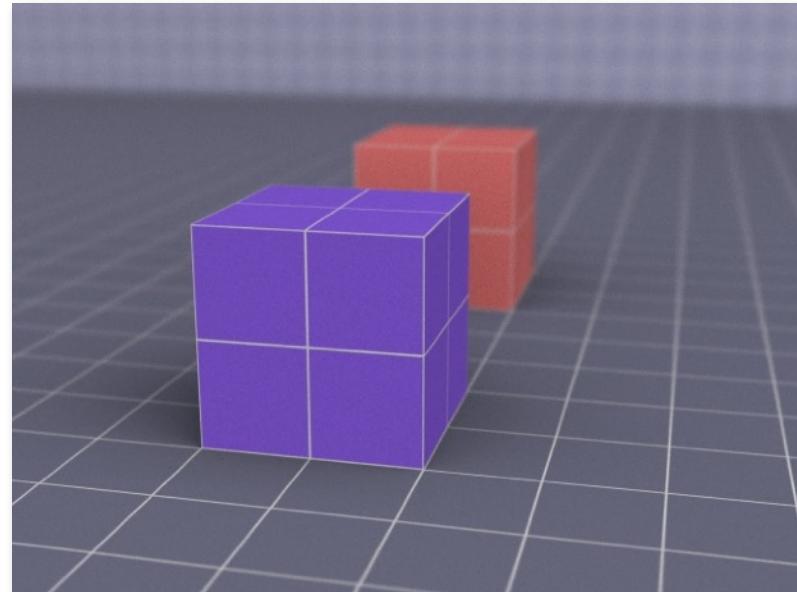
- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel
- ▶ Glossy Reflection
 - Several rays are reflected
- ▶ Motion blur
 - Several rays during shutter opening



Even more rays

« Distributed Ray Tracing » Cook et al. (1984)

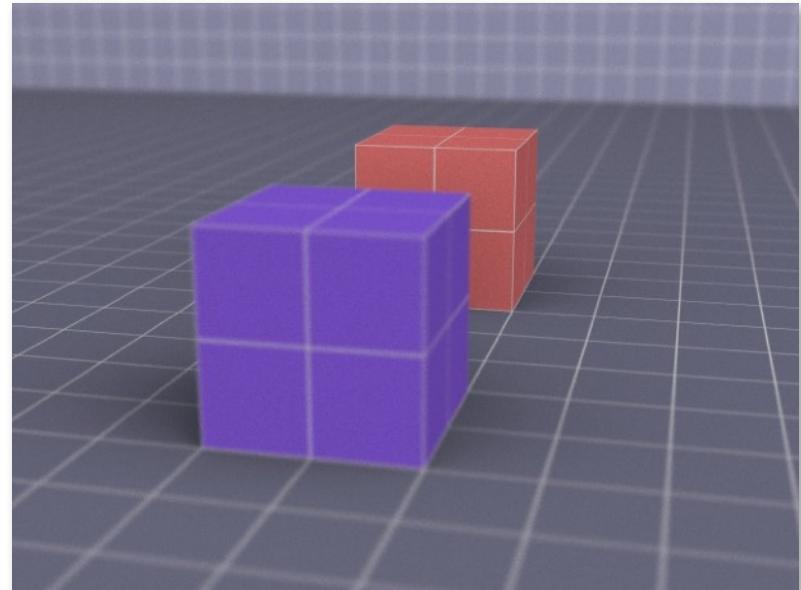
- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel
- ▶ Glossy Reflection
 - Several rays are reflected
- ▶ Motion blur
 - Several rays during time
- ▶ Depth of field
 - Several rays per pixel,
focusing with a lens



Even more rays

« Distributed Ray Tracing » Cook et al. (1984)

- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel
- ▶ Glossy Reflection
 - Several rays are reflected
- ▶ Motion blur
 - Several rays during time
- ▶ Depth of field
 - Several rays per pixel,
focusing with a lens







Ray-tracing

- ▶ Main operation:
 - Ray-primitive intersection
- ▶ Speed-up:
 - Scene structure

Ray–Primitive intersection

- ▶ Ray: starting point P , direction u
- ▶ Primitives:
 - Sphere
 - Cylinder
 - Plane (infinite)
 - Triangle

Ray–sphere intersection

- ▶ Ray: starting point P , direction u
- ▶ Sphere : center C , radius r
- ▶ \exists intersection: Boolean

$$\Delta = (\vec{u} \cdot \overrightarrow{CP})^2 - CP^2 + r^2 > 0$$

- ▶ Intersection point: $Q = P + \lambda u$
 - Several potential intersection points
 - $\lambda > 0$
 - First point only

Ray–cylinder intersection

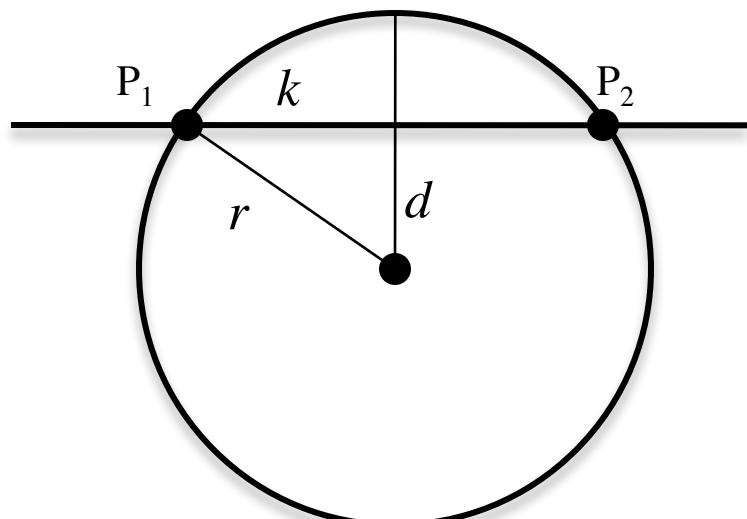
- ▶ Ray: starting point P , direction u
- ▶ Cylinder: axis $D(Q, v)$, radius r

- ▶ \exists intersection : Boolean

- $n = (u \times v) / \|u \times v\|$
 - $d = |PQ \cdot n| < r$

- ▶ 2 intersection points:

- $\lambda_{\text{middle}} = (PQ \times v) \cdot n / \|u \times v\|$
 - + Pythagore for P_1, P_2 : $k = \sqrt{r^2 - d^2}$



“Intersecting a ray with a cylinder”, Joseph M. Cychosz
& Warren N. Waggenspack, Jr., Graphics Gems IV, 1994

Ray–plane intersection

- ▶ Ray: starting point P , direction u
- ▶ Plane: point Q , normal n
- ▶ Intersection point:
 - ▶ $\lambda = - (PQ \cdot n) / (u \cdot n)$

Ray–triangle intersection

- ▶ Ray: starting point P , direction u
- ▶ Triangle (V_0, V_1, V_2)
- ▶ [Badouel 90]:
 - Ray–plane intersection
 - Projection on 2D plane
 - (remove one coordinate)
 - Compute barycentric coordinates in 2D

Ray–triangle intersection

- ▶ Ray: starting point P , direction \vec{u}
- ▶ Triangle (V_0, V_1, V_2)
- ▶ [Möller–Trumbore 97]: fast+minimal storage
- ▶ Solve for (λ, α, β) in a single step:

$$P + \lambda \vec{u} = (1 - \alpha - \beta) V_0 + \alpha V_1 + \beta V_2$$

$$\begin{pmatrix} \lambda \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\vec{u} & V_1 - V_0 & V_2 - V_0 \end{pmatrix}^{-1} (P - V_0)$$

Ray–triangle intersection

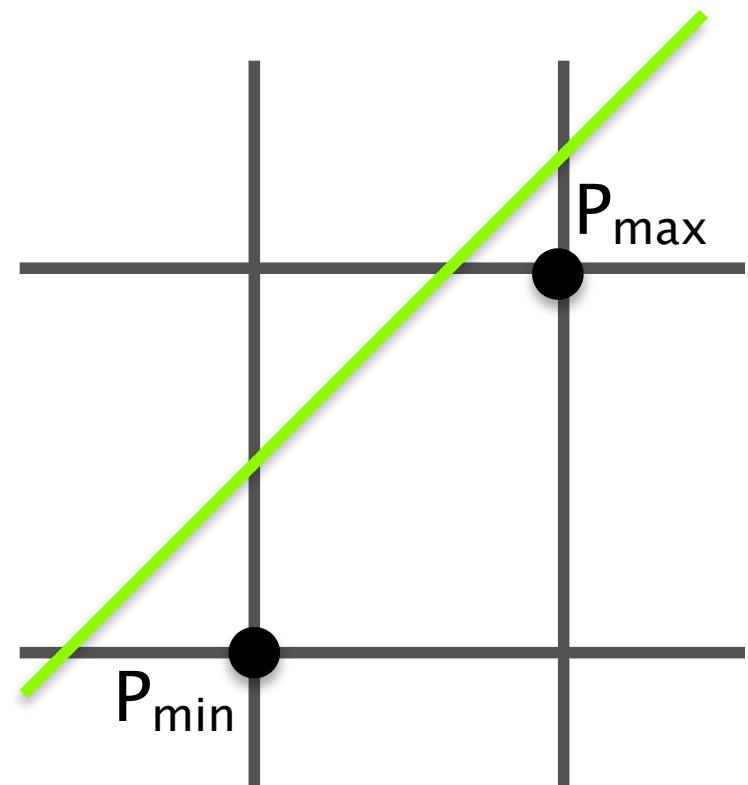
- ▶ Ray: starting point P , direction u
- ▶ [Möller–Trumbore 97]: fast+minimal storage
- ▶ $e_i = V_i - V_0 \quad t = P - V_0$

$$\begin{pmatrix} \lambda \\ \alpha \\ \beta \end{pmatrix} = \frac{1}{(\vec{u} \times \vec{e}_2) \cdot \vec{e}_1} \begin{pmatrix} (\vec{t} \times \vec{e}_1) \cdot \vec{e}_2 \\ (\vec{u} \times \vec{e}_2) \cdot \vec{t} \\ (\vec{t} \times \vec{e}_1) \cdot \vec{u} \end{pmatrix} = \frac{1}{\vec{p} \cdot \vec{e}_1} \begin{pmatrix} \vec{q} \cdot \vec{e}_2 \\ \vec{p} \cdot \vec{t} \\ \vec{q} \cdot \vec{u} \end{pmatrix}$$

Speed/memory tradeoff: precompute the e_i
Still the fastest algorithm, even with GPUs

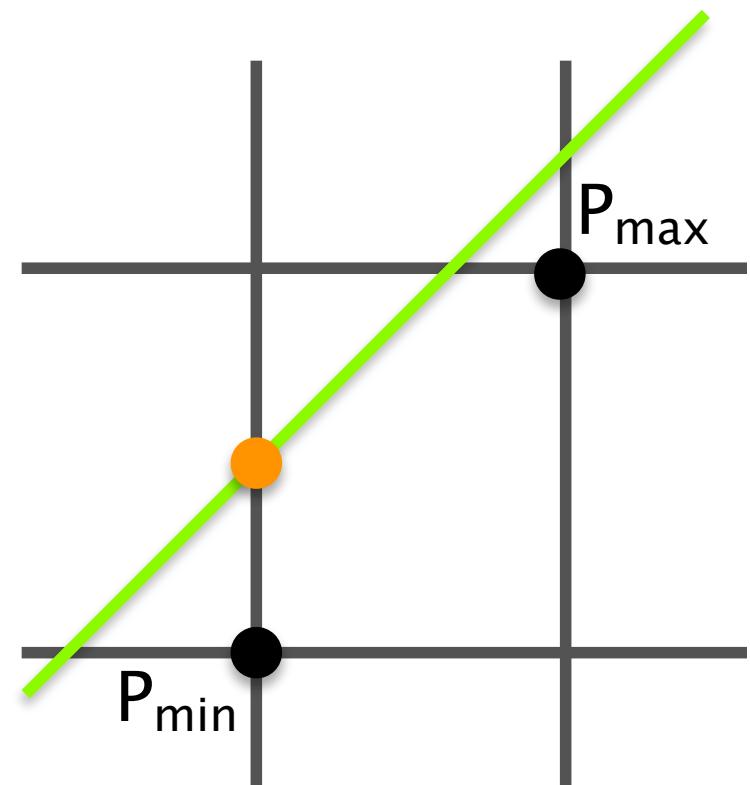
Axis-Aligned Bounding Box

- ▶ Bounding box defined by P_{\min} , P_{\max}
- ▶ Ray intersection?
 - ▶ $\lambda_{x\min} = (x_{\min} - P_x) / \mathbf{u}_x$
- ▶ Think parallel!
 - ▶ $t_{\min} = (P_{\min} - P) / \mathbf{u}$
 - ▶ $t_{\max} = (P_{\max} - P) / \mathbf{u}$
 - ▶ Component-wise div
 - ▶ 2 GPU operations



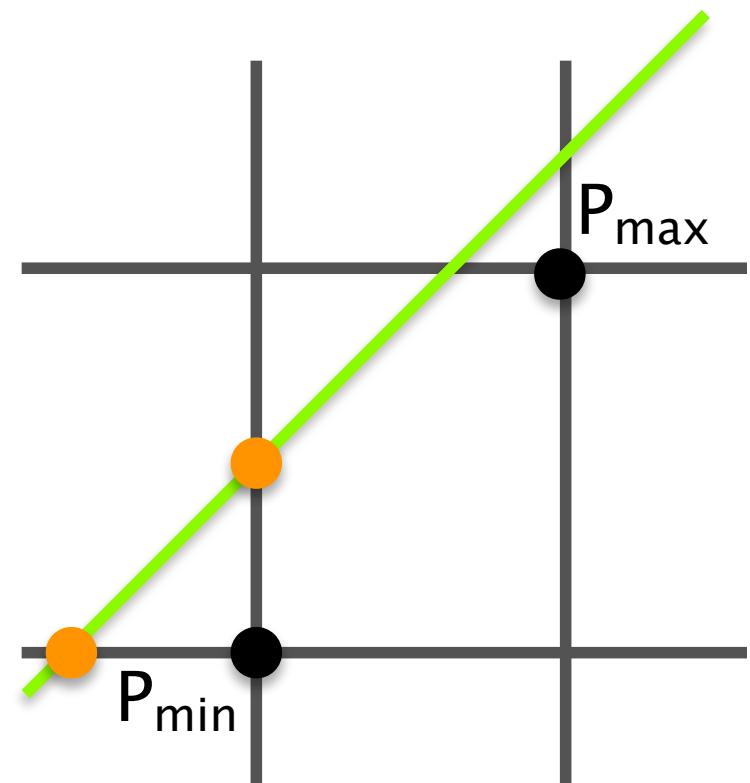
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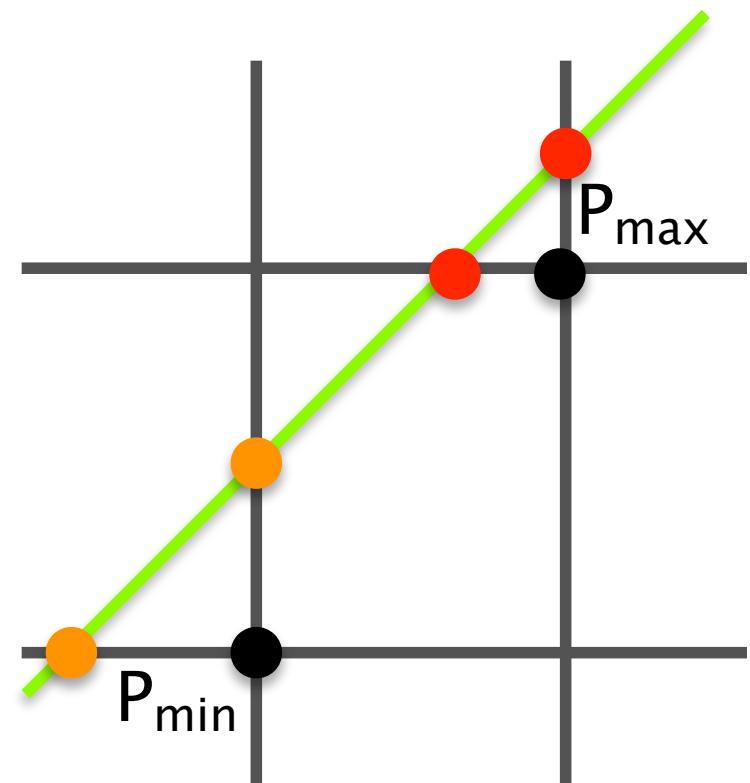
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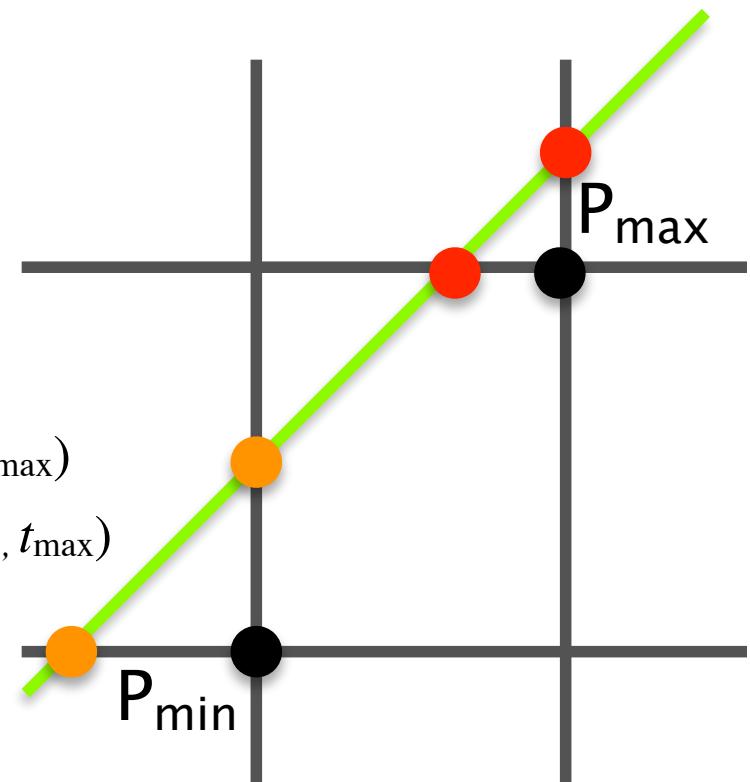
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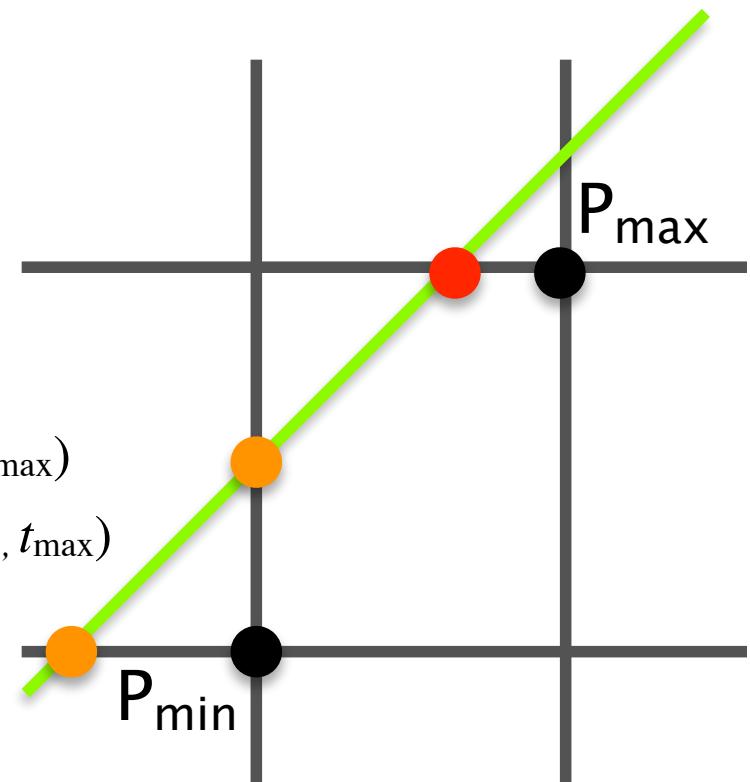
Axis-Aligned Bounding Box

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- ▶ Ray intersection?
 - ▶ $t_{\min} = (P_{\min} - P) / \mathbf{u}$
 - ▶ $t_{\max} = (P_{\max} - P) / \mathbf{u}$
- ▶ Needs sorting:
 - ▶ In = max value of $\min(t_{\min}, t_{\max})$
 - ▶ Out = min value of $\max(t_{\min}, t_{\max})$
- ▶ **Out > In = intersection**
 - ▶ Otherwise: no intersect



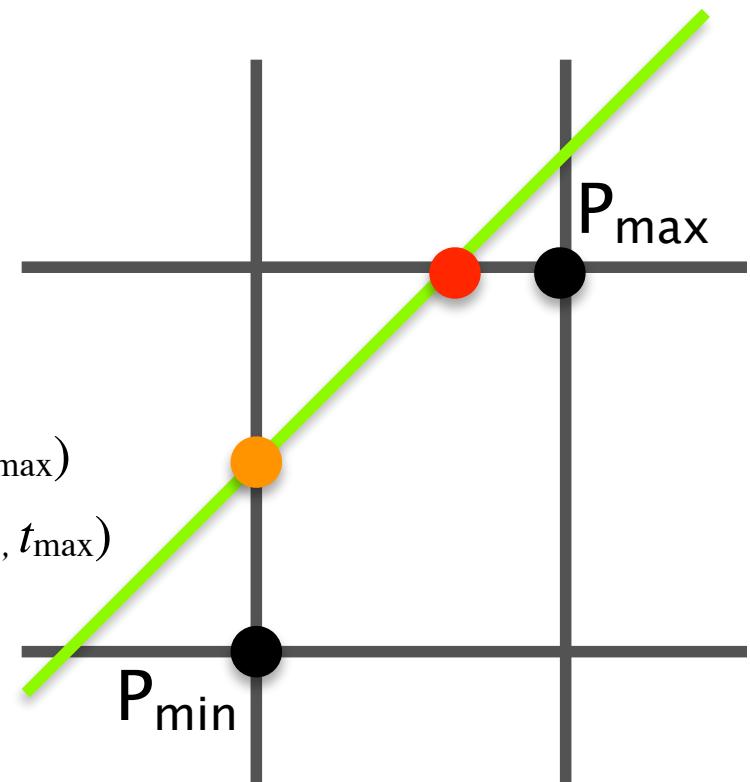
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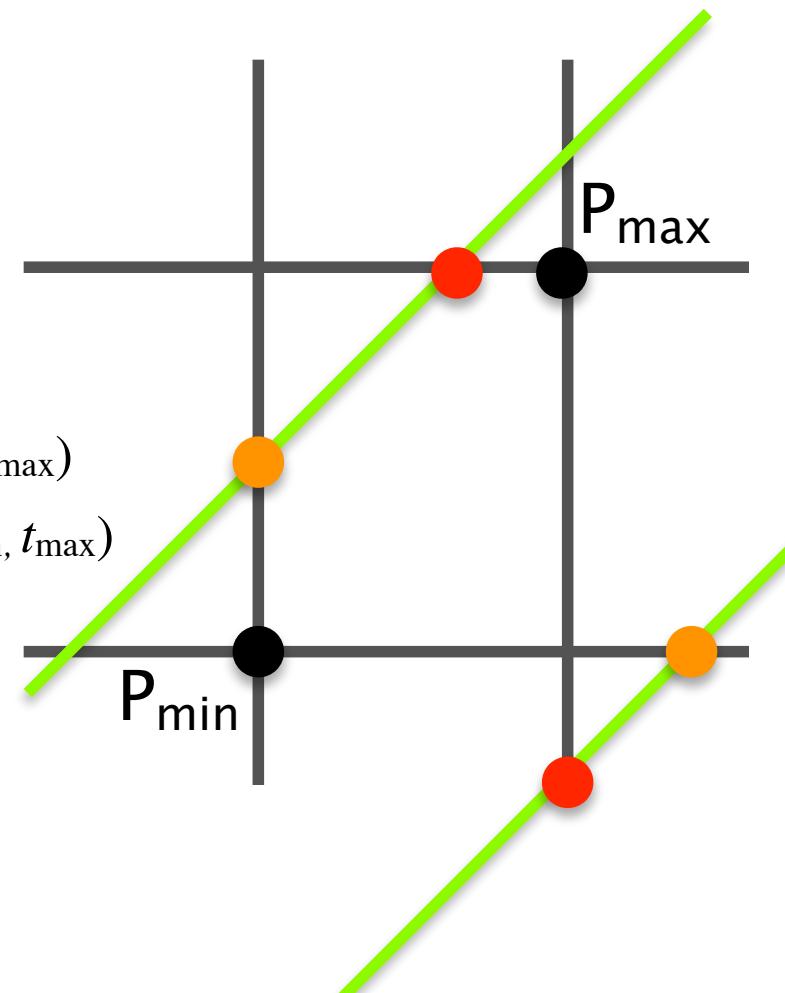
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Counting operations

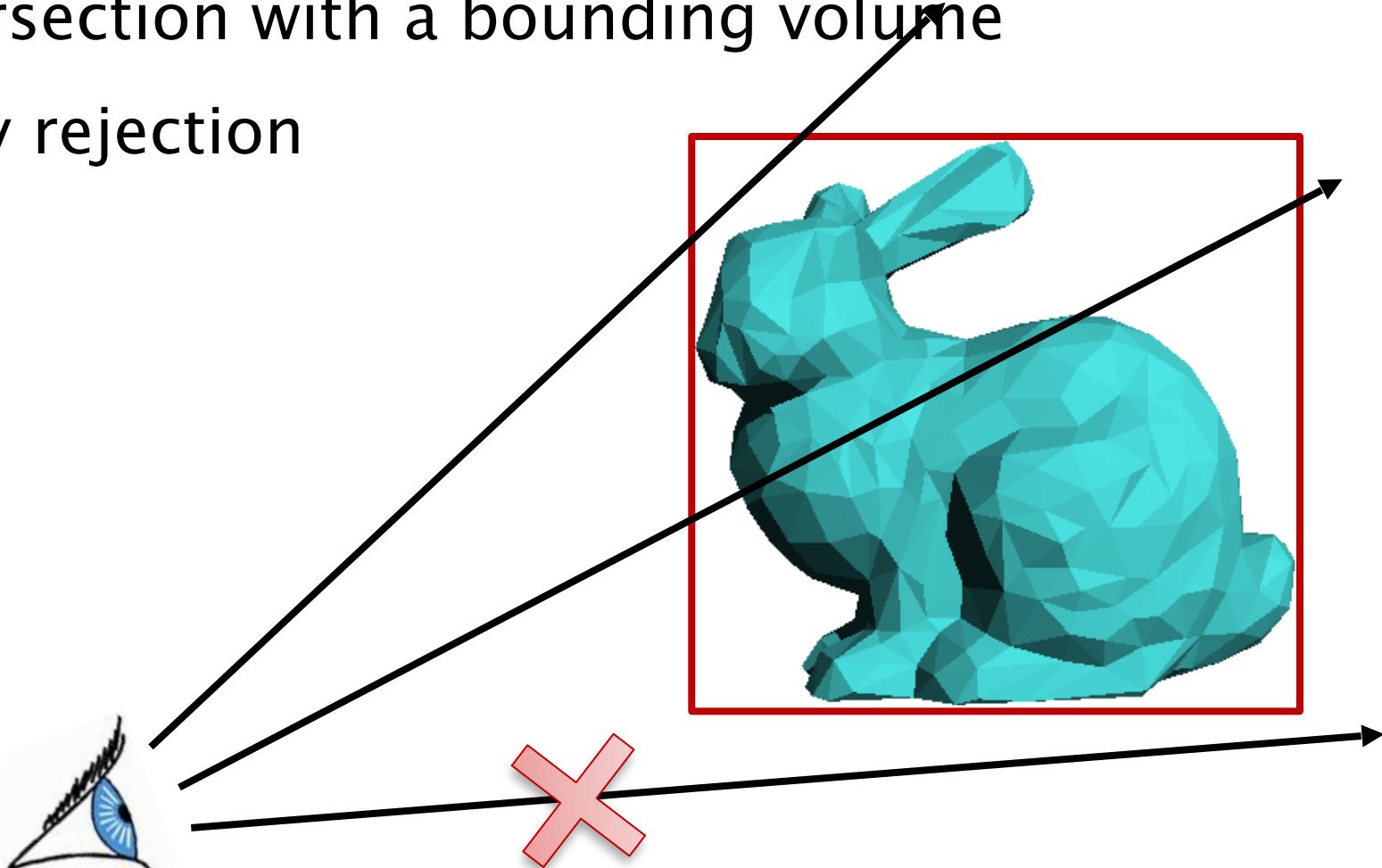
- ▶ Ray–sphere intersection:
 - ▶ 2 dot products, 3 vec4 ops, 3 float ops
- ▶ Ray–triangle intersection:
 - ▶ 4 dot products, 2 cross products, 3 vec4 ops, 3 float ops
- ▶ Ray–AABB intersection:
 - ▶ 6 vec4 ops, 5 float ops
- ▶ Ray–bounding volume ≈ 1 ray–triangle

Ray-scene intersection

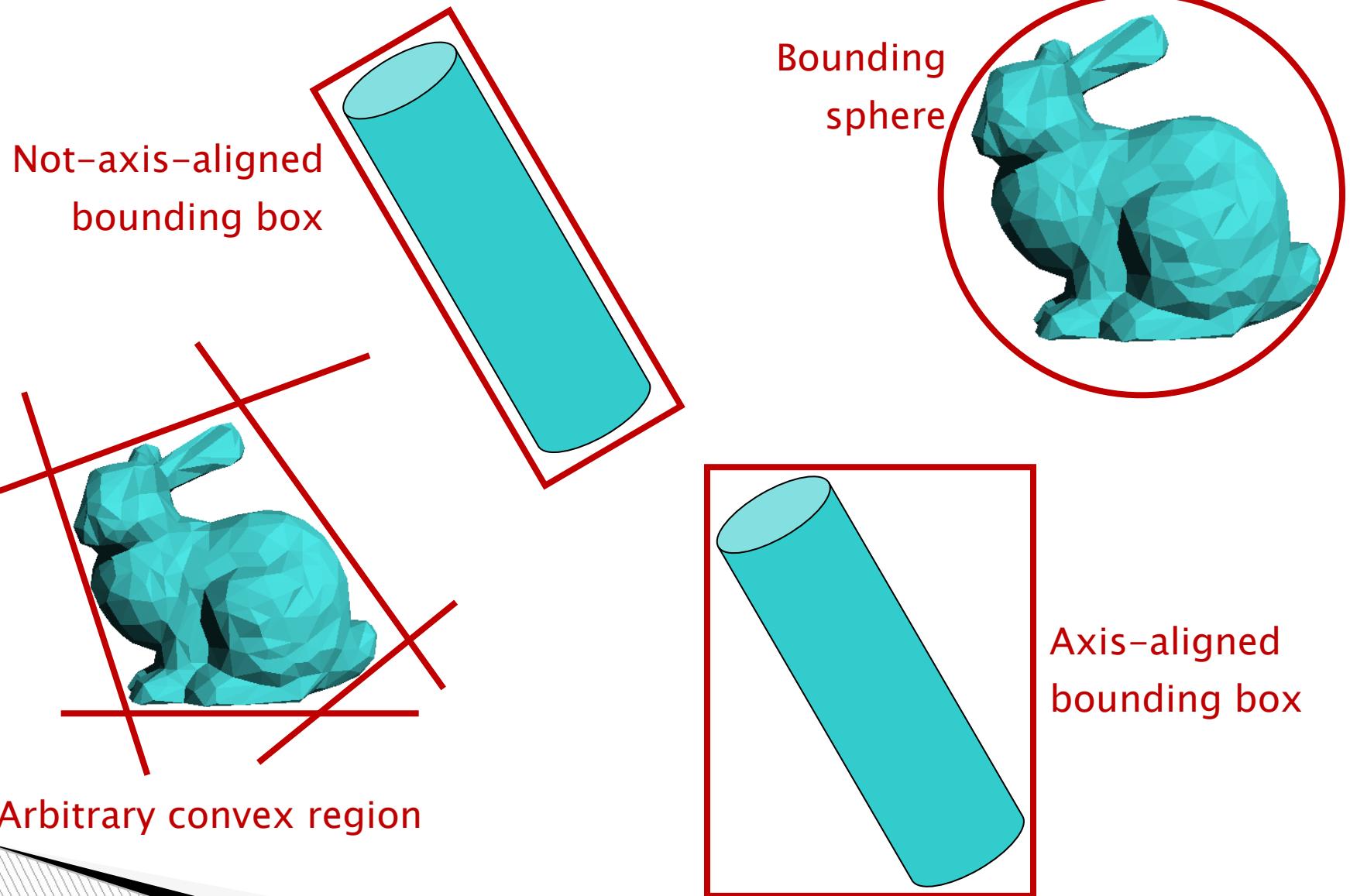
- ▶ 99 % of the time is spent doing intersections
- ▶ >100 K triangles
- ▶ Need for accelerations:
 - Bounding volumes
 - Hierarchies of bounding volumes
 - kD-tree
 - Octree, uniform grid
 - *Building* hierarchies: offline, CPU
 - *Intersecting* hierarchies: GPU

Bounding volumes

- ▶ Intersection with a bounding volume
- ▶ Early rejection



Bounding volumes



Spatial hierarchies

- ▶ 1 object = 100K+ triangles
- ▶ Need spatial hierarchy beyond AABB
- ▶ At the hierarchy leaves:
 - ▶ Intersect leaf, then list of triangles
 - ▶ 1 triangle per leaf: wasteful
 - ▶ 10 triangles per leaf: too much
 - ▶ Threshold: usually 4 triangles
- ▶ BVH, kD-trees, octrees, uniform grid...

Bounding Volume Hierarchy

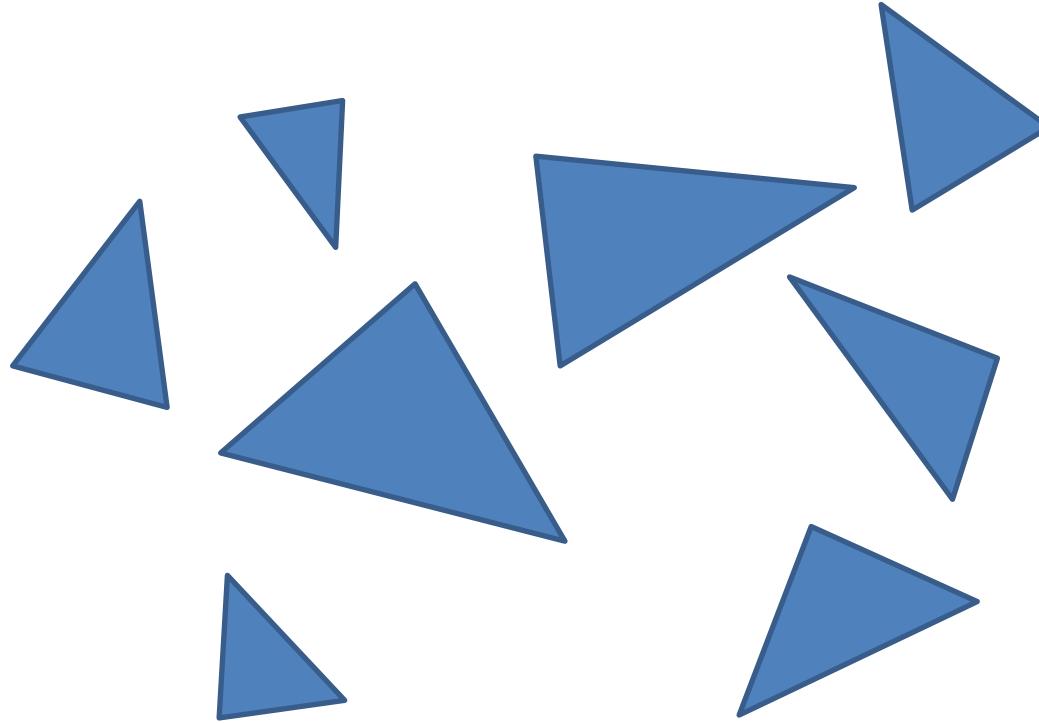
- ▶ Top-down or bottom-up
- ▶ Top-down:
 - ▶ Start with object AABB
 - ▶ Subdivide recursively until limit
 - ▶ Easier to implement
- ▶ Bottom-up:
 - ▶ Start with triangles
 - ▶ Find spatially close triangles
 - ▶ Group and recurse
 - ▶ More compact

BVH top-down construction

- ▶ Slides from Magnus Andersson (LTH)
- ▶ [http://fileadmin.cs.lth.se/cs/Education/
EDAN30/lectures/S2-bvh.pdf](http://fileadmin.cs.lth.se/cs/Education/EDAN30/lectures/S2-bvh.pdf)

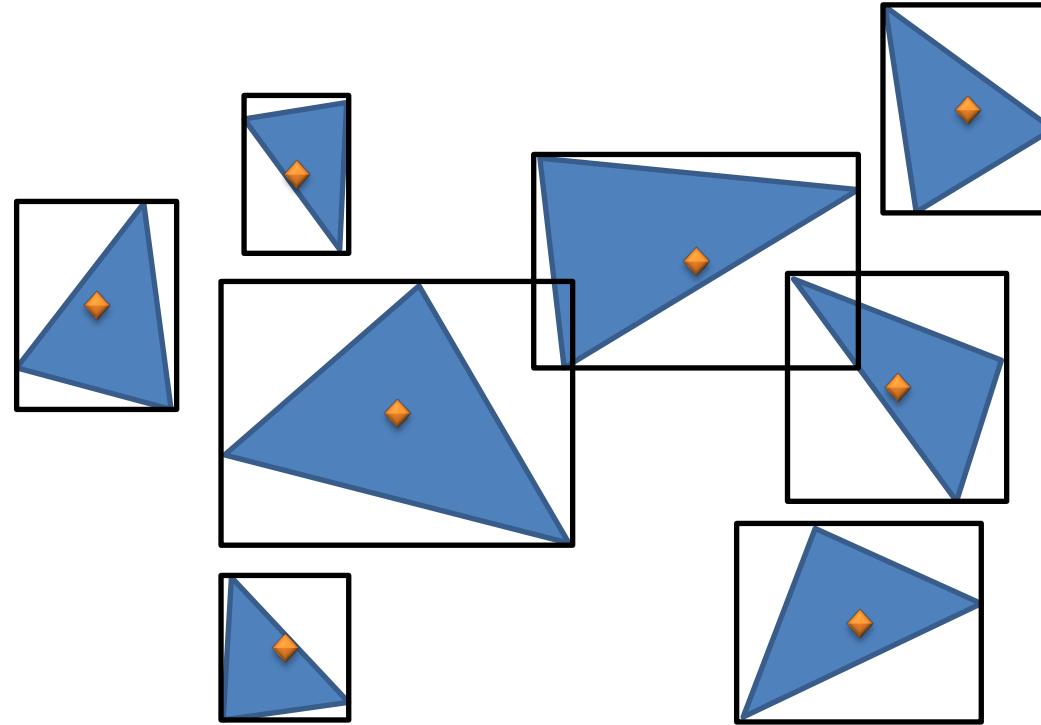
Construction

Construction, brief overview



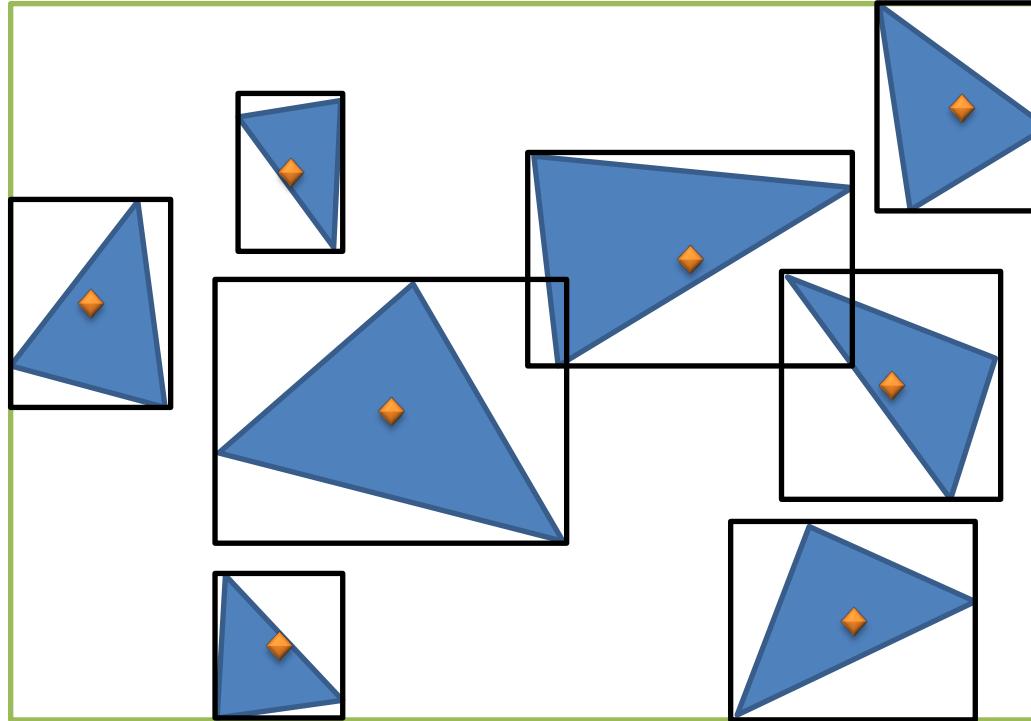
We begin with a bunch of Intersectables

Construction , brief overview



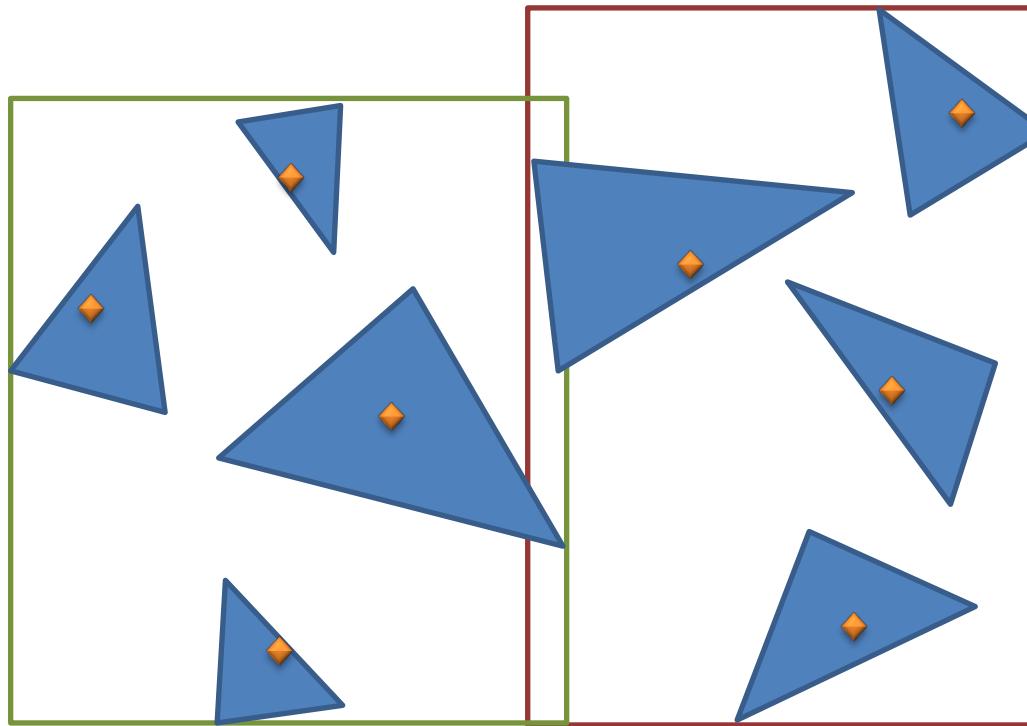
Find bounding box centroids of all intersectables

Construction , brief overview



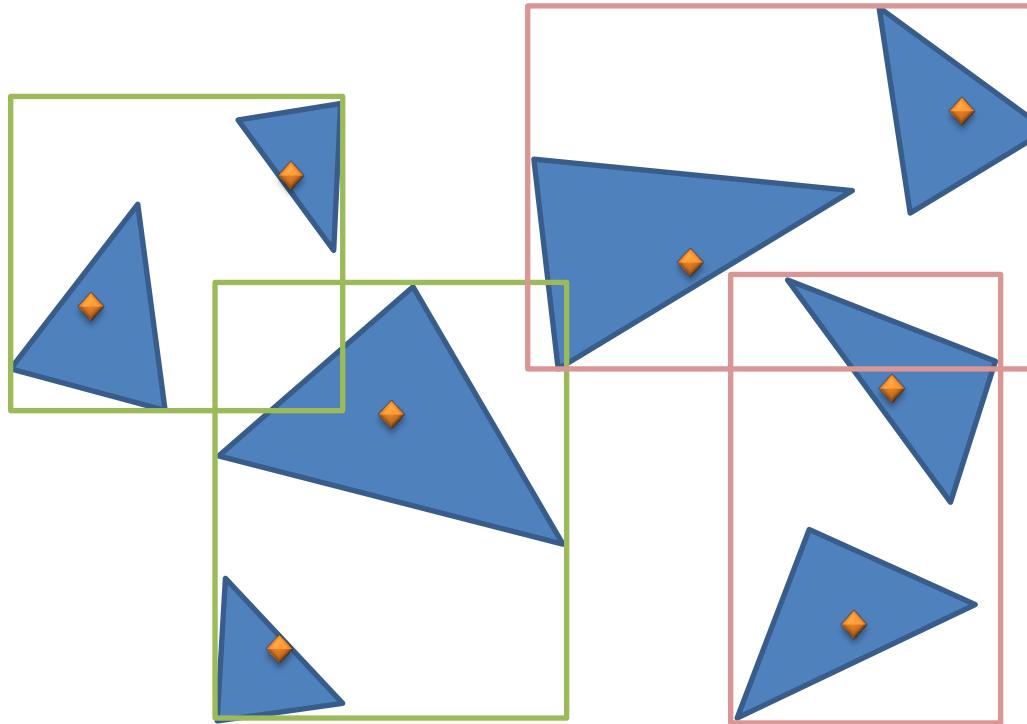
Find the *world* bounding box and create a root node

Construction , brief overview



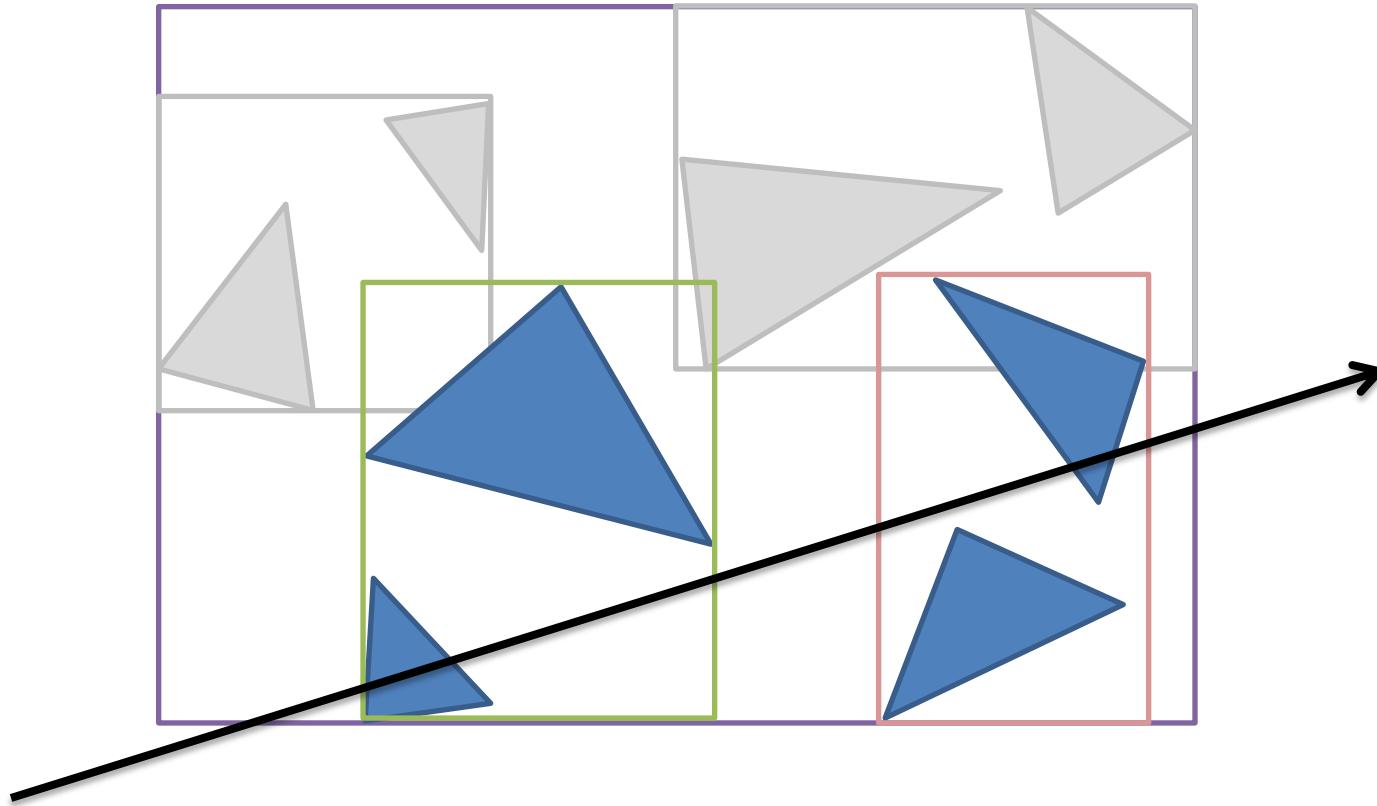
Use some splitting criteria to find a sensible division of the elements into new child nodes

Construction , brief overview



Continue to split recursively until each node contains only *one* or *a few* elements

Construction , brief overview



Now, when shooting rays we don't have to test all Intersectables anymore!

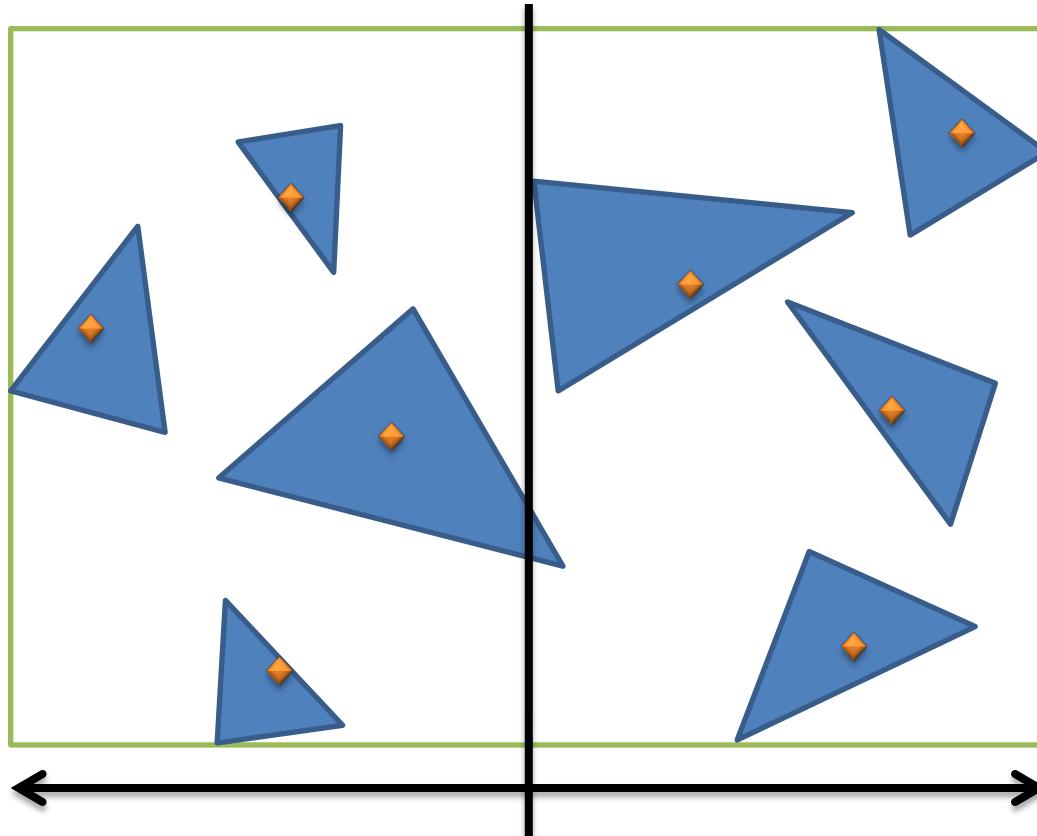
Construction

- So what is a sensible splitting criteria?
- Why not use *mid-point* splitting, since it's easy to understand and implement
 - *Works well when primitives are fairly evenly distributed*

Construction

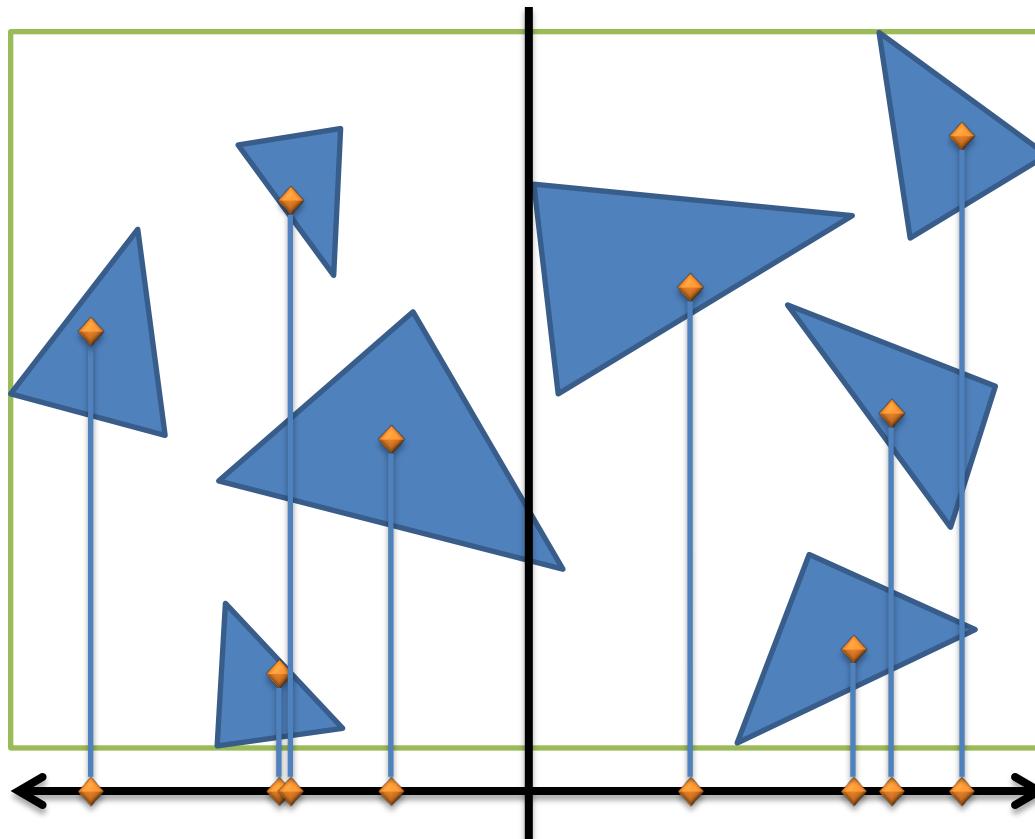
- So what is a sensible splitting criteria?
- Why not use *mid-point* splitting, since it's easy to understand and implement
 - *Works well when primitives are fairly evenly distributed*
- You can try to come up with a different criteria if you want to
 - *I tried splitting on the mean and median. Both were outperformed by mid-point splitting*

Construction



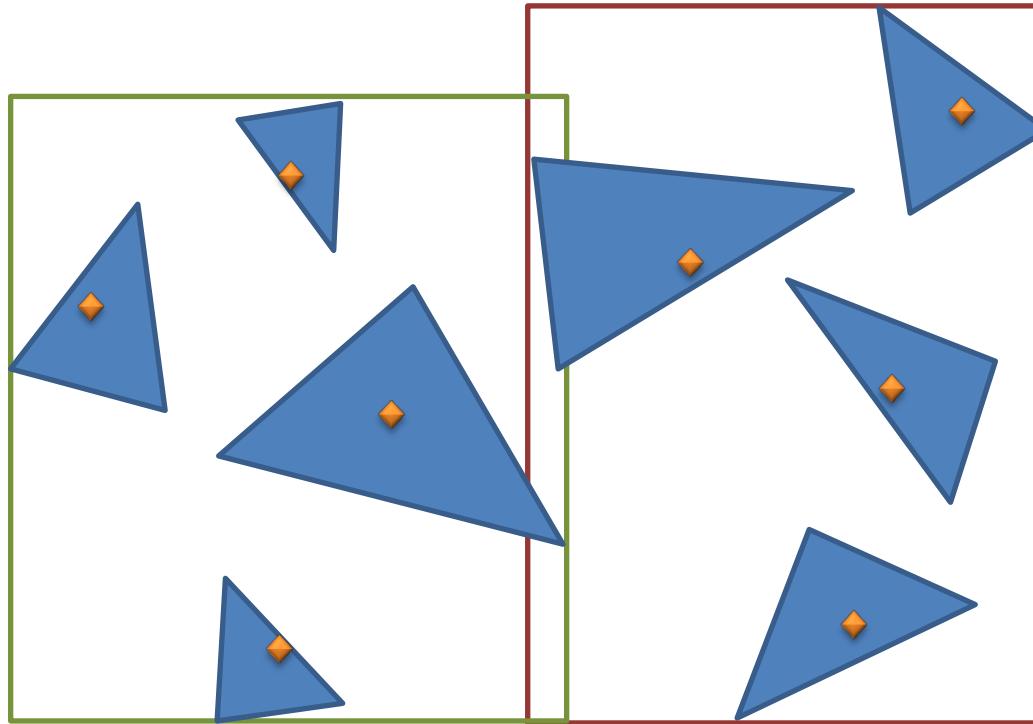
Find the mid point of the largest axis

Construction



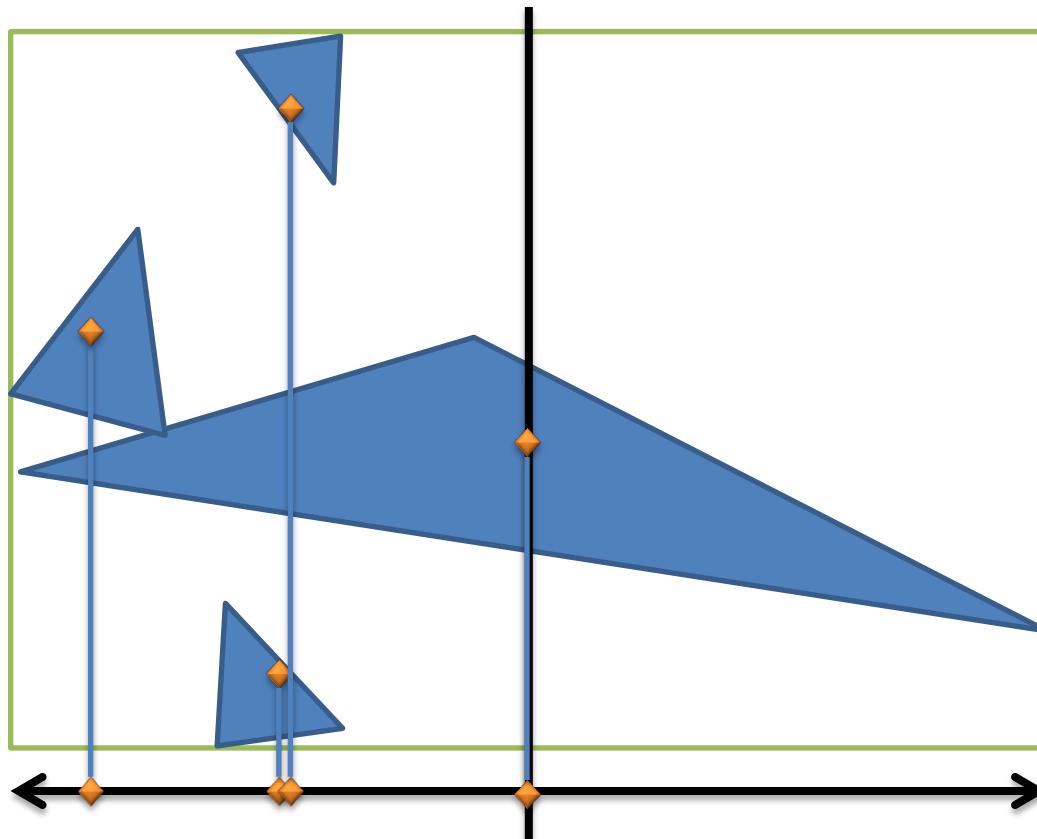
Sort the bounding box *centroids* in the largest axis direction. Then split into a *left* and a *right* side

Construction



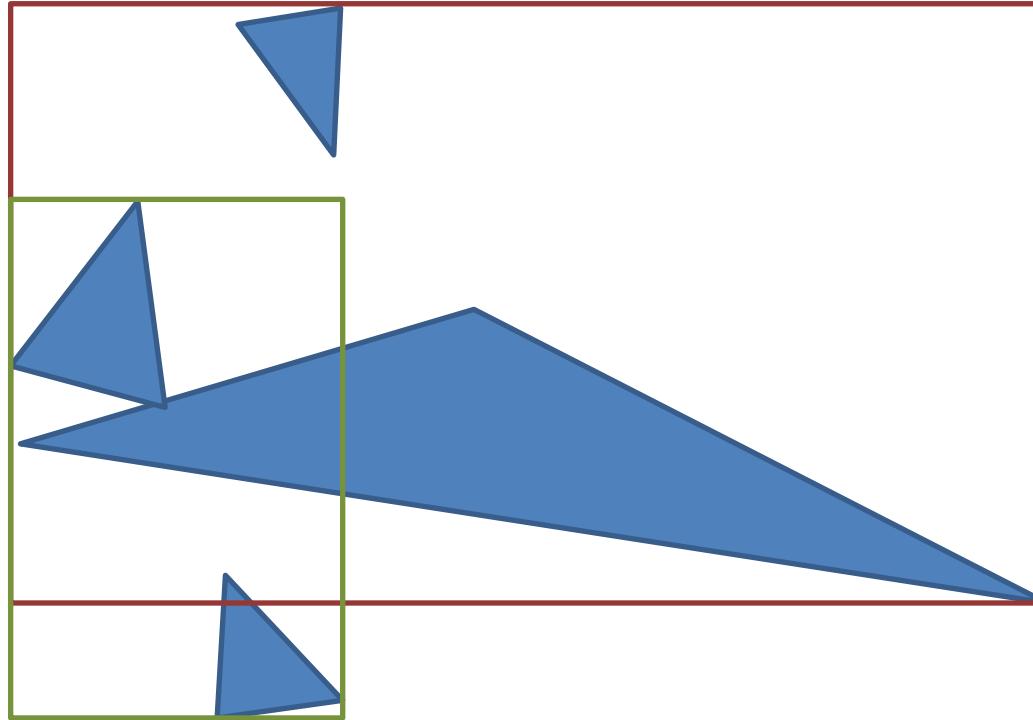
Lather, rinse and repeat. Terminate when a node contains few intersectables (I used 4, which worked well)

Construction



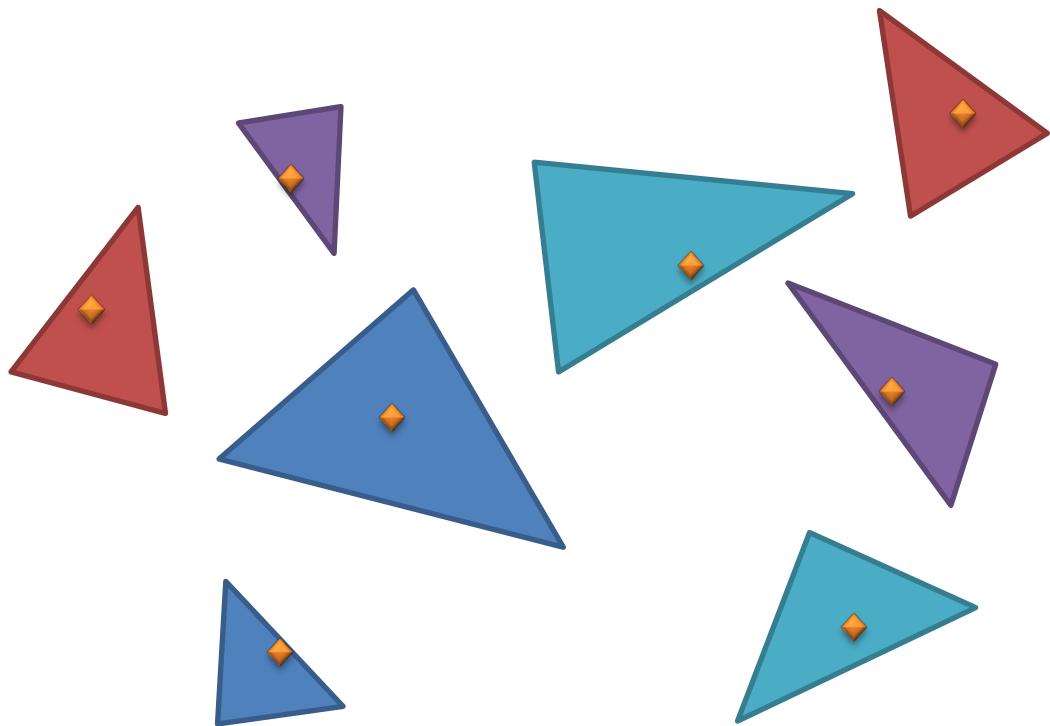
There is a hazard in getting all intersectables on one side – we could end up with empty nodes!

Construction



If this happens, you can, for example revert to median or mean splitting (*median split is depicted above*)

Construction



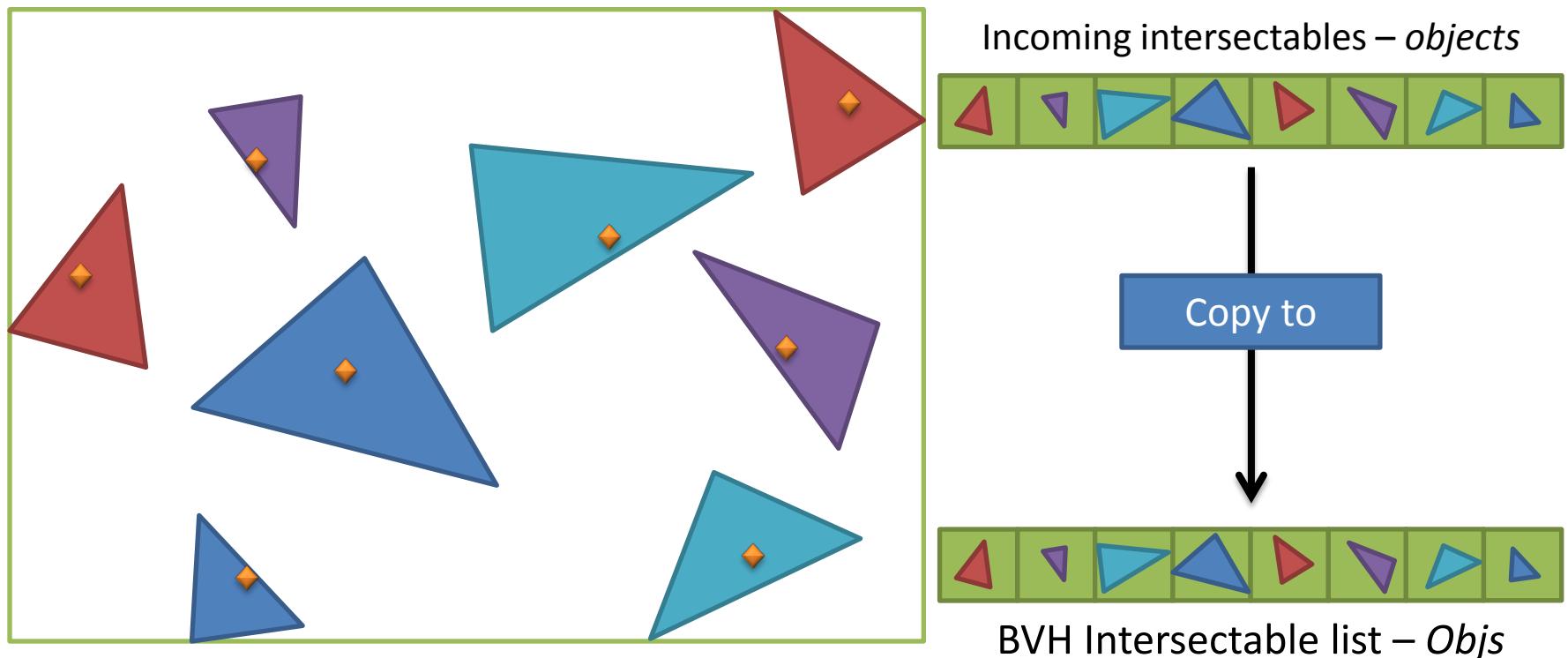
Now that you know the general concepts of a BVH, we will discuss *in-depth* how we keep track of our nodes and intersectables throughout the construction process.

Node class

- BVH node class (inner class of *BVHAccelerator*)

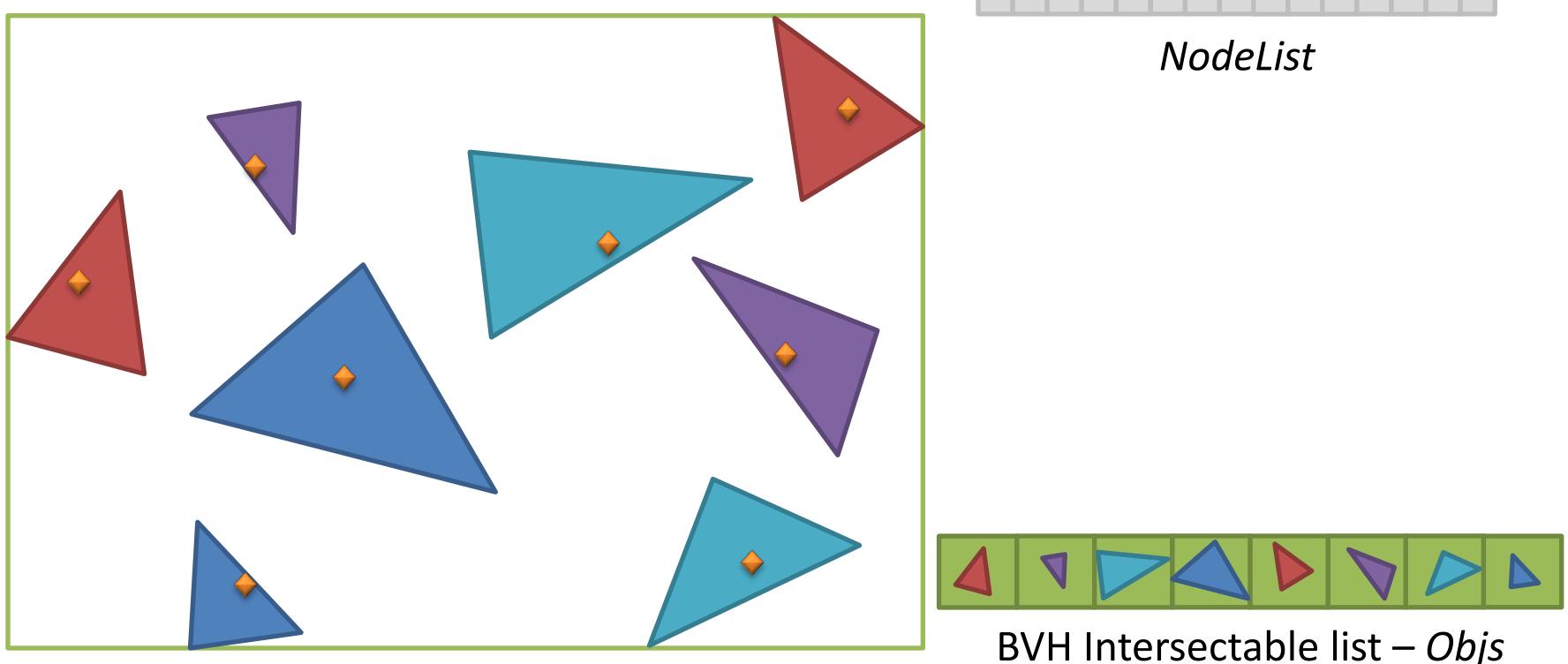
```
class BVHNode {  
    private:  
        AABB bbox;  
        bool leaf;  
        unsigned int n_objs;  
        unsigned int index;    // if leaf == false: index to left child node,  
                               // else if leaf == true: index to first Intersectable in Objs vector  
    public:  
        void setAABB(AABB &bbox_) {...}  
        void makeLeaf(unsigned int index_, unsigned int n_objs_) {...}  
        void makeNode(unsigned int left_index_, unsigned int n_objs) {...}  
            // n_objs in makeNode is for debug purposes only, and may be omitted later on  
  
        bool isLeaf() { return leaf; }  
        unsigned int getIndex() { return index; }  
        unsigned int getNOBJs() { return n_objs; }  
        AABB &getAABB() { return bbox; };  
};
```

Construction



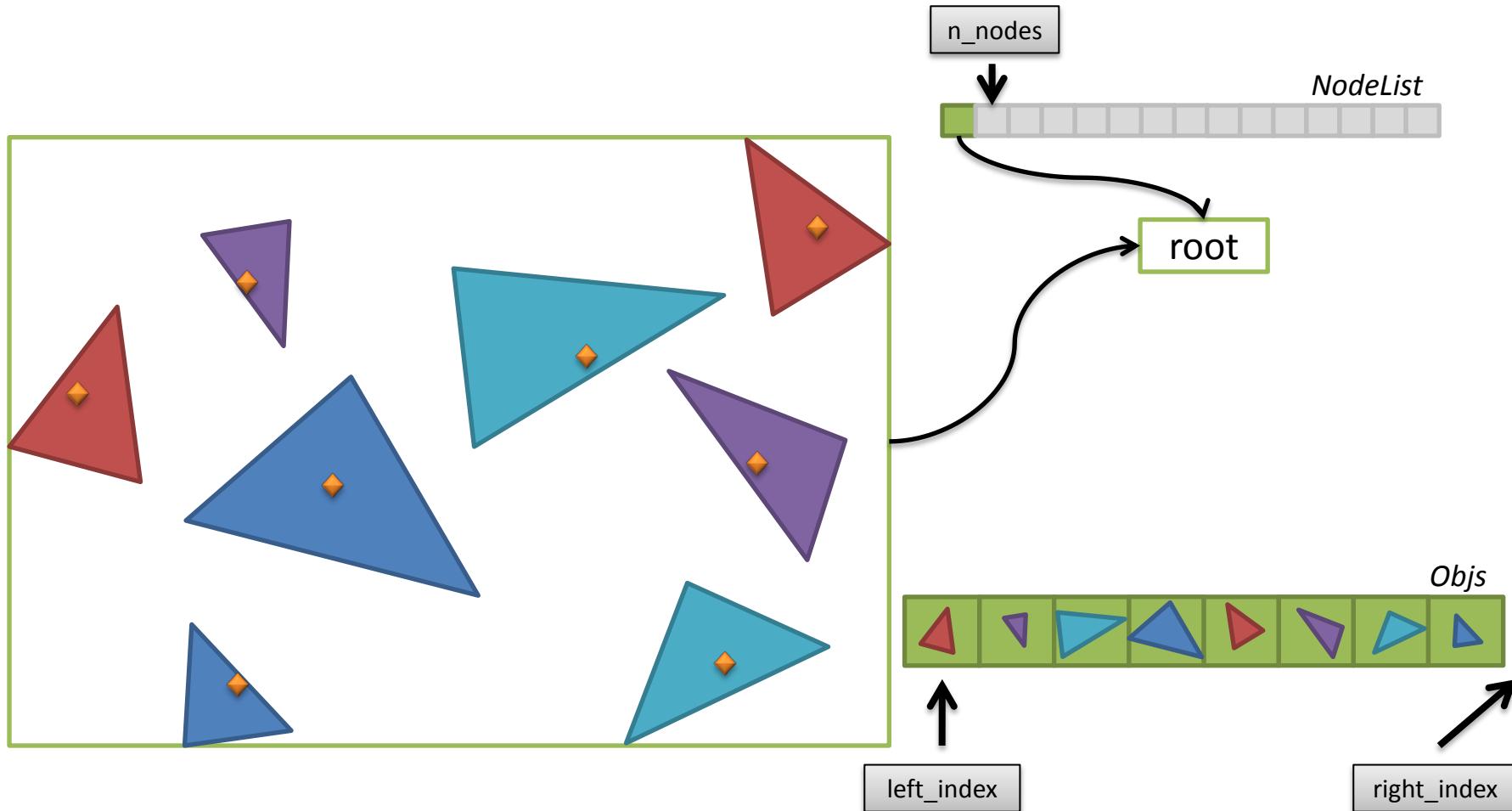
In the **build()**-function we get a list of unsorted *Intersectable* pointers, which we copy to a local vector. At the same time we calculate the **world bounding box**.

Construction



Set up a *NodeList* vector, which will hold our nodes. (We also happen to know that the number of nodes needed will be at most $2n - 1$ nodes, if the leaf nodes contain 1 element each).

Construction

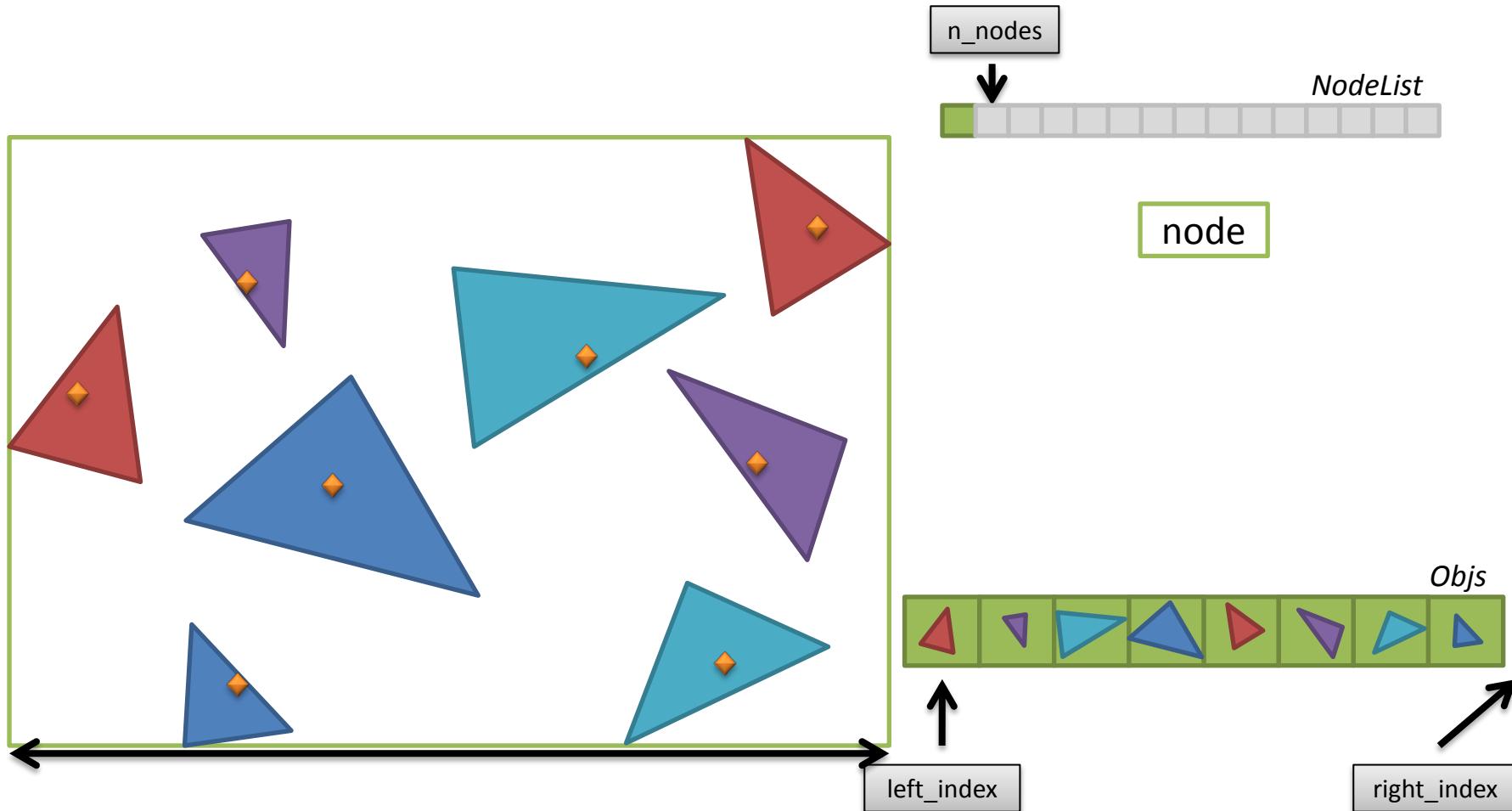


Set *left_index* = 0, *right_index* = *Objs.size()* and *n_nodes* = 1.

Set the world bounding box to the root node using `BVHNode::setAABB(box)`.

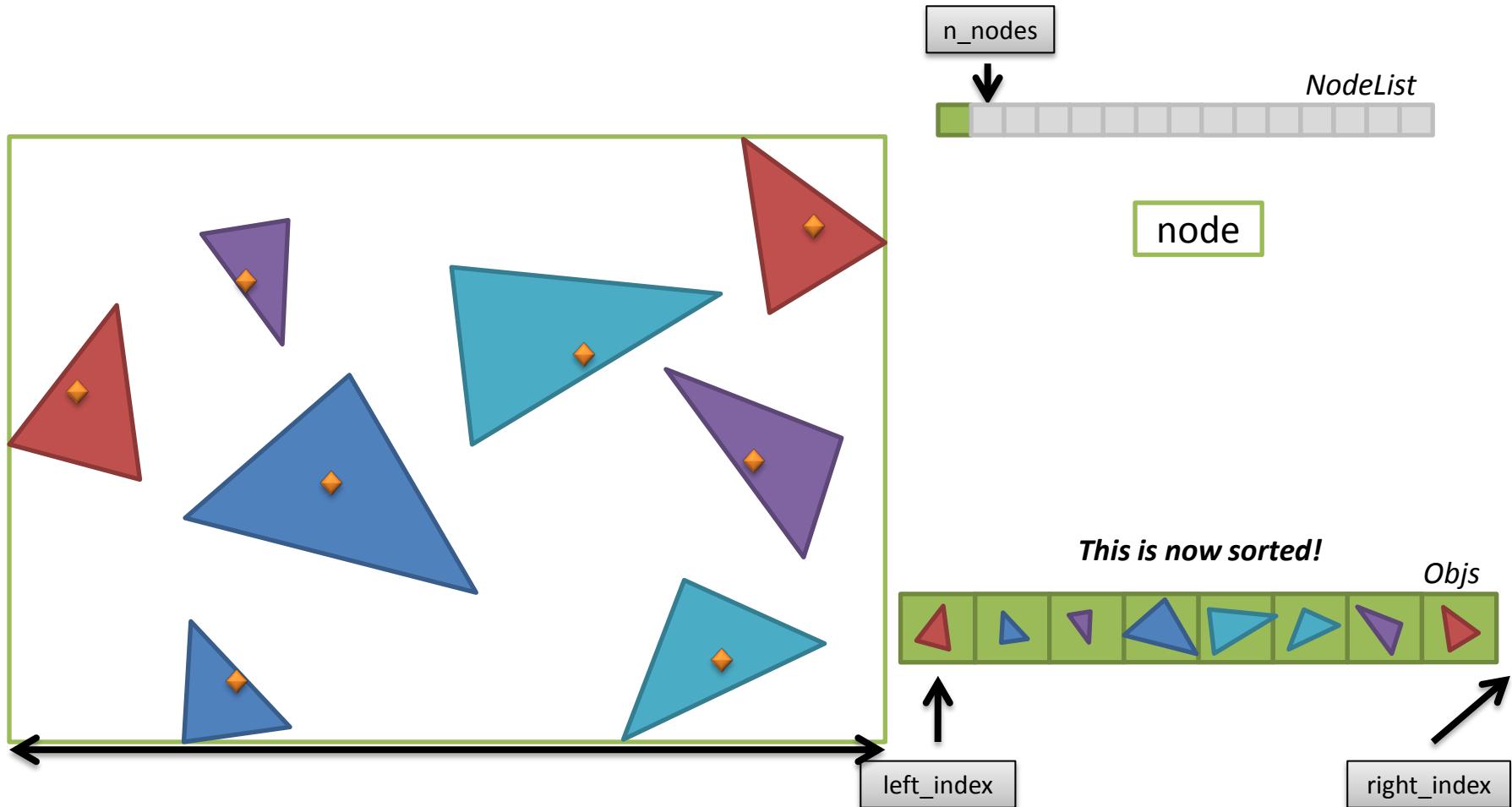
Then start building recursively.

Construction



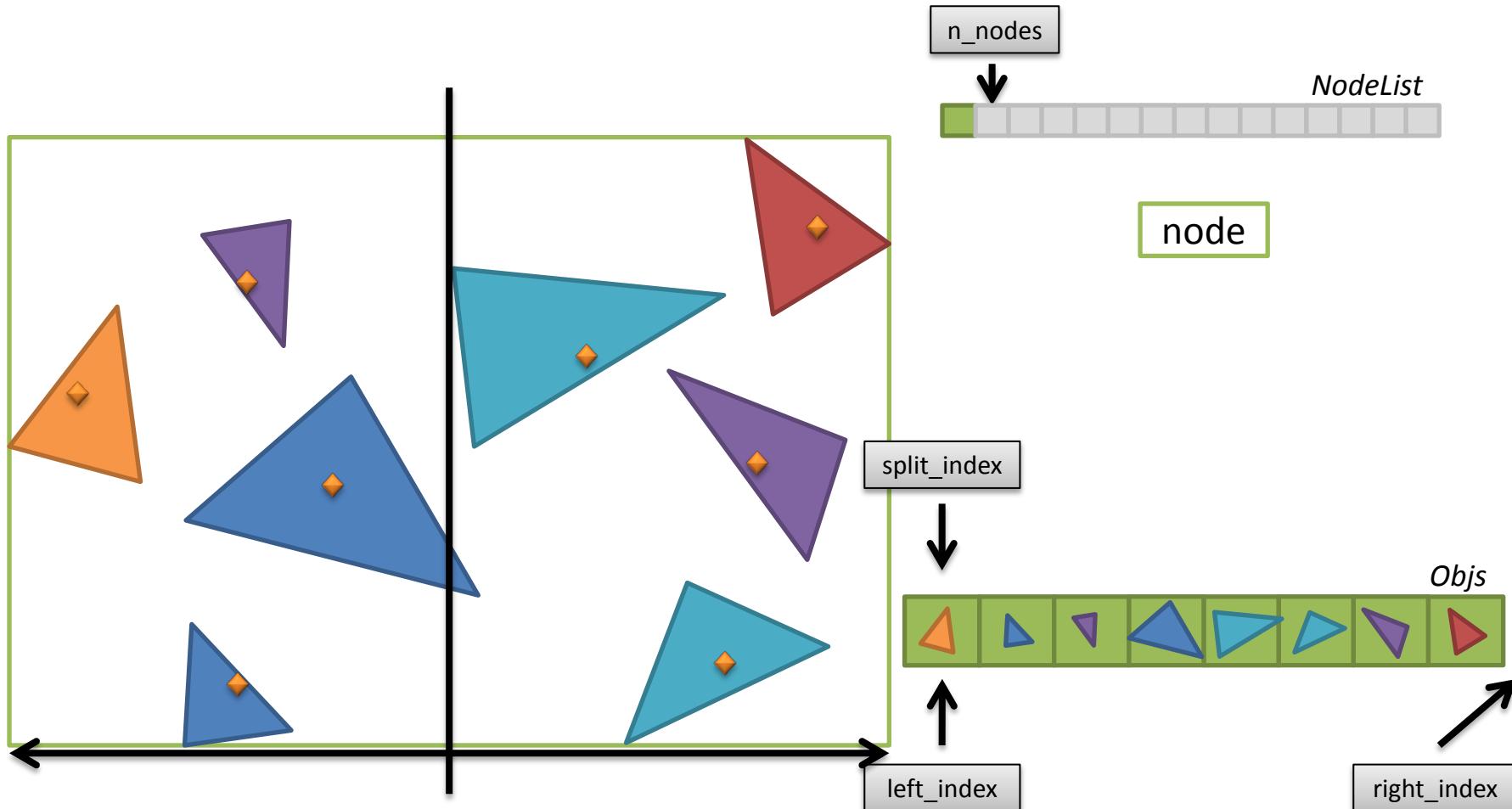
First, check if the number of intersectables is fewer than the threshold (let's say 2 in this case). It isn't.

Construction



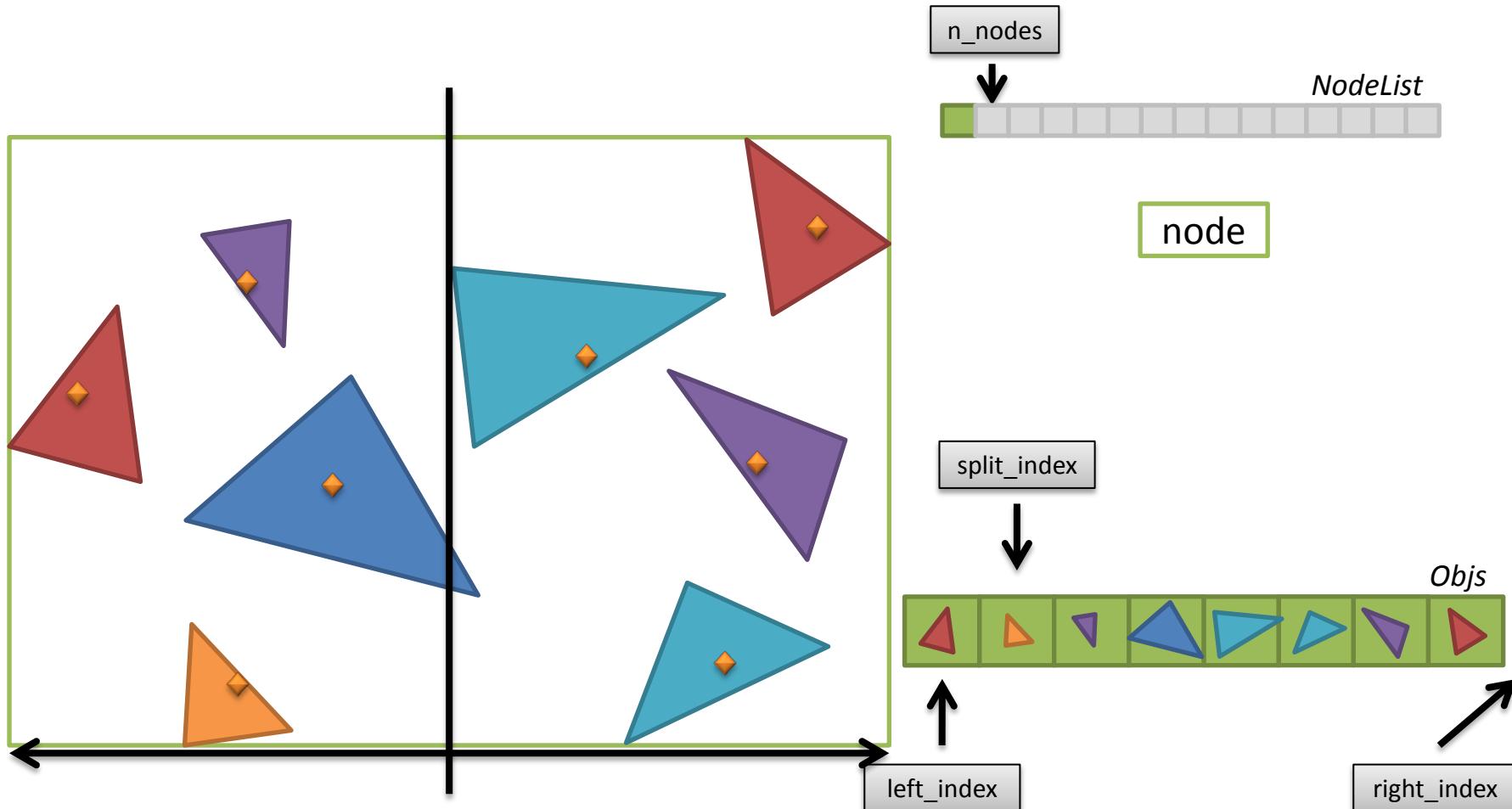
Find largest dimension d and sort the elements in that dimension.

Construction



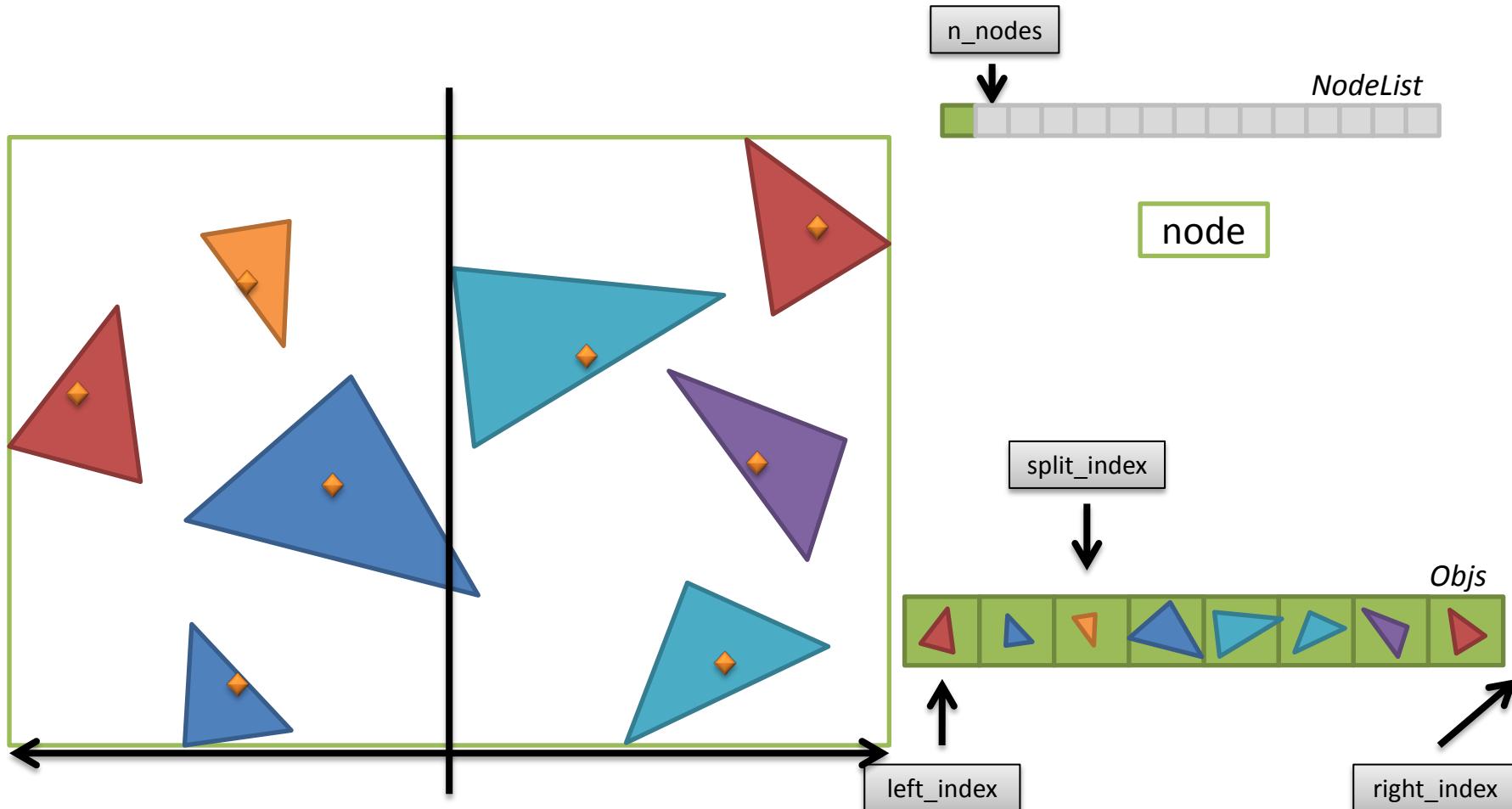
Find the *split_index*, where the mid point divides the primitives in a *left* and *right* side

Construction



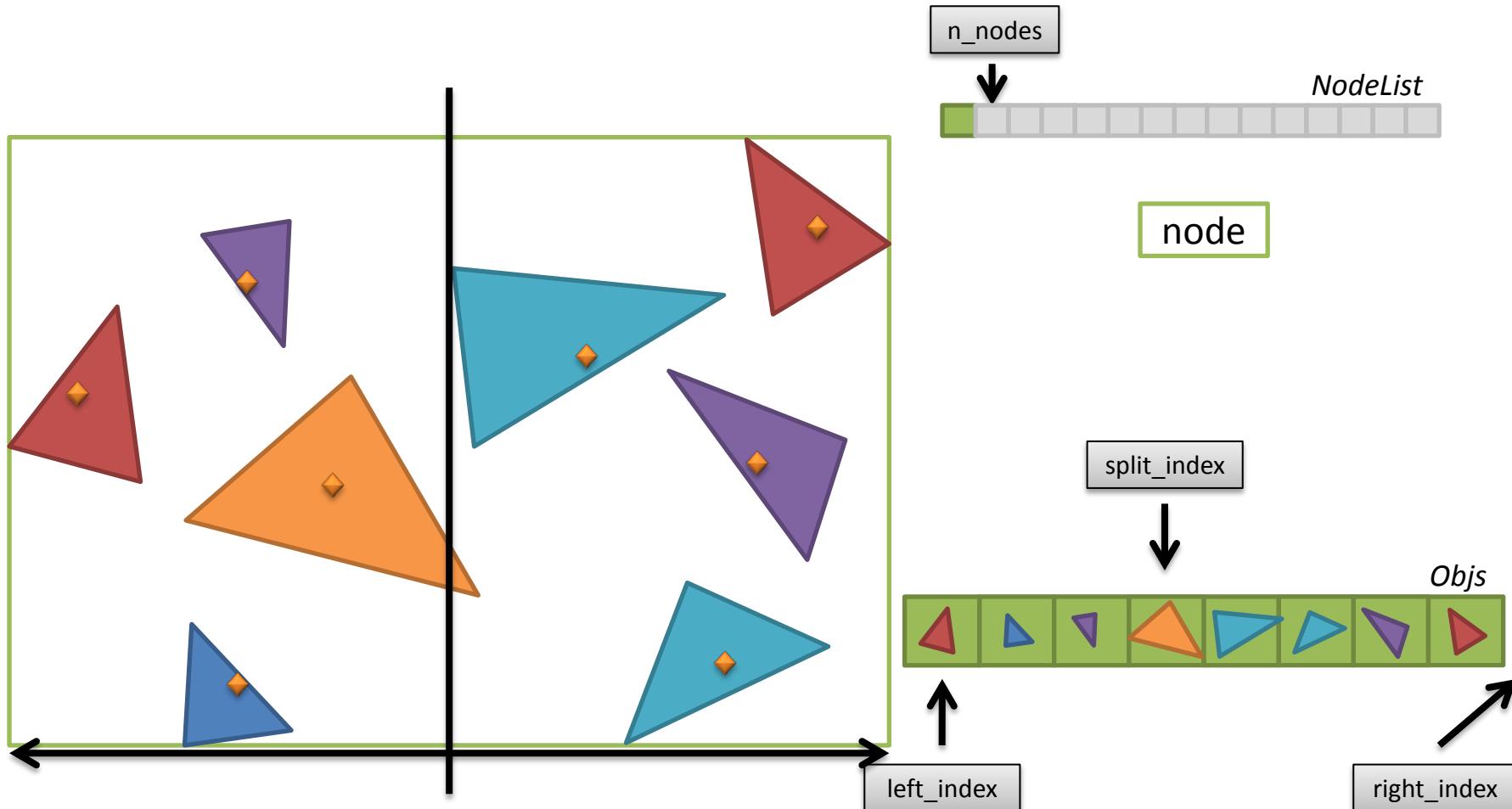
Find the *split_index*, where the mid point divides the primitives in a *left* and *right* side

Construction



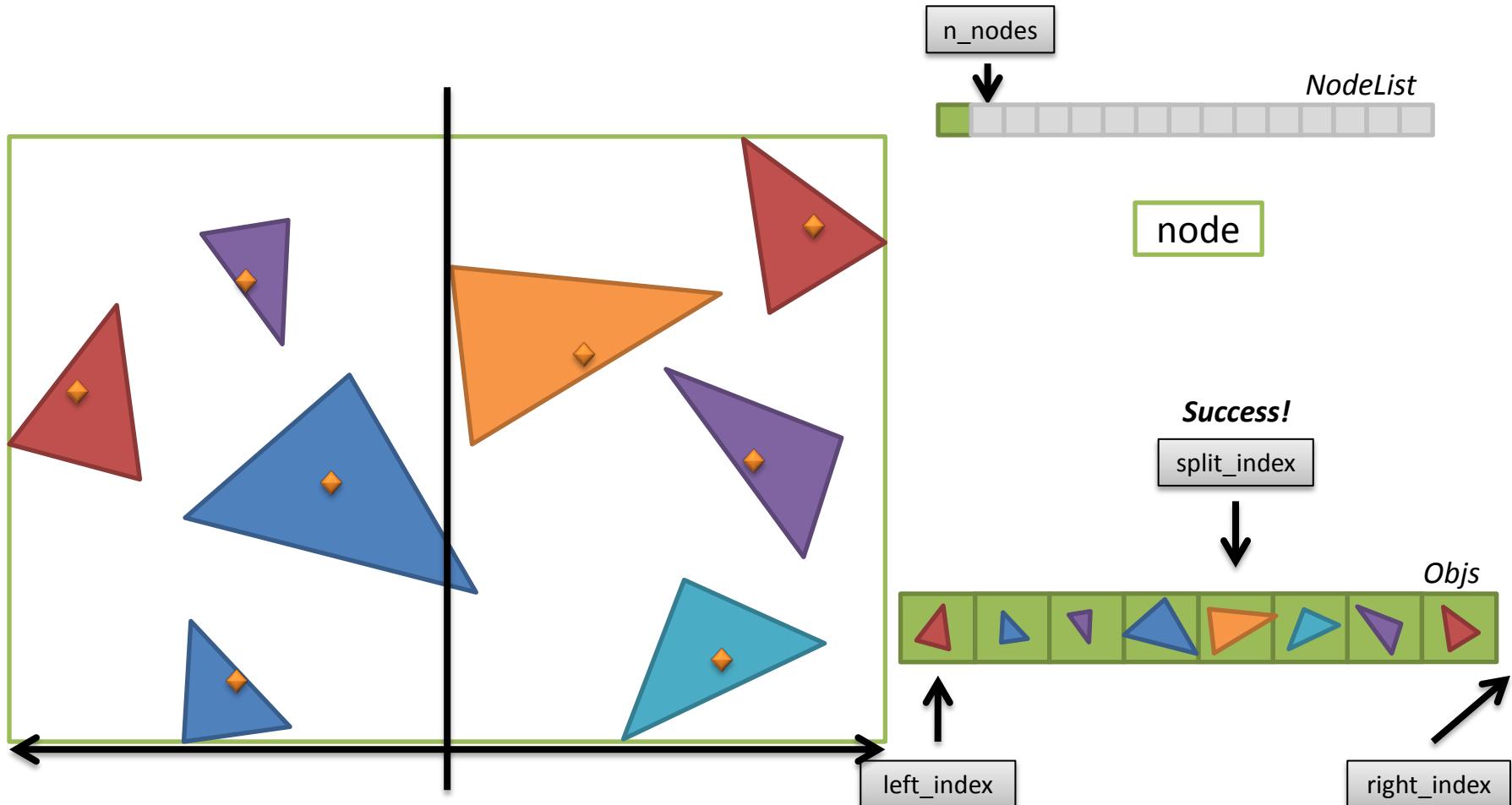
Find the *split_index*, where the mid point divides the primitives in a *left* and *right* side

Construction



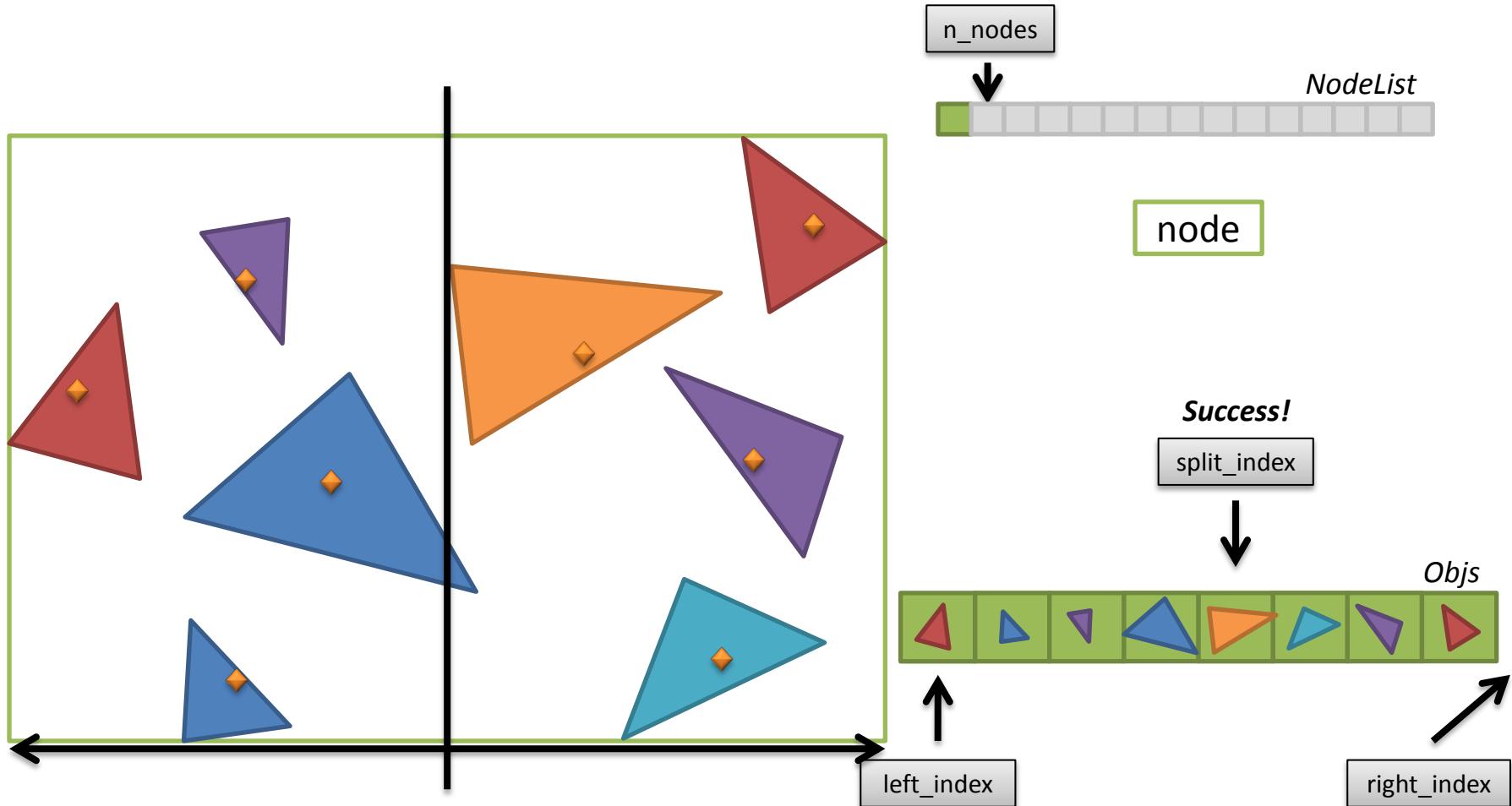
Find the *split_index*, where the mid point divides the primitives in a *left* and *right* side

Construction



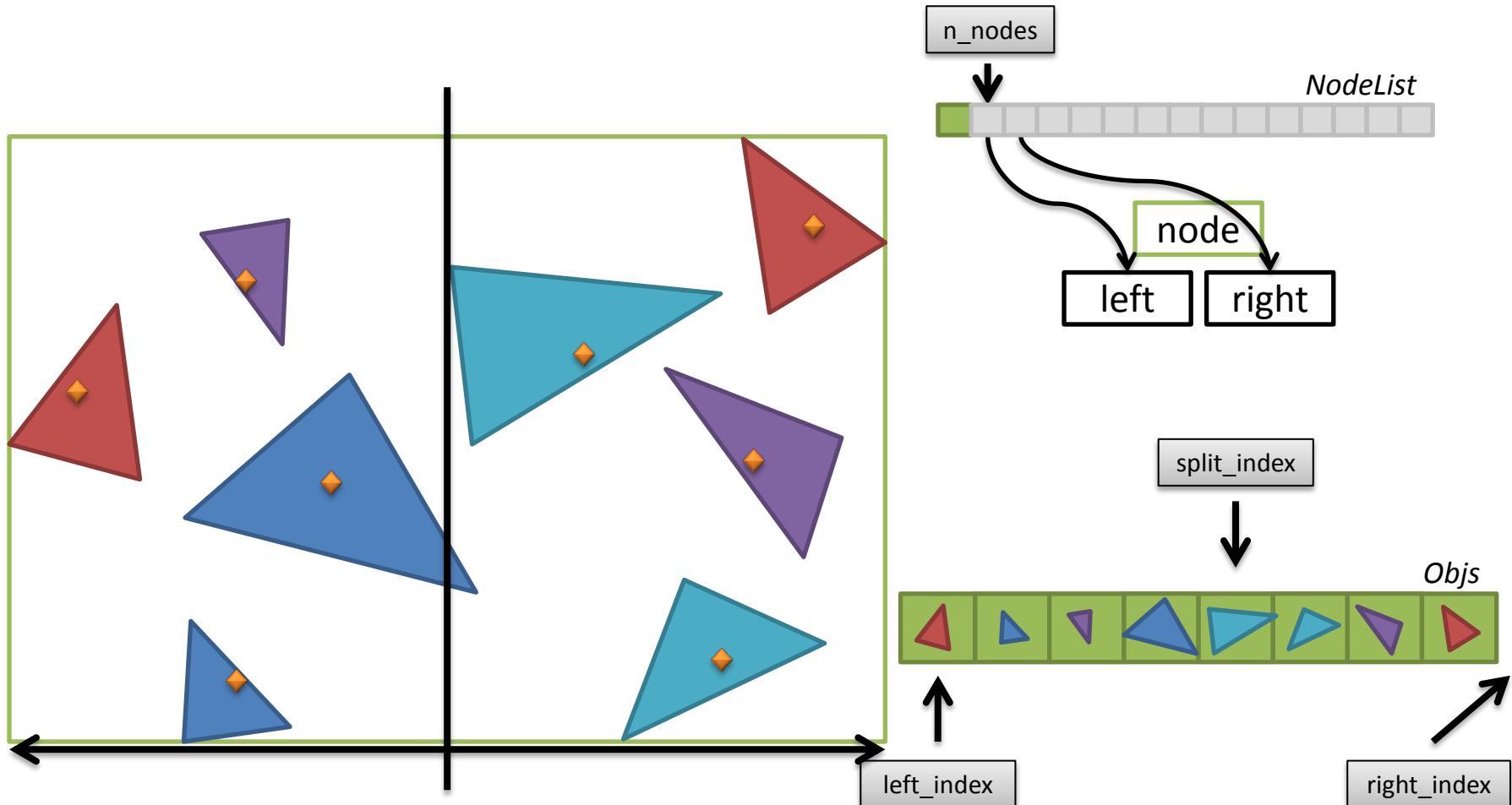
Find the *split_index*, where the mid point divides the primitives in a *left* and *right* side

Construction



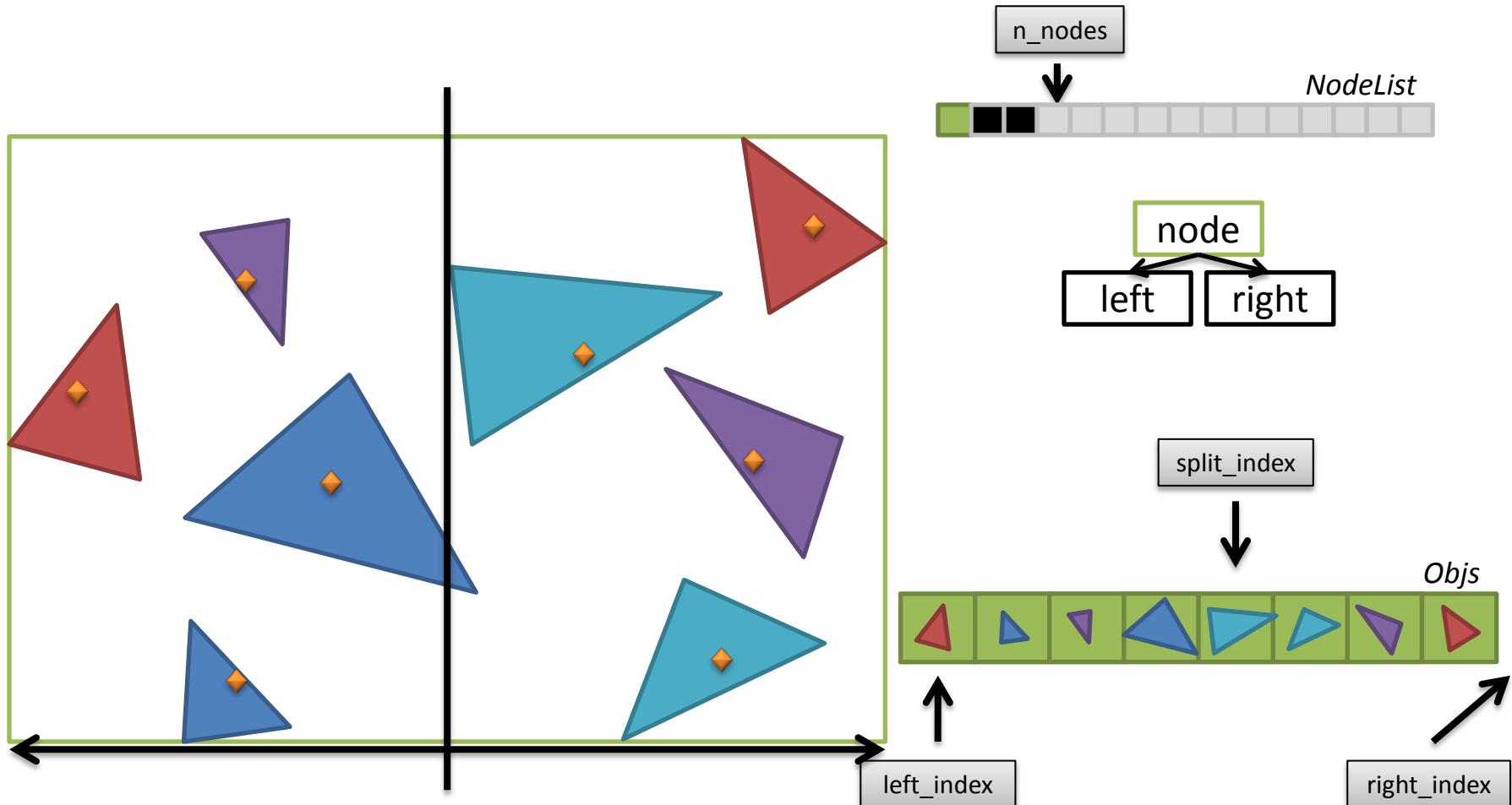
(We could have used binary search)

Construction



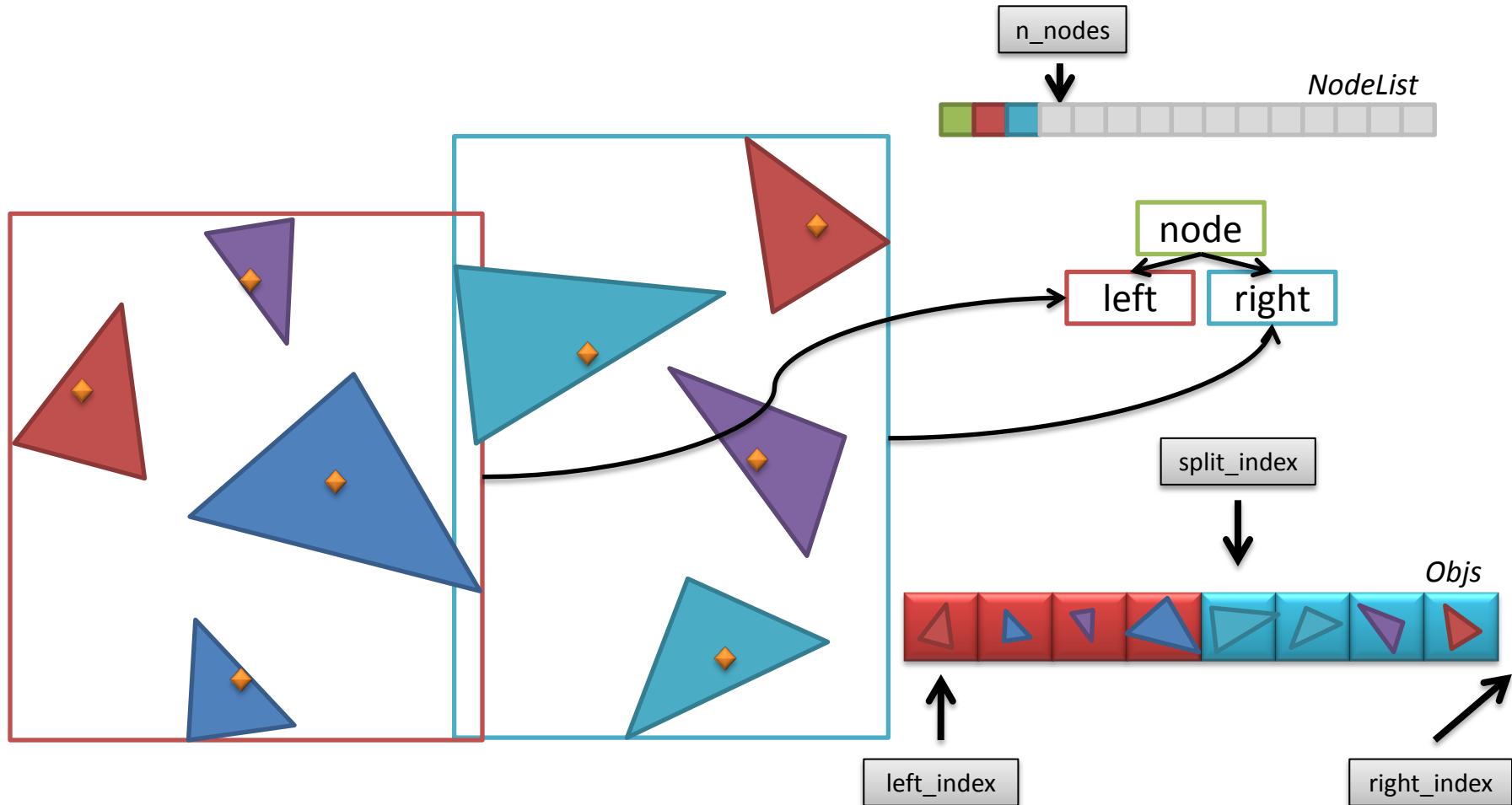
Allocate two new nodes (*left* and *right*) from the *NodeList*. The left node will have index ***n_nodes*** and the right one ***n_nodes + 1***.

Construction



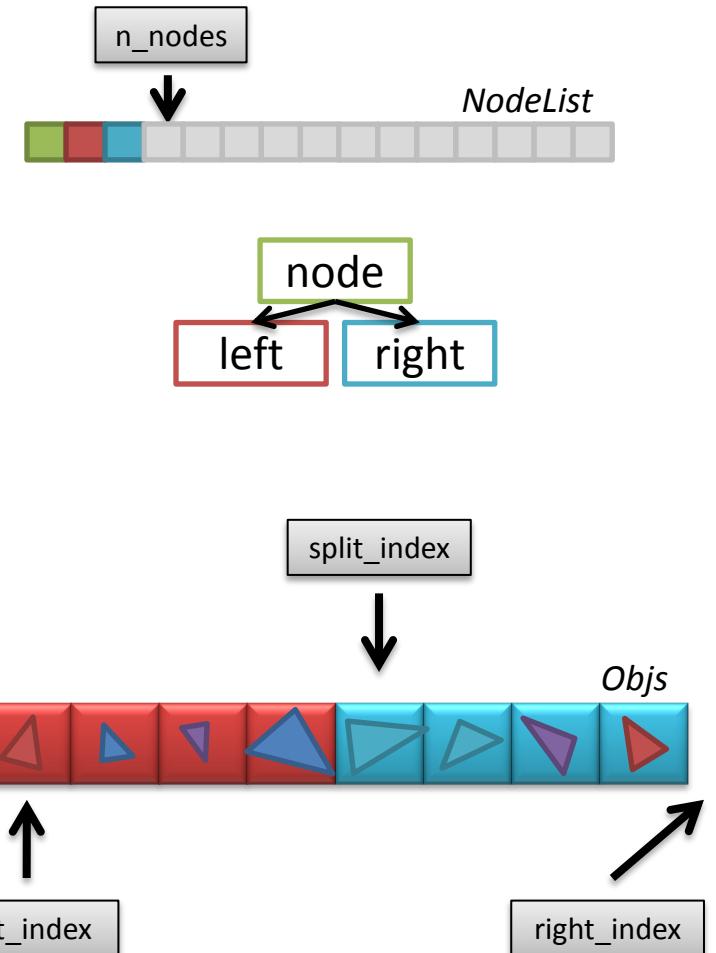
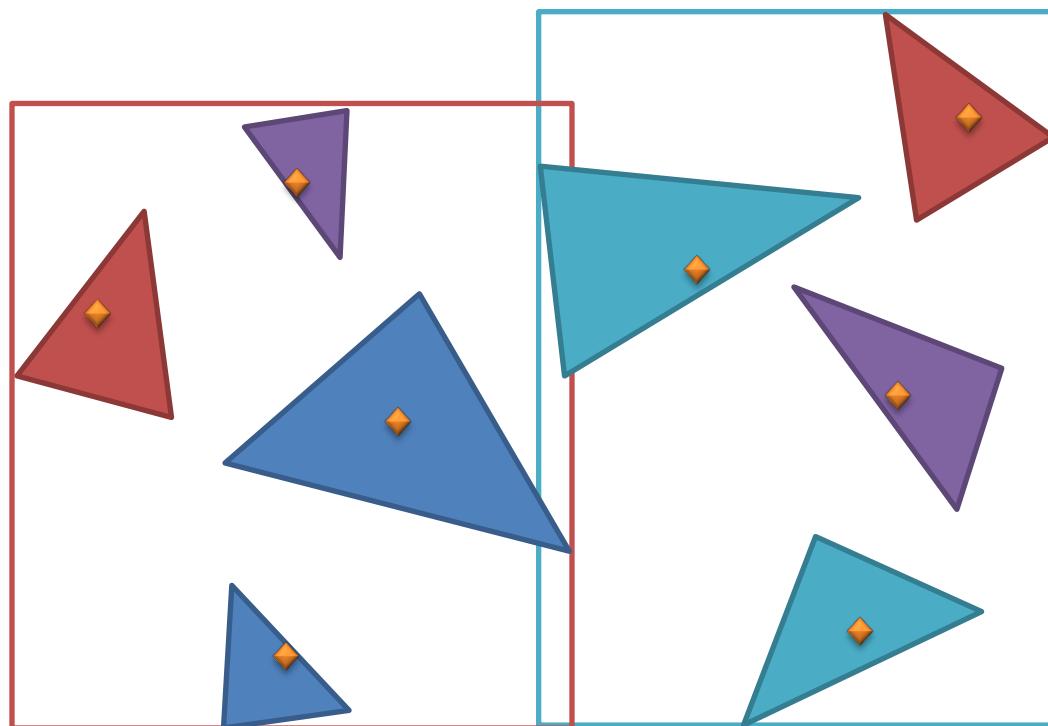
Initiate **node** (which is currently the root node) with
`BVHNode::makeNode(n_nodes)`.
(The index to the right node is always $n_nodes + 1$)

Construction



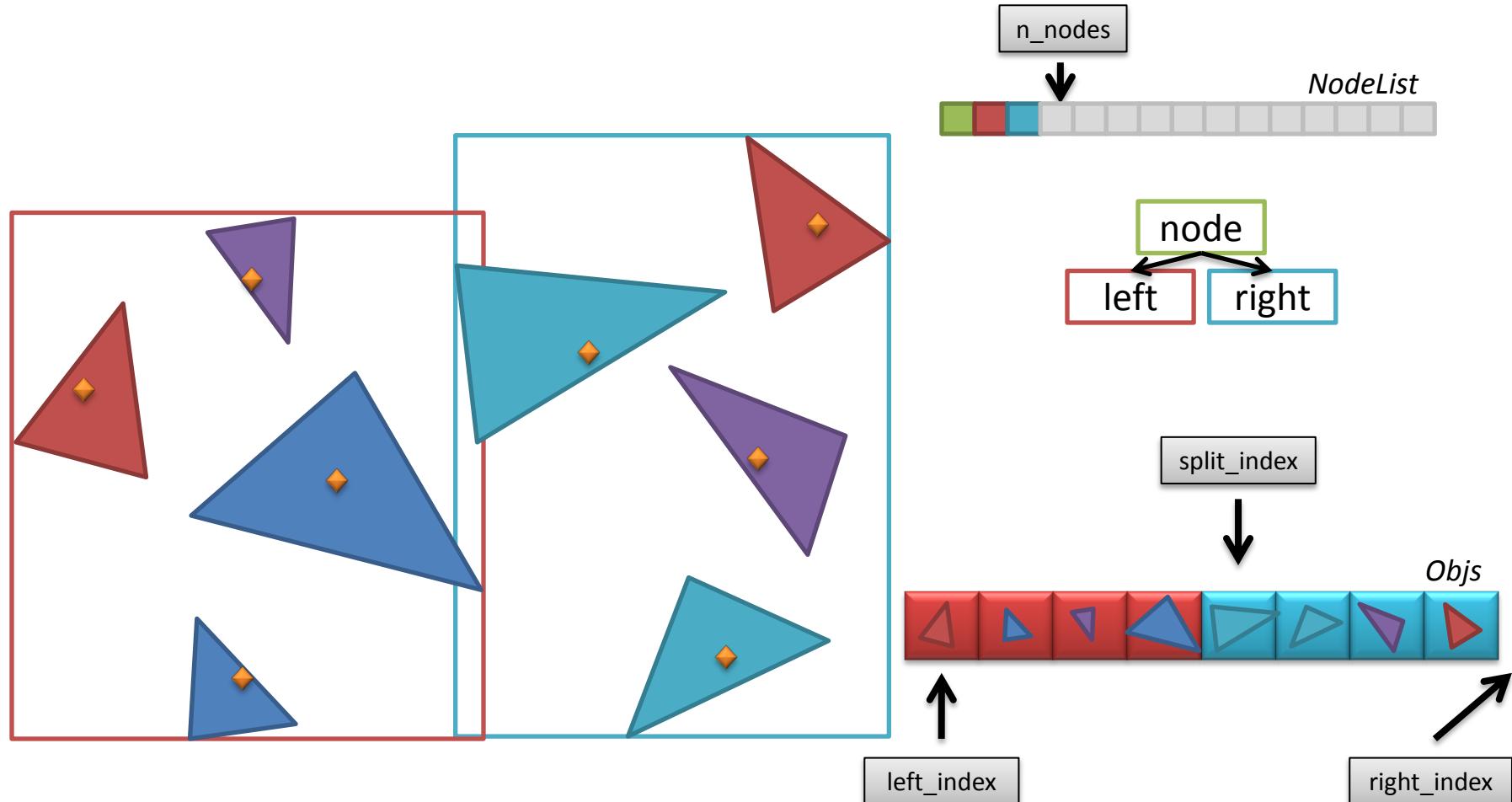
Calculate the bounding boxes for *left* and *right* and assign them to the two newly created nodes

Construction



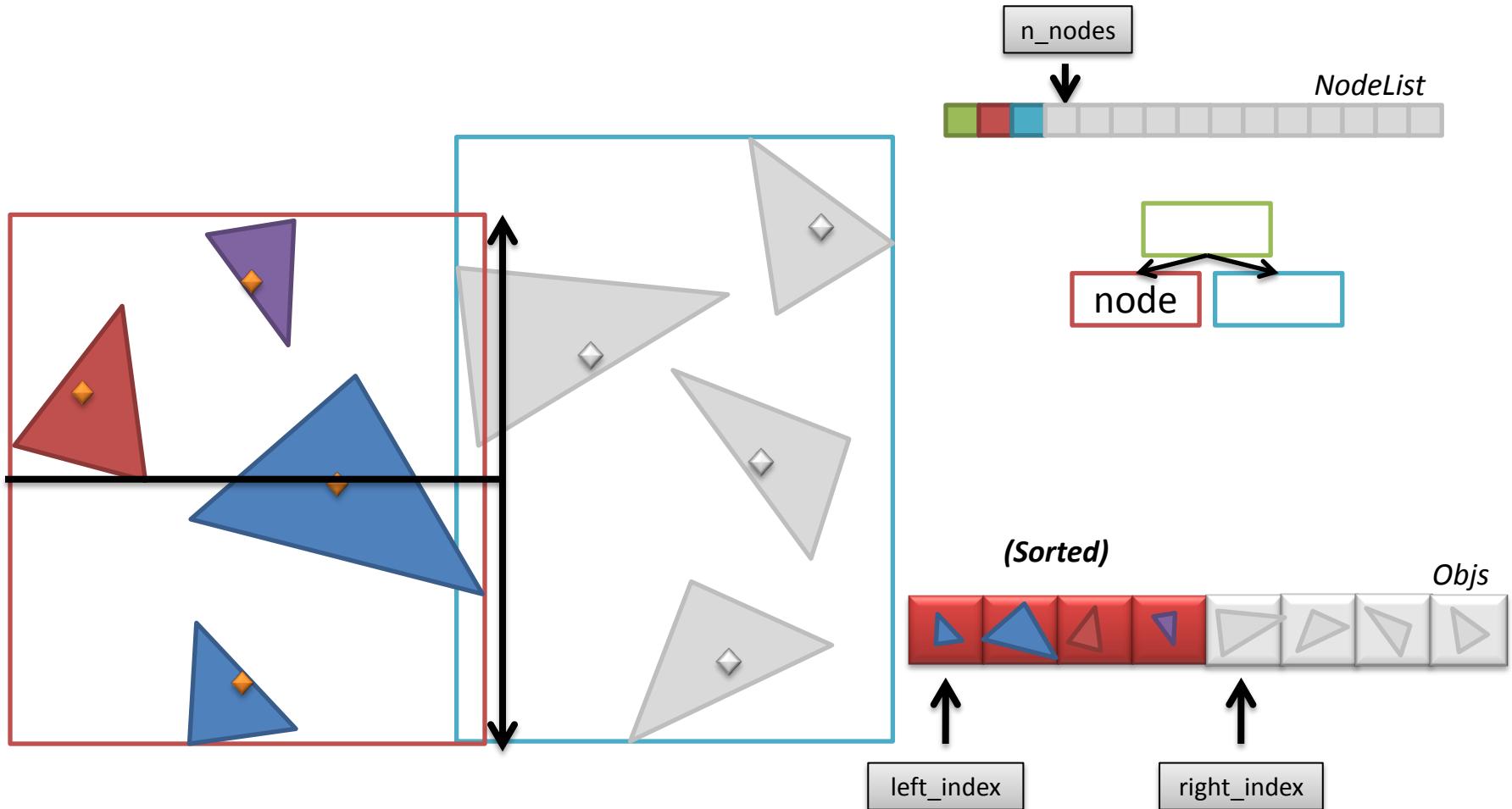
Call the **build_recursive()**-function for the *left* and then the *right* node.

Construction



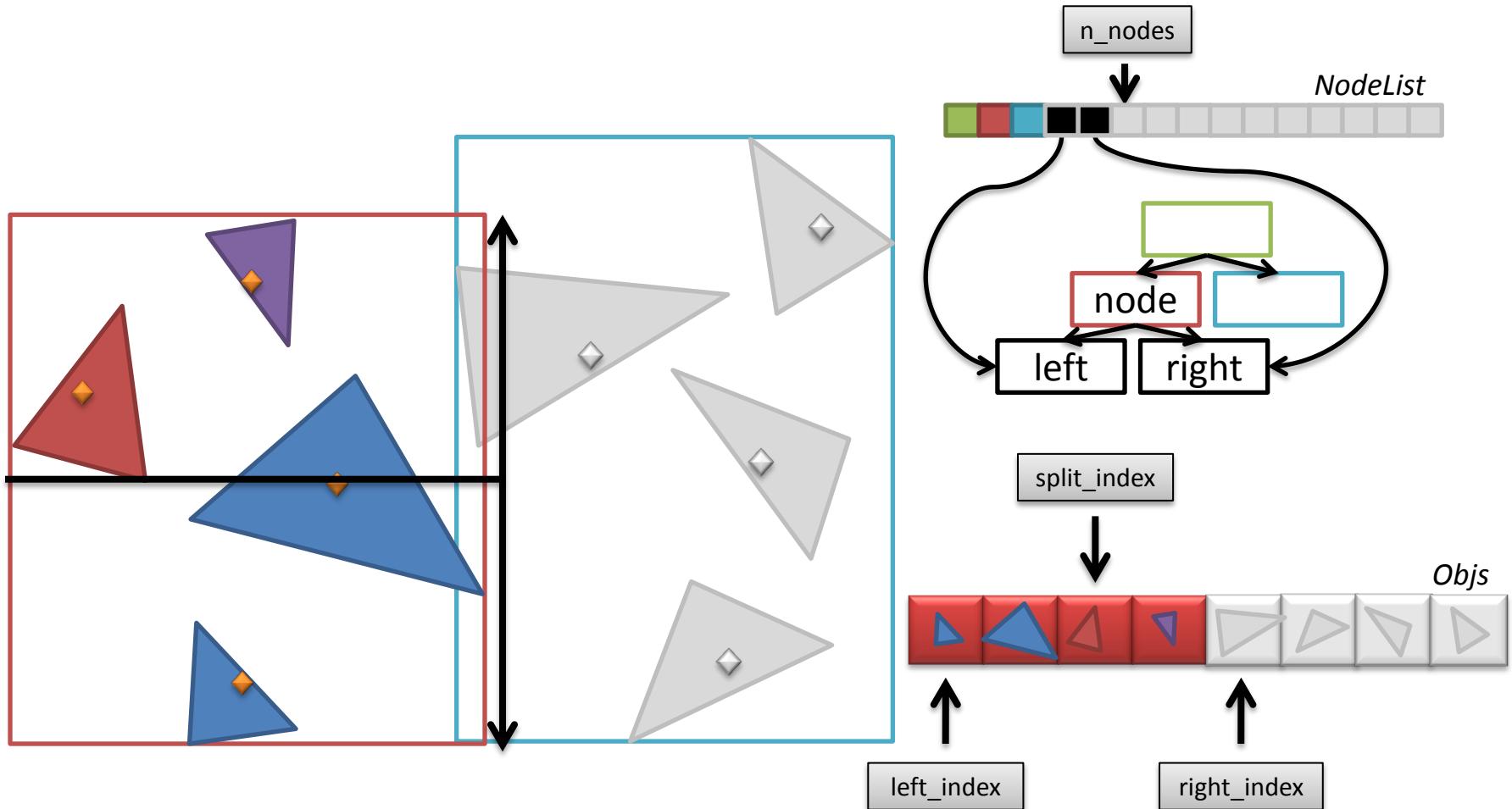
Be sure to pass the correct **Obj**-vector **indices** to the *left* and *right* nodes. The *left* node is now responsible for $[left_index, split_index)$ and the *right* node for $[split_index, right_index)$.

Construction

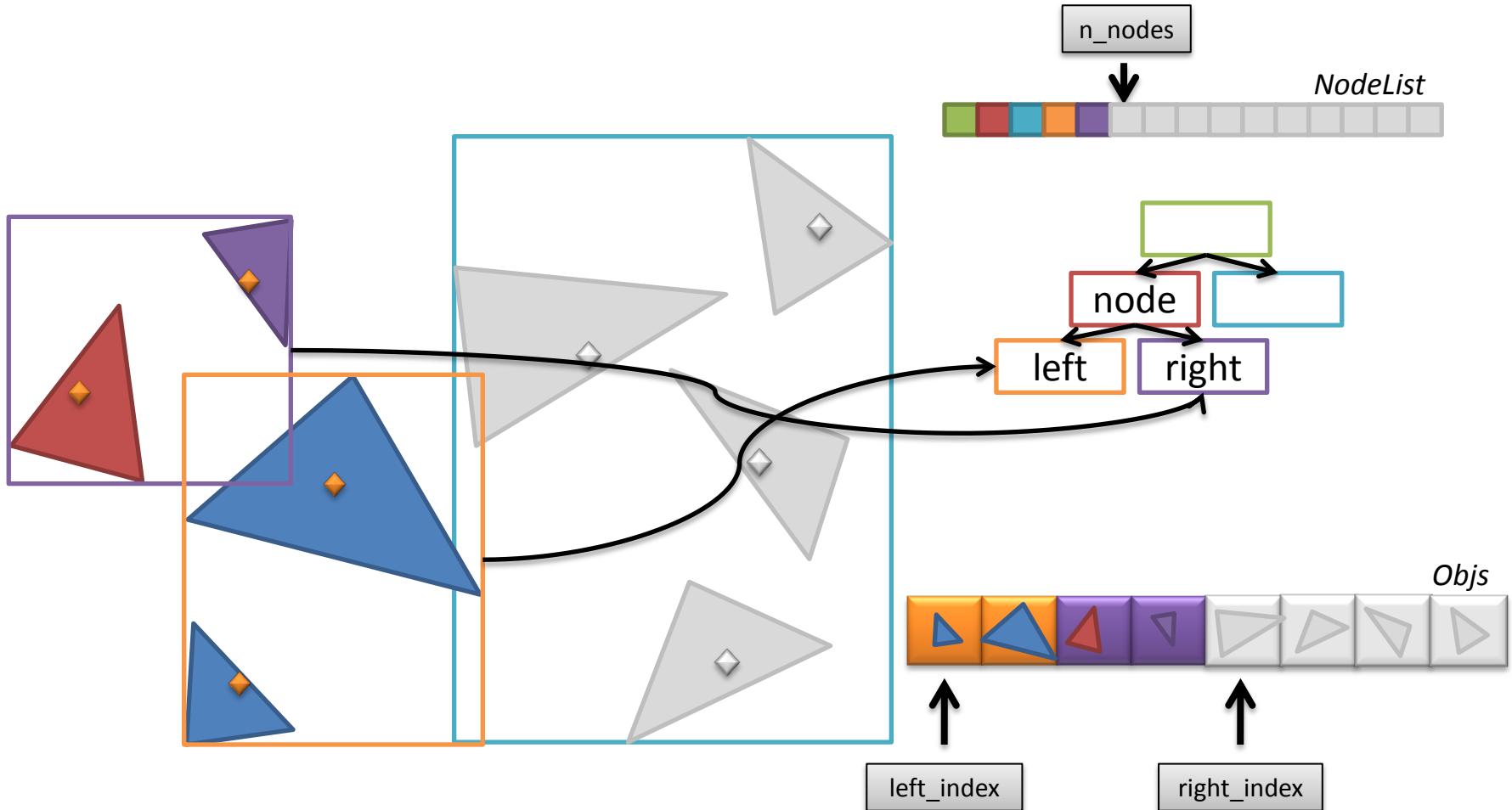


Processing of the left node yields the following result...

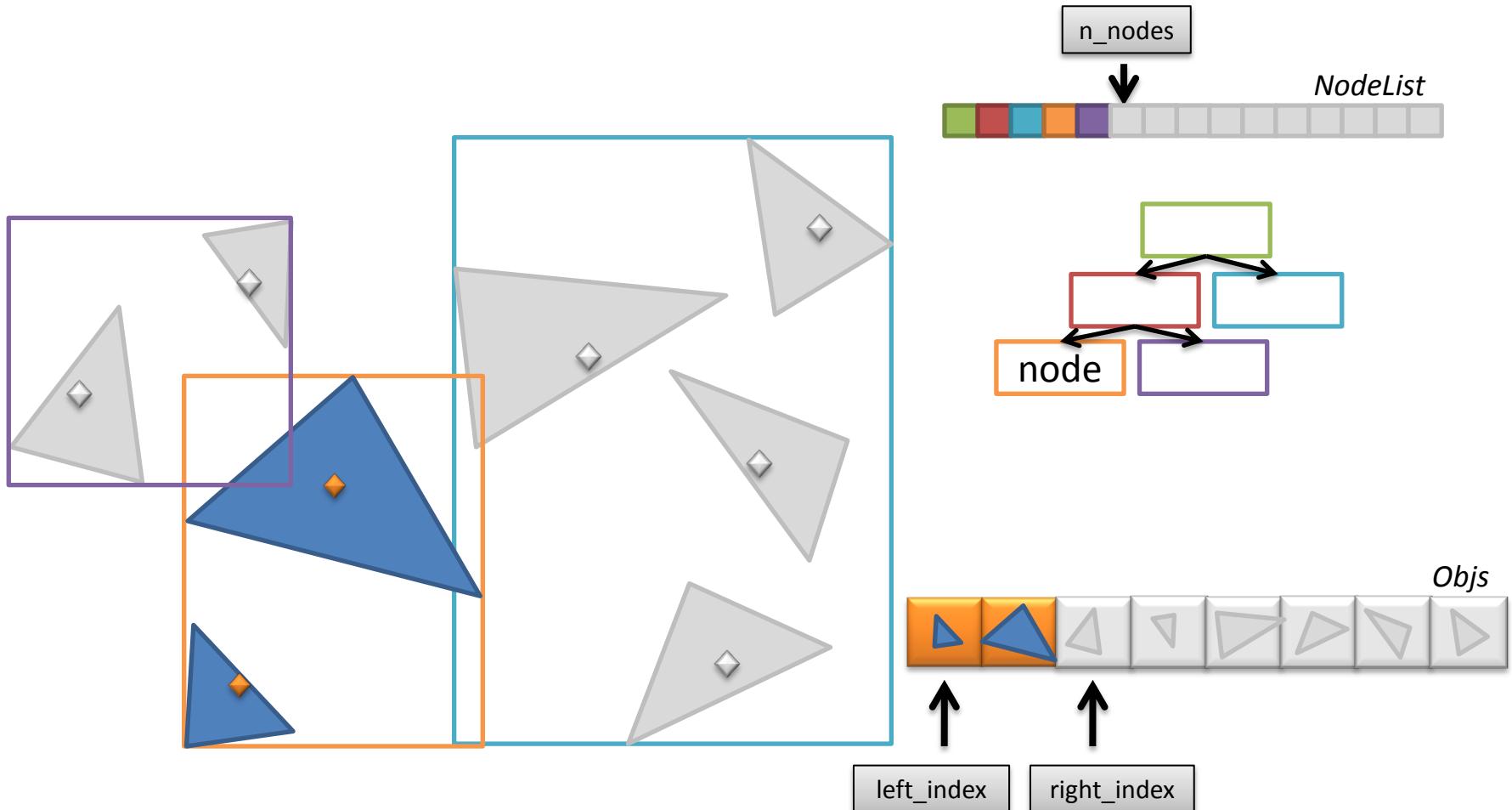
Construction



Construction

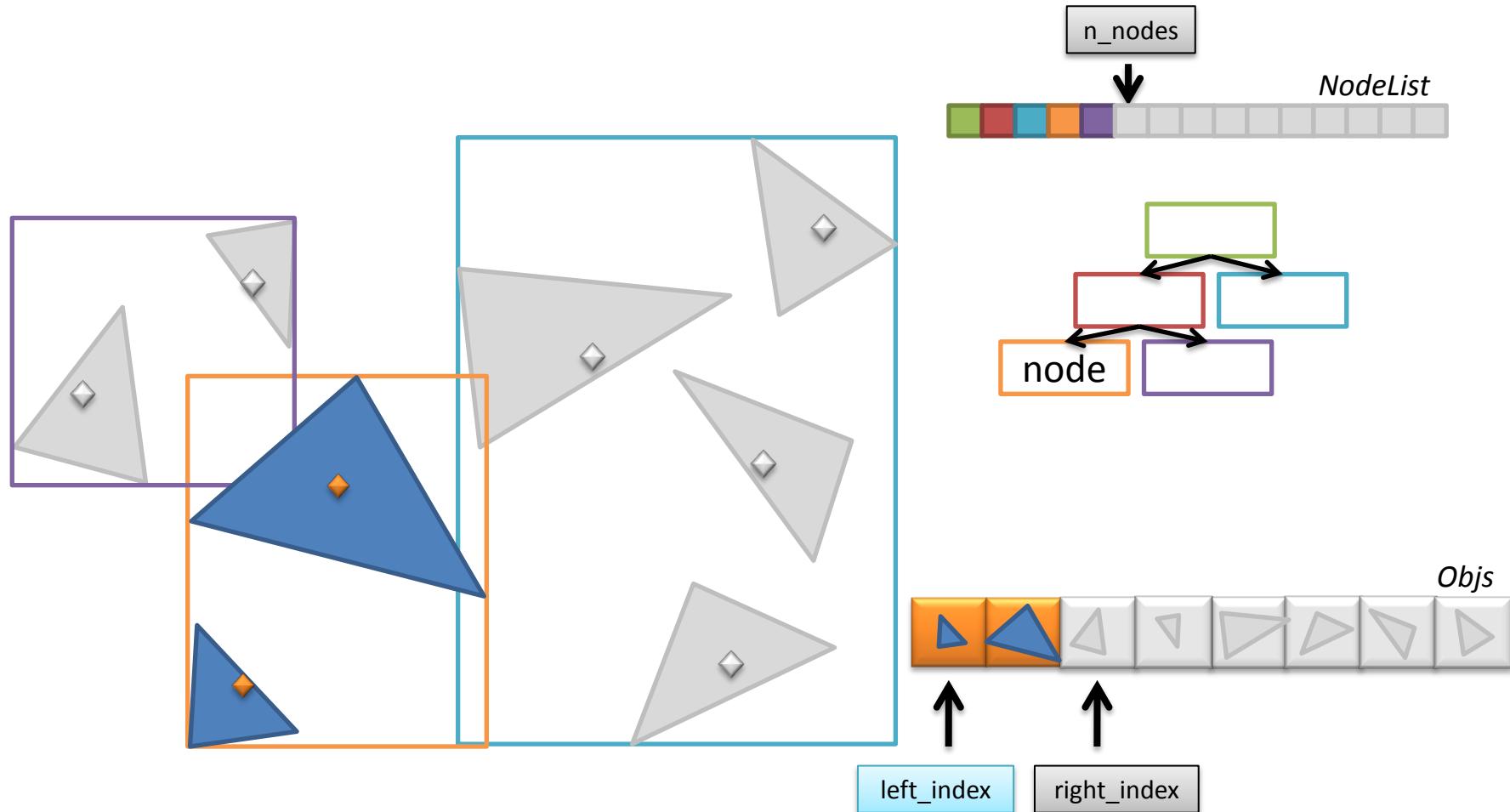


Construction



Finally we have only 2 primitives in the node:
(right_index – left_index <= 2)

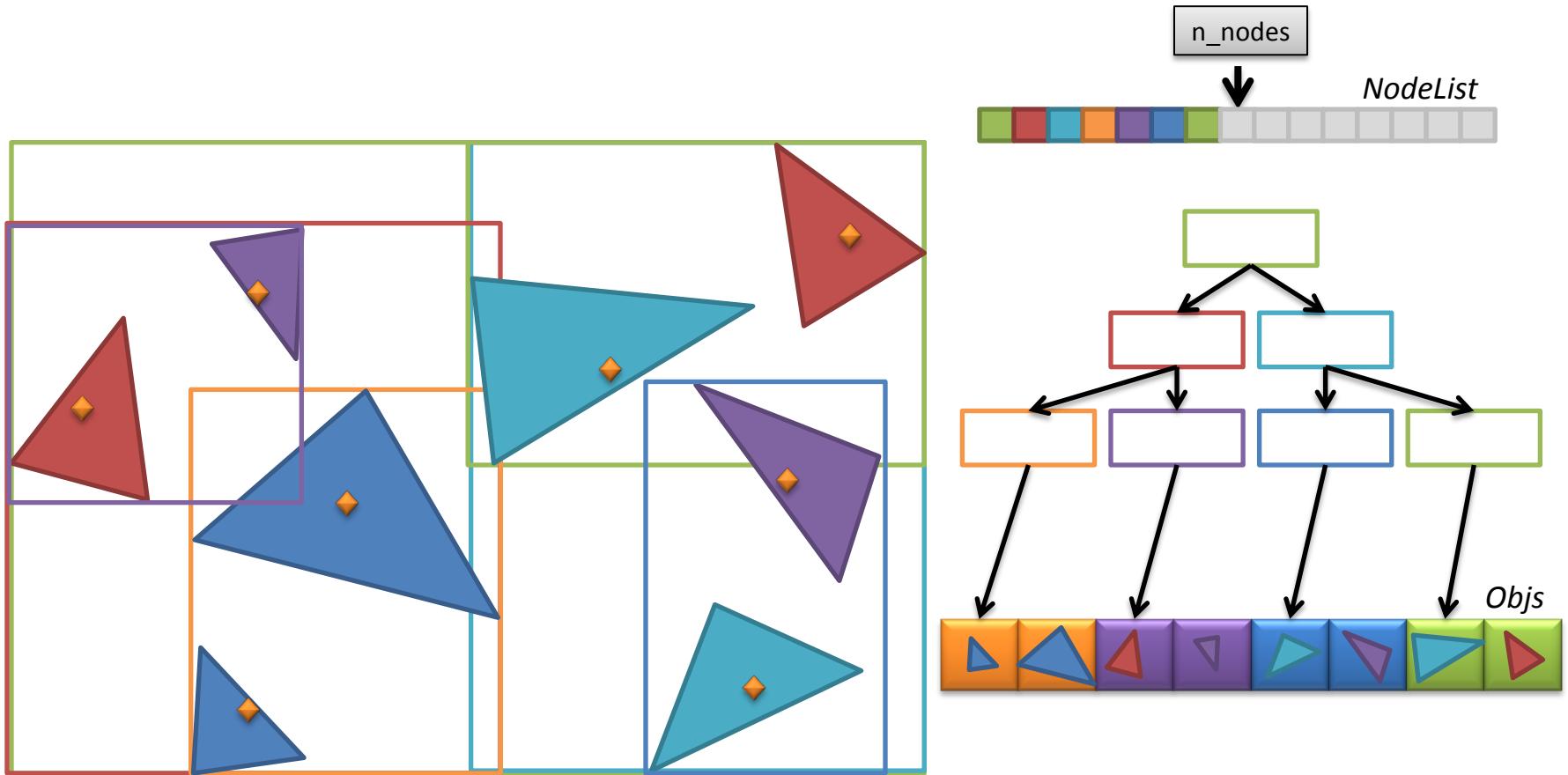
Construction



We initiate the current node as a leaf using

```
void BVHNode::makeLeaf(left_index, right_index - left_index);
```

Construction



This is what we end up with when we're done.

Construction, Pseudo code

Setup

```
void build(const std::vector<Intersectable *> &objects)
```

- Create new vector for *Intersectable* pointer copies
- Create new vector for the nodes
- Create Root node
- worldBox = AABB(); // world bounding box
- For each intersectable[i] in *objects*
 - worldBox.include(intersectable[i] bounding box)
 - Obj.push_back(intersectable[i])
- EndFor
- Set world bounding box to root node
- build_recursive(0, Obj.size(), root, 0);

The declaration was: `void build_recursive(int left_index, int right_index, BVHNode *node, int depth);`

Construction, Pseudo code

Recursion

```
void build_recursive(int left_index, int right_index, BVHNode *node, int depth)
```

- If ((right_index – left_index) <= Threshold || (*other termination criteria*))
 - Initiate current node as a leaf with primitives from Objs[left_index] to Objs[right_index]
- Else
 - Split intersectables into *left* and *right* by finding a *split_index*
 - Make sure that neither *left* nor *right* is completely empty
 - Calculate bounding boxes of *left* and *right* sides
 - Create two new nodes, *leftNode* and *rightNode* and assign bounding boxes
 - Initiate current node as an interior node with *leftNode* and *rightNode* as children
 - build_recursive(left_index, split_index, leftNode, depth + 1)
 - build_recursive(split_index, right_index, rightNode, depth + 1)
- EndIf

Construction

- Sorting in C++
 - *This is what I did at least...*

```
#include <algorithm>  
// ...  
ComparePrimitives cmp;  
cmp.sort_dim = 0;           // x = 0, y = 1, z = 2  
std::sort(objs.begin() + from_index, objs.begin() + to_index, cmp);
```

- *ComparePrimitives??*

Construction

- Sorting in C++

```
class ComparePrimitives {  
public:  
    bool operator() (Intersectable *a, Intersectable *b) {  
        AABB box;  
        a->getAABB(box);  
        float ca = (box.mMax(sort_dim) + box.mMin(sort_dim)) * 0.5f;  
        b->getAABB(box);  
        float cb = (box.mMax(sort_dim) + box.mMin(sort_dim)) * 0.5f;  
        return ca < cb;  
    }  
  
    int sort_dim;  
};
```

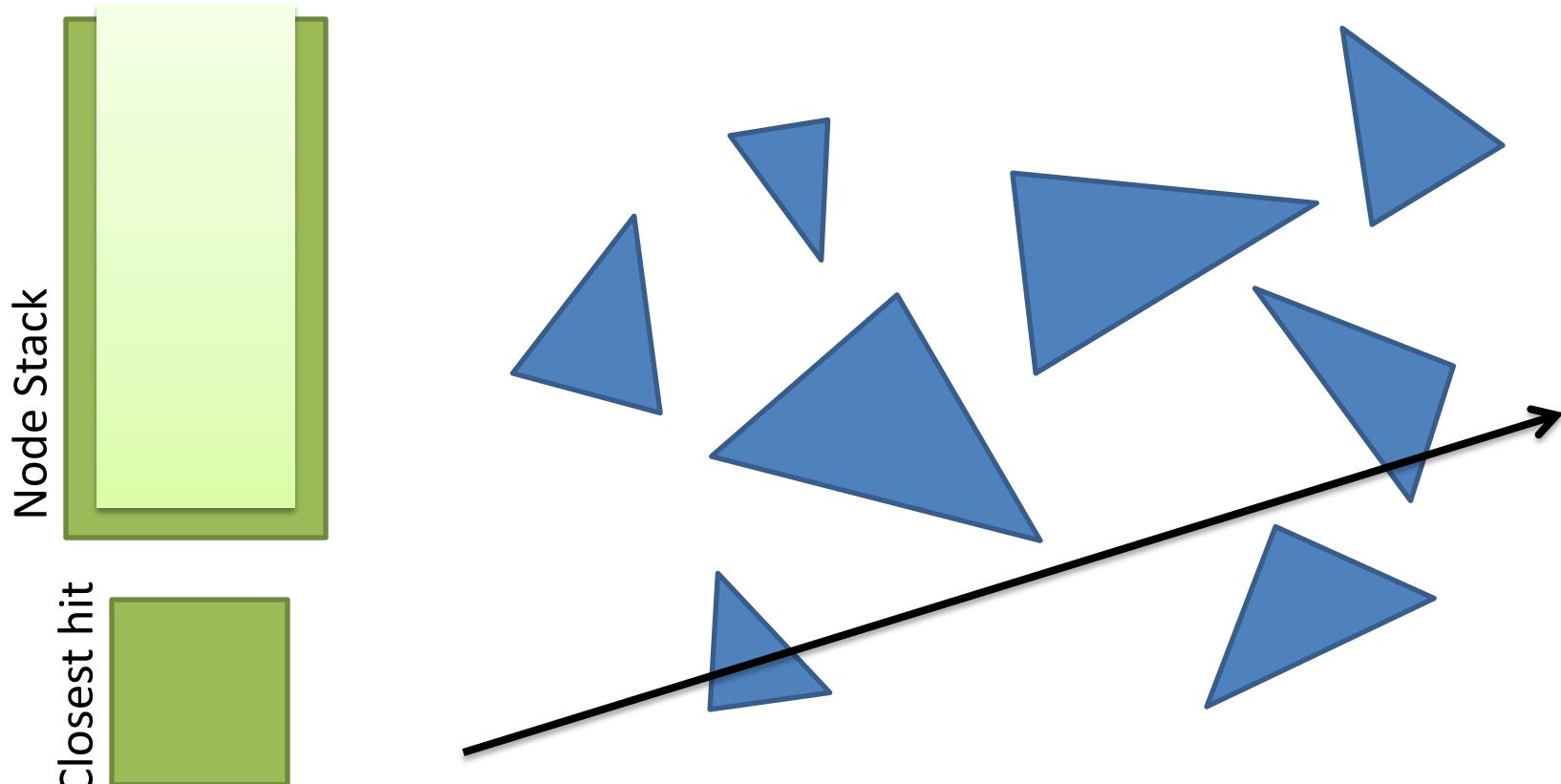
BVH intersection

- ▶ Construction: offline process, usually CPU
- ▶ Intersection: rendering process, can be GPU
- ▶ Slides from Magnus Andersson (LTH)
- ▶ [http://fileadmin.cs.lth.se/cs/Education/
EDAN30/lectures/S2-bvh.pdf](http://fileadmin.cs.lth.se/cs/Education/EDAN30/lectures/S2-bvh.pdf)

Intersection

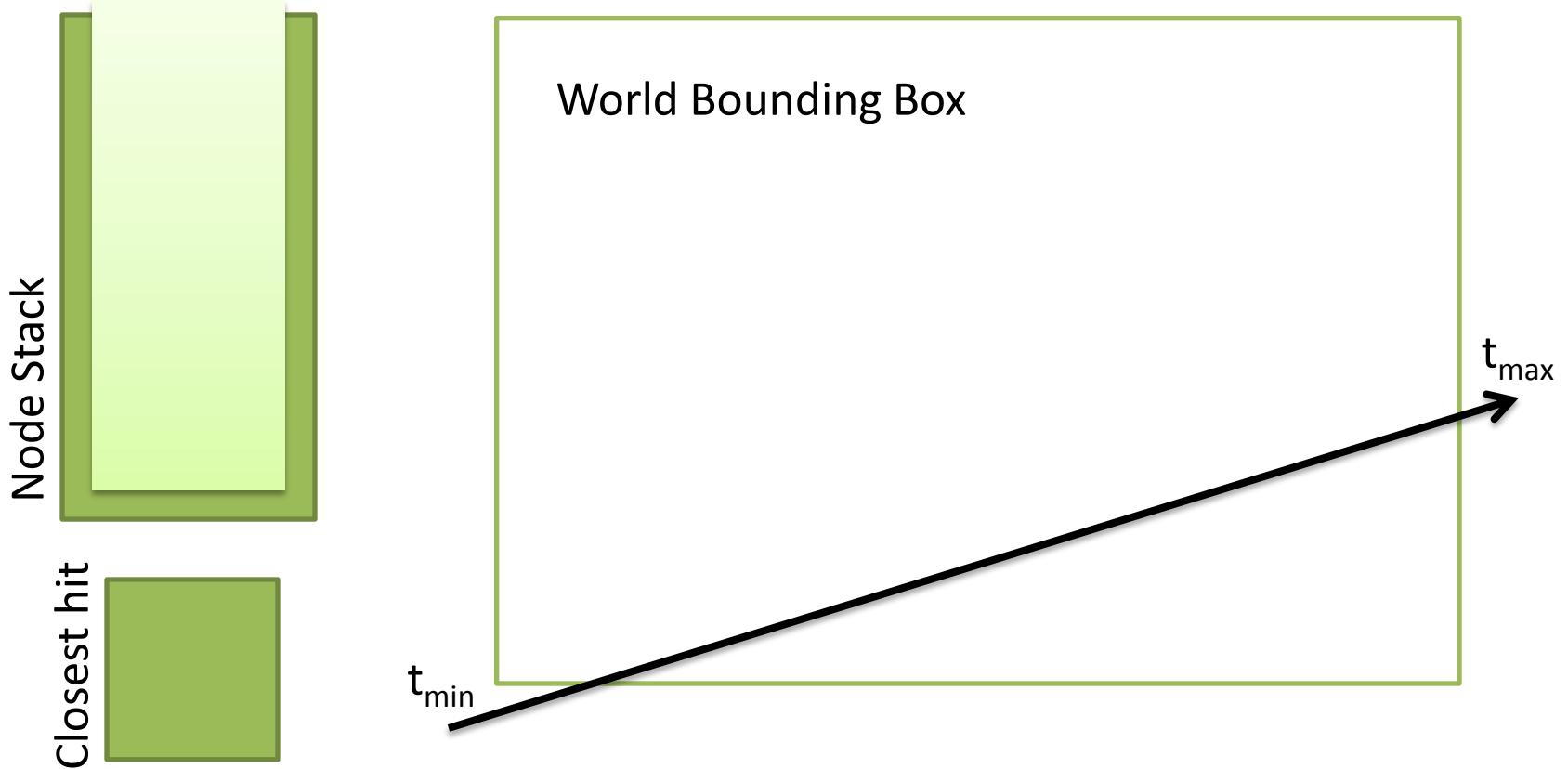
- For this assignment you must implement
 - *Boolean test*
`bool BVHAccelerator::intersect(const Ray& ray);`
 - *Closest hit*
`bool BVHAccelerator::intersect(const Ray& ray, Intersection& is);`
- The two functions are very similar. If you have one of them, you can easily implement the other.

Closest-Hit Intersection



Find closest intersection point

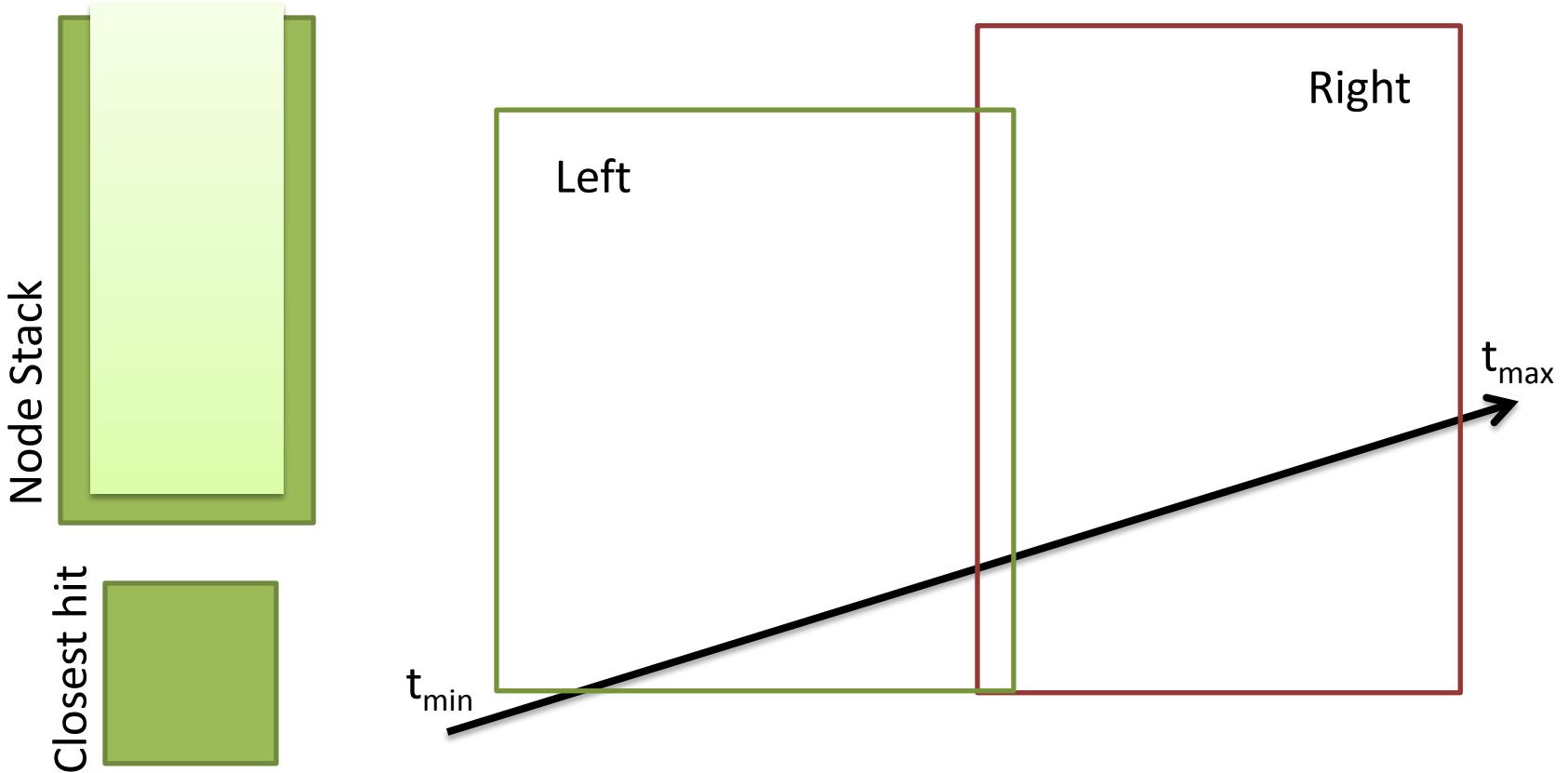
Closest-Hit Intersection



First check if we even hit the world bounding box.

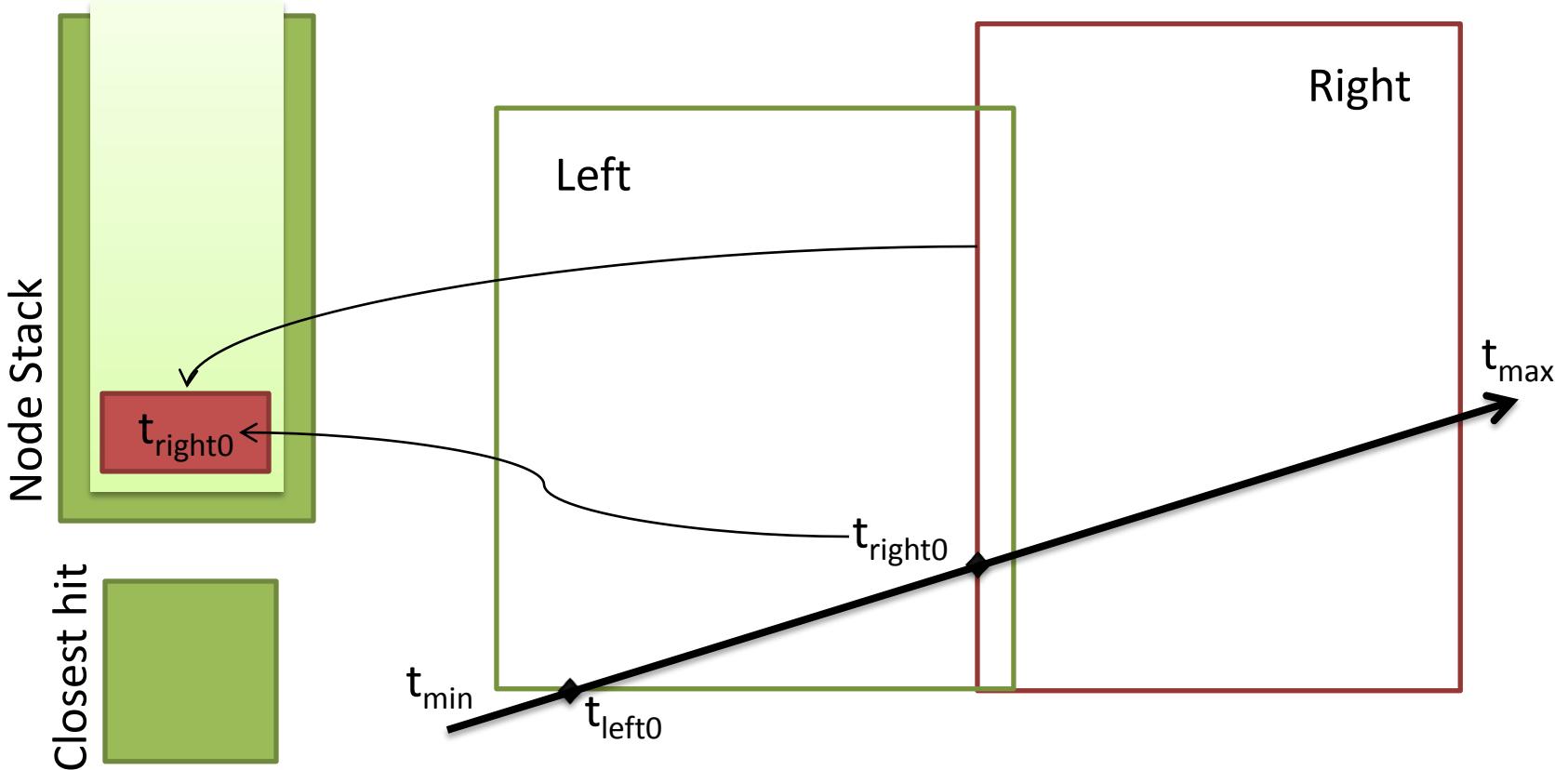
```
bool AABB::intersect(const Ray& r, float& tmin, float& tmax) const;
```

Closest-Hit Intersection



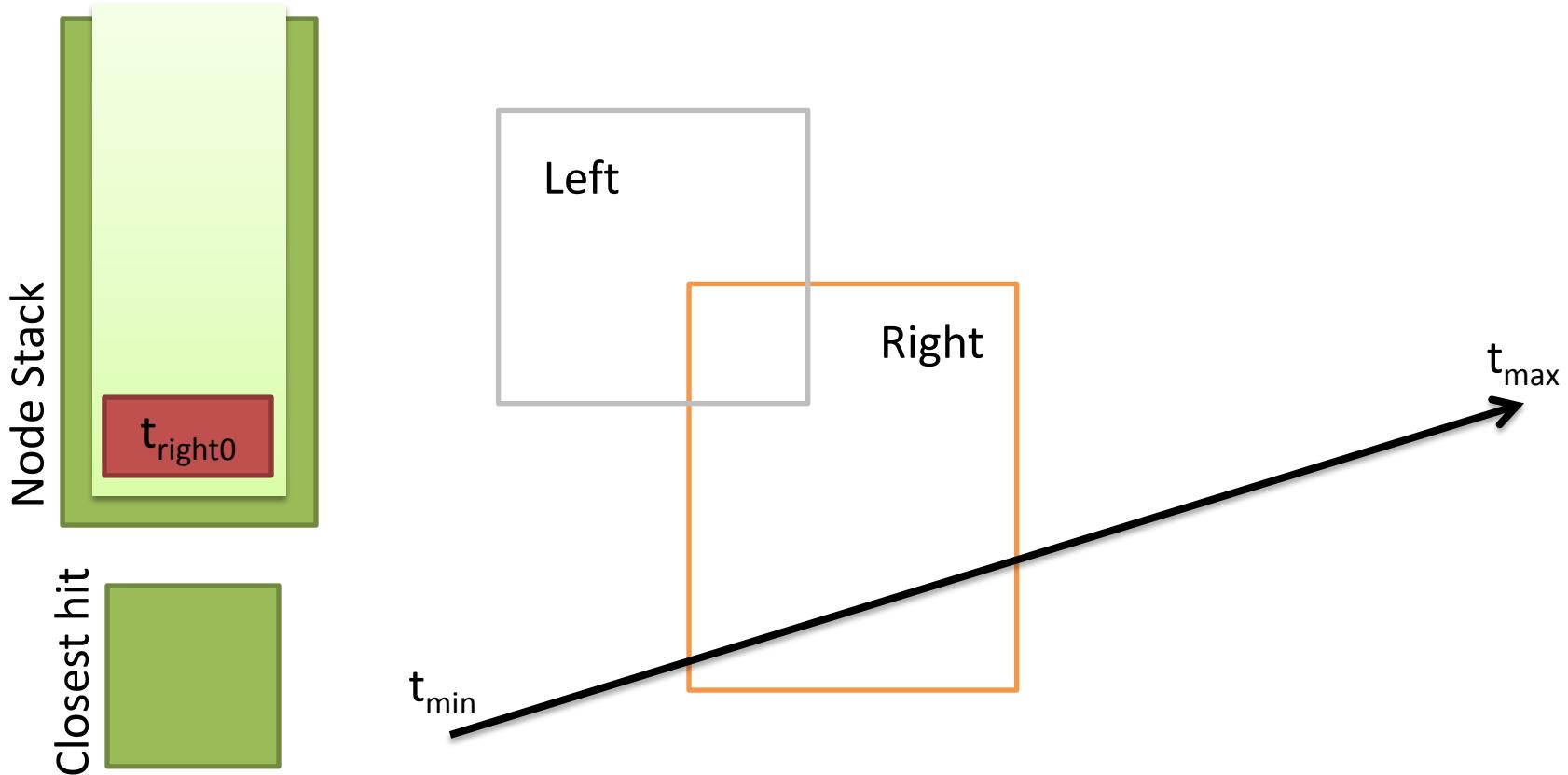
Check the two children for intersection (again using **AABB::intersect(...)**). In this case, both boxes were hit.

Closest-Hit Intersection



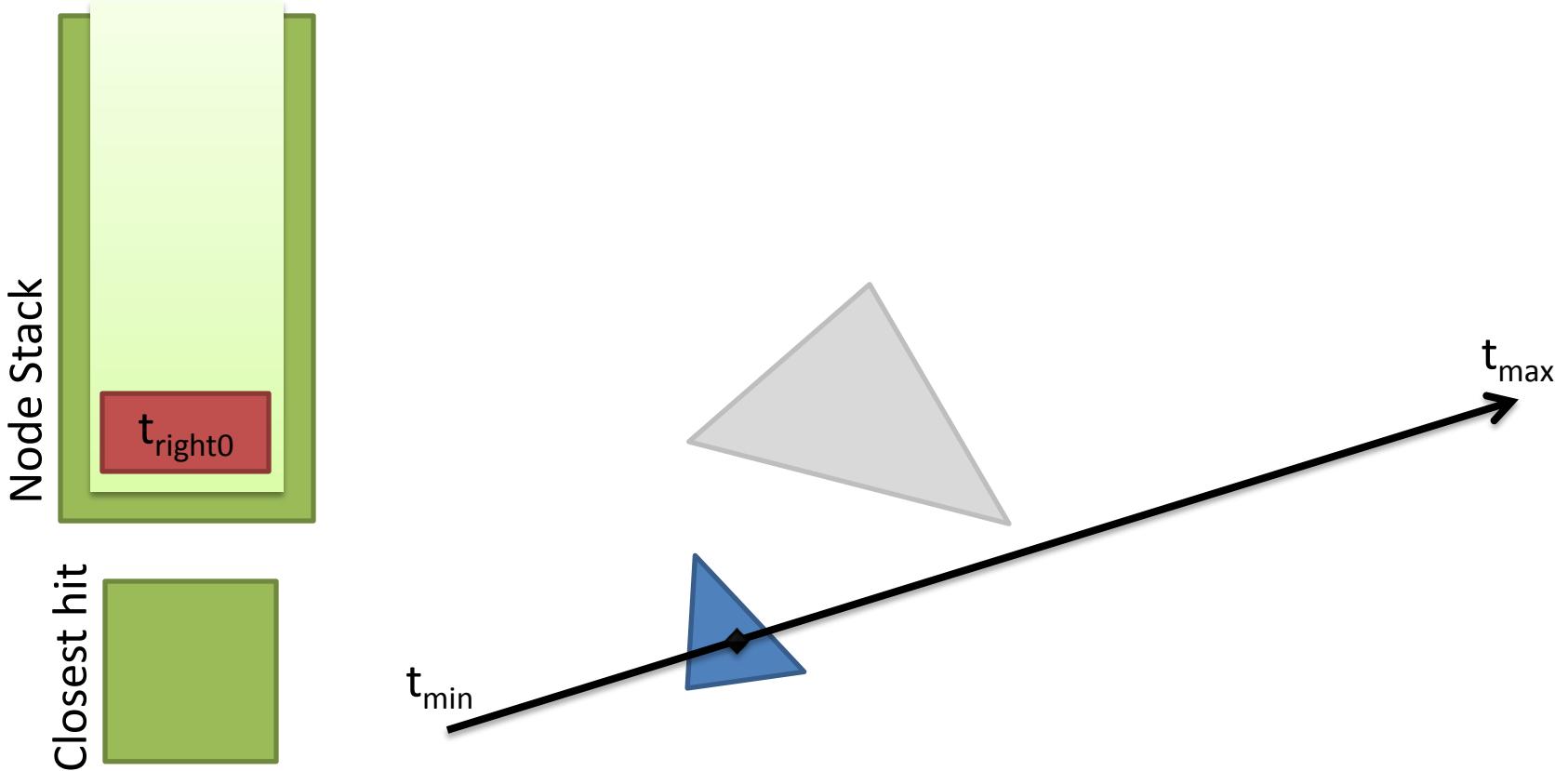
Put the node furthest away on the stack along with it's hit parameter t . Traverse the closest node

Closest-Hit Intersection



This time we only hit one node, which happens to be a *leaf node*

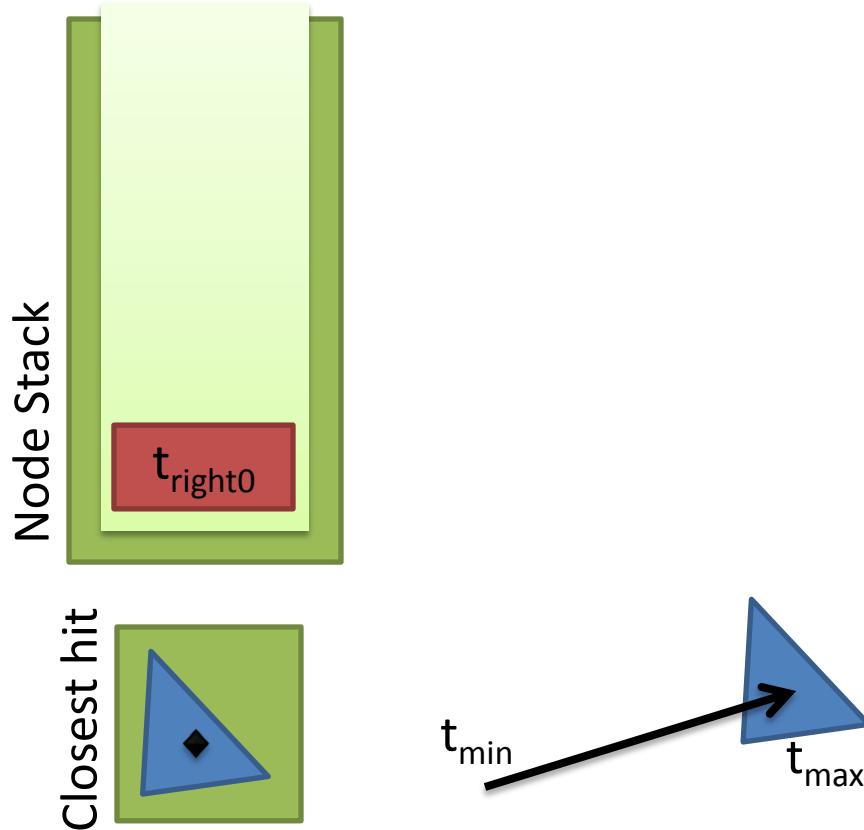
Closest-Hit Intersection



Intersection test with each primitive in the leaf.

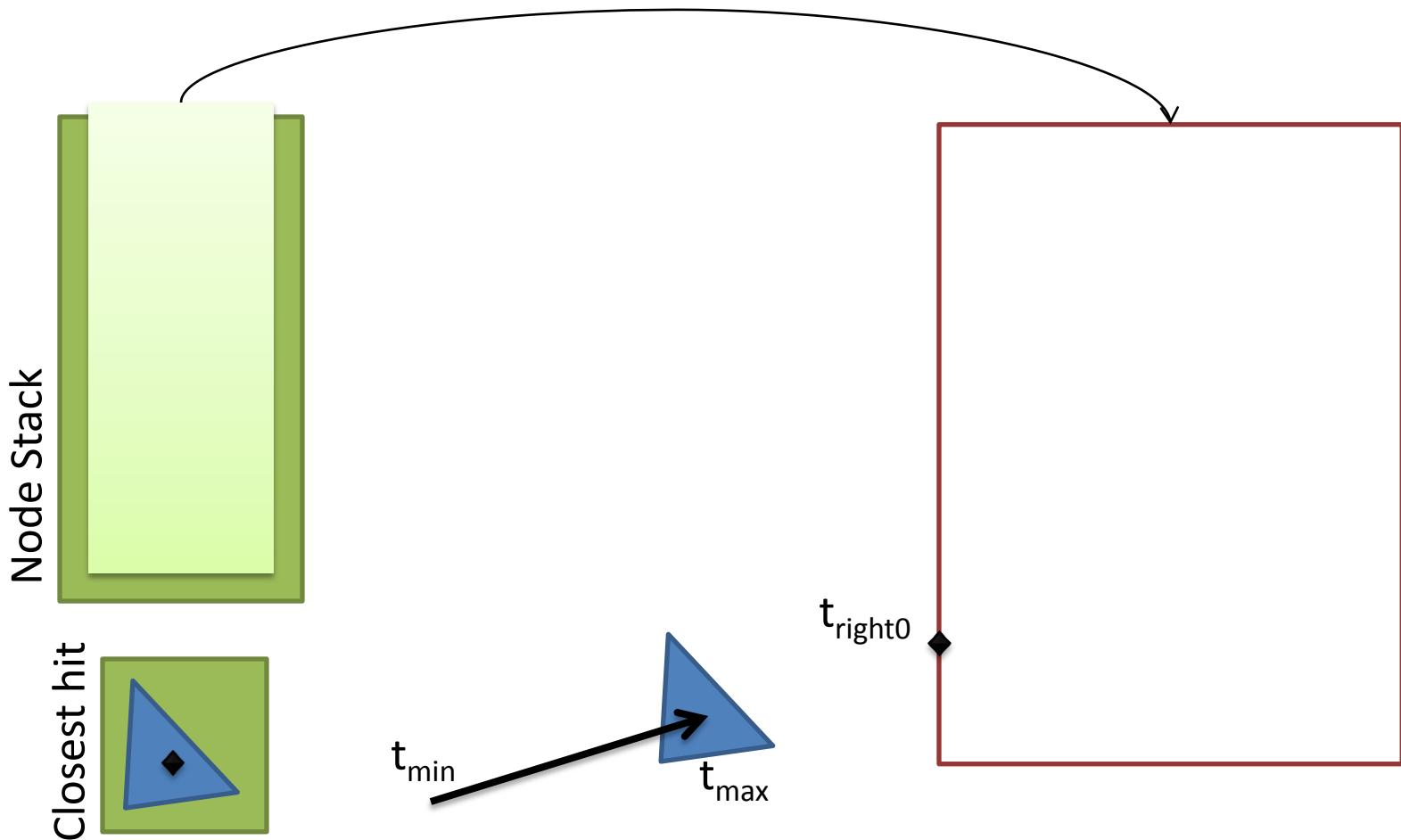
```
bool Intersectable::intersect(const Ray& ray, Intersection& is) const;
```

Closest-Hit Intersection



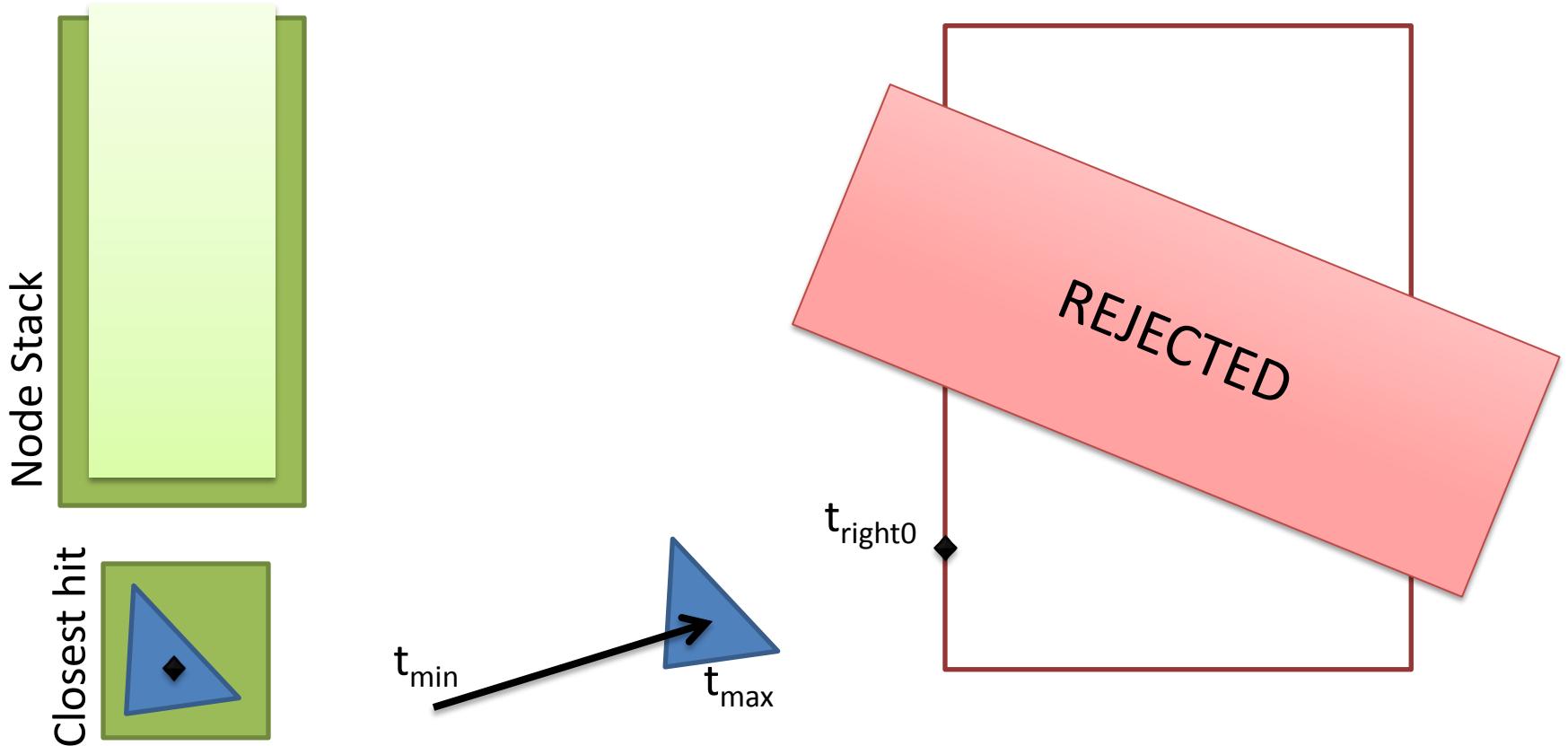
Store intersection and shorten ray.

Closest-Hit Intersection



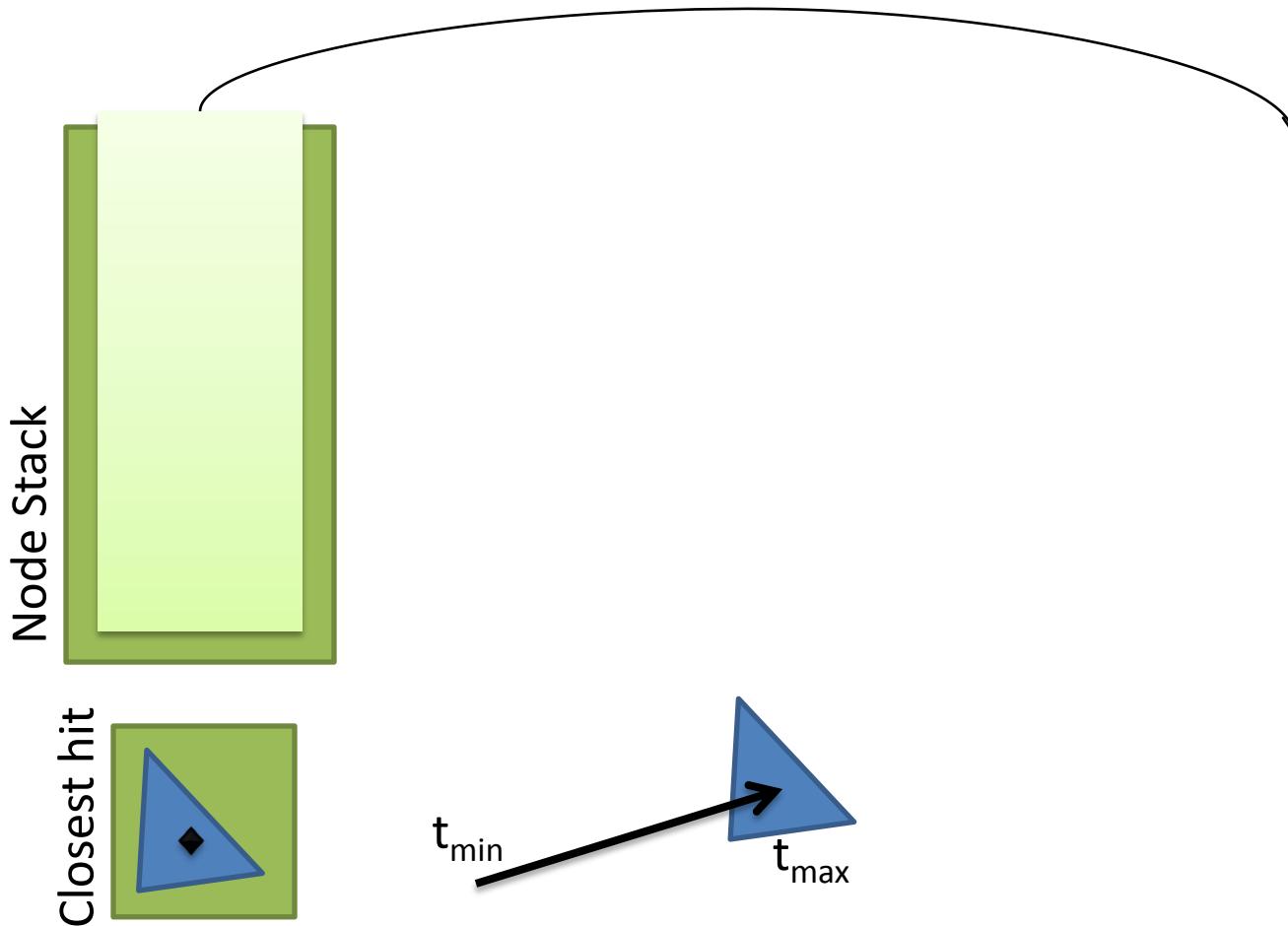
Pop the stack and recursively intersection test with the node.

Closest-Hit Intersection



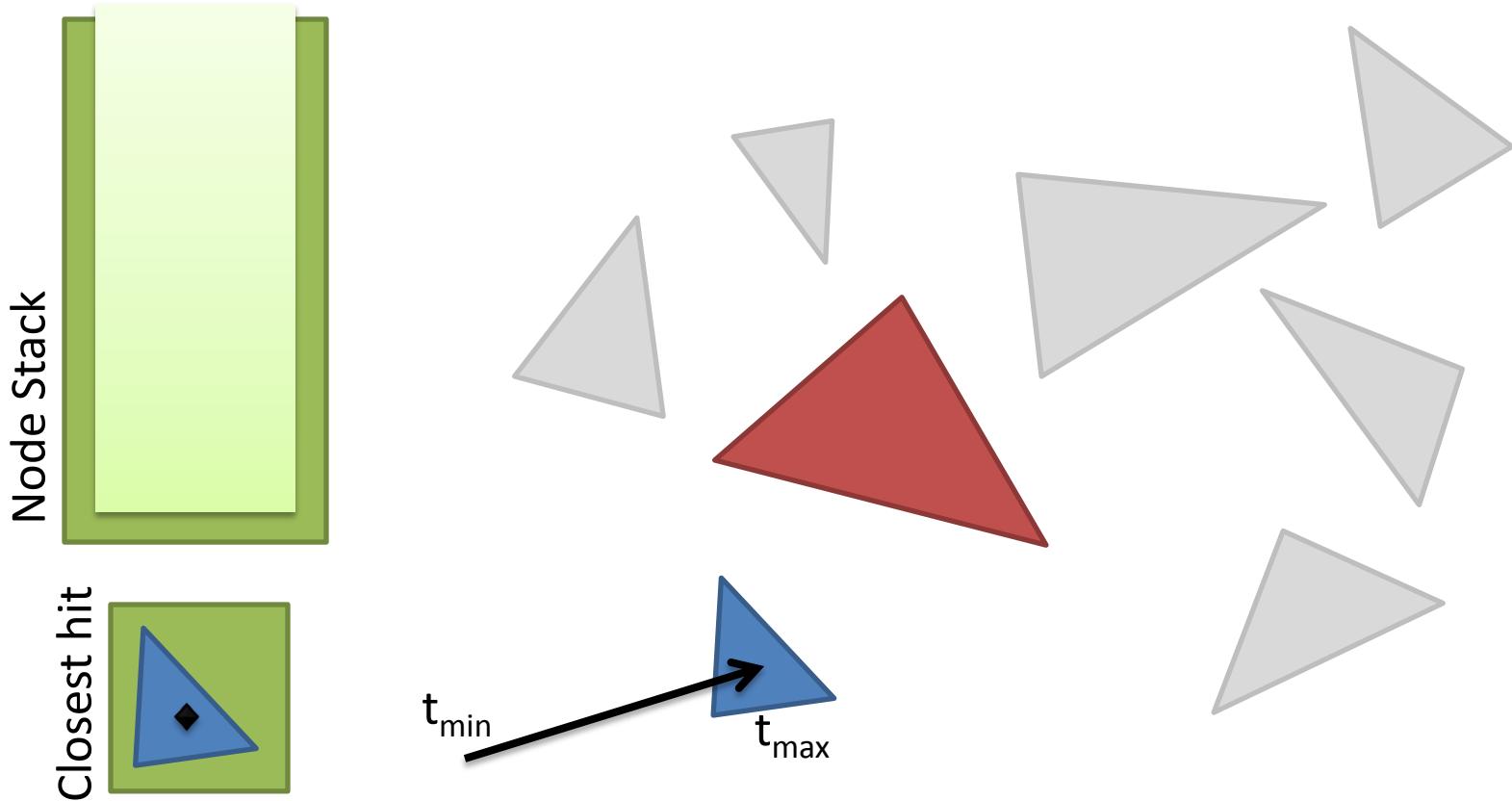
Optimization – We can trivially reject the pop'd node since its t -value is now further away than t_{max} of the ray.

Closest-Hit Intersection



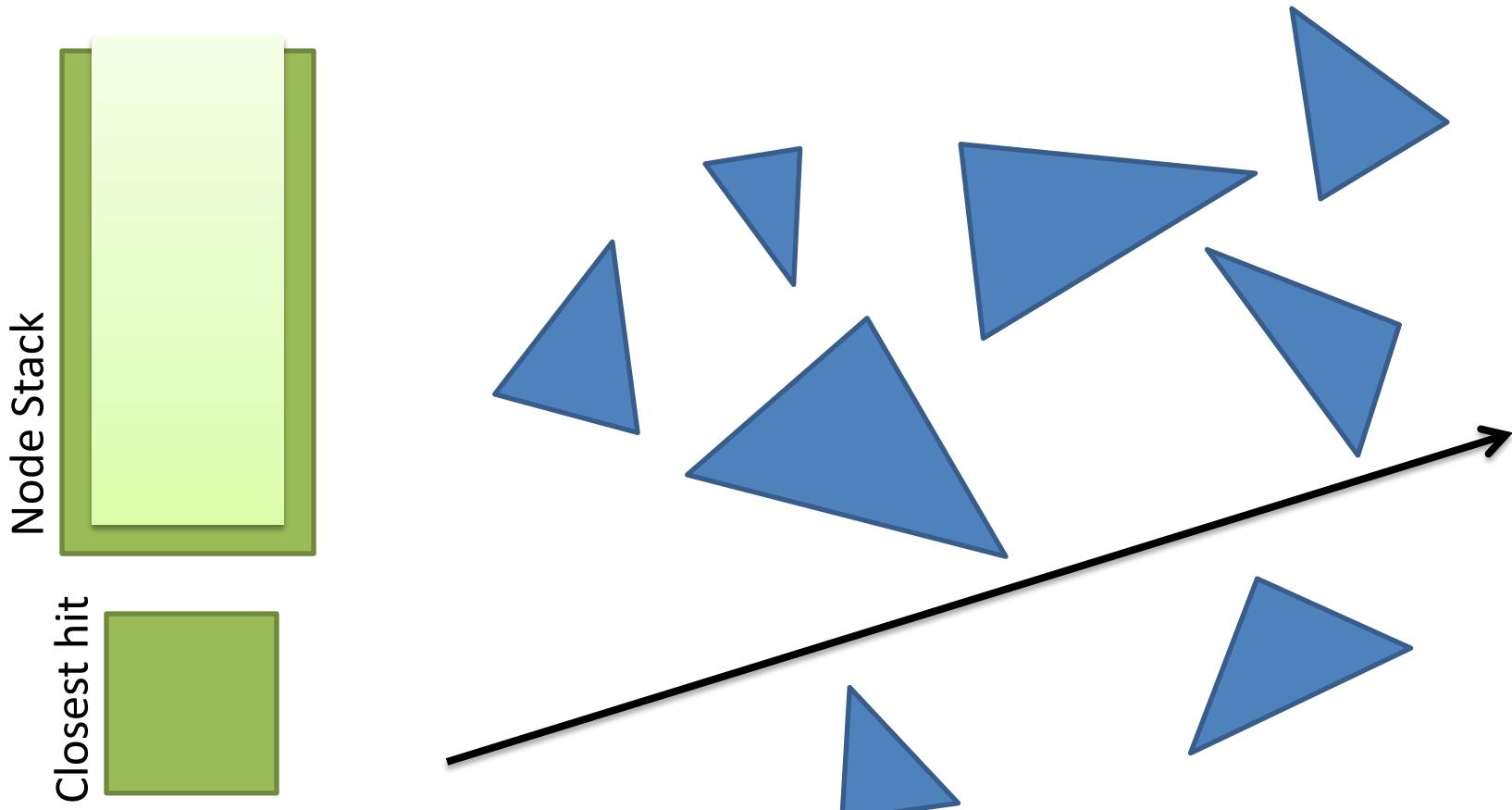
Try to pop the stack again to fetch the next node... but now it's empty, which means we're done!

Closest-Hit Intersection



We found the closest hit with little effort!

No Intersection



If there is no intersection, the *Closest hit* will of course be empty – return *false*

Closest-Hit Intersection, Pseudo code

- LocalRay = Ray, CurrentNode = Root
- Check LocalRay intersection with Root (world box)
 - No hit => return *false*
- For (infinity)
 - If (**NOT** CurrentNode.isLeaf())
 - Intersection test with both child nodes
 - Both nodes hit => Put the one furthest away on the stack. *CurrentNode* = closest node
 - » continue
 - Only one node hit => *CurrentNode* = hit node
 - » continue
 - No Hit: Do nothing (let the stack-popping code below be reached)
 - Else // Is leaf
 - For each primitive in leaf perform intersection testing
 - Intersected => update *LocalRay.maxT* and store *ClosestHit*
 - EndIf
 - Pop stack until you find a node with $t < \text{LocalRay}.maxT \Rightarrow \text{CurrentNode} = \text{pop'd}$
 - Stack is empty? => return *ClosestHit* (no closest hit => return false, otherwise return true)
- EndFor

Intersection

- Stack element

```
struct StackItem {  
    BVHNode      *ptr;  
    float        t;  
};
```

- Use either a C-style vector or C++ Stack class
 - StackItem stack[MAX_STACK_DEPTH];
 - Stack<StackItem> stack;

Boolean Intersection, Pseudo code

- LocalRay = Ray, CurrentNode = Root
- Check LocalRay intersection with Root (world box)
 - No hit => return *false*
- For (infinity)
 - If (**NOT** CurrentNode.isLeaf())
 - Intersection test with both child nodes
 - Both nodes hit => Put **right** one on the stack. *CurrentNode* = **left** node
 - » Goto LOOP;
 - Only one node hit => *CurrentNode* = **hit** node
 - » Goto LOOP;
 - No Hit: Do nothing (let the stack-popping code below be reached)
 - Else // Is leaf
 - For each primitive in leaf perform intersection testing
 - Intersected => **return true;**
 - EndIf
 - Pop stack, *CurrentNode* = pop'd node
 - Stack is empty => **return false**
 - EndFor

BVH traversal on the GPU

- ▶ No pointers!
 - ▶ Tree structure with no pointers
 - ▶ Tree = index into these arrays
 - ▶ Array of Structures / Structure of Arrays
- ▶ Efficient traversal: avoid recursion
 - ▶ Previous slide
- ▶ Precompute traversal: MTBVH
 - ▶ <http://www.ci.i.u-tokyo.ac.jp/~hachisuka/tdf2015.pdf>

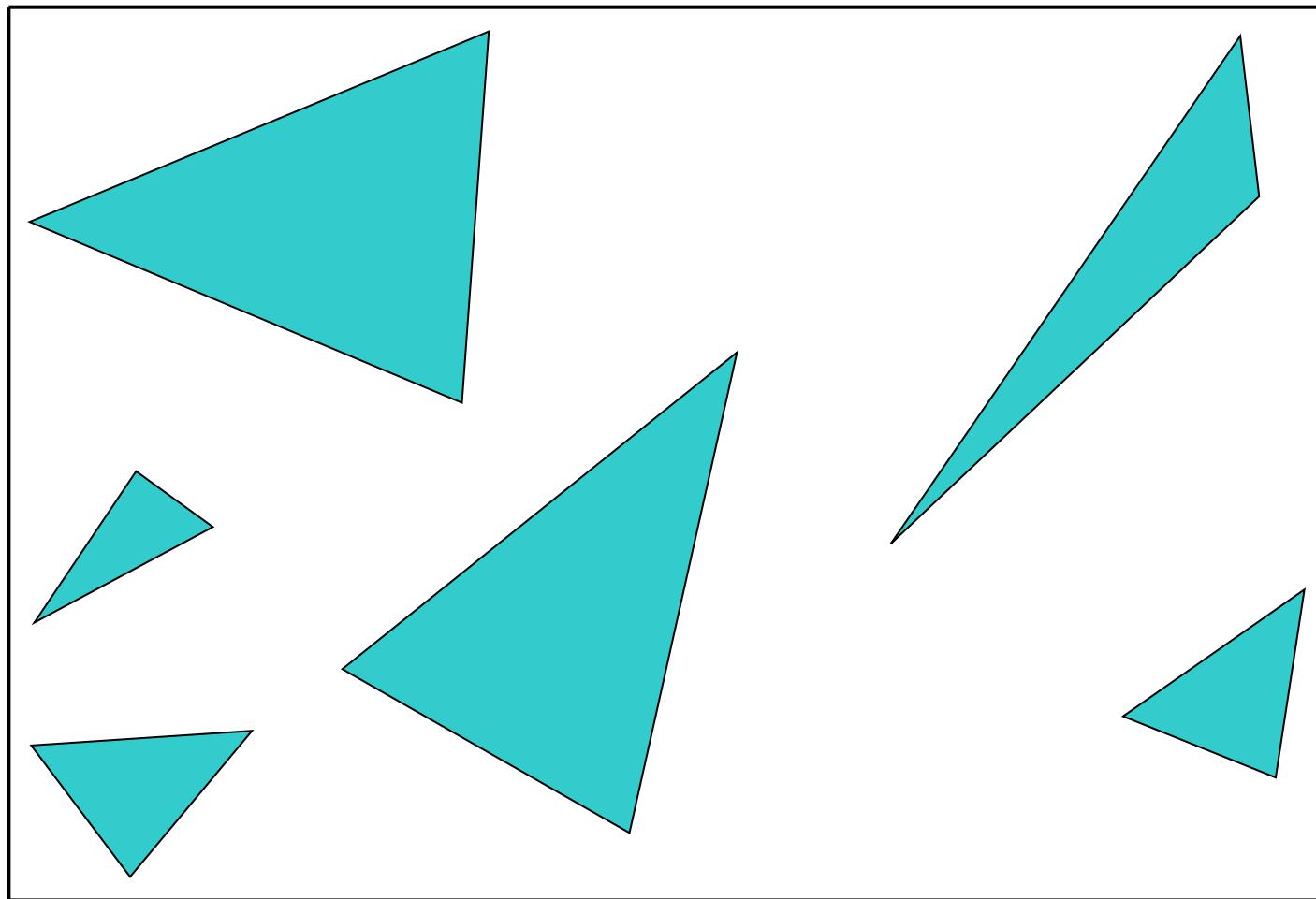
Ray-tracing on the GPU

- ▶ Same problem: avoid recursive calls
- ▶ Naïve approach:
 - ▶ `out = direct_light + color * trace(reflected_ray)`
 - ▶ Problem: recursive calls to `trace(ray)`
- ▶ Better approach:
 - ▶ `for (s=0..max_depth)`
 - ▶ Store `reflected_ray[s]` (`origin, direction`)
 - ▶ `for (s=max_depth..0)`
 - ▶ `out = direct_light(reflected ray[s]) + color * out`

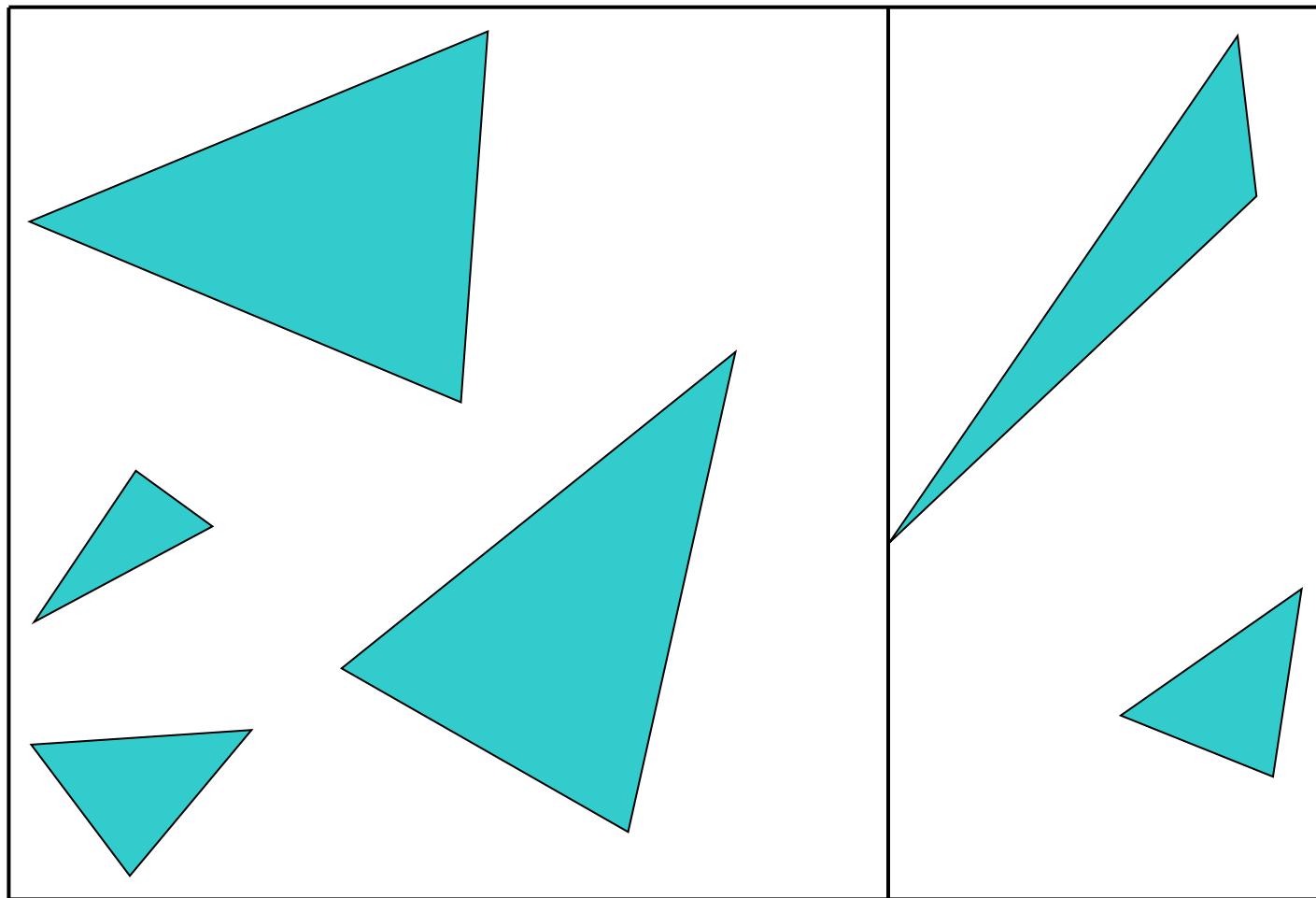
Other structures

- ▶ Uniform grid
 - ▶ Adaptive grid: octree
 - ▶ k-D tree
-
- ▶ Keep nearest intersection found
 - ▶ Check if cells are closer than this intersection

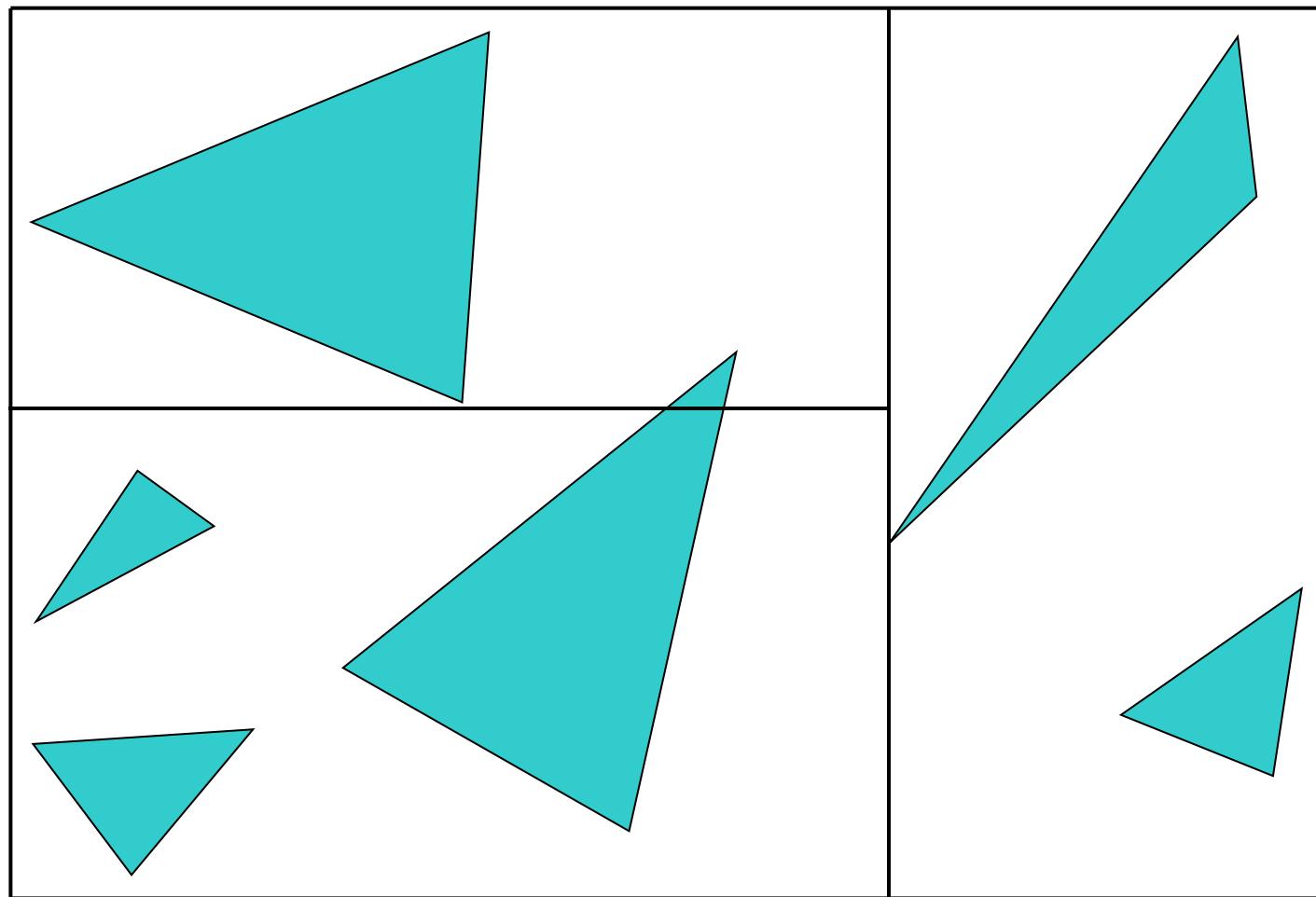
k-D tree



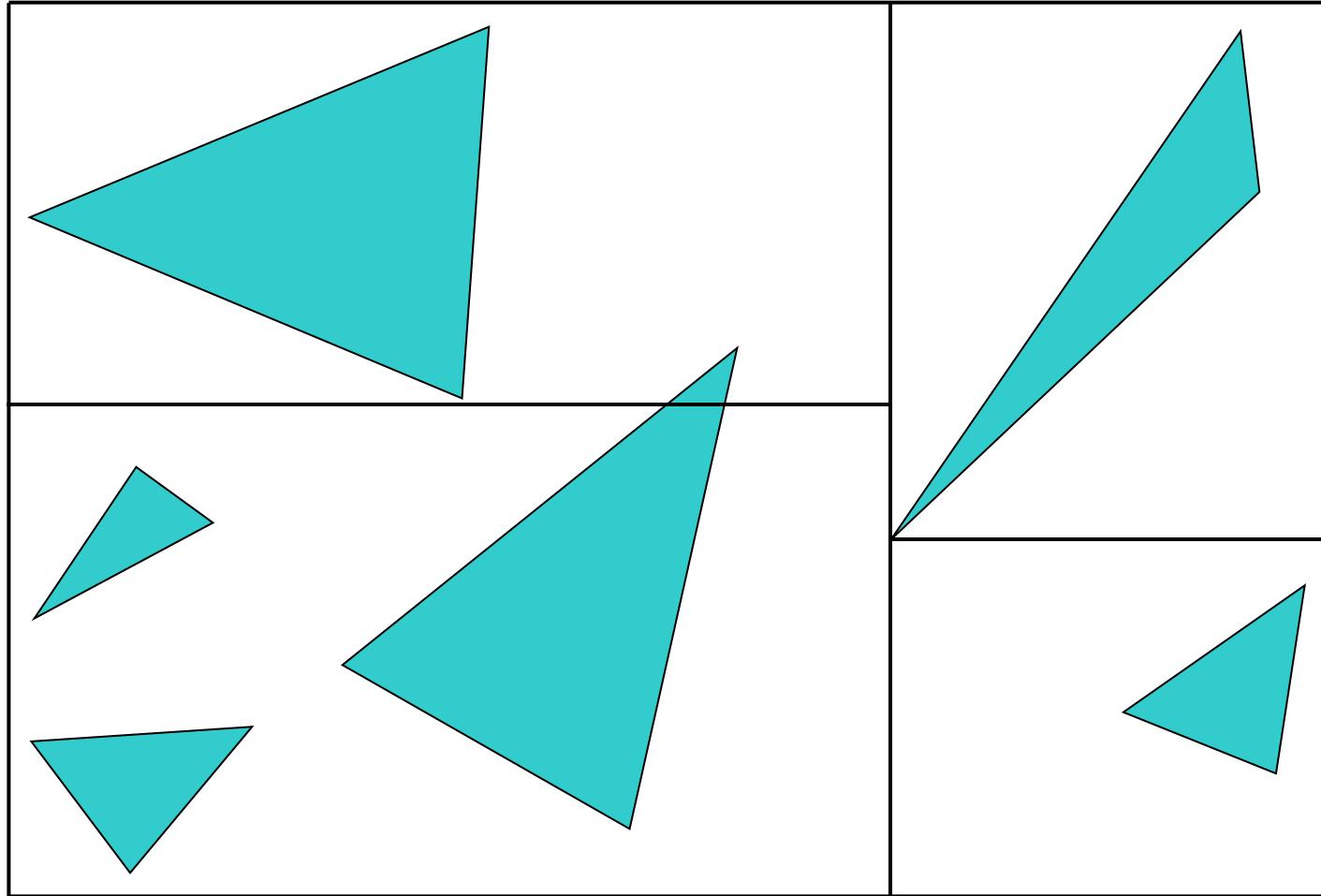
k-D tree



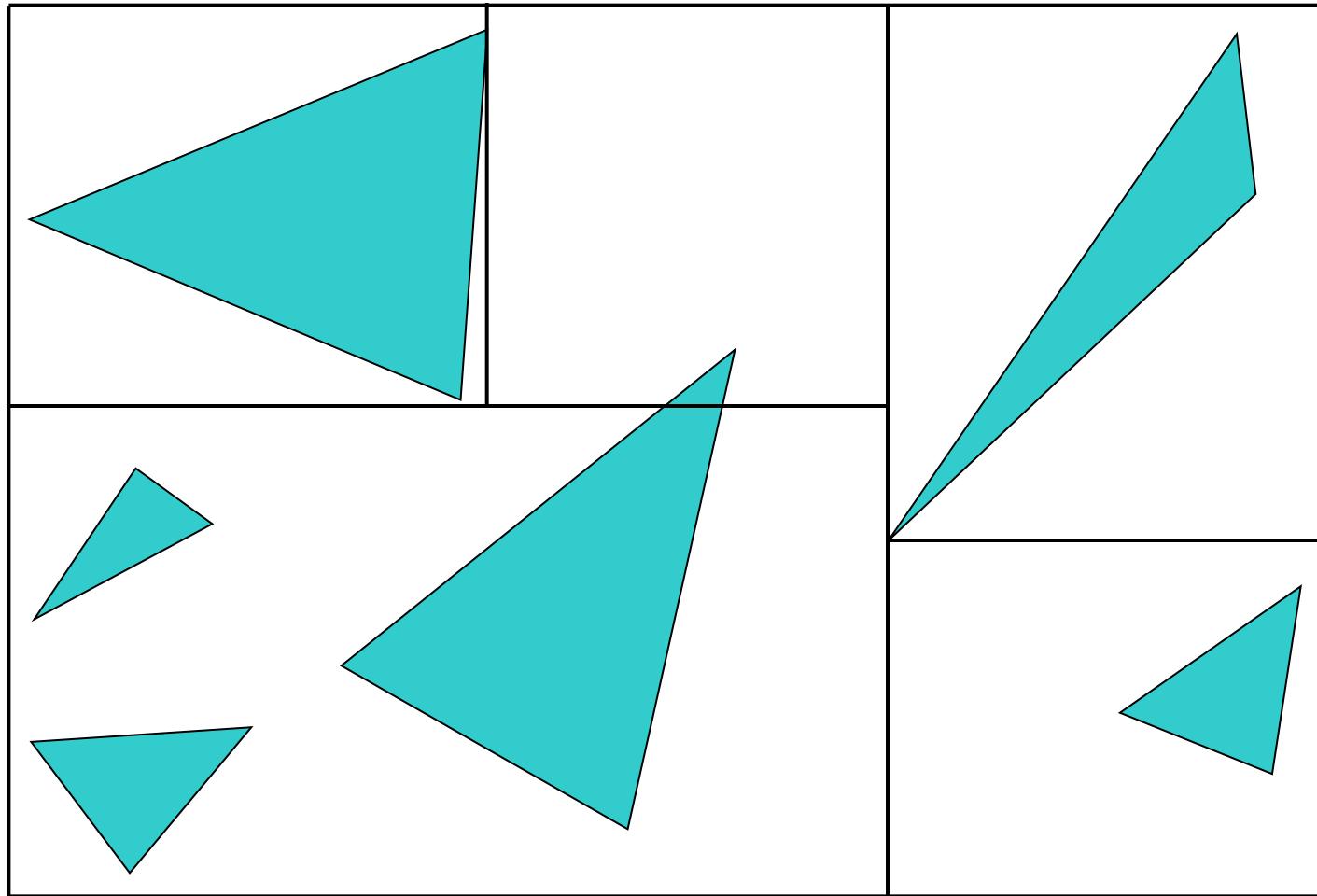
k-D tree



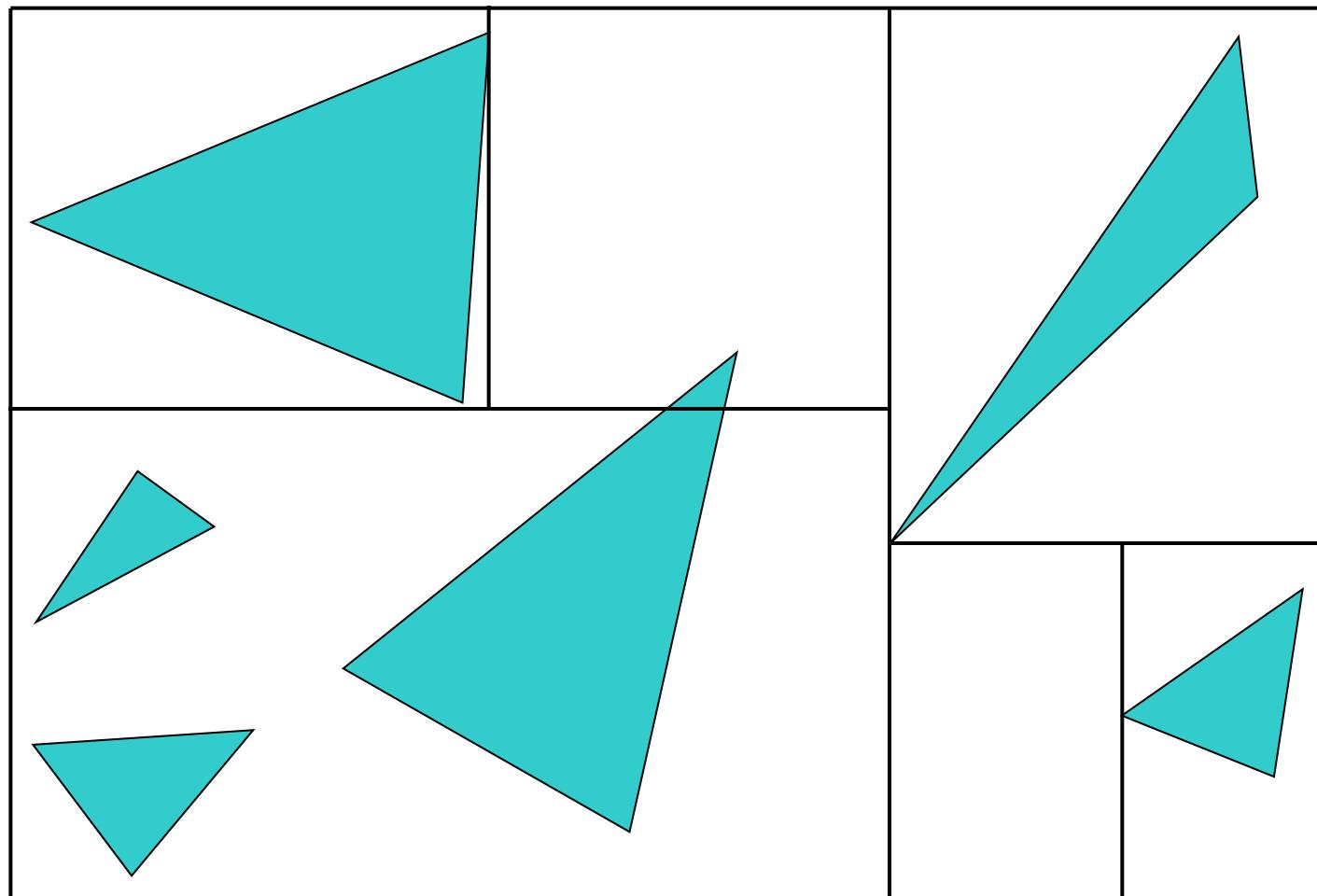
k-D tree



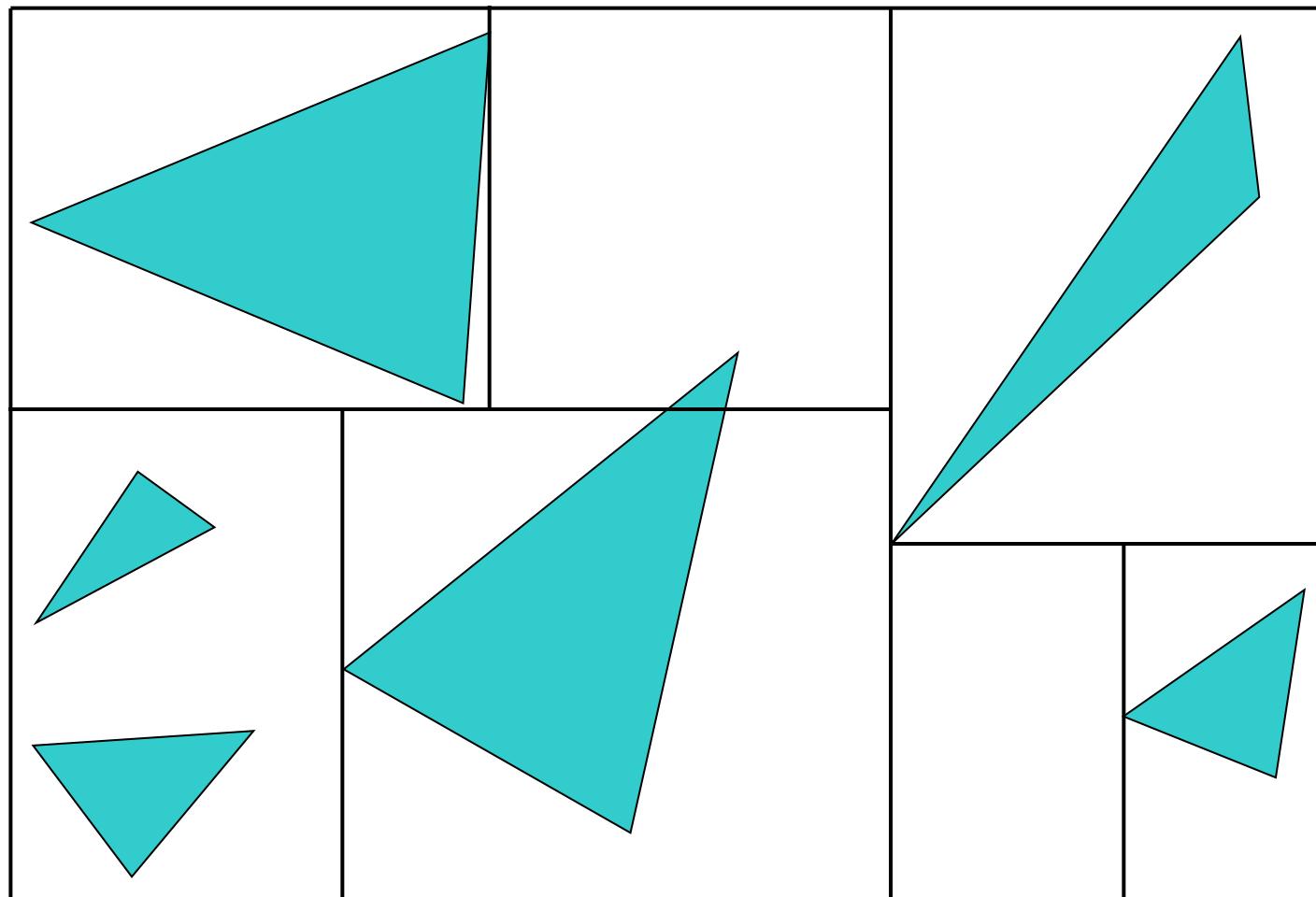
k-D tree



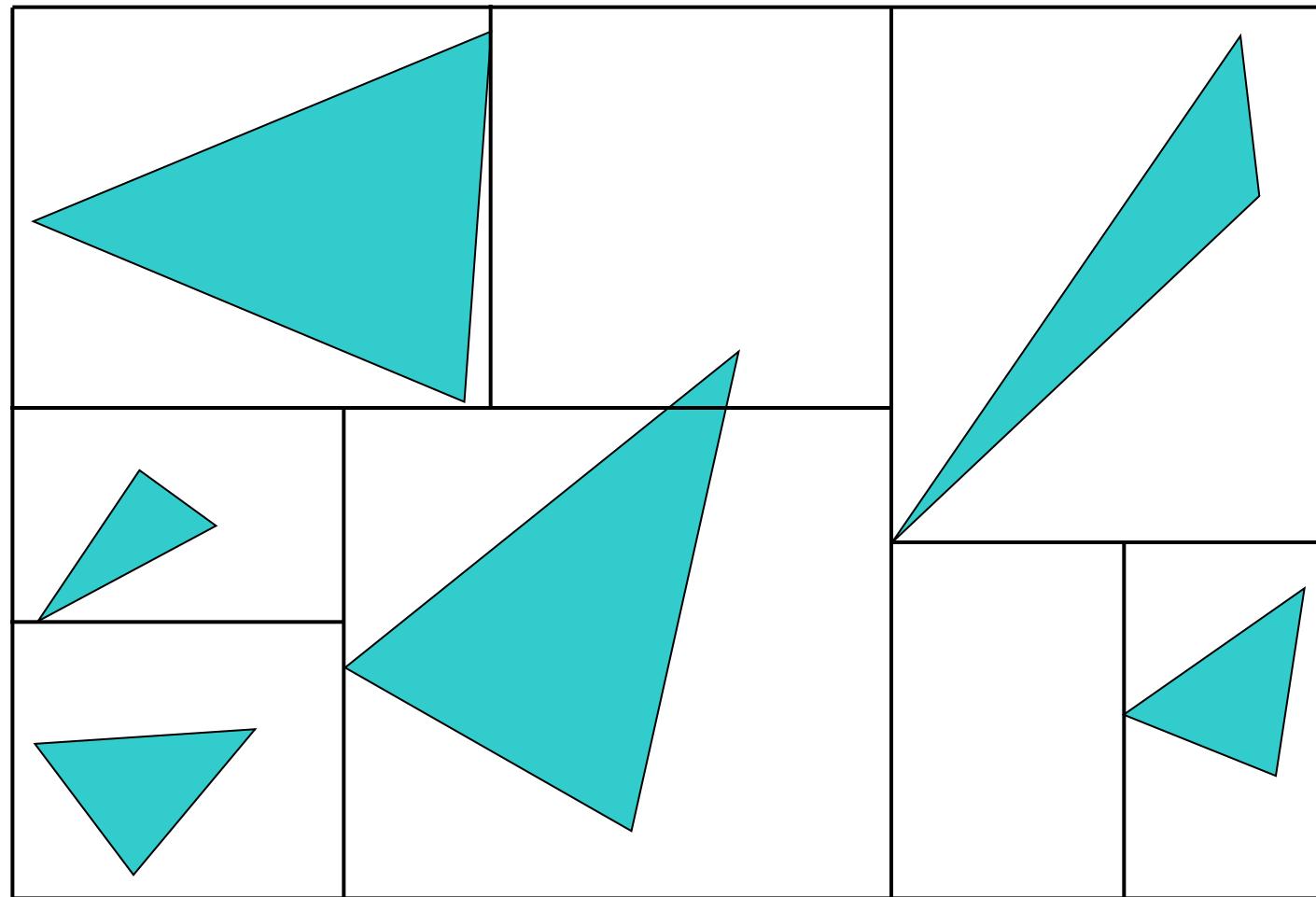
k-D tree



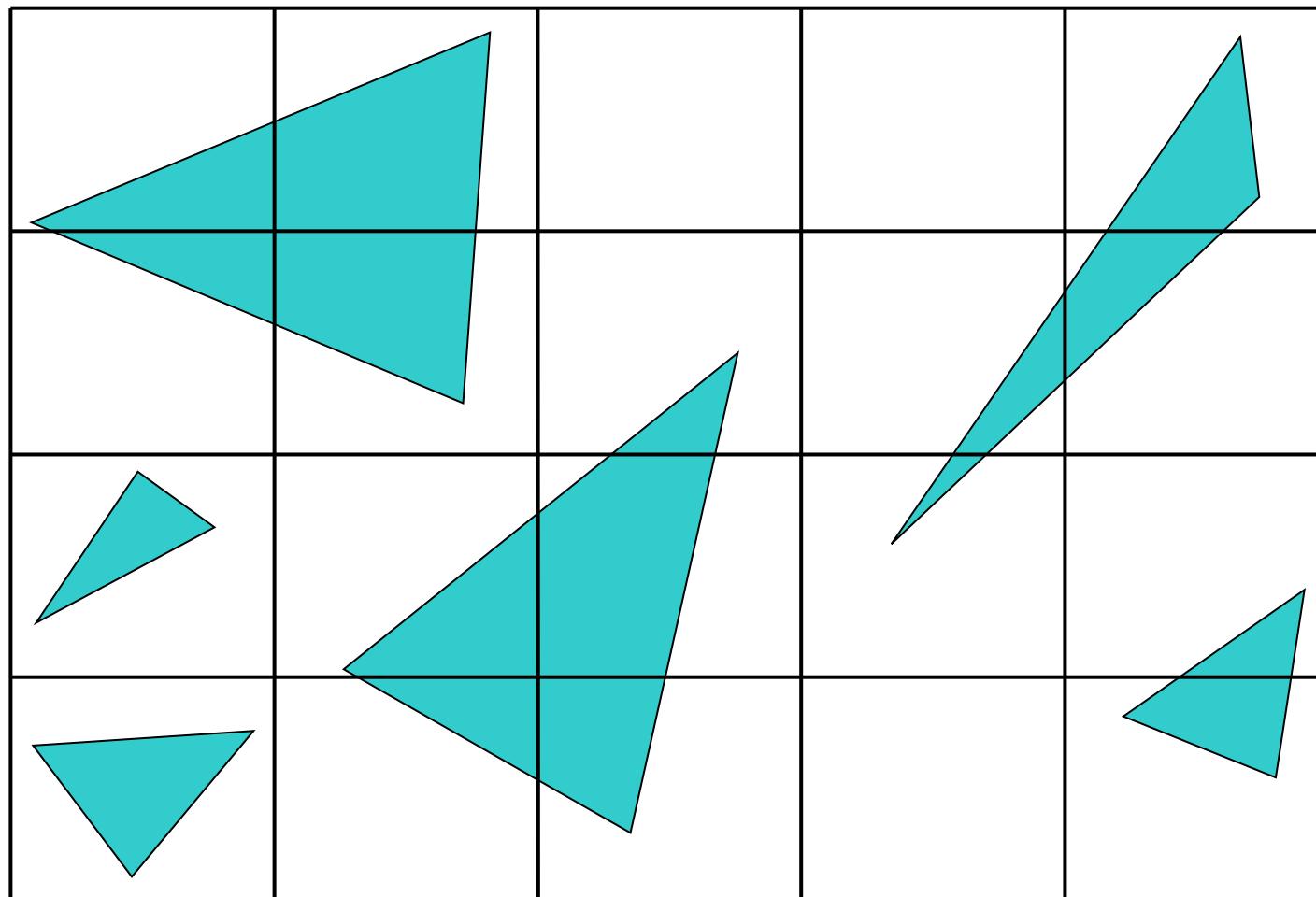
k-D tree



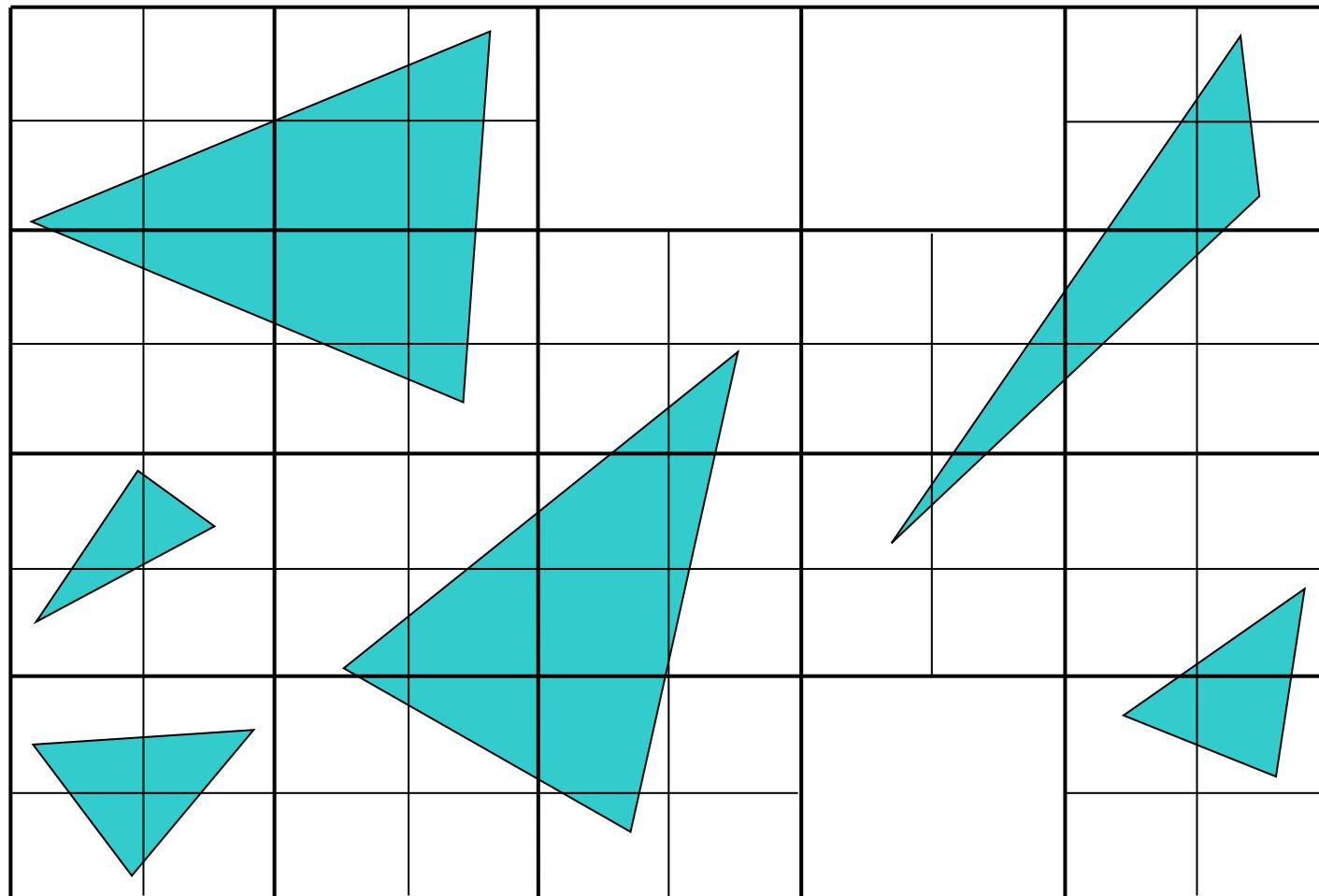
k-D tree



Uniform grid



Adaptive grid: Octree



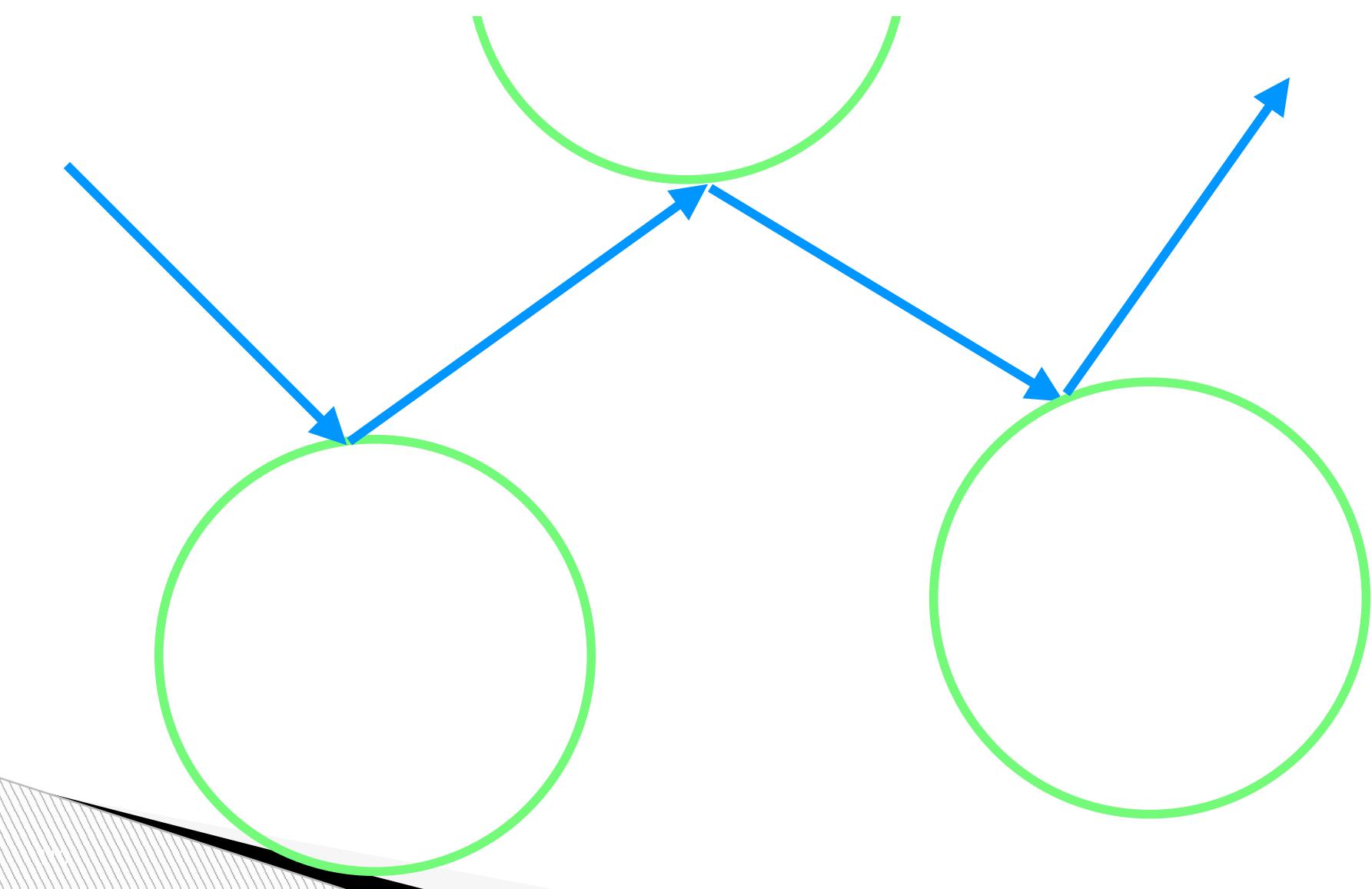
GPU ray-tracing (fragment shader)

- ▶ Ray-tracing: embarrassingly parallel
 - ▶ Each ray independent from the others
- ▶ GPU: massively parallel computer
 - ▶ Fragment shader: each fragment, independent
- ▶ Use GPU for ray-tracing
 - ▶ It works
 - ▶ Issues: data input, memory replication, cache misses, synchronicity

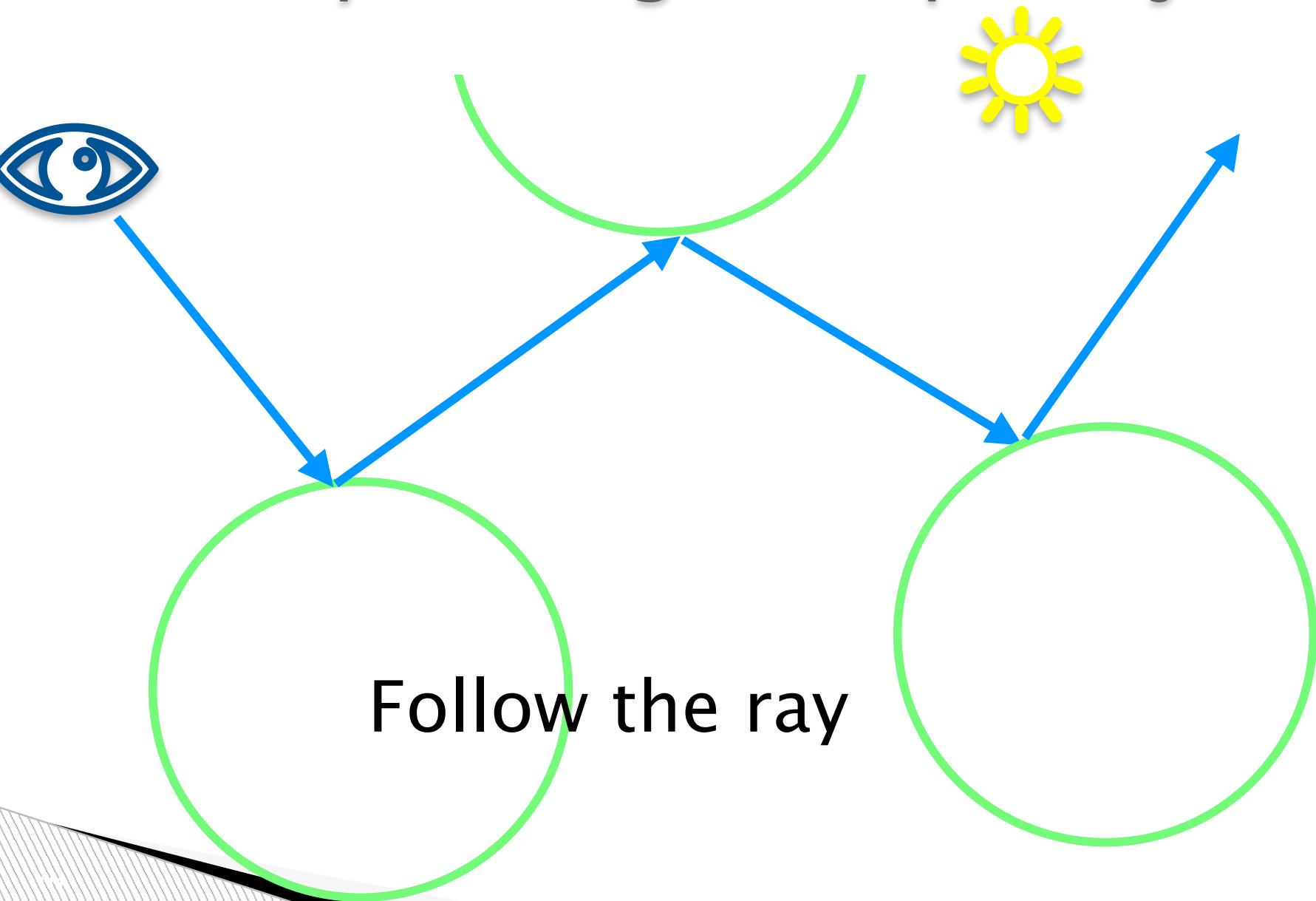
GPU ray-tracing (fragment shader)

- ▶ How?
- ▶ 1 fragment shader call = 1 ray
- ▶ Fragment shader needs:
 - ▶ Pixel position
 - ▶ Ray: starting point P , direction u
 - ▶ These are connected (use transform matrix)
- ▶ *How do we cover all pixels?*
 - ▶ Render a single quad, covering the screen
 - ▶ Add a parameter for screen position
 - ▶ Fragment shader gets interpolated value: pixel pos.

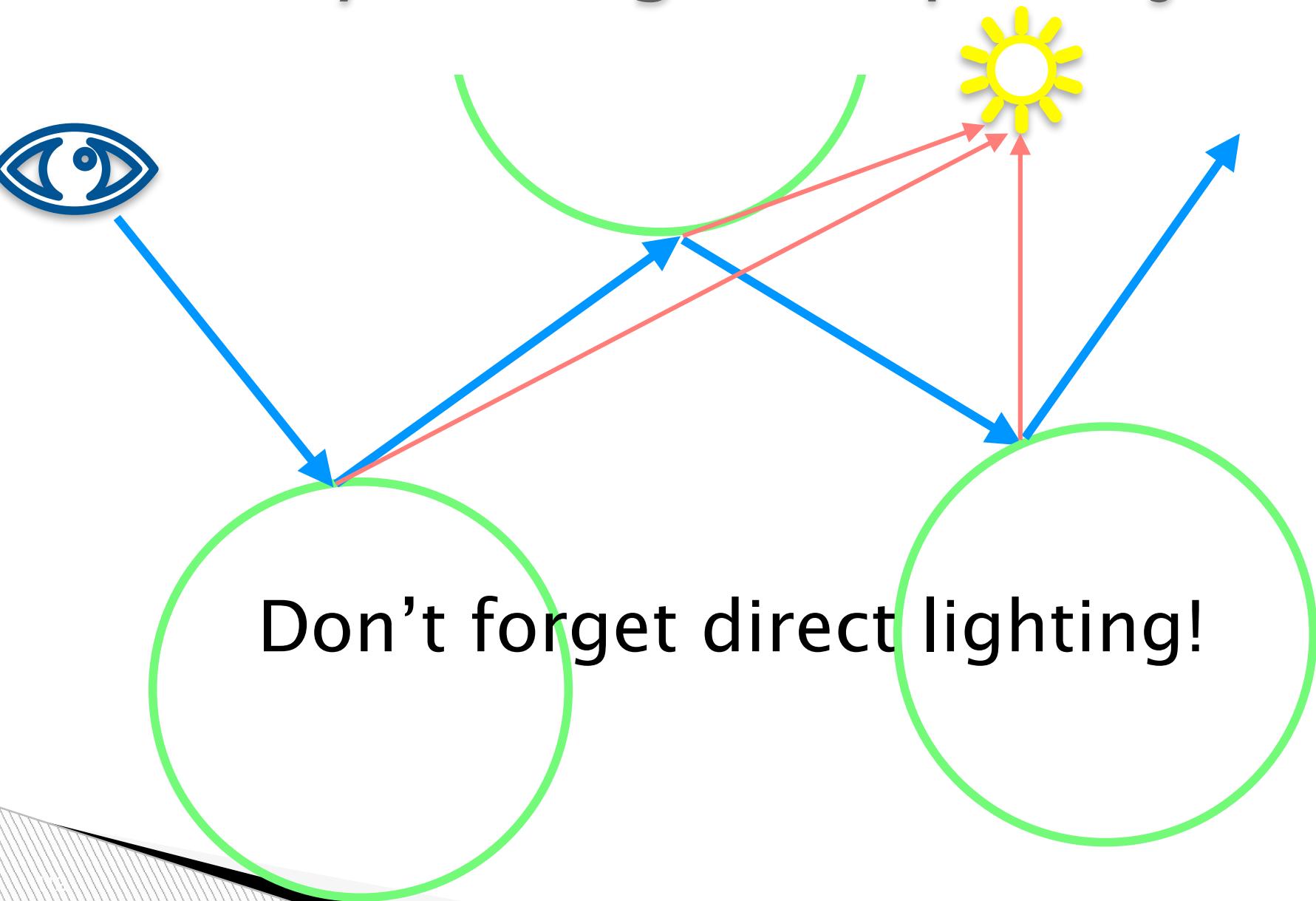
GPU ray-tracing: multiple objects



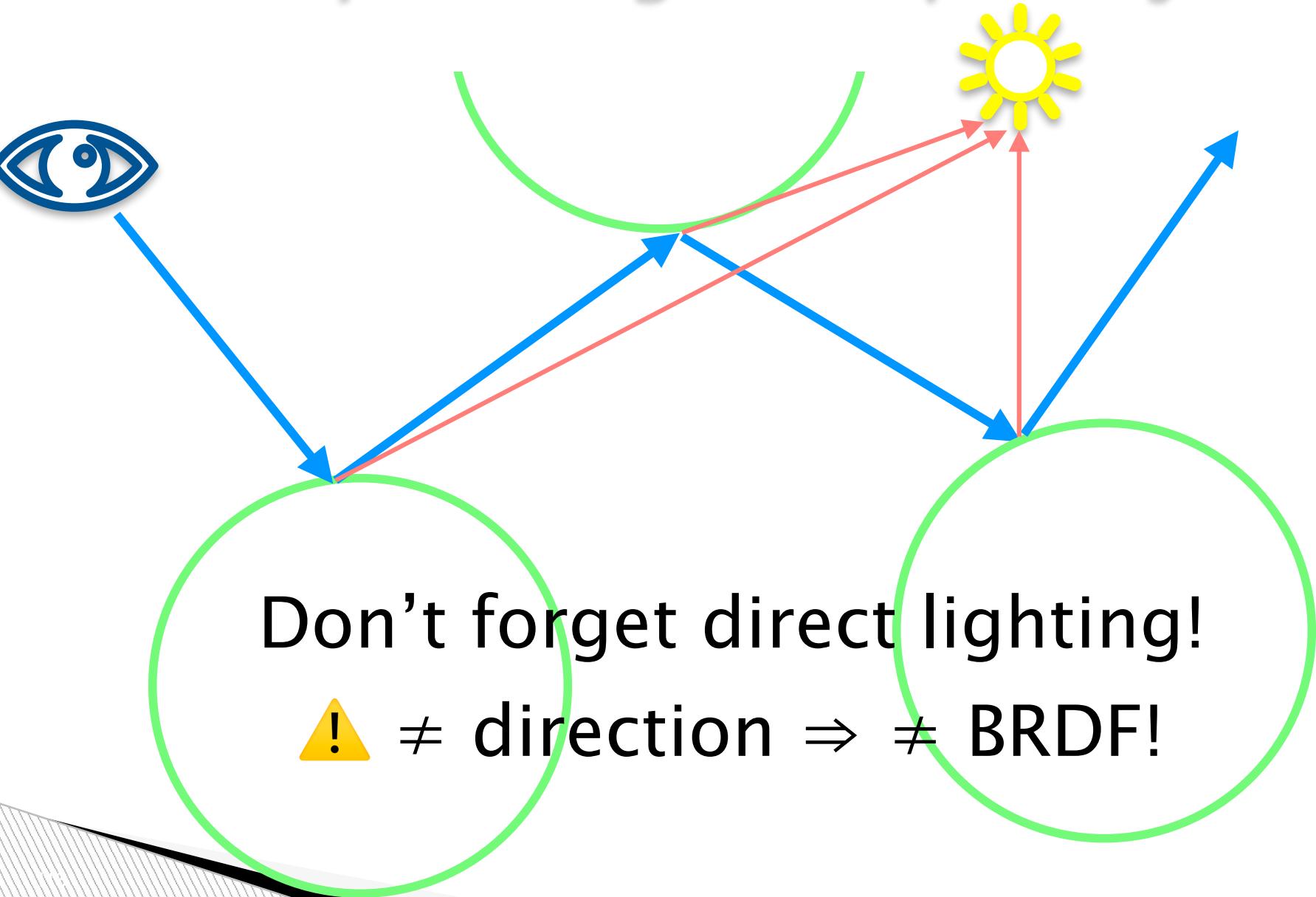
GPU ray-tracing: multiple objects



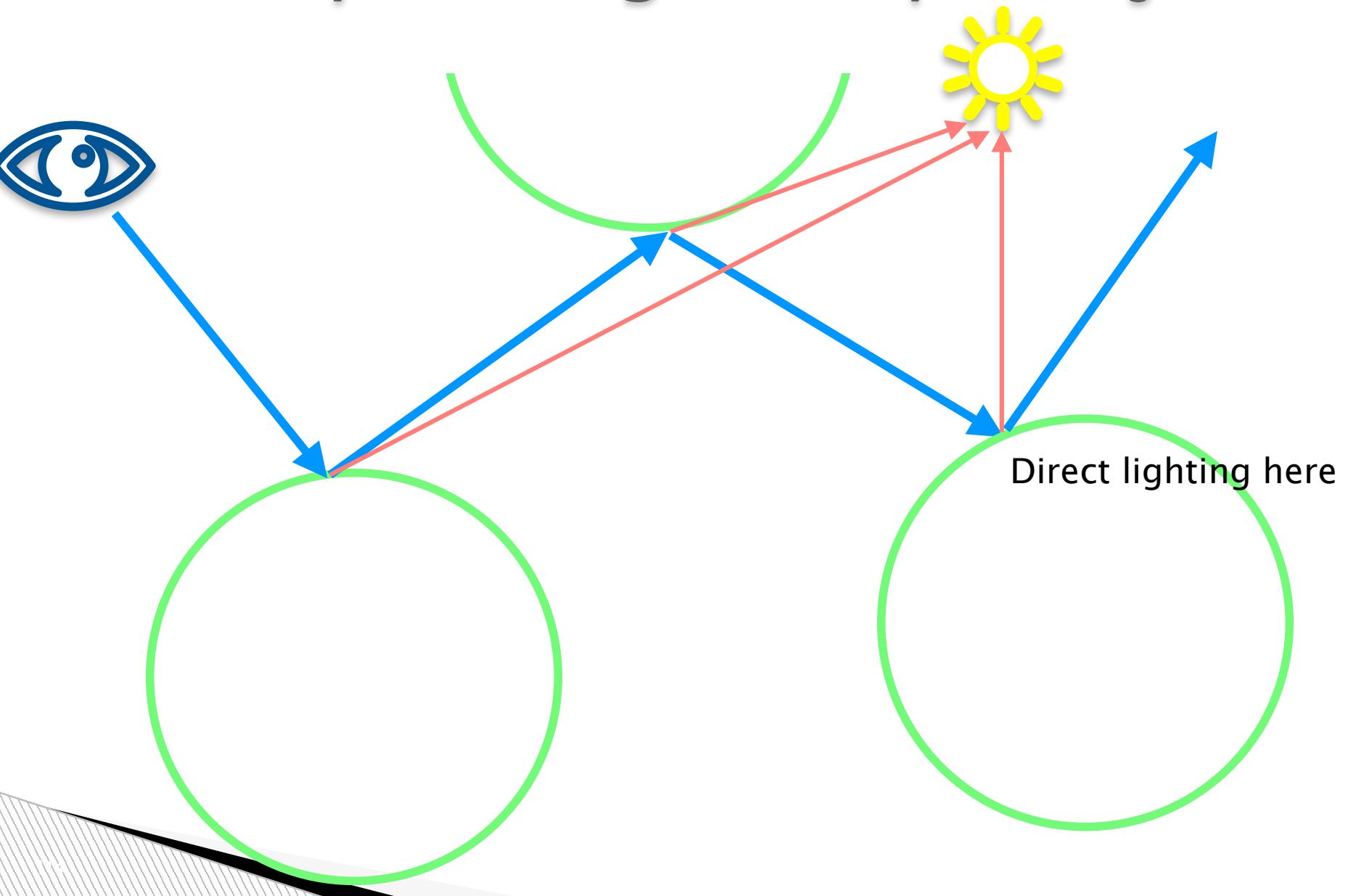
GPU ray-tracing: multiple objects



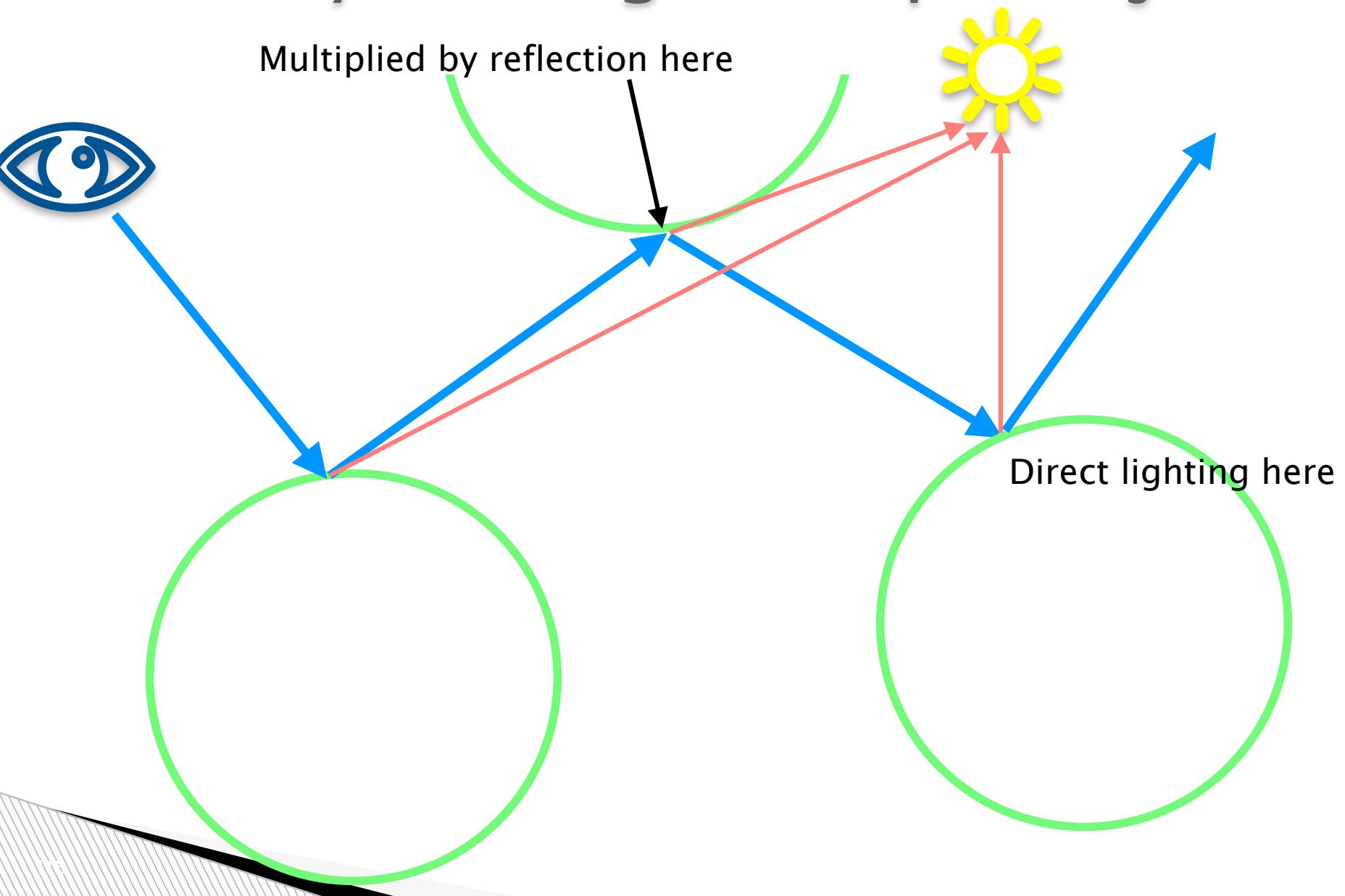
GPU ray-tracing: multiple objects



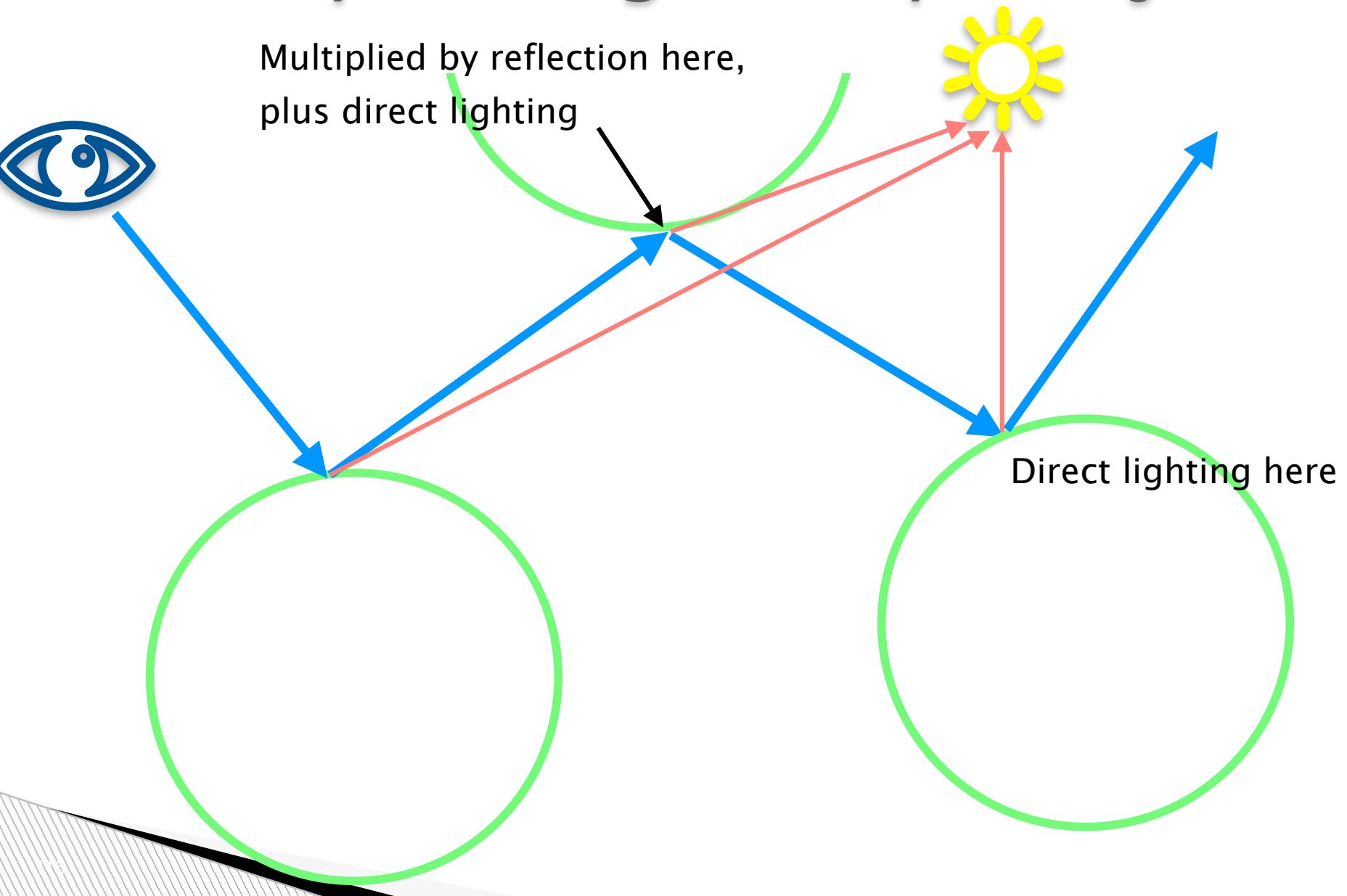
GPU ray-tracing: multiple objects



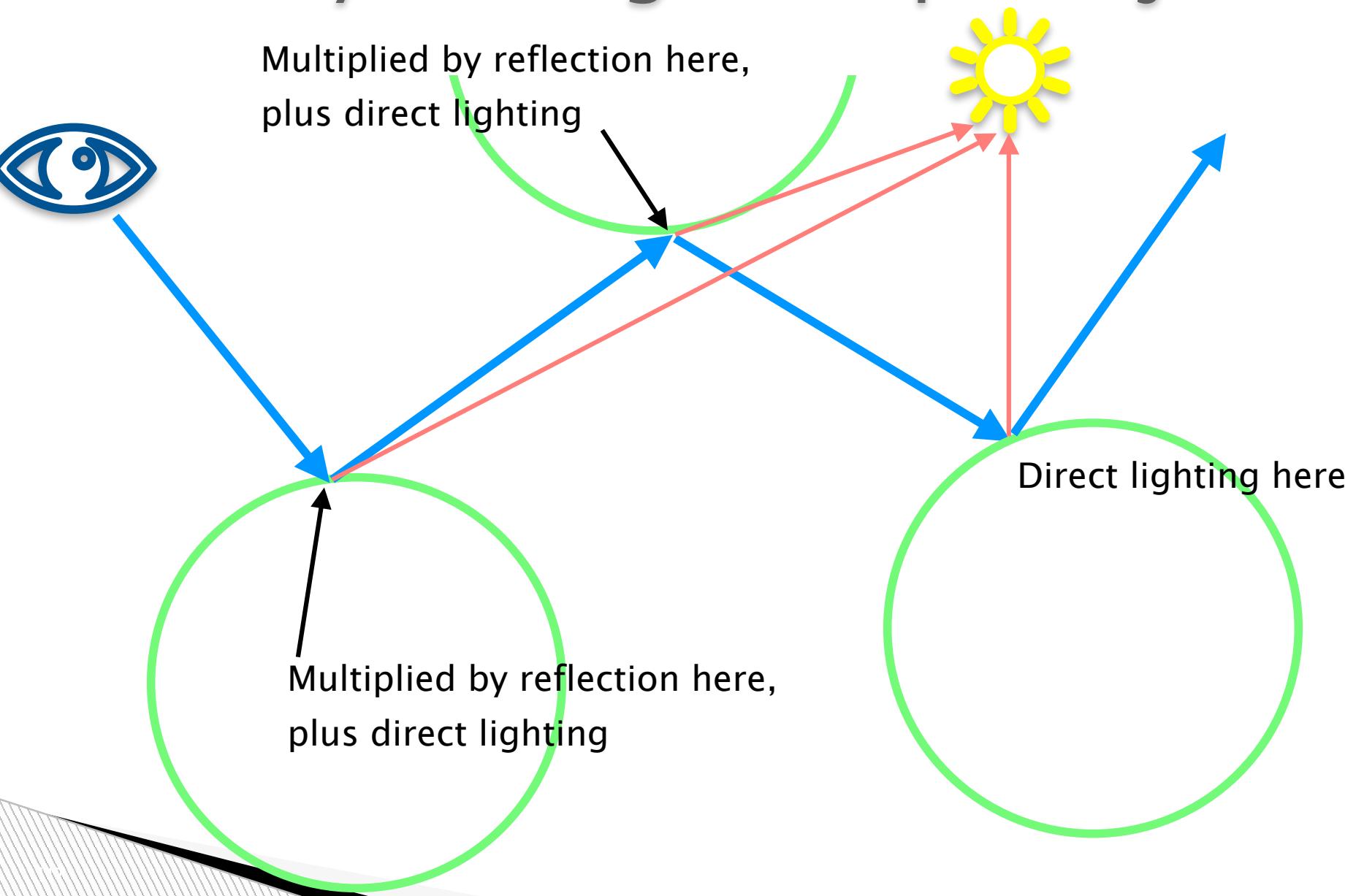
GPU ray-tracing: multiple objects



GPU ray-tracing: multiple objects



GPU ray-tracing: multiple objects



Computing pixel color:

- ▶ Pixel color:

$$C_p = d_0 + \rho_0 (d_1 + \rho_1 (d_2 + \rho_2 \dots \rho_{n-2} (d_{n-1} + \rho_{n-1} d_n)))$$

- ▶ How can we compute that?
 - ▶ Incrementally, from the end

Computing pixel color

- ▶ Need to work from the end of the ray
- ▶ Ray is created from the start
- ▶ Two-step process:
 - ▶ Follow the ray from the camera
 - ▶ Store intersection points in a queue
 - ▶ Follow the ray from the end
 - ▶ Compute illumination at each point
 - ▶ ≠ BRDF for direct and indirect lighting

Chromatic aberration

- ▶ Refractive index depends on wavelength
- ▶ Cauchy rule (dielectrics only):

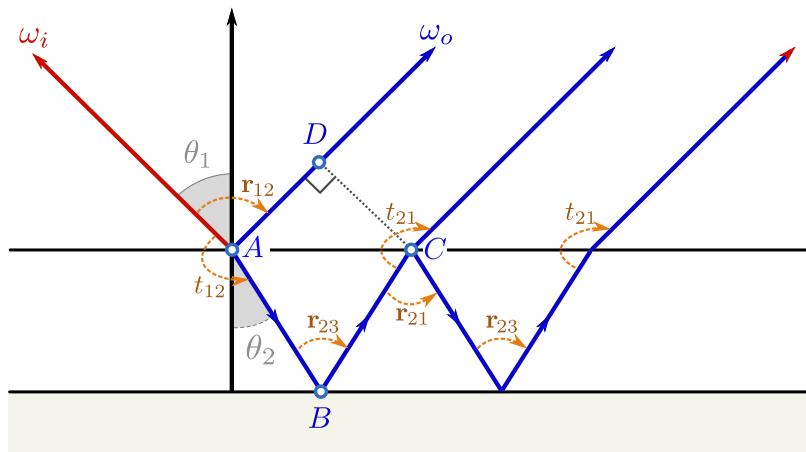
$$\eta = B + \frac{C}{\lambda^2}$$

- ▶ 3 *different* rays (R, G, B)
 - ▶ x3 computation time
 - ▶ Subtle effect



Diffraction

- ▶ Soap bubbles = thin layer \approx wavelength
- ▶ Diffraction caused by the layer + reflection
- ▶ Can be encoded in an RGB function



Ray-tracing: advantages

- ▶ Slow, but no extra charge for:
 - hidden surface removal,
 - shadows,
 - transparency,
 - texture-mapping (including procedural).
- ▶ Inter-reflexions between objects,
- ▶ Any graphics primitives,
- ▶ Global illumination model.

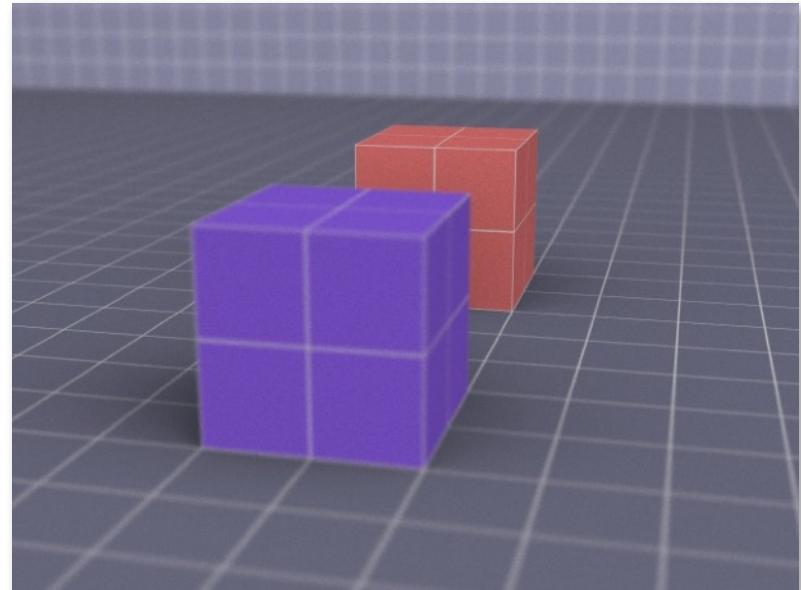
Ray-tracing: issues

- ▶ Limited to Snell–Descartes:
 - all objects appear metallic.
- ▶ Ray tree limited to a certain depth:
 - complex objects may be a problem (diamonds, cristal glass)

Even more rays

« Distributed Ray Tracing » Cook et al. (1984)

- ▶ Soft shadows
 - Several rays for each extended light source
- ▶ Anti-aliasing
 - Several rays for each pixel
- ▶ Glossy Reflection
 - Several rays are reflected
- ▶ Motion blur
 - Several rays during time
- ▶ Depth of field
 - Several rays per pixel,
focusing with a lens



Further readings

- ▶ Morgan McGuire: Ray-tracing tips
- ▶ Peter Shirley: Ray-tracing in a week-end
- ▶ Eric Haines: Ray-tracing resources